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Optimal Relay Location for Resource-Limited Energy-Efficient Wireless Communication

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Abstract

In the design of wireless networks, techniques for improving energy efficiency and extending network lifetime have great importance, particularly for defense and civil/rescue applications where resupplying transmitters with new batteries is not feasible. In this paper we study a method for improving the lifetime of wireless networks by minimizing the length of the longest edge in the interconnecting tree with deploying additional relay nodes. Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of n terminals in the Euclidean plane. For a positive integer k, the *bottleneck Steiner tree problem* ($BSTP$) asks to find a Steiner tree with at most k Steiner points such that the length of the longest edge in the tree is minimized. We give a ratio-$\sqrt{3} + \epsilon$ polynomial time approximation algorithm for BSTP, where ϵ is an arbitrary positive number.

Keywords: wireless networks, power efficient, approximation algorithms, Steiner tree, bottleneck Steiner tree.

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1 Introduction

Recent advances in affordable and efficient electronics have had a dramatic impact on the availability and performance of radio-frequency wireless communication equipment. A considerable number of defense and civil applications involve deployment of computing devices or sensors able to communicate digital information through wireless connections. Ad-hoc wireless networks require no infrastructure, central access points or wired switches. The wireless nodes run communication protocols that enable on-the-fly organization of traffic routing, so that all nodes achieve end-to-end connectivity by forwarding data packets from one node to a destination node across multiple hops. The lack of any infrastructure simplifies rapid network deployment, especially useful in situations where human presence is not appropriate or even possible, because of a dangerous environment. One representative ad-hoc wireless network consists of sensors capable of monitoring the environment and sending acoustic, video or seismic information to a data collection node. In most cases the sensors are battery powered and therefore operate for a limited time before they consume all power and stop working. For radio-frequency wireless communication, the transmission power required for a radio signal to be received at a destination node located at distance r from the source is proportional to r^k, with $k \in [2,4]$. So, in order to prolong the network lifetime in general, it is desirable to minimize the distance between nodes. When the node positions are fixed, there are several different methods to extend the network lifetime, mainly based on power-aware routing and transmission scheduling.

This paper contributes an algorithm for extending the lifetime of a wireless network when n nodes have fixed locations and a number of up to k additional nodes can be placed at arbitrary positions. The objective of the algorithm is to build a spanning tree that connects the n fixed points and up to k additional nodes in the Euclidean plane, so that the length of the longest tree edge is minimized. Hence, the power required to transmit on the longest link is minimized also, and the network lifetime, in terms of connectivity, is extended.

The problem described above is a variation of a Steiner tree problem, named *bottleneck Steiner tree problem* (BSTP for short). A Steiner tree is an acyclic network interconnecting a set P of terminals and some other points. Every vertex in a Steiner tree other than a terminals is called a *Steiner point*. The *bottleneck Steiner tree problem* is defined as follows: given a set P of n terminals and a positive integer k, find a Steiner tree with at most k Steiner points such that the length of the longest edge in the tree is minimized. Contrary to the classic Steiner tree problem, degree-2 Steiner points are allowed in BSTP. Instead of minimizing the total length of the tree, here we want to minimize the length of the longest edge.

The BSTP is NP-hard. The work in [10] shows that BSTP cannot be approximated in polynomial time with performance ratios less than 2 and less than $\sqrt{2}$ in the rectilinear plane and the Euclidean plane, respectively. Moreover, a ratio-2 approximation algorithm was introduced for both the rectilinear plane and the Euclidean plane in [10]. For the rectilinear plane, this performance ratio is the best possible. A ratio-1.866 approximation algorithm for the Euclidean plane has been described in [11]. In this paper, we give a randomized approximation algorithm with performance ratio $\sqrt{3}+\epsilon$ for the Euclidean plane, where ϵ is an arbitrary positive number.

As mentioned above, this problem has an immediate application in the design of wireless networks for extending their lifetime. A typical scenario where the algorithm can be used, consists of n units (combat units, rescue crews or sensors) that need to communicate. The algorithm determines the number and location of maximum k communication relay nodes (mobile nodes, unmanned aerial vehicles) deployed to improve connectivity and save power for the wireless network. The computed tree spanning at most $n+k$ nodes could be used to route traffic between nodes.

This paper continues in section 2 with a presentation of other techniques to optimize power consumption in wireless networks. Section 3 presents the main theorem that proves the $\sqrt{3}+\epsilon$ approximation performance and continues with the algorithm description. Section 4 concludes the paper with some final remarks.

2 Related Work

The recent advances in wireless technology have stimulated a strong research current in power efficiency for wireless networks. In [2], Chang and Tassiulas formulate the maximum lifetime routing problem for a wireless network as a linear program, similar to the maximum flow problem with node capacities. Their goal is to maximize the time until the network partitions, which is similar to our goal. Their algorithm computes optimal data flows for the single and the multi-commodity cases and they also consider a version for power-efficient routing with delay constraints, where the delay is given by the number of intermediary hops. Furthermore, Chang and Tassiulas extend their model in [3], and introduce a new class of *flow augmentation* and *flow redirection* algorithms that employ shortest paths and power consumption balance across nodes, proportional to their energy reserves, in order to maximize the network lifetime. Their approaches consider static networks and compute flow-based optimal routes, while our algorithm works on optimal placement of Steiner points - communication relays - that reduce the power consumption for the bottleneck edges in the network.

In [9], Slijepcevic and Potkonjak study the problem of placement of wireless nodes (sensors) into a monitored area and transmission scheduling to achieve full coverage with minimal power utilization. Their heuristic solution for the *Set K-Cover* problem partitions the wireless nodes into mutually exclusive sets, where nodes in each set fully cover the monitored area. Since at one time only one set of nodes is active, and the disjoint sets are rotated, significant power savings are achieved and the network lifetime is extended. The proposed technique works well for applications that do not need continuous connectivity for all nodes, sensor networks, for instance.

A novel power-aware routing method is described by Li et al. in [6]. The authors model the network lifetime as the earliest time when a message cannot be transmitted and propose a routing algorithm, named $max-min\ zP_{min}-path$, that consumes at most $z\ P_{min}$ power while maximizing the minimal residual power fraction.

Steiner trees have been a constant source of interesting problems with relevant applications in the wireless networking domain. For variations of Steiner tree problems and their applications please refer to [1, 4, 5, 8].

3 Ratio-$\sqrt{3}$ approximation algorithm for BSTP

In this section, we present a ratio-$\sqrt{3}$ approximation algorithm for BSTP in the Euclidean plane. We start by defining some key notions.

Definition 1 *A full component of a Steiner tree is a subtree in which each terminal is a leaf and each internal node is a Steiner point.*

Definition 2 *A Steiner tree for n terminals is a k-restricted Steiner tree if each full component spans at most k terminals.*

The next theorem characterizes the performance of our approximation algorithm for BSTP:

Theorem 1 *Let T be an optimum Steiner tree for BSTP. Then, there exists a 3-restricted Steiner tree with the same number of Steiner points as T such that the longest edge in the tree is at most $\sqrt{3}$ times the optimum.*

Proof. We assume that T is rooted by arbitrarily selecting a Steiner point as its root. We will modify T bottom up into a 3-restricted Steiner tree without increasing the number of Steiner points such that the length of the longest edge is at most $\sqrt{3}$ times the optimum. Without loss of generality, we assume that T is a full Steiner tree, i.e., every internal node in T is a Steiner point and every leaf in T is a terminal.

We organize the nodes in T level by level (ignoring degree-2 Steiner points). Level 1 is the lowest level. Level i is the level above level $i-1$. Let v be a node at level 3 that has some grandchildren. Let v' be a child of v. If v' is a Steiner point, we can assume that the degree of v' is 3, i.e., v' has two children that are terminals. Otherwise, suppose that v' has 3 or more children that are terminals, say, a, b, and c. Assume that a, b, c are positioned clockwise around v'. Then the three angles $\angle av'b$, $\angle bv'c$ and $\angle cv'a$ form 360^o. Thus, at least one of the three angles $\angle av'b$, $\angle bv'c$ and $\angle cv'a$ is at most 120^o. Without loss of generality, assume that $\angle av'b \leq 120^o$ and av' is not shorter than bv'. Then $|ab| \leq \sqrt{3}|av'|$.

Let m be the number of degree-2 Steiner points (not including v') in the path from a to v'. We construct a new Steiner tree T' by removing all the edges on T, and directly connecting a and b with m degree-2 Steiner points so that the length of each edge in the segment ab is at most $\sqrt{3}$. Now, we need only to consider the tree obtained from T' by removing a and all the degree-2 nodes on the path connecting a and b in T'.

From now on, we assume that the degree of v' is at most 3. We consider two cases.

Case 1. Every edge below v in T has length no more than 1. We consider the case where v has 4 grandchildren. The case where v has 3-children is simpler and is left to the interested readers.

We first consider that v is a degree-3 node. (See Figure 1.a.) In this case, we assume that $\angle bv'c > 120°$ and $\angle dfe > 120°$. Otherwise, assume that $\angle dfe \leq 120°$, then we can directly connect d and e. The length of edge de is at most $\sqrt{3}$. Therefore, we have

$$\min\{\angle dfv, \angle efv\} < \frac{360° - 120°}{2} = 120° \quad \text{and} \quad \min\{\angle cv'v, \angle bv'v\} < 120°,$$

i.e.,

$$\min\{|dv|, |ev|\} < \sqrt{3} \quad \text{and} \quad \min\{|cv|, |bv|\} < \sqrt{3}. \tag{1}$$

Without loss of generality, assume that

$$\angle dfv = \theta = \min\{\angle vv'v, \angle vv'c, \angle dfv, \angle efv\} \tag{2}$$

and

$$\angle vv'c \leq \angle vv'b. \tag{3}$$

We will find a point h on edge vv' such that $\max\{|ch|, |bh|, |dh|\} \leq \sqrt{3}$, and construct a new tree by removing nodes v' and f, adding edges ch, cv, dh, bh, and connecting d and e directly with a Steiner point w on the middle of de. (See Figure 1.b.) Then, we can continue the modification process with $n-3$ terminals in $P \cup \{v\} - \{b, c, d, e\}$.

By (1) and (3), we know that $|ch| \leq \sqrt{3}$ for any h on the edge vv'. So, we need only to choose an h to guarantee $|bh| \leq \sqrt{3}$ and $|dh| \leq \sqrt{3}$.

First we suppose that $\theta < 90°$, then we take h to be the point on edge vv' such that $|vh| = 2 - \sqrt{3}$. It is clear that $|bh| \leq \sqrt{3}$. Note that $|dv| \leq 1$ and $|fv| \leq 1$. By triangle inequality, $|dh| \leq |dv| + |vh|$. It is easy to see that $|dv| \leq \sqrt{2}$ when $\angle dfv < 90^o$. (dv is the third edge in Δdfv.) Thus,

$$|dh| \leq |dv| + |vh| \leq \sqrt{2} + 2 - \sqrt{3} < \sqrt{3}.$$

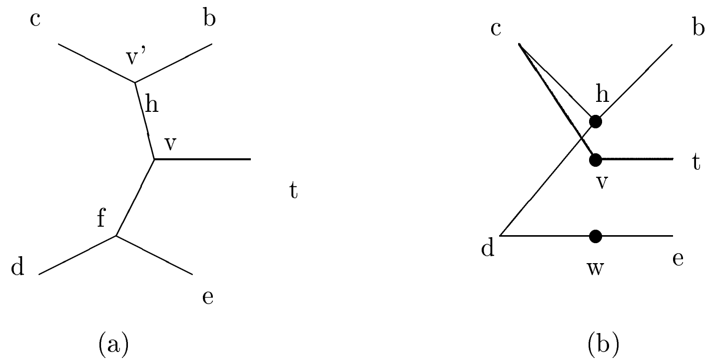

Figure 1: a. The original tree. b. The modified tree.

Now we suppose that $90° \le \theta < 120°$. By (2) and (3), we know that $\angle bv'v < 360° - 120° - \theta = 240° - \theta$. We choose h to be the point on edge vv' such that $|vh| = \sqrt{3} - |dv|$. By (1), $|vh| > 0$. It is easy to see that

$$\begin{aligned} |v'h| &\le 1 - |vh| \\ &= 1 - (\sqrt{3} - |dv|) \qquad (4) \\ &= 1 - \sqrt{3} + \sqrt{|df|^2 + |fv|^2 - 2|df||fv|\cos\theta)} \\ &\le 1 - \sqrt{3} + \sqrt{2 - 2\cos\theta}. \qquad (5) \end{aligned}$$

By triangle inequality,

$$|dh| \le |dv| + |vh| = \sqrt{3}. \qquad (6)$$

Using (5), we have

$$\begin{aligned} |bh|^2 &= |bv'|^2 + |v'h|^2 - 2|bv'||v'h|\cos\angle bv'v \\ &\le 1 + (1 - \sqrt{3} + \sqrt{2 - 2\cos\theta})^2 \\ &\quad -2(1 - \sqrt{3} + \sqrt{2 - 2\cos\theta})\cos(240° - \theta) \\ &= 1 + (1 - \sqrt{3} + \sqrt{2 - 2\cos\theta})^2 \\ &\quad +2(1 - \sqrt{3} + \sqrt{2 - 2\cos\theta})\sin(\theta + 30°) \\ &= 1 + (1 - \sqrt{3} + 2\sin\frac{\theta}{2})^2 \\ &\quad +2(1 - \sqrt{3} + 2\sin\frac{\theta}{2})\sin(\theta + 30°) = G(\theta). \end{aligned}$$

Set $H(\theta) = G(\theta) - (\sqrt{3})^2$.

$$H(\theta) = 1 + (1 - \sqrt{3} + 2\sin\frac{\theta}{2})^2 + 2(1 - \sqrt{3} + 2\sin\frac{\theta}{2})\sin(\theta + 30°) - 3$$

$$= \quad -2 + (1 - \sqrt{3} + 2\sin\frac{\theta}{2})^2 + 2(1 - \sqrt{3} + 2\sin\frac{\theta}{2})\sin(\theta + 30^\circ).$$

We will show that $H(\theta) \le 0$ for $\theta \in [90^\circ, 120^\circ]$. Then, combined with (6), h is certainly a correct choice.

$$\begin{aligned}
\frac{dH}{d\theta} &= 2(1 - \sqrt{3} + 2\sin\frac{\theta}{2})\cos\frac{\theta}{2} + 2(1 - \sqrt{3} + 2\sin\frac{\theta}{2})\cos(\theta + 30^\circ) \\
&\quad +2\sin(\theta + 30^\circ)\cos\frac{\theta}{2}, \\
\frac{d^2H}{d\theta^2} &= (\sqrt{3} - 1)\sin\frac{\theta}{2} + 2(\sqrt{3} - 1)\sin(\theta + 30^\circ) + 2\cos\theta \\
&\quad +5\cos(\frac{3\theta}{2} + 30^\circ) - \cos\frac{\theta}{2}\cos(\theta + 30^\circ), \\
\frac{d^3H}{d\theta^3} &= \frac{\sqrt{3} - 1}{2}\cos\frac{\theta}{2} + 2(\sqrt{3} - 1)\cos(\theta + 30^\circ) - 2\sin\theta \\
&\quad -7\sin(\frac{3\theta}{2} + 30^\circ) + \frac{1}{2}\cos\frac{\theta}{2}\sin(\theta + 30^\circ), \\
\frac{d^4H}{d\theta^4} &= -\frac{\sqrt{3} - 1}{4}\sin\frac{\theta}{2} - 2(\sqrt{3} - 1)\sin(\theta + 30^\circ) - 2\cos\theta \\
&\quad -\frac{21}{2}\cos(\frac{3\theta}{2} + 30^\circ) - \frac{1}{4}\sin\frac{\theta}{2}\sin(\theta + 30^\circ) + \frac{1}{2}\cos\frac{\theta}{2}\cos(\theta + 30^\circ).
\end{aligned}$$

If $90^\circ \le \theta \le 120^\circ$, then $-\frac{\sqrt{3}}{2} \ge \cos(\frac{3\theta}{2} + 30^\circ) \ge -1$, so it is easy to see that

$$\frac{d^4H}{d\theta^4}(\theta) > 0.$$

This means that $\frac{d^3H}{d\theta^3}(\theta)$ is strictly an increasing function on $[90^\circ, 120^\circ]$. By

$$\frac{d^3H}{d\theta^3}(90^\circ) < 0 \quad \text{and} \quad \frac{d^3H}{d\theta^3}(120^\circ) > 0,$$

we know that $\frac{d^3H}{d\theta^3}(\theta) = 0$ has a unique solution, say $\theta_0 \in (90^\circ, 120^\circ)$. $\frac{d^2H}{d\theta^2}(\theta)$ is decreasing on $(90^\circ, \theta_0)$, and is increasing on $(\theta_0, 120^\circ)$. Therefore, the maximum value of $\frac{d^2H}{d\theta^2}(\theta)$ on $[90^\circ, 120^\circ]$ should be either $\theta = 90^\circ$ or $\theta = 120^\circ$, i.e.,

$$\frac{d^2H}{d\theta^2}(\theta) \le \max\{\frac{d^2H}{d\theta^2}(90^\circ), \frac{d^2H}{d\theta^2}(120^\circ)\} < 0 \quad \text{for} \quad \theta \in [90^\circ, 120^\circ].$$

So, $\frac{dH}{d\theta}(\theta)$ is strictly decreasing on $[90^\circ, 120^\circ]$, and then we have for $\theta \in [90^\circ, 120^\circ]$,

$$\frac{dH}{d\theta}(\theta) \ge \frac{dH}{d\theta}(120^\circ) > 0.$$

Now, we know that $H(\theta)$ is a strictly increasing continuous function on $[90^\circ, 120^\circ]$. Therefore,

$$H(\theta) \le H(120^\circ) = 0.$$

Case 2. Some edges below v have length greater than 1. Let u be a Steiner point which is a child of v and has degree 3, x and y the two terminals connected to u.

Without loss of generality, suppose that $|ux| \le |uy|$, ux and uy have l and k Steiner points (both including u), respectively. Let z be the point on uy such that $|uz| = l$. Then, we can assume that zy contains $k - l$ Steiner points (including z), and ux and uz contain totally $2(l-1)$ Steiner points (not including u and z). We directly connect x and z and equally insert $\lceil 1.155l \rceil - 1$ Steiner points into xz. Then, the length of each edge on xz is

$$\frac{2l}{\lceil 1.115l \rceil} \le \sqrt{3}.$$

After that, we still have $2(l-1) - (\lceil 1.115l \rceil - 1) = 2l - 1 - \lceil 1.115l \rceil = \lfloor 0.885l \rfloor - 1$ Steiner points which can be used to equally break ux into smaller edges. Then, by inserting $\lfloor 0.885l \rfloor - 1$ Steiner points into ux, each edge on ux has length at most

$$\frac{l}{\lfloor 0.885l \rfloor} \le \sqrt{3} \quad \text{if } l \ge 3.$$

By this operation, u is changed into a vertex of degree 2 in the new tree, then we can continue the process with $n-1$ terminals in $P \cup \{u\} \setminus \{x, y\}$.

Now we turn into the situation when $l \le 2$. If $k > l$, we directly connect x and y and insert $k-1$ Steiner points into xy, then u becomes a vertex of degree 2 and each edge on xy is at most $\frac{5}{3} < \sqrt{3}$.

Next, we assume that $k = l$. Let m be the number of Steiner points on vu (not including v).

(1) $l = 2$. If $\angle xuy \le 120°$, $|xy| \le 2\sqrt{3}$, we can connect x and y by inserting a unique Steiner point to break xy into two pieces of length at most $\sqrt{3}$, then u is changed into a vertex of degree 2. If $\angle xuy > 120°$, then one of $\angle xuv$ and $\angle yuv$ is less than $120°$. Assume $\angle xuv < 120°$. Then, $|vx| \le m + \sqrt{3}$. We directly connect x and v and select a point z in xv such that $|vz| = 1$, and then insert $m-1$ Steiner points into xz to break it into equally pieces, and connect x and y and insert two Steiner points to break xy into equally pieces. Thus, z becomes a vertex of degree 2 and each edge below z has length at most $\sqrt{3}$ (note that u is no longer in the new tree). We can now continue the process with $n-1$ terminals $(P \cup \{z\}) \setminus \{x, y\}$.

(2) $l = 1$, i.e., $|ux| \le 1$ and $|uy| \le 1$. In this case, $m \ge 2$ (the case $m = 1$ has been discussed in Case 1). If $\angle xvy \le 120°$, then we connect x and y directly. If $\angle xuy > 120°$, then we can assume $\angle xuv < 120°$. We (1) directly connect x and v and select a point z in xv such that $|vz| = 1$, and then insert $m-2$ Steiner points into xz to break it into equally pieces, and (2) connect x and y and insert a Steiner point to break xy into two pieces. We can continue the process with $n-1$ terminals $(P \cup \{z\}) \setminus \{x, y\}$. □

The algorithm for finding the approximation of an optimal 3-restricted Steiner tree is the same as that of [11]. It uses the notion of *hypergraph*, defined as $H = (V, F)$, where V is a set of vertices and a F is the set of edges, which is an arbitrary family of subsets of V. A *weighted hypergraph* $H = (V, F, w)$ is a hypergraph such that each edge e in F has a weight $w(e)$. An r-*hypergraph* $H_r(V, F, w)$ is a weighted hypergraph, each edge having cardinality at most r.

The following theorem, introduced in [7], proves the existence of a randomized algorithm for computing a minimum spanning tree for a weighted 3-hypergraph:

Theorem 2 *There exists with probability at least* 0.5 *a randomized algorithm for the minimum spanning tree problem for* 3*-hypergraphs, running in* $poly(n, w_{max})$ *time, where* n *is the number of nodes in the hypergraph and* w_{max} *is the largest weight of edges in the hypergraph.*

We construct a weighted 3-hypergraph $H_3(V, F, w)$ from the set P of terminals. Here the vertex set for the hypergraph, $V = P$, and and the edge set $F = \{(a, b)|a \in P \text{ and } b \in P\} \cup \{(a, b, c)|a \in P \text{ and } b \in P \text{ and } c \in P\}$. To obtain the weight of each edge in F, we need to know B, the length of the longest edge in an optimal solution for BSTP. It is hard to find the exact value of B. However, we can find an approximate value, B', that is at most $(1 + \epsilon)B$ for any ϵ, in time $poly(n, \epsilon)$, as illustrated in steps 1 and 2 in the algorithm listed below. Interested readers can find more details for determining B' in [11].

Theorems 1 and 2 prove the existence of the $\sqrt{3} + \epsilon$ approximation algorithm and its performance:

Theorem 3 *For any given* $\epsilon > 0$*, there exists with probability at least* 0.5 *a randomized algorithm that computes a Steiner tree with* n *terminals and at most* k *Steiner points such that the length of the longest edge in the approximated tree is at most* $\sqrt{3} + \epsilon$ *multiplied with the length of the longest edge in the optimum tree. The algorithm running time is* $\frac{1}{\epsilon} \times poly(n, k)$.

Next we present the Bottleneck Steiner tree approximation algorithm:

Input: A set P of n terminals in the Euclidean plane, an integer k and a positive number ϵ.
Output: A 3-restricted Steiner tree T with at most k Steiner points.
Step 1. Call the ratio-2 approximation algorithm for BSTP from [10] and obtain a number X as the length of the longest edge.
Step 2. For $B = \frac{X}{2}, \frac{X}{2}(1 + \epsilon), \frac{X}{2}(1 + 2\epsilon), \cdots, \frac{X}{2}(1 + \epsilon \times \lceil\frac{1}{\epsilon}\rceil)$ **do**:
Step 2.1. Construct a weighted hypergraph $H_3(V, F, w)$ ([11]).

Step 2.2. Call the randomized algorithm from [7] to compute a minimum spanning tree T for $H_3(V, F, w)$.
Step 3. Consider the solution T' of the smallest B such that $w(T') \leq k$.
Step 4. Replace every edge f of the minimum spanning tree T' on $H_3(V, F, w)$ with a Steiner tree with $w(f)$ Steiner points such that the maximum length of each edge in the tree is at most B, and output the obtained tree.

Thee work in [11] analyses the polynomial running time of this algorithm.

4 Conclusions

Efficient energy management is an important issue in the design of wireless networks with battery-powered nodes. For applications where replacing drained batteries is not feasible, extending the network lifetime by prolonging the network connectivity, may signify the success or failure of a mission. Existing methods for improving energy consumption are based on computing optimal flows, transmission scheduling or power-aware routing.

In this paper we present an approximation algorithm for the NP-complete bottleneck Steiner tree problem in the Euclidean plane, and we prove a $\sqrt{3} + \epsilon$ performance ratio. The output of the polynomial-time algorithm consists of a Steiner tree with n fixed terminal nodes and up to k Steiner nodes such that the length of the longest edge in the tree is minimized. This algorithm helps designing power-efficient wireless networks by computing the location of maximum k additional communication relay nodes so that the resulting spanning tree of at most $n + k$ nodes minimizes the length of the longest edge. Thus, the transmission power for the longest link is minimized, and, as a result, the time until the first node drains its battery and stops transmitting, breaking the network connectivity, is prolonged.

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