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A PRACTICAL TEACHING OF MATHEMATICS
IN OUR
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A Practical Teaching of Mathematics
in Secondary Schools.

"Scientific principles are the back-bone of knowledge of teaching, but concrete exercises are its flesh and blood" - Thorndike's "Principles of Teaching."

The somewhat chaotic condition of the mathematical texts and therefore instruction in secondary schools, prominent at present, has appealed strongly to the writer.

A condition produced largely by the inefficiency of the present instruction, and the severe, tho just criticisms, to which the whole field of secondary mathematics has recently been subjected.

Among the more prominent features may be mentioned the absence of correlation or continuity between the grade and high school mathematics; the unrelated subject matter; the abstract manipulation of symbols and formulae; detrimental to interest, power, or growth; the lack of accuracy, efficiency and power of thought - effective tools for future service - manifested by high school products; the ancient and impractical subject matter; the basing of our teaching upon traditions of the past; and especially the insistent demand from the business world for more practical results from this field. Our own experience as student and teacher has confirmed many of the criticisms offered. Realizing furthermore, that there must be a "better way," this paper has been written in expectation and hope that a thought or suggestion might find lodgment in some constructive brain and bear practical and tangible fruit.

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The aim has been to confine the theme to one phase - the content of the mathematical instruction. The argument if it possesses any merit is debtor to the (a), Department of Education of the University of Minnesota, (b), Pedagogical Department of University of Chicago, (c), Report of the Committee on Entrance Requirements for Secondary Schools in the Northwest, (d), "First year Mathematics" by Prof. Slaughter, and (e), various other sources.

The present age, commercial, industrial and cosmopolitan, has been enabled, by these same tokens, to place the public high school within the reach of practically all of the common people.

I start out, then, with this premise:

That the high school of today is the school for the common people.

And therefore, as the common people are the practical work-a-day people, the chief function of this school is to prepare the child of such parentage for life's activities.

That life is real - that it is a compendium of activities - that to many it is a struggle for existence, needs no proof nor comment. Ethical and intellectual attributes may appeal strongly to us. The soul and the mind may yearn for the higher ideals, yet man is fashioned out of human clay and the ever present human wants must be satisfied. The bread-and-butter proposition may be a sordid one, but it is a very practical one. Therefore our schools, our teaching, curricula, etc. etc., should be practical, first, foremost, lastly, and all the time. There must be a direct and consistent relation existing between the subject matter taught in our high school and life's activities.

The fundamentals, especially, must be practical. Mathematics - the foundation upon which the whole school curriculum rests, should be as rock-ribbed, practically, as Gibraltar. For years it has been considered so. So much so, that a self-assured, satisfied condition has come to prevail concerning this subject, that has fostered an inertia, conducive to dry-rot, which threatens to undermine the whole fabric. Our self-assurance has

been so great concerning mathematical instruction, its value, utility, etc. etc., that not until recently has any one had the temerity to question its value, much less its right, to a place in our public school system. Happily, however, the last decade, while iconoclastic, perhaps, has vigorously assailed the mathematical instruction from a practical stand-point.

First of all - What should be the aims of our mathematical instruction? From the Report as given by the Committee of the North Central Ass'n thru the Committee on Mathematical Entrance Requirements, the following aims are cited:-

- (a), To gain knowledge of an important field of facts and methods of utmost use to man-kind;
- (b), Power to apply this knowledge in practical and theoretical investigations;
- (c), To develop and strengthen power of perceiving exact relations and to induce and make permanent the habits of casual thinking.

Educators, in general, subscribe to these aims. May not, however, the content of the subjects taught, be so artificial in character, so divorced from the realities of life, that the preceding aims are diverted, submerged, and often lost sight of altogether?

Instead of concrete applications of principles, there abounds a preponderance of manipulations of abstract symbols. The problems are artificial in character. The abstruse reasoning and theoretical work required of immature pupils, not only has little practical value, but discourages and overwhelms the pupil. Again, the subject-matter is unconnected and unrelated - (1) From year to year - e.g., Elementary Algebra, Plane Geometry, (no mathematics the third year) and Higher Algebra and Solid Geometry the fourth year. (2) The various phases of any subject pursued any particular year are diverse and unrelated. Any standard text in elementary algebra will prove the assertion.

• Report of Committee on Mathematical Entrance Requirements, North Central Association of Colleges.

Again, the present arrangement whereby Elementary Algebra makes a distinct and irreconcilable break in the mathematical chain, is greatly to be deplored.

The weaknesses of the present algebra teaching may be briefly stated as follows:

* 1. Young pupils required to take up difficult abstract algebraic subjects not practical.

2. An entire year (an immature one) given to abstract work unrelated to the life activities of boys and girls.

3. It separates and isolates various mathematical subjects.

4. It is isolated from concrete subjects that make realities of life.

5. Many obsolete, impractical subjects receive an undue prominence.

The present teaching is, therefore, discouraging, unscientific, non-pedagogical and non-utilitarian.

What, then, should constitute a practical course in high school mathematics? Briefly stated as follows:-

- I. (a) Practical Arithmetic - two or three months;
(b) Algebraic Arithmetic;
(c) Arithmetical Algebra.
- II. Mensuration and Plane and Solid Geometry.
- III. Plane Trigonometry and Surveying.
- IV. Commercial Arithmetic.

There exists a break in our mathematical instruction between the grades and the high school. From 8th grade arithmetic, the pupil jumps abruptly to abstract algebra. It is illogical and inconsistent.

To avoid the break in the mathematical chain and to make the work of more practical value, some time, say two or three months, should be devoted at the beginning of the first year -

1, to collecting, evening up, systematizing and intensifying the knowledge of entering pupils;

* Prof. Meyer - First Year Mathematics - University of Chicago Press.

2, to strengthening the current labial and lingual exercises - the overspiralized exercises, simplified and clothed in juvenile vocabulary have sapped the problems of their vitality.

3, to teaching the language of arithmetical science. Law, medicine, theology, commerce, etc., - each has a language of its own that adds clearness, diction, force and eclat. So should it be with mathematics. There is a language of the science of arithmetic, clear, distinct and forcible, but foreign to the mind of the high school student. It should be taught.

4, to a critical review of entire field of grade mathematics.

5, to the equation - its uses, value, convenience and possibilities should be taught.

6, to a careful, systematic, arithmetical treatment of topics preliminary to their algebraic or geometric consideration.

"From every view-point - from the view-point of pedagogics, - culture, economy, efficiency, or utility, the demand is insistent, this important step in elementary teaching must be made and made at the proper psychological time."

Lastly, algebra - not the traditionally classical algebra, but abridged, enriched, and developed from the view-point of the beginner.

But "Qui Bono"? Many affirm that algebra is not taught for its practical but for its cultural or ethical value. - That it has only disciplinary value. To such the answer can be made that the utilitarian can be intellectual and ethical, and the ethical and cultural should be practical.

Napoleon, the most practical of men, said: -
L'avancement, le perfectionnement des mathematiques sont lies a la prosperite de l'Etat."

* School Review - Oct. 1907 - P/380 - Results of first year's work - Art. by Prof. Meyer.

The advancement, the perfecting of mathematics are bound up with the prosperity of the State.

Again, if not for its utility to the average citizen for (1) future mathematical work, (2) knowledge of astronomy, (3) physics, (4) mechanics, (5), science of gunnery, (6), engineering, etc., a knowledge of algebra is necessary.

Moreover, algebra is a science. It is developed from unimpeachable axioms. Its conclusions are logically developed. The student can plant each step firmly before proceeding to the next. It gives, clearness, power, vigor, and strength. It forms an indisputable continuation of arithmetic. It is generalized arithmetic. It has immense practical value in its ability to abridge and simplify computations. Its teaching is pedagogical for it proceeds from the special to the general.

But the subject matter must be abridged - abridged of abstract notions - abridged by the omission of all those elements that perplex and exhaust without adding anything to the sum total of the child's intellectual or practical knowledge - viz., jagged fractions, complicated and compounded factoring, repeated divisions for highest common factor, signs of aggregation, difficult combinations of indices, more than three unknowns, theory of exponents, cube root, complicated radicals, rationalizing denominators, more than one way of completing square in quadratics, etc. etc.

Enriched by the aid of inductive arithmetic - by the use of the equation and its possibilities - by practical problems of a constructional or mensurational nature, such as the drawing of squares, perpendiculars, triangles, parallelograms, etc., under specified conditions. Enriched by concrete problems based upon the child's experience drawn from arithmetic, geometry, physics, mechanics, book-keeping, commerce, etc. Enriched by proceeding systematically, developing conclusions logically.

By enabling the student to plant each step firmly before proceeding to next he gains clearness, confidence and strength. Thus about an algebraic center radiate and correlate an extensive, intensive and varied body of mathematical ideas.

Progress would be accelerated by begetting greater clearness and stability of thinking. Such work may not be considered pure algebra, but its influence would be vitalizing and practical. Algebra is thus enriched, abridged, made useful and based upon child's experience.

* To sum up: "the cardinal purpose of the first year of high school mathematics is to lay a broad foundation of mathematical concepts and elemental truths and to build solidly upon them in various directions, completing very definitely a considerable amount of practical algebra, and accomplishing meanwhile on the side a rounding out of elementary school mathematics and inducting beginners into the ideas, method and spirit of geometry."

There is a two-fold value attached to the study of geometry. (a) Its cultural value which assumes the form of logic - This may be practical but only in a secondary sense. (b) Its practical value seen chiefly in mensuration.

The chief criticisms upon our geometry instruction are as follows: (a) It is too formal or set; (b) Its content too general and meaningless to the student; and (c) Its teaching too mechanical.

For the purpose of unification and practicality, it would seem well to continue the algebraic and arithmetical elements, but instead of algebra have geometry - give direction, organization and unity to all. It would also seem profitable to at least imitate the European plan of carrying algebra and geometry side by side. The break between the two would not be so abrupt, the depending of one upon the other would be demonstrated and students would not so soon "forget their algebra."

Besides demonstrations, concrete problems should be based upon geometric principles and thinking but solvable arithmetically and algebraically. The practical phases to be emphasized are the constructional, mensurational and metrical. As to scope, Plane and Solid should be completed in a year's time.

In geometry, "the more haste, the less speed" is very true. At the beginning rapidity should be avoided. Slowly but accurately attacked according to a logical

* Paper - First year Mathematics - Prof. Meyer in School Review - Oct. 1907.

procedure, the so-called insurmountable obstacles will yield to the assault. A method of attack which has proven effective in our immediate schools is briefly stated as follows:

1. The figure is sketched in rough, estimates, etc. are made;
2. The figure is then carefully drawn and appropriate parts are accurately measured;
3. All possible inferences drawn as to conclusions that follow from conforming to given conditions;
4. The elimination of the non-essential;
5. The simplifying and systematizing of the essential;
6. The correct enunciation of the principle to be established;
7. A deductive proof in standard form.

The foregoing method, modified or abridged to meet the class-room requirements has been followed in two schools under the writer's supervision. A somewhat careful and systematic inspection of the work, based upon the practical results accruing from such a plan of procedure convinces that such a method trains the sense of number, leads to a perfect understanding of conditions, develops power to estimate and approximate, shows dependence of form and magnitude upon imposed limitations, and trains the faculties of observation, inference and judgment.

Furthermore, it develops a logical line of reasoning towards required result, gives training in ways of discovering new geometrical truths, as well as in using reasoning to prove or disprove suspected truths. Incidentally the scheme divorces the pupil from the text-book and develops logic, common sense and technique. In a word, the plan reaches the virtues of geometric education.

The mode of conducting the recitation must of necessity be an elastic one, more or less informal, combining the essentials of the experimental, Socratic, laboratory, and the old style of the set recitation.

Here as elsewhere, the teacher must be a leader,

capable of guiding and must be in full possession of the sound theory of mathematics and the pedagogy of the subject. The writer has observed the workings of the plan and believes it to be logical, economical, practical and most productive of interest and profit to the learner.

The utility of the preceding two years' work develops an interest which leads to a demand for applied mathematics.

This demand should be met in the normal high school by an elective course of a year's duration in (a) Plane Trigonometry, and (b) Plane Surveying. Is it not a sad commentary upon our public school system that such instruction is in no wise provided for? Every farmer-boy, to say nothing of the prospective engineer, contractor or artisan, should receive such instruction, elementary to his future vocation, in our public schools.

The fourth year should be devoted to a substantial, solid, well-proportioned course in industrial arithmetic. The field to be broad, deep and comprehensive - the pulsing arteries of commerce should vitalize it. The experience and efficiency of shop, countinghouse and mercantile office should dignify it. The aim to be accuracy, rapidity, self-reliance, efficiency. The method of instruction to be practical, direct, precise and as insistent as the business world. Eliminated of all frills or fads. As practical as the industrial age can make it. The subject matter to be concrete and utilitarian.

No argument seems necessary for the establishment of such a course. The pupils are three or four years more mature than when arithmetic was last pursued. The principles and customs of the commercial world are better understood. In a few weeks these same young men and women will be ground out by the graduating machine - thrown upon an unfeeling world - the law of the "survival of the fittest" seizes them. Does not the "bread-and-butter" proposition demand that they receive training that will enable them to survive until they can at least re-adjust themselves to their changed conditions? Such a course will supply efficient tools and eventually spell Success for them.

What should be taught?

1. The fundamentals - Drill until they become reflex
2. The underlying principles - developed naturally and in a business-like manner.
3. Practical Countinghouse methods - not antique, ancient and obsolete, but 20th century commercial methods.
4. Mental work - Train the head as well as the hand.

Mental arithmetic serves a two-fold purpose. It has value as a medium of acquiring knowledge. It insists upon a directness of approach and an economy of space. But its chief value lies in close, clear, concise, rapid work. The ability to seize the salient facts and to state deductions in a concise logical manner. Logical analysis, passing from known to unknown, accuracy, the power of holding the essential factors in grasp and disregarding the non-essential - of weighing, reasoning, judging - it is these which constitute the value of this branch. Mental arithmetic is like the ozone of our Northland to the rugged pioneer - it clarifies; it vivifies.

5. Concrete business problems - addition, grouping, short methods, business forms, trade and bank discounts, freights, tariffs, the various forms of paper, etc.

6. Practical measurements - invade the shop of the mechanic, the office of the contractor, the consulting desk of the architect, and the inner sanctum of the business man and take those actual problems which they are called upon to master.

7. The various and varied commercial transactions, multitudinous in name and character, but which sound the whole gamut of successful business experience.

8. The equation - its uses and possibilities.

9. Mensuration, practical, accurate, and exhaustive in its scope.

10. Graphing - So common today in the business and scientific world that no course should be complete without its various forms, uses, adaptability and effectiveness being thoroughly understood.

11. The method of attack - not the least attention should be given to this important branch of work.

There is a skill, a science of computation, beyond the comprehension of the uninitiated. There must be (1) a proper understanding of premises and conditions, (2) a foundation must be established upon which the analysis rests, (3) Emphasize that one must proceed from the known to unity - from unity to the unknown, (4) the elimination of all unnecessary work, (5) checking work carefully for (a) accuracy and (b) certainty of student.

These, we believe, will constitute practical mathematics for high schools.

To sum up: -

1. Practical arithmetic leading up to a combination of algebraic arithmetic and arithmetical algebra the first year.

2. Mensuration, Plane and Solid Geometry the second year.

3. An optional course in Plane Trigonometry and Plane Surveying the third year.

- (4), A year's work in Industro-Commercial arithmetic the fourth year. All related to the life activities and taught in a practical pedagogical way.

There may be no royal road to success, but there should be a direct highway leading from the class-room to the concrete realities of life. There may be no "open sesame" to the intricacies of life's mazy and devious manipulations, but there should be practical instruction of sufficient scope and strength to enable the high school graduate to walk uprightly, manfully thru life's highways and by-ways, conscious of his powers, knowing his limitations, beholding his rights, beholden to his obligations, not led astray by every false light or will-o-the-wisp abstraction or theory. It is the province, nay, the duty of our high schools to give such instruction.

The chief essential must ever be the teacher. Without a practical master-mind to guide, direct, lead and counsel, the work must be inefficient and barren of results.

In conclusion may we say our "plaint" has been an endeavor:

1. To correlate all branches.
2. To lay more stress upon fundamentals.
3. To eliminate non-essentials.
4. To make content of more utility.
5. To make work more concrete and practical.
6. To teach thoroughly what is taught.
7. To put the child in possession of facts, processes, habits, and strength of mentality and powers of discernment, that will enable him in the tangled web of life to
 - (a) separate essential from the non-essential;
 - (b) to classify and arrange the essential;
 - (c) to gradate;
 - (d) to relate cause and effect;
 - (e) to keep alert the mental faculties.
8. To industrialize, modernize and harmonize all to meet the demands of Twentieth Century activities.

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