# INCOME AND SUBSTITUTION EFFECTS IN LABOR FORCE PARTICIPATION AND HOURS OF WORK

by

H. Gregg Lewis

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Center for Economic Research
University of Minnesota
Department of Economics
Minneapolis, Minnesota 55455

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## A. Introduction

Recent developments in the economic analysis of the allocation of the time of persons among alternative activities, especially by Professors Gary Becker and Jacob Mincer, have stimulated much empirical work on the determinants of hours of work and labor force participation rates. Thus we are accumulating a substantial stock of empirical hours of work and labor force participation rate equations incorporating ideas suggested by these recent analytical developments. Much of this work is characterized by close attention to the economic analysis in the formulation of equations to be fitted to data and by interpretation of the coefficients of variables in the fitted equations in terms of income and substitution effects suggested by the analysis.

Let $\underline{\theta}$ be the fraction of a population of persons who choose to work on the labor market during some specified period and $\underline{h}^*$ the mean hours of work of this fraction during this period. Broadly speaking, the variables regarded as the determinants of the labor force participation rate $\underline{\theta}$ are those regarded as the determinants of $\underline{h}^*$. Denote these variables by $\underline{v_1}$, $\underline{v_2}$, . . ., and write the participation and hours of work equations as follows:

$$\theta = \theta(v_1, v_2, \ldots,); \quad h^* = h^*(v_1, v_2, \ldots,)$$

Differentiate these equations logarithmically:

$$d\ln\theta = \eta_{\theta 1} d\ln v_1 + \eta_{\theta 2} d\ln v_2 + \ldots + \; ; \; d\ln h^* = \eta_{h*1} d\ln v_1 + \eta_{h*2} d\ln v_2 + \ldots + \; ;$$

where the $\eta$'s are partial elasticities. Question: Do $\eta_{\theta i}$ and $\eta_{h*i}$ $(i = 1, 2, \ldots,)$ estimate the same thing so that $\eta_{\theta i}$ is an estimate of $\eta_{h*i}$, and conversely? It is sometimes, perhaps often, assumed that the answer to this question is affirmative,[1] yet I know of no demonstration that the affirmative answer is correct. The analysis that follows indicates that the question should be answered negatively.[2] But then what is the correct relation between the labor force participation rate equation and the corresponding hours of work equation?

## B. Textbook Hours of Work Theory and Its Labor Force Participation Implications

I consider first an elementary hours of work theory rather similar to that often presented in introductory textbooks in price theory. Each household, I assume, consists of an adult person whose utility function is

$$u = \left(\alpha \ell^{\frac{\sigma-1}{\sigma}} + x^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \qquad (1)$$

where $u$ is the person's utility, $\ell$ is his rate of consumption of his own time in household activities, $x$ is his rate of consumption of market goods, $\sigma$ (a positive number) is the elasticity of substitution between $x$ and $\ell$, and $\alpha$ (a positive number) is a taste or technique parameter. Both $x$ and $\ell$, of course, are nonnegative. To simplify notation, I express $\ell$ as a fraction of his total time; hence $\ell$ cannot exceed unity, a critical restraint from the point of view of his labor force participation. The marginal rate of substitution in the utility function between $x$ and $\ell$ is

$$\rho(x/\ell\colon u) \equiv -\left.\frac{dx}{d\ell}\right|_u = \alpha\left(\frac{x}{\ell}\right)^{\frac{1}{\sigma}} \qquad (2)$$

I assume that any time not spent by the person in household activities is spent working on the labor market at a given wage rate $w$ greater than zero. The fraction of the person's time spent working on the market is

$$h = 1-\ell; \quad 0 \le h \le 1 \quad . \tag{3}$$

The person's market earnings are

$$g \equiv wh = w(1-\ell) \tag{4}$$

I denote by $y$ the person's income other than his earnings and I assume that $y$ exceeds zero and is given to the household. I use market goods as numeraire so that $y$, $w$, and $g$ are measured in units of market goods. The household's budget restraint then is

$$f \equiv y + w = x + w\ell \tag{5}$$

where $f$ , the household's "full" income, exceeds zero. The marginal rate of substitution in the budget restraint between $x$ and $\ell$ is

$$\rho(x/\ell{:}f) \equiv -\left.\frac{dx}{d\ell}\right|_f = w \quad . \tag{6}$$

Now assume that the household maximizes the utility function (1) subject to the budget restraint (5). The first order condition for this maximum, if there is an interior solution, is the familiar one: Equate the marginal rate of substitution $\rho(x/\ell{:}\ u)$ in the utility function with the corresponding marginal rate of substitution $\rho(x/\ell{:}\ f)$ in the budget restraint. Thus from (2) and (6):

$$\alpha\left(\frac{x}{\ell}\right)^{\frac{1}{\sigma}} = w \quad \text{or} \quad x = \ell(w/\alpha)^{\sigma} \tag{7}$$

Substitute the last result in the budget restraint and solve for $\ell$ :

$$\ell = \frac{f}{p}\left(\frac{w}{\alpha p}\right)^{-\sigma} = \frac{y+w}{\left(\frac{w}{\alpha}\right)^{\sigma} + w} > 0; \quad p^{1-\sigma} \equiv \alpha^{\sigma}w^{1-\sigma} + 1 \ ; \tag{8}$$

where $p$ is the composite price to the household of market goods and household time. Thus $f/p$ is real full income and $w/p$ is the real wage. Because $h = 1 - \ell$, it follows from (8) that

$$h = 1 - \frac{f}{p}\left(\frac{w}{\alpha p}\right)^{-\sigma} = \frac{\left(\frac{w}{\alpha}\right)^{\sigma} - y}{\left(\frac{w}{\alpha}\right)^{\sigma} + w} < 1 \quad . \tag{9}$$

Equation (8) is the household's demand equation for the household time of its member, provided that the right-hand side of the equation does not exceed unity. The right-hand side is greater than unity if and only if $0 < (w/\alpha)^{\sigma} < y$. In that event the correct utility maximizing value of $\ell$ is unity and then

$$h = 0,\ x = y,\ u = \left(\alpha + y^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \quad \text{and} \quad \rho(x/\ell:u) = \alpha y^{\frac{1}{\sigma}} \quad .$$

For values of $y$, $w$, and $\alpha$ such that $y \leq (w/\alpha)^{\sigma}$ equation (8) holds and the equation implies that the real full income elasticity of demand for household time is unity, and the own-price (real wage) elasticity of demand is $-\sigma$. Similarly, equation (9) is the household's hours of work equation if and only if the right-hand side of (9) is not negative -- i.e., the right-hand side of (8) does not exceed unity. If the right-hand side of (9) is not positive, then the utility-maximizing value of $h$ is zero. But if $h$ is zero, the person is not a labor force participant. It follows directly from (9) or (8) that the person is not a labor force participant if and only if $y \geq (w/\alpha)^{\sigma} > 0$ -- that is, if and only if $\alpha y^{\frac{1}{\sigma}} = \rho(x/\ell:u,\ \ell=1) \geq w > 0$. But $\alpha y^{\frac{1}{\sigma}} = \rho(x/\ell:u,\ \ell=1)$ is the marginal rate of substitution between $x$ and $\ell$ in the utility function, or the household's shadow price of household time in terms of market goods, for a nonparticipant person. Thus the condition for nonparticipation in the labor force is that this shadow price be greater than or equal to the market price. For a participant the elasticity of $h$ with respect to real full income $f/p$ is $-\ell/h$ and the elasticity of $h$ with respect to the real wage $w/p$ is $\sigma\ell/h$ .

In order to focus attention on issues of economics rather than of statistical inference, I assume that the preceding theory is completely correct, that though persons may differ in their values of $y$, $w$, and $\alpha$, they all have the same CES utility function (1) with the same value of $\sigma$, and that, except as explicitly noted, precise data are available for each person on $\ell = 1-h$, $y$, $w$, and $\alpha$ and therefore on $f \equiv y + w$ and $x = y + wh$ . Under these circumstances the substitution elasticity $\sigma$ can be calculated easily from the data for any labor force participant with the help of equation (7) as follows:

$$\sigma = \frac{\ell n(x/\ell)}{\ell n(w/\alpha)} \tag{10}$$

Once $\sigma$ has been calculated, the composite price $p = (\alpha^{\sigma} w^{1-\sigma} + 1)^{\frac{1}{1-\sigma}}$ can be calculated for each person.

Consider now a population of persons some of whom, I assume, are labor force participants and others are not, so that in the population $0 < \theta < 1$ . I use as the labor force participation criterion for each person the variable

$$P \equiv \frac{\lambda}{s} \tag{11}$$

where $\lambda$ is a continuous function of $f/p$ and $w/\alpha p$ (or of $y$, $w$, and $\alpha$) that exceeds zero for a labor force participant and is zero or negative for a nonparticipant and $s$ is the standard deviation of $\lambda$ in the population. It is clear from the preceding analysis that $\lambda$ can be expressed as a function of these variables, but in order to establish some general results first, I postpone consideration of the specification of the function. I denote the mean of $P$ in the population of $\bar{P}$ and the corresponding mean of $\underline{\lambda}$ by $\bar{\lambda}$ . The variance of $P$ in the population of course is unity.

Array the persons in the population by their values of the criterion $\underline{P}$ and let

$$N = N(\underline{P}:\underline{\bar{P}},\gamma) \tag{12}$$

be the frequency density function for $\underline{P}$ in the population where $\underline{\gamma}$ denotes any relevant parameters of the density function other than its mean $\underline{\bar{P}}$ . For simplicity in the argument that follows, I assume that the density function is continuous. Then the labor force participation rate $\underline{\theta}$ in the population

$$\theta = \int_0^{\infty} N(P:\bar{P},\gamma)\ dP = \theta(\bar{P},\gamma) = \theta\left(\frac{\lambda}{s},\lambda\right) \quad . \tag{13}$$

Thus in general $\underline{\theta}$ depends upon the form (normal, rectangular, etc.) of the density function $\underline{N(P:\bar{P},\gamma)}$ and hence of its cumulative frequency function $\underline{\theta(\bar{P},\gamma)}$ and upon its parameters $\underline{P} = \underline{\bar{\lambda}/s}$ and $\underline{\gamma}$ . I assume, however, that both the form of $\underline{\theta(P,\gamma)}$ and its parameters $\underline{\gamma}$ are fixed. Then $\underline{\theta}$ is a fixed function of $\underline{\bar{P}} = \underline{\bar{\lambda}/s}$ and it follows from (13) that

$$d\ell n\theta = \frac{N_o}{\theta}\ d\ \underline{\bar{P}} = \frac{N_o}{\theta}\ d\ (\bar{\lambda}/s) \tag{14}$$

where $\underline{N_o}$ is the frequency density at $\underline{P} = \underline{\lambda/s} = 0$ . The coefficient $\underline{N_o/\theta}$ depends only on $\underline{\theta}$ (or equivalently on $\underline{\bar{P}}$) and in the normal distribution and many other distributions declines rapidly as $\theta$ increases.

It is usually true that data on the wage variable $\underline{w}$ and hence on the participation criterion $\underline{\lambda}$ are not available for persons who are not labor force participants. Let $\underline{P}^*$ denote the mean of $\underline{P}$ and $\underline{\lambda}^*$ the mean of

$\underline{\lambda}$ among the labor force participants in the population. Then $\underline{P}^* = \underline{\lambda^*/s}$ . Furthermore, when the parameters $\underline{\gamma}$ are fixed,

$$\underline{P}^* = \frac{1}{\theta(\underline{\bar{P}})} \int_0^\infty \underline{P}N(P:\underline{\bar{P}})\ d\underline{P} = \underline{P}^*(\underline{\bar{P}}) \quad . \tag{15}$$

That is, when the parameters $\underline{\gamma}$ are fixed, $\underline{P}^*$ depends only on $\underline{\bar{P}}$ and inversely $\underline{\bar{P}}$ depends only on $\underline{P}^*$ . But then it follows from (13) that $\underline{\theta}$ depends only on $\underline{P}^*$ . Differentiate equation (15) and use equation (14) to obtain

$$dP = \frac{\theta}{\theta - N_o P^*}\, dP^* \quad . \tag{16}$$

Substitute this result into equation (14):

$$d\ell n\theta = sAdP^* = sAP^* d\ell nP^* = A\lambda^* d\ell n(\lambda^*/s); \qquad A \equiv N_o/s(\theta - N_o P^*) \tag{17}$$

The mean $\underline{P}^* = \underline{\lambda^*/s}$ , of course, is always positive. The elasticity coefficient $\underline{A\lambda}^*$ , like the coefficient $\underline{N_o/\theta}$ in (14), depends only on $\underline{\theta}$ and in the normal distribution and numerous other distributions declines as $\underline{\theta}$ increases.

Of course, when data on $\underline{\lambda}$ are available only for the labor force participants in the population, the standard deviation $\underline{s}$ cannot be calculated and used to "deflate" $\underline{\lambda}^*$ in (17). However, under appropriate circumstances the standard deviation $\underline{s}^*$ of $\underline{\lambda}$ among the labor force participants in the population can be substituted for $\underline{s}$ . Define

$$P^{**} \equiv \lambda^*/s^*;\ q \equiv \left(\frac{s^*}{s}\right) \equiv \left(\frac{P^*}{P^{**}}\right)^2;\ d\ell nq = 2(d\ell ns^* - d\ell ns) = 2(d\ell nP^* - d\ell nP^{**}) \ . \tag{18}$$

By definition

$$q \equiv \left(\frac{s^*}{s}\right)^2 \equiv \left(\frac{P^*}{P^{**}}\right)^2 = \frac{1}{\theta}\int_0^\infty \underline{P}^2 N(\underline{P}:\underline{\bar{P}})\ d\underline{P} - (\underline{P}^*)^2 \tag{19}$$

when the parameters $\underline{\gamma}$ are fixed. Since both $\underline{\theta}$ and the integral in (19) depend only on $\bar{P}$ and $\bar{P}$ in turn depends only on $P^*$, it follows that $q \equiv (P^*/P^{**})^2$ depends only on $P^*$ and hence inversely that $P^*$ depends only on $P^{**} \equiv \lambda^*/s^*$. But then $\theta$ depends only on $P^{**} \equiv \lambda^*/s^*$.

Differentiate (19) and use (16) and (17) to obtain

$$d\ell nq = \eta_{qP^*} d\ell nP^*; \quad \eta_{qP^*} \equiv A\lambda^* \left[\frac{(P^*)^2 - q}{q}\right] = A\lambda^*[(P^{**})^2 - 1] \tag{20}$$

Now substitute $2\,(d\ell nP^* - d\ell nP^{**})$ for $d\ell nq$ in (20) and solve for $d\ell nP^*$:

$$d\ell nP^* = \frac{2}{2-\eta_{qP^*}}\, d\ell nP^{**} \quad . \tag{21}$$

If the elasticity $\eta_{qP^*}$ of $q \equiv (P^*/P^{**})^2$ with respect to $P^* \equiv \lambda^*/s$ is equal to $2$ -- that is, if $q \equiv (P^*/P^{**})^2$ is proportional to $(P^*)^2$, then $P^{**} \equiv \lambda^*/s^*$ is constant, so that $s^*$ is proportional to $\lambda^*$ and does not depend at all upon $s$.[3] In that event $s^*$ cannot be substituted for $s$. Hence I assume that $\eta_{qP^*}$ is not equal to 2. Replace $d\ell nP^*$ in (17) by the right-hand side of (21)

$$d\ell n\theta = \frac{2s^* AP^{**}}{2-\eta_{qP^*}}\, d\ell nP^{**} = \frac{2A\lambda^*}{2-\eta_{qP^*}}\, d\ell n(\lambda^*/s^*) \tag{22}$$

where the elasticity coefficient depends only on $\theta$ and in the normal distribution and some other distributions declines as $\theta$ increases.

The preceding results regarding the relation of the participation rate to the distribution of the participation criterion $\lambda$ are easily summarized. The participation rate $\theta$ is the cumulated frequency over positive values of $\lambda$. Thus the form of the participation rate function is simply that of the cumulative frequency (or probability) function. Furthermore, when the form of the function and its parameters, other than its mean and variance, are fixed, the participation rate $\theta$ depends only on the ratio $P \equiv \lambda/s$ of the population mean of $\lambda$ to its standard deviation, or alternatively on the ratio $\underline{P}^* \equiv \underline{\lambda^*/s}$ of the mean of $\underline{\lambda}$ among labor force participants to the population standard deviation, or on the ratio $\underline{P}^{**} \equiv \underline{\lambda^*/s^*}$ of the mean to the standard deviation of $\underline{\lambda}$ among labor force participants.

For example, suppose that the distribution of $\underline{P} \equiv \underline{\lambda/s}$ is normal. Then

$$\left.\begin{aligned}
&\text{(a)} \quad \theta = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{1}{2}(P-\bar{P})^2} dP \quad ; \\
&\text{(b)} \quad N_o = e^{-\frac{1}{2}(\bar{P})^2} / \sqrt{2\pi} \quad ; \\
&\text{(c)} \quad \frac{d\ell n\theta}{dP} = \frac{N_o}{\theta} \\
&\text{(d)} \quad P^* = \bar{P} + N_o/\theta \\
&\text{(e)} \quad \eta_{\theta P^*} \equiv \frac{d\ell n\theta}{d\ell nP^*} = sAP^* = A\lambda^* = \frac{N_o P^*}{\theta - N_o P^*} \quad ; \\
&\text{(f)} \quad q \equiv \left(\frac{s^*}{s}\right)^2 = \left(\frac{P^*}{P^{**}}\right)^2 = \frac{\theta - N_o P^*}{\theta} \quad ; \\
&\text{(g)} \quad P^{**} = P^*/\sqrt{q} \quad ; \\
&\text{(h)} \quad \eta_{qP^*} = \eta_{\theta P^*} \left[(P^{**})^2 - 1\right] \quad ; \\
&\text{(i)} \quad \eta_{\theta P^{**}} = 2\eta_{\theta P^*}/(2-\eta_{qP^*}) \quad .
\end{aligned}\right\} \quad (23)$$

Tables are readily available showing the values of $\underline{\theta}$ and $\underline{N_o}$ for various values of $\underline{\bar{P}}$ and from these the other concepts listed above can be calculated. Table 1 below shows for selected values of $\underline{\theta}$ the corresponding values of concepts (23c) - (23i) for the normal distribution.

# TABLE 1

## COEFFICIENTS IN THE NORMAL PARTICIPATION RATE FUNCTION

| Coefficient | | Participation Rate $\theta$ | | | | | |
|---|---|---|---|---|---|---|---|
| | | 0.10 | 0.30 | 0.50 | 0.70 | 0.90 | 0.95 |
| (23c) | $\frac{d\ell n\theta}{dP} = \frac{N_o}{\theta}$ | 1.76 | 1.16 | 0.80 | 0.50 | 0.20 | 0.11 |
| (23d) | $P^* = \overline{P} + N_o/\theta$ | 0.47 | 0.63 | 0.80 | 1.02 | 1.48 | 1.75 |
| (23e) | $\eta_{\theta P^*} = N_oP^*/(\theta-N_oP^*)$ | 4.91 | 2.78 | 1.75 | 1.03 | 0.40 | 0.24 |
| (23f) | $q = (\theta-N_oP^*)/\theta$ | 0.17 | 0.26 | 0.36 | 0.49 | 0.71 | 0.81 |
| (23g) | $P^{**} = P^*/\sqrt{q}$ | 1.15 | 1.23 | 1.32 | 1.45 | 1.75 | 1.95 |
| (23h) | $\eta_{qP^*} = \eta_{\theta P^*}[(P^{**})^2-1]$ | 1.59 | 1.45 | 1.32 | 1.15 | 0.83 | 0.66 |
| (23i) | $\eta_{\theta P^{**}} = 2\eta_{\theta P^*}/(2-\eta_{qP^*})$ | 24.1 | 10.2 | 5.13 | 2.42 | 0.69 | 0.35 |

Notice especially how rapidly the coefficients $N_o/\theta$, $\eta_{\theta P^*}$, and $\eta_{\theta P^{**}}$ decline as $\theta$ increases. The table indicates that one should not be surprised to observe much larger coefficients, $N_o/\theta$, $\eta_{\theta P^*}$, and $\eta_{\theta P^{**}}$, of the response of $\theta$ to changes in the participation rate variables $\bar{P}$, $P^*$, and $P^{**}$ respectively for populations with relatively low participation rates (such as married women and other "secondary" labor force participants) than for populations with high participation rates ("prime age" males). Such differences, of course, depend on the form of the frequency distribution of the participation criterion $P$ and have no implications whatsoever for differences among populations in the underlying income and substitution elasticities of the hours of work analysis.

The preceding analysis assumes that not only the form of the distribution of the criterion $\lambda$, but also all of its parameters $\gamma$ ($\gamma$ excludes the mean and variance) are fixed. Of course, if these parameters $\gamma$ change, in general the participation rate $\theta$ also will change. For example, suppose that the density distribution of $P \equiv \lambda/s$ is triangular with a fixed mean $\bar{P}$. Then the participation rate $\theta$ depends only on the skewness of the distribution. In the extreme cases of skewness, the density distribution is a right triangle. Each line of Table 2 -- $\bar{P}$ varies between lines -- compares the participation rate $\theta_o$ in the symmetric triangle (hence no skewness) with $\theta_1$ the participation rate in the positively skewed right triangle and with $\theta_2$ the participation rate in a less highly positively skewed triangle. Skewness, as measured by the third moment of the distribution of $P$, is 0.57 in the right triangle and 0.31 in the less highly skewed triangle. The table shows that for triangular distributions, increasing the amount of skewness raises $\theta$ when $\theta$ is low or high and reduces $\underline{\theta}$ for values of $\underline{\theta}$ close to 0.5. Although the differences, in decimal points, in the participation rates associated with differences in skewness are not large, the table suggests that substantial differences in skewness among populations may have nonnegligible effects on the population participation rates.

I turn now to the specification of the function relating the participation criterion $\underline{\lambda}$ to the variables affecting hours of work. It is only through such a specification, of course, that participation rate equations and hours of work equations can be related to each other. In principle, the form of the $\underline{\lambda}$-function and the variables entering it surely should be derived from the underlying hours of work analysis.

I return then to the hours of work analysis. Denote by $\underline{W}$ the shadow price of household time to a person when he devotes all of his time to household activities $(\underline{\ell} = 1, \ \underline{h} = 0)$ . The person is a labor force participant if and only if $\underline{w}$ exceeds $\underline{W}$ . Hence one possible specification, call it $\underline{\lambda_o}$ , of $\underline{\lambda}$ is $\underline{\lambda} = \underline{\lambda_o} = \underline{\ell n(w/W)^\sigma} = \underline{\sigma \ell n(w/W)}$.[4] Furthermore, according to the admittedly oversimplified theory presented earlier $\underline{W^\sigma} = \underline{y\alpha^\sigma}$ , so that

$$\lambda = \lambda_o = \sigma \ell n(w/W) = \sigma \ell n w - \sigma \ell n \alpha - \ell n y \quad . \qquad (24)$$

Moreover, equations (4), (5), and (7) imply that

$$\sigma \ell n w - \sigma \ell n \alpha = \ell n x - \ell n \ell = \ell n(y+g) - \ell w \ell \quad , \qquad (25)$$

<u>for labor force participants</u>.

Denote the geometric mean of a subscripted variable among the labor force participants in a population by $\underline{G}^*$ and the corresponding variance or covariance by $\underline{V}^*$ . Then it follows from (24) and (25) that

## TABLE 2

EFFECTS OF SKEWNESS ON THE PARTICIPATION RATE $\underline{\theta}$

| $\theta_o$ | $\theta_1$ | $\theta_2$ | $\bar{P}$ |
|---|---|---|---|
| 0.05 | 0.07 | 0.06 | -1.67 |
| 0.10 | 0.12 | 0.11 | -1.35 |
| 0.30 | 0.29 | 0.29 | -0.55 |
| 0.50 | 0.44 | 0.46 | 0.00 |
| 0.70 | 0.63 | 0.67 | 0.55 |
| 0.90 | 0.97 | 0.92 | 1.35 |
| 0.95 | 1.00 | 0.97 | 1.67 |

$$\lambda^{*} = \lambda_{o}^{*} = \sigma \ell n G_{w}^{*} - \sigma \ell n G_{\alpha}^{*} - \ell n G_{y}^{*} = - \ell n G_{y/x} - \ell n G_{\ell}^{*} \quad ;$$

$$(s^{*})^{2} = (s_{o}^{*})^{2} = \sigma^{2} V_{\ell n w}^{*} + \sigma^{2} V_{\ell n \alpha}^{*} + V_{\ell n y}^{*} - 2\sigma^{2} V_{\ell n w, \ell n \alpha}^{*} - 2\sigma V_{\ell n w, \ell n y}^{*} + 2\sigma V_{\ell n \alpha, \ell n y}^{*}$$

$$= V_{\ell n (y/x)}^{*} + V_{\ell n \ell}^{*} + 2V_{\ell n \ell, \ell n (y/x)}^{*} \quad ; \qquad (26)$$

where $y/x$ is the ratio of "other" income $\underline{y}$ to conventionally measured income $\underline{x} = \underline{y + g}$. Now substitute these results into the labor force participation rate equation (22) to obtain

$$\text{(a)} \quad d\ell n\theta = \frac{\eta_{\theta P**}}{sP^{*}} (\sigma d\ell n G_{w}^{*} - \sigma d\ell n G_{\alpha}^{*} - d\ell n G_{y}^{*}) - \eta_{\theta P**}\, d\ell n s_{o}^{*}; \; \eta_{\theta P**} \equiv \frac{2sAP^{*}}{2-\eta_{q P*}} \; ;$$

$$\text{(b)} \quad d\ell n\theta = \frac{-\eta_{\theta P**}}{sP^{*}} (d\ell n G_{y/x}^{*} + d\ell n G_{\ell}^{*}) - \eta_{\theta P**} d\ell n s_{o}^{*} \quad . \qquad (27)$$

Equations (27) imply that the independent variables in the labor force participation rate equation should include not only the means ($\underline{G_{w}^{*}}$, $\underline{G_{\alpha}^{*}}$, and $\underline{G_{y}^{*}}$ or alternatively $\underline{G_{y/x}^{*}}$ and $\underline{G_{\ell}^{*}}$) of the underlying variables ($\underline{w}$, $\underline{\alpha}$, and $\underline{y}$ or alternatively $\underline{y/x}$ and $\underline{\ell}$) but also the standard deviation $\underline{s_{o}^{*}}$ -- that is, the variances and covariances of $\underline{\ell n w}$, $\underline{\ell n \alpha}$, and $\underline{\ell n y}$ or alternatively $\underline{\ell n (y/x)}$ and $\underline{\ell n \ell}$. I do not know of any actual empirical participation rate equations in which the variance of the participation criterion was included as a control variable. Moreover, Table 1 suggests that partial elasticities of $\underline{\theta}$ in equation (27) with respect to the means $\underline{G_{w}^{*}}$, $\underline{G_{\alpha}^{*}}$, and $\underline{G_{y}^{*}}$, or alternatively $\underline{G_{y/x}^{*}}$ and $\underline{G_{\ell}^{*}}$, not only are not constant, but may be substantially higher numerically for populations with low participation rates than for populations with high participation rates.

Equation (27a) implies that the partial elasticities of $\underline{\theta}$ with respect to $\underline{G^*_w}$, $\underline{G^*_\alpha}$, and $\underline{G^*_y}$ are neither equal nor proportional to the corresponding elasticities in the hours equations (8) and (9) even when the latter elasticities are evaluated at $\underline{w} = \underline{G^*_w}$, $\underline{\alpha} = \underline{G^*_\alpha}$, and $\underline{y} = \underline{G^*_y}$ . This proposition may be verified by differentiating equations (8) and (9):

$$d\ell nh = -\frac{\ell}{h}\, d\ell n\ell = \frac{\ell}{h}\,[\sigma d\ell n(w/p) - \sigma d\ell n\alpha - d\ell n(f/p)]$$
$$= \frac{\ell}{hf}\,\{[(y+g)\sigma - g]d\ell nw - (y+g)\sigma d\ell n\alpha - yd\ell ny\} \tag{28}$$

and comparing the elasticities with those in (27a). Indeed, the wage elasticity, which is positive in the participation rate equation (27a), may be negative in the corresponding hours equation (28) in which the elasticity is calculated holding "other" income $\underline{y}$ , rather than full income $\underline{f} \equiv \underline{y + w}$, constant. Thus if the specification $\underline{\lambda_o}$ of $\underline{\lambda}$ is correct, the question asked at the outset of the paper must be answered negatively.

However, if the hypothesis that $\underline{\lambda} = \underline{\lambda_o}$ is correct, it is possible to interpret the participation rate equation in terms of underlying "substitution and income" effects -- that is, it is possible to go from the participation rate equation to the corresponding hours equation. Let me be more precise. Define the following partial elasticities:

$\underline{\eta_{hw}}$ : of $\underline{h}$ with respect to $\underline{w/p}$ holding $\underline{\alpha}$ and $\underline{f/p}$ constant;

$\underline{\eta_h}$ : of $\underline{h}$ with respect to $\underline{f/p}$ holding $\underline{w/\alpha p}$ constant;

$\underline{\eta_{\theta w,\theta}}$ : of $\underline{\theta}$ with respect to $\underline{G^*_w}$ holding $\underline{G^*_\alpha}$, $\underline{G^*_y}$, and $\underline{s^*_o}$ constant in (27a);

$\eta_{\theta y,o}$ : of $\underline{\theta}$ with respect to $\underline{G^*_y}$ holding $\underline{G^*_w}$, $\underline{G^*_\alpha}$, and $\underline{s^*_o}$ constant in (27a).

Then if the hypothesis that $\underline{\lambda} = \underline{\lambda_o} = \underline{\sigma \ell n(w/W)} = \underline{\sigma \ell n w} - \underline{\sigma \ell n \alpha} - \underline{\ell w y}$ is correct, it follows from (27a) and (28) that

$$\frac{\eta_{\theta w, o}}{\eta_{\theta y, o}} = \frac{\eta_{hw}}{\eta_{hf}} = -\sigma \quad , \tag{29}$$

which is a testable implication of this hypothesis.

The formulation of the participation rate equation suggested by (27b) is, I think, unusual in that it includes the (geometric) mean household hours $\underline{G^*_{\ell}} = \underline{G^*_{1-h}}$, an independent variable and does not require data on the "taste or technique" variable $\underline{\alpha}$. An empirical participation rate equation according to this formulation, of course, would not provide an estimate of the substitution elasticity $\underline{\sigma}$. Rather the purpose of this formulation is to provide an additional test of the hypothesis that $\underline{\lambda} = \underline{\lambda_o}$. Assume that the hypothesis is correct. Then it follows from (27b) that

$$\eta_{\theta G^*_{\ell}} = \eta_{\theta G^*_{y/x}} \tag{30}$$

where $\underline{\eta_{\theta G^*_{\ell}}}$ is the partial elasticity in (27b) of $\underline{\theta}$ with respect to $\underline{G^*_{\ell}}$ and $\underline{\eta_{\theta G_{y/x}}}$, is the corresponding partial elasticity with respect to $\underline{G^*_{y/x}}$.

The hypothesis $\underline{\lambda} = \underline{\lambda_o}$, of course, is only one of many possible hypotheses about the $\underline{\lambda}$-function. Each alternative hypothesis about $\underline{\lambda}$ leads to an alternative pair of hypotheses about the participation rate equation comparable to (27) for the specification $\underline{\lambda} = \underline{\lambda_o}$ and these in turn lead to an alternative way of relating the participation rate equation to the hours equations (8) and (9), comparable to (29) and (30). The specification $\underline{\lambda} = \underline{\lambda_1} = \underline{a\lambda_o}$, where $\underline{a}$ is a positive constant such as $1/\sigma$, of course, is the same as the hypothesis $\underline{\lambda} = \underline{\lambda_o}$. The following specifications

$$\lambda = \lambda_2 = e^{\lambda_o} - 1 = \left(\frac{w}{W}\right)^{\sigma} - 1 = \left(\frac{w/\alpha}{y}\right)^{\sigma} - 1$$

$$\lambda = \lambda_3 = e^{\lambda_o/\sigma} - 1 = \frac{w}{W} - 1 = \frac{w}{\alpha y^{\frac{1}{\sigma}}} - 1$$

$$\lambda = \lambda_4 = 1 - e^{-\lambda_o} = 1 - \left(\frac{W}{w}\right)^{\sigma} = 1 - \left(\frac{y}{w/\alpha}\right)^{\sigma}$$

$$\lambda = \lambda_5 = 1 - e^{-\lambda_o/\sigma} = 1 - \frac{W}{w} = 1 - \frac{\alpha y^{\frac{1}{\sigma}}}{w}$$

and a number of other obviously similar ones such as $\underline{\lambda} = \underline{\lambda_6} = e^{\lambda_o/2} - 1$ are not the same as $\underline{\lambda} = \underline{\lambda_o}$, but, because they rank the persons in a population in the same way as does $\underline{\lambda} = \underline{\lambda_o}$, I suspect that in empirical work it will be difficult to distinguish between any of them and the specification $\underline{\lambda} = \underline{\lambda_o}$.

On the other hand, the specification $\underline{\lambda} = \underline{\lambda_7} = \underline{W\lambda_3} = \underline{w\lambda_5}$ will tend to rank the persons in a population in a different way than do the preceding specifications, so that the differences between $\underline{\lambda_7}$ and the preceding specifications in implications for the participation rate equation may be quite enough to be distinguished in the empirical data. The same is true of a specification suggested by equation (27b). Suppose that in the empirical test of the hypothesis $\underline{\lambda} = \underline{\lambda_o}$ indicated by equations (27b) and (30) the elasticity $\underline{\eta_{\theta G^*_{y/x}}}$ of $\underline{\theta}$ with respect to $\underline{G^*_{y/x}}$ consistently were to be estimated as equal to zero. This would imply that the correct specification of $\underline{\lambda}$ is

$$\lambda = \lambda_8 = -\ell n\left[\frac{f}{p}\left(\frac{w}{\alpha p}\right)^{-\sigma}\right] = -\ell n\left[\frac{y + w}{\left(\frac{w}{\alpha}\right)^{\sigma} + w}\right] = \ell n\left[\frac{y\left(\frac{w}{W}\right)^{\sigma} + w}{y + w}\right] = \ell n\,\frac{ye^{\lambda_o} + w}{y + w} \qquad (31)$$

Furthermore, for labor force participants $\underline{\lambda} = \underline{\lambda_8} = -\underline{\ell n \ell} = \underline{\ell n\left(\frac{1}{1-h}\right)}$ so that

$\underline{\lambda}^* = \underline{\ell n G^*_\ell}$ . Thus, equation (27b) implies,

$$d\ell n\theta = - \frac{\eta_{\theta P**}}{sP^*} d\ell n G^*_\ell - \eta_{\theta P**} d\ell n s^*_8 \tag{32}$$

where $\underline{s^*_8}$ is the standard deviation of $\ell n\ell = \ell n(1-h)$ among the labor force participants in a population. The obvious implication of the hypothesis that $\underline{\lambda} = \underline{\lambda_8}$ is that the partial elasticities in the labor force participation rate equation with respect to the wage, income, and taste or technique variables are proportional to the corresponding elasticities in the hours equations (8) and (9). The factor of proportionality with respect to (8) is $\underline{\eta_{\theta P**}/\ell n G^*_\ell}$ , which is negative because $\underline{\ell n G_\ell}$ is negative, and that for (9) is $\underline{\eta_{\theta P**}(G^*_{\ell-1})/G^*_\ell \ell n G^*_\ell}$ , which is positive. Notice that the specification $\underline{\lambda} = \underline{\lambda_9} = \underline{1 - e^{-\lambda_8}}$ ( = $\underline{h}$ for labor force participants) is very similar to $\underline{\lambda} = \underline{\lambda_8}$ so that the two specifications may be difficult to distinguish empirically.

It may be argued that the hypothesis $\underline{\lambda} = \underline{\lambda_8}$ and similar ones such as $\underline{\lambda} = \underline{\lambda_9}$ already have been shown to be false. I have the impression that wage elasticities typically have been found to be negative in empirical hours of work equations and positive in participation rate equations, while "other income" elasticities typically have been negative in both equations. Indeed, it was this impression that initially set me to work on the subject of this paper. There are several problems, however, in using the existing stock of equations to test hypothesis about the $\underline{\lambda}$-functions. First, none of the participation rate equations that I know of has included $\underline{s}^*$ , the standard deviation of $\underline{\lambda}$ among labor force participants, among the explanatory variables and this omission may lead to biased estimates of the wage and income elasticities. Second, many of the hours equations have no "matching"[5] participation rate equations, and conversely. Third, in many of the hours equations errors of measurement in the dependent hours of work variable may very well be negatively correlated with the errors of measurement in the independent wage variable and this may lead to estimates of the wage elasticity that are negative even though the true elasticity is positive. (This negative correlation of errors of measurement may arise when the wage is estimated by dividing earnings by the estimate of hours of work used as the dependent variable.)

The task of testing hypotheses about the relation of the participation criterion $\underline{\lambda}$ to the underlying economic variables in the hours of work analysis remains to be done. Until it is done, the relation of participation rate equations to hours of work equations will remain ambiguous This is the chief negative conclusion of this paper.

## FOOTNOTES

* This paper corrects and extends my two earlier papers on the same subject (Lewis, 1968, 1969). See also the recent (1971) paper by Professor Yoram Ben-Porath.

1. See Kosters (1966, 1968) and Cain (1966), for example. Bowen and Finegan (1969), however, answer the question negatively, though without stating their view of the correct relation between the $\eta_\theta$'s and the $\eta_h$'s.

2. This conclusion is one in which Ben-Porath concurs.

3. The elasticity $\eta_{qP^*}$ is equal to 2, for example, in a rectangular density distribution and in several other linear distributions.

4. In the first of my two earlier papers (Lewis, 1968, 1969) I used the specification $\lambda = w/W - 1$ and in the second $\lambda = \ell n(w/W)$, both of which are quite similar to the specification $\lambda = \ell n(w/W)^\sigma$.

5. The equations do not match in the sense that they pertain to different population groups, different dates, include different independent variables, and so on.