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Modeling the Infiltration Component of the Rainfall-Runoff Process

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SYMBOLS USED

SYMBOL	MEANING
a	Parameter in Kostyakov and Holtan equations
A	Parameter in Philip equation
[A]	Tridiagonal matrix in finite difference scheme
b	Parameter in Kostyakov equation
f	Infiltration rate
f_c	Equilibrium or steady infiltration capacity
f_p	Infiltration capacity
f_o	Initial infiltration capacity (Horton)
F	Infiltration volume
F_p	Potential infiltration volume (Holtan)
F_s	Infiltration volume to start of runoff
i	Subscript denoting depth node
I	Rainfall intensity
IMC	Initial moisture content
IMD	Initial moisture deficit
j	Subscript denoting time node
k	Parameter in Horton equation
k_r	Relative conductivity
K	Capillary conductivity
K_s	Saturated capillary conductivity
L	Depth to equivalent wetting front
LHS	Vector in finite difference scheme
m	Number of depth increments used at each time step
n	Wettable porosity in Green Ampt equation
RAIN	Rainfall intensity used in finite difference program

SYMBOLS USED (cont)

SYMBOL	MEANING
s	Sorptivity in Philip equation
S	Capillary suction ($= -\Psi$)
S_{av}	Capillary suction at the wetting front
SL	Slope of the S-Θ curve
t	Time from start of rainfall
t_s'	Adjusted time to allow for rainfall infiltration
t_s	Time to start of runoff
v	Darcy fluid velocity
z	Distance below soil surface (positive downwards)
Θ	Moisture content (vol/vol)
Θ_i	Initial moisture content
Θ_s	Saturated moisture content
Ψ_{av}	Average capillary potential at wetting front
Ψ	Capillary potential
Φ	Total potential

FOREWORD

This Bulletin is published in furtherance of the purposes of the Water Resources Research Act of 1964. The purpose of the Act is to stimulate, sponsor, provide for, and supplement present programs for the conduct of research, investigations, experiments, and the training of scientists in the field of water and resources which affect water. The Act is promoting a more adequate national program of water resources research by furnishing financial assistance to non-Federal research.

The Act provides for establishment of Water Resources Research Institutes or Centers at Universities throughout the Nation. On September 1, 1964, a Water Resources Research Center was established in the Graduate School as an interdisciplinary component of the University of Minnesota. The Center has the responsibility for unifying and stimulating University water resources research through the administration of funds covered in the Act and made available by other sources; coordinating University research with water resources programs of local, State and Federal agencies and private organizations throughout the State; and assisting in training additional scientists for work in the field of water resources through research.

This report is the forty-third in a series of publications designed to present information bearing on water resources research in Minnesota and the results of some of the research sponsored by the Center. In this study, a simple infiltration model is proposed for the idealized case of a constant intensity rainfall infiltrating into a homogeneous soil at uniform initial moisture content.

This bulletin represents a research effort completed under Project 12-55 of the Minnesota Agricultural Experiment Station, entitled "Hydrology of Small Watersheds", in the Department of Agricultural Engineering. Russell G. Mein served as Research Assistant and Curtis L. Larson is Project Leader. Computer time was furnished by the University of Minnesota Computer Center.

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MODELING THE INFILTRATION COMPONENT OF THE RAINFALL-RUNOFF PROCESS

I. INTRODUCTION

Hydrology is a broad science which deals with those processes of the hydrologic or water cycle which occur on land. The elements of the hydrologic cycle have been qualitatively understood for several centuries (10), but an adequate quantitative description of each component and its interaction with other components has yet to be achieved.

The classic problem in hydrology is the prediction of both amount and rate of watershed runoff caused by rainfall. Such prediction is vital for the correct design of all types of hydraulic structures, large or small. For medium and large watersheds the hydrologist frequently has rainfall and stream flow data to work with, but for small watersheds, most commonly there is no streamflow data. To make predictions of runoff for small watersheds, design engineers have been forced to use one of the many empirical formulas found in the literature (9). These formulas are generally entirely empirical, giving the peak flow as a function of the rainfall intensity, watershed area, and perhaps other watershed factors. Physical representation of the actual rainfall-runoff process is neither attempted nor achieved.

Watershed Models

The advent of the high speed digital computer has been of tremendous significance in hydrology. On one hand it has enabled fast storage and retrieval of hydrological records, whose sheer bulk had long before reached unmanageable proportions. On the other hand, computational techniques, previously impractical because of the number of calculations involved, can now be used for the simulation of hydrologic processes.

It is not surprising, therefore, that computers should be applied in hydrology to simulate the actual processes involved in rainfall-runoff on a watershed. The best known mathematical model is the Stanford Watershed Model (11), which takes hourly rainfall data as input and accounts for interception, infiltration, interflow, groundwater flow, evaporation, and overland flow, and gives the runoff hydrograph as output. Unfortunately this model has several empirical constants which must be "fitted" to the watershed; thus, historical records of rainfall and runoff are necessary. If some way could be found of predicting these parameters from watershed characteristics then the model could be applied to ungaged watersheds, which have no records.

In recent years many other watershed models have been proposed, e.g., (12), (29), (48) and tested against measured runoff hydrographs. Almost invariably, there are several parameters to be fitted, a severe limitation for any watershed model, again because of the lack of data available for most watersheds.

The Infiltration Component

For the continental United States, approximately 30% of the total precipitation becomes streamflow (10). From this figure one can argue that more than 70% of the annual precipitation infiltrates into the soil, for both interflow and groundwater contribute to streamflow after infiltration. But for individual storms, the percentage varies widely, ranging from 100% when all of the rainfall infiltrates, to perhaps 30-50% for a high runoff storm. It is apparent, therefore, that a watershed model must describe infiltration accurately to have any chance of producing valid and useful results. The watershed model must also include soil moisture redistribution, drainage, and evapotranspiration, since the moisture level prior to a storm is a dominant factor in the infiltration rates observed. Recent work in redistribution, drainage, and evapotranspiration models shows promise (see References (1), (17), and (18)) and may well lead to a good soil accounting moisture model.

Despite its importance, the infiltration component in all of the current watershed models is usually represented by empirical relationships of one form or another. Typically, an equation needing one or more fitted parameters is used. Huggins and Monke (29) showed that the choice of infiltration parameter employed in their watershed model had more influence than any of his other parameters on the outflow hydrograph, which again emphasizes the desirability of a valid infiltration model.

It is evident that there is a pressing need for an improved infiltration model, one which will better represent the actual infiltration process, utilizing parameters that are physically meaningful and capable of being measured. The model must be sufficiently flexible to be useable under different rainfall conditions, and should not require a large amount of computing time. The study reported herein was carried out with these objectives in mind. This report is essentially the same as the Ph.D. dissertation by Mein (34), with the junior author as major adviser.

II. THE INFILTRATION PROCESS

The infiltration process is a complex one, for it is influenced by many soil and water properties. Even when the soil is completely homogeneous (a rare occurrence in nature) and the fluid properties are known, it is a difficult phenomenom to describe. This chapter is devoted to a discussion of the major factors which influence infiltration, and of some of the problems involved in the formulation of any infiltration model.

Definitions

Throughout this bulletin, the following terminology and units will be used:

Infiltration: The process of water entry into a soil through the soil surface.

Infiltration rate (f): The rate at which water moves through the soil surface. (Units: cm/sec)

Infiltration capacity (f_p): The maximum rate at which the soil can absorb water through the soil surface. (Units: cm/sec)

Infiltration volume (F): The total volume of infiltration from the beginning of the rainfall event. (Units: cm)

Capillary potential (Ψ): Hydraulic head due to capillary forces. (Units: cm. of water)

Capillary suction (S): The same as capillary potential, but with opposite sign, i.e., a positive suction represents a negative hydraulic head. (Units: cm. of water)

Capillary conductivity (K): The volume rate of flow of water through the soil under a unit gradient, which is dependent on the soil moisture content. Sometimes referred to as conductivity, or hydraulic conductivity. (Units: cm/sec)

Saturated conductivity (K_s): The capillary conductivity when the soil is saturated. (Units: cm/sec)

Relative conductivity (k_r): Capillary conductivity for a given moisture content divided by the saturated conductivity. (Dimensionless)

Infiltration - The Ideal Soil

For the purpose of discussion, the ideal soil is considered to be one which is homogeneous throughout the profile, and in which all of the pores are interconnected by capillaries. In addition it is assumed that the applied rainfall falls uniformly over the soil surface (natural rainfall is discussed later). Because the movement of water into the soil is areally uniform, the infiltration process can be considered to be one-dimensional.

For this ideal case, perhaps the most important factors which affect the infiltration capacity are soil type and moisture content. The soil type determines the size and number of the capillaries through which the water must flow. The moisture content determines the capillary potential in the soil and the relative conductivity as shown for a typical soil in Fig. 1. For a low moisture content the suctions are high and the conductivity low. At high moisture contents, near saturation, the soil suction is low, and the relative conductivity high.

One can now examine a typical infiltration event - rainfall falling on a dry soil. Because the initial moisture content is low the relative conductivity (Fig. 1) is small. This implies that the moisture level must be higher before water will move further into the soil mass. Because of this, a distinct wetting front will form with the moisture content ahead of the front still low, and the soil behind the front virtually saturated. A considerable capillary suction exists at the wetting front.

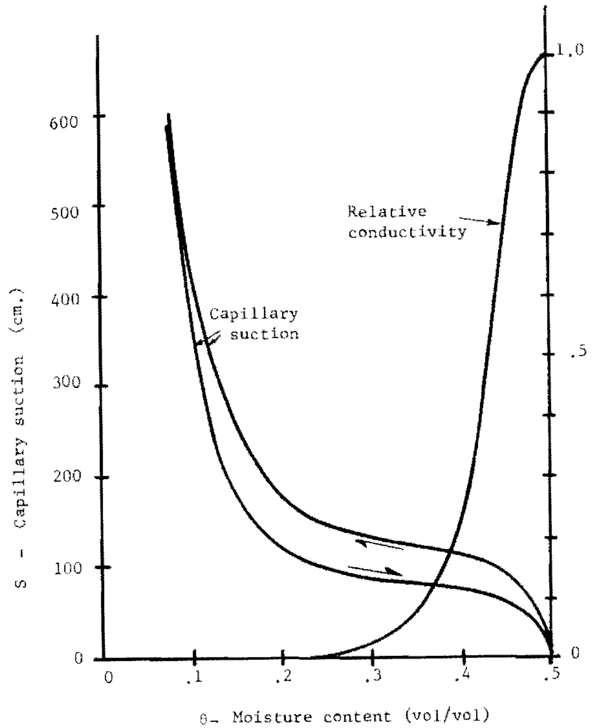

Fig. 1. Capillary suction and relative conductivity as a function of moisture content for a typical soil (Saturated moisture content = .5)

At the onset of rainfall, the potential gradient causing moisture movement is very high, for the potential difference between the wetting front and the soil surface acts over a very small distance. At this stage, the infiltration capacity is higher than the rainfall rate, so the infiltration rate is limited to the rainfall rate. Figs. 2 and 3 illustrate the phenomenom, which corresponds to curves 1, 2 and 3.

As more and more water enters the soil, the thickness of the wet zone becomes larger and the potential gradient is reduced. The infiltration capacity decreases until it is just equal to the rainfall rate - this moment corresponds to curve 4 in Fig. 2, and point 4 in Fig. 3, and occurs at the moment when the surface becomes saturated.

The potential gradient continues to lessen as the wetted depth increases, so that the infiltration capacity is reduced with time (corresponding to 5 and 6 in Figs. 2 and 3). Theoretically, the potential gradient will be unity after an infinite time, for then the case is the same as saturated flow under the influence of gravity, and the infiltration capacity is equal to the saturated conductivity. In practice, the infiltration rate is asymptotic to the saturated conductivity line (Fig. 3), but does not reach it.

The prediction of point 4 (Fig. 3) is of vital concern to the hydrologist, for it is then that runoff will begin. Also, it is the beginning of $f = f_p$, instead of $f = I$. However, for a given soil type this moment depends not only on the initial moisture content of the soil, but also on the rainfall rate. The shape of the subsequent curve 4-5-6 likewise depends on the initial moisture content and rainfall rate.

Hysteresis

The capillary vs. moisture content curve is different for wetting the soil than it is for dewatering the soil. The curves shown in Fig. 1 demonstrate this phenomenom--these are called the boundary wetting and boundary drying curves applicable for continuous wetting or continuous drying, respectively. However, between these curves there are an infinite number of possible paths, or "scanning curves", depending on the wetting and drying, history of the soil. Thus, even though infiltration is a wetting phenomena, it cannot invariably be assumed that the suction is following the boundary wetting curve.

There are several methods of handling the hysteresis complication in soil moisture computations. A suitable numerical scheme is given by Ibrahim and Brutsaert (30), in the form of a triangular suction table. However, in this study, because the scanning curves were not available, hysteresis is not be considered and the starting point is assumed to be on the wetting curve.

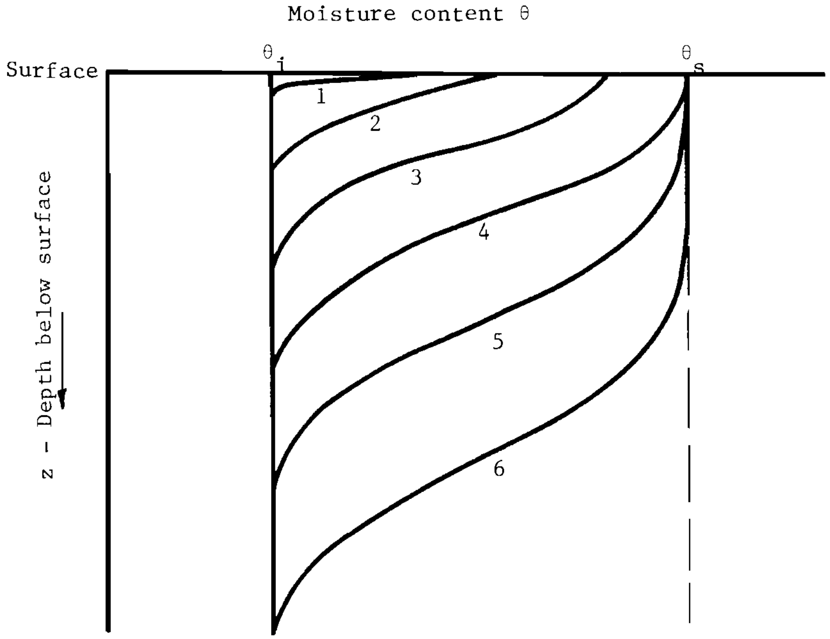

Fig. 2. Hypothetical moisture profile development under a constant rainfall rate (After Childs, Ref. 7, Fig. 12.13)

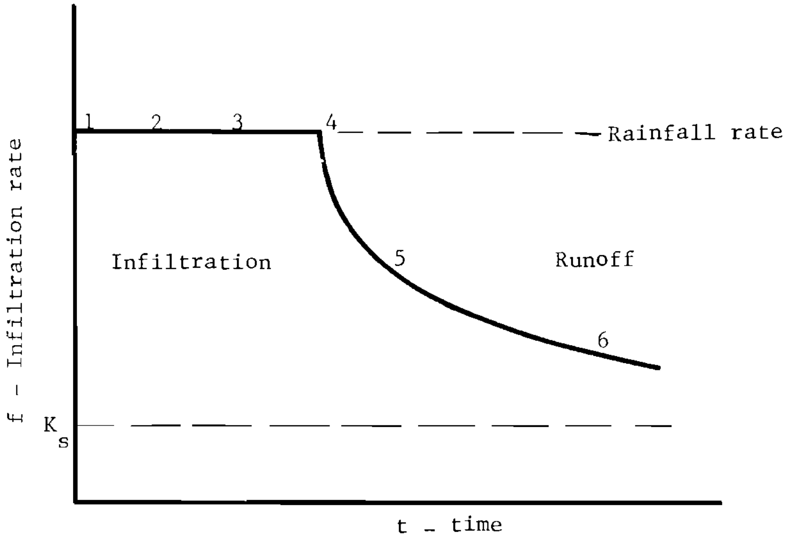

Fig. 3. Infiltration rate vs. time corresponding to Fig. 2.

Infiltration - The Natural Soil

The variability in natural soils makes a difficult situation virtually impossible, as far as soil profile descriptions are concerned. The surface condition, which has a vital influence on infiltration capacities, can vary over a watershed from complete vegetative cover to bare soil. The latter case is subject to surface sealing by raindrop compaction, shown to significantly alter the infiltration characteristics (13). A plowed surface results in much greater infiltration, almost 20 times more in one test (6).

A complete description of all the factors which influence infiltration on a natural watershed (cracking, vegetation, layering, etc.) will not be included here. For that the reader is referred to articles by Parr and Bertrand (38), or Erie (15). Suffice it to say that the variability is so significant, even in seemingly uniform soils, that soil samples from the same site often do not exhibit the same properties (22).

It follows then that a simple infiltration model which will be accurate for natural soils is not possible, for the complete description of the soil itself is impracticable. One can only hope to produce a model which will give a reasonable estimate of the infiltration behavior. That is the goal of this study.

Rainfall Intensity

Most textbook infiltration curves (e.g., Ref. 33), are of the form shown in Fig. 4, which assumes a saturated surface at time zero. While in flood irrigation this condition can occur, under rainfall or sprinkler irrigation it rarely occurs. Almost always, there is at least a brief period when all the rainfall infiltrates, as shown in curves, 1, 2, 3, in Fig. 5.

There are 3 general cases of infiltration under rainfall:

Case A: $I < K_s$. The rainfall rate is less than the saturated conductivity; curve 4 in Fig. 5 shows this condition. For a deep homogeneous soil, runoff will never occur under this condition, for all of the rainfall will infiltrate, regardless of the duration. As far as a watershed model is concerned, such a rain must still be accounted for because the soil moisture level is being altered.

Case B: $K_s < I < f_p$. The rainfall rate is greater than the saturated conductivity, but less than the infiltration capacity. This condition corresponds to the horizontal portion of curves 1, 2, 3 in Fig. 5. Note that the period of time from zero to surface saturation (t_s) is different for the different intensities.

Case C: $I > f_p$. The rainfall intensity is greater than the infiltration capacity. Only under this condition can runoff occur. Fig. 4 and the decreasing portions of curves, 1, 2, and 3, (Fig. 5) illustrate this and show the dependence of the curve shape on the initial rainfall intensity.

Because the major interest of the hydrologist is to predict runoff, most work has involved study of Case C. It is the contention of the writers that Case B is just as vital, since it almost always precedes and influences the behavior in Case C. This argument will be strengthened further in later sections.

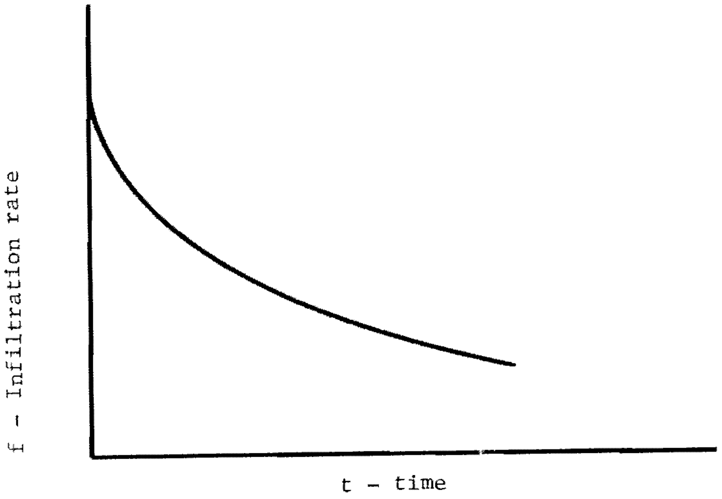

Fig. 4. Typical textbook infiltration curve.

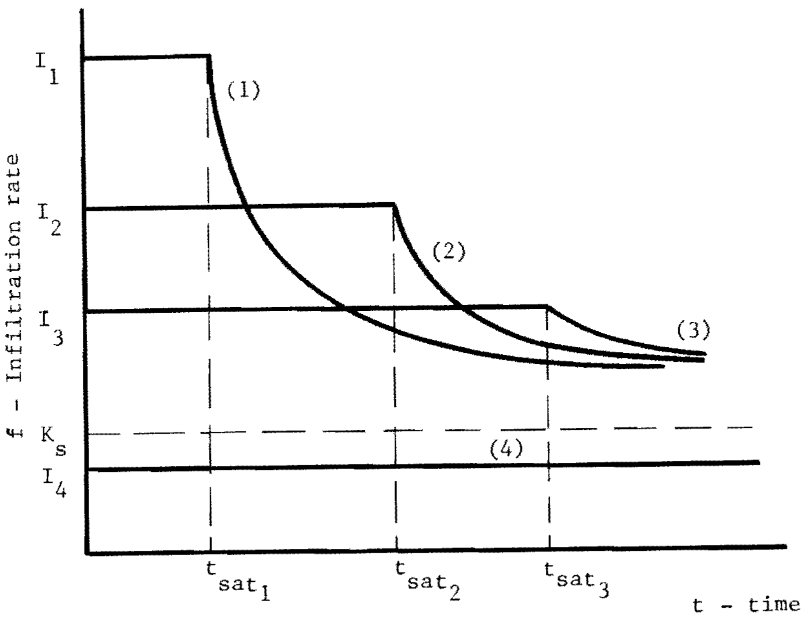

Fig. 5. Infiltration curves for different rainfall intensities I_1, I_2, I_3 and I_4.

III. EXISTING INFILTRATION MODELS

Many different infiltration equations have been published in the literature. In this chapter, the more important of these are presented with a brief discussion of the derivation, attributes and deficiencies of each. Because infiltration can be thought of as the surface boundary condition of soil moisture movement, it can also be predicted from the equations which describe soil moisture flow. Development of this latter approach suggests an alternative method of deriving a useful infiltration model.

Empirical Algebraic Equations

As mentioned in the previous chapter, the infiltration capacity decreases as the volume of water in the profile increases. So it is not surprising that early attempts to produce a field infiltration equation merely fitted the characteristic curve shape by a decay-type function.

The Kostyakov Equation

The general form of the infiltration expression put forward by Kostyakov in 1932 as cited by Rode (42) is

$$F = at^b \tag{1}$$

where F is the total infiltration volume, t is the time from the onset of infiltration, and a and b are constants ($0<b<1$). The form of this expression is simple, but the constants a and b cannot be predicted in advance. There is no provision for predicting when runoff will begin under a rainfall input (the transition from Case B to Case C given in Chapt. 1).

The simplicity has encouraged the use of this equation in flood irrigation studies, e.g., Fok (16), for which the normal procedure is to fit a and b from test data on the plot concerned. A modified time variable was proposed by Kincaid et al (31) to use this equation for sprinkler irrigation for which the initial infiltration capacity of the soil exceeded the application rate. The modification improved the fit markedly but the constants a (which depends on the initial moisture content as well as the soil type) and b (which depends on the soil type) still had to be fitted.

The Horton Equation

Possibly the best known infiltration expression is that known as Horton's equation (7), which he proposed in 1940. The equation is

$$f_p = f_c + (f_o - f_c)\, e^{-kt} \tag{2}$$

where f_p is the infiltration capacity at any time t, f_o is the initial infiltration capacity, f_c is the final or equilibrium infiltration capacity, f_c is the final or equilibrium infiltration capacity, and k is a constant dependent on soil type and the initial moisture content. The feature of Eq. 2 is its simplicity, and good fit to experimental data, but f_o, f_c and k have all to be fitted. Theoretically f_c should be a constant for a particular location, but Burman (5) found better fits to experimental data by varying it too, from one storm to another. This would indicate that the good fit possible with Eq. 2 might be due more to the fact that it has three parameters, rather than because of its representation of the infiltration process. The Horton equation, likewise, cannot be used for an event beginning as Case B.

The Holtan Equation

A more recent expression is that of Holtan (27) developed from a substantial volume of field data.

$$f_p = f_c + aF_p^n \tag{3}$$

where f_p is the infiltration capacity, f_c is again the final steady infiltration capacity, F_p is the potential infiltration volume (equal to the initial available moisture storage minus the volume of water already infiltrated), a is a constant between 0.25 and 0.80, and n is a constant dependent on soil type. This expression is well suited for inclusion in a watershed model, because it links infiltration capacity to the soil moisture level and is not directly time dependent. An expansion of this idea into an infiltration-drainage-evaporation-runoff model was made by Holtan et al (28) by conceptually dividing the soil storage into capillary storage and free water storage.

Use of this formula has been made by Huggins and Monke (29) who used the effective storage depth of the soil as a parameter to be fitted. The depths used ranged from 3" to 16", without logical explanation. This illustrates the major difficulty of the Holtan model, for the control depth specified has a large influence on the prediction of infiltration.

Theoretically Derived Algebraic Equations

There are many equations derived from applying Darcy's Law to the wetted zone in the soil, using the fact that a distinct wetting front exists. Green and Ampt (20) were the first with this approach, and their equation will be given here.

The Green and Ampt Equation

The equation derived by Green and Ampt in 1911 assumes that the soil surface is covered by a pool of water whose depth can be neglected. Referring to Fig. 6, and assuming that the conductivity in the wetted zone is K_s, Darcy's law can be applied to give

$$f_p = K_s \frac{L + S}{L} \tag{4}$$

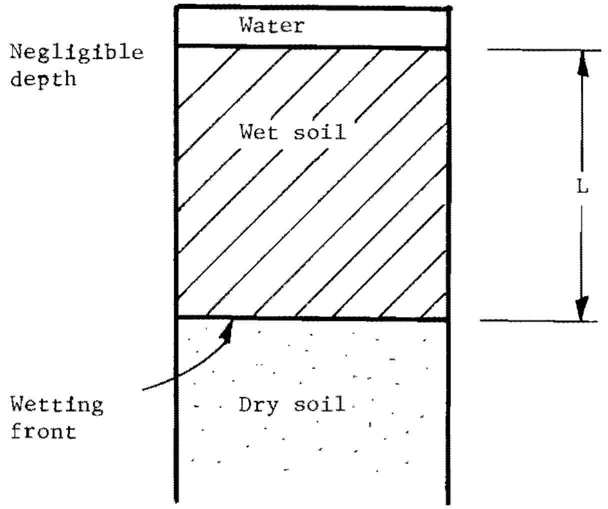

Fig. 6. Definition diagram for Green and Ampt formula

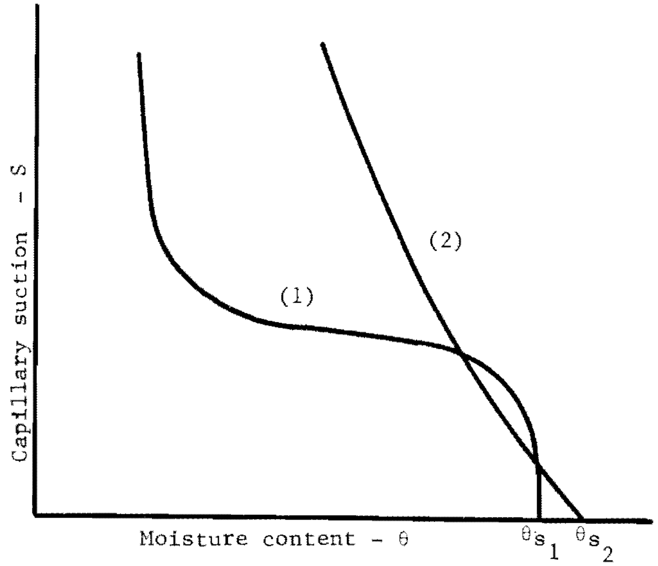

Fig. 7. Typical capillary suction vs. moisture content for (1) sandy soil, (2) clay soil

where L is the distance from the soil surface to the wetting front and S is the capillary suction at the wetting front. By continuity, nL = F, where n is the initial moisture deficit ($\Theta_s - \Theta_i$) and F is the volume of infiltration. Thus, writing $f_p = dF/dt$,

$$\frac{dF}{dt} = K_s \left(1 + \frac{nS}{F}\right) \tag{5}$$

Integrating, and substituting F = 0 at t = 0

$$f - Sn \log_e \left(\frac{F + nS}{nS}\right) = K_s t \tag{6}$$

This form of the G - A law is more convenient than Eq. 4 for watershed modeling, for it relates the infiltration volume to the time from the start of infiltration. From it one can obtain the volume for any given time or vice versa. The advantage of Eq. 6 is that all of the parameters are physical properties of the soil-water system and can be measured. Estimation of S for a soil represented by curve 2 (Fig. 7) is difficult, however, since the capillary suction always varies widely with moisture content. The resultant suction for such a soil will be some kind of average taken over the whole wetting front. This point is discussed in later sections. Note that the equations do not account for the circumstance of rainfall intensity being less than the infiltration capacity (Case B).

Several other workers have derived equations similar to Eqs. 4 and 6 using Darcy's law and the law of conservation of mass. Rode (42) lists Alekseev (1948), Budagovskii (1955) and Subnitsyn (1964), to which we can add Philip (1954), and Hall (1956), although the latter author failed to include the capillary force at the wetting front (23, 25, 39).

Interest in the Green and Ampt formula has revived, and the results have been very encouraging. Bouwer has developed apparatus to measure S in the field (2) and shown how to extend the analysis to a layered soil (3). Laboratory tests and the G - A theory for layered soils have shown excellent agreement (8). None of these tests, however, were made for the rainfall condition at the surface (Case B).

The Philip Equation

For a homogeneous soil with a uniform initial moisture content and an excess water supply at the surface, Philip has solved the partial differential equation of soil moisture flow (Eq. 8 - discussed in the next section). The solution is in the form of an infinite series, but because of the rapid convergence only the first two terms need to be considered (41), giving

$$F = st^{\frac{1}{2}} + At \tag{7}$$

where F is the volume of infiltration to time t, and s and A are constants depending on the soil and its initial moisture content.

The form of this equation is convenient and s and A can be predicted in advance, but the method of finding their correct values is complex and beyond the scope of a simple watershed model. The rainfall boundary condition (Case B) is not simulated by Eq. 7.

Tests of the above equations have been made against field data. Skaggs et al (44) fitted all of the equations and found them all satisfactory, particularly for short times. Such a test may show that the equation has the right form if the constants are correct, but still does not answer the question of predicting the constants. A test of the Green-Ampt (Eq. 6) and Philip (Eq. 7) equations was made by Whisler and Bouwer (47), who concluded that the G - A equation was not only simpler to use, but also gave better results. They experienced considerable difficulty in predicting s and A for Eq. 7.

The Soil Moisture Flow Equation

A combination of Darcy's law (applied to unsaturated flow) and the equation of continuity results in the second-order, non-linear, partial differential equation, e.g., Ref. (7).

$$\frac{\partial \Theta}{\partial t} = -\frac{\partial}{\partial z}\left(K(\Theta)\,\frac{\partial S(\Theta)}{\partial z}\right) - \frac{\partial K(\Theta)}{\partial z} \tag{8}$$

where Θ is volumetric moisture content, t is time, z is the distance below the surface, $S(\Theta)$ is the capillary suction, and $K(\Theta)$ is the unsaturated conductivity. Equation 8 is sometimes referred to as the Richards equation, and also as the diffusion equation.

The solution of this equation for unsteady flow is difficult; the analytic solution proposed by Philip (40) is complex and applicable only to homogeneous soils with a uniform initial moisture content and an excess water supply at the surface. For other boundary and initial conditions, numerical analysis techniques (usually finite differences) are necessary, and the required computation is far from simple, as shown in the following section.

It remains to be shown here that Eq. 8 is a valid model for soil moisture flow, one which is suitable to generate infiltration data under a variety of input conditions.

There are many examples of solutions of Eq. 8 being compared with field tests. Neilson et al (36), after stating that Eq. 8 is valid for laboratory soils, compared results given by Eq. 8 with observed field moisture profiles in Monona silt loam and Ida silt loam. Agreement was termed adequate, especially when the depth of water penetration was in the homogeneous soil layer. Whisler and Bouwer (47), in the tests cited earlier, judged Eq. 8 good; better than the other infiltration equations tested.

Seepage of water from a pond was simulated by Green et al (21), using Eq. 8, and compared with a field test. A non-homogeneous soil was modeled by Wang and Lakshminarayana (46) and good agreement found with a field trial.

The above examples are cited as validation of the Richards equation. It is unfortunate that its complexity, and its requirement of the Ψ-Θ and K-Θ curves as input data rule it out for inclusion in a practical watershed model.

The Need for a Simple Model - The Research Plan

It was shown in Section II that there are three distinct cases of rainfall intensity as related to infiltration and runoff. There were, as given previously, cases:

(A) $I<K_s$, (B) $K_s<I<f_p$, (C) $I>f_p$

For the hydrologist concerned with predicting the volume and timing of runoff (A) need not be considered, except as a source of soil moisture for future events. For most natural runoff events the infiltration process starts in Case B, and when the surface becomes saturated moves to Case C and runoff begins. The important point here is that the behavior in (C) is markedly influenced by the volume infiltrated in (B). For this, there are no simple infiltration equations.

The principal objective of this study was to develop a simple method of predicting the time between the start of rainfall and the beginning of runoff, using measurable properties of the soil and rainfall intensity. The infiltration "decay" portion of the curve was studied with the intent of modifying the Green and Ampt equation to allow for the infiltration volume prior to surface saturation, and the hope of determining the appropriate capillary suction value for the soil.

The proposed model was tested against solutions of the Richards equation for several soils, each with several different initial moisture contents and a number of rainfall intensity levels. Experimental and field data were used for testing to the extent available.

IV. DEVELOPMENT OF AN INFILTRATION MODEL

Both saturated and unsaturated fluid flow through soils can be described by Darcy's law, providing the appropriate conductivity value is used. However, as shown in Sec. II, the capillary conductivity is highly dependent on the moisture content, which is a considerable complication. To derive simple expressions for infiltration based on Darcy's law, some assumptions about the shape and movement of the wetting front are necessary. These assumptions are explained in the development of the infiltration model.

The situation to be examined here is that of a constant intensity rainfall which initially completely infiltrates into the soil, but which eventually produces runoff. The two stages in the process, previously designated Case B and Case C, are considered separately in the following discussion.

Prediction of the Infiltration Volume Prior to Runoff (Case B)

As stated in Sec. II, the moisture content at the surface increases during the rainfall until surface saturation is reached. Although Darcy's law is applicable at any time during the process, it is at the moment of saturation that a useful relationship can be developed, for at that moment, the surface moisture content and conductivity are known.

A typical profile at the moment of surface saturation is shown in Fig. 8. (It should be noted that the shape and extent of the profile depends on the soil properties and the rainfall intensity.) The shaded area is equal in volume to the volume of infiltration up to surface saturation, F_s. Darcy's law will be applied to the shaded area; in other words, we are assuming an abrupt wetting front with the soil beyond at the initial moisture content, and the soil behind at saturation.

Darcy's law can be written

$$q = -K\ (\Theta)\ \frac{\partial \Phi}{\partial z} \tag{9}$$

where q is the flow rate in depth per unit of time, K(Θ) is the capillary conductivity, Φ is the potential, and z is the depth.

In finite difference form this can be written

$$q = -K(\Theta)\ \frac{\Phi_2-\Phi_1}{z_2-z_1} \tag{10}$$

in which the subscripts 1 and 2 refer to the surface and wetting front respectively. At the moment of saturation the infiltration rate is still equal to the rainfall rate, so we can put q = I. Applying Eq. 10 to Fig. 8, we see $z_2 - z_1 = L_s$. The potential at the surface Φ_1 is arbitrarily taken as zero, so the potential at the wetting front will be $-(L_s + S_{av})$ where S_{av} is the average capillary potential, discussed later.

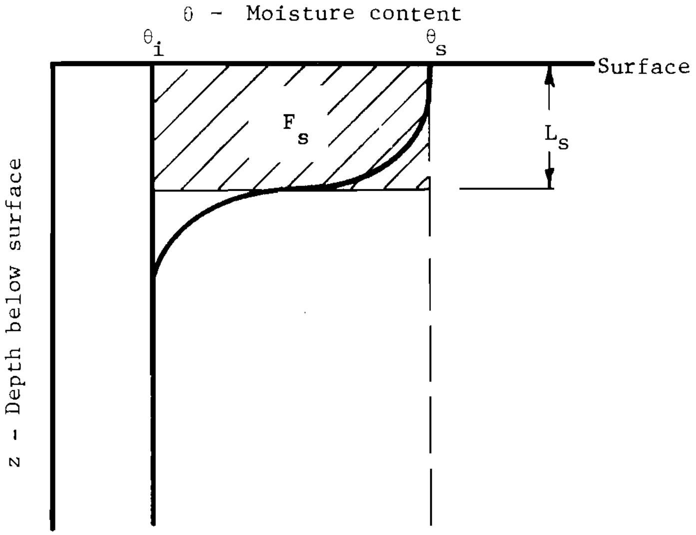

Fig. 8. A typical moisture content profile at the moment of saturation.

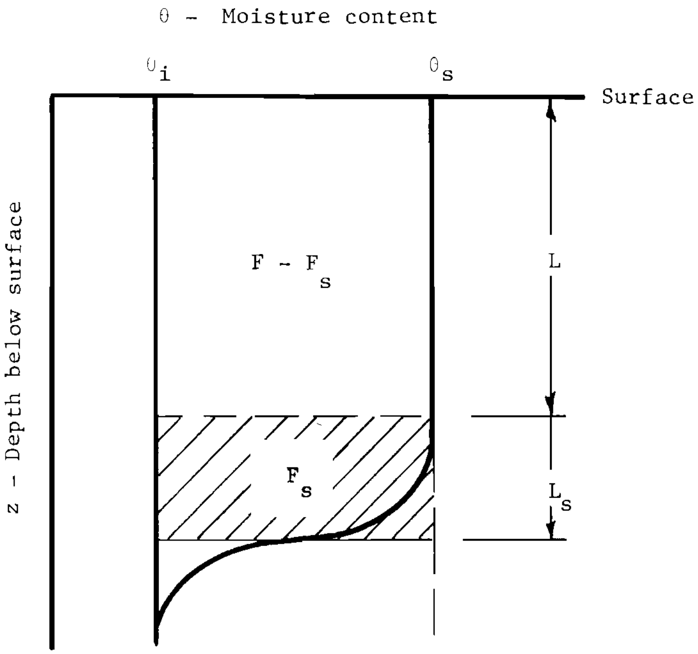

Fig. 9. The assumed moisture content profile for derivation of the infiltration capacity equation.

The capillary conductivity is assumed to be equal to the saturated conductivity. Making these substitutions, Eq. 10 becomes

$$I = K_s \left(\frac{S_{av} + L_s}{L_s} \right) \tag{11}$$

From Fig. 8,

$$L = \frac{\text{volume infiltrated}}{\text{initial moisture deficit}} = \frac{F_s}{IMD} \tag{12}$$

where the initial moisture deficit, $IMD = \theta_s - \theta_i$.

Substituting Eq. 12 into Eq. 11, and rearranging terms, we obtain

$$F_s = \frac{S_{av}\,(IMD)}{(I/K_s)-1} \tag{13}$$

which can be used to predict the volume of infiltration prior to runoff. Intuitively,(Eq. 13) is satisfactory, for if IMD = 0 (soil saturated), there is no infiltration prior to runoff ($F_s = 0$). When $I/K_s \rightarrow 1$, $F_s \rightarrow \infty$, for all of the rainfall at this low intensity will infiltrate.

A relationship similar to equation 13 was proposed as part of the predicted intake volume to saturation by Rubin and Steinhardt (43) for a soil with a well defined "bubbling pressure" (the capillary potential at which the soil begins to desaturate). In their case S_{av} was equal to the bubbling pressure. Since many soils do not exhibit a well defined bubbling pressure, the determination of S_{av} in the following way is considered substantially more general.

The Capillary Potential at the Wetting Front (S_{av})

There would appear to be two alternative ways of determining the average suction at the wetting front, from the S - θ relationship or from the S - K relationship for the soil in question. Examination of these curves for a soil shows that the former is not acceptable. Take Fig. 1 for example. It shows that the capillary conductivity is effectively zero at a moisture content of about 30%. For moisture contents below this level, soil moisture movement is comparitively very slow, despite the high suctions in this region. Since we are interested in the moving front, it would seem that the average capillary suction is best determined from the S - K curve. It is proposed, therefore, that S_{av} be determined by

$$S_{av} = \frac{\int_{K(\theta_i)}^{K(\theta_s)} S.dK(\theta)}{K(\theta_s) - K(\theta_i)} \tag{14}$$

It is more convenient to use the relative conductivities, k_r, instead of the absolute conductivity $K(\Theta)$.

$$k_r(\Theta) = \frac{K(\Theta)}{K(\Theta_s)} \qquad (15)$$

Substituting Eq. 15 into Eq. 14, and noting that $K(\Theta_s) = K_s$,

$$S_{av} = \frac{\int_{k_r(\Theta_i)}^{1} S . dk_r(\Theta)}{1 - k_r(\Theta_i)} \qquad (16)$$

Typically the initial moisture content before a rainfall event is such that $k_r(\Theta_i) << 1$, ($\Theta \leq$ field capacity), so that we can write $k_r(\Theta_i) \simeq 0$ with negligible error. Then

$$S_{av} = \int_0^1 S . dk_r \qquad (17)$$

or simply, the area under the $S - k_r$ curve, between $k_r = 0$ and $k_r = 1$. In practice the $S - k_r$ relationship near $k_r = 0$ is hard to define, so, in this study, the area in the range $k_r = .01$ to $k_r = 1$ was used.

<u>Prediction of the Infiltration Capacity After Runoff Begins</u> (Case C)

Let us assume that some time after the surface has saturated the moisture profile can be represented by Fig. 9. Darcy's law can be applied again, noting that the infiltration rate is now equal to the infiltration capacity (f_p), to give

$$f_p = K_s \frac{S_{av} + L_s + L}{L_s + L} \qquad (18)$$

As in Eq. 12, $L_s = F_s/IMD$. Similarly, $L = (F - F_s)/IMD$

Hence

$$f_p = K_s \left(1 + \frac{IMD \cdot S_{av}}{F}\right) \qquad (19)$$

Note that in this equation, the infiltration capacity is not influenced by the infiltration volume up to surface saturation F_s. In other words, Eq. 19 predicts a relationship as shown in Fig. 10, in which the infiltration capacity curve is constant in position for differing rainfall intensities. Experimental data presented by Rubin and Steinhardt (43) is very similar to Fig. 10.

It may sometimes be more useful to have the cumulative infiltration as a function of the time since the beginning of the rainfall. By placing $f_p = dF/dt$ in Eq. 19 and separating the variables we obtain

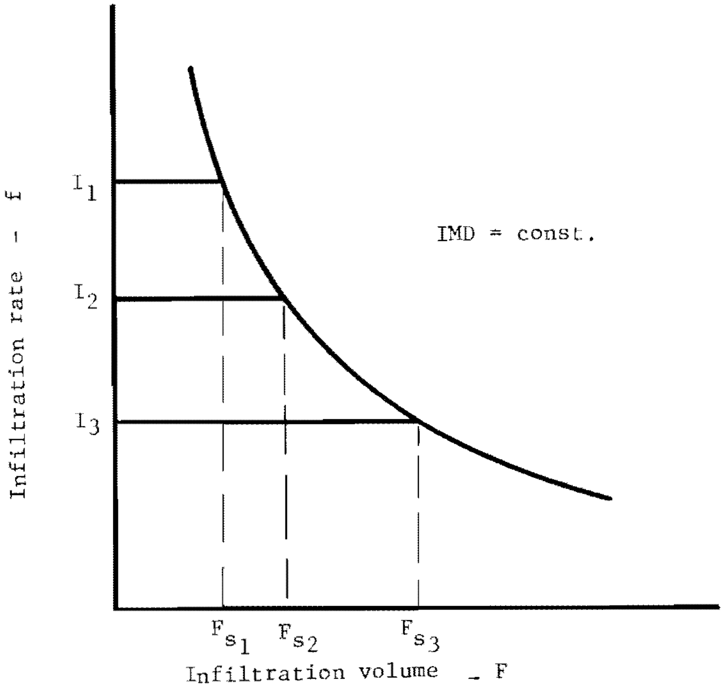

Fig. 10. Behaviour of infiltration capacity vs. infiltration volume relationship as predicted by Eq. 19. For const. IMD. (I_1, I_2, and I_3 are different rainfall intensities)

$$\int_{F_s}^{F} \frac{F.dF}{F + (IMD)\, S_{av}} = \int_{t_s}^{t} K_s . dt \qquad (20)$$

which can be reduced to

$$K_s\,(t - t_s) = F - (IMD) \cdot S_{av} . \log_e[F_s + IMD \cdot S_{av}]$$
$$- F_s + (IMD) . S_{av} . \log_e [F_s + IMD . S_{av}] \qquad (21)$$

If we call t_s' the equivalent time to infiltrate volume F_s under saturated surface conditions, Eq. 21 can be manipulated to read

$$K_s\,(t - t_s + t_s') = F - (IMD)\, S_{av} \log_e\left(\frac{F + (IMD) . S_{av}}{(IMD) . S_{av}}\right) \qquad (22)$$

which is similar to the Green and Ampt expression given in Eq. 6, but with an adjusted time scale. Eq. 22 can be solved directly for the actual time, t, or by iteration for F.

Summary of Model

It has been proposed in this section that a suitable model for infiltration of a constant intensity rainfall ($I>K_s$) into a soil of uniform moisture content can be expressed in the following form

$f = I$ until $F = F_s$, given by

$$F_s = \frac{S_{av}\ (IMD)}{(I/K_s) - 1} \tag{23}$$

and

for $F > F_s$, $f = f_p$ given by

$$f_p = K_s\ (1 + \frac{S_{av}\ (IMD)}{F}) \tag{24}$$

V. NUMERICAL SOLUTION OF THE MOISTURE FLOW EQUATION

The soil moisture flow equation (Eq. 8) is a second - order non-linear partial differential equation. An ideal solution would be an analytic one, but except for a few special cases, no general analytic solution is available. To obtain solutions for situations which approximate real-life conditions, several researchers have employed finite difference methods of one form or another, and programmed a computer for the repetitive and tedious computations involved.

Smith (45) examined many of the published methods in an attempt to develop one which both satisfied the law of continuity and predicted a smooth infiltration curve whose shape agreed with field observations. The numerical scheme developed below is a method based on the Crank - Nicholson finite difference scheme, and is similar to that adopted by Smith. Additional refinements and modifications have been made and the resulting program is suitable for infiltration under any constant rainfall rate into any homogeneous soil type, and for any initial moisture distribution. Layered soils and rainfall of non-constant intensity can be simulated with simple modifications to the program.

Formulation of Basic Finite Difference Scheme

Interior Nodes

For convenience, Eq. 8 will be rewritten here

$$\frac{\partial\Theta}{\partial t} = \frac{\partial}{\partial z}\left(K(\Theta)\ .\ \frac{\partial S(\Theta)}{\partial z}\right) - \frac{\partial K(\Theta)}{\partial z} \tag{25}$$

The conductivity $K(\Theta)$ can be written as $K\ (\Theta) = K_s\ k_r$, where K_s is the saturated hydraulic conductivity, and k_r is the relative conductivity. (This form is more convenient for interpretation, since k_r represents the fraction of the maximum conductivity possible for the soil in question). Also, using the chain rule of calculus

$$\frac{\partial\Theta}{\partial t} = \frac{\partial\Theta}{\partial S} \cdot \frac{\partial S}{\partial t} = - K_s \frac{\partial}{\partial z}\ (k_r \frac{\partial S}{\partial z}) - K_s \frac{\partial k_r}{\partial z} \tag{26}$$

Replacing the differential expressions by their corresponding finite difference approximations, one obtains

$$\frac{\Delta\Theta}{\Delta S} \cdot \frac{\Delta S}{\Delta t} = - K_s \frac{1}{\Delta z} \Delta\ (k_r \frac{\Delta S}{\Delta z}) - K_s \frac{\Delta k_r}{\Delta z} \tag{27}$$

There are two dependent variables in this equation, Θ and S. It can be reduced to an expression with S as the only dependent variable by noting that $\Delta\Theta/\Delta S$ is the slope of the Θ vs. S curve at some appropriate point. Writing $\Delta\Theta/\Delta S$ as SL and multiplying both sides by Δz,

$$(SL)\ \frac{\Delta z \Delta S}{\Delta t} = - K_s\ [\Delta\ (k_r \frac{\Delta S}{\Delta z}) - \Delta k_r\ ] \tag{28}$$

To evaluate the difference expression, the notation demonstrated in Fig. 11 will be used. All nodes on the time - distance plane are numbered with the subscript i referring to the depth dimension, and the subscript j referring to the time dimension. Thus, i = 1 and j = 1 refer to zero depth and zero time respectively. The time and depth increments are not necessarily constant (discussed later) so the node spacings shown in Fig. 11 are uneven. The depth node i is assumed to represent the soil thickness Δz_i which extends from the midpoint of z_i and z_{i-1} to the midpoint of z_i and z_{i+1}. These midpoints will be designated minus (-), and plus (+), respectively, in the notation. In the finite difference approximation, the distance-averaged quantities will be evaluated at the midpoint of the time step, Δt. Eq. 28 becomes

$$(SL)_i^{j-\frac{1}{2}} \frac{\Delta z_i}{\Delta t} (S_i^j - S_i^{j-1}) = -K_s \left(k_{r+}^{j-\frac{1}{2}} \frac{\Delta S_+^{j-\frac{1}{2}}}{\Delta z_+} - k_r^{j-\frac{1}{2}} \frac{\Delta S_-^{j-\frac{1}{2}}}{\Delta z_-} + k_{r+}^{j-\frac{1}{2}} - k_{r-}^{j-\frac{1}{2}} \right) \qquad (29)$$

Now we can write

$$\Delta S_+^{j-\frac{1}{2}} = (S_{i+1}^j + S_{i+1}^{j-1} - S_i^j - S_i^{j-1})/2$$

$$\Delta S_-^{j-\frac{1}{2}} = (S_i^j + S_i^{j-1} - S_{i-1}^j - S_{i-1}^{j-1})/2 \qquad (30)$$

The terms $k_{r+}^{j-\frac{1}{2}}$ and $k_{r-}^{j-\frac{1}{2}}$ could be evaluated as being conductivities corresponding to the suctions $S_{i+\frac{1}{2}}^{j-\frac{1}{2}}$ and $S_{i-\frac{1}{2}}^{j-\frac{1}{2}}$ respectively, or the average conductivity taken from the four surrounding node points. A third alternative is to select $k_r^{j-\frac{1}{2}}$ as a time-averaged conductivity from the node from which flow to node i is originating. It was found that the latter assumption improves convergence, although there is no appreciable difference from the results obtained by the other two methods. Therefore, let

$$k_{r-}^{j-\frac{1}{2}} = (k_{r_{i-1}}^{\ j} + k_{r_{i-1}}^{\ j-1})/2$$

and

$$k_{r+}^{j-\frac{1}{2}} = (k_{r_i}^j + k_{r_i}^{j-1})/2 \qquad (31)$$

Substituting Eq. 30 into Eq. 29, we obtain

$$(SL)_i^{j-\frac{1}{2}} \frac{\Delta z_i}{\Delta t} \left(S_i^j - S_i^{j-1} \right) = -K_s \left[k_{r+}^{j-\frac{1}{2}} \left(\frac{S_{i+1}^j + S_{i+1}^{j-1} - S_i^j - S_i^{j-1}}{2\Delta z_+} \right) - k_{r-}^{j-\frac{1}{2}} \left(\frac{S_i^j + S_i^{j-1} - S_{i-1}^j - S_{i-1}^{j-1}}{2\Delta z_-} \right) + k_{r+} - k_{r-} \right] \qquad (32)$$

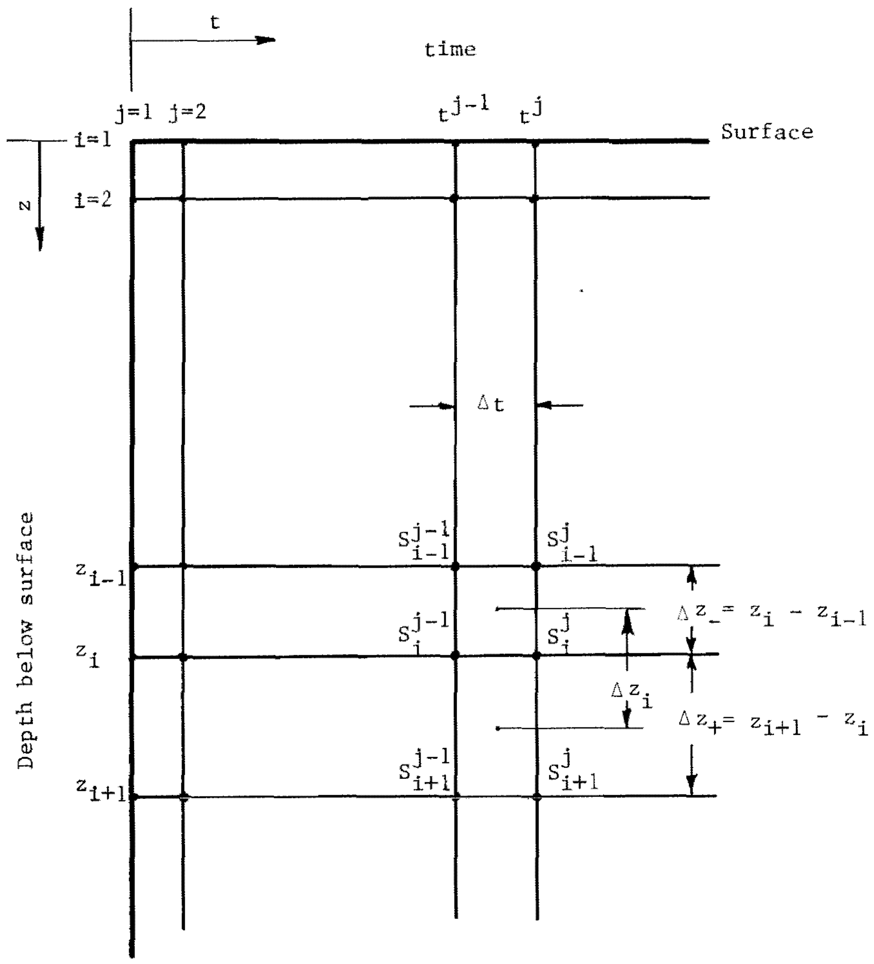

Fig. 11. Definition diagram for finite difference scheme.

The unknown terms are the S^j'$_s$ - these will be placed on the right hand side of the equation.

$$-(SL)_i^{j-\frac{1}{2}}\frac{\Delta z_i}{\Delta t}S_i^{j-1} + K_s\,k_{r+}^{j-\frac{1}{2}}\left(\frac{S_{i+1}^{j-1} - S_i^{j-1} + 2\Delta z_+}{2\Delta z_+}\right) -$$

$$K_s\,k_{r+}^{j-\frac{1}{2}}\left(\frac{S_i^{j-1} - S_{i-1}^{j-1} + 2\Delta z_-}{2\Delta z_-}\right) = -\,S_{i-1}^j\left(\frac{K_s\,k_{r-}^{j-\frac{1}{2}}}{2\Delta z_-}\right) +$$

$$S_i^j\left(\frac{K_s k_{r+}^{j-\frac{1}{2}}}{2\Delta z_+} + \frac{K_s k_{r-}^{j-\frac{1}{2}}}{2\Delta z_+} + \frac{K_s k_{r-}^{j-\frac{1}{2}}}{2\Delta z_-} - (SL)_i^{j-\frac{1}{2}}\frac{\Delta z_i}{\Delta t}\right) -$$

$$S_{i+1}^j\left(\frac{K_s k_{r+}^{j-\frac{1}{2}}}{2\Delta z_+}\right) \tag{33}$$

which can be written more simply in the form

$$LHS_i = a_i\,S_{i-1}^j + b_i\,S_i^j + c_i\,S_{i+1}^j \tag{34}$$

An equation of the form of Eq. 34 can be written for the upper and lower boundary conditions (see next section) and for each interlying depth mode, resulting in a set of $\underline{m}$ equations for $\underline{m}$ unknown values of S. In matrix form, this can be written as

$$[A]\,\vec{S} = \overrightarrow{LHS} \tag{35}$$

where [A] is a tri-diagonal matrix, a form which can be solved rapidly with a simple numerical algorithm. The upper and lower boundary conditions should now be examined. For the upper boundary there are two possibilities; either the surface is unsaturated (rainfall condition - Case B) or the surface is saturated (Case C).

Upper Boundary Condition-Case B

For this case, the surface layer is unsaturated, and the extra term, the rainfall rate, must be added to the continuity equation for the nodes i = 1. Following the same procedure as before, an equation analagous to Eq. 32 can be written for the surface node,

$$(SL)_1^{j-\frac{1}{2}}\frac{\Delta z_1}{\Delta t}(S_1^j - S_1^{j-1}) = -\,K_s k_{r+}\left(\frac{S_2^j + S_2^{j-1} - S_1^j - S_2^{j-1}}{2\Delta z_+} + 1\right) + RAIN \tag{36}$$

where RAIN is the rainfall intensity.

Placing the unknown values on the right hand side as before results in

$$-(SL)_1^{j-\frac{1}{2}}\frac{\Delta z_1}{\Delta t}S_1^{j-1} + K_s k_{r+}\frac{(S_2^{j-1} - S_1^{j-1} + 2\Delta z_+)}{2\Delta z_+} - RAIN$$

$$= S_1^j\left(\frac{K_s k_{r+}}{2\Delta_+} - (SL)_1^{j-\frac{1}{2}}\frac{\Delta z_1}{\Delta t}\right) - S_2^j\left(\frac{K_s k_{r+}}{2\Delta z_+}\right) \tag{37}$$

or

$$LHS_1 = b_1 S_1^j + c_1\,S_2^j \tag{38}$$

This is the first equation of the tridiagonal matrix (Eq. 35) for the rainfall condition at the surface.

Upper Boundary Condition-Case C

When the surface layer is saturated (the transition from unsaturated to saturated is discussed later) the capillary suction S_1^j is set equal to zero, which assumes there is no significant ponding on the surface. Thus we skip node 1 and start computation at node 2. Because S_1^j is known (= 0), Eq.34 becomes

$$LHS_2 = b_2\,S_2^j + c_2\,S_3^j \tag{39}$$

which is the first equation of the tridiagonal matrix (Eq. 35) for a saturated surface.

Lower Boundary Condition

In the early stages of infiltration, the wetting front has not penetrated far into the profile. It is more economical in computation time if the nodes which have not yet been reached by the wetting front are excluded from the calculation. Therefore, the matrix [A] is truncated. For this condition at the appropriate node m, one can say $S_m^j = S_m^{j-1}$. Hence, Eq. 34 becomes

$$LHS_m = a_m\,S_{m-1}^j + b_m\,S_m^j \tag{40}$$

which is the final equation in the m x m tridiagonal matrix.

Initial Conditions

The required initial conditions are the initial moisture content for each node, and whether or not the surface is to be instantaneously saturated at time zero. All cases simulated in the study involved infiltration under rainfall into a soil with a uniform initial moisture content.

Solution of Eq. 29 for Each Time Step

Soil Properties

It is necessary at every step to compute the values of k_r and SL corresponding to the capillary suction at each node. To expedite this the Θ vs S and k_r vs S relationships for the soil are stored in the computer as tables of data points. These points are chosen such that an insignificant error is incurred by assuming a straight line between successive points. By this method, the appropriate values of Θ and k_r can be determined efficiently by linear interpolation. To compute $(SL)^{j-\frac{1}{2}}$, the Θ-S table is scanned for the data point nearest the value of the midpoint of S_i^{j-1} and S_i^j. The slope is determined by fitting a parabola to this and its adjacent data points. If the difference between S_i^{j-1} and S_i^j is greater than a specified value, then the slope of the chord joining $(\Theta_i^{j-1}, S_i^{j-1})$ and (Θ_i^j, S_i^j) is used.

Requirement of Iteration

To solve Eq. 29 for a given time step it is necessary to use the correct values of k_r and SL at each node. However, to obtain these values, the corresponding S-values (which it is required to find) have to be known. Thus, an iterative procedure is needed, whereby the S-values are estimated and k_r and (SL) determined for these estimated values. Equation 33 is then solved for the new S-values which are compared with the estimated values. If the estimated and new S-values at each node do not correspond to within a specified error tolerance, the process is repeated, using the new S-values as the new estimates.

Transition from Unsaturated to Saturated Surface

As infiltration under a rainfall rate $>K_s$ proceeds, the surface layer becomes wetter, and the transition from Case B to Case C occurs. A constant check is kept of the moisture level in the surface node, and when saturation is reached, the time step is adjusted so that the phenomenom occurs just at the end of the time increment, and the step recomputed. Subsequent time steps are then assumed to have a saturated surface condition prevailing.

Acceleration of Convergence

The convergence for the iterative procedure described above can be accelerated using Aitken's Δ^2 - method (26), which predicts an improved value from the result of the previous three iterations. The formula can be stated thus

$$S_{n+2} = S_n - \frac{(S_{n+1} - S_n)^2}{(S_{n+2} - 2S_{n+1} + S_n)} \tag{41}$$

where S_n, S_{n+1}, S_{n+2}, are the values of S from the previous three iterations. S_{n+2} hopefully is an improved estimate which will jump several iterations. Some tests of the numerical solution showed that, when the Aitken Δ^2 - method was applied to those nodes which had not converged, the average saving in computation was two iteration steps for each time step. The method was not applied to the first iteration step,but to every three thereafter, for it is not suitable unless convergence is occurring.

Computation of Infiltration

After convergence is achieved the cumulative infiltration is computed from the change in the soil moisture level since the beginning of the event. That is

$$F^j = \sum_{i=1}^{m} \Delta z_i \, (\Theta_i^j - \Theta_i^1) \tag{42}$$

The average infiltration rate over the time interval is simply

$$f = \frac{F^j - F^{j-1}}{\Delta t} \tag{43}$$

For the unsaturated surface condition, a continuity check is possible by comparing the cumulative infiltration volume from Eq. 42 with the rainfall volume in the same time. Such a check was made for each computer run and the maximum error was of the order of 1%, but generally much less.

Depth Increment

Many published finite difference schemes for solving Eq. 25 have a fixed depth increment (e.g., Ref. 24). Smith (45) showed that the choice of depth increment has a considerable influence on the results, and proposed that increments be small near the soil surface where the suction gradients are higher and where it is necessary to accurately predict surface saturation, and larger at points deeper in the soil where gradients are smaller. Palmer (37) reached the same conclusion.

In the work reported here, the same pattern of depth increments was used for all soils. These values are shown in Table 1.

Table 1. Depth increments used in the finite difference scheme

Depth Range (cm)	Increment Size (cm)	No. of Increments
0 - 1	.20	5
1 - 8	.25	28
8 - 10	.50	4
10 - 15	1.00	5
15 - 25	2.00	5
25 -	5.00	3

These depth increments are a compromise based on the results of Smith (45), Palmer (37), and several tests by the senior author. It was found that by halving the depth increment a smaller time to surface saturation was predicted. A second halving of the increment resulted in a yet smaller predicted time but the discrepancy was approximately half of the previous difference. Thus, the depth increment must be reduced until the difference between successive predicted values is an acceptable one. For the choice given above the estimated error is of the order of 5%. (Note that if the depth increment is halved, the computation time is more than doubled).

Time Increment

Eq. 25 is non-linear because SL and k_r are functions of S. As mentioned before it is solved by assuming that k_r and SL are constant over the particular time step. Tests with the numerical solution showed that errors in computation of SL produced continuity errors, and that the solution was far more sensitive to SL than k_r.

The initial time step was arbitrarily chosen as the time needed to fill 10% of the available storage in the uppermost soil layer. Larger initial steps caused instability in the solution. Hence, the time step was computed from a prediction of the slope change for each node over the new time step. This prediction was based on the change over the previous time step and the curvature of the Θ-S curve at that point. A secondary factor in the choice of the time step was the number of iterations involved. Too large a time step means many iterations and poorer accuracy, while too small a step means few iterations, but many more time steps for the same event time. An increase of the time step by 20% was made if the iterations on the previous step were less than four, and the time step was halved if more than twenty iterations were needed.

Generally, for each run the time step increased reasonably uniformly to a predetermined maximum value.

The Computer Program

The program was written in FORTRAN IV and was run on the CDC 6600 computer at the University of Minnesota Computer Center. The complete program was given by Mein (34). It is written as a series of subroutines for maximum flexibility. Briefly, the data input is handled in DATIN, and the initial conditions are computed in CALC. Subroutine MAIN solves the basic soil moisture equation, and CONS computes the infiltration, elapsed time, depth of wetting, and the new time step. PLOTT is an option to plot the moisture content or capillary potential as a function of depth, while INTERP and SLOPE are used to interpolate the soil data curves, and calculate the slope of the Θ-S curve respectively.

For an average run the compile time was 4 seconds and the run time 15 seconds.

VI. GENERATION OF INFILTRATION DATA

Discussion of the Richards Equation in Sec. III showed that it is considered the best available mathematical description of moisture flow in soils. With numerical solution of this equation, one can therefore generate any number of infiltration events, simply by using the appropriate soil properties and the desired values of initial moisture content and rainfall intensity. In this manner, the influence of any factor on the infiltration behavior can be studied by systematically changing its value. Such a technique is extremely difficult in both field and laboratory experiments.

The Soils Used

It was desired to generate infiltration data for a range of soil types. The soils, selected from published data and listed below in Table 2, vary in texture from a sand to a light clay.

Table 2. Soils selected for the study

Soil	Reference	K_s(cm/sec)	Porosity
Plainfield Sand (disturbed sample)	(1)	3.44×10^{-3}	.477
Columbia Sandy Loam (disturbed sample)	(32)	1.39×10^{-3}	.518
Guelph Loam (air dried, sieved)	(14)	3.67×10^{-4}	.523
Ida Silt Loam (undisturbed sample)	(19)	2.92×10^{-5}	.530
Yolo Light Clay (disturbed sample)	(35)	1.23×10^{-5}	.499

The published data for the first 3 soils in Table 2 were desorption data, but because infiltration is an absorption process it is important to use the absorption or wetting curve of the soil characteristic curves. A reasonable estimate of the wetting curve can be obtained by dividing the drying curve S - values by a factor approximately equal to 1.6 (D. A. Farrell, Research Soil Scientist, Agricultural Research Service, private communication). For spherical particles Rode (42) shows that the ratio of S_{des} to S_{abs} is about 1.8, but it would be somewhat less for non-spherical particles. Examination of published hysteresis data showed different values of the ratio of S_{des} to S_{abs} ranging from 1.5 to 1.8 over most of the curve. In this study a constant ratio of 1.6 was used, i.e., for the sand, sandy loam, and loam, the S values for the S-Θ and S-K curves were divided by 1.6 to obtain wetting data.

The S-Θ and S-k_r curves for the five soils listed in Table 2 are shown in Figs. 12 and 13. Note that a wide variability in soil properties is represented by these curves. Complete details of the input data for each soil are given in Appendix A.

Choice of Initial Moisture Contents and Rainfall Intensities

The moisture contents used in the study for each soil were somewhat arbitrarily chosen, but in all cases, the highest initial moisture content was such that the relative conductivity at that moisture level was small (<5%). With this as the upper limit a number of values spread over the remaining soil data range were selected for each soil. (In natural soil, drainage after rainfall can reduce the moisture content fairly rapidly to a point where the conductivity is low, so the choice of the upper limit above is realistic).

Rainfall intensities were chosen as fixed multiples of the saturated conductivity for each soil; values of 4 and 8 were used for all soils.

Columbia Sandy Loam and Ida Silt Loam were singled out for more extensive testing, both to save a substantial number of computer runs on the other 3 soils, and because they represent two completely different soil types. Additional rainfall intensities (2 K_s and 6 K_s) and extra moisture levels of moisture content were used for these two soils.

Results

A number of graphs and figures representative of the infiltration data generated for each soil will be presented in this chapter. Additional data are given in Sec. VII and the complete results are listed in Appendix B.

Graphs of infiltration rates vs time for Columbia Sandy Loam are given in Figs. 14 and 15, showing the effect of initial moisture content on the shape of the infiltration capacity curve. As one would expect, the time to surface saturation is larger when the initial moisture level is less. Note also that the infiltration curve is steeper the sooner that surface saturation is reached, and that the slope of the tail of each curve becomes small even though the infiltration rate is significantly greater than K_s.

Figs. 16 and 17 are different ways of plotting the results for Columbia Sandy Loam. Fig. 16 clearly shows the effect that rainfall intensity has on both the shape and position of the resulting infiltration capacity curve. However, as shown in Fig. 17, by plotting infiltration volume on the abscissa, the infiltration capacities for each run fall on the same curve. Similar data for Ida Silt Loam are shown in Figs. 18 and 19, although in the latter figure, the infiltration capacity curves do not quite coincide. The implication of the common infiltration capacity vs infiltration volume curve and a possible explanation of the Ida Silt Loam discrepancy is discussed in the next section

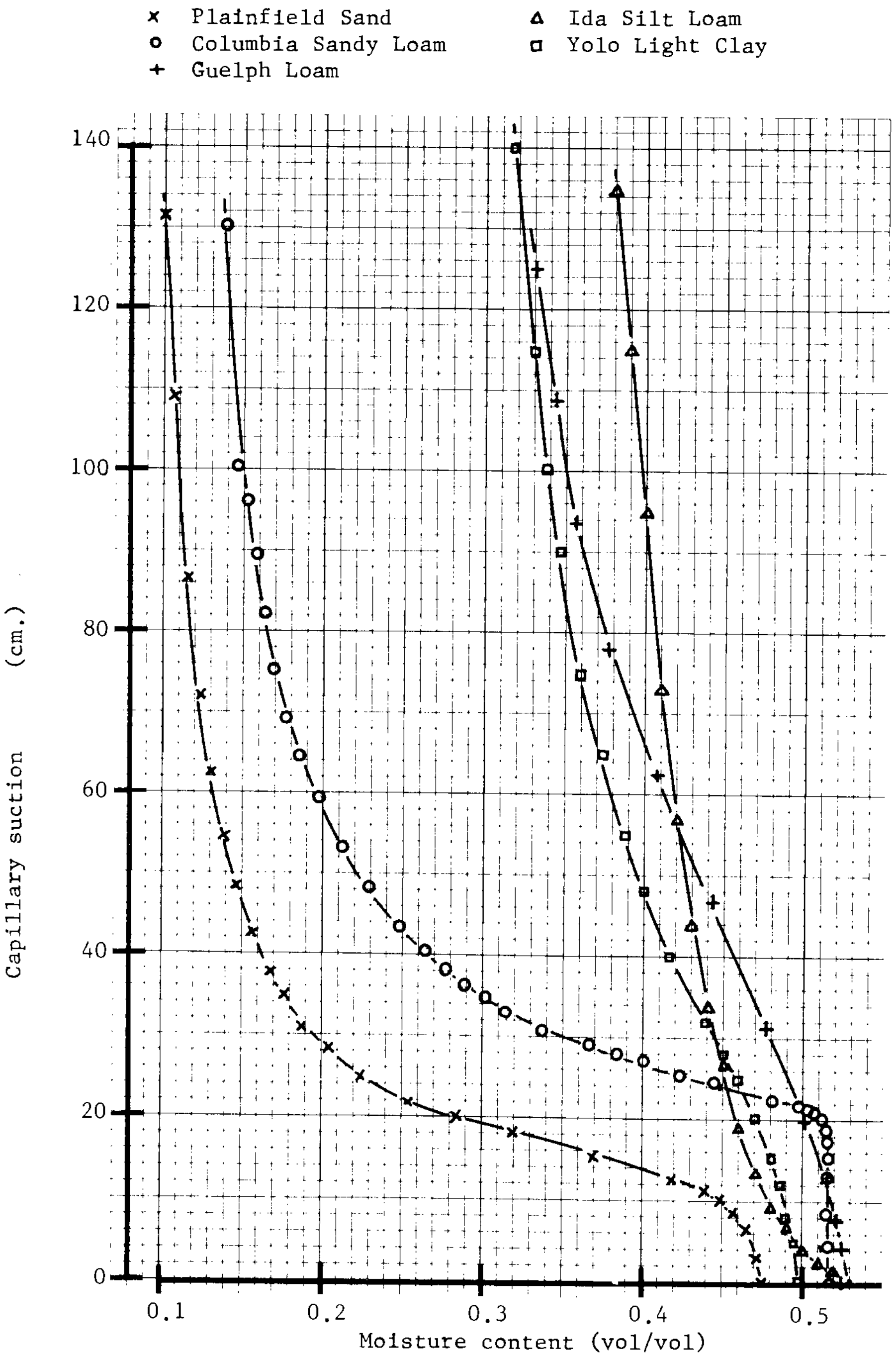

Fig. 12. Capillary suction vs. moisture content for the selected soils. (Absorbtion data - points listed in Appendix A)

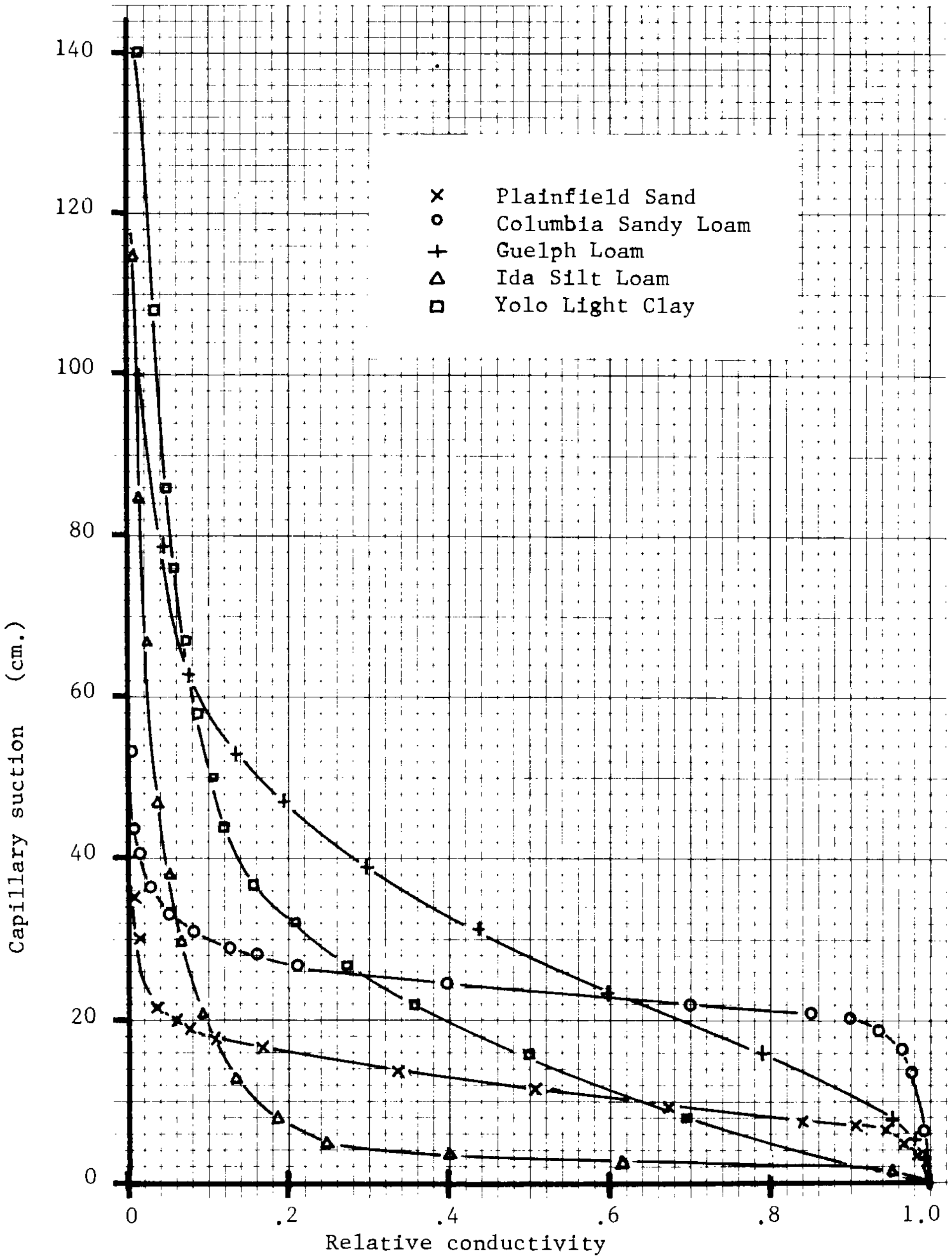

Fig. 13. Capillary suction vs. relative conductivity for the selected soils. (Absorbtion data - points listed in Appendix A)

(page 39). It should also be mentioned that of the five soil types used, the coincidence of the infiltration capacity vs. infiltration volume curves was best for Columbia Sandy Loam and poorest for Ida Silt Loam, the examples shown in Figs. 17 and 19. These results illustrate the fact that infiltration capacity is best expressed in terms of prior infiltration volume and initial moisture content, rather than by the usual empirical equations which give infiltration capacity as a function of time.

Along with the infiltration data, the soil moisture profile at each time step was obtained. Figs. 20 and 21 show the profile at the instant of surface saturation and at a later time for Columbia Sandy Loam and Ida Silt Loam respectively.

A general observation on the depth of wetting is worth noting here. The program to generate the infiltration event halted when a specified event time had elapsed or the wetting front reached 30 cm, whichever happened first. For the loam, silt loam, and light clay the former criterion stopped the run, whilst the latter criterion caused many of the sand and sandy loam runs to terminate. In all cases, the infiltration rate had fallen to the portion of the curve where the infiltration capacity was decreasing slowly. The corresponding depths of the wetting front ranged from around 8 cm for the clay to 30 cm for the sand. This observation has important implications which are given in Sec. VIII.

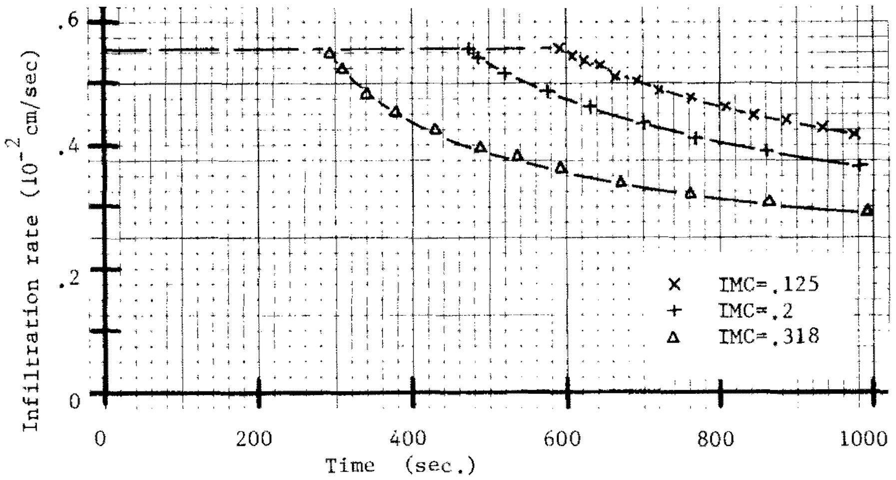

Fig. 14. Infiltration rate vs. time for Columbia Sandy Loam showing effect of initial moisture content. ($I = 4K_s$)

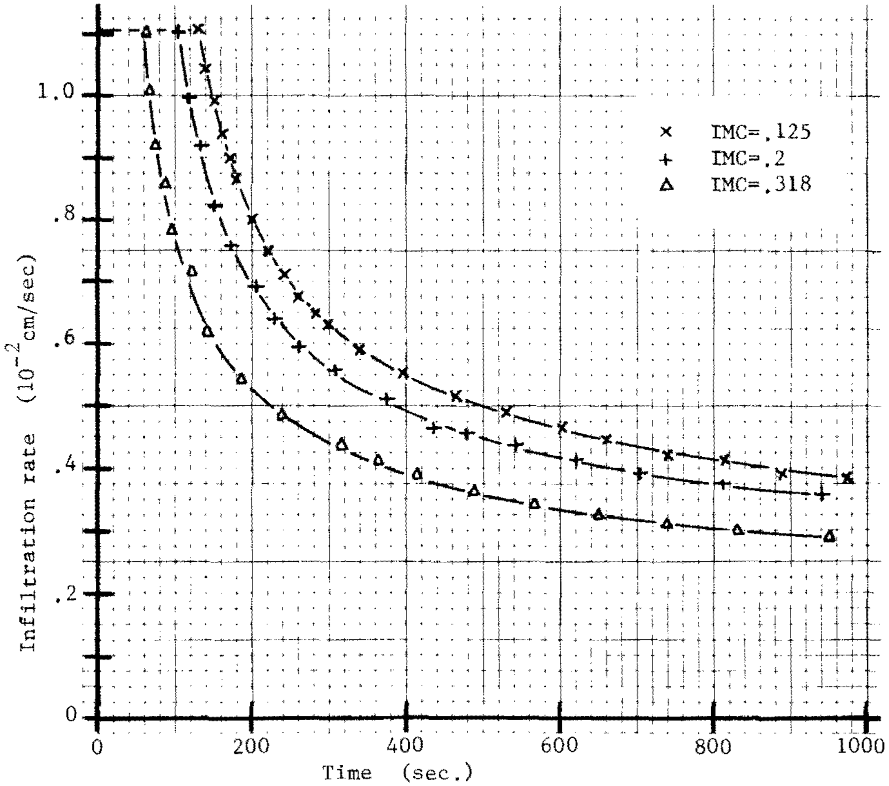

Fig. 15. Infiltration rate vs. time for Columbia Sandy Loam showing effect of initial moisture content. ($I = 8K_s$)

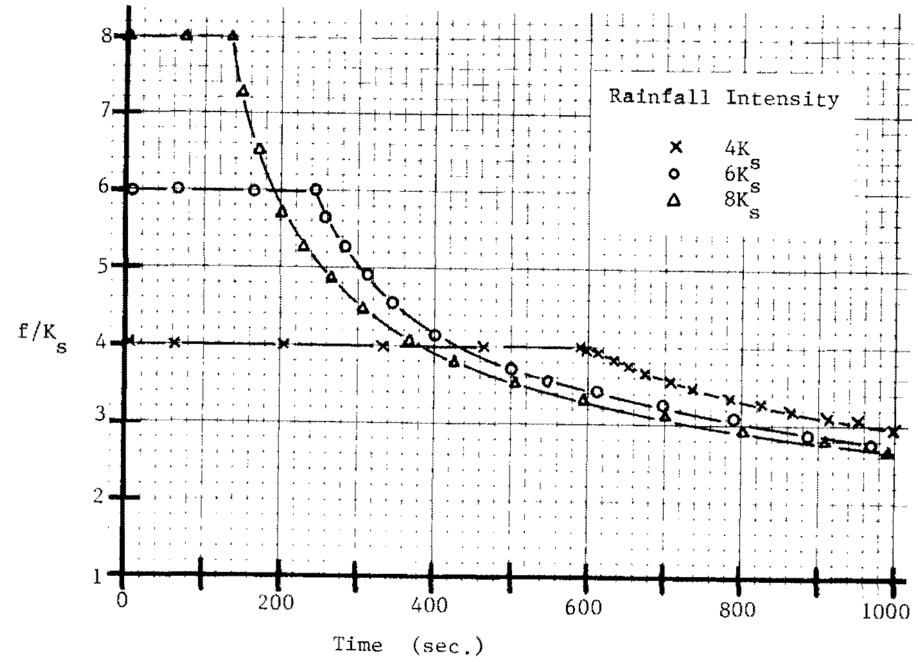

Fig. 16. Dimensionless infiltration rate vs. time for Columbia Sandy Loam showing effect of rainfall intensity. (IMC=.125)

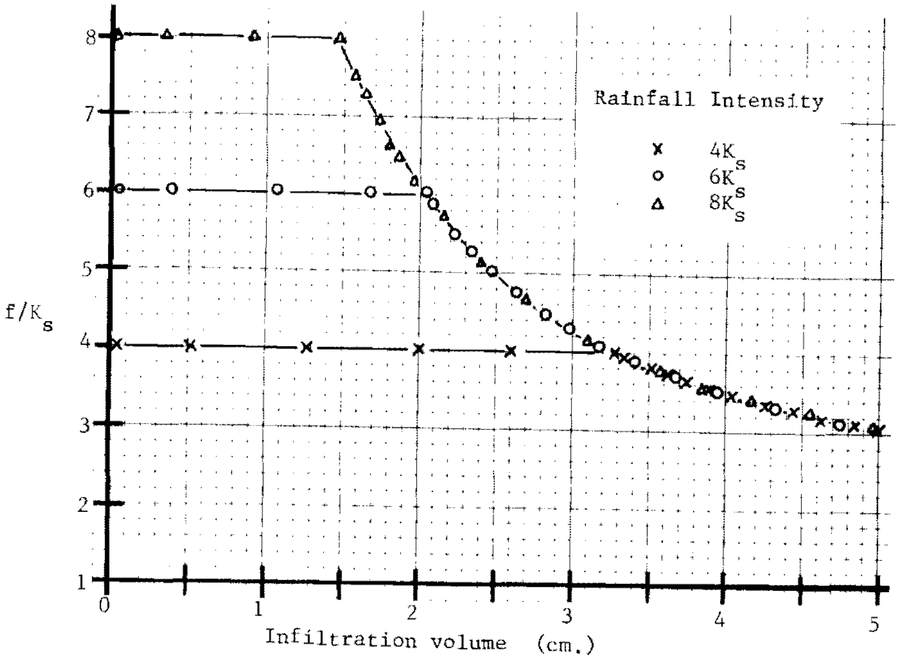

Fig. 17. Dimensionless infiltration rate vs. infiltration volume for Columbia Sandy Loam showing effect of rainfall intensity. (IMC=.125)

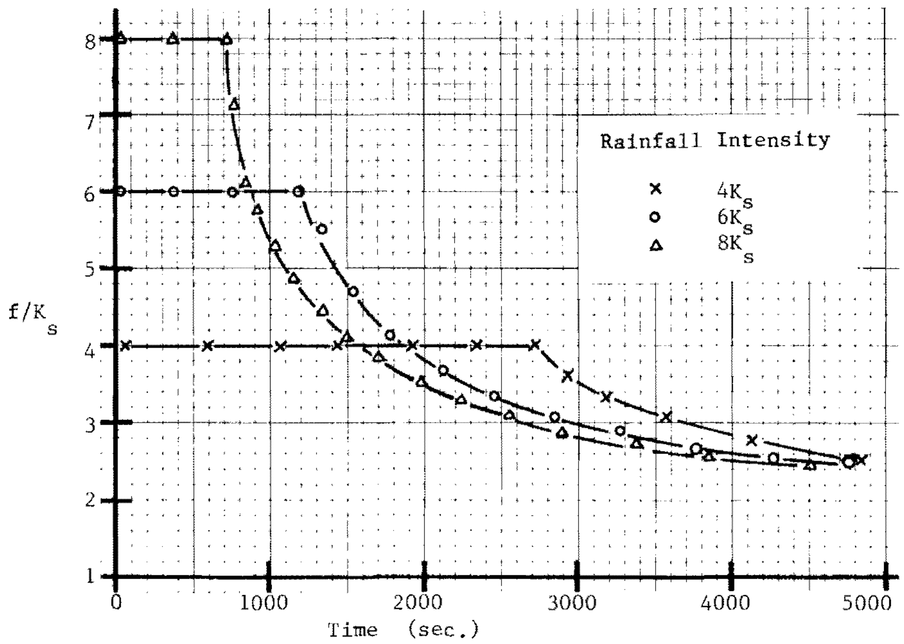

Fig. 18. Dimensionless infiltration rate vs. time for Ida Silt Loam showing effect of rainfall intensity. (IMC =.4)

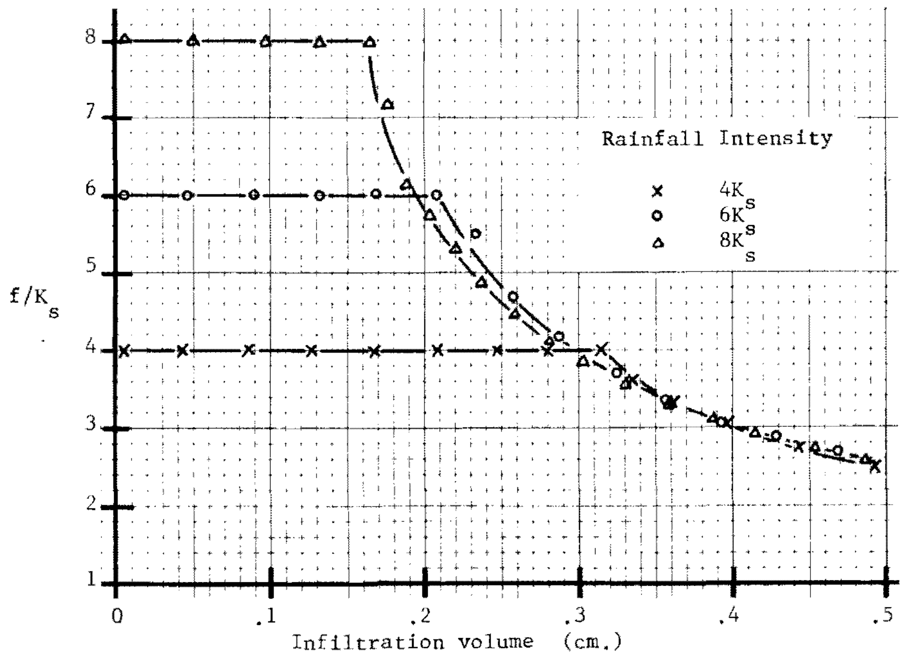

Fig. 19. Dimensionless infiltration rate vs. infiltration volume for Ida Silt Loam showing effect of rainfall intensity. (IMC=.4)

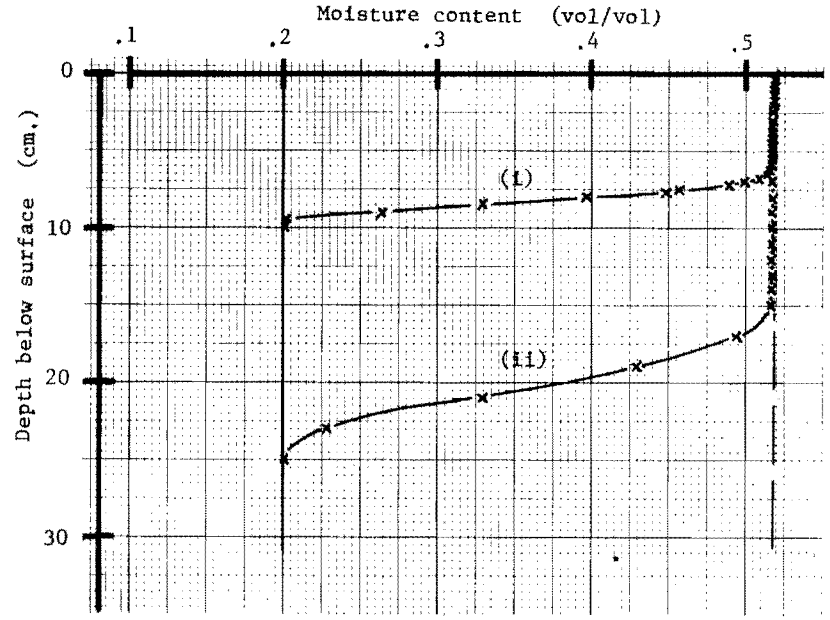

Fig. 20. Soil moisture profiles for Columbia Sandy Loam at
(i) 472 secs. - the moment of surface saturation
(ii) 1450 secs.
(IMC = .2, $I = 4K_s$)

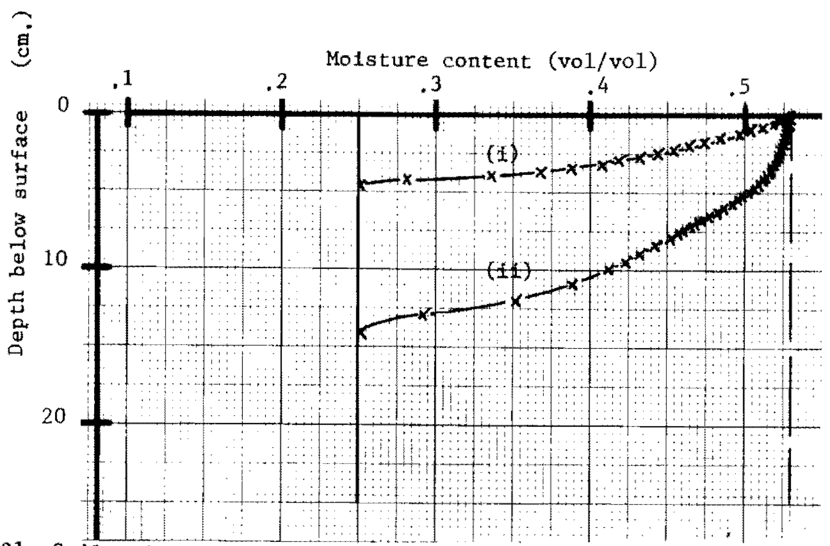

Fig. 21. Soil moisture profiles for Ida Silt Loam at
(i) 7277 secs. - the moment of surface saturation
(ii) 40340 secs.
(IMC = .25, $I = 4K_s$)

In this section, the infiltration behavior predicted independently by the proposed model (Eqs. 23 and 24) is compared with the data generated by the finite difference approximation to the Richards equation (Eq. 28). There are two parts to the model: prediction of the moment of surface saturation, which is the end of the stage for which infiltration rate equals the rainfall intensity, and prediction of the infiltration capacity of the soil when the infiltration rate falls below the rainfall rate. A further test of the former stage is made using published experimental data.

Prediction of F_s

To predict the infiltration volume prior to surface saturation (F_s) Eq. 23 is used. However, the parameter S_{av} must first be evaluated using Eq. 17 and the S-k_r curve for each soil, shown in Fig. 13. The values obtained for each soil are given in Table 3.

Table 3. Values of S_{av} obtained from Eq. 17 and Fig. 13

Soil	S_{av}(cm)
Plainfield Sand	11.73
Columbia Sandy Loam	23.83
Guelph Loam	31.38
Ida Silt Loam	7.43
Yolo Light Clay	22.36

With the appropriate value of S_{av} in Eq. 23, the volume of water uptake to the point of surface saturation can be predicted. The values obtained for the five soils for a variety of rainfall intensities and initial moisture contents are compared with the corresponding values computed by Eq. 28 in Tables 4 to 8.

In view of the assumptions involved in the derivation of Eq. 23, the agreement obtained between Eq. 23 and Eq. 28 for all of the soils except Ida Silt Loam is very good. If one considers the error in the results from the Richards equation due to a finite depth increment (discussed in Sec. V) and realizing that this tends to increase the value of F_s obtained, the results look better still.

Table 7 shows comparatively large percentage discrepancies for Ida Silt Loam, although absolute differences are small. The explanation may be that the different soil properties near saturation (See Fig. 12) render less valid the assumption of a rectangular wetting front profile, made in deriving part of Eq. 23. Certainly the soil moisture profile for this soil at saturation (Fig. 21) is vastly different from that of Columbia Sandy Loam (Fig. 20). Further investigation should be directed at

Table 4. Predicted and Computed Infiltration Volume to Surface Saturation (cm) for Plainfield Sand

I/K_s	IMD	F_s(Pred) Eq. 13	F_s(Calc) Eq. 28	Difference Eq.28-Eq.13	% Diff
4	.347	1.35	1.43	.08	5.6
	.247	.96	.99	.03	3.0
8	.347	.581	.677	.096	14.2
	.247	.413	.463	.050	10.8

Table 5. Predicted and Computed Infiltration Volume to Surface Saturation (cm) for Columbia Sandy Loam

I/K_s	IMD	F_s(Pred) Eq. 13	F_s(Calc) Eq. 28	Difference Eq.28-Eq.13	% Diff
2	.393	9.43	10.10	.67	6.6
	.268	6.38	6.75	.37	5.5
4	.393	3.12	3.28	.26	7.9
	.368	2.92	3.06	.14	4.6
	.318	2.52	2.62	.10	3.8
	.268	2.13	2.19	.06	2.7
	.200	1.59	1.61	.02	1.2
6	.393	1.87	2.01	.14	7.0
	.268	1.28	1.34	.06	4.5
8	.393	1.34	1.47	.13	8.8
	.368	1.25	1.37	.12	8.8
	.318	1.08	1.18	.10	8.5
	.268	.913	.795	.062	6.4
	.200	.681	.712	.031	4.4

Table 6. Predicted and Computed Infiltration Volume to Surface Saturation (cm) for Guelph Loam

I/K_s	IMD	F_s(Pred) Eq. 13	F_s(Calc) Eq. 28	Difference Eq.28-Eq.13	% Diff
4	.223	2.23	2.37	.04	1.7
	.173	1.81	1.78	-.03	1.7
8	.223	.999	1.07	.07	6.5
	.173	.775	.801	.026	3.2

Table 7. Predicted and Computed Infiltration Volume to Surface Saturation (cm) for Ida Silt Loam

I/K_s	IMD	F_s(Pred) Eq. 13	F_s(Calc) Eq. 28	Difference Eq.28-Eq.13	% Diff
2	.28	2.08	1.85	-.23	-12.4
	.13	.965	.727	-.238	-32.7
4	.28	.691	.850	.159	17.7
	.23	.568	.673	.105	15.7
	.18	.446	.513	.067	13.1
	.13	.321	.315	-.006	- 1.9
	.10	.248	.215	-.033	-15.3
6	.28	.416	.607	.191	31.4
	.13	.193	.209	.016	7.7
8	.28	.298	.478	.180	37.7
	.23	.244	.367	.123	33.6
	.18	.191	.251	.060	23.9
	.13	.138	.163	.025	15.4
	.10	.106	.105	-.001	- 1.0

this apparent anomaly.

All of the results in Tables 4 to 8 (except for two large values in Table 5) are plotted in Fig. 22 for a visual comparison between results from Eqs. 23 and 28. Generally, agreement is good.

A further test of Eq. 23 can be made against data published by Rubin and Steinhardt (43) for rainfall infiltration into columns of air-dry Rehovot Sand. Because the actual moment of surface saturation (defined in this study as the moment when the suction at the surface is just zero) is difficult to observe, Rubin and Steinhardt noted three stages: visible retardation of rainfall just apparent (Stage A), about 1/3 of surface just covered by water (Stage P), and surface just completely covered (Stage C). They reasoned that surface saturation should occur somewhere between stages A and P. The experimental points corresponding to stages A and P are plotted for several rainfall intensities in Fig. 23.

From their published data for Rehovot sand, the following properties were noted. The porosity is .387 (vol/vol), the saturated conductivity is 47.9 cm/hr, the initial moisture content (air dry) was taken to be .025 (vol/vol), and S_{av} was computed to be 16.1 cm. The values predicted by Eq. 23 for the appropriate rainfall intensities are plotted with the experimental data in Fig. 23. The good agreement between the observed and predicted infiltration volume to surface saturation is further proof of the validity of Eq. 23.

Table 8. Predicted and Computed Infiltration Volume to Surface Saturation (cm) for Yolo Light Clay

I/K_s	IMD	F_s(Pred) Eq. 13	F_s(Calc) Eq. 28	Difference Eq. 28-Eq. 13	% Diff
4	.249	1.86	1.90	.04	2.1
	.149	1.11	.966	-.14	-1.5
8	.249	.797	.918	.121	13.1
	.149	.477	.459	-.018	3.9

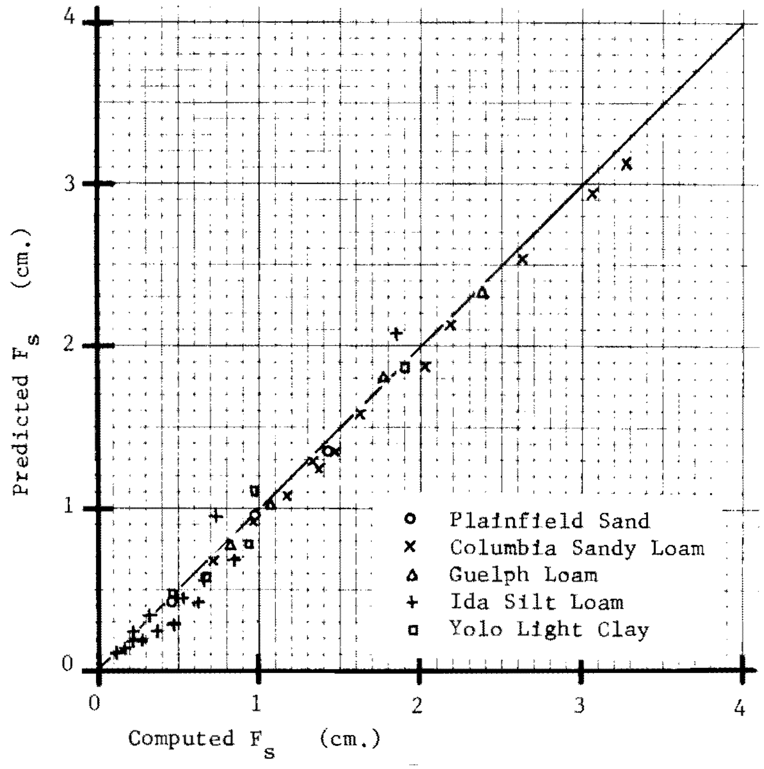

Fig. 22. Predicted volume to saturation (Eq. 23) vs. computed volume to saturation (Eq. 27) for all soils.

<u>Prediction of Infiltration Capacity After Surface Saturation</u>

There are two possible methods for testing this portion of the proposed model (Eq. 24). The model predictions could be calculated using the point of saturation from Eq. 26 as the starting point i.e., independently of the first stage of the model given above, or could be computed from the predicted point of saturation. Because Eq. 23 is a necessary part of the model, and also because it would be a more severe test, it was decided to use the latter approach.

For each value of I/K_s and IMD given in Tables 4 to 8 an infiltration capacity curve was obtained. The data points for these curves are given in Appendix B. A point by point comparison of the data points against values predicted by Eq. 24 has been made, but would be too voluminous to present here. Instead, two or three points were picked randomly from each curve and the predicted and computed values plotted in Figs. 24 and 25. The scatter in the f_p/K_s plot in Fig. 24 is small, but is even less on the plot of infiltration volumes in Fig. 25.

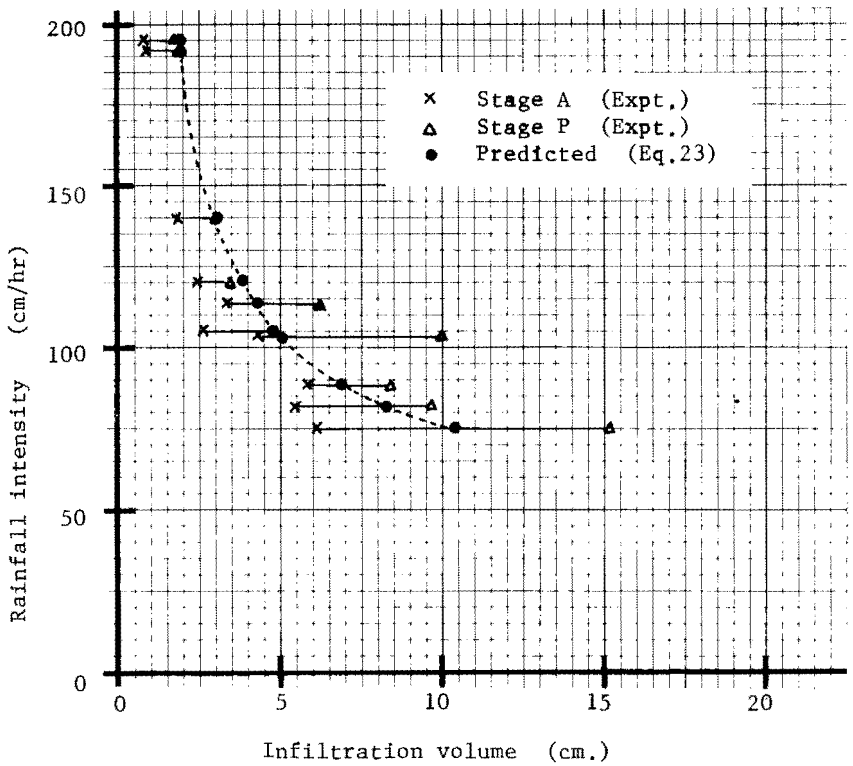

Fig. 23. Comparison of predicted (Eq. 23) and observed values of the infiltration volume at surface saturation (F_S) for columns Rehovot Sand. (Experimental data from Rubin and Steinhardt, Ref. 43, Fig. 2)

Stage A - the moment at which visible retardation of the rainfall is first observed.

Stage P - the point at which 1/3 of the soil surface is covered by water.

A general assessment of the comparison of observed and predicted values for each soil is presented in Table 9. Errors up to about 10% were considered acceptable.

It should be noted that the error increased slowly with time. In practice, the rainfall events would not persist for the long times simulated, so that the increasing error is not considered to be a major objection to the model. A further point to note here is that because the prediction from the first stage of the model is used as the starting point, the error here is actually the combined error from both steps, i.e., the complete model.

Table 9. General Comparison of Infiltration Volumes given by Eq. 24 and the Finite Difference Solution to the Richards Equation (Eq. 28).

Soil	Comments
Plainfield Sand	Agreement excellent-largest error obtained was 3.6%
Columbia Sandy Loam	Agreement excellent-for most runs less than 2% difference; largest 3.4%
Guelph Loam	Agreement excellent - largest error 4.4%
Ida Silt Loam	Agreement fair - error generally less than 10%, but some as high as 20%
Yolo Light Clay	Agreement - satisfactory - errors up to maximum of 11%

Again, the Ida Silt Loam was the soil for which the proposed model gave the poorest results. Examination of Fig. 21 may show why. In the development of Eq. 24 it was assumed that the shape of the wetting front was essentially unchanged after surface saturation occurred. For Columbia Sandy Loam (Fig. 20) this appears to be a good assumption, but for Ida Silt Loam (Fig. 21) it is not. Consequently the infiltration rate vs infiltration volume data points for different intensities for Ida Silt Loam (Fig. 19) do not fall quite on the same curve as predicted by Eq. 24 (see Fig. 10). Contrast this result with those obtained for Columbia Sandy Loam (Fig. 17) for which the points do fall on the same curve.

The authors are not aware of any field data with which the proposed model can be tested. Although many many infiltration tests have been published, it is uncommon for a range of rainfall intensities to be applied, and rare indeed for the suction - conductivity curve to be available for the soils. In 1911, Green and Ampt (20) urged that soils be classified in terms of the parameters permeability, porosity, and capillary potential. Unfortunately, this excellent advice has not been followed.

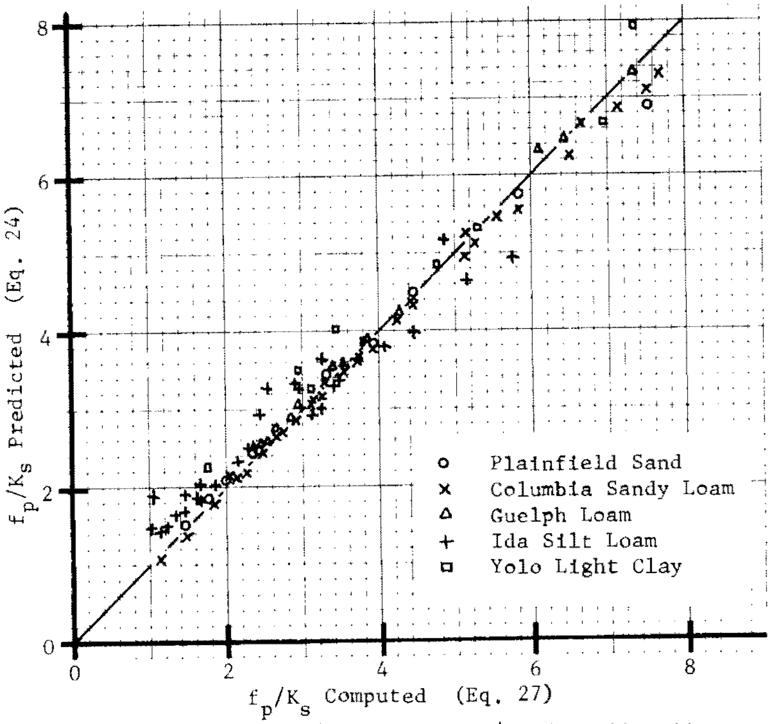

Fig. 24. Predicted vs. Computed values of f_p/K_s for all soils

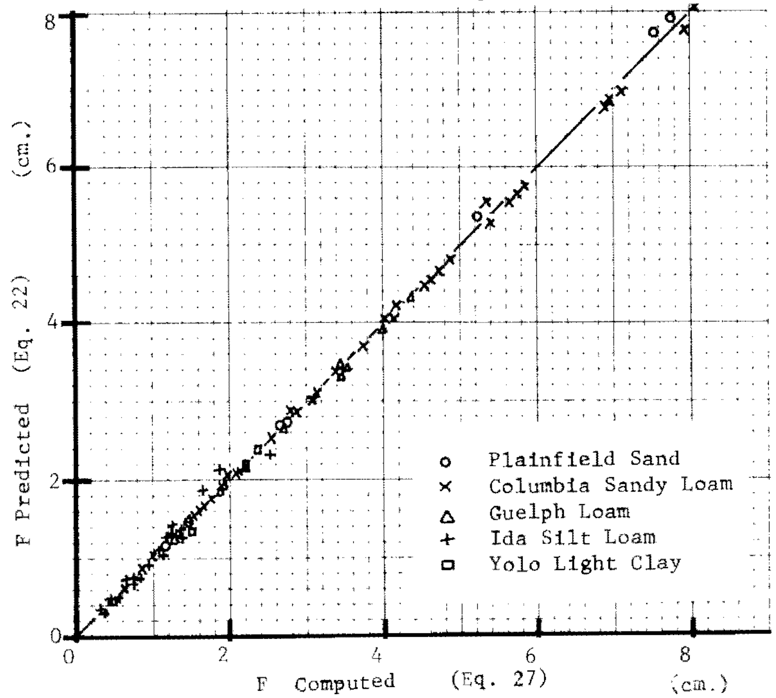

Fig. 25. Predicted vs. Computed values of F for all soils.

VIII. CONCLUSION

Summary

Infiltration is probably the most important hydrologic process affecting watershed runoff from a storm, but to date it has been the weakest component of most current watershed models. The major problem has been the prediction of the start of runoff during a storm, and of the infiltration capacity vs. time relationship after runoff has begun. Both of these stages in the rainfall-infiltration-runoff process depend strongly on soil type, rainfall intensity, and the initial soil moisture content.

Based on this study, a simple infiltration model is proposed for the idealized case of a constant intensity rainfall infiltrating into a homogeneous soil at uniform initial moisture content. The infiltration volume prior to the beginning of runoff is predicted from a modified form of Darcy's Law. The succeeding infiltration capacity "decay" curve is described by the Green and Ampt equation with a correction included to allow for the volume of infiltration prior to surface saturation. All parameters in the model can be obtained from the rainfall intensity and measurable properties of the soil; no fitting is required.

The model equations were tested against numerous solutions of the diffusion equation for soil moisture flow. Comparisons were made for five different soil types (sand, sandy loam, loam, silt loam, light clay) and for a variety of rainfall intensities and initial soil moisture levels. A further independent test of the prediction of the water uptake before runoff was made against published experimental observations. Good to excellent agreement was obtained for all tests except in the case of the silt loam, for which agreement was fair.

The proposed model for $I > K_s$ is as follows:

$f = I$ until $F = F_s$, given by

$$F_s = \frac{S_{av}\ (IMD)}{I/K_s - 1} \tag{44}$$

and, following this, when $F > F_s$, $f = f_p$, given by,

$$f_p = K_s\ (1 + \frac{S_{av}\ (IMD)}{F}) \tag{45}$$

where
- f is the infiltration rate (cm/sec)
- f_p is the infiltration capacity (cm/sec)
- F is the infiltration volume from start of rainfall (cm)
- F_s is the infiltration volume to time of surface saturation (cm)
- IMD is the initial moisture deficit = porosity − initial moisture content (vol/vol)

K_s is the saturated hydraulic conductivity (cm/sec)
I is the rainfall intensity (cm/sec)
S_{av} is the average capillary suction at the wetting front, obtained from the S - K relationship for the soil (cm)

Conclusions

For the idealized conditions assumed for this study (constant rainfall intensity, homogeneous soil, uniform initial moisture content) the following specific conclusions can be drawn.

1. For a rainfall intensity greater than the saturated conductivity but less than the infiltration capacity, the volume of infiltration prior to the commencement of runoff (surface just saturated) can be reasonably predicted by Eq. 44 for soil types ranging from sand to at least a light clay.

2. For a soil with a saturated surface condition (infiltration capacity less than rainfall intensity), the decline of infiltration capacity can be reasonably predicted by Eq. 45 for a wide range of soil types (sand to light clay).

3. The appropriate value of the capillary suction at the wetting front can be determined by simply taking the area under the S - K_r curve for the range $0.01 \leq k_r \leq 1.0$ if the initial capillary conductivity is small compared to the saturated conductivity.

A more general conclusion is that it is far more meaningful to relate infiltration capacity to the prior infiltration volume and the initial moisture content rather than to express it as a function of time.

General applicability of results

It would be a rare circumstance for which one would find a rainfall of constant intensity falling onto a truly homogeneous soil of uniform moisture content as was assumed in this study. But examination of each assumption shows that they are less restrictive than it would first appear.

The derivation of the first stage of the model (Eq. 44) assumed a constant rainfall intensity. After that, so long as the intensity exceeds the infiltration capacity ($I \geq f_p$), it makes no difference what the rainfall intensity is (a note on surface sealing is made later). Now we see that many storms could fit the requirements - all that is needed is a period of constant or perhaps nearly intensity rainfall until the surface is saturated.

The next assumption to look at is that of homogeneity. It was noted in Section VI that the depth reached by the wetting front during the infiltration event was comparitively small - 5 cm for the clay to 35-30 cm. for the sand. That is, it is the upper layer only which determines the infiltration behavior of a soil. It is this layer only which we have assumed to be homogeneous, an assumption which can be made for many soils. For example, the long term effect of tillage tends to mix the topsoil into a more homogeneous layer. (The short term effect of tillage can often be thought of as a creation of depression storage -- possibly one could apply the infiltration model to the soil below the plow depth).

The biggest difficulty with any soil is to determine the appropriate S - K relationship to use. (Note that it is difficult to experimentally determine this relationship even for an ideal soil). There are several reported mathematical models to predict the S - K curve from the S - Θ curve, e.g., Ref. 4, and recent work with this goal shows promise (Farrell, private communication). Because of the scarcity of S - K data equations 44 and 45 may have to be fitted to find S_{av}. Note however that once S_{av} is determined for the soil, the model is applicable for different moisture contents and intensities. There is no need to redetermine parameters as is the case with most empirical infiltration equations.

The assumption of uniform moisture content is also softened somewhat by the discussion of the influencing depth of soil above. It may well be that the moisture content in the upper soil is non-uniform in many cases. Thus, influence of non-uniform initial moisture content should be studied further.

The condition of the soil surface is known to be an important factor; in particular, surface sealing by raindrop compaction can have a big influence. Soil cracking, root holes, etc., can also affect the infiltration capacity. It is not claimed here that the model proposed above will account for this kind of natural variability. It is only hoped that this model, with measurable parameters, is an improvement over many of the existing models.

Suggestions for Further Study

Several possibilities exist for extending the applicability of the model, discussed in the previous section.

1. It should be possible to extend the analysis to layered soils as has been done for the Green-Ampt equation (Refs. 3, 8). Further work is needed on this.

2. The assumption of uniform initial moisture content is limiting for the model, but a good possibility exists that the division of the soil mass into layers of uniform initial moisture content would give good results.

3. Some investigation of non-uniform rainfall intensity should be made for the first stage of the model to see how a reasonable estimate of the moment of surface saturation could be made for this case.

Finally, a series of field tests would be desirable to test the model further.

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APPENDIX A

SOIL PROPERTIES USED TO SIMULATE INFILTRATION

SOIL PLAINFIELD SAND BLACK ET AL (1969)
SAT. CONDUCTIVITY= 3.440E-03 CM/SEC (4.876E+00 IN/HR)
POROSITY= .477 PERCENT(VOL/VOL)
ORIGINAL DATA WERE OBTAINED BY DESORBTION. DATA BELOW
WERE OBTAINED BY DIVIDING BY 1.6 TO ALLOW FOR HYSTERESIS

PSI (CM)	MOIS. CONT. (PERCENT)	PSI (CM)	RELATIVE HYD. CONDY.
-131.3	.1000	-131.3	0.
-109.4	.1080	-125.0	3.372E-05
-86.3	.1170	-93.8	1.009E-04
-71.9	.1250	-62.5	1.346E-04
-62.5	.1310	-52.5	3.343E-04
-54.4	.1400	-43.8	9.419E-04
-48.1	.1470	-35.0	3.372E-03
-42.5	.1580	-25.0	1.581E-02
-37.5	.1690	-21.9	3.372E-02
-34.4	.1770	-20.0	6.047E-02
-31.3	.1890	-18.7	7.733E-02
-28.1	.2050	-17.5	1.076E-01
-25.0	.2250	-16.3	1.683E-01
-21.9	.2550	-13.8	3.372E-01
-20.0	.2850	-11.3	5.058E-01
-18.1	.3200	-9.4	6.715E-01
-15.6	.3700	-7.5	8.401E-01
-12.5	.4200	-6.6	9.099E-01
-11.3	.4400	-6.3	9.419E-01
-10.0	.4500	-4.7	9.680E-01
-8.1	.4580	-3.1	9.855E-01
-6.3	.4650	0.0	1.000E+00
-3.1	.4720	3.1	1.009E+00
0.0	.4770	6.3	1.009E+00
3.1	.4820		
6.3	.4870		

APPENDIX A (CONT.)

SOIL PROPERTIES USED TO SIMULATE INFILTRATION

SOIL COLUMBIA SANDY LOAM. LALIBERTE,AL (1966)
SAT. CONDUCTIVITY= 1.390E-03 CM/SEC (1.970E+00 IN/HR)
POROSITY= .518 PERCENT(VOL/VOL)
ORIGINAL DATA WERE OBTAINED BY DESORBTION. DATA BELOW
WERE OBTAINED BY DIVIDING BY 1.6 TO ALLOW FOR HYSTERESIS

PSI (CM)	MOIS. CONT. (PERCENT)	PSI (CM)	RELATIVE HYD. CONDY.
-281.3	.1210	-218.8	1.640E-07
-218.8	.1240	-156.3	1.560E-06
-156.3	.1320	-104.8	2.280E-05
-130.0	.1390	-95.9	4.140E-05
-104.4	.1450	-89.4	6.640E-05
-95.9	.1520	-82.0	1.180E-04
-89.4	.1600	-75.2	2.120E-04
-82.0	.1650	-64.4	5.970E-04
-75.2	.1700	-53.5	2.080E-03
-69.4	.1780	-43.8	7.940E-03
-64.4	.1860	-40.4	1.360E-02
-59.4	.1970	-36.2	2.870E-02
-53.5	.2120	-33.1	5.250E-02
-48.1	.2300	-31.0	8.090E-02
-43.8	.2490	-28.9	1.280E-01
-40.4	.2630	-28.0	1.600E-01
-38.1	.2770	-26.9	2.110E-01
-36.2	.2880	-24.4	3.990E-01
-34.8	.3010	-22.0	7.000E-01
-33.1	.3150	-21.1	8.500E-01
-31.0	.3380	-20.1	9.000E-01
-28.9	.3670	-18.7	9.370E-01
-27.9	.3830	-16.3	9.650E-01
-26.9	.4010	-13.8	9.800E-01
-25.7	.4230	-6.3	9.920E-01
-24.7	.4450	0.0	1.000E+00
-22.8	.4810	3.1	1.000E+00
-21.9	.4960	6.3	1.000E+00
-21.3	.5040		
-20.7	.5090		
-20.2	.5130		
-19.4	.5145		
-18.7	.5155		
-17.6	.5160		
-15.6	.5162		
-13.1	.5165		
-8.6	.5170		
-4.1	.5175		
0.0	.5180		
3.1	.5185		
6.3	.5190		

APPENDIX A (CONT.)

SOIL PROPERTIES USED TO SIMULATE INFILTRATION

SOIL GUELPH LOAM ELRICK ET AL (1964)
SAT. CONDUCTIVITY= 3.670E-04 CM/SEC (5.202E-01 IN/HR)
POROSITY= .523 PERCENT(VOL/VOL)
ORIGINAL DATA WERE OBTAINED BY DESORBTION. DATA BELOW
WERE OBTAINED BY DIVIDING BY 1.6 TO ALLOW FOR HYSTERESIS

PSI (CM)	MOIS. CONT. (PERCENT)	PSI (CM)	RELATIVE HYD. CONDY.
-312.5	.2760	-353.1	4.550E-04
-250.0	.2900	-207.5	1.954E-03
-218.8	.2970	-145.0	4.550E-03
-187.5	.3050	-100.0	1.452E-02
-156.3	.3140	-78.1	4.360E-02
-125.0	.3300	-62.5	7.357E-02
-109.4	.3420	-53.1	1.362E-01
-93.8	.3560	-46.9	1.962E-01
-78.1	.3770	-39.1	2.997E-01
-62.5	.4060	-31.3	4.360E-01
-46.9	.4410	-23.4	5.995E-01
-31.3	.4760	-15.6	7.902E-01
-20.0	.5000	-7.8	9.537E-01
-12.5	.5140	-5.6	9.809E-01
-7.8	.5200	-3.1	9.946E-01
-4.4	.5220	0.0	1.000E+00
0.0	.5230	3.1	1.000E+00
4.4	.5240	6.3	1.000E+00
6.3	.5250		

APPENDIX A (CONT.)

SOIL PROPERTIES USED TO SIMULATE INFILTRATION

SOIL IDA SILT LOAM GREEN (1962)
SAT. CONDUCTIVITY= 2.920E-05 CM/SEC (4.139E-02 IN/HR)
POROSITY= .530 PERCENT(VOL/VOL)
ORIGINAL DATA ARE ABSORBTION DATA

PSI (CM)	MOIS. CONT. (PERCENT)	PSI (CM)	RELATIVE HYD. CONDY.
-950.0	.2300	-870.0	5.719E-06
-820.0	.2400	-775.0	9.521E-06
-720.0	.2500	-550.0	3.801E-05
-630.0	.2600	-445.0	9.521E-05
-550.0	.2700	-320.0	3.801E-04
-490.0	.2800	-255.0	9.521E-04
-440.0	.2900	-170.0	3.801E-03
-390.0	.3000	-115.0	9.521E-03
-350.0	.3100	-85.0	1.199E-02
-310.0	.3200	-65.0	2.226E-02
-275.0	.3300	-47.0	3.801E-02
-240.0	.3400	-38.0	5.137E-02
-215.0	.3500	-30.0	6.849E-02
-190.0	.3600	-21.0	9.521E-02
-160.0	.3700	-13.0	1.370E-01
-135.0	.3800	-8.0	1.884E-01
-115.0	.3900	-5.0	2.483E-01
-95.0	.4000	-4.0	3.801E-01
-73.0	.4100	-2.6	6.164E-01
-57.0	.4200	-1.7	9.521E-01
-44.0	.4300	0.0	1.000E+00
-34.0	.4400	5.0	1.000E+00
-27.0	.4500	10.0	1.000E+00
-19.0	.4600		
-13.5	.4700		
-9.4	.4800		
-6.3	.4900		
-4.0	.5000		
-2.6	.5100		
-1.7	.5200		
0.0	.5300		
5.0	.5300		
10.0	.5300		

APPENDIX A (CONT.)

SOIL PROPERTIES USED TO SIMULATE INFILTRATION

SOIL YOLO LIGHT CLAY MOORE (1939)
SAT. CONDUCTIVITY= 1.230E-05 CM/SEC (1.743E-02 IN/HR)
POROSITY= .499 PERCENT(VOL/VOL)
ORIGINAL DATA ARE ABSORPTION DATA

PSI (CM)	MOIS. CONT. (PERCENT)	PSI (CM)	RELATIVE HYD. CONDY.
-700.0	.2300	-700.0	0.
-593.0	.2390	-592.0	9.756E-04
-550.0	.2420	-440.0	1.789E-03
-500.0	.2470	-366.0	3.333E-03
-450.0	.2530	-278.0	5.203E-03
-400.0	.2600	-244.0	7.398E-03
-350.0	.2670	-172.0	1.081E-02
-300.0	.2760	-140.0	1.748E-02
-250.0	.2860	-108.0	3.236E-02
-200.0	.2990	-86.0	4.715E-02
-165.0	.3090	-76.0	5.691E-02
-140.0	.3180	-67.0	7.073E-02
-115.0	.3300	-58.0	8.130E-02
-100.0	.3380	-50.0	1.016E-01
-90.0	.3460	-44.0	1.203E-01
-75.0	.3600	-37.0	1.553E-01
-65.0	.3730	-32.0	2.081E-01
-55.0	.3870	-27.0	2.683E-01
-48.0	.4000	-22.0	3.577E-01
-40.0	.4150	-16.0	5.000E-01
-32.0	.4400	-8.0	6.992E-01
-28.0	.4500	0.0	1.000E+00
-25.0	.4580	5.0	1.138E+00
-20.0	.4700	10.0	1.138E+00
-15.0	.4800		
-12.0	.4850		
-8.0	.4900		
-5.0	.4950		
0.0	.4990		
5.0	.5030		
10.0	.5070		

APPENDIX B GENERATED INFILTRATION DATA (EQN 27)

IN THE TABLES LISTED BELOW, THE TIME FROM THE START OF RAINFALL IN SECS (T) AND THE CORRESPONDING INFILTRATION VOLUME IN CM. OF WATER (F) ARE GIVEN. THE VALUES REPRESENT SUCCESSIVE TIME STEPS FROM THE BEGINNING OF SURFACE SATURATION (EXCEPTION: SOME TABLES FOR IDA SILT LOAM MARKED *** REPRESENT EVERY OTHER STEP).

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PLAINFIELD SAND, SOIL DATA FROM BLACK ET AL. (1969)
POROSITY= .477 (VOL/VOL), SAT. CONDY. = 3.44E-3 (CM/SEC)

****

RAINFALL RATE= 1.380E-02 CM/SEC INIT M/C= .130 VOL/VOL

T	F		T	F		T	F
103.8	1.431	:	104.0	1.434	:	104.3	1.438
104.8	1.444	:	105.4	1.453	:	106.4	1.466
107.8	1.485	:	109.8	1.510	:	112.2	1.540
115.6	1.583	:	120.5	1.644	:	126.5	1.714
133.6	1.796	:	141.5	1.882	:	148.7	1.958
155.8	2.033	:	163.1	2.106	:	170.3	2.176
177.4	2.244	:	184.3	2.309	:	192.3	2.382
201.9	2.468	:	212.1	2.557	:	220.5	2.629
229.1	2.702	:	239.4	2.787	:	251.4	2.885
263.6	2.981	:	277.3	3.089	:	293.2	3.210
311.3	3.344	:	327.1	3.460	:	345.2	3.591
360.6	3.701	:	382.2	3.850	:	407.6	4.023
426.9	4.151	:	449.5	4.300	:	474.2	4.459
503.2	4.640	:	532.1	4.818	:	565.1	5.018
597.6	5.212	:	632.1	5.415	:	679.2	5.686
716.4	5.897	:	762.7	6.160	:	812.7	6.435
850.8	6.641	:	900.8	6.907	:	945.2	7.141
995.2	7.399	:	1045.2	7.654	:	1095.2	7.905
1145.2	8.154	:	1195.2	8.404	:		

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RAINFALL RATE= 1.380E-02 CM/SEC INIT M/C= .230 VOL/VOL

T	F		T	F		T	F
71.8	.990	:	76.2	1.049	:	82.6	1.128
91.8	1.234	:	104.6	1.371	:	118.3	1.507
132.1	1.635	:	150.3	1.794	:	168.3	1.944
188.5	2.105	:	215.2	2.307	:	242.5	2.501
272.9	2.710	:	308.6	2.944	:	348.2	3.194
390.6	3.451	:	439.2	3.734	:	489.2	4.015
539.2	4.293	:	589.2	4.564	:	639.2	4.827
689.2	5.085	:	739.2	5.337	:	789.2	5.584
839.2	5.828	:					

....

RAINFALL RATE= 2.750E-02 CM/SEC INIT M/C= .130 VOL/VOL

T	F	:	T	F	:	T	F
24.6	.677	:	24.9	.684	:	25.3	.694
25.6	.708	:	26.5	.728	:	27.8	.756
29.4	.794	:	31.8	.847	:	34.6	.905
37.2	.957	:	40.4	1.016	:	43.8	1.076
47.1	1.131	:	51.6	1.203	:	55.1	1.258
59.5	1.322	:	63.7	1.381	:	68.1	1.441
72.1	1.493	:	76.8	1.554	:	81.9	1.617
87.0	1.677	:	91.4	1.729	:	96.7	1.789
102.3	1.851	:	107.8	1.910	:	112.5	1.959
118.1	2.018	:	124.4	2.082	:	130.2	2.139
136.5	2.200	:	143.9	2.270	:	151.1	2.338
157.1	2.392	:	162.6	2.441	:	169.3	2.500
177.2	2.569	:	185.4	2.639	:	193.1	2.703
204.2	2.795	:	212.9	2.865	:	222.8	2.945
234.8	3.038	:	246.9	3.132	:	258.1	3.216
270.4	3.309	:	286.3	3.426	:	302.5	3.543
321.2	3.675	:	335.8	3.778	:	353.4	3.899
369.9	4.011	:	391.5	4.156	:	416.2	4.317
436.0	4.445	:	458.0	4.581	:	484.5	4.747
513.8	4.927	:	543.4	5.104	:	585.2	5.351
624.4	5.578	:	658.5	5.773	:	690.5	5.954
736.6	6.209	:	780.5	6.449	:	826.5	6.698
870.0	6.929	:	920.0	7.191	:	970.0	7.450
1020.0	7.704	:	1070.0	7.955	:	1120.0	8.204

RAINFALL RATE= 2.750E-02 CM/SEC INIT M/C= .230 VOL/VOL

T	F	:	T	F	:	T	F
16.8	.463	:	19.0	.519	:	21.8	.580
25.3	.650	:	29.4	.725	:	33.2	.787
37.2	.850	:	42.0	.921	:	48.4	1.007
55.4	1.095	:	63.7	1.194	:	71.8	1.282
81.3	1.381	:	91.7	1.484	:	102.5	1.596
114.1	1.691	:	127.1	1.804	:	142.4	1.930
158.4	2.056	:	177.7	2.206	:	204.2	2.399
235.0	2.614	:	271.2	2.855	:	310.9	3.111
351.5	3.366	:	396.3	3.629	:	446.3	3.919
496.3	4.202	:	546.3	4.473	:	596.3	4.737
646.3	4.995	:	696.3	5.249	:	746.3	5.494
796.3	5.740	:	846.3	5.980	:		

COLUMBIA SANDY LOAM, SOIL DATA FROM LALIBERTE ET AL (1966)
POROSITY= .518 (VOL/VOL), SAT. CONDY. = 1.39E-3 (CM/SEC)

RAINFALL RATE= 2.780E-03 CM/SEC INIT M/C= .250 VOL/VOL

T	F	:	T	F	:	T	F
2436.9	6.749	:	2636.9	7.293	:	2833.5	7.812
3033.5	8.315	:	3233.5	8.809	:		

RAINFALL RATE= 5.560E-03 CM/SEC INIT M/C= .125 VOL/VOL

T	F	:	T	F	:	T	F
590.1	3.282	:	600.4	3.338	:	607.5	3.377
616.0	3.423	:	626.1	3.477	:	638.2	3.541
646.3	3.584	:	658.1	3.645	:	670.1	3.707
684.6	3.780	:	701.9	3.867	:	712.5	3.920
727.8	3.994	:	749.8	4.100	:	773.0	4.210
795.3	4.314	:	814.0	4.400	:	836.3	4.501
853.7	4.579	:	870.4	4.653	:	894.5	4.759
928.6	4.905	:	944.2	4.972	:	960.5	5.040
983.8	5.137	:	1017.5	5.274	:	1038.7	5.359
1057.2	5.433	:	1083.8	5.538	:	1115.7	5.662
1161.6	5.837	:	1189.2	5.941	:	1212.5	6.028
1246.1	6.152	:	1294.3	6.327	:	1347.4	6.518
1379.4	6.632	:	1425.4	6.793	:	1488.1	7.008
1547.3	7.209	:	1583.7	7.331	:		

RAINFALL RATE= 5.560E-03 CM/SEC INIT M/C= .150 VOL/VOL

T	F	:	T	F	:	T	F
550.0	3.059	:	553.0	3.075	:	557.2	3.099
563.3	3.132	:	572.1	3.179	:	584.8	3.248
603.0	3.343	:	624.3	3.456	:	643.8	3.552
666.5	3.664	:	687.5	3.766	:	718.0	3.910
754.4	4.080	:	780.7	4.199	:	812.2	4.339
841.9	4.468	:	884.5	4.650	:	917.8	4.788
956.5	4.945	:	1003.0	5.129	:	1037.7	5.264
1087.8	5.455	:	1140.2	5.651	:	1199.2	5.868
1282.9	6.166	:	1339.5	6.364	:	1392.4	6.545
1468.3	6.800	:	1532.9	7.013	:	1603.5	7.241
1705.0	7.563	:	1782.4	7.802	:	1869.8	8.067
1988.1	8.417	:	2087.5	8.705	:	2193.1	9.010

RAINFALL RATE= 5.560E-03 CM/SEC INIT M/C= .200 VOL/VOL

T	F	T	F	T	F
472.2	2.624	489.7	2.719	507.5	2.814
531.5	2.939	559.8	3.083	582.3	3.193
605.4	3.321	644.1	3.482	682.9	3.659
714.7	3.798	751.1	3.954	783.9	4.090
831.2	4.280	883.3	4.483	951.7	4.742
1008.5	4.950	1081.5	5.213	1142.5	5.427
1230.5	5.724	1357.2	6.139	1450.6	6.432
1647.3	6.730	1686.6	7.145	1793.2	7.454

RAINFALL RATE= 5.560E-03 CM/SEC INIT M/C= .250 VOL/VOL

T	F	T	F	T	F
396.9	2.190	420.3	2.328	449.3	2.478
476.9	2.615	512.6	2.785	558.3	2.996
602.4	3.189	652.6	3.400	713.3	3.642
774.6	3.875	836.9	4.102	926.4	4.420
1015.5	4.722	1110.9	5.037	1236.5	5.432
1353.7	5.783	1507.0	6.235	1707.0	6.793

RAINFALL RATE= 5.560E-03 CM/SEC INIT M/C= .318 VOL/VOL

T	F	T	F	T	F
291.2	1.607	320.4	1.760	358.9	1.948
399.3	2.131	457.4	2.380	509.2	2.596
559.0	2.776	621.9	3.005	712.6	3.317
867.6	3.626	916.0	3.963	1058.2	4.384
1199.6	4.777	1399.6	5.291		

RAINFALL RATE= 8.340E-03 CM/SEC INIT M/C= .125 VOL/VOL

T	F	T	F	T	F
241.5	2.013	243.8	2.032	247.0	2.059
250.9	2.090	255.5	2.127	259.0	2.154
263.1	2.187	268.1	2.225	272.3	2.256
277.2	2.293	283.2	2.337	288.7	2.377
293.9	2.414	298.1	2.443	303.2	2.478
309.2	2.520	314.5	2.555	320.7	2.597
325.2	2.627	330.5	2.662	336.8	2.703
342.6	2.740	349.2	2.781	353.1	2.805
358.6	2.840	363.3	2.881	369.6	2.907
374.8	2.938	381.0	2.975	388.5	3.020
395.0	3.058	400.9	3.092	407.9	3.132
416.2	3.180	423.7	3.222	430.6	3.261
439.0	3.307	449.1	3.363	455.3	3.397
462.7	3.437	471.7	3.485	482.5	3.542
489.8	3.580	498.7	3.626	508.2	3.676
521.9	3.745	537.0	3.821	546.7	3.869
558.4	3.927	570.6	3.987	588.2	4.072

T	F	T	F	T	F
612.8	4.189	623.8	4.240	639.6	4.314
658.6	4.402	682.2	4.509	698.9	4.593
712.7	4.644	732.5	4.731	756.3	4.834
771.6	4.899	789.7	4.978	806.3	5.046
830.0	5.144	861.4	5.272	884.8	5.366
903.0	5.439	929.2	5.542	967.0	5.689
1014.0	5.867				

RAINFALL RATE= 8.340E-03 CM/SEC INIT M/C= .250 VOL/VOL

T	F	T	F	T	F
160.9	1.341	168.7	1.404	180.0	1.491
196.3	1.611	216.9	1.752	233.3	1.857
249.5	1.957	265.6	2.052	288.6	2.182
312.7	2.311	338.5	2.445	367.8	2.592
400.3	2.747	437.5	2.920	489.5	3.149
541.5	3.369	594.7	3.544	646.9	3.787
709.2	4.021	786.6	4.208	891.9	4.523
974.4	4.938	1073.8	5.254	1187.2	5.604
1350.6	6.088	1484.5	6.465	1650.2	6.916

RAINFALL RATE= 1.110E-02 CM/SEC INIT M/C= .125 VOL/VOL

T	F	T	F	T	F
132.9	1.472	135.6	1.502	138.8	1.537
142.8	1.578	145.3	1.605	149.0	1.642
152.5	1.676	156.5	1.716	160.4	1.752
163.0	1.776	166.1	1.805	169.9	1.839
172.0	1.858	174.5	1.881	178.3	1.913
182.7	1.952	185.5	1.975	188.8	2.003
193.5	2.042	197.0	2.070	200.7	2.100
205.1	2.135	208.7	2.162	213.7	2.201
219.8	2.247	224.2	2.281	228.7	2.314
232.2	2.339	236.3	2.369	241.3	2.405
244.7	2.428	248.7	2.455	253.4	2.489
257.0	2.514	261.2	2.542	266.3	2.577
269.9	2.600	274.1	2.629	279.1	2.661
285.1	2.700	291.7	2.743	298.0	2.782
301.9	2.806	307.4	2.841	314.1	2.882
318.4	2.908	323.5	2.939	329.8	2.976
337.3	3.021	342.8	3.053	348.8	3.088
355.9	3.129	364.5	3.177	371.6	3.218
379.2	3.260	390.1	3.321	402.2	3.387
414.9	3.456	425.4	3.511	437.9	3.577
446.1	3.619	457.8	3.680	474.4	3.764
494.3	3.863	505.3	3.917	519.8	3.984
538.4	4.078	560.0	4.180	572.7	4.240
591.0	4.325	612.9	4.426	636.4	4.533
650.6	4.596	671.0	4.686	700.3	4.814
728.1	4.933	751.2	5.030	777.2	5.137
805.9	5.254	824.9	5.331	847.8	5.423
872.6	5.520	908.4	5.659	959.9	5.856
989.4	5.967	1017.5	6.071		

RAINFALL RATE= 1.110E-02 CM/SEC   INIT M/C= .150 VOL/VOL

T	F		T	F		T	F
123.9	1.374	:	126.2	1.399	:	129.5	1.435
134.3	1.484	:	139.6	1.539	:	143.7	1.579
147.5	1.615	:	153.0	1.666	:	157.7	1.709
162.7	1.754	:	167.3	1.793	:	173.8	1.849
178.9	1.891	:	184.9	1.940	:	190.6	1.985
197.5	2.039	:	203.9	2.088	:	211.6	2.145
217.5	2.189	:	223.1	2.229	:	229.8	2.276
235.9	2.318	:	243.2	2.368	:	251.5	2.423
262.0	2.493	:	269.6	2.541	:	276.0	2.582
283.7	2.630	:	292.2	2.683	:	302.3	2.744
310.9	2.795	:	323.3	2.867	:	332.5	2.920
345.3	2.992	:	360.7	3.078	:	371.5	3.137
384.4	3.207	:	398.7	3.284	:	419.8	3.392
433.8	3.464	:	452.0	3.556	:	473.8	3.664
489.6	3.741	:	512.3	3.849	:	538.1	3.970
556.1	4.054	:	581.2	4.171	:	610.9	4.301
631.3	4.390	:	660.0	4.514	:	691.1	4.646
713.5	4.739	:	745.6	4.871	:	784.0	5.025
811.5	5.133	:	851.0	5.287	:	908.0	5.503
953.4	5.672	:	1001.7	5.849	:	1066.3	6.091
1111.8	6.241	:	1172.2	6.451	:	1243.9	6.694
1329.8	6.980	:	1393.2	7.186	:	1484.4	7.475
1558.5	7.719	:	1632.3	7.936	:	1738.5	8.254
1822.1	8.498	:	1930.5	8.819	:	2057.9	9.172

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RAINFALL RATE= 1.110E-02 CM/SEC   INIT M/C= .200 VOL/VOL

T	F		T	F		T	F
105.9	1.176	:	109.8	1.218	:	115.5	1.277
121.7	1.339	:	127.4	1.394	:	134.3	1.457
141.1	1.517	:	146.7	1.566	:	154.9	1.633
162.2	1.700	:	171.5	1.764	:	179.3	1.823
187.9	1.886	:	197.7	1.955	:	206.7	2.017
215.8	2.079	:	224.9	2.138	:	235.8	2.208
245.5	2.268	:	256.5	2.335	:	267.1	2.399
280.5	2.477	:	296.2	2.566	:	317.7	2.685
332.0	2.762	:	349.1	2.853	:	366.7	2.944
389.4	3.058	:	407.8	3.149	:	434.3	3.277
465.5	3.422	:	494.7	3.556	:	522.2	3.678
561.8	3.850	:	602.3	4.020	:	641.2	4.180
679.7	4.334	:	730.7	4.523	:	785.8	4.740
838.7	4.937	:	913.2	5.204	:	971.5	5.409
1055.4	5.693	:	1176.4	6.069	:	1275.5	6.405
1342.3	6.612	:	1438.4	6.902	:	1576.9	7.310
1709.6	7.686	:	1993.1	8.196	:		

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RAINFALL RATE= 1.110E-02 CM/SEC   INIT M/C= .250 VOL/VOL

T	F		T	F		T	F
87.9	.976	:	89.6	.994	:	92.0	1.020
95.5	1.057	:	100.5	1.107	:	107.8	1.177
118.2	1.271	:	133.2	1.395	:	145.1	1.487
156.3	1.570	:	168.7	1.656	:	183.6	1.756
200.5	1.865	:	219.3	1.980	:	238.8	2.093
256.3	2.191	:	280.8	2.324	:	308.0	2.466
337.6	2.614	:	369.6	2.766	:	415.8	2.977
459.5	3.169	:	508.5	3.375	:	563.0	3.594
620.3	3.813	:	686.2	4.059	:	781.1	4.397
865.8	4.688	:	956.8	4.988	:	1051.6	5.290
1157.3	5.617	:	1309.4	6.067	:	1442.1	6.440
1619.5	6.930	:					

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RAINFALL RATE= 1.110E-02 CM/SEC   INIT M/C= .318 VOL/VOL

T	F		T	F		T	F
64.1	.712	:	73.0	.803	:	81.6	.883
93.9	.987	:	105.5	1.078	:	118.6	1.172
137.4	1.298	:	155.2	1.408	:	172.8	1.516
195.8	1.637	:	223.9	1.782	:	256.0	1.922
290.5	2.097	:	340.1	2.313	:	387.7	2.506
451.1	2.756	:	527.1	3.031	:	612.1	3.325
695.0	3.594	:	785.4	3.877	:	881.0	4.164
1018.6	4.560	:	1216.8	5.095	:		

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GUELPH LOAM.   SOIL DATA FROM ELRICK AND BOWMAN (1964)
POROSITY= .523 (VOL/VOL).   SAT. CONDY. = 3.67E-4 (CM/SEC)

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RAINFALL RATE= 1.470E-03 CM/SEC   INIT M/C= .300 VOL/VOL

T	F		T	F		T	F
1610.6	2.370	:	1688.1	2.480	:	1799.6	2.629
1878.6	2.731	:	1975.2	2.851	:	2069.7	2.966
2156.0	3.067	:	2280.3	3.209	:	2406.8	3.342
2533.1	3.486	:	2594.8	3.645	:	2796.4	3.759
2957.2	3.919	:	3115.0	4.073	:	3287.0	4.235
3457.0	4.419	:	3578.8	4.591	:	3878.8	4.766

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APPENDIX B (CONT.) GUELPH LOAM

RAINFALL RATE= 1.470E-03 CM/SEC INIT M/C= .350 VOL/VOL

T	F	T	F	T	F
1218.2	1.780	1242.5	1.815	1277.4	1.864
1327.7	1.932	1400.2	2.025	1504.5	2.153
1654.7	2.329	1854.7	2.549	2054.7	2.757
2254.7	2.954	2454.7	3.143	2654.7	3.324
2854.7	3.498	3054.7	3.667		

RAINFALL RATE= 2.940E-03 CM/SEC INIT M/C= .300 VOL/VOL

T	F	T	F	T	F
363.6	1.070	375.5	1.104	389.9	1.143
410.6	1.197	434.8	1.256	459.2	1.313
476.2	1.352	492.6	1.388	512.3	1.430
535.9	1.480	558.9	1.526	586.2	1.580
617.7	1.640	652.4	1.705	683.4	1.760
721.2	1.826	753.1	1.897	801.0	1.960
849.2	2.037	909.5	2.130	953.8	2.197
1017.6	2.290	1070.2	2.365	1145.7	2.470
1205.5	2.551	1291.6	2.663	1369.3	2.762
1465.5	2.881	1569.4	3.005	1671.4	3.124
1812.9	3.284	1931.7	3.414	2076.7	3.568
2209.2	3.705	2356.6	3.864	2529.0	4.023
2659.2	4.148	2846.7	4.323	3021.4	4.482
3169.9	4.614	3369.9	4.787		

RAINFALL RATE= 2.940E-03 CM/SEC INIT M/C= .350 VOL/VOL

T	F	T	F	T	F
273.8	.801	302.4	.879	337.5	.964
373.8	1.045	412.7	1.125	453.7	1.205
507.4	1.303	561.9	1.397	624.6	1.498
701.8	1.617	780.3	1.730	859.0	1.840
956.7	1.967	1082.0	2.123	1226.8	2.294
1375.7	2.460	1515.5	2.609	1715.6	2.813
1915.6	3.007	2112.9	3.191	2312.9	3.371
2512.9	3.542	2712.9	3.708		

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IDA SILT LOAM. SOIL DATA FROM GREEN(1962)
POROSITY= .530 (VOL/VOL). SAT. CONDY. = 2.92E-5 (CM/SEC)

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RAINFALL RATE= 5.840E-05 CM/SEC INIT M/C= .250 VOL/VOL

T	F	T	F	T	F
31561.3	1.851	32564.5	1.892	33060.6	1.919
33529.6	1.945	34204.9	1.981	34968.7	2.022
35545.3	2.049	36192.5	2.085	37017.4	2.127
37716.2	2.162	38504.5	2.201	39457.6	2.248
40830.2	2.315				

RAINFALL RATE= 5.840E-05 CM/SEC INIT M/C= .400 VOL/VOL

T	F	T	F	T	F
12591.5	.727	14210.6	.815	15983.9	.902
17714.7	.983	19714.7	1.072	21714.7	1.157
23714.7	1.240	25714.7	1.321	27714.7	1.399
29714.7	1.476	31714.7	1.551	33714.7	1.626
35714.7	1.699	37714.7	1.772	39714.7	1.843
41714.7	1.914				

RAINFALL RATE= 1.170E-04 CM/SEC INIT M/C= .250 VOL/VOL

T	F	T	F	T	F
7277.2	.850	7442.4	.869	7626.3	.889
7891.2	.916	8058.0	.933	8238.4	.951
8498.1	.976	8655.3	.991	8798.1	1.004
8989.5	1.021	9210.7	1.041	9476.2	1.065
9794.8	1.092	9998.4	1.108	10147.3	1.121
10376.0	1.140	10558.9	1.155	10778.5	1.172
11094.6	1.197	11306.9	1.214	11496.4	1.228
11769.3	1.249	11988.3	1.265	12251.0	1.284
12566.3	1.307	12788.3	1.323	13054.7	1.342
13374.4	1.365	13597.7	1.380	13854.3	1.398
14223.9	1.423	14471.5	1.439	14739.5	1.457
15125.4	1.482	15362.8	1.499	15691.8	1.519
16109.9	1.545	16611.6	1.577	17018.1	1.602
17505.8	1.631	17926.9	1.656	18432.1	1.686
18891.0	1.713	19204.3	1.731	19580.3	1.753
20031.4	1.779	20572.7	1.809	20935.2	1.829
21415.9	1.856	22108.2	1.893	22646.9	1.922
23044.2	1.943	23616.4	1.974	24440.4	2.017
24988.9	2.045	25575.9	2.075	26418.9	2.118
27431.5	2.169	28048.1	2.199	28935.9	2.243
30214.3	2.305	31102.8	2.348	31904.1	2.385
32851.9	2.430	33989.2	2.483	35627.0	2.558
36590.3	2.602	37620.4	2.648	38856.5	2.703

RAINFALL RATE= 1.170E-04 CM/SEC INIT M/C= .300 VOL/VOL

T	F	T	F	T	F
5747.9	.673	5886.7	.688	6086.7	.710
6374.6	.739	6720.0	.772	7073.3	.805
7497.1	.843	8005.7	.886	8523.9	.928
9145.8	.976	9798.1	1.025	10365.9	1.065
10933.1	1.105	11506.1	1.144	12100.0	1.183
12606.3	1.215	13148.7	1.249	13717.9	1.284
14497.0	1.331	15282.8	1.377	15914.4	1.413
16669.4	1.456	17276.3	1.489	18111.5	1.534
18841.3	1.573	19581.4	1.612	20400.5	1.654
21530.7	1.711	22593.2	1.764	23978.2	1.831
25583.8	1.908	26657.8	1.959	27651.1	2.005
28751.9	2.055	29818.4	2.103	31008.5	2.157
32330.9	2.215	33644.2	2.272	34612.6	2.314
35822.6	2.366	37466.6	2.434	38995.7	2.499
40422.1	2.558				

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RAINFALL RATE= 1.170E-04 CM/SEC INIT M/C= .350 VOL/VOL

T	F	T	F	T	F
4393.0	.513	4971.4	.574	6004.8	.662
7265.6	.770	9049.1	.900	10976.0	1.027
12976.0	1.149	14976.0	1.263	16976.0	1.371
18976.0	1.474	20976.0	1.573		

RAINFALL RATE= 1.170E-04 CM/SEC INIT M/C= .400 VOL/VOL

T	F	T	F	T	F
2716.9	.315	2905.2	.335	3176.3	.361
3566.7	.396	4128.8	.442	4849.5	.495
5618.0	.547	6501.3	.602	7543.9	.663
8728.8	.729	9921.3	.791	11307.1	.852
12979.1	.939	14447.8	1.006	16447.8	1.094
18447.8	1.179	18697.8	1.189	19057.8	1.204
19576.2	1.225	20322.6	1.255	21397.6	1.298
22945.5	1.359	23135.5	1.369	23555.5	1.392
24073.9	1.402	24920.4	1.430	25895.4	1.471
27443.3	1.530	27943.3	1.549	28663.3	1.575
29181.7	1.595	29928.2	1.621	30465.7	1.640
31239.7	1.667	31796.3	1.686	32599.4	1.714
33754.9	1.758	35418.3	1.817	35668.9	1.826
36028.9	1.838	36547.3	1.856	37293.7	1.883
38368.7	1.920	39916.6	1.974	40916.6	2.003

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RAINFALL RATE= 1.170E-04 CM/SEC INIT M/C= .430 VOL/VOL

T	F	T	F	T	F
1846.3	.215	2393.5	.266	3303.9	.337
4303.9	.403	5303.9	.464	6303.9	.520
7303.9	.572	8303.9	.622	9303.9	.669
10303.9	.714	11303.9	.759	12303.9	.802
13303.9	.844	14303.9	.886	15303.9	.926
16303.9	.966	17303.9	1.006	18303.9	1.044
19303.9	1.082	20303.9	1.120		

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RAINFALL RATE= 1.750E-04 CM/SEC INIT M/C= .250 VOL/VOL

T	F	T	F	T	F
3470.6	.608	3773.6	.657	3991.8	.689
4249.4	.726	4556.5	.766	4944.6	.815
5256.1	.852	5678.5	.899	6324.8	.969
6839.0	1.021	7483.3	1.083	8124.2	1.142
8929.7	1.213	9751.5	1.282	10825.9	1.369
11847.5	1.448	12917.9	1.528	14120.0	1.615
15716.2	1.725	17530.7	1.849	19195.3	1.950

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RAINFALL RATE= 1.750E-04 CM/SEC INIT M/C= .400 VOL/VOL

T	F	T	F	T	F
1337.8	.231	1774.4	.287	2448.0	.357
3281.3	.430	4281.3	.506	5281.3	.575
6281.3	.639	7281.3	.698	8281.3	.756
9281.3	.810	10281.3	.862	11281.3	.913
12281.3	.961	13281.3	1.008	14281.3	1.054
15281.3	1.100	16281.3	1.143	17281.3	1.187
18281.3	1.231	19281.3	1.271	20281.3	1.313

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RAINFALL RATE= 2.330E-04 CM/SEC INIT M/C= .250 VOL/VOL

T	F	T	F	T	F
2049.9	.478	2252.1	.519	2449.5	.555
2675.9	.594	2929.5	.635	3277.8	.696
3479.1	.716	3828.5	.762	4127.7	.800
4478.3	.842	4989.9	.900	5482.5	.953
5850.5	.991	6425.2	1.049	7116.4	1.114
7938.5	1.188	8754.9	1.257	9635.6	1.330
11016.3	1.439	12037.1	1.515	13206.8	1.599
14937.2	1.720	16338.6	1.812	17856.7	1.909
19393.4	2.005				

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RAINFALL RATE= 2.330E-04 CM/SEC   INIT M/C= .300 VOL/VOL

T	F		T	F		T	F
1574.4	.367	:	1663.7	.387	:	1783.1	.410
1942.1	.438	:	2061.6	.461	:	2194.1	.484
2294.9	.495	:	2440.1	.517	:	2595.2	.540
2753.8	.561	:	2880.2	.573	:	3062.3	.591
3259.5	.626	:	3438.5	.647	:	3627.7	.669
3900.2	.699	:	4176.1	.729	:	4412.1	.753
4727.5	.785	:	5181.8	.829	:	5574.7	.866
5909.6	.896	:	6311.6	.931	:	6739.0	.967
7254.3	1.009	:	7996.9	1.061	:	8509.7	1.107
9100.2	1.152	:	9839.8	1.206	:	10595.2	1.259
11585.2	1.327	:	12585.2	1.393	:	13354.2	1.442
14183.8	1.494	:	15183.8	1.556	:	16183.8	1.615
17035.2	1.665	:	18035.2	1.722	:	19035.2	1.778
20035.2	1.833	:					

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RAINFALL RATE= 2.330E-04 CM/SEC   INIT M/C= .350 VOL/VOL

T	F		T	F		T	F
1074.9	.251	:	1238.7	.287	:	1429.6	.323
1690.6	.361	:	1971.1	.401	:	2349.5	.448
2778.3	.497	:	3374.1	.559	:	3941.8	.612
4693.5	.678	:	5405.5	.736	:	6282.4	.803
7277.1	.875	:	8277.1	.943	:	9277.1	1.006
10277.1	1.071	:	11277.1	1.130	:	12277.1	1.189
13277.1	1.245	:	14277.1	1.300	:	15277.1	1.353
16277.1	1.405	:	17277.1	1.455	:	18277.1	1.504
19277.1	1.553	:	20277.1	1.602	:		

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RAINFALL RATE= 2.330E-04 CM/SEC   INIT M/C= .400 VOL/VOL

T	F		T	F		T	F
704.6	.163	:	762.9	.175	:	822.4	.186
908.1	.201	:	1031.6	.220	:	1151.2	.237
1322.6	.259	:	1495.4	.280	:	1686.2	.301
1960.9	.330	:	2236.5	.357	:	2553.4	.386
2887.1	.414	:	3367.5	.453	:	3832.5	.498
4496.4	.536	:	5279.5	.589	:	6051.0	.638
6993.0	.695	:	7925.9	.748	:	8925.9	.803
9925.9	.856	:	10925.9	.907	:	11925.9	.956
12925.9	1.005	:	13925.9	1.052	:	14925.9	1.099
15925.9	1.143	:	16925.9	1.187	:	17925.9	1.231
18925.9	1.273	:	19925.9	1.315	:	20925.9	1.356

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RAINFALL RATE= 2.330E-04 CM/SEC   INIT M/C= .430 VOL/VOL

T	F		T	F		T	F
449.8	.105	:	575.8	.128	:	794.9	.166
1115.2	.198	:	1599.9	.245	:	2258.5	.300
3113.8	.362	:	4113.8	.425	:	5113.8	.484
6113.8	.539	:	7113.8	.590	:	8113.8	.639
9113.8	.686	:	10113.8	.730	:	11113.8	.776
12113.8	.818	:	13113.8	.859	:	14113.8	.901
15113.8	.941	:	16113.8	.982	:	17113.8	1.021
18113.8	1.059	:	19113.8	1.097	:	20113.8	1.134

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YOLO LIGHT CLAY. SOIL DATA FROM MOORE (1939)
POROSITY= .499 (VOL/VOL). SAT. CONDY. = 1.23E-5 (CM/SEC)

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RAINFALL RATE= 4.920E-05 CM/SEC   INIT M/C= .250 VOL/VOL

T	F		T	F		T	F
38617.8	1.897	:	38761.9	1.904	:	38969.2	1.914
39267.9	1.928	:	39697.9	1.948	:	40317.2	1.977
41208.9	2.017	:	42493.0	2.074	:	44342.0	2.152
46342.0	2.234	:	48342.0	2.314	:	50342.0	2.391

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RAINFALL RATE= 4.920E-05 CM/SEC   INIT M/C= .350 VOL/VOL

T	F		T	F		T	F
19983.6	.966	:	20680.1	.998	:	21683.0	1.042
23127.2	1.103	:	24627.2	1.162	:	26127.2	1.219
27627.2	1.274	:	29127.2	1.327	:	30627.2	1.378

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APPENDIX B (CONT.)   YOLO LIGHT CLAY

RAINFALL RATE= 9.840E-05 CM/SEC   INIT M/C= .250 VOL/VOL

T	F		T	F		T	F
9341.5	.918	:	9896.4	.961	:	10475.9	1.019
11306.6	1.086	:	12118.0	1.167	:	12821.2	1.197
13761.4	1.251	:	14847.0	1.332	:	15936.1	1.399
16910.4	1.456	:	18099.2	1.523	:	19508.6	1.600
21140.0	1.685	:	22954.7	1.775	:	24693.8	1.857
26693.8	1.948	:	28556.5	2.030	:	30129.7	2.097
32017.0	2.175	:	34017.0	2.255	:	36017.0	2.333
38017.0	2.409	:	40017.0	2.484	:	42017.0	2.557
44017.0	2.628	:	46017.0	2.698	:	48017.0	2.766
50017.0	2.833	:					

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RAINFALL RATE= 9.840E-05 CM/SEC   INIT M/C= .350 VOL/VOL

T	F		T	F		T	F
4704.0	.459	:	5003.8	.486	:	5435.4	.522
6056.9	.570	:	6950.0	.632	:	7902.5	.692
9070.5	.761	:	10549.4	.840	:	12049.4	.913
13549.4	.982	:	15049.4	1.045	:	16549.4	1.108
18049.4	1.167	:	19549.4	1.223	:	21049.4	1.277
22549.4	1.330	:	24049.4	1.381	:	25549.4	1.430
27049.4	1.478	:	28549.4	1.526	:	30049.4	1.572

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