INTERNATIONAL EVIDENCE ON MONETARY FACTORS IN MACROECONOMIC FLUCTUATIONS

by

Christopher A. Sims

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Center for Economic Research
Department of Economics
University of Minnesota
Minneapolis, Minnesota 55455

Sudden upward jumps in the interest rate tend to precede sharp slides in output by 6 months to a little over a year. They also tend to precede slightly drops in the rate of growth in the money stock. In an earlier paper (1980) I pointed out that these empirical generalizations apply to both interwar and postwar data in the U.S. I argued in that paper that it was difficult to account for this persistent pattern under a monetarist interpretation of the business cycle.

This paper shows that the pattern persists in data for West Germany, France, and Britain. It argues again that a monetarist interpretation of the results is strained -- moreso in the light of the paper's international evidence -- and suggests that the results are most naturally explained by passive response of the monetary system to non-monetary cyclical influences. The quick response of money stock to interest rates, together with the anticipation of cyclical developments by the financial markets which set interest rates, can account for the apparently strong causal role of the money stock in some countries and periods when interest rates are omitted from the system.

The Theory of Interest Rates and the Cycle

Simple Keynesian theory links the interest rate most strongly to investment and the demand for money. Consumption and saving are treated as tracking the time path of income without help from interest rate incentives. Current dynamic equilibrium macroeconomic theories, perhaps partly because of the analytical difficulties in doing otherwise, frequently treat the interest rate as either fixed over time or identically equal to the marginal product of a homogeneous, perfectly elastically supplied, capital stock. In none of these theories does the interest rate receive emphasis as a causal link in the dynamics of consumption decisions. However, it is clear that the interest rate could in principle play an important role in controlling the allocation of consumption over time.

In an economy with well-developed financial markets, consumers (or at least a substantial fraction of them) will see the term structure of interest rates as describing opportunities for transferring purchasing power over time. Given interest rates, the time path of income itself is irrelevant to the time path of consumption except insofar as the discounted present value of income fixes the level, but not the shape, of the consumption path. If investment, the gap between current income and consumption, can be costlessly varied, then a constant interest rate may be efficient. For in that case the lack of relation between consumption and income, creating an arbitrary path for investment, exacts no costs. More generally, though, it must be expected that fluctuations in investment are not costless; in fact, in the short run (over a few months) unanticipated shifts in the level of investment may be substantially more expensive than unanticipated shifts in consumption. To reconcile consumers to the need to adapt the time path of their consumption to that of output, a systematic connection of interest rates to output is required.

There seems to be no way to avoid the need, in modeling consumer behavior with a time-varying interest rate, to employ models which involve dynamic optimization of non-quadratic structures. This is because the central constraint the consumer faces -- that discounted consumption must equal discounted income -- involves the product of the decision variable (consumption) with one of the main variables exogenous to the consumer (the discount factor). As a result, mathematical models in this area become messy, producing a bewildering variety of results. Though I will present a model or two below, I first give a verbal description of the theory which may be clearer and more reliable.

A high interest rate which is expected to persist for some time makes consumption later cheap relative to consumption now. It can, therefore, be expected to lower current consumption. A drop in permanent income will also lower current consumption. However an anticipated drop in income which is expected to last only a year or two has a small effect on permanent income. It will not lower consumption by more than a small fraction of the anticipated drop in current income. If consumers are to have an incentive to lower their consumption by a substantial fraction of the drop in current income, an interest rate rise, providing an incentive to postpone consumption, will be required.

Any sort of cyclical bad news about the economy, therefore, is likely to produce a rise in the interest rate. The source of the bad news could be a conventional "supply shock" -- realization that for some reason excess demand for raw materials has developed -- or a realization that some other category of final demand had expanded more rapidly than expected and would continue to do so for a while. The latter type of news could fit the picture of an investment boom hitting a capacity constraint. Since the consumption theory generating this conclusion depends on freely functioning markets only for intertemporal allocation of consumption, it could apply also when there are price rigidities in labor or product markets. Impending cyclical declines in output which originate in demand shocks and Keynesian multiplier effects also generate the type of cyclical bad news which should generate an interest rate rise.

This latter possibility runs counter to the usual Keynesian story and it may therefore deserve some elaboration. Suppose there is a decline in government expenditure with no corresponding decline in taxation. Suppose the eliminated expenditure was on labor, so that the immediate impact of the expenditure decline is to reduce total employment. With sticky wages, the workers are not immediately re-employed. Total consumer income drops, and the temporarily unemployed increase demand for credit by borrowing against future income. (Remember, the market for loans is perfect -- this part of the story is not very convincing, but it is also not essential to the conclusion.) The drop in permanent income of the unemployed workers leads to a decline in consumption demand. This leads to accumulation of inventories, layoffs of employees, and decline in production. Once production actually begins to decline, it becomes necessary for consumers in the aggregate to consume less, and interest rates must rise. This situation becomes apparent to the banking system as consumers apply for loans and reduce the inflow of deposits, trying to finance consumption during their period of reduced income. At the same time businesses will be trying to borrow to finance the unintended expansion of inventories. If the banking system had held interest-earning government debt, the reduced deficit or increased surplus in the government budget provides a flow of support to the banking system. The full amount of the incomes of the employees laid off in the "impact" round of this scenario could be financed by the reduction in the deficit -- the banks could sell off their government debt and issue consumer loans in the same amount. But once the multiplier effect has caused an actual inefficient use of resources, the decline in the availability of product cannot be financed away. Credit will become scarce and interest rates will rise.

This entire scenario obviously depends on the impossibility of quickly reducing investment expenditures. If capital expenditure were to drop immediately as excess inventories began to accumulate, and if this immediately freed resources for production of consumption goods, production of consumption goods might not decline. But in the spirit of this type of analysis it seems reasonable to suppose that this is not possible within a time frame of a few months.

The discussion to this point has ignored the distinction between real and nominal interest rates. If the price level begins to rise because of supply effects -- e.g. rising prices of imported raw materials -- the nominal interest rate might rise to keep the real rate constant; and when the effects of the restricted supplies of inputs showed up in output levels we would find an output drop following the interest rate rise. One might expect that as the output drop occurred interest rates would have to rise because consumption would drop. But this is not necessarily true if the supply constriction is well-enough anticipated that it can be accomodated primarily by a temporary drop in investment. This provides an essentially different mechanism by which it might occur that nominal interest rate rises precede output drops.

Of course it should be understood that the foregoing discussion, though phrased in terms of interest rate rises and income and consumption drops, is largely symmetric and could be rephrased in terms of interest rate drops and consumption rises.

Some Mathematical Models

What has now become the conventional modern approach to the dynamics of saving and consumption turns out to be inadequate to capture the intuitive economics of the preceding section. The conventional approach assumes the consumer maximizes the integral over time of discounted utility, $U(C(t))\exp(-\delta t)$, where $C(t)$ is current consumption, δ is the consumer's discount rate, and U is an arbitrary utility function. He faces the constraint that discounted consumption must equal discounted income, or equivalently that "terminal" wealth must be non-zero. Formally, his problem is

$$1) \qquad \int_{t_0}^{\infty} U(C(t))\exp[-\delta t]dt \qquad \text{subject to}$$

$$\int_{t_0}^{\infty} R_t C(t)dt = \int_{t_0}^{\infty} R_t Y_{(t)}, \quad \text{where} \quad R_t = \exp[-\int_0^t \rho(s)ds]$$

and ρ is the interest rate. This problem generates the first-order condition for a maximum,

$$2) \qquad -(U''/U')\dot{C} = \rho - \delta$$

The derivative of consumption is positively (if U implies risk aversion) related to the interest rate. It is common practice to assume forms for U which make (2) imply a simple proportionality between $\dot{C}$ and ρ or between $\dot{C}/C$ and ρ. A derivative leads the level of the same variable by a quarter of a cycle and ρ is therefore implied to lead C or its logarithm by a quarter of a cycle. This is consistent with the discussion of the preceding section.

The model also looks reasonable if we ask what effect it implies for an unanticipated increase in the level of the interest rate which is expected to persist. If we begin in an equilibrium with the interest rate equal to the discount rate and the level of income constant, consumption will have been constant and equal to income. If the interest rate now rises to a new constant level, consumption must drop instantly, then slowly rise. An example of the implied time path is displayed as the solid line in Chart 1, for the case of quadratic $U(C) = C - C^2$. An unrealistic aspect of this response, however, is the instantaneous reaction of consumption. The effect which appears in the data clearly involves a delayed response, or at least a response which at first changes only the derivative, not the level, of consumption.

There is no way to avoid this unrealistic aspect of the result by manipulating the form of U or the discount rate δ. The instantaneous response of C to changes in the level of ρ is a characteristic of any model which makes the dynamic utility function separable. That is, we can avoid this unrealistic prediction only by abandoning the assumption that utility over time is a simple adding up of utilities at different times. The most direct route to a more realistic model is to introduce a quadratic penalty in the utility function for rapid changes in the level of consumption. If we assume, say $U(C) = C - C^2 - 2\theta\dot{C}^2$, the initial equilibrium with constant income and $\rho = \delta$ will be the same as the one we computed before. The same rise of .02 in the discount rate, however, now implies the time path for consumption plotted as the dashed line in Chart 1. This looks much more like the type of response found in the data.[1/]

Steps Remaining to a Complete Theory

Perhaps the most interesting and serious gap in the discussion to this point is the conflict between the theory described here and that used by Hall (1978) in his recent work on consumption. Hall uses the same type of mathematical

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[1/] Though the optimization problem described here is simple as a description of economic behavior, its solution is surprisingly messy. The solution is described in the appendix. Because of the model's non-quadratic structure, response to an interest rate change depends on the nature of the initial equilibrium, most critically to whether ρ is larger or smaller than δ initially and whether $Y(t)$ is anticipated to be constant or changing systematically. When anticipated Y is constant, "permanent income" remains constant as ρ changes. This is no longer true if Y is anticipated to be steadily rising or falling. I have made a considerable number of computational experiments to verify that the qualitative pattern displayed in Chart 1 holds up under a wide range of plausible variations in the initial equilibrium.

model discussed in the preceding section, maintaining the assumption of separability. The implication of the theory which he tests is essentially an implication of the separability assumption: that changes of consumption should be unpredictable. Hall finds that these changes are largely unpredictable. Furthermore, the results reported below for U.S. quarterly data are consistent with Hall's results; the variance of changes in consumption is mostly variance of consumption's own innovation, which is unpredictable.

Now as we have seen, a theory which maintains the assumption of intertemporally separable utility cannot account for the smooth, negative, delayed response of consumption to interest rate surprises. It is equally true that a theory which maintains the assumption of non-separable utility, with a quadratic penalty on the derivative of consumption, cannot account for the change in consumption's being unpredictable. Consumers should smooth the time path of consumption's response to changes in expected future income to avoid paying the penalty of rapidly changing consumption.

There is no neat resolution of this contradiction that I can see. This paper's quarterly U.S. results do show a substantial predictable component for changes in non-durable consumption (and even more for durable consumption). Table 1 displays the equation for consumption of non-durables in this paper's quarterly U.S. model. The coefficients on lagged M1, CPI, RGNP, and CPR (money, prices, real GNP and interest rate) are significant as a group with an $F(16, 107)$ of 3.46.

Omitting these variables increases the equation's standard error of estimate by 15 percent. Nonetheless consumption changes remain too unpredictable to fit comfortably with a theory which includes a quadratic penalty in the utility function for $\dot{C}$.

The most reasonable suggestion, it seems to me, is that the cost of rapid changes in consumption is not a direct impact on utility of rapid changes, but instead the cost of frequent reconsideration of intertemporal allocation decisions. Most consumers do not audit their financial status each month and readjust consumption plans accordingly; this is probably true even of wealthy consumers who do have access to capital markets for consumption smoothing. Interest rates have a delayed impact on consumption because consumers only at intervals consider the possible gains from reacting to changes in interest rates. This does not in itself explain why changes in consumption show so little persistence -- it might be taken as an argument for inertia, and hence persistence in the rate of change. But if a substantial fraction of consumers do not have frictionless access to credit markets while income changes are themselves largely unpredictable (which seems to be the case), then consumption might contain a large, unpredictable component reacting immediately to income together with another large component, corresponding to the consumption of people with access to capital markets, which shows some inertia but reacts within a few months to changes in interest rates.[2/]

I hope it is clear that the foregoing paragraph is speculative. Other explanations of the facts are certainly possible, and I have not formalized the argument sufficiently to be sure it could work as suggested.

A second major gap in the theory presented here is that it concerns consumers who do not hold money. The spirit of the way we apply the theory below to interpretation of the data is that money does not play a large role in intertemporal consumption decisions so that a model which treats deposits as just negative loans, paying the same rate, is a good approximation. Then to explain the observed response of money to interest rates we invoke the common sense conclusion that any reasonable theory of money demand will imply a negative response to interest rates. To improve this situation in a way consistent with this paper's approach would require treating seriously the distinction between money and other assets the consumer might hold. If this were done by assuming a "real yield" on money and zero interest on money, the effects on the conclusions and on the difficulty of the analysis would be small. If we tried

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[2/] Note that it is essential to this story that the "costs of adjustment" in the utility function correspond to information-gathering or computation costs, not to an actual cost or disutility of changing $\dot{C}$. Otherwise the potential gain from providing access to credit for consumers without such access and subject to erratically fluctuating income would be so great that the existence of credit-constrained consumers would be in doubt.

instead to capture the nature of the value of money as a flexible, divisible asset, the analysis would become more difficult and conclusions might change.

The theory presented here is deterministic. It ought to be stochastic, but as already mentioned the intertemporal allocation problem with uncertain interest rate is formidable because of its non-quadratic nature. The deterministic results we have presented ought to be qualitatively reliable for analysis on a monthly scale.

Finally, it would be better if we could model explicitly the reasons why consumption might have to fluctuate more than is consistent with a constant interest rate. That is, it would be nice to have a model with an explicit dynamic production technology so that we could see how costs of adjusting investment plans might lead to responses of interest rates to cyclical shocks. Such a model would be most useful, however, if it could be stochastic, allowing for realistic fluctuations. A stochastic equilibrium model of this type seems extremely difficult to solve.

Statistical Results

This paper is based on the results of analysis of monthly time series on short interest rates, money stock, production, and prices for Germany, France, Britain, and the U.S. In addition, a larger model using quarterly U.S. data on these same variables plus nine GNP components was explored. In the typical case the model used was an unconstrained vector autoregression using a constant term and twelve lags of each of the four variables in the system on the right-hand-side of each equation of the system. Four time series of, say, 270 observations each, are modeled with a system containing 206 free parameters (49 regression coefficients per equation plus 10 parameters for the residual covariance matrix.) There is little of the character of a succinct data summary in a model parameterized so densely, and this creates problems in determining a reasonable way to report results. Reporting each parameter estimate and the variance-covariance matrix of the parameters of each estimated regression equation would involve reporting more real numbers than appeared in the original data set.

It is not reasonable to display all the estimated regression coefficients, because the estimated vector-autoregressive system really plays the role of a preliminary data transformation. The economic content of the statistical work emerges only when we take the further step of confronting the data with specific hypotheses or interpretations. Thus we will describe the methods used in estimating the model, hopefully in enough detail to make the results reproducible, but display only those aspects of the model which relate to specific economic interpretations.

The estimated systems have the form

3) $$Y(t) - A(L)Y(t) + c + u(t) ,$$

where Y is a vector (typically of length 4) of economic time series, A is a 12th order polynomial in positive powers of the lag operator (for the monthly data), and u is a residual identified by the property that $u(t)$ is uncorrelated with $Y(s)$ for all $s < t$. c is a constant. That Y satisfies (3) is a weak restriction on its character as a stochastic process. The coefficients of A can be estimated by least squares and the usual estimated covariances for the estimated parameters can be justified without assuming normality of u or stationarity of Y. Estimates of A in such a system prove to yield little insight, however, without a further transformation. We can always formally invert (3) to obtain

$$Y(t) = (I - A(L))^{-1} u(t) \quad . \tag{4}$$

If the inverse in (4) is taken as the one in positive powers of L (obtainable by, e.g., polynomial long division), then (4), representing Y as a linear combination of current and past one-step-ahead forecast errors (or innovations) is termed the moving average representation of Y.

Letting $B = (I - A(L))^{-1}$, the moving average coefficients in B are interpretable as responses of Y to "typical shocks". The coefficient $b_{ij}(t)$, the coefficient on the t'th power of L in the i'th row and j'th column of B, represents the response after t periods to an initial condition $u_j = 1$, by the i'th component of Y, Y_i. When the elements of u show strong contemporary correlations, it becomes helpful to replace u by a transform, say Vu, such that Vu has the identity as covariance matrix. Then (4) becomes $Y = B(L)V^{-1}Vu$ or, letting $C(L) = B(L)V^{-1}$ and $e = Vu$, $Y = C(1)e$. The coefficients in C then display the response of the system to "shocks" of one-standard-error in elements of e. Each column of C, treated as a vector time series, describes a "mode of vibration" of the system, typical in both size and shape.

The central statistical result of this paper is that, across four countries, we find the response of industrial production and money stock to interest rate innovations remarkably similar. Taking V to be lower triangular with the variables ordered with interest rate on top, followed by money, prices, and production, the elements of C corresponding to the response of production to interest rate innovations have the coefficients plotted in Chart 2.[3/] In the U.K. and the U.S., industrial production has begun a persistent negative response to the interest shock within 7 months. In both cases the response is significantly negative at one, two and three year horizons. In Germany and France, the response is more delayed, with the negative component only beginning to take over firmly at the one year horizon. In both cases, however, the response is

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[1/] We should observe that the U.K. results are based on a three-variable system excluding money stock, due to the lack of a conveniently available money stock series in the sources used for the paper. Data are for 1955 or 56 through the end of 1979 for the European countries, and for 1947 through 1978 for the U.S. More details about the data appear in the data appendix.

significantly negative at the two year horizon.[4/] In each country, the largest component of forecast error variance other than that due to industrial production's own innovation is that due to the innovation in interest rate. The proportion accounted for by interest rate innovations at the 36-month horizon is 22 per cent for France and Germany, 36 per cent for the U.S., and 53 per cent for the U.K. The maximum proportion of forecast error variance in industrial production accounted for by money stock across the U.S., Germany, and France occurs at the 15-month horizon for the U.S., at 7.7 per cent.

Chart 3 displays the estimates and standard errors for responses of money stock to interest rate innovations in the three countries for which money stock data were used. In each case the response is a smooth, strongly significant decline in money stock over a period of a year or more after the interest rate shock. In each case there is negative contemporaneous association between interest rate shocks and money stock, but the absolute size of this association is in each case tiny. Translating it into a regression coefficient (as our choice of V in forming C does) implies that a one-standard-error shock in the interest rate is associated with a drop in the money stock below

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4/ Intercorrelations among the residuals in these monthly data are generally small in absolute value (though statistically significant), so that the form of C is not sensitive to the ordering of variables in choosing V.

its expected level of less than .06 per cent in each country. This compares with a one-step ahead forecast standard error for money stock of more than .8 percent in France and Germany and of .26 per cent in the U.S.

Chart 4 shows the response of interest rates to their own innovations in each of the four countries. In each case the innovation is persistent, remaining of the same sign for months after the initial disturbance.

A monetarist countergument to this paper's interpretation of the data might insist that interest rate surprises are generated by anticipated, policy-induced periods of steady contraction in the money stock. As I pointed out in my earlier paper (1980), this position requires abandoning one version of rational-expectations monetarism, which argues that anticipated contractions in the money stock should have no real effects. Furthermore, in any case it implies that the money stock itself is not a good indicator of monetary policy -- it is only these periods of steady contraction (or expansion) of the money stock, which are preceded by interest rate surprises, which have strong real effects. Most money stock variation is not of this type and does not have strong real effects.

But it seems reasonable to go further, and admit that the most plausible explanation of the results is passive reaction of the money stock to the effects of non-monetary forces on the interest rate. In the U.S. postwar monthly data, the responses of prices to interest rate shocks are small and insignificant. The responses are even smaller in absolute size in Germany. But in France and the U.K. price responses to interest rate shocks are strongly significant and of the same sign as the interest rate shocks. For the first several months after the shock they remain small in absolute value, but in these two countries toward the end of the first year the price responses become strong enough to bring the "real interest rate", computed as the interest rate minus the annualized monthly inflation rate, down close to zero. Positive price responses sufficient to bring the real rate back to zero much faster than the nominal rate occur also in the U.S. quarterly model discussed in more detail below.

Inflation in the wake of an interest rate shock, despite the accompanying contraction in the money supply, is easy to reconcile with an interpretation of the shock as a non-monetary piece of "bad news". If the interest rate shock represents an anticipated contraction of the money stock, though, it is hard to see why it should tend to be followed by inflation. The usual monetarist analysis would suggest the opposite.

Table 2 presents summary statistics for each of the four countries to give some indication of aspects of the statistical results not given detailed attention.

A rather different system was estimated for U.S. quarterly postwar data. For this system industrial production was replaced by an exhaustive 9-element list of

GNP components and the autoregressive operator A(L) was constrained to be zero except for terms corresponding to lagged money stock, interest rate, real GNP, and price level, and to lagged values of the left-hand-side variable. Thus the system constrains all lagged effects of GNP components on other variables in the system to operate through their effects on aggregate GNP. The initial motivation for examining this system was to see if the interest rate innovations which play such a large role in the monthly system might themselves be explained by innovations in the distribution of GNP by components. They are not. The regression of interest rate innovations on innovations in all nine GNP components has an R^2 of only .12, and the proportion of GNP variance accounted for by interest rate innovations at the 3-year horizon is 56 per cent. However as is shown by Chart 5, this system does confirm that consumption, even its non-durable component, is sharing in the negative response of production to interest rates.

The quarterly system also produces one anomaly which I have not yet resolved: the estimated response of price to interest rates is so quick that nominal interest rate surprises are estimated to be associated with opposite-signed surprises in the real rate of interest. The statistical significance of these results is not yet clear because the required (expensive) Monte Carlo study has not been performed. But they conflict sharply with the monthly U.S.

results, where the response of prices to interest rate innovations was negligible, using the same price and interest rate series. One possible explanation is that the GNP components, while not explaining all interest rate surprises, do explain some of the surprises associated with an "end-of-investment-boom" situation, and that the remaining components of the interest rate surprise time series are more heavily dominated by "raw material supply" phenomena. The latter sort of interest rate surprise ought to be more unambiguously associated with inflation.

Conclusions

One conclusion from this paper's results ought to be non-controversial. For forecasting purposes in the postwar period, interest rates have been at least as important as money stock and contain information not contained in the history of the money stock. As is in fact usual with macroeconomic statistical studies, however, the policy implications of this one leave room for controversy.

A modified monetarist model, allowing for the possibility that non-monetary factors have a very substantial role in generating the business cycle, might be able to rationalize this paper's results in a way consistent with a large role for monetary policy. One might argue, for example, that the monetary authority is insufficiently aggressive in preventing monetary growth from responding passively in the wake of interest surprises, and that this systematic tendency plays a major role in generating fluctuations.

Because of the pattern of smooth, delayed reaction to a sudden movement in a financial variable, the results leave room for interpretation in terms of frictions and price stickiness, as I emphasized in the theoretical section above. They certainly leave room, therefore, for a view that stabilization policy might be important in moderating the economy's natural tendency toward inefficient fluctuations.

But there is also no difficulty in accounting for the observed results with an "equilibrium theory" of the business cycle. In fact, the position, which has some adherents among economists, that no instrument of "demand management" policy, including monetary policy, has important real effects, is quite consistent with this paper's results. The paper has not explored in any depth the question of which policy instruments might have real impacts, given that money stock does not. Interest rates, while sometimes treated as policy instruments in discussions of macroeconomic policy, are not obviously controllable. We have presented a theoretical interpretation which rationalizes the observed response to an interest rate rise without alloting any necessary role to monetary policy in generating interest rate surprises.

In summary, while the results cast strong doubt on monetarism in all its conventional forms, they do not provide evidence to distinguish between two non-monetarist views with very different implications. They are consistent on the one hand with the view that countercyclical demand management policies of all types are ineffectual, and on the other hand with the view that fiscal policy and monetary policy, if the latter is defined as interest rate policy, are both effective and valuable.

Appendix 1

Solutions for the behavioral model.

Recall from the text that the simpler model considered, with separable utility function, was

$$1) \qquad \text{Max} \int_{t_o}^{\infty} U(C(t))e^{-\delta t}dt$$

subject to $\int_{t_o}^{\infty} R_t C(t)dt = \int_{t_o}^{\infty} R_t Y(t)dt$, where $R_t = \exp[-\int_o^t \rho_s ds]$ and ρ_s is the interest rate. As noted in the text, this generates the first-order condition

$$2) \qquad U'e^{-\delta t} = \lambda_o R_t \ .$$

With the quadratic utility function $U(C) = C - \gamma C^2$, this becomes

$$A1) \qquad 1 - 2\gamma C = \lambda_o \exp[-\int_o^t (\rho_s - \delta)ds].$$

Thus the gap between C and its utility-maximizing level $1/2\gamma$ closes exponentially at the rate $\rho - \delta$ when there is a constant $\rho > \delta$. For $\rho < \delta$, and assuming $Y(t) < 1/2\gamma$ $C \to -\infty$ as $t \to \infty$. Clearly the quadratic utility function makes sense only if we assume that $\rho_t \to \delta$ as $t \to \infty$.

If we begin with $Y(t)$ constant and $\rho = \delta$, the equilibrium will, from (A1), have $C(t)$ constant, and from the constraint in (1) it is clear that this implies

$C(t) \equiv Y(t) = \bar{Y}$. Keeping $Y(t) \equiv \bar{Y}$ and considering the solution for constant $\rho \neq \delta$, (A1) and the constraint yield

A2) $$1 - 2\gamma C = \lambda_o e^{-(\rho-\delta)(t-t_o)}$$

A3) $$\int_{t_o}^{\infty} C(t) e^{-\rho(t-t_o)} dt = \bar{Y}/\rho \ .$$

Solving (A2) for C and substituting in (A3) yields

A4) $$1/(2\gamma\rho) - \lambda_o/(4\gamma\rho - 2\gamma\delta) = \bar{Y}/\rho \ ,$$

or

A5) $$\lambda_o = 2\gamma(2 - \frac{\delta}{\rho}) \ [(1/2\gamma) - \bar{Y}] \ ,$$

which yields from (A2)

A6) $$C(t_o) = \frac{1}{2\gamma} - (2 - \frac{\delta}{\rho})[(1/2\gamma) - \bar{Y}] \ .$$

If the model is to make sense, $\bar{Y} < (1/2\gamma)$, so $C(t_o)$ is clearly decreasing in ρ. But since $\lambda_o > 0$ and $\dot{C} = +(\rho-\delta)\lambda_o(1-2\gamma C)$, $\dot{C}$ is increasing in ρ. The plot in Chart 1 is generated from these solutions with $\rho = \delta = .10$, $\gamma = 1$, $\bar{Y} = .25$ initially and a change to $\rho = .12$ at t_o.

As we observed earlier, it is not realistic to assume $\rho \neq \delta$ permanently, but results would be very similar if we assumed that ρ converged back to δ after several years.

Replacing U in the forgoing exercise with

A7) $$U(C, \dot{C}) = C - \gamma C^2 - \theta \dot{C}^2$$

yields the first order condition

A8) $$e^{-\delta t}D_1U - \frac{d}{dt}(D_2Ue^{-\delta t}) = \lambda_o R_t ,$$

or

A9) $$1 - 2\gamma C + 2\theta\ddot{C} - 2\theta\delta\dot{C} = \lambda_o R_t e^{\delta t} ,$$

or

A10) $$(2\theta D^2 - 2\theta\delta D - 2\gamma)C = \lambda_o R_t e^{\delta t} - 1$$

The operator applied to C in (A10) has two roots of opposite sign. Since we are interested in the case of an unanticipated change in ρ, (A10) applies from t_o onward in time. Thus we can factor out the positive root, invert the corresponding operator as a "distributed lead", and apply it to the right hand side of (A10).

The two roots in question are

A11) $$(\delta \pm \sqrt{\delta^2 + 4\gamma/\theta})/2 .$$

We let r_2 be the positive root and r_1 be minus the negative root. Then applying to (A10) the operator $(D - r_2)^{-1}/(2\theta)$ produces

A12) $$(D + r_1)c = 1/2\theta r_2 - [\lambda_o/2\theta(r_2 + \rho - \delta)]e^{-(\rho-\delta)t} .$$

This implies in turn (using $r_1 r_2 = \gamma/\theta$)

A13) $$C(t) = 1/2\gamma + A_1 e^{-(\rho-\delta)(t-t_o)} + A_2 e^{-r_1(t-t_o)} ,$$

where A_1 and A_2 are constants fixed by side conditions. (A_1 is determined by λ_o). The equations determining A_1 and A_2 are

A14) $$C(t_o) = 1/2\gamma + A_1 + A_2$$

and

A15) $$1/2\gamma\rho + A_1/(2\rho-\delta) + A_2/(\rho + r_1) = \bar{Y}/\rho$$

Equation (A14) is justified by the assumption that, because $\dot{C}^2$ enters the utility function, C will not jump in response to the change in ρ. Equation (A15) is just the constraint relating discounted consumption to discounted income. The hatched line in Chart 2 is obtained by assuming that, starting from initial equilibrium with $\rho = \delta = .10$, $C = \bar{Y} = .25$, $\gamma = 1$, $\theta = .1$, there is an unanticipated shift to $\rho = .12$. Note that the dramatic difference between the response of C to ρ between the "$\theta = 0$" and "$\theta = .1$" case apparent in Chart 1 occurs with a very small weight given to $\dot{C}^2$. To produce an effect on utility equal to that produced by a decline in the level of C from .25 to .24, $\dot{C}$ has to reach $\pm .23$, i.e. a rate of change sufficient to produce the corresponding change in level in less than a month. Thus so long as the time path of C is smooth within the year, the $\dot{C}$ term in the utility function has a negligible effect.

DATA AND METHODOLOGY APPENDIX

I. Data

A. OECD Main Economic Indicators (Paris)

Historical Statistics 1955-71
Historical Statistics 1960-75
Monthly Reports January 1976 to April 1980

West Germany

IP: Index of industrial production
55-59 Total, excluding construction
60-80 Total
CPI: Consumer price index all items
M1: End of month money stock
R: Treasury Bill Rate (2 to 3 month; end of month)

France

IP: Index of industrial production
56-59 Total, excluding construction
60-79 Total
CPI: Consumer price index all items
M1: End of month money stock
R: Call Money Rate - end of month

United Kingdom

IP: Index of industrial production
56-59 Total, excluding construction
60-80 Total
CPI: All goods and services
R: Treasury Bill Rate, 91 days, average of last issue

B. Citibase Economic Time Series Data Base (Citibank, New York)

United States

IP: Index of industrial production, total, Federal Reserve Board
CPI: Consumer price index, all items, Bureau of Labor Statistics
M1: Currency plus demand deposits, Federal Reserve Board, monthly average of daily figures.
R: Commercial paper rate (prime 4-6 months), Federal Reserve Board, monthly average of daily figures.

The quarterly U.S. data are from the GNP accounts, in constant dollars.

In each country CPI and IP series for the full historical period had to be prepared by splicing overlapping series with different base years.

In France, huge deviations from historical patterns occurred for industrial production in May 1968 and for the money stock in April 1974. Least squares estimation of the system lets these errors (in each case a residual of over 10 "standard errors" in the OLS system) dominate the fit. A careful treatment of this problem would lead into expensive iterative estimation methods. We used instead a rough method.

Letting s be the period of the outlying observation on Y_i, the equation

B1) $$Y_i(t) = \sum_{v=0}^{12} G(v)Y(t-v) + e_i(t) \ ,$$

with the i'th row of $G(0)$ constrained to zero, was fit by OLS for the sample $t = t_1,\ldots,s-1$, where t_1 is the beginning of the regression sample. $Y_i(s)$ was then replaced by the predicted value $\hat{Y}_i(s)$ from (B1). Since there were two outliers for France, this procedure was applied twice, with the estimate of (B1) for the later s using the modified data for the earlier s.

If this procedure were iterated, so that with the final estimated system the implied forms for (B1) had zero residuals at the outlying points, we would in effect have maximum likelihood estimates treating the $e_i(s)$ at the two outlying points as nuisance parameters. Since the $e_i(s)$

would not be consistently estimated, however, the maximum likelihood procedure has nothing particularly to recommend it over the procedure actually used. Also, it is not clear that a procedure like that used here, which tries to ignore the outlier as if it were a pure measurement error, is appropriate. There is some indication in the data that the later of the two outliers, at least, fed through the dynamics of the system in a way not too different from the way ordinary-sized residuals did. If we were confident that the outliers fed through the dynamics like ordinary innovations the appropriate procedure would be some multivariate version of robust linear regressions.

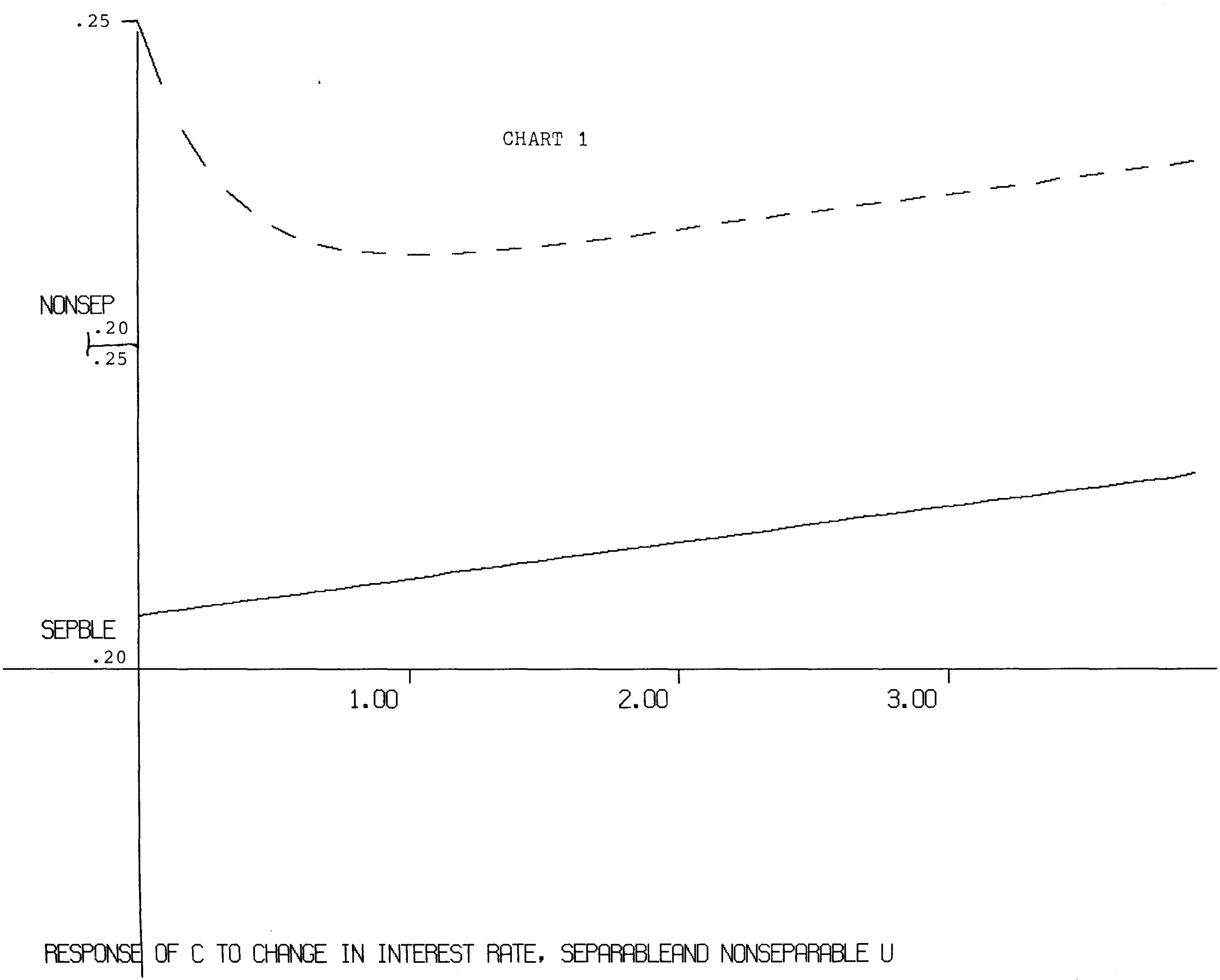
All data were seasonally adjusted, except interest rates, which show little seasonal. The unadjusted German production and money data show very strong seasonals. Since the adjustment required was so strong, and since the system estimated for the officially adjusted German data showed weak cross-variable interrelations, these data were given direct linear seasonal adjustment, starting with unadjusted data.

The seasonal adjustment method consisted of extracting a quadratic trend from each series, Fourier transforming the residuals at 384 equally spaced points, setting to zero the 5 elements of the Fourier transform surrounding each of the seasonal frequencies $2\pi j/12$, $j=1,\ldots,11$, inverse Fourier transforming, and adding the result back onto the initially estimated trends. The implied seasonal bandwidth of about .08 radians means the seasonal is flexible enough to reverse itself over a period of about 6 years. Results with data adjusted this way were quite similar to those with officially adjusted data.

II. Statistical Methods

The estimation and analysis of the linear system follows the methods used in my earlier papers, as described, e.g., in (1980). The references in the text to percentages of variance accounted for by various innovations at various horizons use a methodology described carefully in the cited reference. Briefly, the moving average gives a representation of the k-step ahead forecast error as a linear combination of orthogonal components consisting of innovations in the separate variables. It is this representation which yields the cited decomposition of variance.

The estimated standard errors presented for the monthly systems are obtained by Monte Carlo methods. They are estimates of the standard deviations of posterior distributions of the corresponding parameters, given a flat prior distribution on the parameters. That is, they treat the normalized likelihood function as the p.d.f. of the parameters. Rather than sample directly from this posterior distribution, which is non-normal, the sampling is from the normal estimated asymptotic distribution, with the random draws weighted so that the estimates reflect the true posterior. This amounts to using the method of Monte Carlo integration described by, e.g., Kloeck and van Dijk (1978).



RESPONSE OF C TO CHANGE IN INTEREST RATE, SEPARABLEAND NONSEPARABLE U

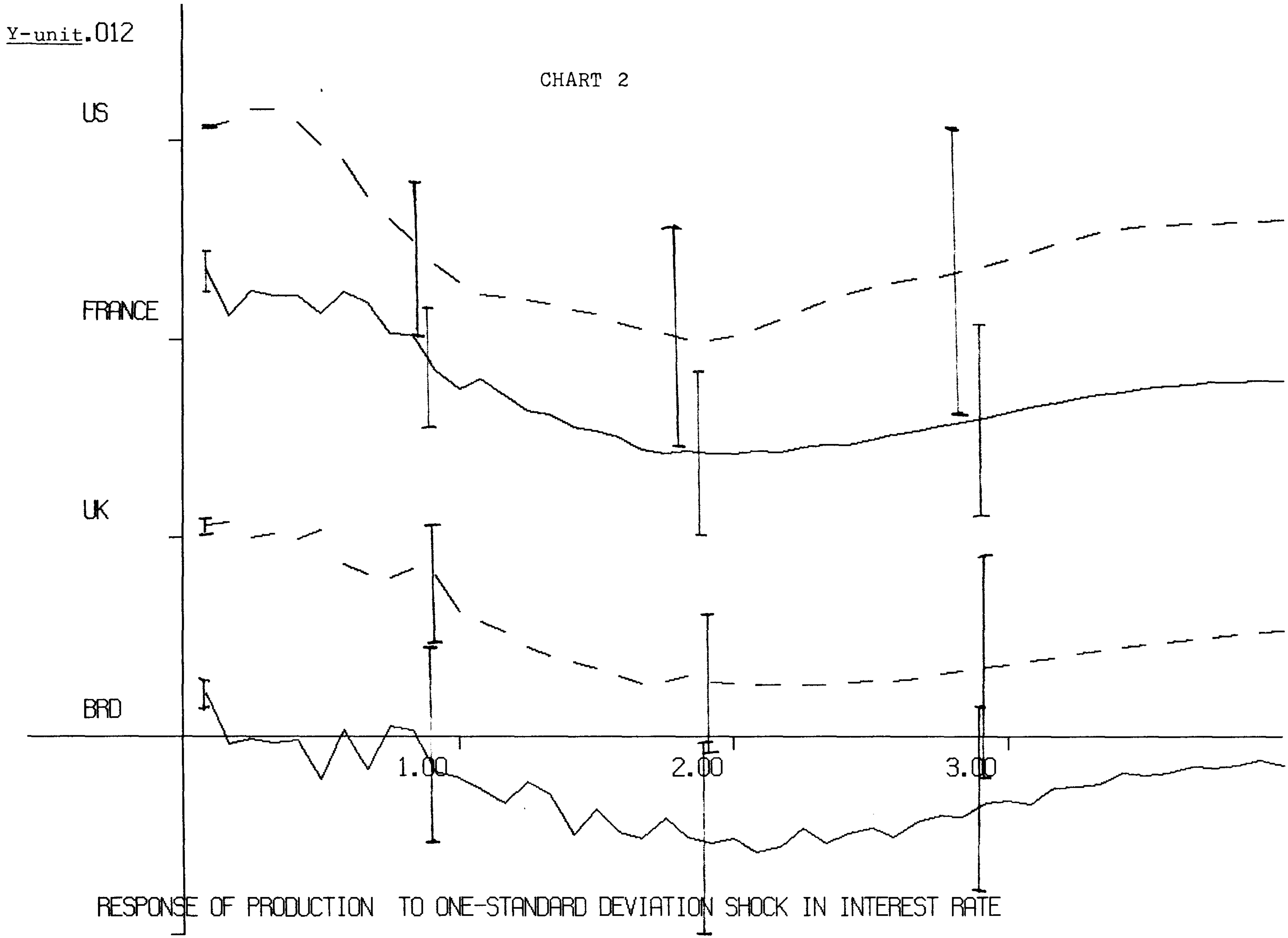

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CHART 2
US
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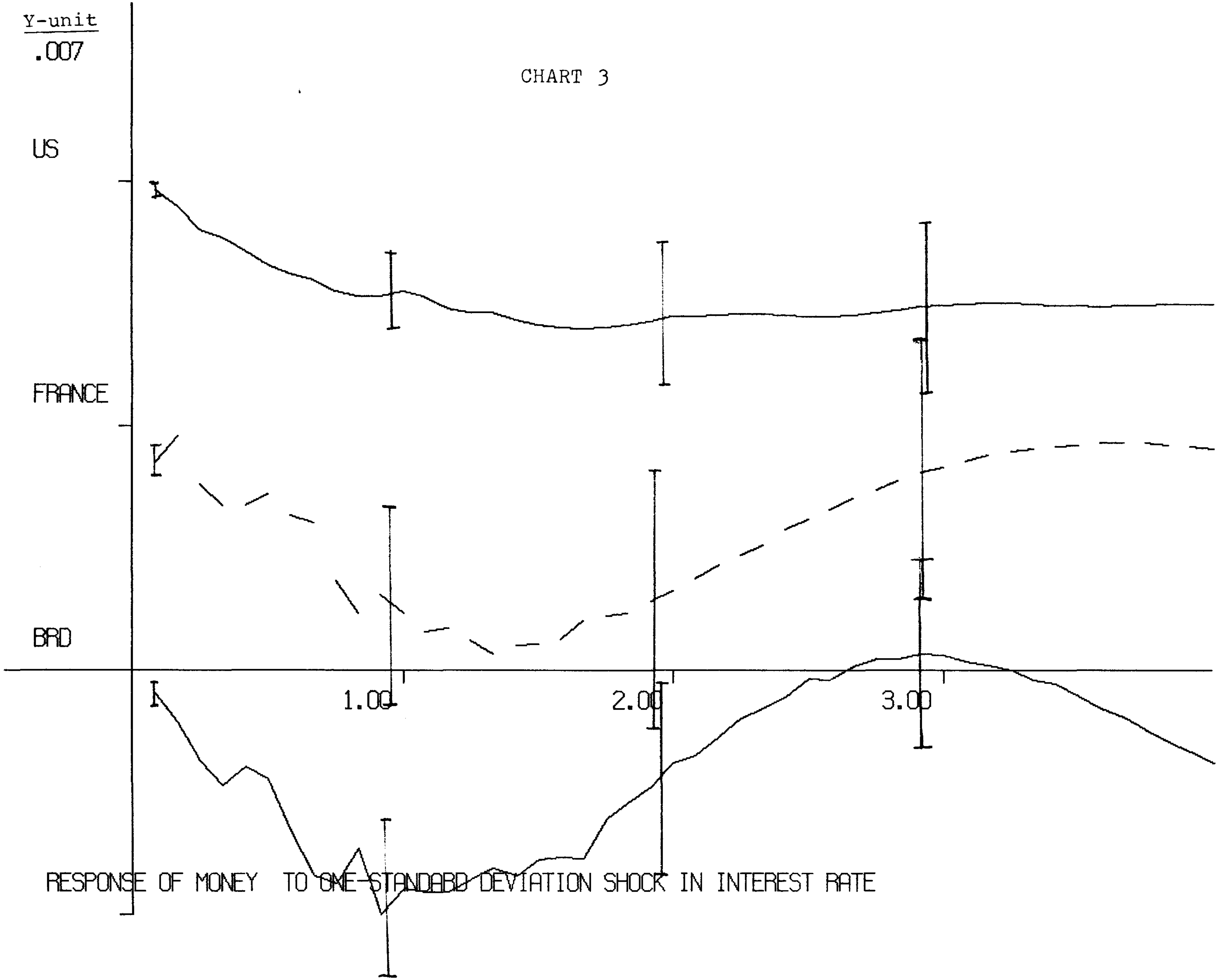
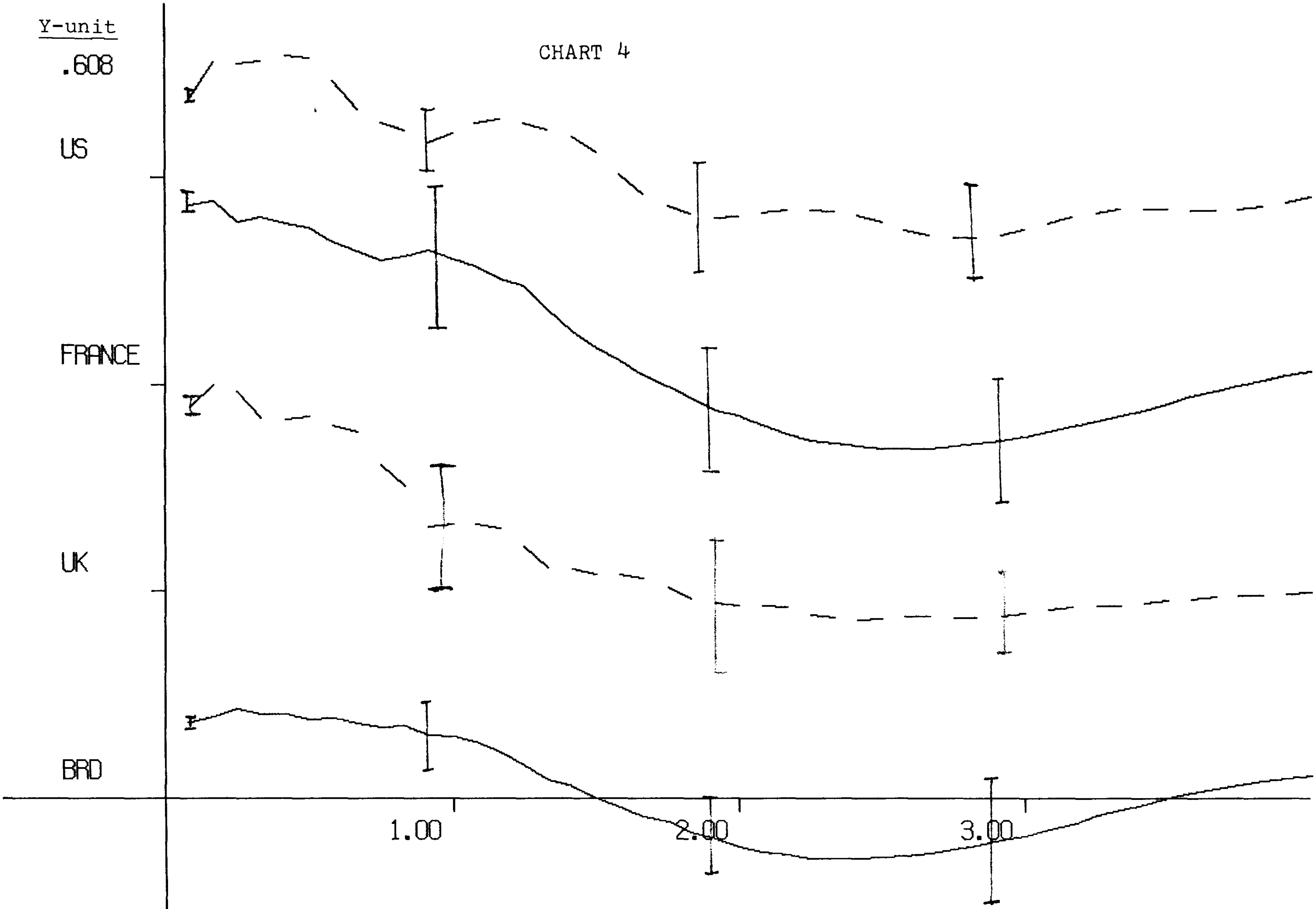
RESPONSE OF PRODUCTION TO ONE-STANDARD DEVIATION SHOCK IN INTEREST RATE

CHART 3



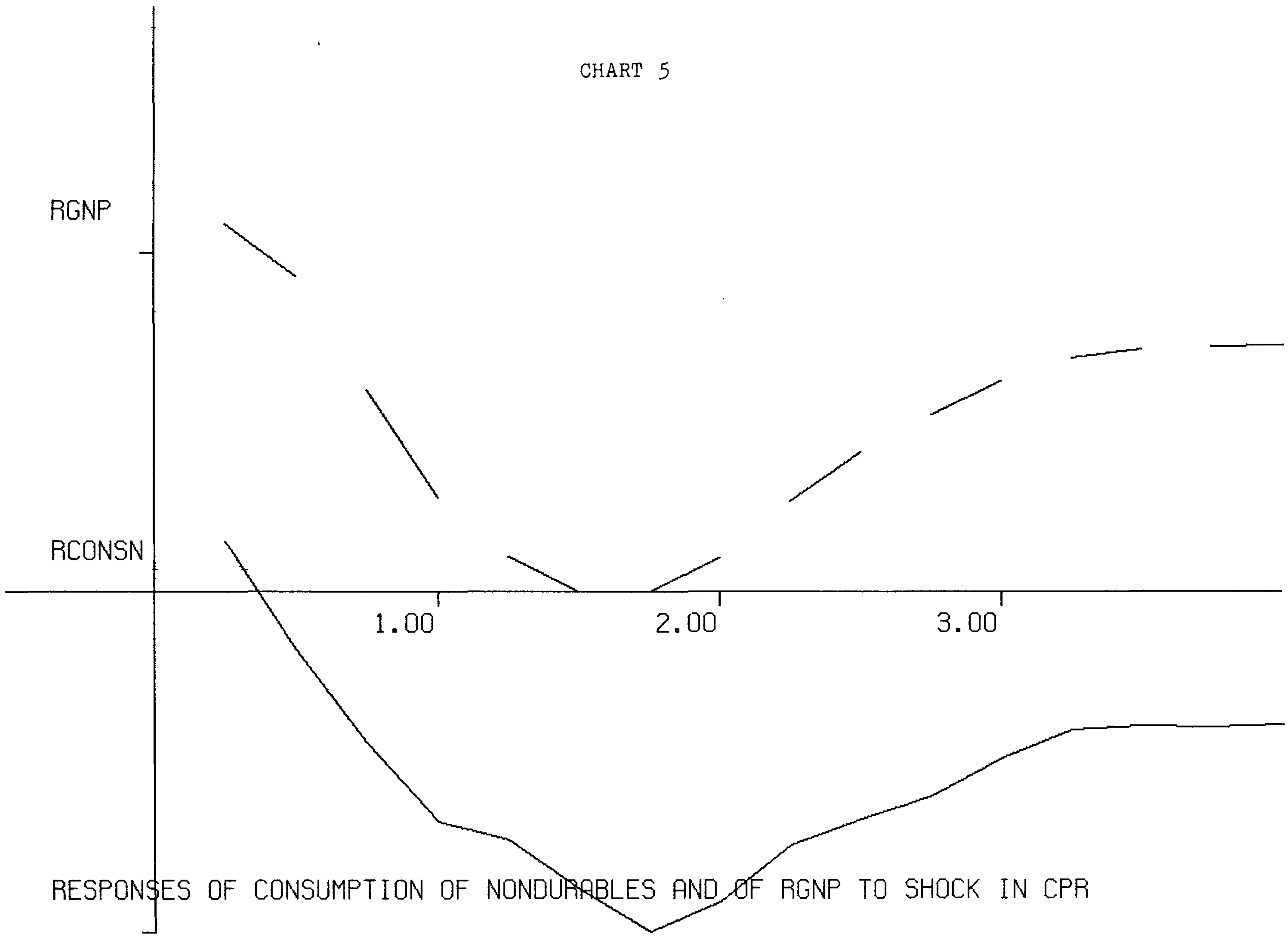
RESPONSE OF MONEY TO ONE-STANDARD DEVIATION SHOCK IN INTEREST RATE

CHART 4



RESPONSE OF INTEREST RATE TO ONE-STANDARD DEVIATION SHOCK IN INTEREST RATE

CHART 5



RESPONSES OF CONSUMPTION OF NONDURABLES AND OF RGNP TO SHOCK IN CPR

Notes on the Charts

The "Y-unit" where indicated is the distance between tickmarks on the vertical axis. Each tickmark is the origin of the plot for the country or variable whose name appears beside it. Tickmarks on the horizontal axis are one year (twelve months) apart. The vertical bars are two-standard-error bands about the OLS-implied responses plotted, computed by Monte Carlo as described in the appendix. These bands were checked against computed probabilities that the estimates would be double or the opposite side of zero from OLS estimates. Treating two-standard error bands as 99 per cent confidence intervals appears roughly justified.

Table 1

Regression Equation Predicting Consumption of Nondurables, Quarterly Data

Dependent Variable	RCONSN		
From 48-1 until 79-4			
Observations	128	Degrees of Freedom	107
R^2 Unadj	.9999	$\bar{R}^2$	.9988
SSR	437.66301	SEE	2.022
Durbin-Watson	1.992		

LABEL	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
RCONSN	1	.8044	.109	7.32	.000
	2	-.0441	.130	-.33	.734
	3	.2044	.130	1.57	.116
	4	-.0079	.106	-.07	.940
CPR	1	-.6620	.338	-1.70	.088
	2	.3664	.631	.58	.561
	3	-.0489	.660	-.07	.940
	4	.1546	.427	.36	.717
M1	1	.4285	.194	2.20	.027
	2	-.5106	.319	-1.59	.110
	3	-.1562	.328	-.47	.634
]	4	.2066	.223	.92	.354
CPI	1	-1.3919	.432	-3.22	.001
	2	2.9499	.767	3.84	.000
	3	-2.2152	.780	-2.83	.004
	4	.7348	.436	1.68	.092
RGNP	1	.0346	.030	1.14	.252
	2	.0379	.038	.99	.322
	3	-.0549	.038	-1.42	.154
	4	.0005	.029	.01	.986
CONSTANT		-.9326	3.043	-.30	.759

Table 2

Covariance Matrices and Correlations of Innovations

U.K.

COVARIANCE MATRIX

VARIABLE	TBILLS	CPI	IP
TBILLS	.29618	-.20727E-03	.45532E-03
CPI	-.73444E-01	.26891E-04	-.37958E-05
IP	.52574E-01	-.45998E-01	.25324E-03

FRANCE

COVARIANCE MATRIX

VARIABLE	CALLRATE			
CALLRATE	.29147	-.14890E-03	-.23393E-04	.81079E-03
M1	-.31130E-01	.78494E-04	-.62823E-06	-.41723E-04
CPI	-.11648E-01	-.19062E-01	.13838E-04	.38100E-05
IP	.74243E-01	-.23281	.50634E-01	.40917E-03

WEST GERMANY

COVARIANCE MATRIX

VARIABLE	TBILLS			
TBILLS	.51211E-01	-.14289E-03	-.90633E-05	.58280E-03
M1	-.78460E-01	.64762E-04	-.96587E-06	.42693E-04
CPI	-.54421E-02	-.16309E-01	.54159E-04	.18471E-05
IP	.11768	.24242	.11469E-01	.47893E-03

U.S.

COVARIANCE MATRIX

VARIABLE	CPR			
CRP	.55991E-01	-.57861E-04	.21108E-04	.19311E-03
M1	-.93644E-01	.68187E-05	.50989E-06	.20258E-05
CPI	.37037E-01	.81074E-01	.58009E-05	.22422E-05
IP	.82005E-01	.77952E-01	.93546E-01	.99043E-04

Note: In each matrix, entries below the diagonal are correlations.

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