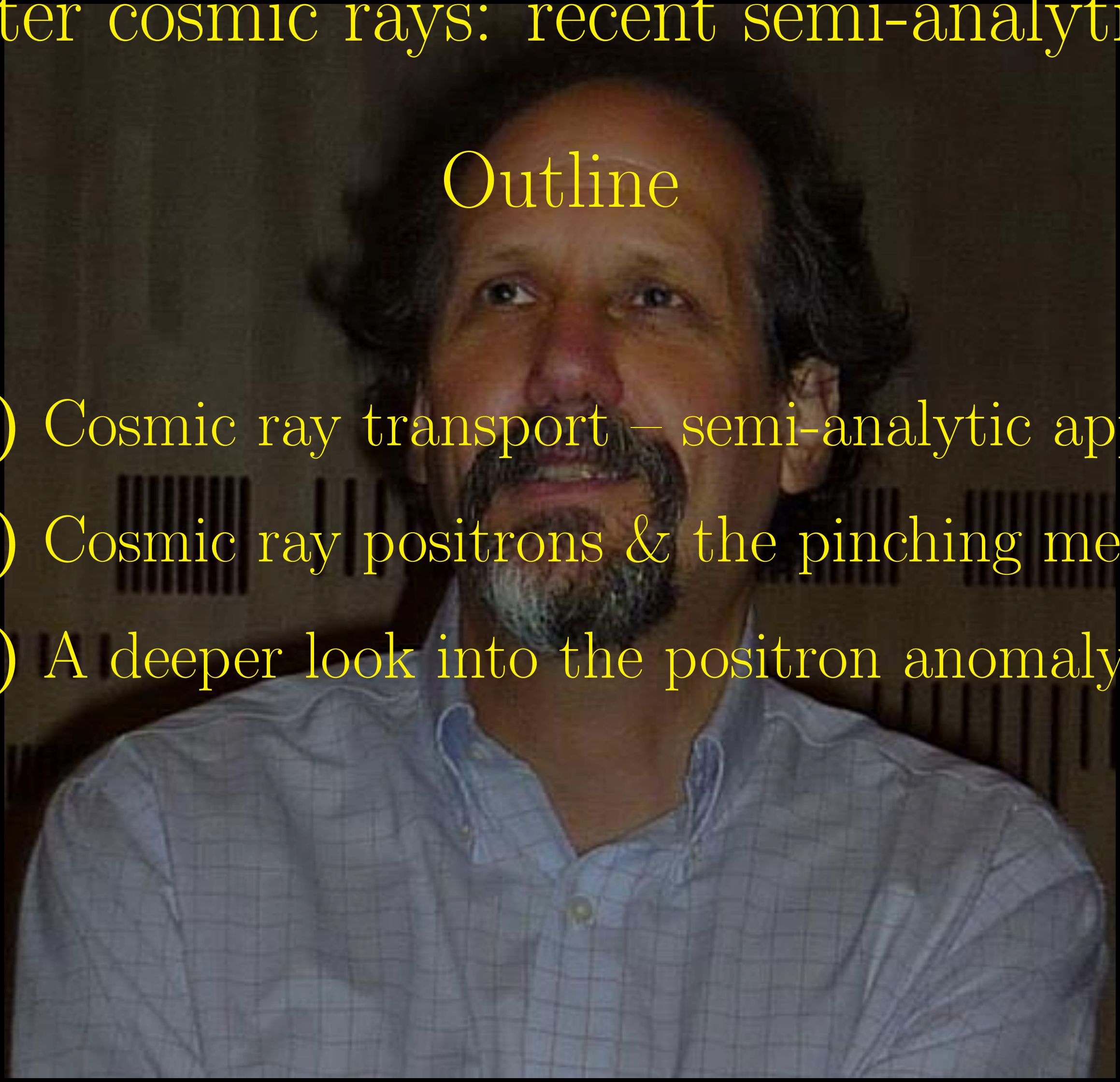
Antimatter cosmic rays: recent semi-analytic results



Outline

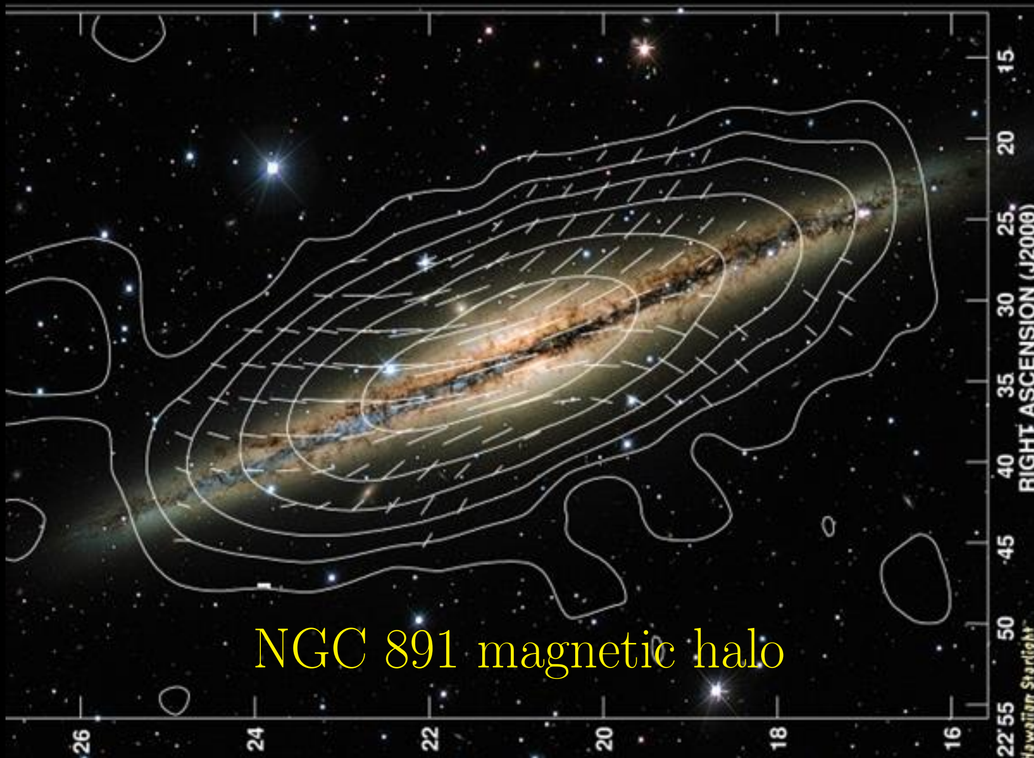
1) Cosmic ray transport – semi-analytic approach

2) Cosmic ray positrons & the pinching method

3) A deeper look into the positron anomaly

Pierre Salati – LAPTh & Université Savoie Mont Blanc

1) Cosmic ray transport – semi-analytic approach



NGC 891 magnetic halo

$$\psi = \frac{dn}{dE} = \frac{d^4N}{d^3\boldsymbol{x}\,dE}$$

$$\Phi = \frac{1}{4\pi}\,v\,\psi$$

$(\mathrm{GeV/nuc})^{-1}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$

ISM spallation

$$\dot{\psi} + \boldsymbol{\nabla}\cdot\{-K\,\boldsymbol{\nabla}\psi + \psi\boldsymbol{V}_C\} + \frac{\partial}{\partial E}\left\{b\,\psi - D_{EE}\frac{\partial\psi}{\partial E}\right\} = q - (\sigma v\, n_{\mathrm{H}})\,\psi$$

convection

E losses

$q = q_{\mathrm{acc}}, q_{\mathrm{sec}}, q_{\mathrm{DM}}$

$\boldsymbol{x}$ diffusion

$$K = K_0\,\beta\,\mathcal{R}^{\delta}$$

E diffusion

$$D_{EE} = \frac{2}{9}\,\frac{V_A^2\beta^4E^2}{K}$$

1) Cosmic ray transport – semi-analytic approach

ISM spallation

$$\dot{\psi} + \boldsymbol{\nabla}\cdot\{-K\,\boldsymbol{\nabla}\psi + \psi \boldsymbol{V}_C\} + \frac{\partial}{\partial E}\left\{ b\,\psi - D_{EE}\frac{\partial \psi}{\partial E}\right\} = q - (\sigma v\, n_{\mathrm{H}})\,\psi$$

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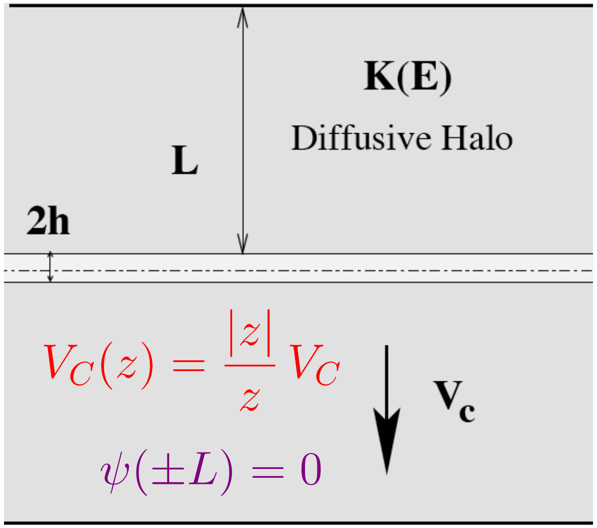
Infinite slab and 1D vertical CR model

A few reasonable assumptions

- Stationary solution $\dot{\psi} = 0$
- CR density described by $\psi(z, E)$
- Spallation reactions confined in the thin disk
- E diffusion (DR) also confined in the thin disk
- Infinitely thin disk with $2\,h\,\delta(z)$ factor



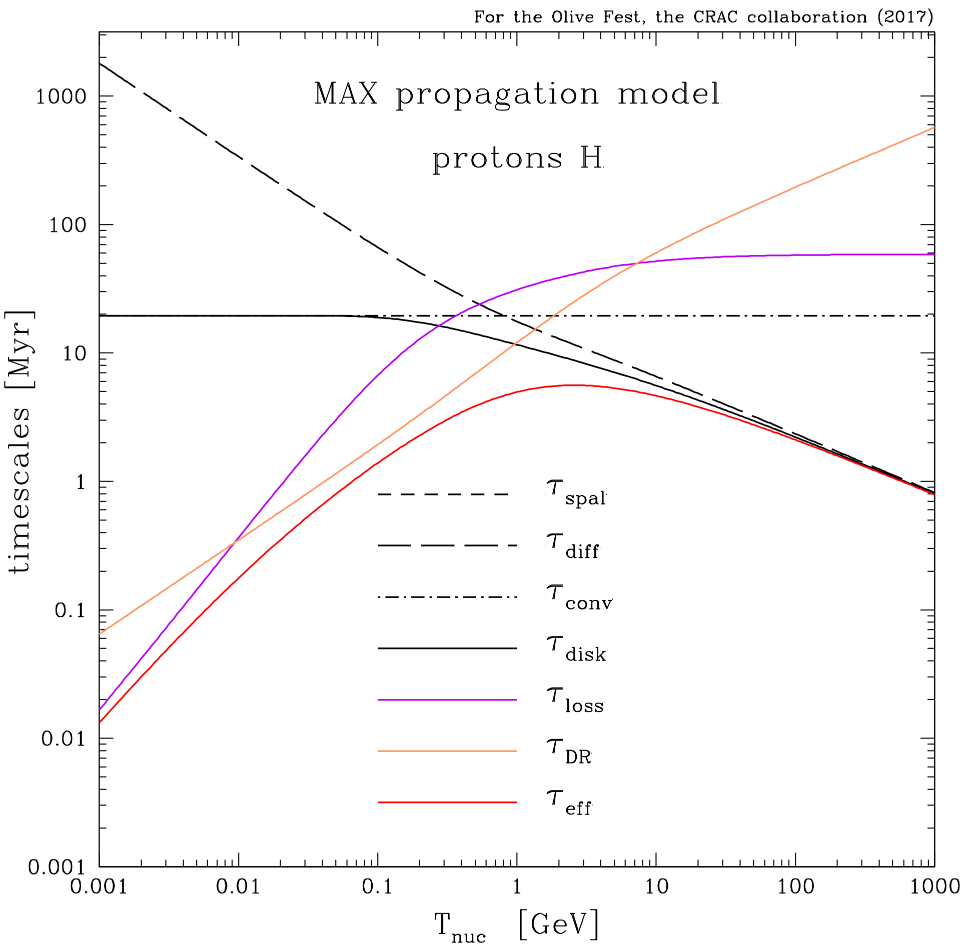
$$\frac{\partial}{\partial z}\left\{-K\frac{\partial \psi}{\partial z} + \psi V_C(z)\right\} + \frac{\partial}{\partial E}\left\{b\,\psi - 2\,h\,\delta(z) D_{EE}\frac{\partial \psi}{\partial E}\right\} = q - 2\,h\,\delta(z)\frac{\psi}{\tau_{\rm spal}}$$

For CR nuclei and antinuclei ($\bar{\rm p}$,$\bar{\rm d}$)

- E losses confined in the thin disk
- $q_{\rm acc}$ (primary nuclei) & $q_{\rm sec}$ (secondaries) as well

$$q_{\rm acc} = \frac{\psi}{\tau_{\rm disk}} + \frac{d}{dE}\left\{b\,\psi - D_{EE}\frac{d\,\psi}{dE}\right\} \quad \text{with} \quad \psi \equiv \psi(z=0, E)$$

Typical primary CR nuclei timescales



- Diffusion, convection & spallations

$$\frac{1}{\tau_{\rm disk}} = \frac{1/\tau_{\rm conv}}{1 - \exp(-\tau_{\rm diff}/\tau_{\rm conv})} + \frac{1}{\tau_{\rm spal}}$$

$$\tau_{\rm diff} = \frac{hL}{K} \qquad \tau_{\rm conv} = \frac{h}{V_C} \qquad \tau_{\rm spal}^{-1} = \sigma v \, n_{\rm H}$$

- Energy losses

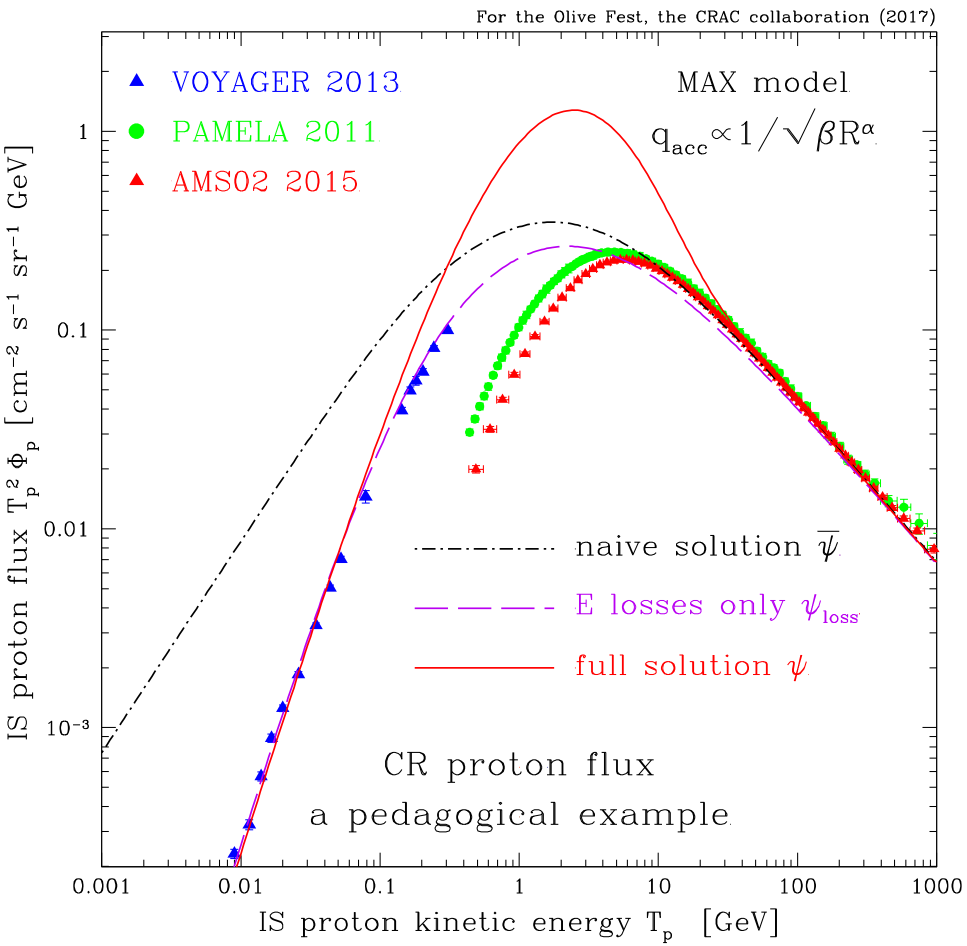
$$\tau_{\rm loss} = \frac{T}{|b|} \text{ where } T \text{ kinetic energy}$$

- Diffusive reacceleration

$$\tau_{\rm DR} = \frac{T^2}{D_{EE}}$$

$$\frac{1}{\tau_{\rm eff}} = \frac{1}{\tau_{\rm disk}} + \frac{1}{\tau_{\rm loss}} + \frac{1}{\tau_{\rm DR}}$$

CR protons as a pedagogical example



- High-energy regime – diffusion – $\tau_{\rm disk}$

$$\psi = \tau_{\rm disk} \times q_{\rm acc} \equiv \bar{\psi}$$

- Low-energy region – E losses – $\tau_{\rm loss}$

$$q_{\rm acc} = \frac{\psi_{\rm loss}}{\tau_{\rm disk}} + \frac{d(b\,\psi_{\rm loss})}{dE}$$

$$b\,\bar{\psi} = b\,\psi_{\rm loss} + \frac{d(b\,\psi_{\rm loss})}{d\tilde{t}} \quad \Longleftarrow \quad d\tilde{t} = -\,\frac{\tau_{\rm loss}}{\tau_{\rm disk}}\,\frac{dT}{T}$$

- From 0.1 to 10 GeV – E diffusion – $\tau_{\rm DR}$

$$q_{\rm acc} = \frac{\psi}{\tau_{\rm disk}} + \frac{d}{dE}\left\{ b\,\psi - D_{EE}\,\frac{d\,\psi}{dE} \right\}$$

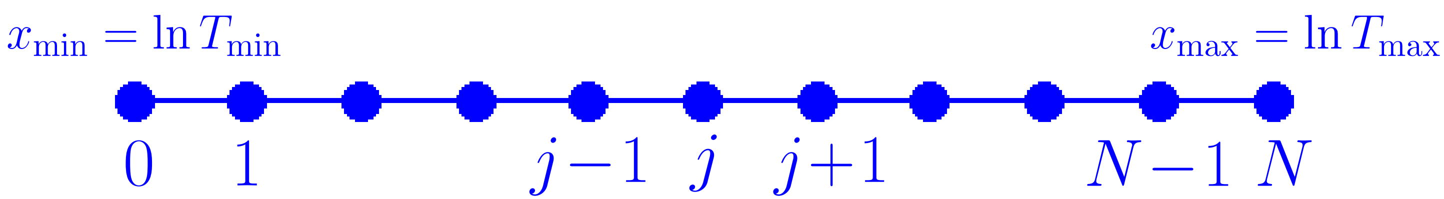
Crank-Nicholson vs Direct Inversion

The transport equation may be recast as an ODE where the variable is $x = \ln T$

$$\alpha\,\bar{\psi} = \alpha\,\psi + \frac{d}{dx}\left\{\beta\,\psi - \gamma\,\frac{d\,\psi}{dx}\right\} \quad \text{where} \quad \alpha = \frac{T}{\tau_{\rm disk}},\ \beta = \frac{-T}{\tau_{\rm loss}}\ \&\ \gamma = \frac{T}{\tau_{\rm DR}}$$

$$\Downarrow$$

Resolution on the x grid with $\Delta x = \frac{1}{N}\ln\left(T_{\rm max}/T_{\rm min}\right)$



- The equation can be discretized and put on the x grid to yield

$$\alpha_j\,\bar{\psi}_j = \alpha_j\,\psi_j + \frac{\beta_{j+1}\psi_{j+1} - \beta_{j-1}\psi_{j-1}}{2\Delta x} - \frac{\gamma_{j+1/2}}{\Delta x^2}\,\psi_{j+1} - \frac{\gamma_{j-1/2}}{\Delta x^2}\,\psi_{j-1} + \frac{\gamma_{j+1/2} + \gamma_{j-1/2}}{\Delta x^2}\,\psi_j$$

$$\Downarrow$$

$$\bar{\psi}_j = a_j\,\psi_{j-1} + b_j\,\psi_j + c_j\,\psi_{j+1} \ \text{or}\ \left[\bar{\psi}\right] = \left[M(a,b,c)\right]\left[\psi\right]$$

$$\Downarrow$$

Direct Inversion : $\left[\psi\right] = \left[M(a,b,c)\right]^{-1}\left[\bar{\psi}\right]$

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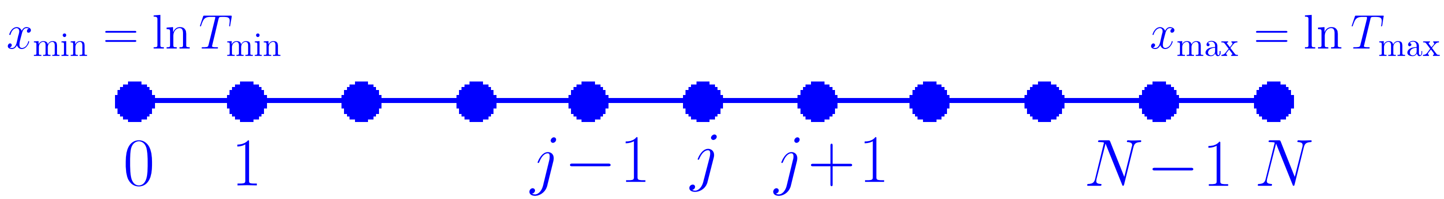
$$\Downarrow$$

Resolution on the x grid with $\Delta x = \dfrac{1}{N}\ln\left(T_{\max}/T_{\min}\right)$



- The DI method may lead to numerical errors related to the inversion of $[M(a,b,c)]$

$$\bar{\psi} = \left(\frac{d\psi}{dt}\right)_{\rm num} + \;\psi\; + \frac{1}{\alpha}\frac{d}{dx}\left\{\beta\,\psi - \gamma\,\frac{d\,\psi}{dx}\right\}$$

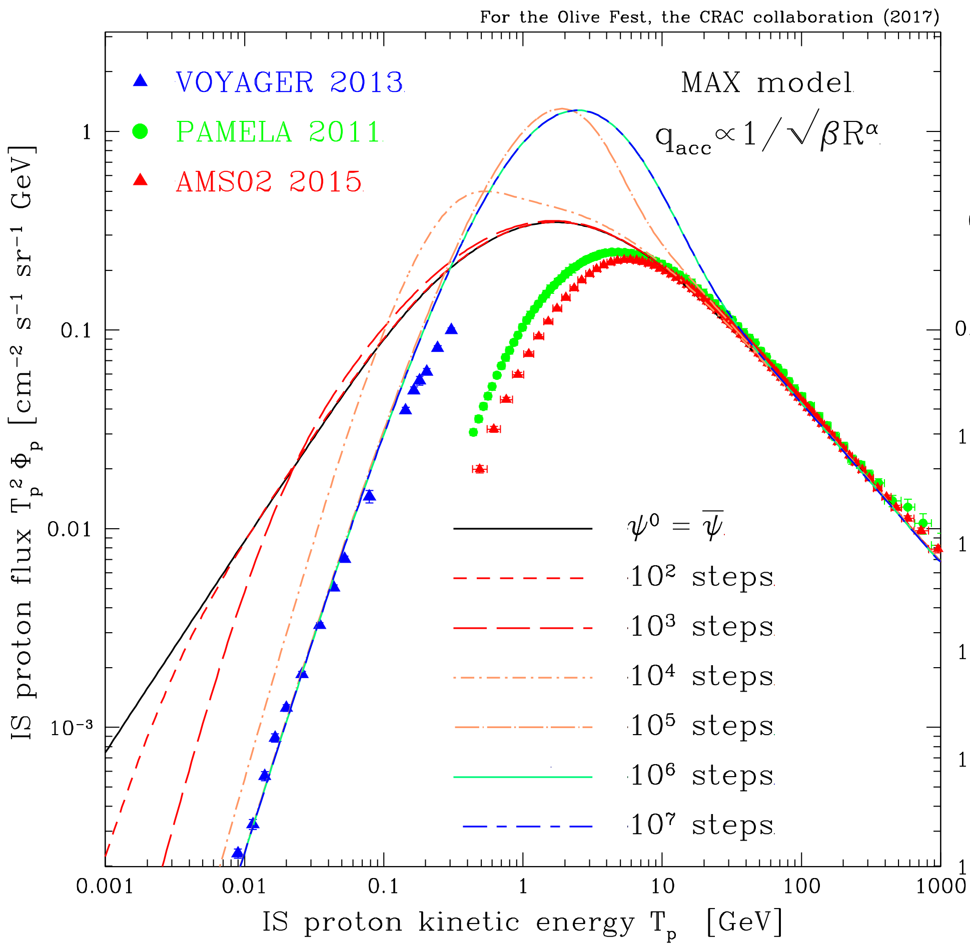
$$\Downarrow$$

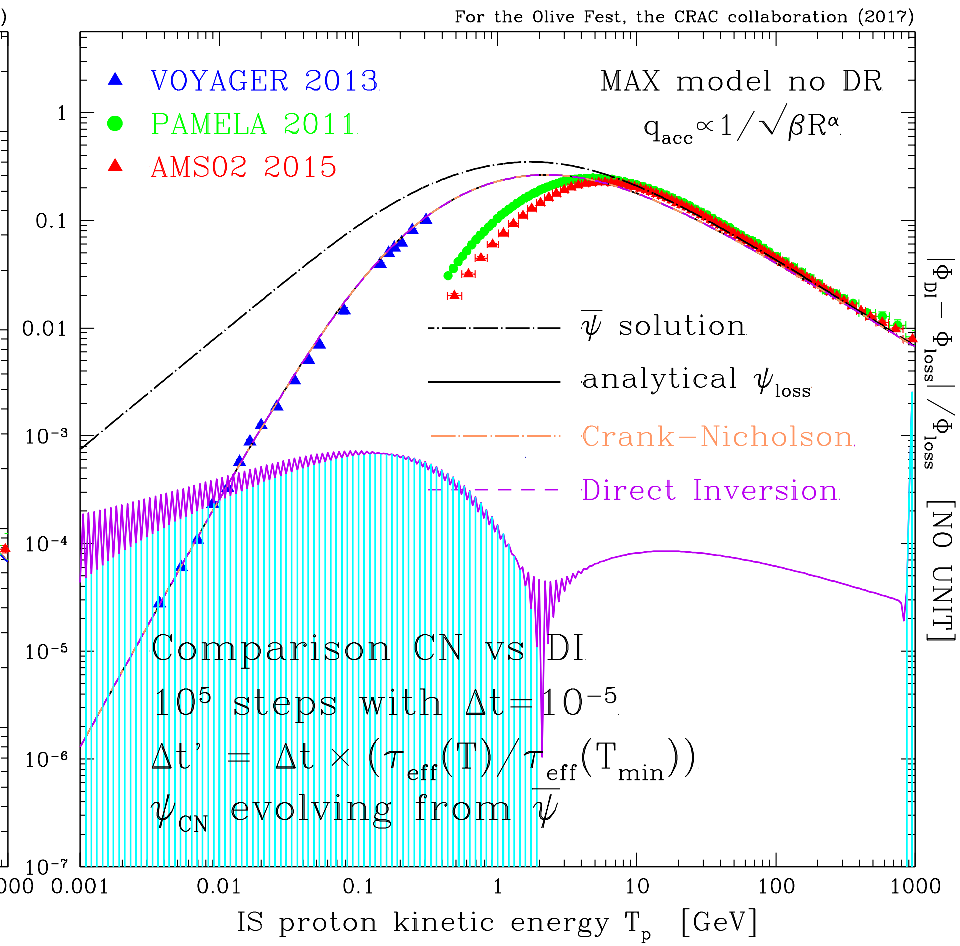
$$\dot{\psi}_{\rm num} \equiv \frac{\psi_j^{\,n+1} - \psi_j^{\,n}}{\Delta t}$$

$$\Downarrow$$

Crank-Nicholson : $\left\{1 + \dfrac{\Delta t}{2}\,[M]\right\}\left[\psi^{\,n+1}\right] = \Delta t\,\left[\bar{\psi}\,\right] + \left\{1 - \dfrac{\Delta t}{2}\,[M]\right\}\left[\psi^{\,n}\right]$

Crank-Nicholson vs Direct Inversion





CN is slow & reliable. **DI** is fast & unstable, unless suitable LEBC as $\ddot{\psi}(T_{\mathrm{min}})=0$.

Direct Inversion : $[\psi] = [M(a,b,c)]^{-1}\,[\bar{\psi}\,]$

Crank-Nicholson : $\left\{1 + \dfrac{\Delta t}{2}\,[M]\right\}[\psi^{\,n+1}] = \Delta t\,[\bar{\psi}\,] + \left\{1 - \dfrac{\Delta t}{2}\,[M]\right\}[\psi^{\,n}]$

2) Cosmic ray positrons & the pinching method

$$\nabla \cdot \left\{ -K \nabla \psi + \psi \mathbf{V}_C \right\} + \frac{\partial}{\partial E} \left\{ b\,\psi - 2\,h\,\delta(z) D_{EE} \frac{\partial \psi}{\partial E} \right\} = q_{e^+} - 2\,h\,\delta(z) \frac{\psi}{\tau_{\rm ann}}$$

The CR positron density ψ fulfills the same transport equation as for nuclei. But this time E losses are spread both in the disk and in the magnetic halo.

$$b(E) = 2\,h\,\delta(z) \left\{ b^{\rm disk} \equiv b^{\rm brem} + b^{\rm C} + b^{\rm ion} \right\} + \left\{ b^{\rm MH} \equiv b^{\rm IC} + b^{\rm S} \right\}$$

Semi-analytic approach is limited – HE regime and disk pinching

(i) – The high-energy regime and the heat equation

$$-K\,\Delta\psi + \frac{\partial}{\partial E} \left\{ (b \simeq b^{\rm MH}) \psi \right\} = q_{e^+}$$

$$\Downarrow$$

$$-\,\Delta\tilde{\psi} + \frac{\partial \tilde{\psi}}{\partial \tilde{t}} = \tilde{q} \equiv \frac{b}{K} q_{e^+} \text{ with } \tilde{\psi} = b\,\psi \text{ and } d\tilde{t} = K\,dE/b.$$

$$\Downarrow$$

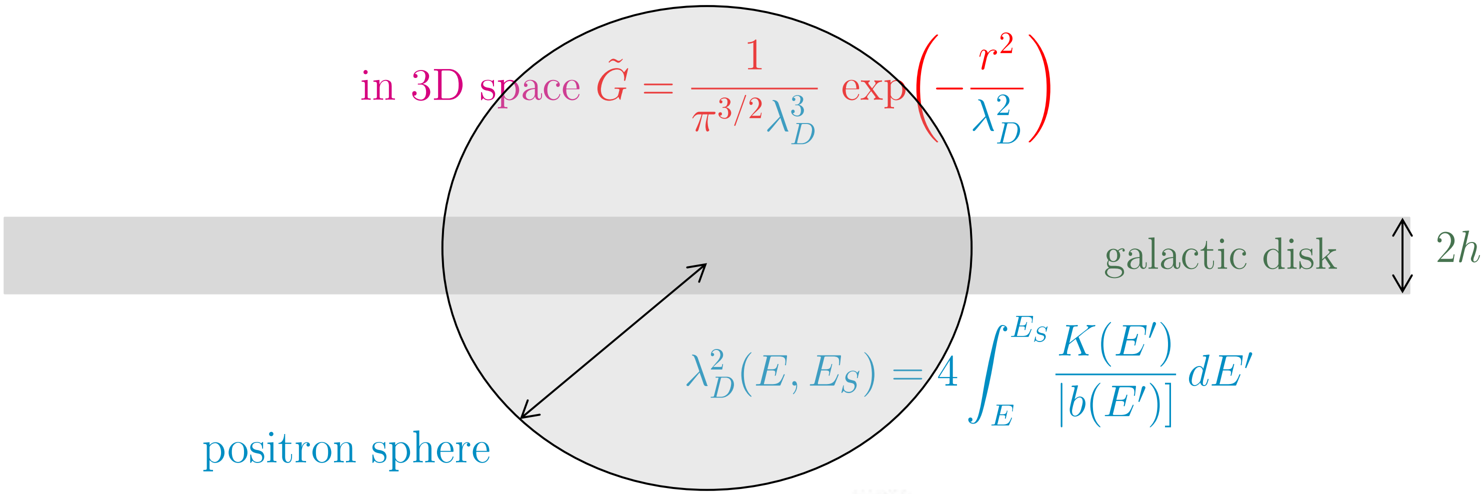
$$\psi_{e^+}(\boldsymbol{x}, E) = \int d^3\boldsymbol{x}_S \int dE_S \left\{ \frac{\tilde{G}(\boldsymbol{x}, \tilde{t}(E) \leftarrow \boldsymbol{x}_S, \tilde{t}_S(E_S))}{|b(E)|} \right\} q_{e^+}(\boldsymbol{x}_S, E_S)$$

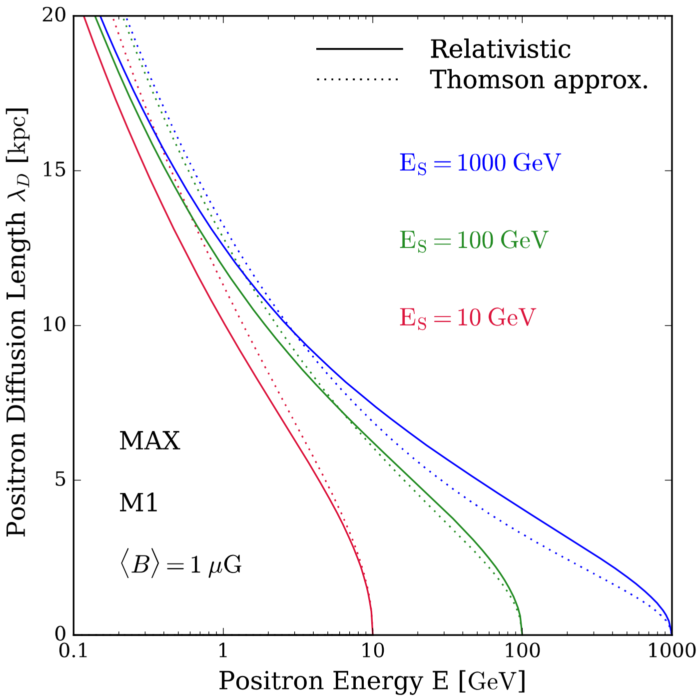
E. Baltz & J. Edsjo, PRD **59** (1998) 023511

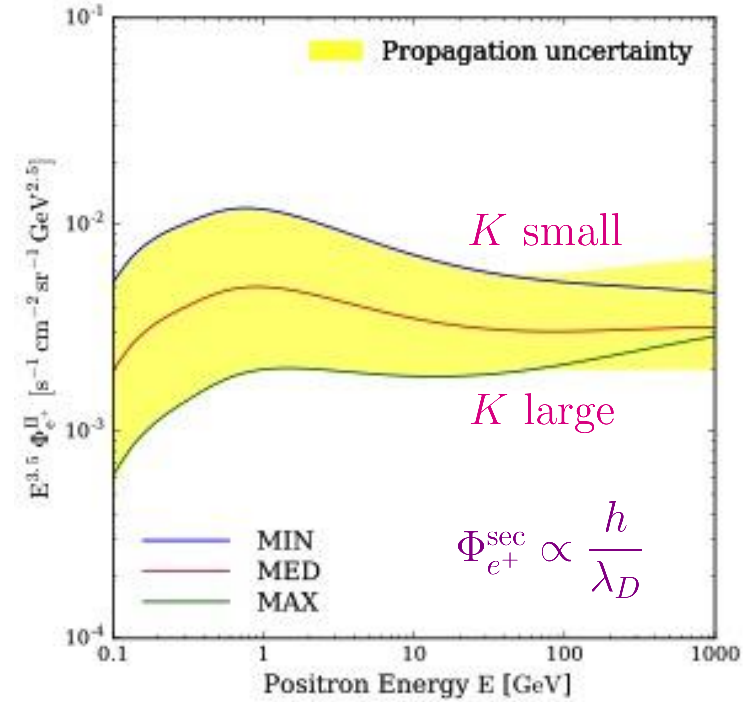
positron propagator $G_{e^+}(\boldsymbol{x}, E \leftarrow \boldsymbol{x}_S, E_S) = \dfrac{1}{|b(E)|}\, \tilde{G}\left(\boldsymbol{x}, \tilde{t}(E) \leftarrow \boldsymbol{x}_S, \tilde{t}_S(E_S)\right)$

in 3D space $\tilde{G} = \dfrac{1}{\pi^{3/2}\lambda_D^3}\, \exp\left(-\dfrac{r^2}{\lambda_D^2}\right)$

$$\lambda_D^2(E, E_S) = 4\int_E^{E_S} \frac{K(E')}{|b(E')|}\, dE'$$







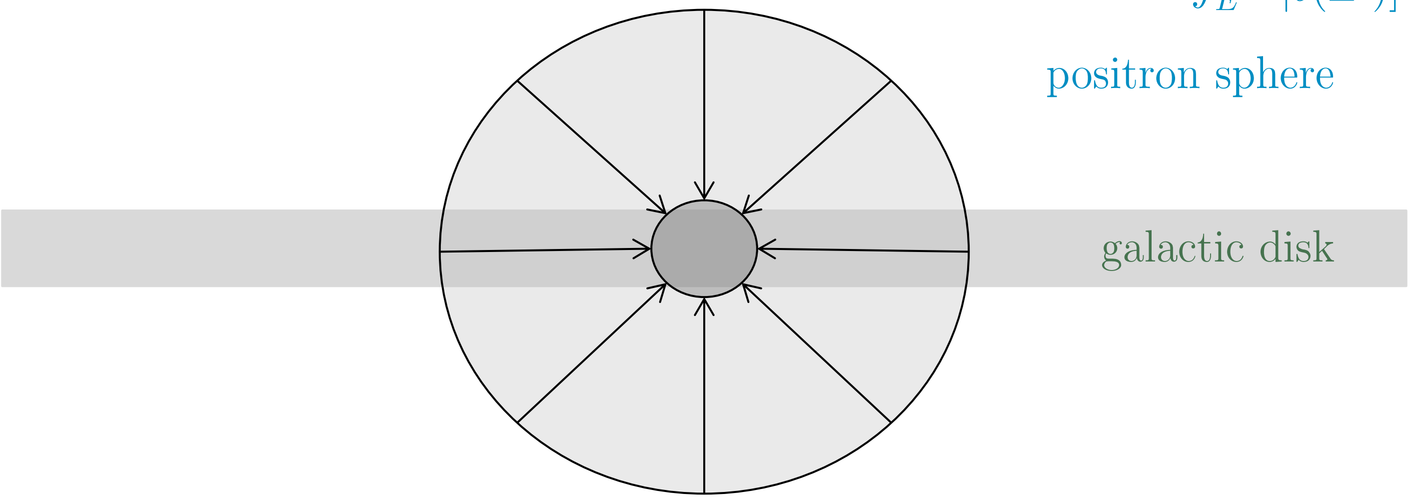
E. Baltz & J. Edsjo, PRD **59** (1998) 023511

(i) – The pinching method – a semi-analytic trick

- The CR transport equation is readily solved if all E losses are in the thin disk

$$b^{\mathrm{MH}}(z) \equiv b^{\mathrm{IC}} + b^{\mathrm{S}} \Longrightarrow 2\,h\,\delta(z)\,b^{\mathrm{MH}}_{\mathrm{eff}}$$

$$\lambda_D^2(E, E_S) = 4\int_E^{E_S} \frac{K(E')}{|b(E')|}\,dE'$$



$$b^{\mathrm{MH}}_{\mathrm{eff}} = \xi \times b^{\mathrm{MH}}$$

naively $\xi \sim \lambda_D / h$

M. Boudaud et al., arXiv:1612.03924, to appear in A&A

(i) – The pinching method – a semi-analytic trick

- The CR transport equation is readily solved if all E losses are in the thin disk

$$b^{\mathrm{MH}}(z) \equiv b^{\mathrm{IC}} + b^{\mathrm{S}} \Longrightarrow 2\,h\,\delta(z)\,b_{\mathrm{eff}}^{\mathrm{MH}}$$

- In the HE regime, b^{MH} leads to the 1D halo solution

$$-K\,\frac{\partial^2\psi}{\partial z^2} + \frac{\partial}{\partial E}\left\{(b \equiv b^{\mathrm{MH}})\,\psi\right\} = q_{e^+}(z,E)$$

$$\Downarrow$$

$$\psi^h(E) = \frac{1}{|b(E)|}\int_E^{+\infty} dE_S \left\{ B(E,E_S) = \sum_{n=2m+1} q_n(E_S)\,\exp\left\{-C_n\,\lambda_D^2(E,E_S)\right\}\right\}$$

where $q_n(E)$ Fourier transforms of $q_{e^+}(z,E)$ while $C_n = n^2 k_0^2/4$ and $k_0 = \pi/2L$

- Pinched E losses with $2\,h\,\delta(z)\,b_{\mathrm{eff}}^{\mathrm{MH}}$ yield the 1D disk solution

$$-K\,\frac{\partial^2\psi}{\partial z^2} + 2\,h\,\delta(z)\frac{\partial}{\partial E}\left\{(b^* \equiv b_{\mathrm{eff}}^{\mathrm{MH}})\,\psi\right\} = q_{e^+}(z,E)$$

$$\Downarrow$$

$$\psi^d(E) = \frac{1}{|b^*(E)|}\int_E^{+\infty} dE_S \left\{ J(E_S) = \int_0^L \frac{dz}{h}\left(1 - \frac{z}{L}\right)\, q_{e^+}(z,E_S)\right\} \exp\left\{-\frac{\lambda_D^{*2}(E,E_S)}{4\,hL}\right\}$$

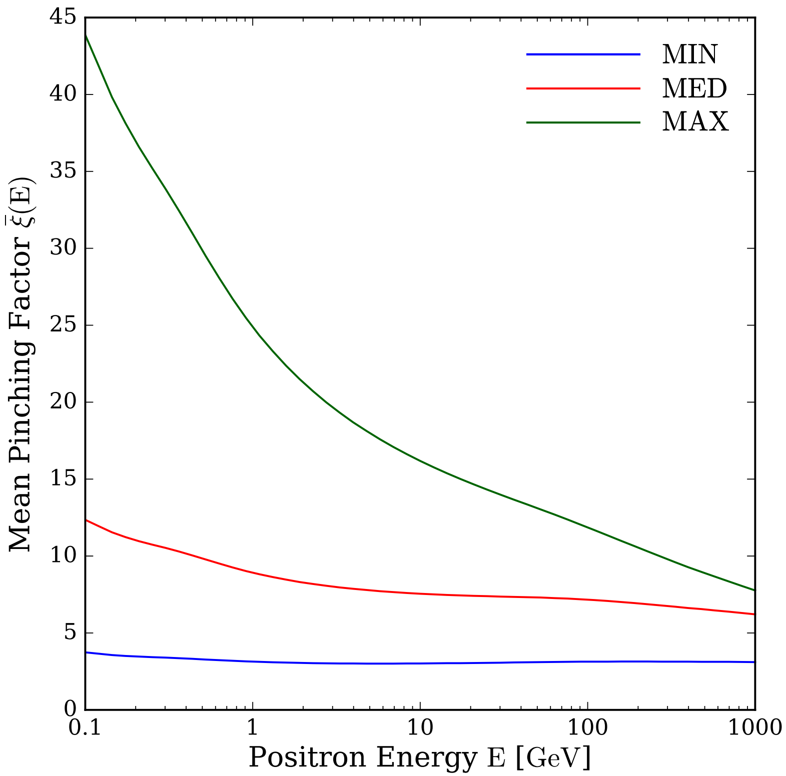
- The pinching factor $\xi(E, E_S)$ is defined as the ratio between b^* and b such that

$$b^*(E) = \xi(E, E_S)\, b(E) \quad \text{with} \quad \psi^h(E, E_S) = \psi^d(E, E_S)$$

$$\Downarrow$$

$$\boxed{\xi(E, E_S) = \frac{J(E_S)}{B(E, E_S)} \, \exp\left\{ -\frac{\lambda_D^{*2}(E, E_S)}{4hL} \right\}}$$

The effective pinching factor $\bar{\xi}(E)$ depends on q_{e^+}



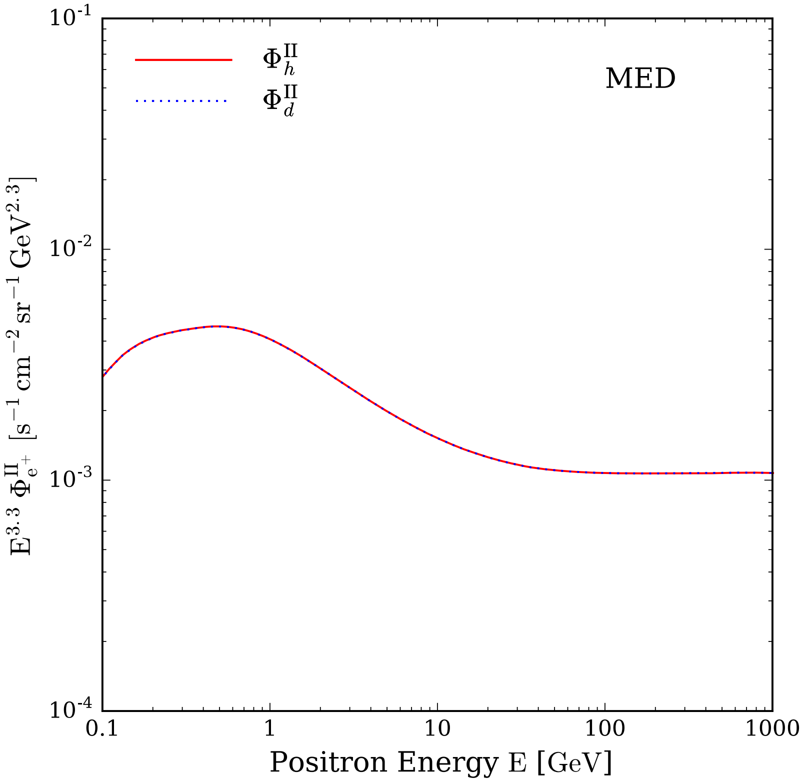
$$\bar{\xi}(E) = \frac{\displaystyle\int_E^{+\infty} dE_S \left[J(E_S) + \frac{1}{hL} \int_E^{E_S} dE' \, \frac{K(E')}{b(E')} \, B(E', E_S) \right]}{\displaystyle\int_E^{+\infty} dE_S \, B(E, E_S)}$$

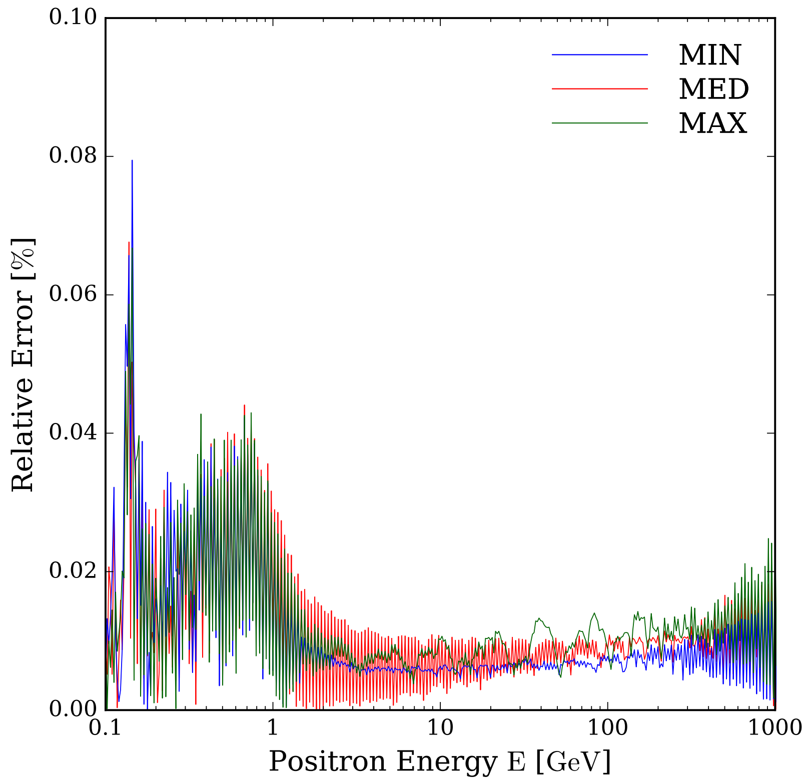
M. Boudaud et al., arXiv:1612.03924, to appear in A&A

(i) – The pinching method – a semi-analytic trick

The effective pinching factor $\bar{\xi}(E)$ depends on q_{e^+}

The **pinched** flux Φ_{e^+} is calculated with $\bar{\xi}$ through Crank-Nicholson on E





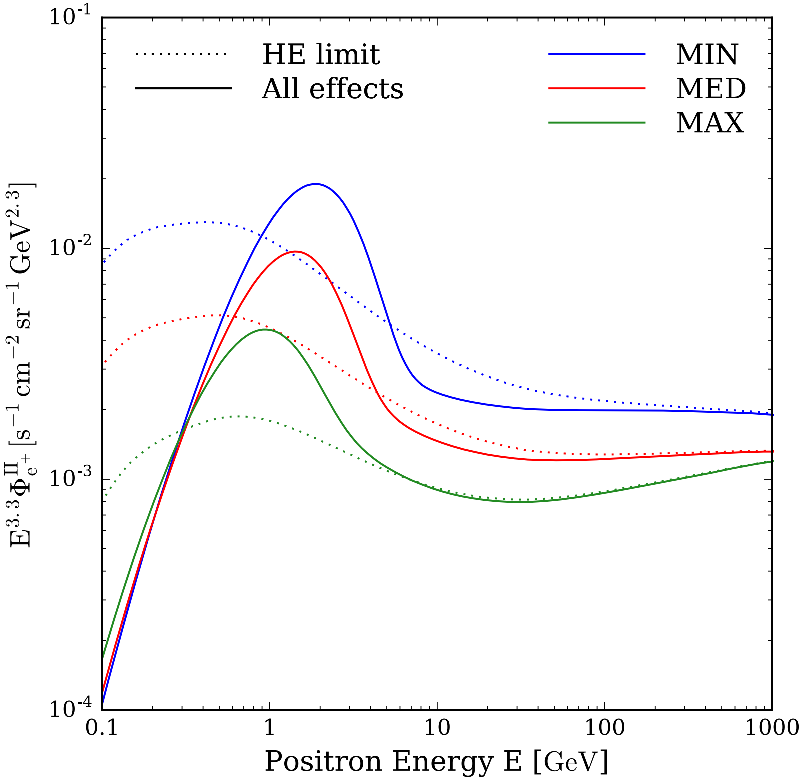
Although $\bar{\xi}$ definition intricate, excellent agreement between Φ^d and Φ^h when **only** diffusion K and halo E losses

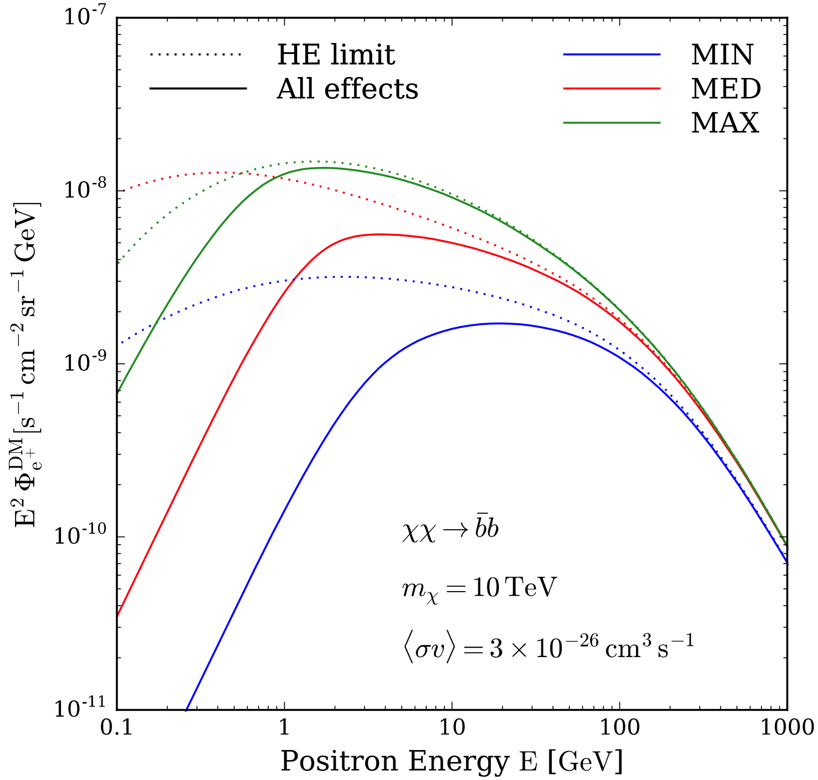
M. Boudaud et al., arXiv:1612.03924, to appear in A&A

(i) – The pinching method – a semi-analytic trick

Taking into account **all** CR transport effects leads
to differences in the final fluxes below 10 GeV

Sizeable differences with previous calculations

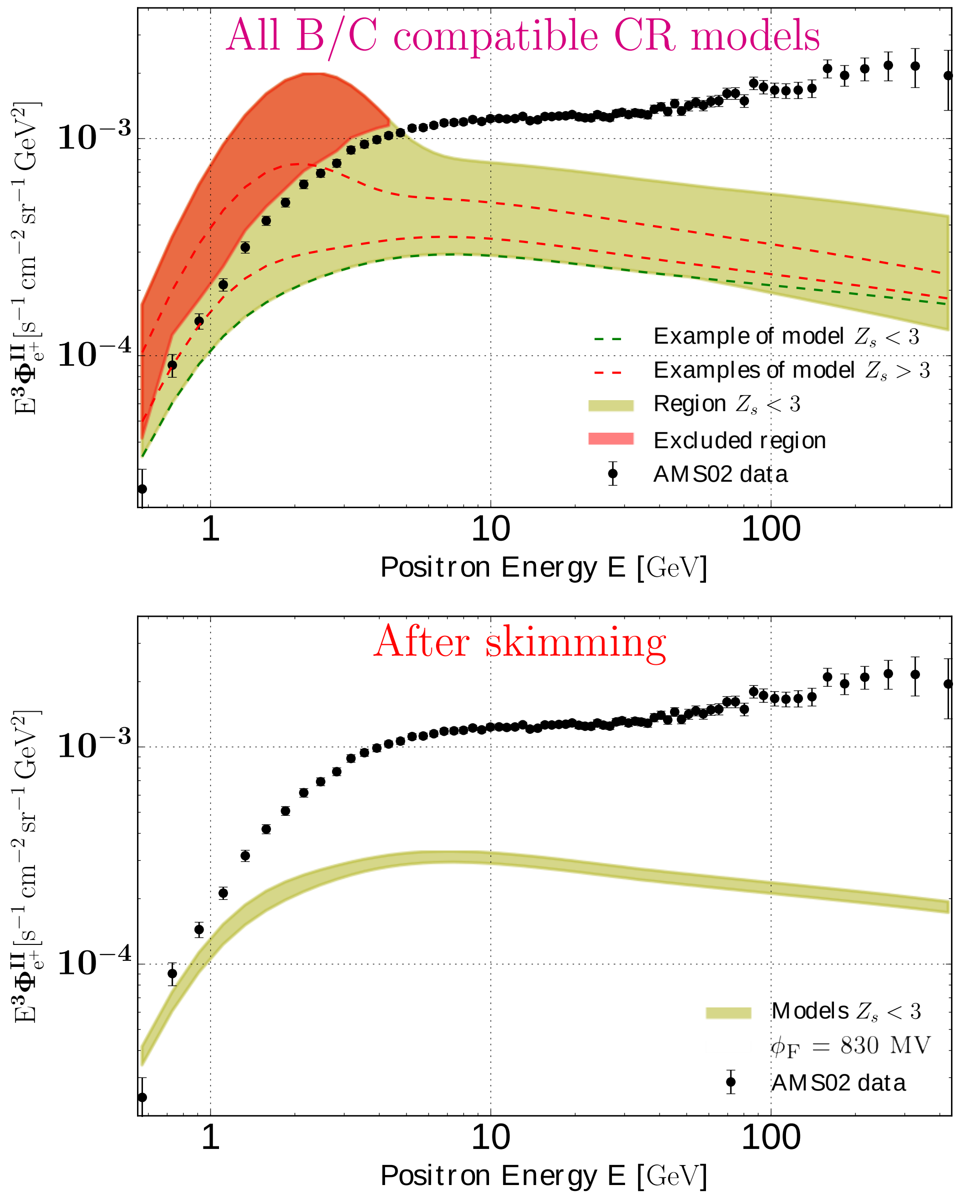




- Galactic convection V_C yields a decrease @ a few GeV
- Diffusive reacceleration V_A creates a bump just below

M. Boudaud et al., arXiv:1612.03924, to appear in A&A

3) A deeper look into the positron anomaly



M. Boudaud et al., arXiv:1612.03924, to appear in A&A

DM annihilating into SM particles

$$\Phi_{e^+} = \Phi_{e^+}^{\rm sec} + \Phi_{e^+}^{\rm DM}\{\chi\chi \to b\,\bar{b} + W^+W^- + e^+e^- + \mu^+\mu^- + \tau^+\tau^-\}$$

For each DM mass, surviving CR model and $\phi_{\rm F}$
a fit is performed on $\langle\sigma_{\rm ann} v\rangle$ and branching ratios

$$\Downarrow$$

$$\boxed{\chi^2_{\rm min}}$$

M. Boudaud et al., A&A **575** (2015) A67

A. Lopez et al., JCAP **1603** (2016) 033

M. Boudaud et al., arXiv:1612.03924, to appear in A&A

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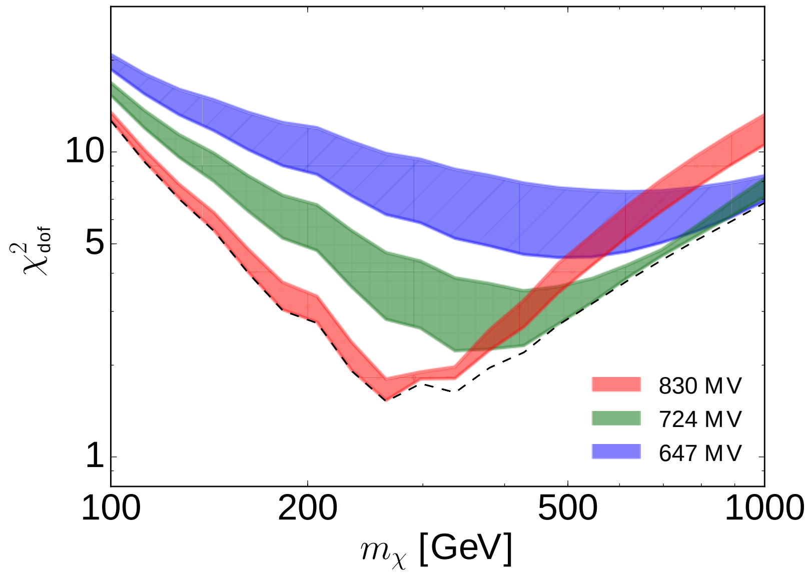
$$\Downarrow$$

$$\chi^2_{\rm min}$$

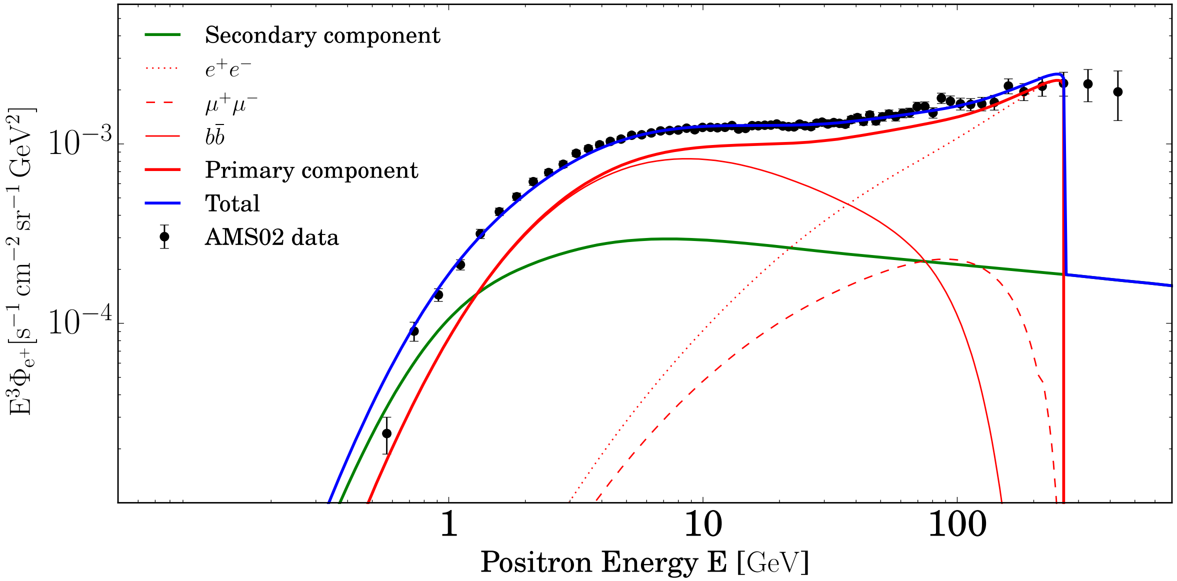


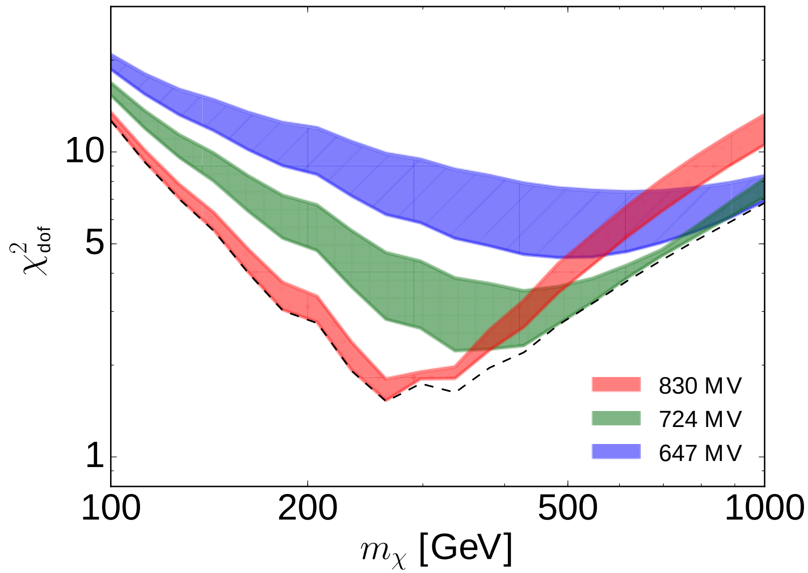
$\chi^2_{\rm dof} = 1.5$

$m_\chi = 264\,\mathrm{GeV}$

p-value 0.4%

DM annihilating into SM particles





$$\chi^2_{\rm dof} = 1.5$$

$$m_\chi = 264\,\mathrm{GeV}$$

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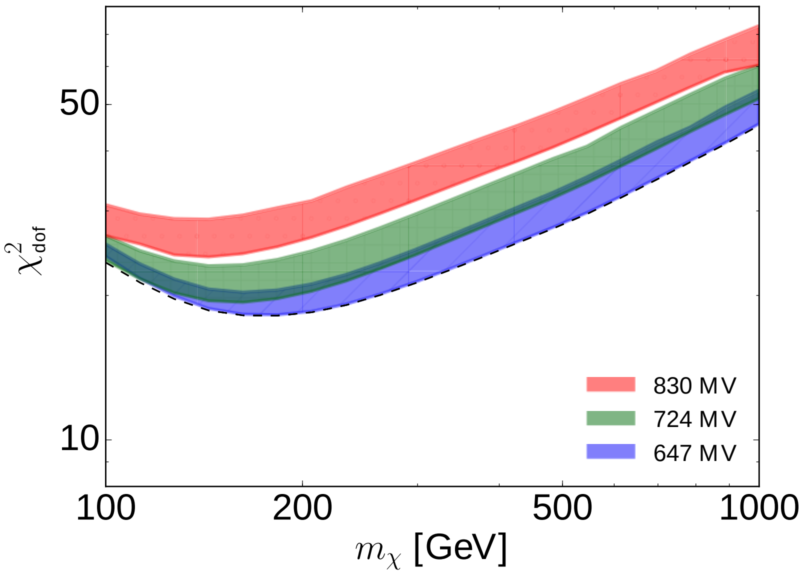
DM annihilating into mediators

$$\Phi_{e^+} = \Phi_{e^+}^{\rm sec} + \Phi_{e^+}^{\rm DM}\{\chi\chi \to \varphi\varphi \,\&\, \varphi \to e^+e^- + \mu^+\mu^- + \tau^+\tau^-\}$$

For each DM mass, surviving CR model and $\phi_{\rm F}$
a fit is performed on $\langle\sigma_{\rm ann} v\rangle$ and branching ratios

$$\Downarrow$$

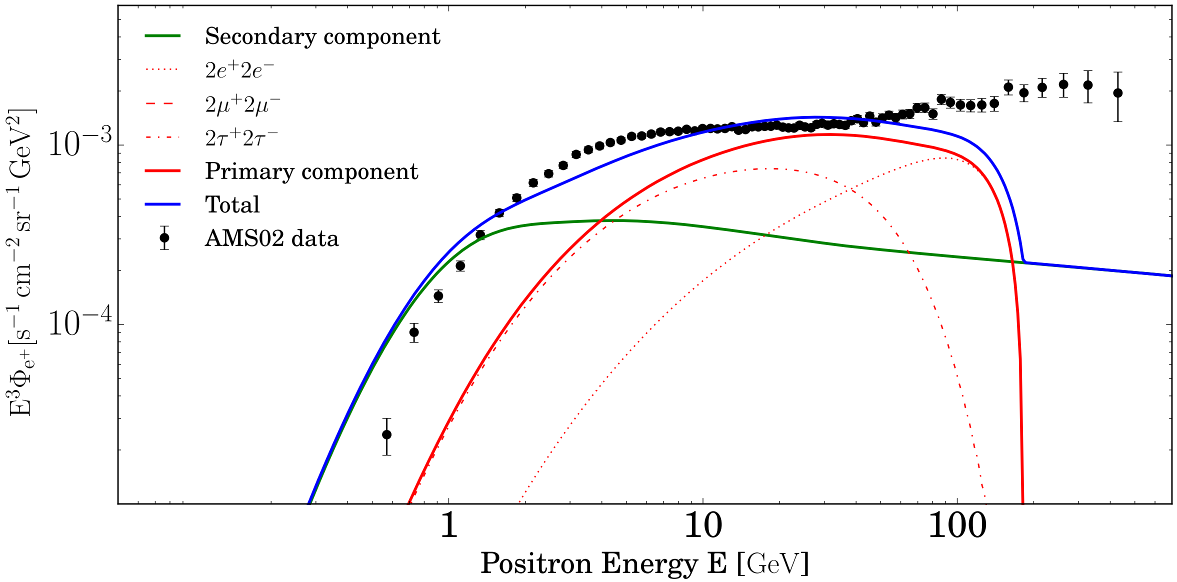
$$\boxed{\chi^2_{\rm min}}$$

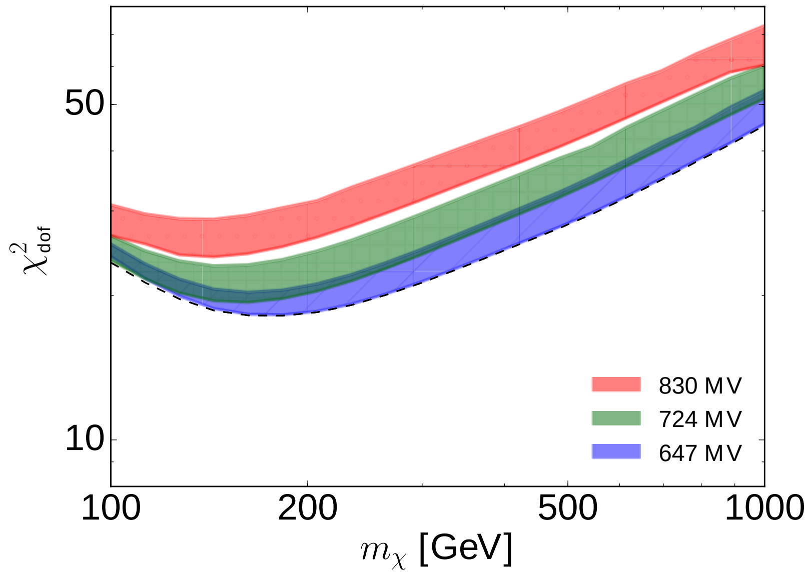


$$\chi^2_{\rm dof} = 18$$

$$m_\chi = 183\,{\rm GeV}$$

DM annihilating into mediators

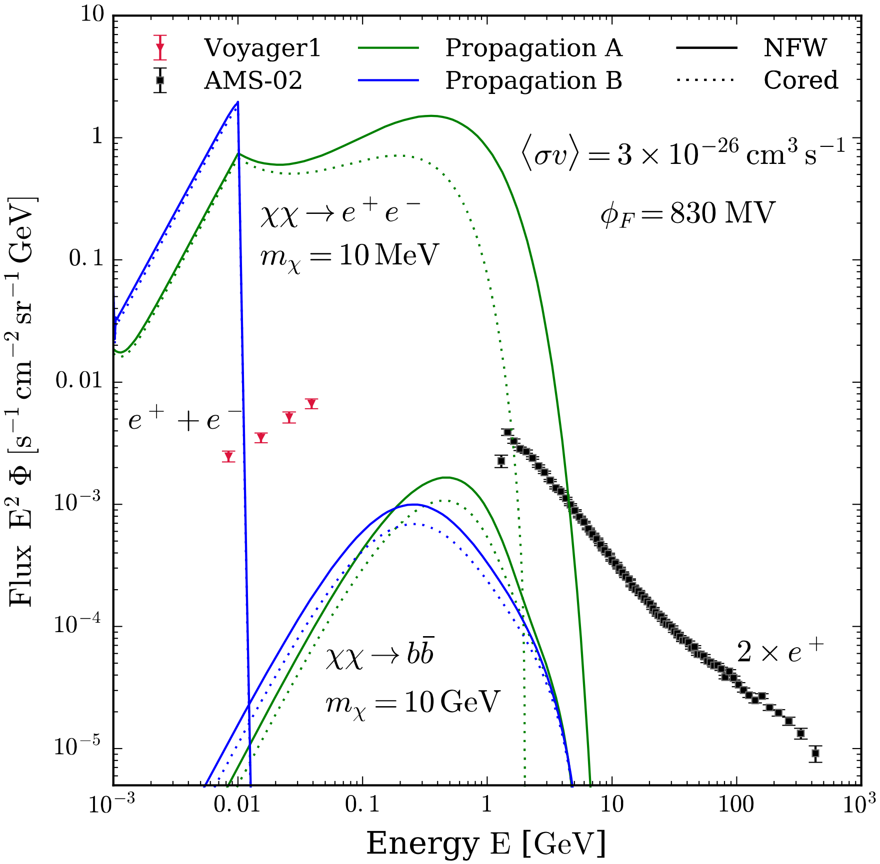


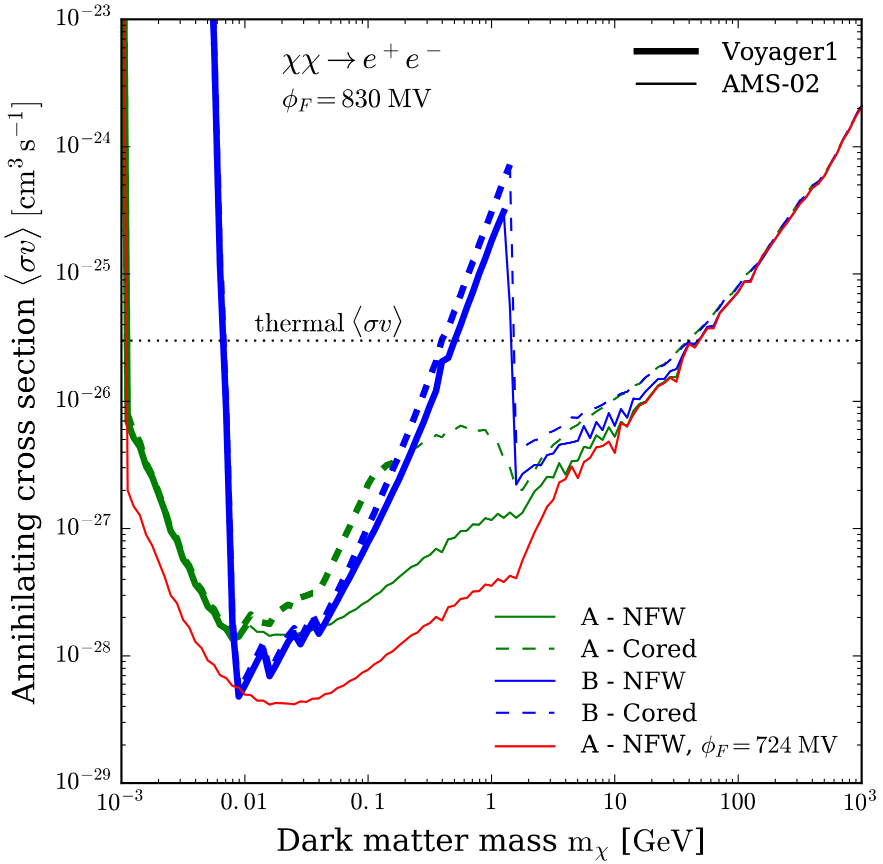


$$\chi^2_{\mathrm{dof}} = 18$$

$$m_\chi = 183\,\mathrm{GeV}$$

Limits on annihilating MeV DM candidates





M. Boudaud et al., arXiv:1612.07698, to appear in PRL

Closing thoughts

• Semi-analytic approaches are nice tools complementing full numerical codes.

faster with a better physical insight

• The **pinching method** allows to derive the positron spectrum, taking into account **all** CR transport processes – notably convection, DR and disk E losses.

e^+ *data constrain CR parameters – small K*

• An excess is found above ~ 2 GeV in the positron spectrum on top of the secondary component that is produced by spallations of the ISM by CR protons and He nuclei.

e^+ *data* **alone** *preclude DM as the source of the anomaly*

• Voyager and AMS $e^\pm$ data yield complementary constraints on MeV DM species.

• No time to discuss antiprotons but stay tuned! B/C analyses are going on.

systematic uncertainties mostly from **cross sections**

HAPPY BIRTHDAY KEITH!

Thank you for your kind invitation
and let us find a CR project for the next 33 years...