Heavy Quark Masses

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SFB TR9

Outline

- Motivation
- Charm quark I:
  pQCD and R^{exp}
- Bottom quark
- Charm quark II:
  pQCD meets LQCD
- Summary

Quark masses

- Fundamental parameters of the Standard Model
- B decays: $\Gamma \sim m_b^5 \ldots$, $\bar{B} \to X_s\gamma$
- Spectroscopy
- Higgs decay ➪ ILC

$$\Gamma(H \to b\bar{b}) = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2 \left(1 + \mathcal{O}(\alpha_s) + \ldots\right)$$

- Yukawa unification needed:
$\frac{\delta m_t}{m_t} \approx \frac{\delta m_b}{m_b}$ ➪ $\delta m_t = 1$ GeV ➪ $\delta m_b = 25$ MeV

Quark mass definitions

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^2_{\mu\nu} + \sum_q \bar{\psi}_q \left(\not{D} - m_q \right) \psi_q$$

- pole mass
- $\overline{\rm MS}$ mass
- kinetic mass [Bigi,Shifman,Uraltsev,Vainshtein'97]
- 1S mass [Hoang,Smith,Stelzer,Willenbrock'99]
- PS mass [Beneke'98]
- RS mass [Pineda'01]
- ...

Choose quark mass definition in theory calculations
➪ this mass is extracted when comparison with experiment is done

Light quark masses

PDG:

$$
\begin{aligned}
m_u &= 1.5 \ldots 3.0 \text{ MeV} \\
m_d &= 3 \ldots 7 \text{ MeV} \\
\overline{m} = \frac{m_u + m_d}{2} &= 2.5 \ldots 5.5 \text{ MeV} \\
m_s &= 95 \pm 25 \text{ MeV}
\end{aligned}
$$

➪ less accurately known than heavy quark masses

Recent results for m_s (2005/2006):

[Chetyrkin,Khodjamirian'06; Jamin et al.'05'06; Narison'05; Gamiz'05; MILC/HPQCD'06;JP-PACS/JLQCD'06]

➪ $m_s(2\text{ GeV}) = 94 \pm 10$ MeV

Top quark masses

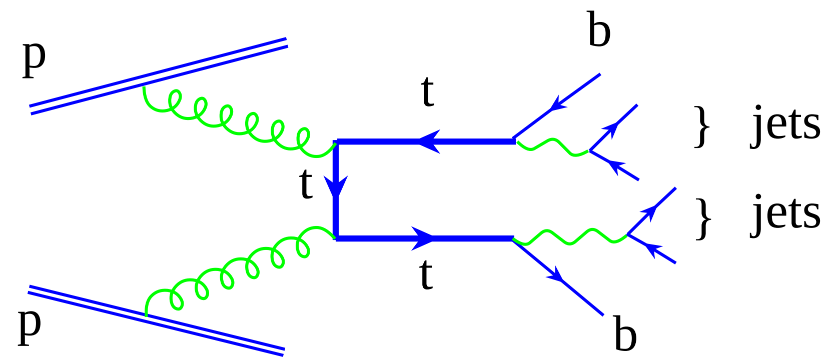
- Tevatron

  $p + \bar{p} \to t\bar{t} \to Wb + Wb \to 4$ jets $+ \mu + \nu$

  ➪ reconstruct top quark ➪ $m_t = 172.6 \pm 0.8 \pm 1.1$ GeV

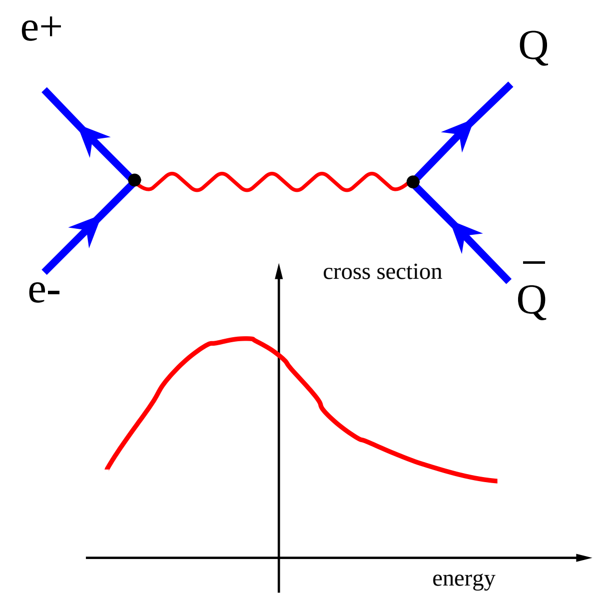
  [CDF+D0]

  - Which mass? Pole mass?



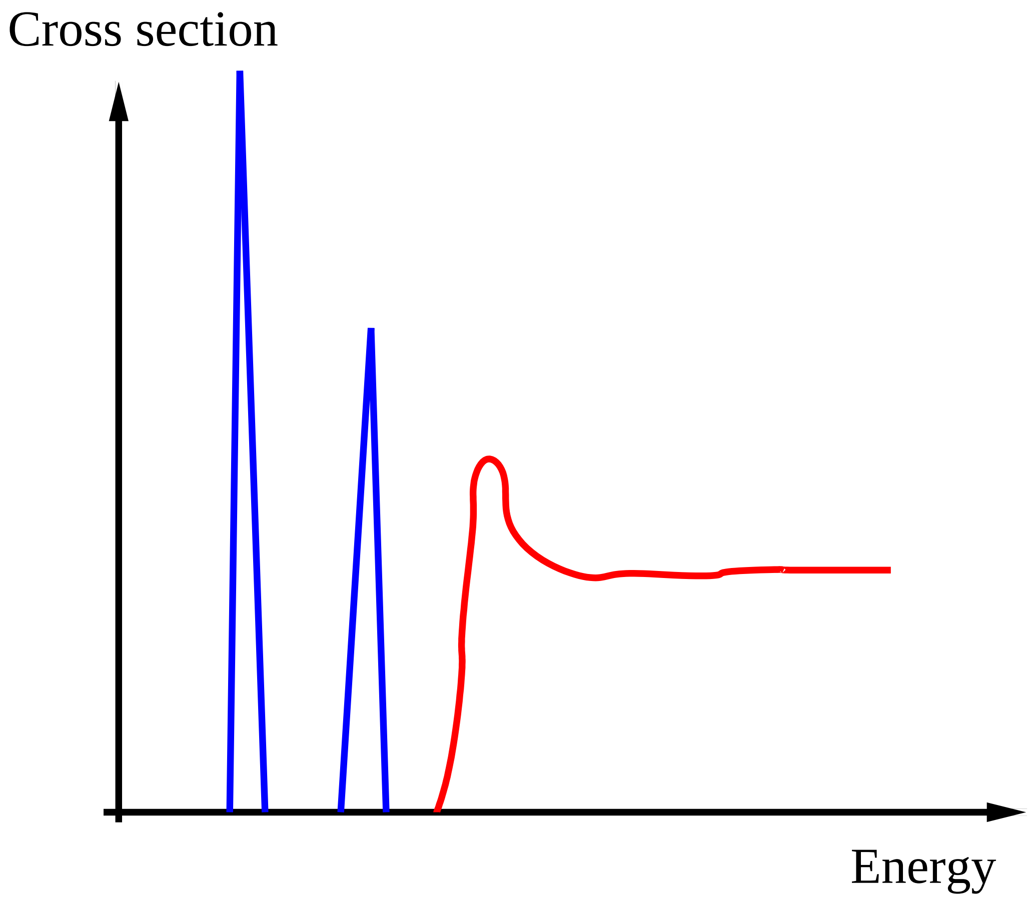
- LHC ➪ $\delta m_t \sim 1$ GeV

- International Linear Collider (ILC)
  - Well-defined c.m. energy
  - Clean initial state
  - Threshold scan
  - Simple counting of $t\bar{t}$
  - $\delta m_t < 100$ MeV



Charm/Bottom

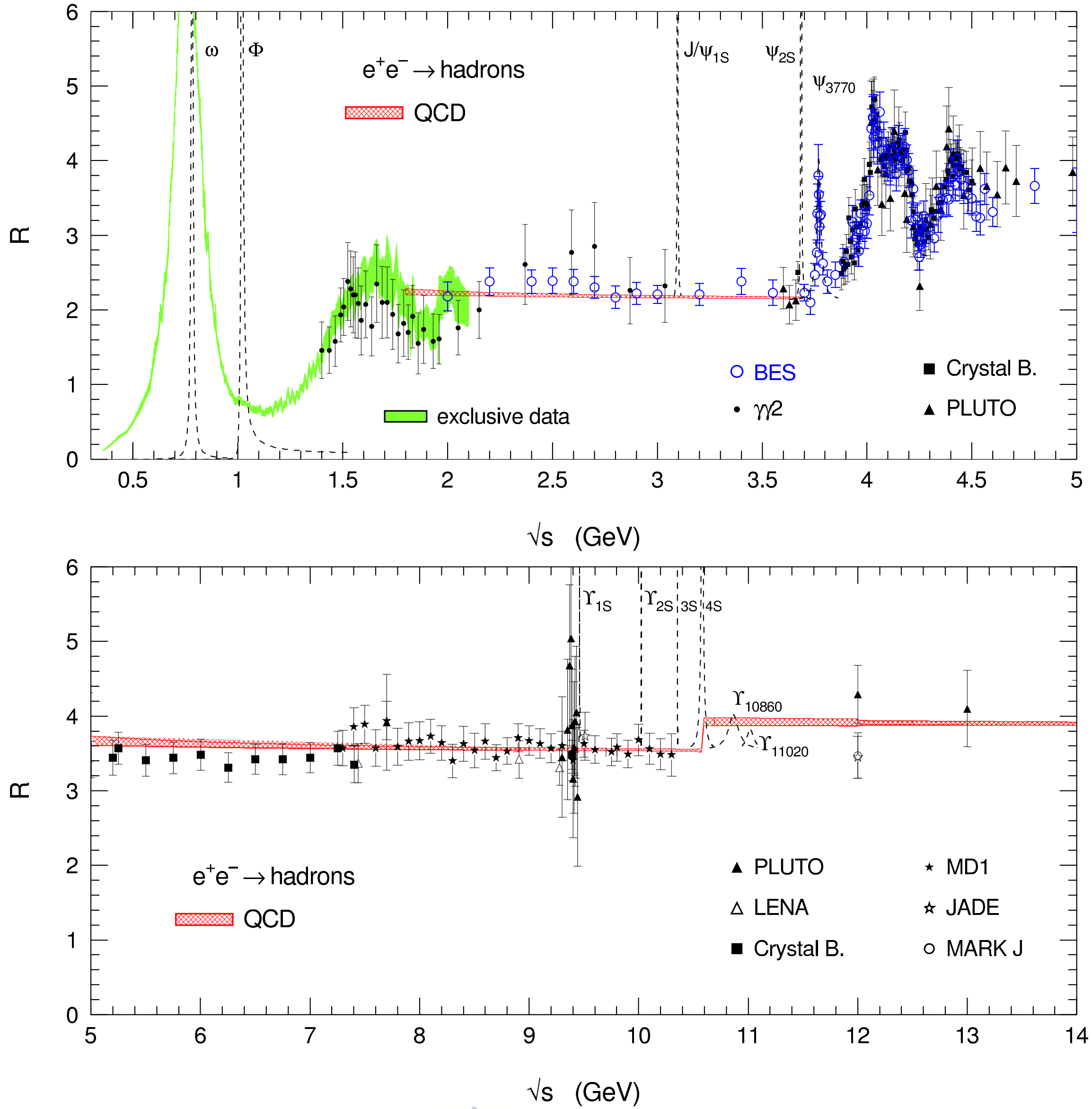
- Consider $\sigma(e^+e^- \to \text{hadrons})$



- m_c, m_b ➪ sum rules, (“SVZ” sum rules)

  [Novikov,Okun,Shifman,Vainshtein,Voloshin,Zakharov’78]

R^{exp}



[Davier,Eidelman,Höcker,Zhang‘02]

Charm quark

Sum rules

$$R_Q = \frac{\sigma(e^+e^- \to Q\bar{Q} + \ldots)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\mathcal{M}_n \equiv \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s)$$ (moments)

$$R_Q = 12\pi \mathrm{Im}\left[\Pi_Q(q^2 = s + i\varepsilon)\right]$$

$$\mathcal{M}_n = \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_Q(q^2)\Bigg|_{q^2=0}$$

(dispersion relation)

Sum rules (2)

$$\mathcal{M}_n \equiv \int \frac{\mathrm{d}s}{s^{n+1}} R_Q(s)$$

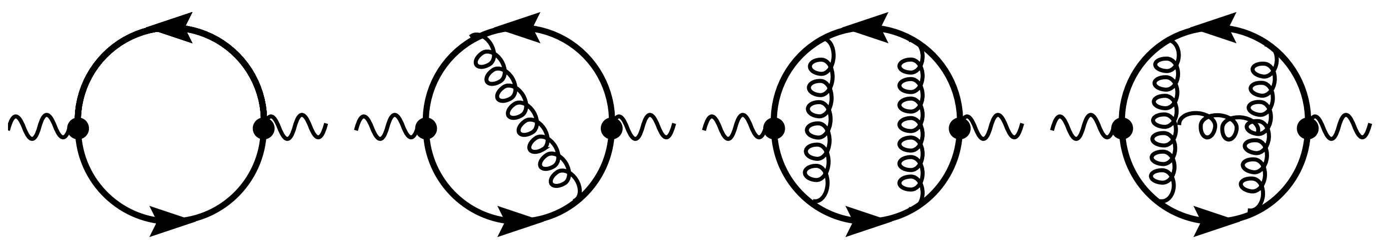
- n large ($\approx 10 \ldots 20$): "Υ sum rules"
  ➪ threshold region important
  ➪ NRQCD, ... [Penin,Pivovarov'98; Melnikov,Yelkhovsky'98;Beneke,Signer'99;Hoang'00]
- here: n small ($n \leq 4$)
  ➪ fixed-order perturbation theory sufficient

$\mathcal{M}_n^{\rm th}$

$$\mathcal{M}_n = \frac{12\pi^2}{n!}\left(\frac{\rm d}{{\rm d}q^2}\right)^n \Pi_Q(q^2)\bigg|_{q^2=0}$$

➪ compute Taylor expansion

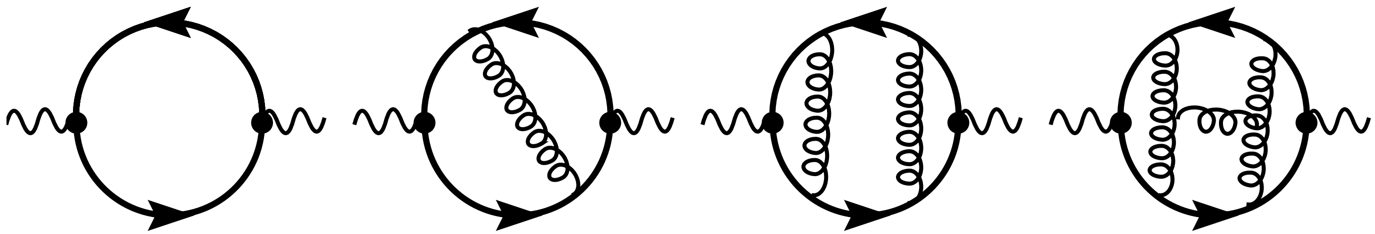
$$\Pi_Q(q^2) = Q_Q^2 \frac{3}{16\pi^2}\sum_{n\geq 0} \bar{C}_n \left(\frac{q^2}{4m_Q^2}\right)^n$$



$$\mathcal{M}_n^{\rm th} = \frac{9}{4}Q_Q^2\left(\frac{1}{4m_Q^2}\right)^n \bar{C}_n \qquad \frac{\delta m_Q}{m_Q} = \frac{1}{2n}\frac{\delta\mathcal{M}_n}{\mathcal{M}_n}$$

C_n to 4 loops

$\bar{C}_n$:



- 3 loops:

  $n <= 8$: [Chetyrkin,Kühn,MS'95'96'97]

  $n \leq 30, ..., 40$ [Boughezal,Czakon,Schutzmeier'06; Maier,Maierhofer,Marquark'07]

- 4 loops:

  $n = 1$: [Chetyrkin,Kühn,Sturm'06; Boughezal,Czakon,Schutzmeier'06]

  $n = 2$: [Maier,Maierhofer,Marquark,Smirnov'08] NEW, PRELIMINARY

C_n to 4 loops

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\left(\bar{C}_n^{(10)} + \bar{C}_n^{(11)} l_{m_c}\right)$$
$$+\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\left(\bar{C}_n^{(20)} + \bar{C}_n^{(21)} l_{m_c} + \bar{C}_n^{(22)} l_{m_c}^2\right)$$
$$+\left(\frac{\alpha_s(\mu)}{\pi}\right)^3\left(\bar{C}_n^{(30)} + \bar{C}_n^{(31)} l_{m_c} + \bar{C}_n^{(32)} l_{m_c}^2 + \bar{C}_n^{(33)} l_{m_c}^3\right)$$

$$l_{m_c} = \ln(m_c^2/\mu^2)$$

n	$\bar{C}_n^{(0)}$	$\bar{C}_n^{(10)}$	$\bar{C}_n^{(11)}$	$\bar{C}_n^{(20)}$	$\bar{C}_n^{(21)}$	$\bar{C}_n^{(22)}$	$\bar{C}_n^{(30)}$	$\bar{C}_n^{(31)}$	$\bar{C}_n^{(32)}$	$\bar{C}_n^{(33)}$
1	1.0667	2.5547	2.1333	2.4967	3.3130	−0.0889	−5.6404	4.0669	0.9590	0.0642
2	0.4571	1.1096	1.8286	2.7770	5.1489	1.7524	—	6.7216	6.4916	−0.0974
3	0.2709	0.5194	1.6254	1.6388	4.7207	3.1831	—	7.5736	13.1654	1.9452
4	0.1847	0.2031	1.4776	0.7956	3.6440	4.3713	—	4.9487	17.4612	5.5856

C_n to 4 loops

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$$-6.0 \le \bar{C}_2^{(30)} \le 7.0\,, -6.0 \le \bar{C}_3^{(30)} \le 5.2\,, -6.0 \le \bar{C}_4^{(30)} \le 3.1$$

C_n to 4 loops

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4-loop, $n = 2$ [Maier,Maierhofer,Marquark,Smirnov'08] NEW, PRELIMINARY

$$-6.0 \le \bar{C}_2^{(30)} \le 7.0\,,\ -6.0 \le \bar{C}_3^{(30)} \le 5.2\,,\ -6.0 \le \bar{C}_4^{(30)} \le 3.1$$

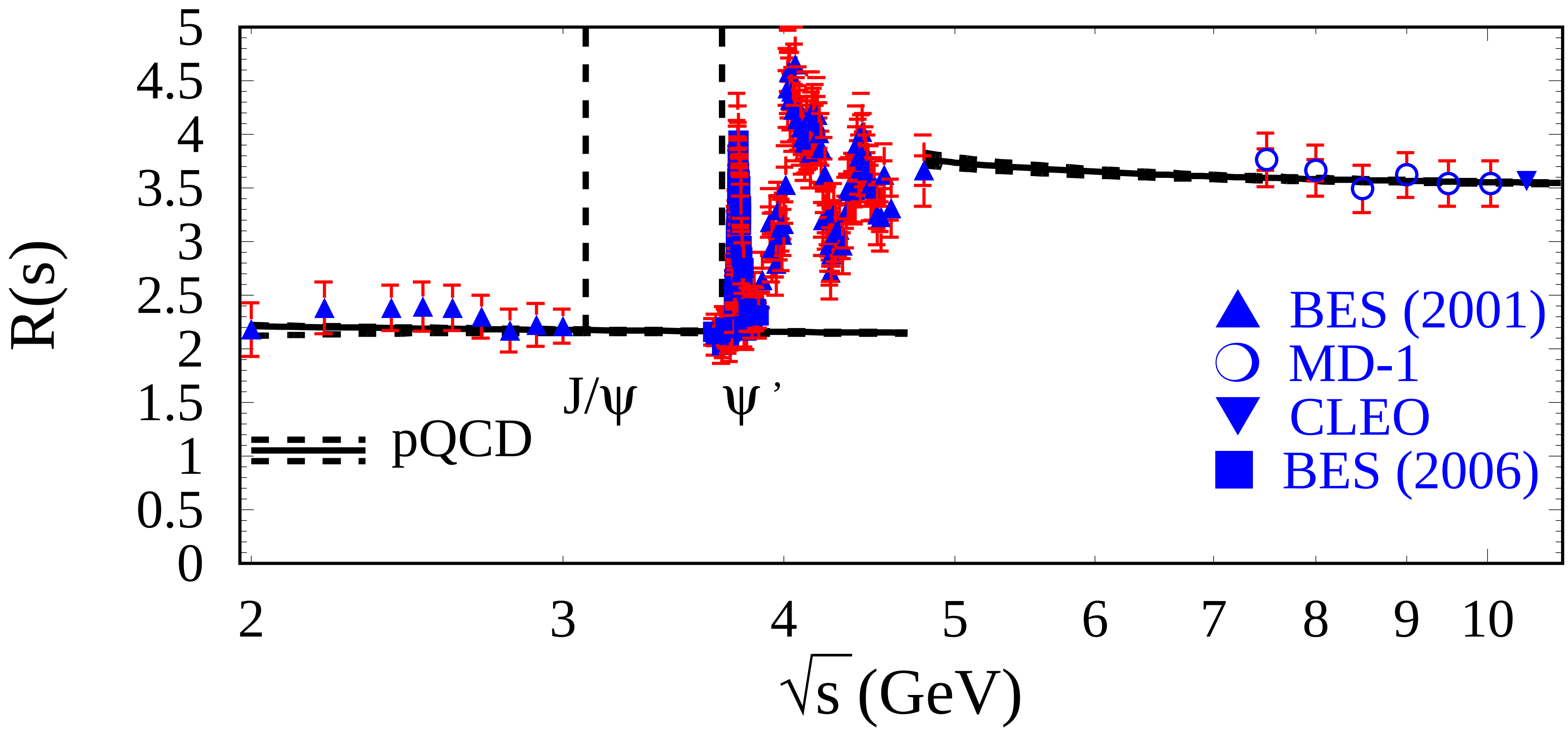
$$\mathcal{M}^{\text{exp}} = \mathcal{M}^{\text{res}} + \mathcal{M}^{\text{thresh}} + \mathcal{M}^{\text{cont}}$$

- $\mathcal{M}^{\text{res}}$: $R^{\text{res}}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$

	J/Ψ	$\Psi(2S)$
M_Ψ(GeV)	3.096916(11)	3.686093(34)
Γ_{ee}(keV)	5.55(14)	2.48(6)
$(\alpha/\alpha(M_\Psi))^2$	0.957785	0.95554

$$\mathcal{M}^{\rm exp} = \mathcal{M}^{\rm res} + \mathcal{M}^{\rm thresh} + \mathcal{M}^{\rm cont}$$

- $\mathcal{M}^{\rm res}$: $R^{\rm res}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2}\left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$
- $\mathcal{M}^{\rm thresh}$:



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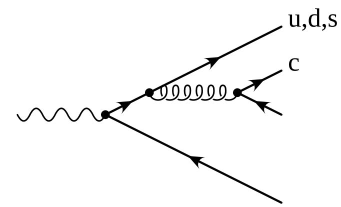
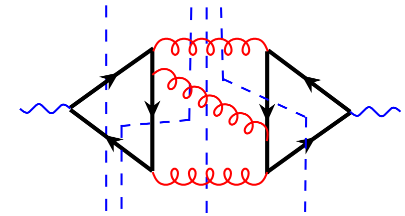
- $\mathcal{M}^{\rm res}$: $R^{\rm res}(s) = \frac{9\pi M_R \Gamma_{ee}}{\alpha^2} \left(\frac{\alpha}{\alpha(s)}\right)^2 \delta(s - M_R^2)$
- $\mathcal{M}^{\rm thresh}$: 3.73 GeV $\le \sqrt{s} \le 4.8$ GeV, BES01,06
- $\mathcal{M}^{\rm cont}$: $\sqrt{s} \ge 4.8$ GeV

no data

$R^{\rm theory}$ ⇨ full mass dependence up to $\mathcal{O}(\alpha_s^2)$ rhad: [Harlander,MS'02]

$\mathcal{M}^{\text{thresh}}$





[Shifman'03]

- subtract R_{uds}
- $\bar{R}$ from data below 3.73 GeV
- $\sqrt{s}$-dependence from theory
- consider deviations from pQCD:
  - oszillations of $R(s)$ around perturbative result
  - from low-lying resonances
  - 2 models: $R(s) = R^{\rm pQCD}(s)\, f_i$
    - $f_1 = 1 + 1.22\, s^{-3/2}\, \sin\left(2\rho\sqrt{s} - \delta\right)$

      ($\rho = 3$ GeV^{-1}, $\delta = 1.32$)
    - $f_2 = 1 - 1.24 \exp\left(-\frac{2\pi s B}{\sigma^2 N_c}\right) \sin\left(\frac{2\pi s}{\sigma^2} - 3.08\right)$

      ($\sigma^2 = 2$ GeV^2, $B = 0.5$, $N_c = 3$)
  - f_1: $\delta m_c(3\text{ GeV}) \approx -1$ MeV
    f_2: smaller contribution

$\mathcal{M}^{\rm exp}$

n	$\mathcal{M}_n^{\rm res}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm thresh}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm cont}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm exp}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\rm np}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

m_c

$$\mathcal{M}_n^{\rm th} + \mathcal{M}_n^{\rm np} \stackrel{!}{=} \mathcal{M}_n^{\rm exp}$$

$$m_c(\mu) \; = \; \frac{1}{2}\left(\frac{\bar{C}_n(\mu/m_c(\mu))}{\mathcal{M}_n^{\rm exp} - \mathcal{M}_n^{\rm np}}\right)^{1/(2n)}$$

- 1. set $\mu = 3$ GeV ⇨ $m_c(3$ GeV$)$
  2. RGE ⇨ $m_c(m_c)$
- Uncertainties
  - $\delta\mathcal{M}_n^{\rm exp}$
  - $\alpha_s(M_Z) = 0.1189 \pm 0.0020$ [Bethke'06]; $\delta\alpha_s \times 2$
  - $\mu = (3 \pm 1)$ GeV
  - $\delta\mathcal{M}_n^{\rm np}$

m_c

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n	$m_c(3\ \text{GeV})$	exp	α_s	μ	np	total	$\delta\bar{C}_n^{(30)}$	$m_c(m_c)$
1	0.986	0.009	0.009	0.002	0.001	0.013	—	1.286
2	0.979	0.006	0.014	0.005	0.000	0.016	0.006	1.280
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

$$m_c(3\ \text{GeV}) = 0.986(13)\ \text{GeV}$$

[Kühn,MS,Sturm'07]

m_c

$$\mathcal{M}_n^{\rm th} + \mathcal{M}_n^{\rm np} \stackrel{!}{=} \mathcal{M}_n^{\rm exp}$$

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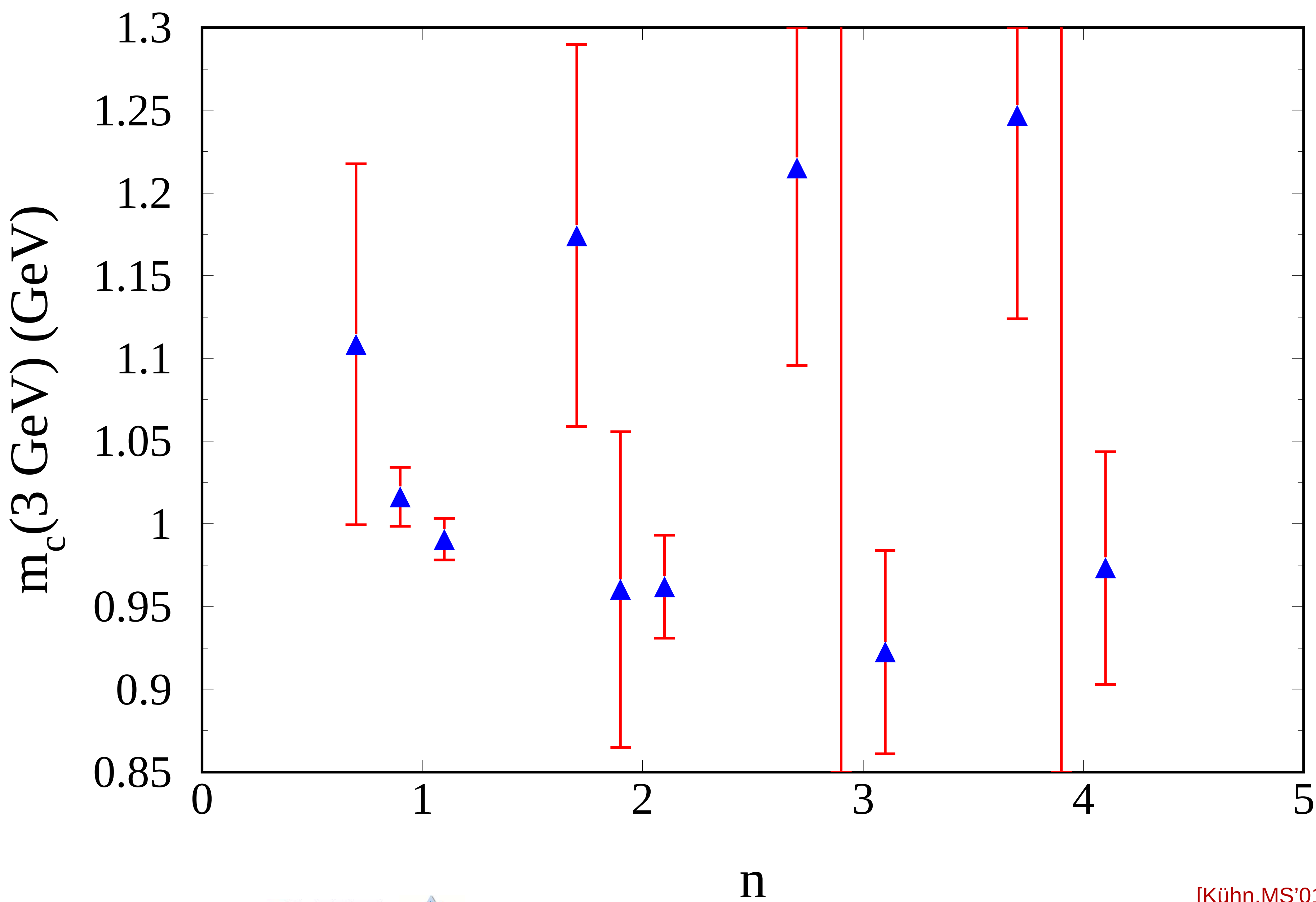
2	0.976	0.006	0.014	0.005	0.000	0.016	—	1.277
3	0.982	0.005	0.014	0.007	0.002	0.016	0.010	1.282
4	1.012	0.003	0.008	0.030	0.007	0.032	0.016	1.309

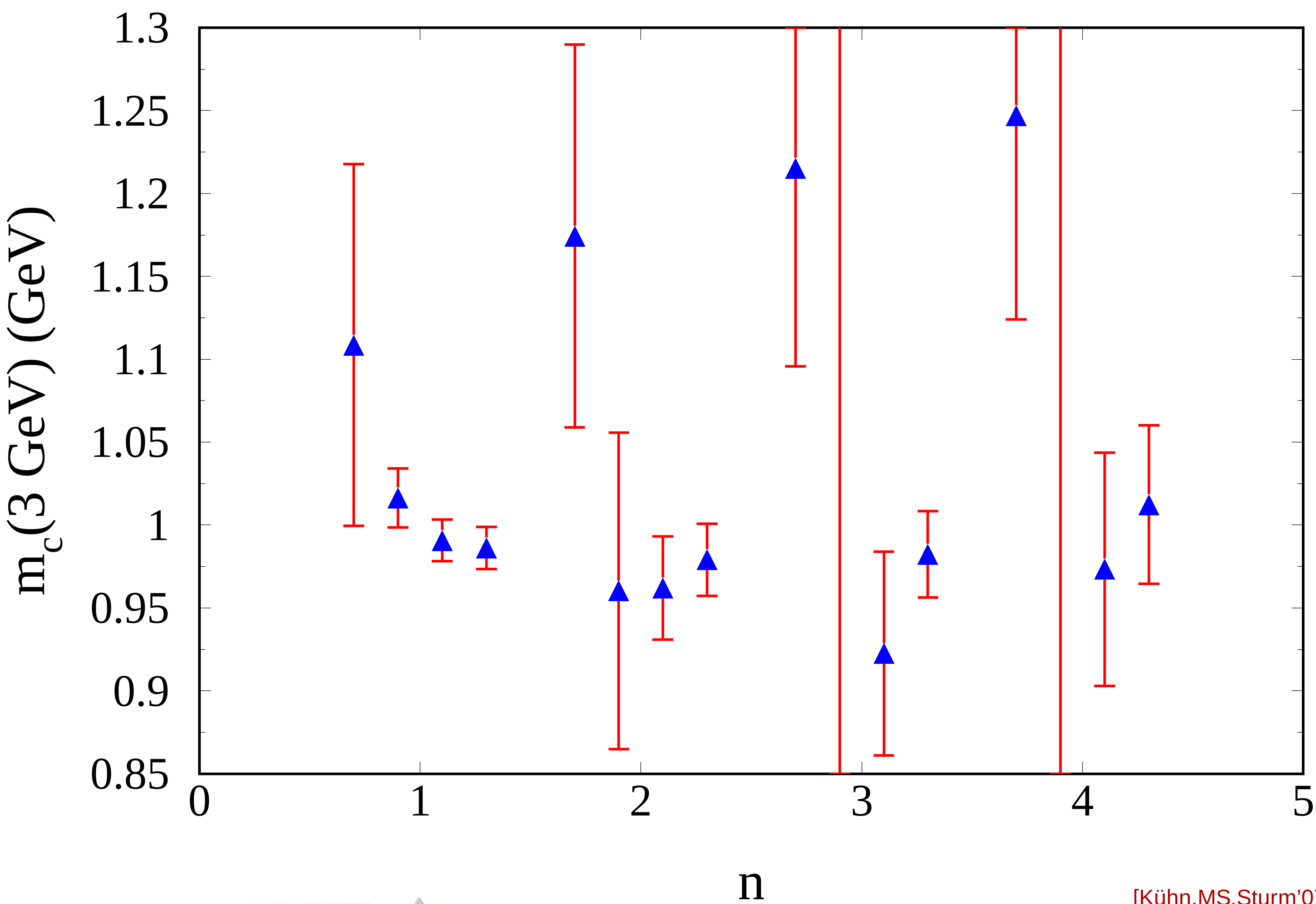
$$m_c(3\text{ GeV}) = 0.986(13)\text{ GeV}$$

$m_c(3\ \mathbf{GeV})$



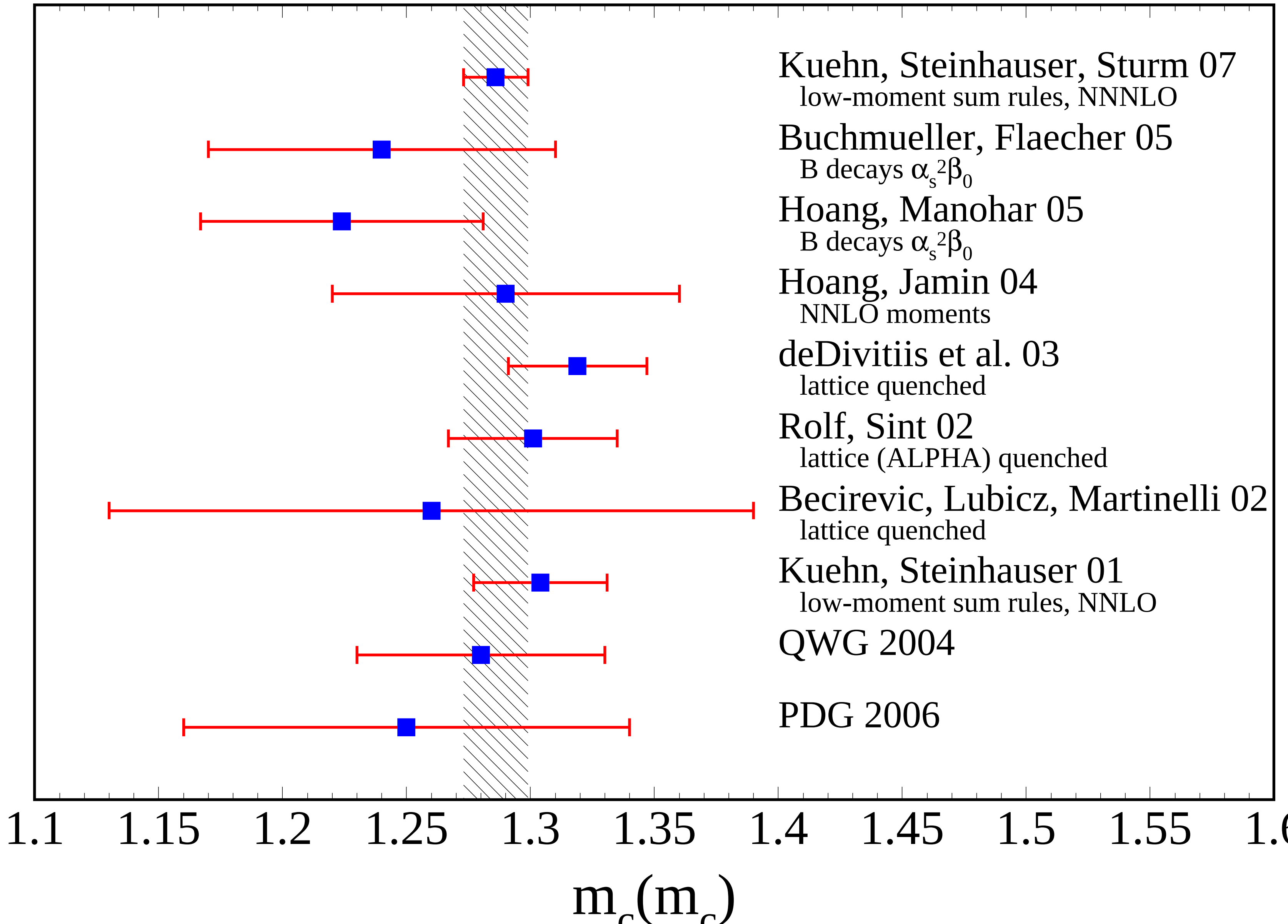
[Kühn,MS'01]

$m_c(3\ \mathbf{GeV})$



[Kühn,MS,Sturm'07]

Charm — comparison



Bottom quark

m_b

$$\mathcal{M}_n^{\rm th} \stackrel{!}{=} \mathcal{M}_n^{\rm exp}$$

$$m_b(\mu) \;=\; \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\rm exp}}\right)^{1/(2n)}$$

- 1. set $\mu = 10$ GeV ➪ $m_b(10$ GeV$)$
  2. RGE ➪ $m_b(m_b)$
- Uncertainties
  - $\delta\mathcal{M}_n^{\rm exp}$
  - $\alpha_s(M_Z) = 0.1189 \pm 0.0020$ [Bethke'06]; $\delta\alpha_s \times 2$
  - $\mu = (10 \pm 5)$ GeV

m_b

$$\mathcal{M}_n^{\rm th} \stackrel{!}{=} \mathcal{M}_n^{\rm exp}$$

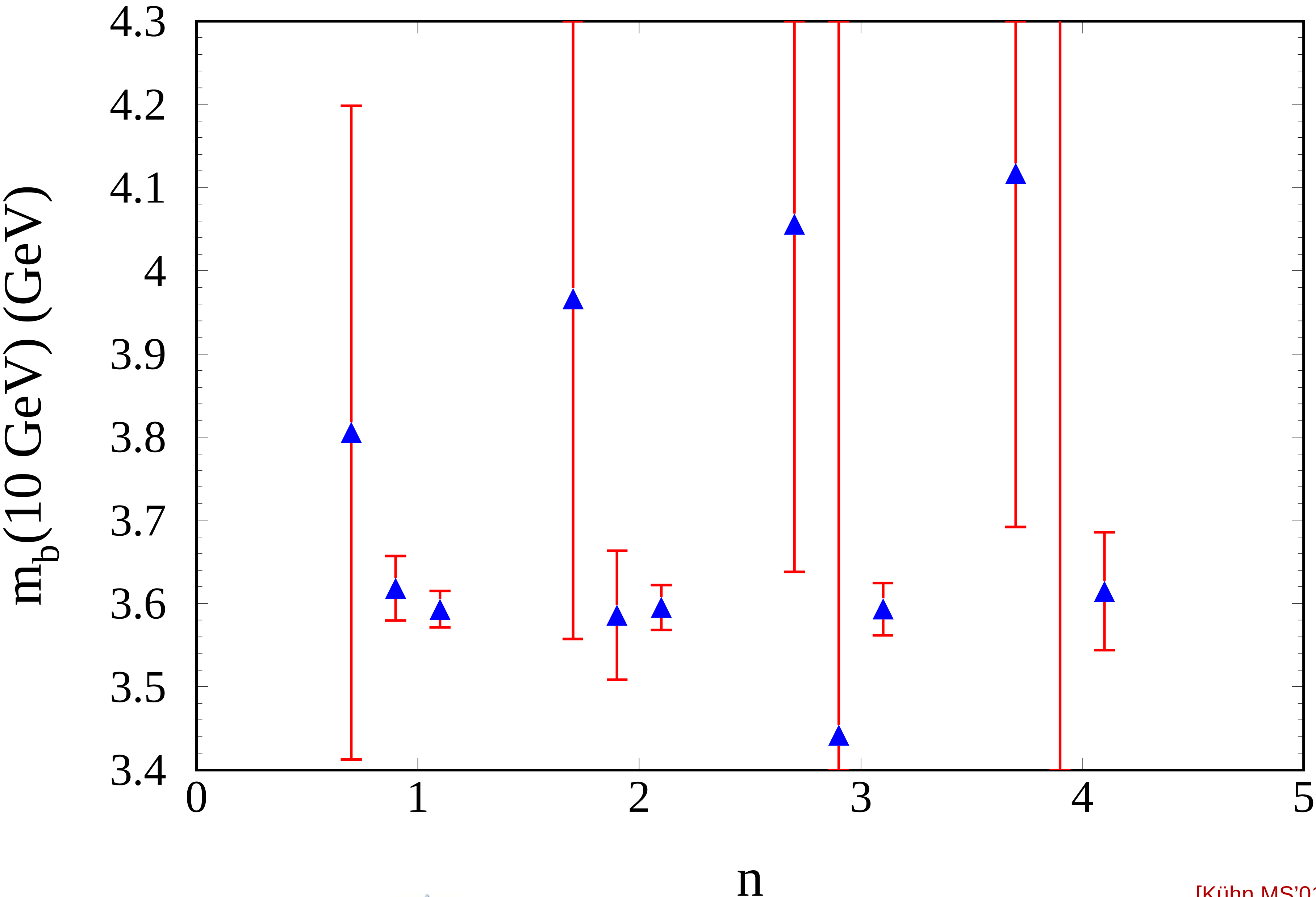
$$m_b(\mu) = \frac{1}{2}\left(\frac{\bar{C}_n}{\mathcal{M}_n^{\rm exp}}\right)^{1/(2n)}$$

n	$m_b(10\ \text{GeV})$	exp	α_s	μ	total	$\delta\bar{C}_n^{(30)}$	$m_b(m_b)$
1	3.593	0.020	0.007	0.002	0.021	—	4.149
2	3.609	0.014	0.012	0.003	0.019	0.006	4.164
3	3.618	0.010	0.014	0.006	0.019	0.008	4.173
4	3.631	0.008	0.015	0.021	0.027	0.012	4.185

$$m_b(10\ \text{GeV}) = 3.609(25)\ \text{GeV}$$

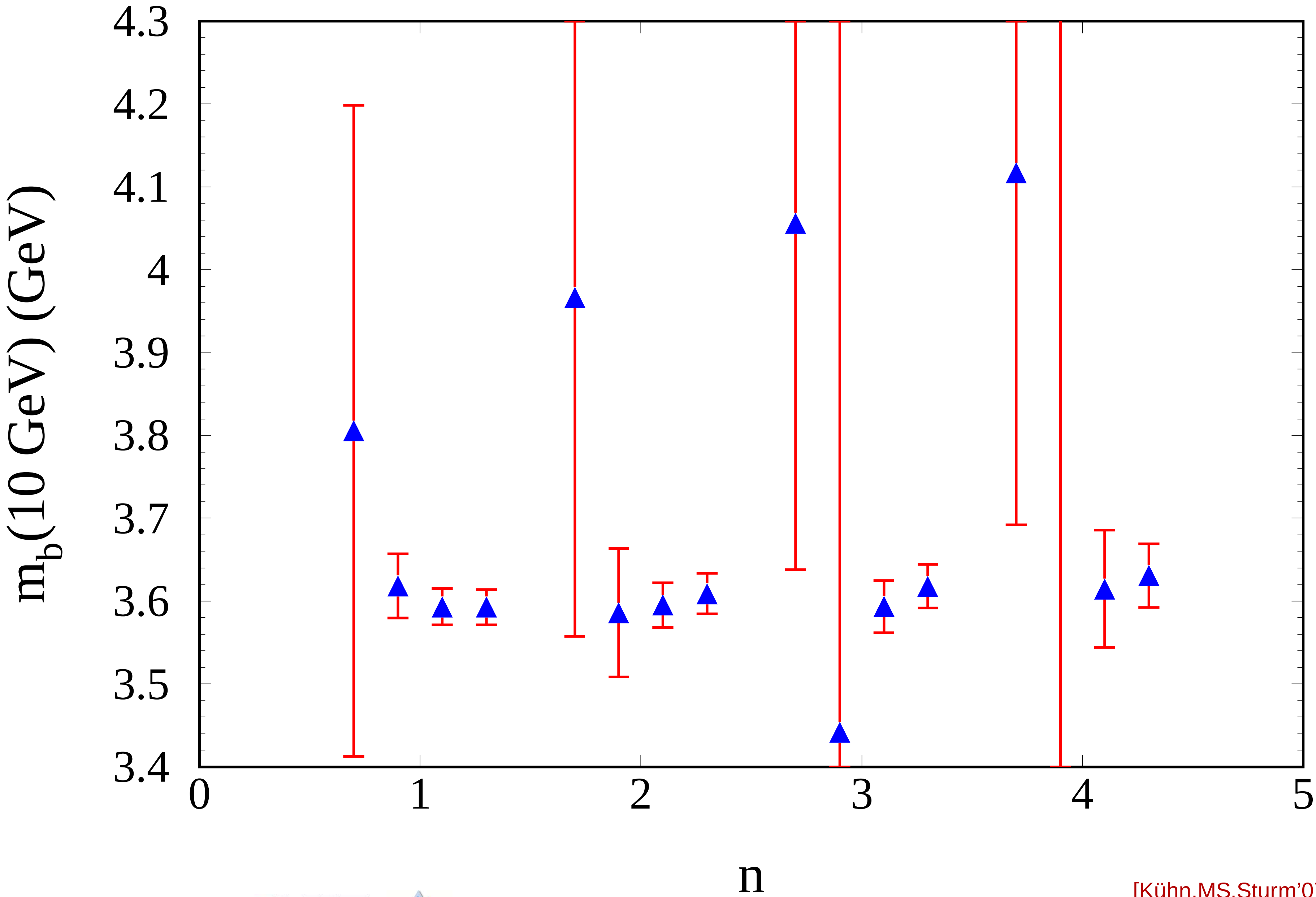
[Kühn,MS,Sturm'07]

$m_b(10\ \mathbf{GeV})$



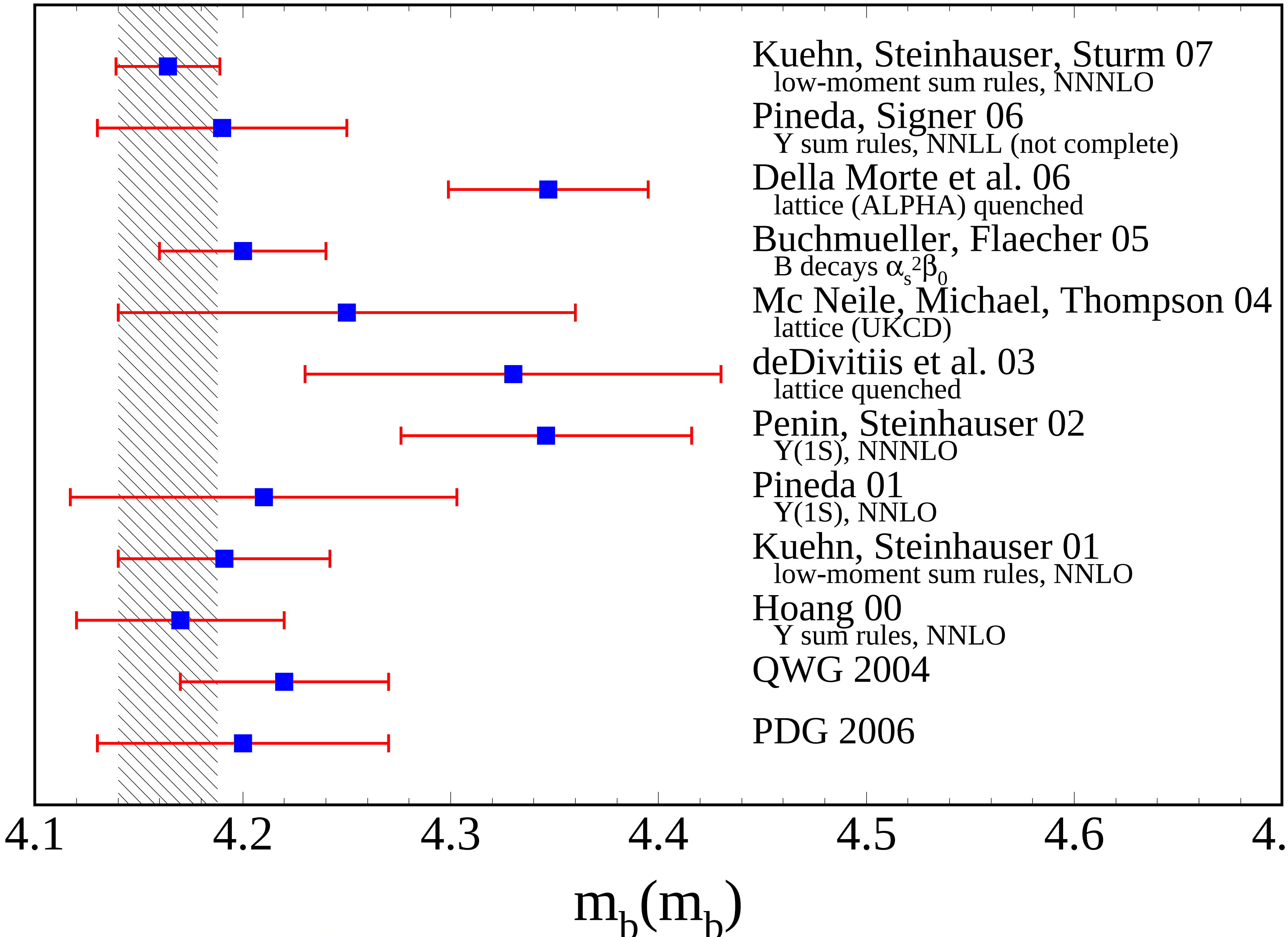
[Kühn,MS'01]

$m_b(10\ \textbf{GeV})$



[Kühn,MS,Sturm'07]

Bottom — comparison



Charm quark II

LQCD and PQCD

Lattice:

[HPQCD Collaboration: Allison et al.]

- Highly Improved Staggered Quark (HISQ)
- simulation of relativistic c quarks
- lattice spacing and quark masses are tuned to reproduce
  - $\Upsilon' - \Upsilon$ meson mass difference
  - $m_\pi^2,\, 2m_K^2 - m_\pi^2,\, m_{\eta_c},\, m_\Upsilon$
- compute moments of the pseudo-scalar current
- vector, axial-vector current ⇨ check

Perturbation theory:

- moments of the pseudo-scalar current
- 4 loops: $n = 1$ and $n = 2$

[Sturm, in preparation]

$m_c(3\ \mathbf{GeV})$

Note:

- $\bar{C}_1$ not sensitive to m_c but to α_s
- $m_c(3\ \text{GeV})$ from $n = 2, 3, 4$

PRELIMINARY

n	$m_c(3\ \text{GeV})$	total	lattice	α_s	h.o.	np
2	0.990	0.016	0.015	0.003	0.004	0.002
3	0.980	0.016	0.012	0.006	0.009	0.000
4	0.978	0.019	0.011	0.010	0.013	0.004

$m_c(3\ \mathbf{GeV})$

Note:

- $\bar{C}_1$ not sensitive to m_c but to α_s
- $m_c(3\ \text{GeV})$ from $n = 2, 3, 4$

PRELIMINARY

n	$m_c(3\ \text{GeV})$	total	lattice	α_s	h.o.	np
2	0.990	0.016	0.015	0.003	0.004	0.002
3	0.980	0.016	0.012	0.006	0.009	0.000
4	0.978	0.019	0.011	0.010	0.013	0.004

$$m_c(3\ \text{GeV}) = 0.984(16)\ \text{GeV}$$

α_s

$\underline{n = 1}$: PRELIMINARY

$\alpha_s^{(4)}(3 \text{ GeV}) = 0.230(18)$ ➪ $\alpha_s^{(5)}(M_Z) = 0.113(4)$

PDG: $\alpha_s^{(5)}(M_Z) = 0.1176(20)$

Conclusions

- $R^{\rm exp}$ and PQCD (vector correlator):
  - $m_c(3~\text{GeV}) = 0.986(13)$ GeV
    $m_b(10~\text{GeV}) = 3.609(25)$ GeV
  - NNNLO analysis
  - $\overline{\rm MS}$ mass
  - Possible improvements: experimental measurements: $R(s)$, Γ_{ee}
- LQCD and PQCD (pseudo-scalar correlator):
  - $m_c(3~\text{GeV}) = 0.984(16)$ GeV PRELIMINARY
  - $\alpha_s^{(5)}(M_Z) = 0.113(4)$
- $\frac{\delta m_s}{m_s} \approx 10\%$; $\frac{\delta m_c}{m_c} \approx 1\%$; $\frac{\delta m_b}{m_b} \approx 0.6\%$; $\frac{\delta m_t}{m_t} \approx 1\%$