Logical Truth and Analyticity in Carnap's "Logical Syntax of Language"

Throughout his philosophical career, Carnap places the foundations of logic and mathematics at the center of his inquiries: he is concerned above all with the Kantian question "How is mathematics (both pure and applied) possible?"[1] Although he changes his mind about many particular issues, Carnap never gives up his belief in the importance and centrality of this question—nor does he ever waver in his conviction that he has the answer: the possibility of mathematics and logic is to be explained by a sharp distinction between formal and factual, analytic and synthetic truth. Thus, throughout his career Carnap calls for, and attempts to provide

> an explication for the distinction between logical and descriptive signs and that between logical and factual truth, because it seems to me that without these distinctions a satisfactory methodological analysis of science is not possible.[2]

For Carnap, it is this foundation for logic and mathematics that is distinctive of logical—as opposed to traditional—empiricism. As he puts it in his intellectual autobiography: "It became possible for the first time to combine the basic tenet of empiricism with a satisfactory explanation of the nature of logic and mathematics."[3] In particular, we can avoid the "non-empiricist" appeal to "pure intuition" or "pure reason" while, at the same time, avoiding the naive and excessively empiricist position of J. S. Mill.[4]

Indeed, from this point of view, Carnap's logicism and especially his debt to Frege become even more important than his empiricism and his connection with the Vienna Circle. The point has been put rather well, I think, by Beth in his insightful article in the Schilpp volume:

> His connection with the Vienna Circle is certainly characteristic of his way of thinking, but by no means did it determine his philosophy. It

seems to me that the influence of Frege's teachings and published work has been much deeper. In fact, this influence must have been decisive, and the development of Carnap's ideas may be considered as characteristic of Frege's philosophy as well.[5]

Carnap endorses this assessment in his reply to Beth,[6] and it is quite consistent with what he says about his debt to Frege elsewhere.[7]

Yet when one looks at *Logical Syntax*,[8] which is clearly Carnap's richest and most systematic discussion of these foundational questions, the idea that Carnap is continuing Frege's logicism appears to be quite problematic. Not only does Carnap put forward an extreme "formalistic" (purely syntactic) conception of the language of mathematics—a conception that, as he explicitly acknowledges, is derived from Hilbert and would be anathema to Frege (§84)—his actual construction of mathematical systems exhibits none of the characteristic features of logicism. No attempt is made to define the natural numbers: the numerals are simply introduced as primitive signs in both of Carnap's constructed languages. Similarly, no attempt is made to derive the principle of mathematical induction from underlying logical laws: in both systems it is introduced as a primitive axiom (in Language I it appears as a primitive [schematic] inference rule [R14 of §12]). In short, Carnap's construction of mathematics is thoroughly axiomatic and, as he explicitly acknowledges (§84), appears to be much closer to Hilbert's formalism than to Frege's logicism.

Carnap's official view of this question is that he is putting forward a reconciliation of logicism and formalism, a combination of Frege and Hilbert that somehow captures the best of both positions (§84).[9] In light of the above, however, it must strike the reader as doubtful that anything important in Frege's position has been retained. For that matter, although Carnap employs formalist rhetoric and an explicitly axiomatic formulation of mathematics, nothing essential to Hilbert's foundational program appears to be retained either. Thus, no attempt is made to give a finitary consistency or conservativeness proof for classical mathematics. Carnap takes Gödel's results to show that the possibility of such a proof is "at best very doubtful," and he puts forward a consistency proof in a metalanguage essentially richer than classical mathematics (containing, in effect, classical mathematics plus a truth-definition for classical mathematics), which, as Carnap again explicitly acknowledges (§§34h, 34i), is therefore of doubtful foundational significance. At this point, then, Car-

nap's claim to reconcile Frege and Hilbert appears hollow indeed. What he has actually done, it seems, is thrown away all that is most interesting and characteristic in both views.

Such an evaluation would be both premature and fundamentally unfair, however. To see why, we must look more closely at the centerpiece of Carnap's philosophy—his conception of *analytic truth*—and how that conception evolves from Frege's while incorporating post-Fregean advances in logic: in particular, advances due to Hilbert and Gödel.

The first point to bear in mind is the familiar one that Frege's construction of arithmetic is not simply the embedding of a special mathematical theory (arithmetic) in a more general one (set theory). Frege's *Begriffsschrift* is not intended to be a mathematical theory at all; rather, it is to function as the logical framework that governs all rational thinking (and therefore all particular theories) whatsoever. As such, it has no special subject matter (the universe of sets, for example) with which we are acquainted by "intuition" or any other special faculty. The principles and theorems of the *Begriffsschrift* are implicit in the requirements of any coherent thinking about anything at all, and this is how Frege's construction of arithmetic within the *Begriffsschrift* is to provide an answer to Kant: arithmetic is in no sense dependent on our spatiotemporal intuition but is built in to the most general conditions of thought itself. This, in the end, is the force of Frege's claim to have established the analyticity of arithmetic.

But why should we think that the principles of Frege's new logic delimit the most general conditions of all rational thinking? Wittgenstein's *Tractatus* attempts to provide an answer: this new logic is itself built in to any system of representation we are willing to call a language. For, from Wittgenstein's point of view, the *Begriffsschrift* rests on two basic ides: Frege's function/argument analysis of predication and quantification, and the iterative construction of complex expressions from simpler expressions via truth-functions. So any language in which we can discern both function/argument structure—in essence, where there are grammatical categories of intersubstitutable terms—and truth-functional iterative constructions will automatically contain all the logical forms and principles of the new logic as well. Since it is plausible to suppose that any system of representation lacking these two features cannot count as a language in any interesting sense, it makes perfectly good sense to view the new logic as delimiting the general conditions of any rational thinking what-

soever. For the new logic is now seen as embodying the most general conditions of meaningfulness (meaningful representation) as such.[10]

Carnap enthusiastically endorses this Wittgensteinian interpretation of Frege's conception of analyticity, and he is quite explicit about his debt to Wittgenstein throughout *Logical Syntax* (§§14, 34a, 52) and throughout his career.[11] Yet, at the same time, Carnap radically transforms the conception of the *Tractatus*, and he does this by emphasizing themes that are only implicit in Wittgenstein's thought. It is here, in fact, that Carnap brings to bear the work of Hilbert and Gödel in a most decisive fashion.

First of all, Carnap interprets Wittgenstein's elucidations of the notions of language, logical truth, logical form, and so on as definitions in *formal syntax*. They are themselves formulated in a metalanguage or "syntax-language," and they concern the syntactic structure either of some particular object-language or of languages in general:

> All questions of logic (taking this word in a very wide sense, but excluding all empirical and therewith all psychological reference) belong to syntax. *As soon as logic is formulated in an exact manner, it turns out to be nothing other than the syntax either of some particular language or of languages in general.* (§62)

This syntactic interpretation of logic is of course completely foreign to Wittgenstein himself. For Wittgenstein, there can be only one language— the single interconnected system of propositions within which everything that can be said must ultimately find a place; and there is no way to get "outside" this system so as to state or describe its logical structure: there can be no syntactic metalanguage. Hence logic and all "formal concepts" must remain ineffable in the *Tractatus*.[12] Yet Carnap takes the work of Hilbert and especially Gödel to have decisively refuted these Wittgensteinian ideas (see especially §73). Syntax (and therefore logic) can be exactly formulated; and, in particular, if our object-language contains primitive recursive arithmetic, the syntax of our language (and every other language) can be formulated within this language itself (§18).

Secondly, Carnap also clearly recognizes that the linguistic or "syntactic" conception of analyticity developed in the *Tractatus* is much too weak to embrace all of classical mathematics or all of Frege's *Begriffsschrift*. For the two devices of function/argument structure (substitution) and iterative truth-functional construction that were seen to underly Frege's distinctive analysis of predication and quantification do not lead us to the rich higher-order principles of classical analysis and set theory. As

Gödel's arithmetization of syntax again decisively shows, all that is forth-coming is primitive recursive arithmetic. Of course, the *Tractatus* is itself quite clear on the restricted scope of its conception of logic and mathematics in comparison with Frege's (and Russell's) conception. Witt-genstein's response to this difficulty is also all too clear: so much the worse for classical mathematics and set theory.[13]

Carnap's own response is quite different, however, for his aim throughout is not to replace or restrict classical mathematics but to pro-vide it with a philosophical foundation: to answer the question "How is classical mathematics possible?" And it is here that Carnap makes his most original and fundamental philosophical move: we are to give up the "ab-solutist" conception of logical truth and analyticity common to Frege and the *Tractatus*. For Carnap, there is no such thing as *the* logical framework governing all rational thought. Many such frameworks, many such systems of what Carnap calls *L-rules* are possible: and all have an equal claim to "correctness." Thus, we can imagine a linguistic framework whose L-rules are just those of primitive recursive arithmetic itself (such as Carnap's Language I); a second whose L-rules are given by set theory or some higher-order logic (such as Carnap's Language II); a third whose L-rules are given by intuitionistic logic; a fourth whose L-rules include part of what is in-tuitively physics (such as physical geometry: cf. §50); and so on. As long as the L-rules in question are clearly and precisely delimited within for-mal syntax, any such linguistic framework defines a perfectly legitimate language (*Principle of Tolerance*):

> *In logic there are no morals.* Everyone is at liberty to build up his own logic, i.e., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clear-ly, and give syntactical rules instead of philosophical arguments. (§17)

Thus, Carnap's basic move is to relativize the "absolutist" and essential-ly Kantian program of Frege and the *Tractatus*.

Carnap's general strategy is then concretely executed as follows. First, within the class of all possible linguistic frameworks, one particular such framework stands out for special attention. A framework whose L-rules are just those of primitive recursive arithmetic has a relatively neutral and uncontroversial status—it is common to "Platonists," "intuitionists," and "constructivists" alike (§16); and, moreover, as Gödel's researches have shown, such a "minimal" framework is nonetheless adequate for for-mulating the logical syntax of any linguistic framework whatsoever—

including its own. So this linguistic framework, Carnap's Language I, can serve as an appropriate beginning and "fixed point" for all subsequent syntactic investigation—including the investigation of much richer and more controversial frameworks.

One such richer framework is Carnap's Language II: a higher-order system of types over the natural numbers including (higher-order) principles of induction, extensionality, and choice (§30). This framework will then be adequate for much of classical mathematics and mathematical physics. Nevertheless, despite the strength of this framework, we can exactly describe its logical structure within logical syntax; and, in particular, we can show that the mathematical principles in question are *analytic-in-Language-II*—in Carnap's technical terminology, they are included in the L-rules ("logical" rules), not the P-rules ("physical" rules) of Language II. So we can thereby explain the "mathematical knowledge" of anyone who adopts (who speaks, as it were) Language II. Such knowledge is implicit in the linguistic framework definitive of meaningfulness for such a person, and it is therefore formal, not factual.

We are now in a position to appreciate the extent to which Carnap has in fact combined the insights of Frege and Hilbert and has, in an important sense, attempted a genuine reconciliation of logicism and formalism. From Frege (and Wittgenstein), Carnap takes the idea that the possibility of mathematics is to be explained by showing how its principles are implicit in the general conditions definitive of meaningfulness and rationality. Mathematics is built in to the very structure of thought and language and is thereby forever distinguished from merely empirical truth. By relativizing the notion of logical truth, Carnap attempts to preserve this basic logicist insight in the face of all the well-known technical difficulties; and this is why questions of reducing mathematics to something else—to "logic" in some antecedently fixed sense—are no longer relevant. From Hilbert (and Gödel), Carnap takes the idea that primitive recursive arithmetic constitutes a privileged and relatively neutral "core" to mathematics and, moreover, that this neutral "core" can be used as a "metalogic" for investigating much richer and more controversial theories. The point, however, is not to provide consistency or conservativeness proofs for classical mathematics, but merely to delimit its logical structure: to show that the mathematical principles in question are analytic in a suitable language. Carnap hopes thereby to avoid the devastating impact of Gödel's Incompleteness Theorems.

Alas, however, it was not meant to be. For Gödel's results decisively undermine Carnap's program after all. To see this, we have to be a bit more explicit about the details of the program. For Carnap, a language or linguistic framework is syntactically specified by its formation and transformation rules, where these latter specify both axioms and rules of inference. The language in question is then characterized by its consequence-relation, which is defined in familiar ways from the underlying transformation rules. Now such a language or linguistic framework will contain both formal and empirical components, both "logical" and "physical' rules. Language II, for example, will not only contain classical mathematics but classical physics as well, including "physical" primitive terms (§40)—such as a functor representing the electromagnetic field— and "physical" primitive axioms (§82)—such as Maxwell's equations. The task of defining analytic-for-a-language, then, is to show how to distinguish these two components: in Carnap's technical terminology, to distinguish L-rules from P-rules, L-consequence from P-consequence (§§51, 52).

How is this distinction to be drawn? Carnap proceeds on the basis of a prior distinction between *logical* and *descriptive* expressions (§50). Intuitively, logical expressions include logical constants in the usual sense (connectives and quantifiers) plus primitive expressions of arithmetic (the numerals, successor, addition, multiplication, and so on). Given the distinction between logical and descriptive expressions, we then define the analytic (L-true) sentences of a language as those theorems (L- or P-consequences of the null set) that remain theorems under all possible substitutions of *descriptive* expressions (§51). In other worlds, what we might call "descriptive invariance" separates the L-consequences from the wider class of consequences *simpliciter*. But how is the distinction between logical and descriptive expressions itself to be drawn? Here Carnap appeals to the *determinacy* of logic and mathematics (§50): logical expressions are just those expressions such that every sentence built up from them alone is decided one way or another by the rules (L-rules or P-rules) of the language. That is, every sentence built up from logical expressions alone is provable or refutable on the basis of these rules. In the case of descriptive expressions, by contrast, although some sentences built up from them will no doubt be provable or refutable as well (in virtue of P-rules, for example), this will not be true for all such sentences—for sentences ascribing particular values of the electromagnetic field to par-

ticular space-time points, for example. In this way, Carnap intends to capture the idea that logic and mathematics are thoroughly a priori.

It is precisely here, of course, that Gödelian complications arise. For, if our consequence-relation is specified in terms of what Carnap calls *definite* syntactic concepts—that is, if this relation is recursively enumerable—then even the theorems of primitive recusive arithmetic (Language I) fail to be analytic; and the situation is even worse, of course, for full classical mathematics (Language II). Indeed, as we would now put it, the set of (Gödel numbers of) analytic sentences of classical first-order number theory is not even an arithmetical set, so it certainly cannot be specified by definite (recursive) means. Carnap himself is perfectly aware of these facts, and this is why he explicitly adds what he calls *indefinite* concepts to syntax (§45). In particular, he explicitly distinguishes (recursive and recursively enumerable) d-terms or rules of derivation from (in general nonarithmetical) c-terms or rules of consequence (§§47, 48).

Moreover, it is here that Carnap is compelled to supplement his "syntactic" methods with techniques we now associate with the name of Tarski: techniques we now call "semantic." In particular, the definition of *analytic-in-Language-II* is, in effect, a truth-definition for classical mathematics (§§34a-34d). Thus, if we think of Language II as containing all types up to ω (all finite types), say, our definition of *analytic-in-Language-II* will be formulated in a stronger metalanguage containing quantification over arbitrary sets of type ω as well. In general, then, Carnap's definition of analyticity for a language of any order will require quantification over sets of still higher order. The extension of analytic-in-L for any L will therefore depend on how quantifiers in a metalanguage essentially richer than L are interpreted; the interpretation of quantifiers in this metalanguage can only be fixed in a still stronger language; and so on (§34d).[14]

But why should this circumstance cause any problems for Carnap? After all, he himself is quite clear about the technical situation; yet he nevertheless sees no difficulty whatever for his logicist program. It is explicitly granted that Gödel's Theorem thereby undermines Hilbert's formalism; but why should it refute Frege's logicism as well? The logicist has no special commitment to the "constructive" or primitive recursive fragment of mathematics: he is quite happy to embrace all of classical mathematics. Indeed, Carnap, in his Principle of Tolerance, explicitly rejects all questions concerning the legitimacy or justification of classical mathematics.

What the logicist wishes to maintain is not a reduction or justification of classical mathematics via its "constructive" fragment (as Hilbert attempts in his finitary consistency and conservativeness proofs), but simply that classical mathematics is analytic: that it is true in virtue of language or meaning, not fact. So why should Gödel's Theorem undermine *this* conception?[15]

To appreciate the full impact of Gödel's results here, it is necessary to become clearer on the fundamental differences between Carnap's conception of analyticity or logical truth and that of his logicist predecessors. For precisely these differences are obscured by the notion of truth-in-virtue-of-meaning or truth-in-virtue-of-language—especially as this notion is wielded by Quine in his polemic against Carnap. Thus, the early pages of "Two Dogmas" distinguish two classes of logical truths.[16] A *general* logical truth—such as "No unmarried man is married"—is "a statement that is true and remains true under all reinterpretations of its components other than the logical particles." An *analytic* statement properly so-called—such as "No bachelor is unmarried"—arises from a general logical truth by substitution of synonyms for synonyms. This latter notion is then singled out for special criticism for relying on a problematic conception of meaning (synonymy); and this is the level on which Quine engages with Carnap.

The first point to notice is that *these* Quinean criticisms are indeed relevant to Carnap, but not at all to his logicist predecessors—for their analytic truths simply do not involve nonlogical constants in this sense. Thus, whereas Carnap's languages contain primitive arithmetical signs (the numerals, successor, addition, and so), Frege's *Begriffsschrift* and Russell's *Principia* do not. In these systems, the arithmetical signs are of course defined via the logical notions of truth-functions and quantifiers (including quantifiers over higher types). So Carnap needs to maintain that arithmetical truths are in some sense true in virtue of the meanings of 'plus' and 'times', say, whereas Frege and Russell do not—these latter signs simply do not occur in their systems.

But what about the logical notions Frege and Russell assume: the notions we now call logical constants and Quine calls logical particles? Does the same problem not arise for them? Do we not have to assume that the general logical truths of the *Begriffsschrift* and *Principia* are true in virtue of the meanings of 'and', 'or', 'not', 'all', and 'some'? According to Wittgenstein's position in the *Tractatus*, the so-called logical constants

do not, properly speaking, have meaning at all. They are not words like others for which a "theory of meaning" is either possible or necessary. Indeed, for Wittgenstein, "there are no . . . 'logical constants' [in Frege's and Russell's sense]" (5.4). Rather, for any language, with any vocabulary of "constants" or primitive signs whatsoever, there are the purely combinatorial possibilities of building complex expressions from simpler expressions and of substituting one expression for another within such a complex expression. These abstract combinatorial possibilities are all that the so-called logical constants express: "Whenever there is compositeness, argument and function are present, and where these are present, we already have all the logical constants" (5.47). Thus, for Wittgenstein, logical truths are not true in virtue of the meanings of particular words—whether of 'and', 'or', 'not', or any others—but solely in virtue of "logical form" the general combinatorial possibilities common to all languages regardless of their particular vocabularies.

Now this conception—that logical truths are true in virtue of "logical form," and not in virtue of "meaning" in anything like Quine's sense—is essential to the *antipsychologism* of the *Tractatus*. For, if logic depends on the meanings of particular words—even "logical words" like 'and', 'not', and so on—then it rests, in the last analysis, on psychological facts about how these words are actually used. It then becomes possible to contest these alleged facts and to argue, for example, that a correct theory of meaning supports intuitionistic rather than classical logic, say. For Wittgenstein, this debate, in these terms, simply does not make sense. Logic rests on no facts whatsoever, and certainly not on facts about the meanings or usages of English (or German) words. Rather, logic rests on the abstract combinatorial possibilities common to all languages as such. In this sense, logic is absolutely presuppositionless and thus absolutely uncontentious.

The problem for this Tractarian conception has nothing at all to do with the Quinean problem of truth-in-virtue-of-meaning or truth-in-virtue-of-language. Rather, the problem is that the logic realizing this conception is much too weak to accomplish the original aim of logicism: explaining how mathematics—classical mathematics—is possible. Frege's *Begriffsschrift* cannot provide the required realization, because of the paradoxes; and neither can Russell's *Principia*, because of the need for axioms like infinity and reducibility. The *Tractatus* itself ends up with a conception of logic that falls somewhere between truth-functional logic and a ramified

type-theory without infinity or reducibility; and it ends up with a conception of mathematics apparently limited to primitive recursive arithmetic.[17] So Wittgenstein may have indeed achieved a genuinely presuppositionless standpoint, but only by failing completely to engage the foundational question that originally motivated logicism.

At this point Carnap has an extremely ingenious idea. We retain Wittgenstein's purely combinatorial conception of logic, but it is implemented at the level of the *metalanguage* and given an explicit subject matter: namely, the syntactic structure of any language whatsoever. At the same time, precisely because logic in this sense is implemented at the level of the metalanguage not the object-language, it no longer has the impoverishing and stultifying effect evident in the *Tractatus*. For, although our purely syntactic metalanguage is to have a very weak, and therefore uncontroversial, underlying logic, we can nonetheless use it to describe—but not to justify or reduce—much stronger systems: in particular, classical mathematics. In this way Carnap hopes to engage, and in fact, to neutralize the basic foundational question. Logic, in the sense of logical syntax, can in no way adjudicate this question. Indeed, from Carnap's point of view, there is no substantive question to be adjudicated. Rather, logic in this sense constitutes a neutral metaperspective from which we can represent the consequences of adopting any and all of the standpoints in question: "Platonist," "constructivist," "intuitionist," and so on.

Corresponding to any one of these standpoints is a notion of logic (analyticity) in a second sense: a notion of analytic-in-L. Sentences analytic-in-L are not true in virtue of the abstract combinatorial possibilities definitive of languages in general, but in virtue of conventions governing this particular L—specifically, on those linguistic conventions that establish some words as "logical" and others as "descriptive." Hence it is at this point, and only at this point, that we arrive at the Quinean problem of truth-in-virtue-of-meaning. And it is at this point, then, that Carnap's logicism threatens to collapse into its dialectical opponent, namely psychologism.

Carnap hopes to avoid such a collapse by rigorously enforcing the distinction between *pure* and *applied* (descriptive) syntax. The latter, to be sure, is an empirical discipline resting ultimately on psychological facts: it aims to determine whether and to what extent the speech dispositions of a given speaker or community realize or exemplify the rules of a given abstractly characterized language L—including and especially those rules

definitive of the notion of analytic-in-L. Pure syntax, on the other hand, is where we develop such abstract characterizations in the first place. We are concerned neither with the question of which linguistic framework is exemplified by a given community or speaker, nor with recommending one linguistic framework over others—classical over "constructive" mathematics, say. Rather, our aim is to step back from all such questions and simply articulate the consequences of adopting any and all such frameworks. The propositions of pure syntax are therefore logical or analytic propositions in the first sense: propositions of the abstract, purely combinatorial metadiscipline of logical syntax.

Here is where Gödel's Theorem strikes a fatal blow. For, as we have seen, Carnap's general notion of analytic-in-L is simply not definable in logical syntax so conceived, that is, conceived in the above "Wittgensteinian" fashion as concerned with the general combinatorial properties of any language whatsoever. *Analytic-in-L* fails to be captured in what Carnap calls the *"combinatorial analysis...* of finite, discrete serial structures" (§2): that is, primitive recursive arithmetic. Hence the very notion that supports, and is indeed essential to, Carnap's logicism simply does not occur in pure syntax as he understands it. If this notion is to have any place at all, then, it can only be within the explicitly empirical and psychological discipline of applied syntax; and the dialectic leading to Quine's challenge is now irresistible.[18] In this sense, Gödel's results knock away the last slender reed on which Carnap's logicism (and antipsychologism) rests.

In the end, what is perhaps most striking about *Logical Syntax* is the way it combines a grasp of the technical situation that is truly remarkable in 1934 with a seemingly unaccountable blindness to the full implications of that situation. Later, under Tarski's direct influence, Carnap of course came to see that his definition of analytic-in-L is not a properly "syntactic" definition at all; and in *Introduction to Semantics*[19] he officially renounces the definitions of *Logical Syntax* (§39) and admits that no satisfactory delimitation of L-truth in "general semantics" is yet known (§§13, 16). Instead, he offers two tentative suggestions: either we can suppose that our metalanguage contains a necessity operator, so that a sentence S is analytic-in -L just in case we have N(S is true) in M^L; or we can suppose that we have been already given a distinction between logical and factual truth in our metalanguage, so that a sentence S is analytic-in-L just in case 'S is true' is analytic-in-M^L (§116).

From our present, post-Quinean vantage point, the triviality and cir-
cularity of these suggestions is painfully obvious; but it was never so for
Carnap. He never lost his conviction that the notion of analytic truth,
together with a fundamentally logicist conception of mathematics, stands
firm and unshakable. And what this shows, finally, is that the Fregean
roots of Carnap's philosophizing run deep indeed. Unfortunately,
however, they have yet to issue in their intended fruit.

Notes

1. I am indebted to helpful suggestions, advice, and criticism from Warren Goldfarb,
Peter Hylton, Thomas Ricketts, and the late Alberto Coffa.
2. R. Carnap, "Reply to E. W. Beth on Constructed Language Systems," in *The
Philosophy of Rudolf Carnap*, ed. P. Schilpp. (La Salle: Open Court, 1963), p. 932.
3. R. Carnap, "Intellectual Autobiography," in *Philosophy of Rudolf Carnap*, p. 47.
4. Ibid.
5. E. W. Beth, "Carnap's Views on the Advantages of Constructed Systems over Natural
Languages in the Philosophy of Science," in *Philosophy of Rudolf Carnap*, pp. 470-71.
6. "Reply to E. W. Beth," pp. 927-28.
7. See, for example, Carnap's "Intellectual Autobiography," pp. 4-6; and the preface
to the second edition of *The Logical Structure of the World*, trans. R. George (Berkeley:
University of California Press, 1967), pp. v-vi.
8. R. Carnap, *The Logical Syntax of Language*, trans. A. Smeaton (London: Routledge,
1937). References are given parenthetically in the text via section numbers.
9. Cf. also Beth, "Carnap's Views on Constructed Systems," pp. 475-82; and Carnap,
"Reply to E. W. Beth," p. 928.
10. For the relationship between Frege and the *Tractatus*, see especially T. Ricketts, "Frege,
the *Tractatus*, and the Logocentric Predicament," *Nous* 19 (1985): 3-15.
11. See especially his "Intellectual Autobiography," p. 25.
12. *Tractatus Logico-Philosophicus*, 4.12-4.128; this is the basis for the "logocentric
predicament" of note 10 above. Further references to Wittgenstein in the text are also to
the *Tractatus*. See also W. Goldfarb, "Logic in the Twenties: the Nature of the Quantifier,"
Journal of Symbolic Logic 44 (1979): 351-68.
13. For the rejection of set theory, see 6.031. Frege's and Russell's impredicative defini-
tion of the ancestral is rejected at 4.1273; the axiom of reducibility is rejected at 6.1232-6.1233.
Apparently, then, we are limited to (at most) predicative analysis—and even this may be
too much because of the doubtful status of the axiom of infinity (5.535). The discussion
of mathematics and logic at 6.2-6.241 strongly suggests a conception of mathematics limited
to primitive recursive arithmetic.
14. Cf. Beth, "Carnap's Views on Constructed Systems," who takes this situation to
undermine both Carnap's logicism and his formalism—and, in fact, to require an appeal
to some kind of "intuition." Carnap's ingenuous response in his "Reply to E. W. Beth,"
pp. 928-30, is most instructive.
15. I am indebted to Thomas Ricketts and especially to Alberto Coffa for pressing me
on this point.
16. W. V. Quine, "Two Dogmas of Empiricism," in *From a Logical Point of View* (New
York: Harper, 1963), pp. 22-23.
17. See note 13 above.
18. See T. Ricketts, "Rationality, Translation, and Epistemology Naturalized," *Jour-
nal of Philosophy* 79 (1982): 117-37.
19. R. Carnap, *Introduction to Semantics* (Cambridge, Mass.: Harvard University Press,
1942); parenthetical references in the text are to section numbers.