Understanding the Estimated Power Distribution in Simulated Seismic Environment

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Project Overview

One of the major contributors to noise faced by the Laser Interferometer Gravitational-wave Observatory (LIGO) detectors is noise from vibrations in the Earth: Seismic Noise. To reduce the Seismic Noise would be to increase the likelihood of the direct detection of gravitational waves.

While data is streaming from an array of seismometers at the Homestake Mine in South Dakota, computer simulations are done to analyze the evolution of a seismic environment as different seismological parameters are varied.

The existing code responsible for conducting the aforementioned seismic simulations is continually being modified in the hopes of obtaining increasingly realistic representations of a seismic environment in which future gravitational wave detectors will be placed. However, one of the aliasing effects seen in the simulations is the presence of negative power in pixels of the produced skymaps.

For the most realistic results—ultimately aiding in the minimization of Seismic Noise—the code must be modified so as to minimize the aliasing effects in the simulated output.

Problem in Question

Does varying the injection frequency or the number of seismometers in the array affect the estimated power distribution across different directions in a simulated seismic environment?

Variables

Controlled variables	Independent variables	Dependent variable
• Seismic wave: P • Wave amplitude: 5 • Injection coordinate: (ϕ, θ) = (60°, 0°) • Observation time: 100 sec.	• Number of seismometers in array • Injection frequency (Hz)	• Power (m^2/Hz)

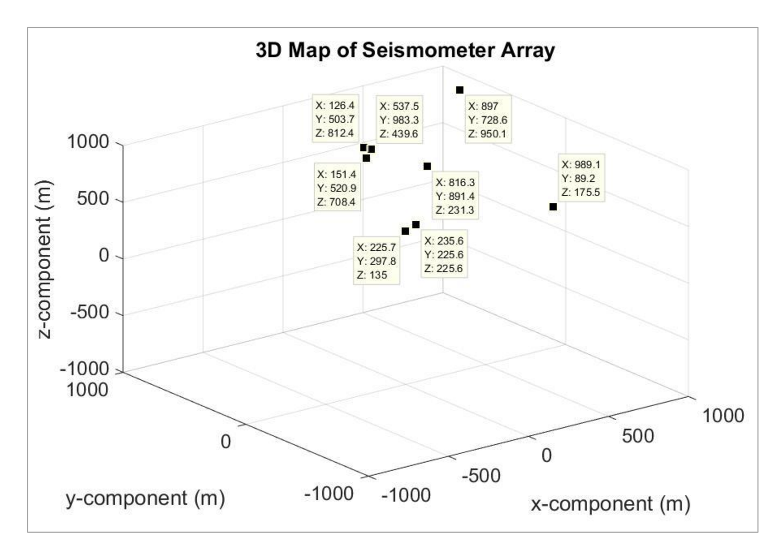
Procedure

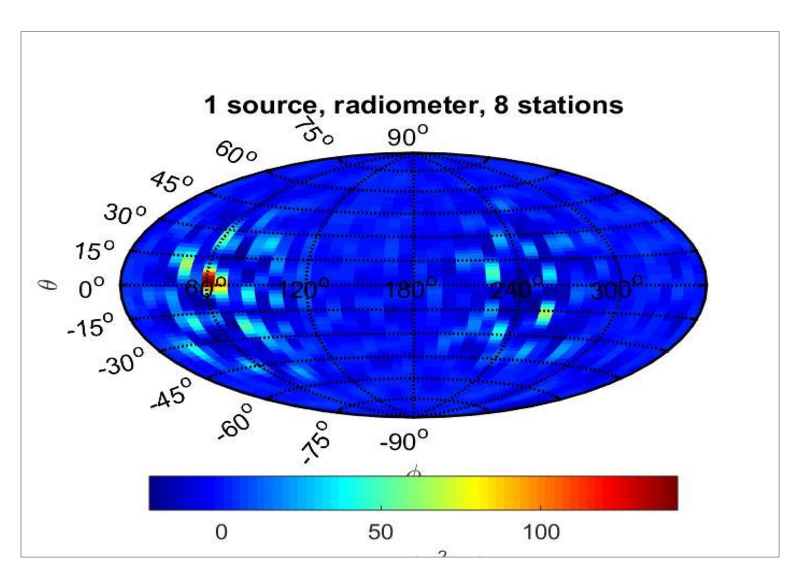
In existing Matlab code:

A random number generator was used to produce Cartesian coordinates for 8 seismometer locations. The injection frequency was determined, and the number of detectors was varied by using the Matlab conventions for (un)commenting the locations. After the array configuration and an injection frequency were specified—and the simulation was run—a skymap of the recovered power was produced, as well as a 3D visualization of the array configuration and a histogram displaying the power contributions from each pixel of the skymap. The array configuration was then kept fixed, and the injection frequency was allowed to vary; after increasing the number of seismometers in the array, the injection frequency was varied in the same manner as before. After a significant number of skymaps and histograms were produced, mathematical models were overlaid on the histograms to aid in the determination of the power contributions.

Figures

An example of the skymaps and histograms produced throughout the experiment are placed below. For context: 8 seismometers were used to recover a simulated P-wave injected at 5.5 Hz.





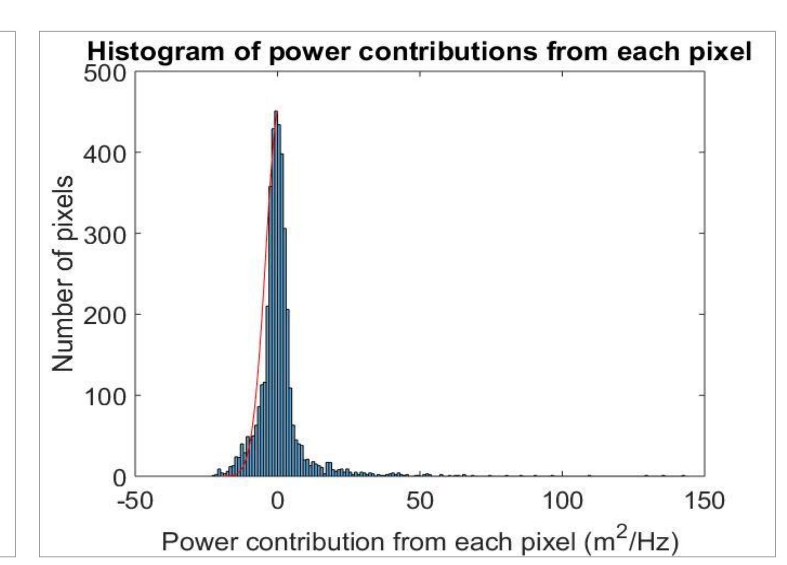


FIG 1: A physically realistic representation is given in the 3D map of the seismometer array (top-center). The skymap (bottom-left) displays the recovered power of a P-wave injected at (φ, θ) = (60º, 0º). Overlaid on the histogram (bottom-right) is an empirically-found Gaussian distribution. This behavior is indicative of nearly all other trials done. Note the magnitude of the power contributions; these power contributions—with an array of 8 seismometers and an injection frequency of 5.5 Hz—are much larger than those with an array of fewer seismometers and smaller injection frequency.

Results

Increasing the number of seismometers or injection frequency did not minimize the negative power pixel contributions; in fact, the (magnitude of) negative power in each pixel increased with each addition of a seismometer in the array and generally increased with increasing frequency.

However—in modelling the behavior of the negative power—the negative power contributions, x, from N pixels are modelled by a Gaussian probability distribution, $G(x)$; namely:

$$G(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{\frac{(x-\mu_x)^2}{2\sigma_x^2}}$$

where σ_x is the standard deviation and empirically found to be:

$$\sigma_x = \sqrt{\frac{1}{4N}\sum_{i=1}^{N}(x_i-\mu)^2}$$

and μ_x is the mean:

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Furthermore, the Gaussian behavior of the negative power pixels becomes more realized as the injection frequency or number of detectors in the array increases; this is reminiscent of the Central Limit Theorem.

Through statistical analysis, it has been shown that decreasing σ_x forces $G(x)$ to become increasingly leptokurtic, that is, increasingly narrow.

Conclusions

While the origin of the observed estimated power distribution is not well understood, it is believed to be caused by the recovery algorithm deployed in the analysis. This work shows that the number of seismometers in an array does not minimize the negative power in pixels.

It is planned to repeat the experiment, but examine how the configuration of seismometers in the array effects the negative power contributions.

Works Cited

Mandic, V., Thrane, E., Tsai, V., *Directional Analysis of Seismic Data,* 2014.