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Accurate ODF Reconstruction in Q-ball Imaging

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Introduction: Q-ball imaging (Tuch 2004) is a high angular resolution diffusion MR imaging technique proven successful in resolving multiple intravoxel fiber orientations. Standard computations of the orientation distribution function (ODF, the probability of diffusion in a given direction) from q-ball use radial projections and do not consider the solid angle, resulting in distributions different from the true ODFs. Moreover, these distributions are not normalized, and tend to be very smooth. This makes it very difficult to identify maxima, and post-processing (sharpening) is generally required. We derive a new formula for the true ODF, which is dimensionless, normalized, and efficiently computable from q-ball acquisition protocols.

Methods: The probability distribution function of the diffusion of water molecules, $P(\vec{r})$, gives the displacement probability $P(\vec{r})dv$ of a molecule, initially placed at the origin, to be in the small volume dv located at $\vec{r}$ after a certain amount of time. This function is represented in spherical coordinates by $\vec{r}=r\hat{u}$, $\hat{u}(\theta,\phi)=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$ being the direction unit vector, and $dv=r^2drd\Omega$ with $d\Omega=\sin\theta\,d\theta d\phi$ being the solid angle element. $ODF(\hat{u})d\Omega$ is the probability of diffusion in the direction $\hat{u}$ in the "small" solid angle $d\Omega$, calculated by integrating the displacement probabilities, $P(\vec{r})dv=P(r\hat{u})r^2drd\Omega$, for all r while keeping $\hat{u}$ constant:

$$ODF(\hat{u})d\Omega=\int_{r=0}^{r=\infty}P(r\hat{u})r^2drd\Omega \qquad \rightarrow \qquad ODF(\hat{u})=\int_0^\infty P(r\hat{u})r^2dr$$

This *correct* definition was used in (Wedeen *et al.* 2005) for diffusion spectrum imaging (DSI), where $P(\vec{r})$ is first computed from the DSI data via Fourier inversion, and then integrated to calculate the ODF. The ODF used in q-ball imaging (Tuch 2004) has so far been different from the above formula, as it has not been weighted by the important factor r^2. We derive a formula for q-ball ODF reconstruction using the correct weighted integral.

Let $E(\vec{q})$ be the 3D Fourier transform of $P(\vec{r})$. We have $E(\vec{q})$ measured on the q-ball, $E(q_0\hat{u})=\tilde{E}(\hat{u})$, and at the origin $E(0)=1$ (zero frequency or the integral of a PDF). Our mathematical derivation is based on the fact that $-\nabla^2E(\vec{q})$ is the Fourier transform of $P(\vec{r})|\vec{r}|^2$. We also use the mono-exponential approximation $E(q\hat{u})\cong E(q_0\hat{u})^{\frac{q^2}{q_0^2}}=\tilde{E}(\hat{u})^{\frac{q^2}{q_0^2}}$, where q_0 is the radius of the q-ball (Stejskal & Tanner 1965; Özarslan *et al.* 2006). We proved that

$$ODF(\hat{u})=\frac{1}{4\pi}+\frac{1}{16\pi^2}\text{Funk\_Radon}\left\{\nabla_b^2\ln\left(-\ln\tilde{E}(\hat{u})\right)\right\}$$

where ∇_b^2 is the Laplace-Beltrami operator. This expression is dimensionless and intrinsically normalized, and to compute it we used an implementation similar to (Descoteaux *et al.* 2007).

Results: Experimental results are presented. Negative values (dark red) are a result of noise in the raw data and the fourth order spherical harmonic approximation used for these experiments. The results on artificial data (top, 76 gradients) show how our approach can better resolve fiber crossings, starting already at 45˚ for our approach vs. 60˚ for the standard ODF computation. The results on real data (middle and bottom) are obtained from a human brain HARDI example of the BrainVISA software (Cointepas *et al.* 2001). It consists of 41 diffusion weighted images with a b-value of 700 s/mm². Note the unique resolution of triple intravoxel configuration (where callosal radiations mingle with the cingulum) obtained with the proposed technique (red frames).

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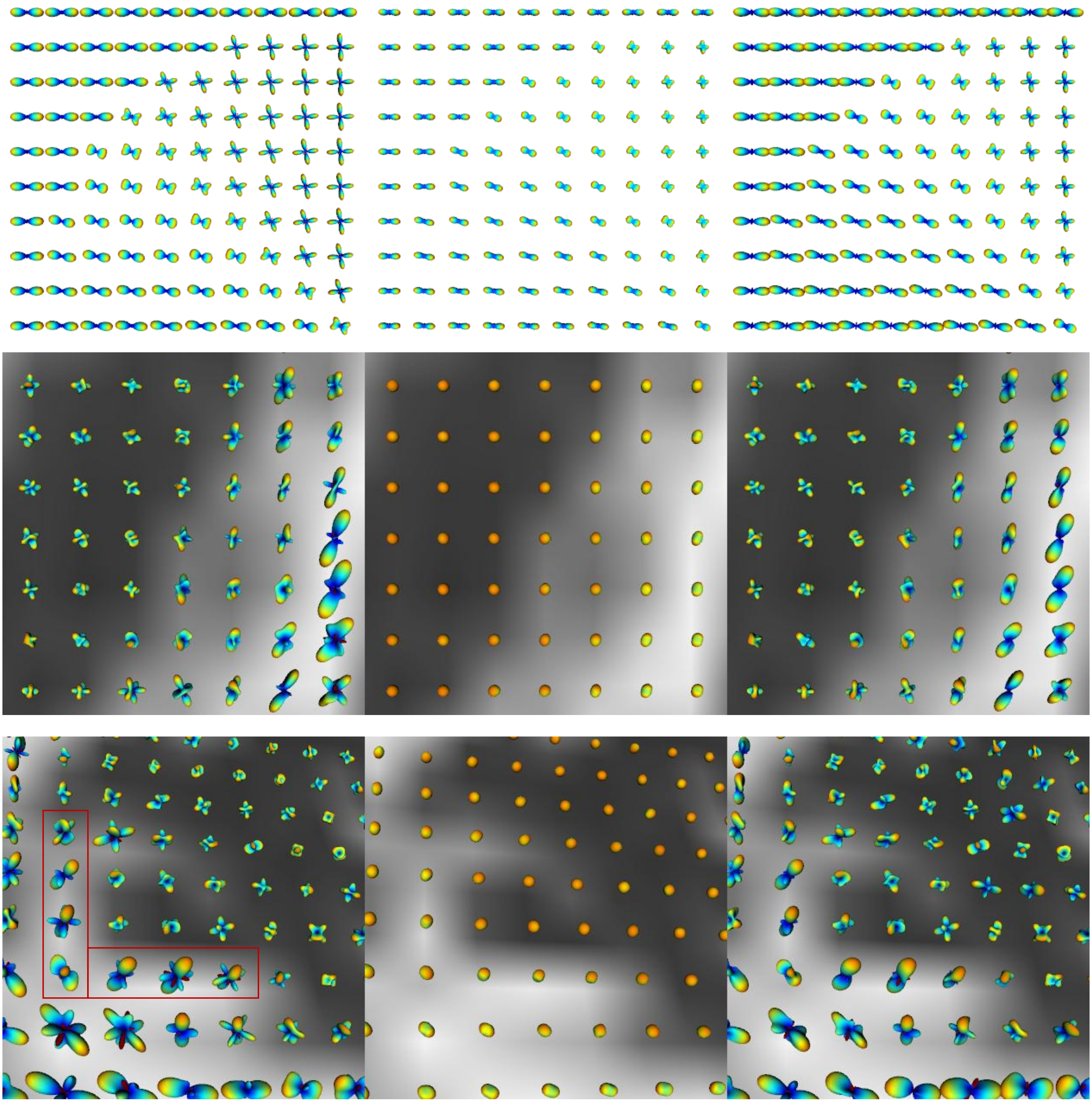

Our ODF reconstruction (with r^2 in the integral) | Original ODF reconstruction (without r^2 in the integral) | Original ODF reconstruction after Laplace-Baltrami sharpening