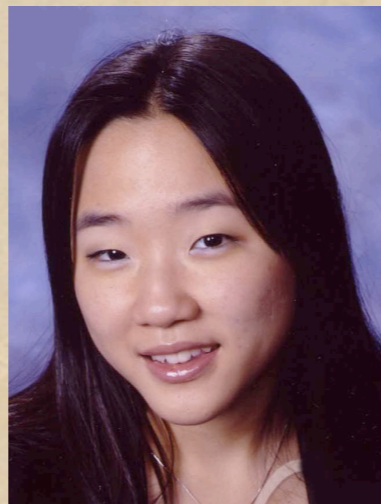


# Electronic quasiparticles and competing orders in the cuprate superconductors



Ettore Vicari  
Pisa



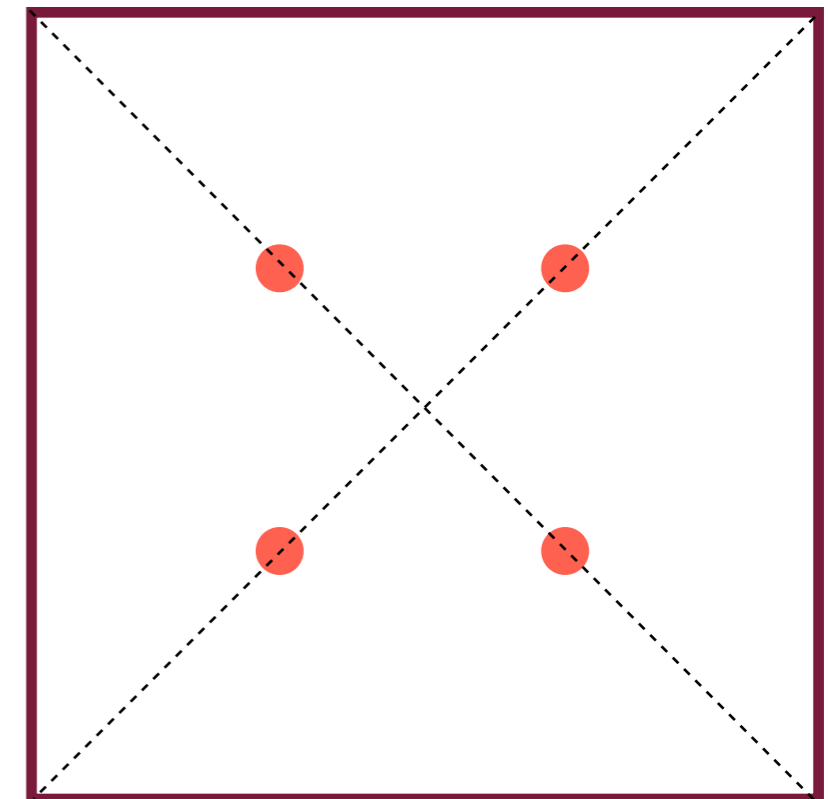
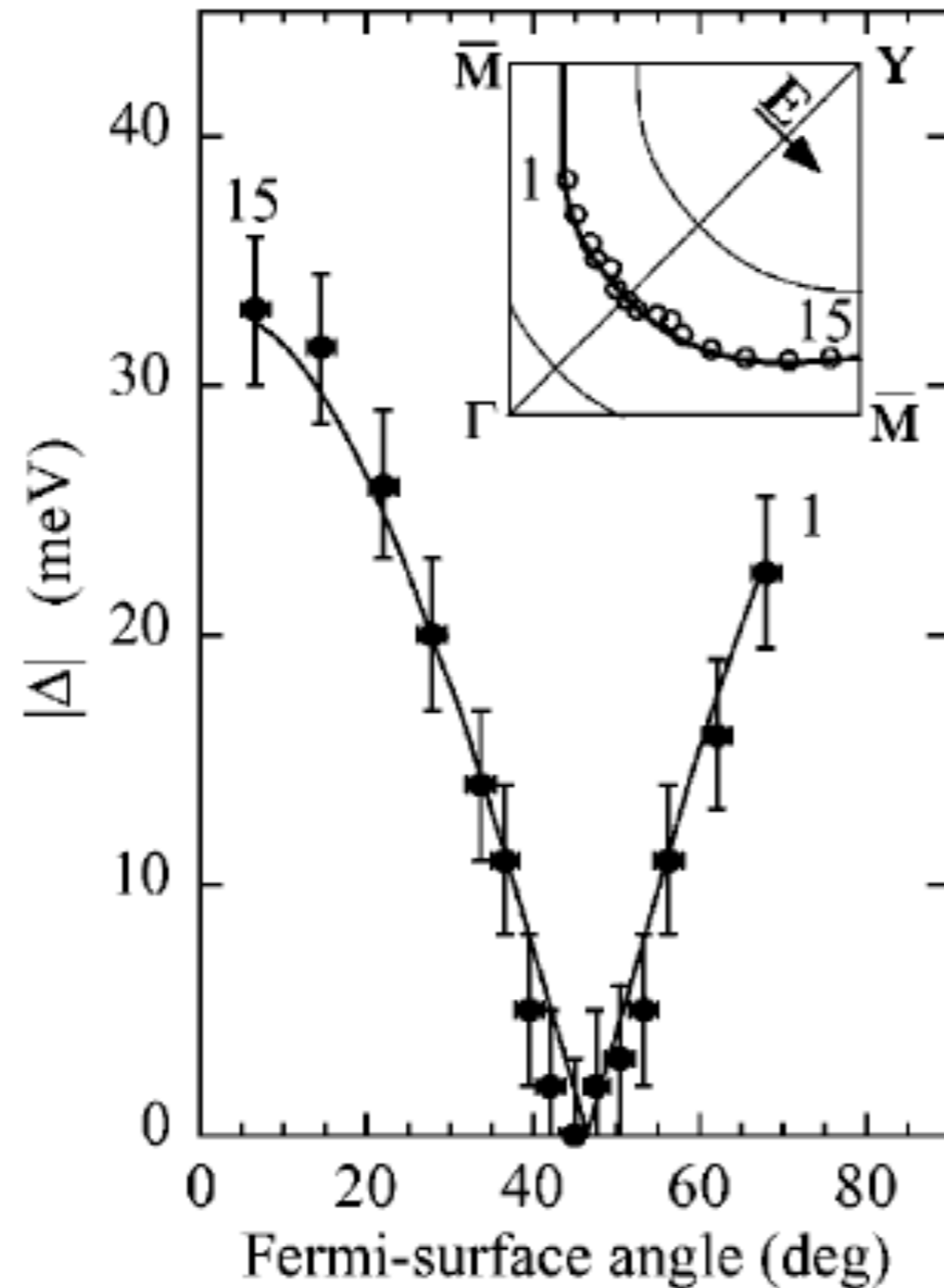
Yejin Huh  
Harvard

Andrea Pelissetto  
Rome

Subir Sachdev  
Harvard



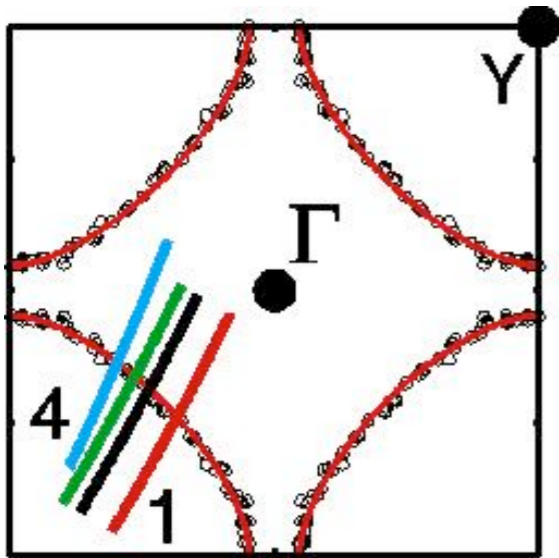
# Gapless nodal quasiparticles in $d$ -wave superconductors



Brillouin zone

FIG. 46. Superconducting gap measured at 13 K on Bi2212 ( $T_c=87$  K) plotted vs the angle along the normal-state Fermi surface (see sketch of the Brillouin zone), together with a  $d$ -wave fit. From Ding, Norman, *et al.*, 1996.

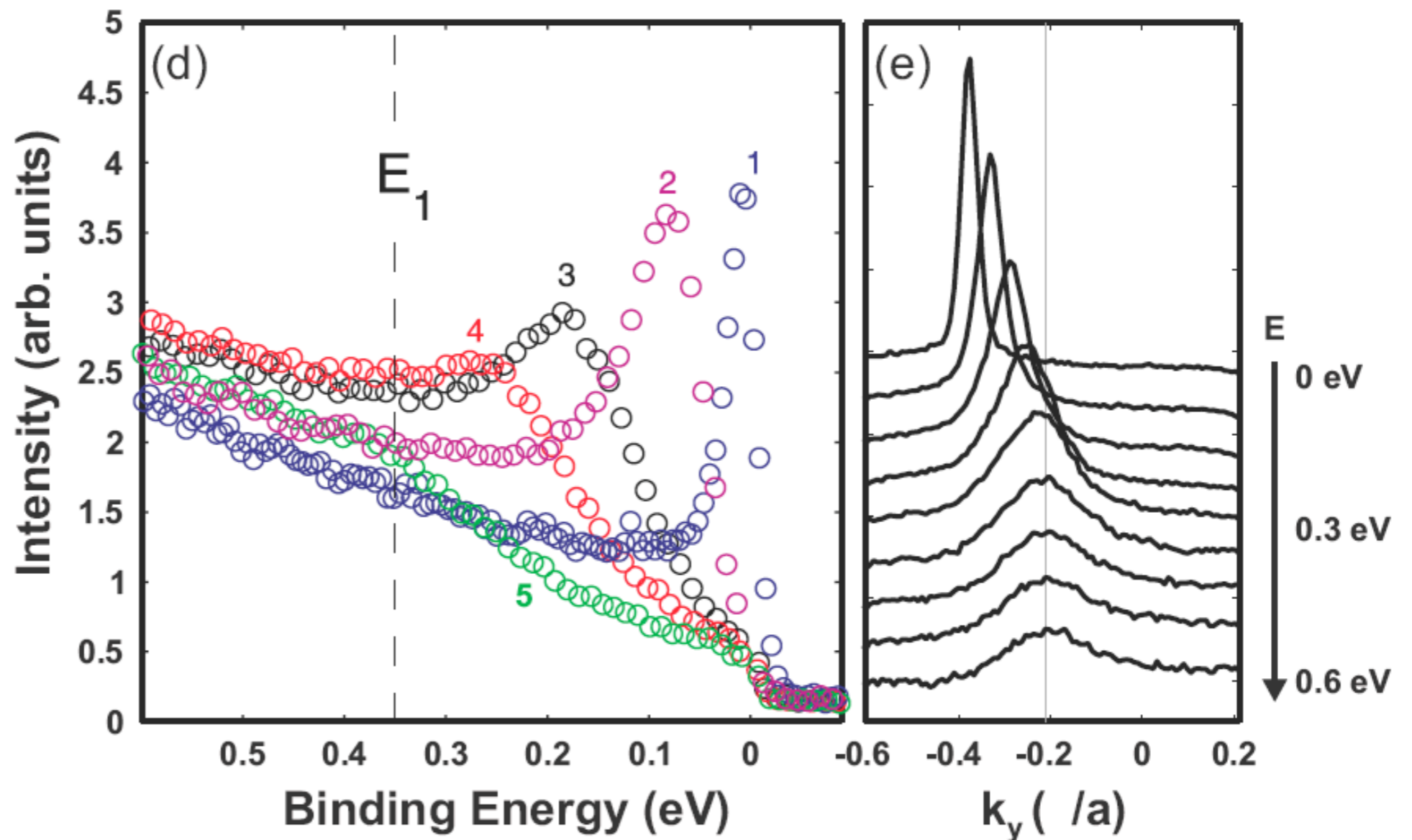
# Photoemission spectra of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$



$x=0.145$

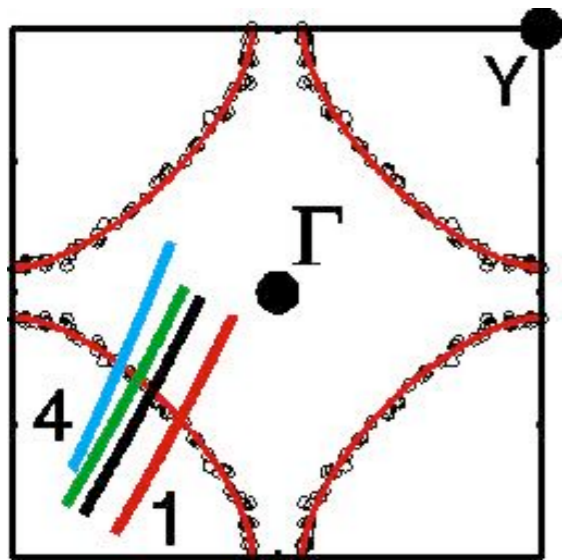
EDC

MDC

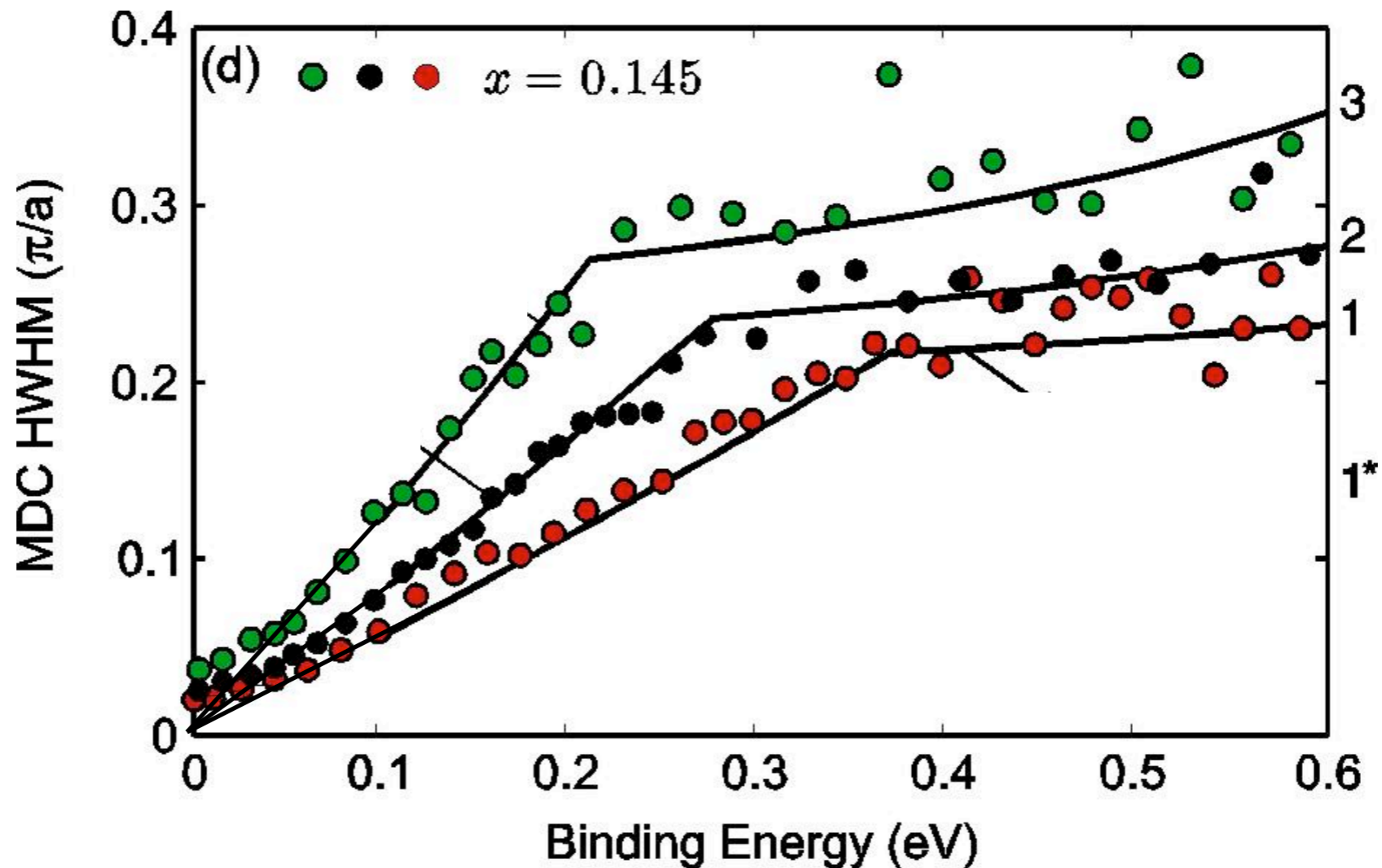


J. Chang, M. Shi, S. Pailhes, M. Maansson, T. Claesson, O. Tjernberg, A. Bendounan, L. Patthey, N. Momono, M. Oda, M. Ido, C. Mudry, and J. Mesot, arXiv:0708.2782

# Photoemission spectra of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$



$x=0.145$



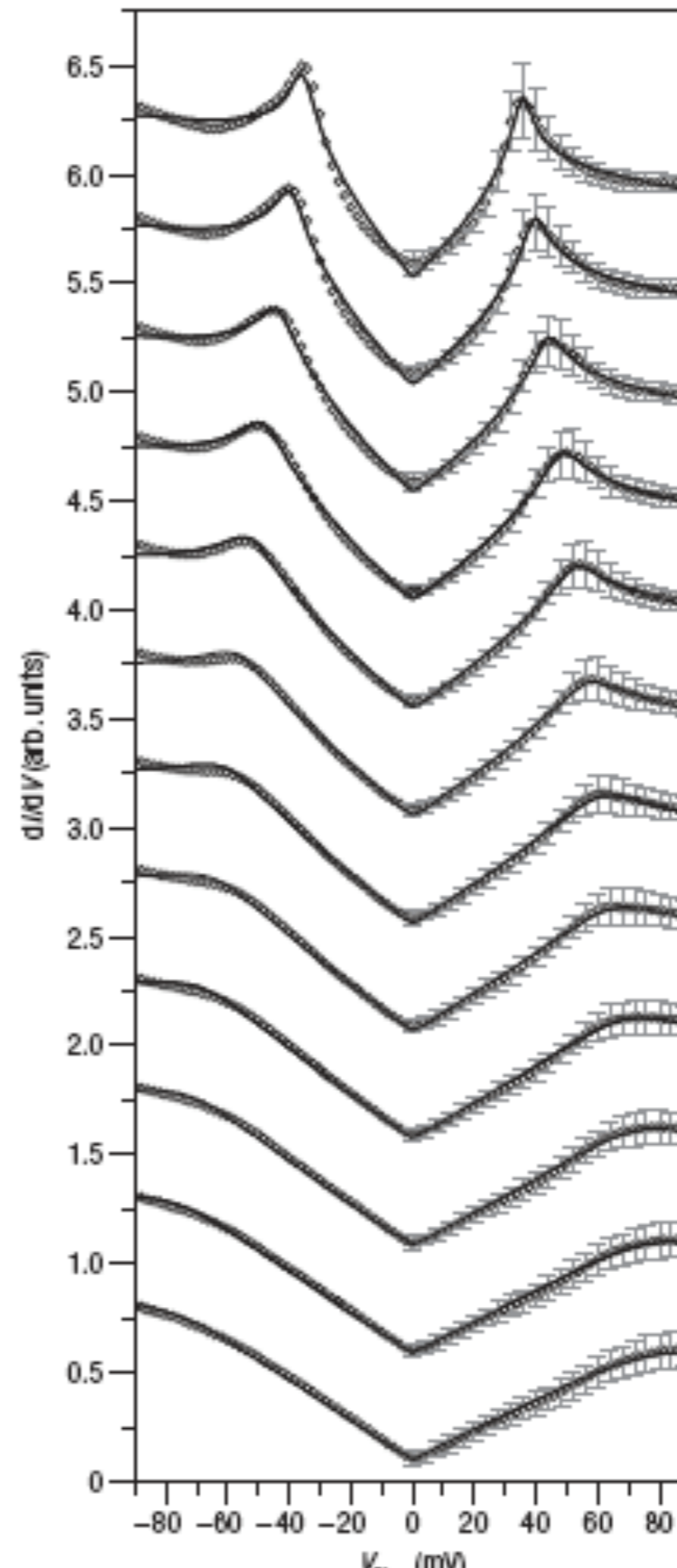
# Scanning tunneling microscopy of BSCCO

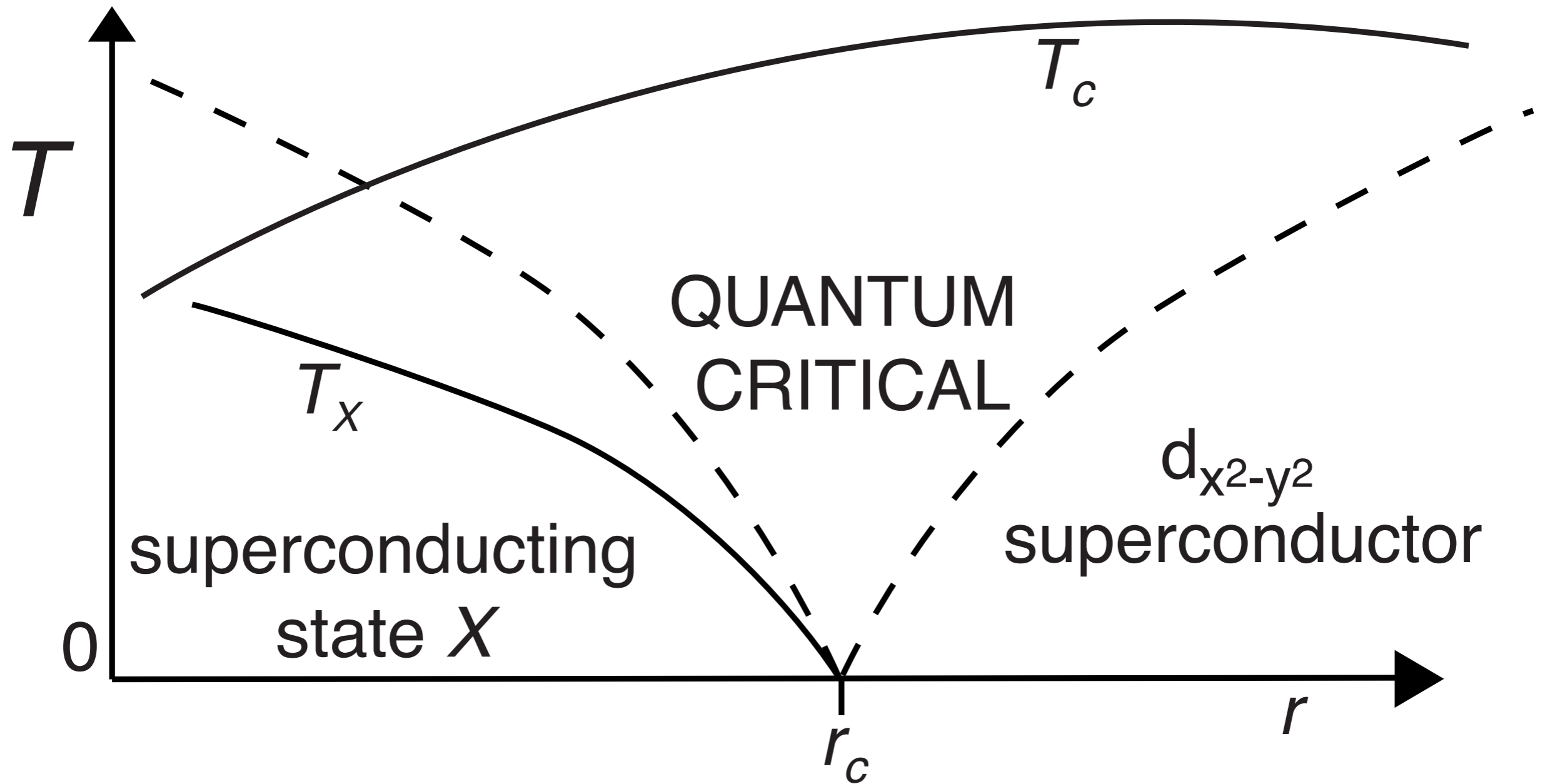
$$N(E, \Gamma_2) = A \times \text{Re} \left( \left\langle \frac{E + i\Gamma_2(E)}{\sqrt{(E + i\Gamma_2(E))^2 - \Delta(k)^2}} \right\rangle_{fs} \right) + I$$

Good fit with  
 $\Gamma_2(E) = \alpha E$

J. W. Alldredge, Jinho Lee,  
K. McElroy, M. Wang,  
K. Fujita, Y. Kohsaka,  
C. Taylor, H. Eisaki,  
S. Uchida, P. J. Hirschfeld,  
and J. C. Davis

*Nature Physics* **4**, 319 (2008)





Needed:  
Quantum critical point with nodal  
quasiparticles part of the critical theory

# Outline

## 1. SDW order in LSCO

*Emergent  $O(4)$  symmetry*

## 2. Nodal quasiparticles at the $O(4)$ critical point

*Unique selection of quasiparticle coupling to  
(composite) nematic order*

## 3. Nematic order in YBCO and BSCCO

*Broken lattice symmetry but no spin order*

## 4. Theory of the onset of nematic order in a d-wave superconductor

*Infinite anisotropy fixed point*

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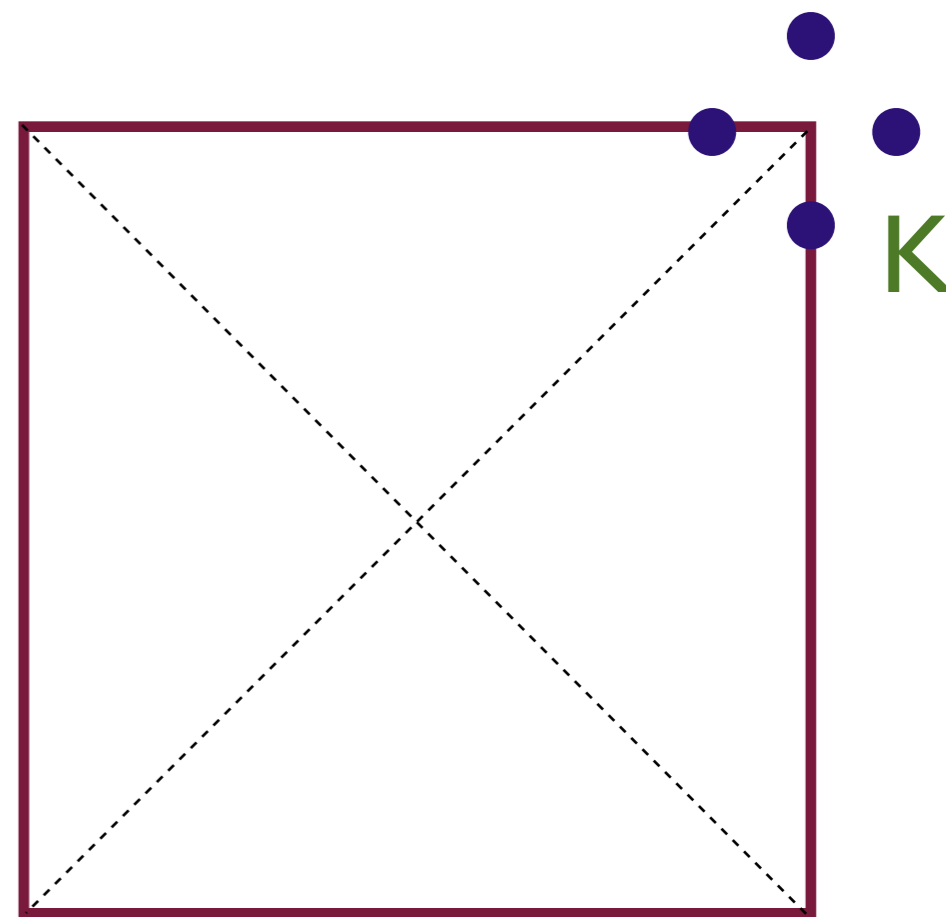
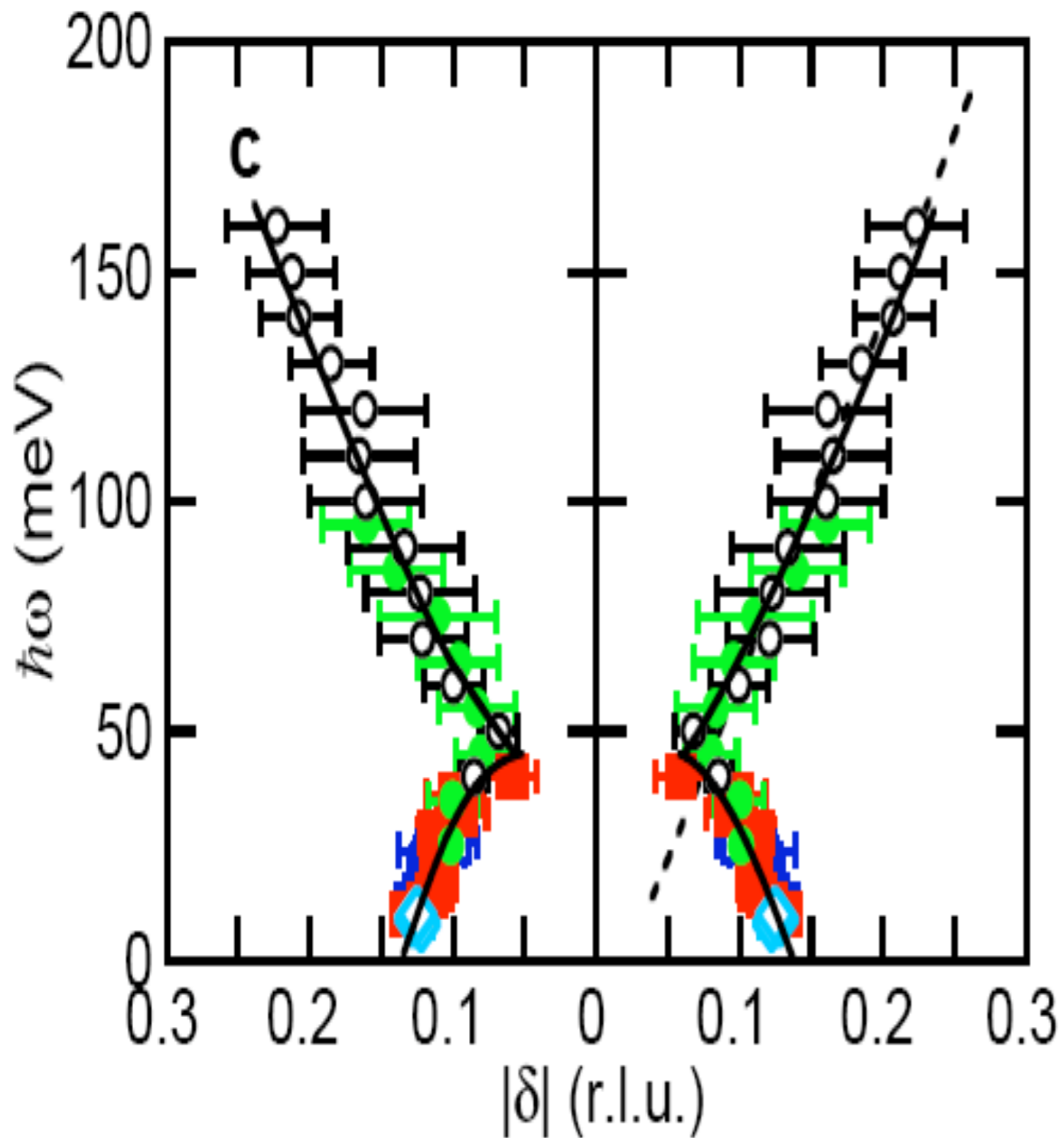
*Broken lattice symmetry but no spin order*

## 4. Theory of the onset of nematic order in a d-wave superconductor

*Infinite anisotropy fixed point*



# Neutron Scattering-LSCO



Brillouin zone

Vignolle *et al.*, Nature Phys. 07

Christensen *et al.*, PRL 04

Hayden *et al.*, Nature 04

Tranquada *et al.*, Nature 04

$T=0$

Elastic scattering intensity

$$I(H) = I(0) + a \frac{H}{H_{c2}} \ln \left( \frac{3H_{c2}}{H} \right)$$

"Normal"  
(Charge order)

SDW

$$H \sim \frac{(s - s_c)}{\ln(1/(s - s_c))}$$

SC+  
SDW

$S = 1$  exciton energy

$$\varepsilon(H) = \varepsilon(0) - b \frac{H}{H_{c2}} \ln \left( \frac{3H_{c2}}{H} \right)$$

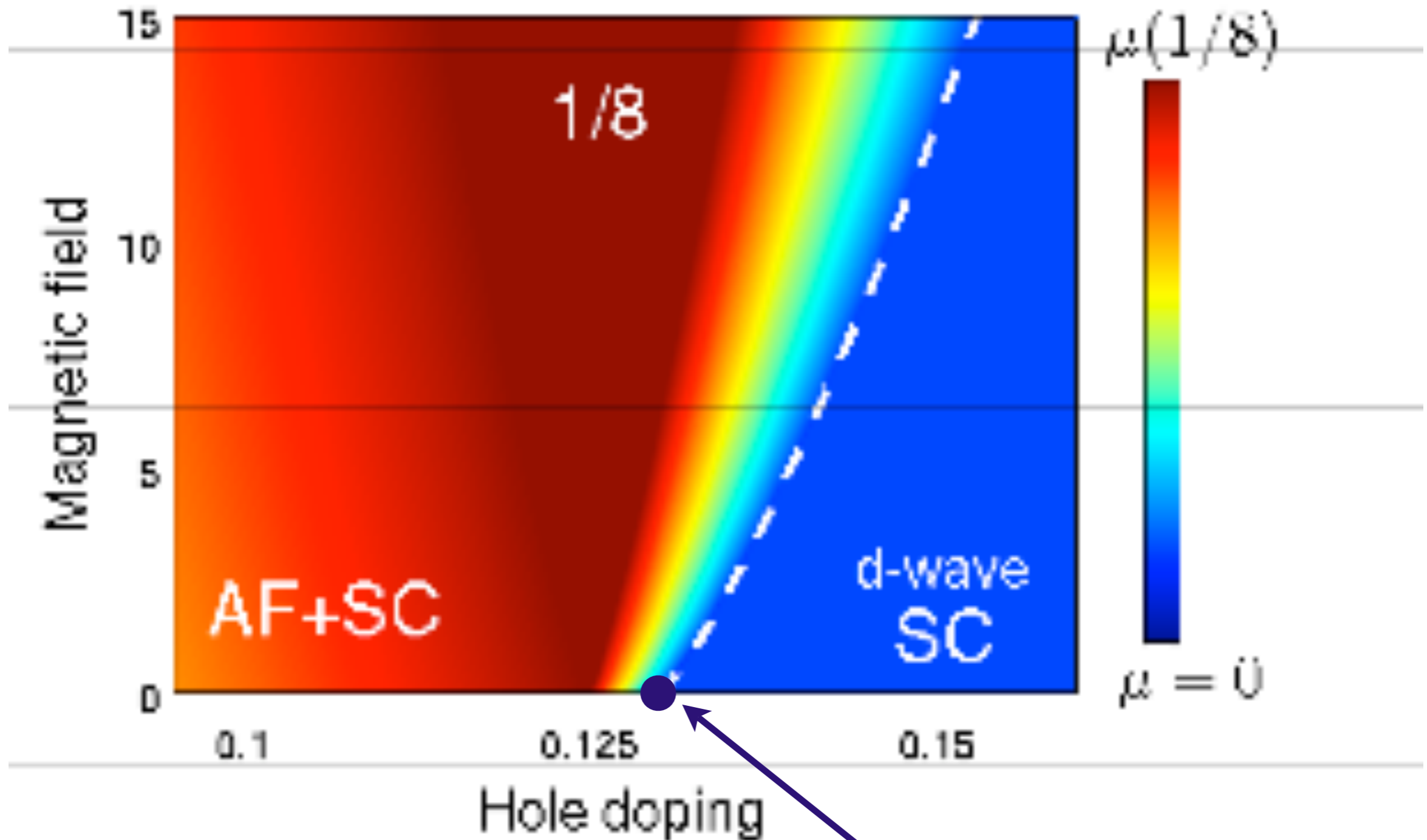
SC

$s_c$

$s$

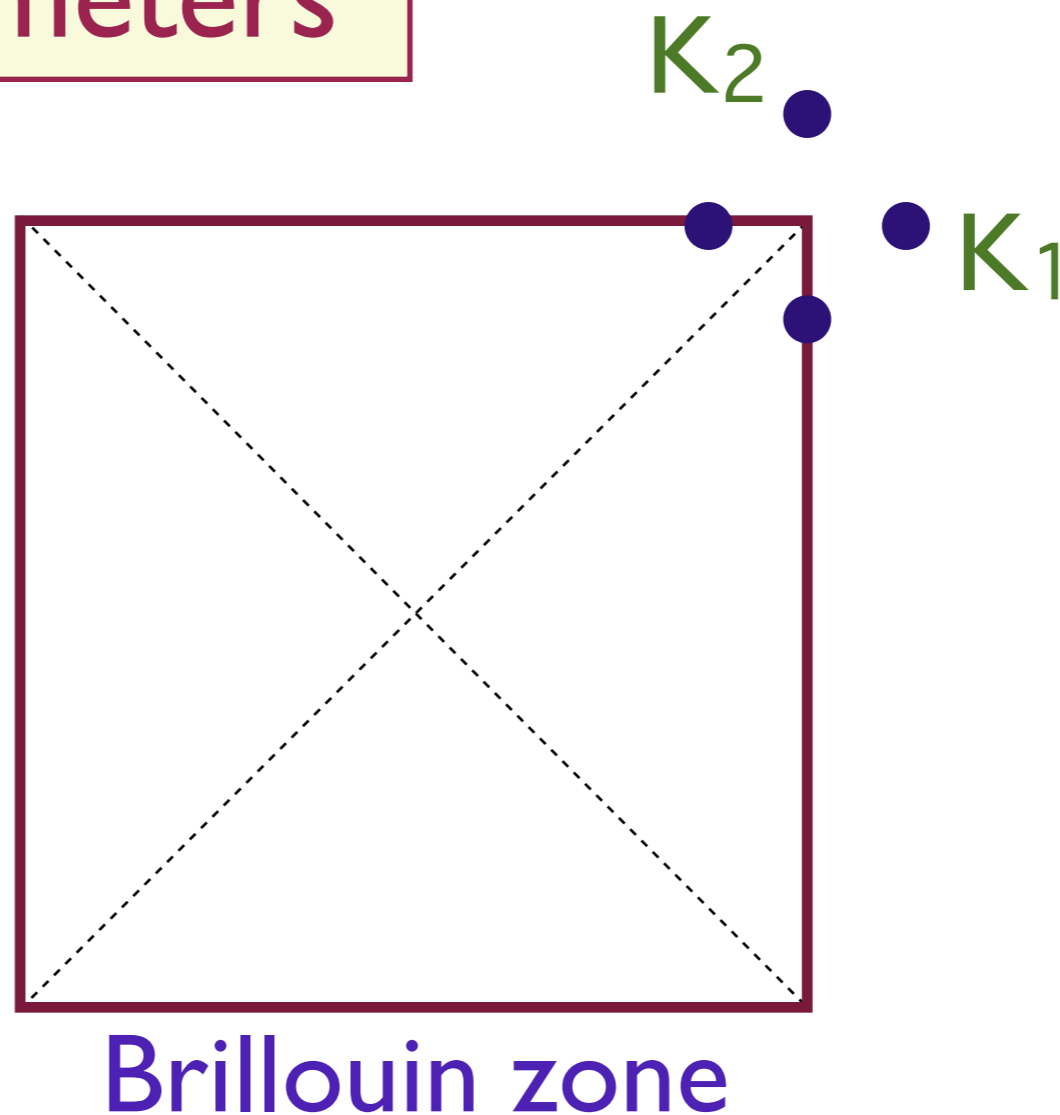
SC to SC+SDW quantum critical point

J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, arXiv:0712.2181



SC to SC+SDW quantum critical point

# SDW order parameters



$$S_i(\mathbf{r}, \tau) = \text{Re} \left[ e^{i\mathbf{K}_1 \cdot \mathbf{r}} \Phi_{1i}(\mathbf{r}, \tau) + e^{i\mathbf{K}_2 \cdot \mathbf{r}} \Phi_{2i}(\mathbf{r}, \tau) \right].$$

$$\mathbf{K}_1 = \left( \frac{2\pi}{a} \right) \left( \frac{1}{2} - \vartheta, \frac{1}{2} \right), \quad \mathbf{K}_2 = \left( \frac{2\pi}{a} \right) \left( \frac{1}{2}, \frac{1}{2} - \vartheta \right),$$

# SDW field theory

$$\begin{aligned}
 \mathcal{L}_\Phi = & |\partial_\tau \Phi_1|^2 + v_1^2 |\partial_x \Phi_1|^2 + v_2^2 |\partial_y \Phi_1|^2 \\
 & + |\partial_\tau \Phi_2|^2 + v_2^2 |\partial_x \Phi_2|^2 + v_1^2 |\partial_y \Phi_2|^2 + r(|\Phi_1|^2 + |\Phi_2|^2) \\
 & + \frac{u_1}{2} (|\Phi_1|^4 + |\Phi_2|^4) + \frac{u_2}{2} (|\Phi_1^2|^2 + |\Phi_2^2|^2) \\
 & + w_1 |\Phi_1|^2 |\Phi_2|^2 + w_2 |\Phi_1 \cdot \Phi_2|^2 + w_3 |\Phi_1^* \cdot \Phi_2|^2
 \end{aligned}$$

Most general theory invariant under spin rotation, square lattice space group, and time-reversal symmetries

	$T_x$	$T_y$	$R$	$I$	$\mathcal{T}$
$\Phi_{1i}$	$-e^{-i\vartheta} \Phi_{1i}$	$-\Phi_{1i}$	$\Phi_{2i}$	$\Phi_{1i}^*$	$-\Phi_{1i}$
$\Phi_{2i}$	$-\Phi_{2i}$	$-e^{-i\vartheta} \Phi_{2i}$	$\Phi_{1i}^*$	$\Phi_{2i}^*$	$-\Phi_{2i}$

# SDW field theory

$$\begin{aligned}\mathcal{L}_\Phi &= |\partial_\tau \Phi_1|^2 + v_1^2 |\partial_x \Phi_1|^2 + v_2^2 |\partial_y \Phi_1|^2 \\ &+ |\partial_\tau \Phi_2|^2 + v_2^2 |\partial_x \Phi_2|^2 + v_1^2 |\partial_y \Phi_2|^2 + r(|\Phi_1|^2 + |\Phi_2|^2) \\ &+ \frac{u_1}{2} (|\Phi_1|^4 + |\Phi_2|^4) + \frac{u_2}{2} (|\Phi_1^2|^2 + |\Phi_2^2|^2) \\ &+ w_1 |\Phi_1|^2 |\Phi_2|^2 + w_2 |\Phi_1 \cdot \Phi_2|^2 + w_3 |\Phi_1^* \cdot \Phi_2|^2\end{aligned}$$

Symmetries:

$$U(1) \otimes U(1) \otimes Z_4 \otimes O(3)$$

x-translations

y-translations

lattice  
rotations

spin  
rotations

# SDW field theory

Stable fixed point in a 6-loop RG analysis:

$$w_1^* = u_1^* - u_2^*, \quad w_2^* = w_3^* = u_2^*, \quad v_1^* = v_2^*$$

$O(4) \otimes O(3)$  invariant theory for  $\varphi_{ai}$ , with  $a = 1 \dots 4$  an  $O(4)$  index, and  $i = 1 \dots 3$  an  $O(3)$  index, and

$$\Phi_{1i} = \varphi_{1i} + i\varphi_{2i}, \quad \Phi_{2i} = \varphi_{3i} + i\varphi_{4i}.$$

M. De Prato, A. Pelissetto, and E. Vicari  
Phys. Rev. B **74**, 144507 (2006).

# SDW field theory

Symmetries:

$$U(1) \otimes U(1) \otimes Z_4 \otimes O(3)$$

x-translations

y-translations

lattice  
rotations

spin  
rotations



# SDW field theory

Symmetries:

$$O(4) \otimes O(3)$$

x-translations

y-translations

lattice  
rotations

spin  
rotations

## Properties of $O(4) \otimes O(3)$ fixed point:

The following 9 order parameters have divergent fluctuations at the spin-density wave ordering transition with the same exponent  $\bar{\gamma}$ :

- The real ‘nematic’ order parameter,  $\phi \equiv \sum_i (|\Phi_{1i}|^2 - |\Phi_{2i}|^2)$  which measures breaking of  $Z_4$  symmetry
- Charge density waves at  $2\mathbf{K}_1$  and  $2\mathbf{K}_2$ :  $\sum_i \Phi_{1i}^2$  and  $\sum_i \Phi_{2i}^2$
- Charge density waves at  $\mathbf{K}_1 \pm \mathbf{K}_2$ :  $\sum_i \Phi_{1i}\Phi_{2i}$  and  $\sum_i \Phi_{1i}^* \Phi_{2i}$

At the quantum critical point, the susceptibilities of these orders *all* diverge as  $\chi \sim T^{-\bar{\gamma}}$ , with

$$\bar{\gamma} = \begin{cases} 0.90(36) & \text{MZM, 6 loops} \\ 0.80(54) & d = 3 \overline{\text{MS}}, 5 \text{ loops} \end{cases}$$

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*Unique selection of quasiparticle coupling to  
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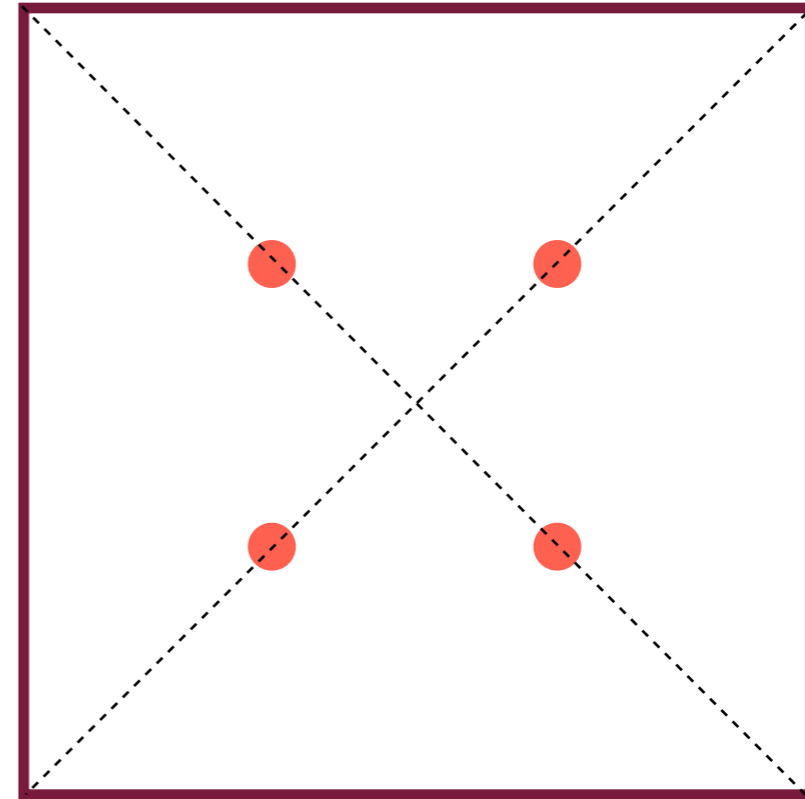
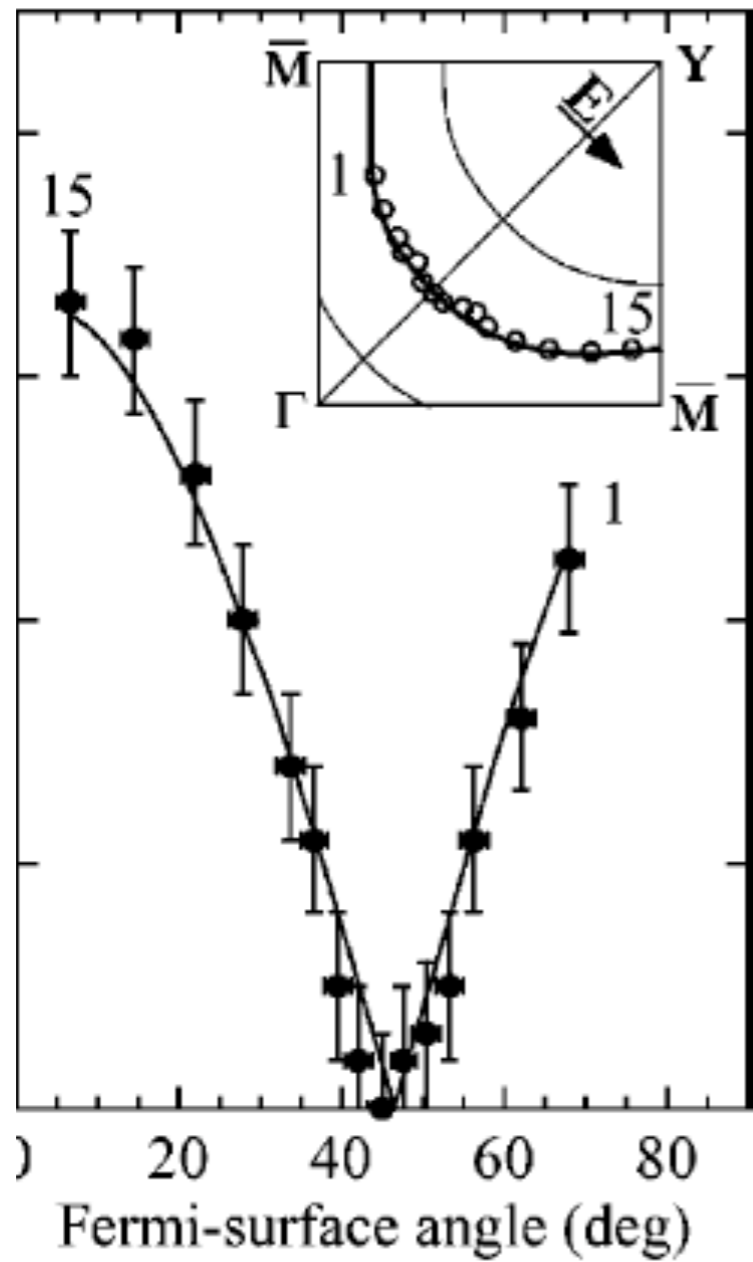
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(composite) nematic order*

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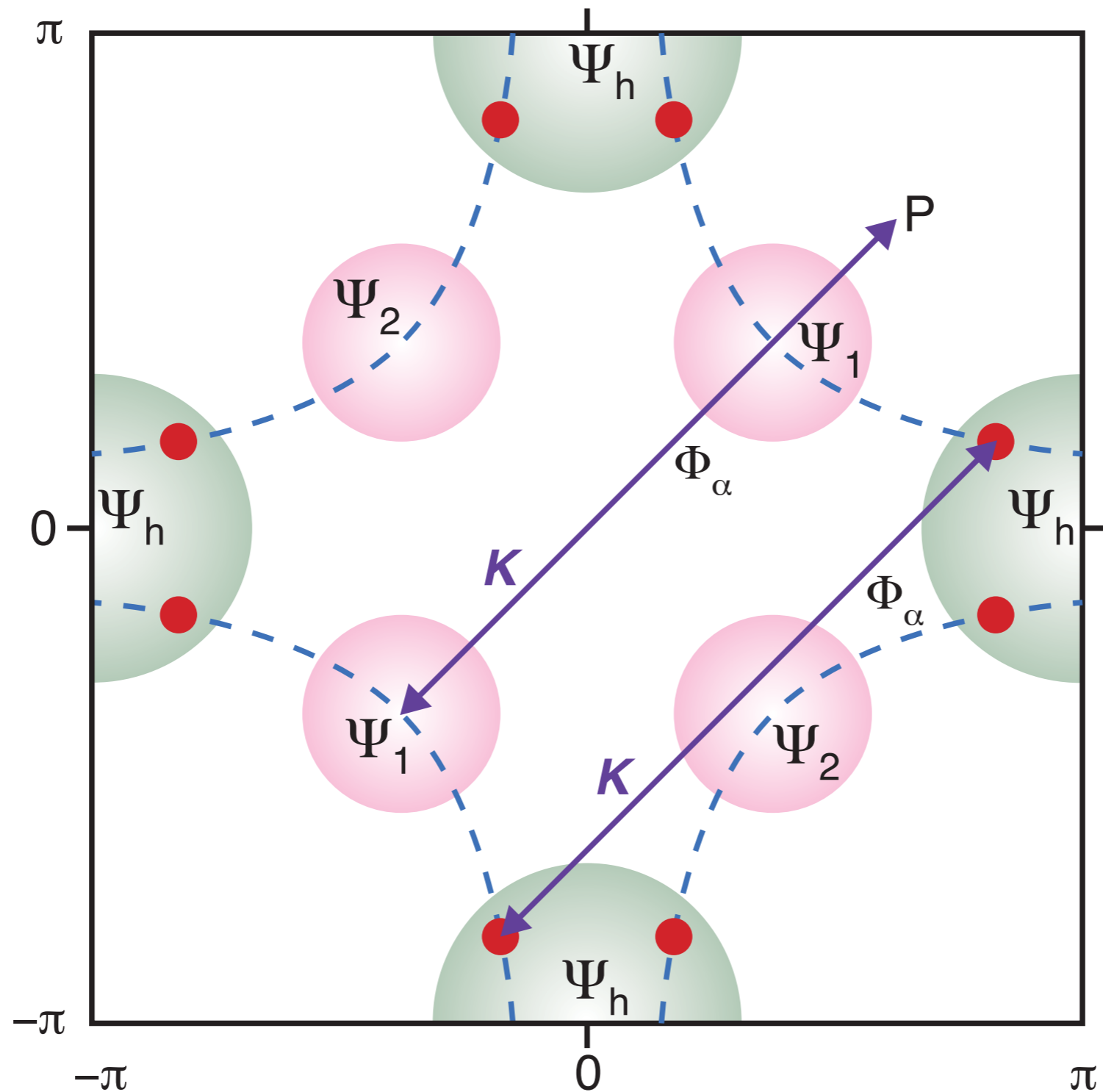
*Infinite anisotropy fixed point*



Brillouin zone

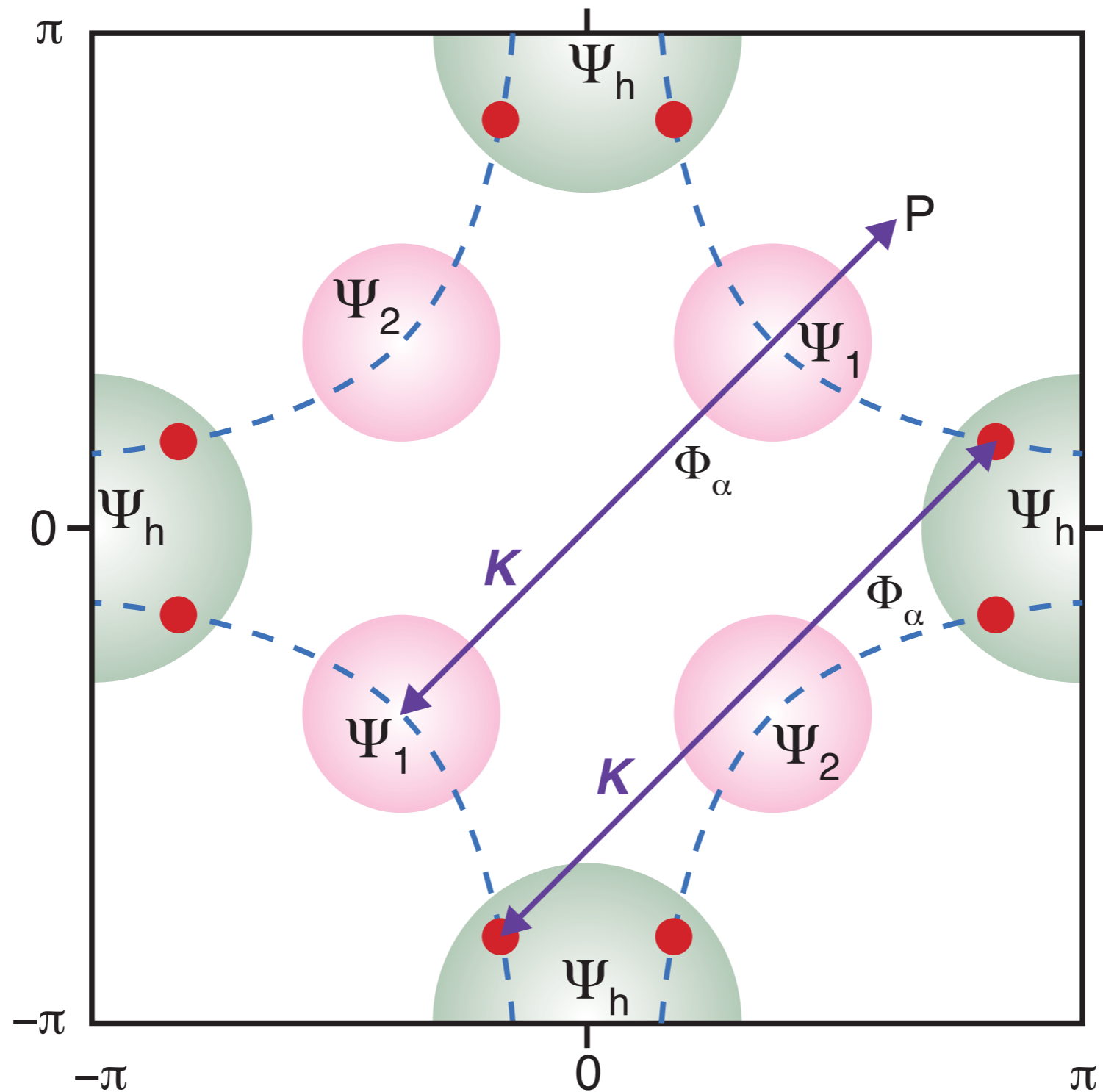
$$\begin{aligned}
 \mathcal{L}_\Psi = & \Psi_1^\dagger \left( \partial_\tau - i \frac{v_F}{\sqrt{2}} (\partial_x + \partial_y) \tau^z - i \frac{v_\Delta}{\sqrt{2}} (-\partial_x + \partial_y) \tau^x \right) \Psi_1 \\
 & + \Psi_2^\dagger \left( \partial_\tau - i \frac{v_F}{\sqrt{2}} (-\partial_x + \partial_y) \tau^z - i \frac{v_\Delta}{\sqrt{2}} (\partial_x + \partial_y) \tau^x \right) \Psi_2.
 \end{aligned}$$

# Coupling of quasiparticles to SDW order



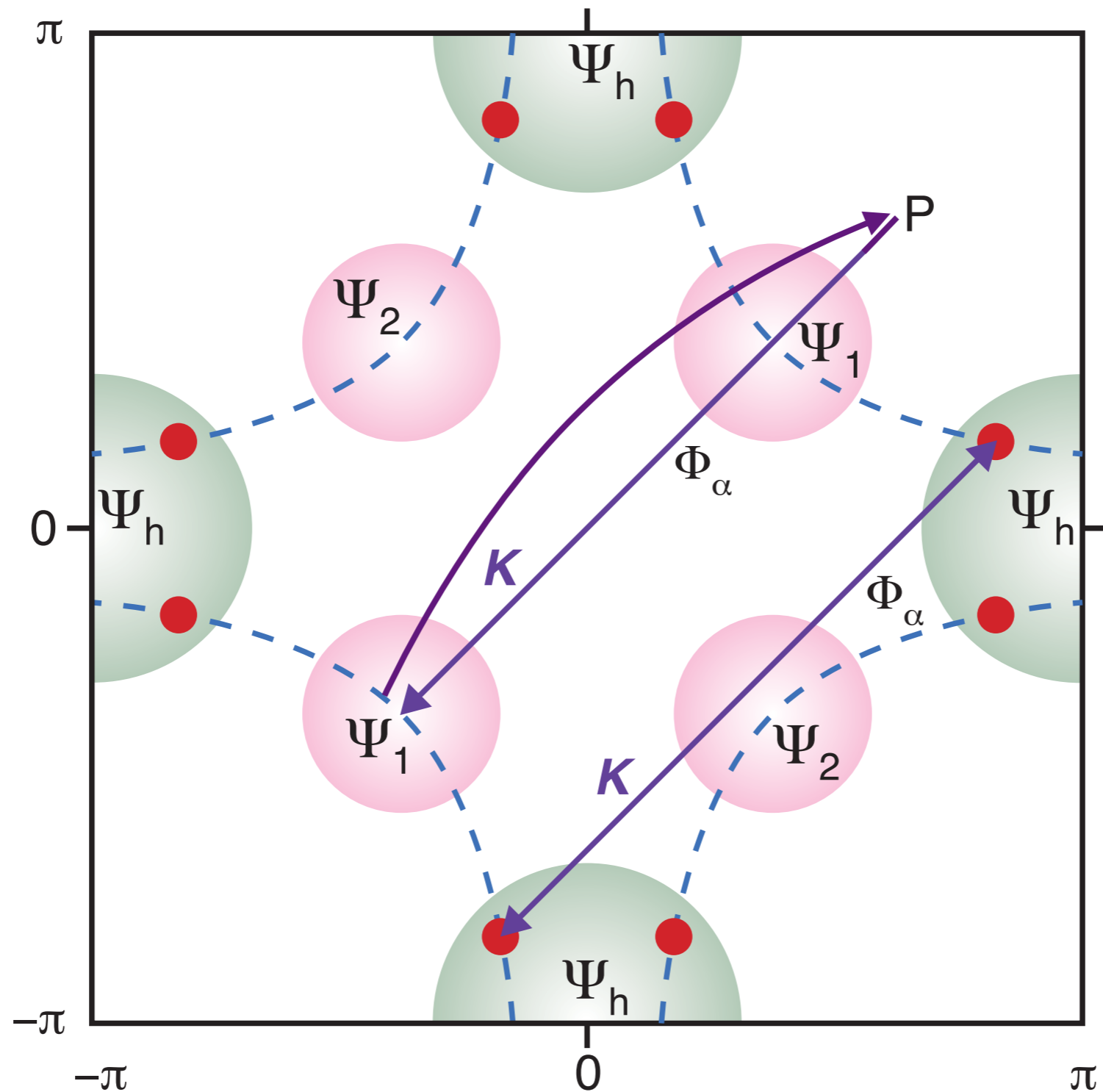
Wavevector mismatch suggests SDW order  
and nodal quasiparticles are decoupled

# Coupling of quasiparticles to SDW order



No “Yukawa” coupling  $\Phi \Psi^\dagger \Psi$

# Coupling of quasiparticles to SDW order



Possible higher order coupling  $\sim |\Phi|^2 \Psi^\dagger \Psi$



# Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_1 = \lambda_1 (|\Phi_1|^2 + |\Phi_2|^2) \left( \Psi_1^\dagger \tau^z \Psi_1 + \Psi_2^\dagger \tau^z \Psi_2 \right)$$

Energy-energy coupling

# Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_2 = \lambda_2 (|\Phi_1|^2 - |\Phi_2|^2) \left( \Psi_1^\dagger \tau^x \Psi_1 + \Psi_2^\dagger \tau^x \Psi_2 \right)$$

Nematic coupling

# Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_3 = \epsilon_{ijk} \left[ \begin{aligned} & (\Phi_{1j}^* \Phi_{1k} + \Phi_{2j}^* \Phi_{2k}) \left( -\lambda_3 \Psi_2^\dagger \tau^x \sigma^i \Psi_2 + \lambda'_3 \Psi_1^\dagger \tau^z \sigma^i \Psi_1 \right) \\ & + (\Phi_{1j}^* \Phi_{1k} - \Phi_{2j}^* \Phi_{2k}) \left( \lambda_3 \Psi_1^\dagger \tau^x \sigma^i \Psi_1 - \lambda'_3 \Psi_2^\dagger \tau^z \sigma^i \Psi_2 \right) \end{aligned} \right].$$

Spiral spin order coupling

# Coupling of quasiparticles to SDW order

Scaling dimensions of these couplings:

$$\dim[\lambda_1] = \frac{1}{\nu} - 2 = -0.9(2)$$

$$\dim[\lambda_2] = \frac{(\bar{\gamma} - 1)}{2} = \begin{cases} -0.05(18) & \text{MZM, 6 loops} \\ -0.10(27) & d = 3 \overline{\text{MS}}, 5 \text{ loops} \end{cases}$$

$$\dim[\lambda_3, \lambda'_3] = \begin{cases} -0.84(8) & \text{MZM, 6 loops} \\ -0.76(8) & d = 3 \overline{\text{MS}}, 5 \text{ loops} \end{cases}$$

# Coupling of quasiparticles to SDW order

Coupling of nematic order is nearly marginal:

Quantum-critical features appear in fermion spectrum via coupling to nematic fluctuations of spin density wave order.

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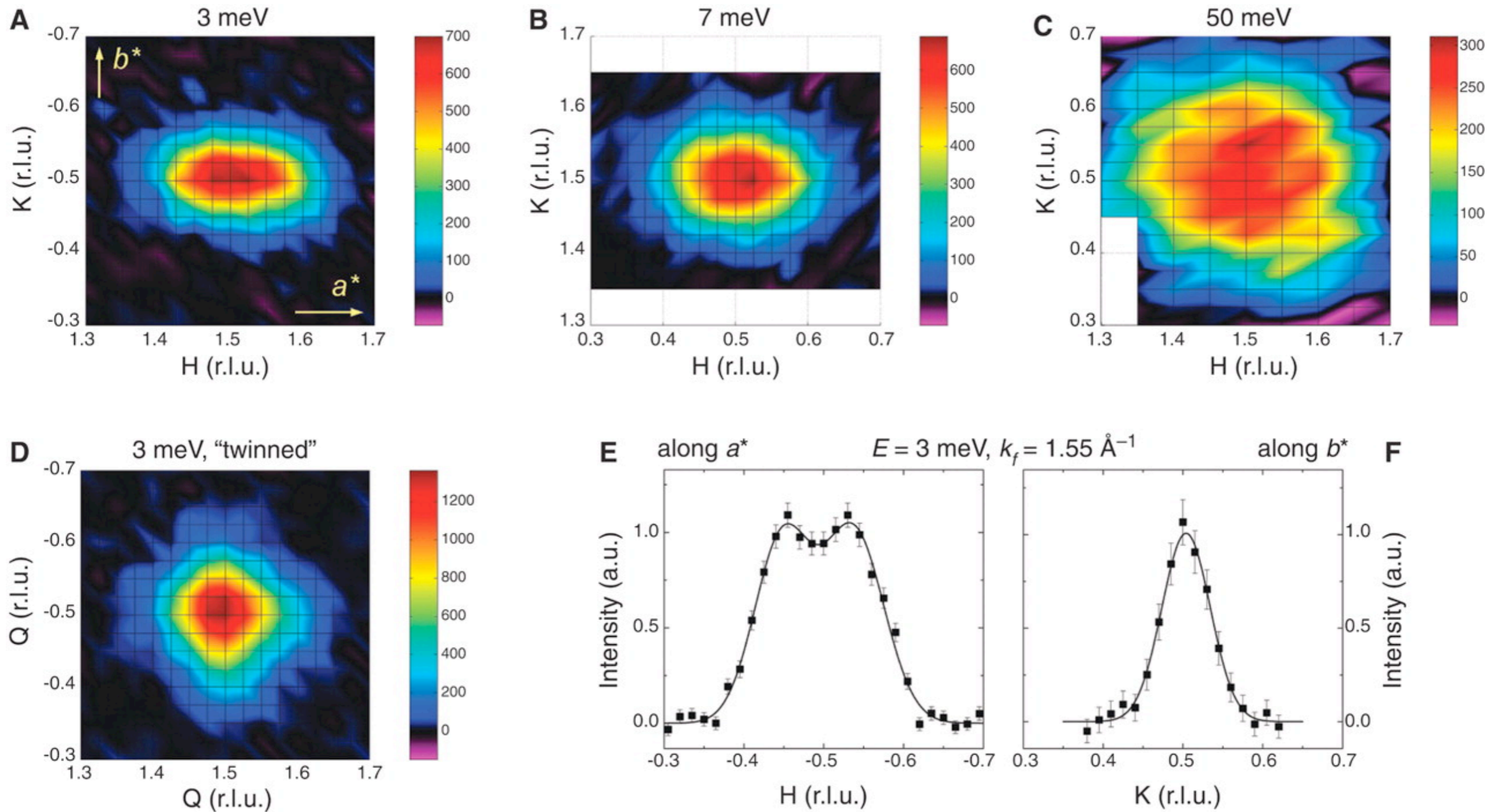
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# Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer, *Science* **319**, 597 (2008)

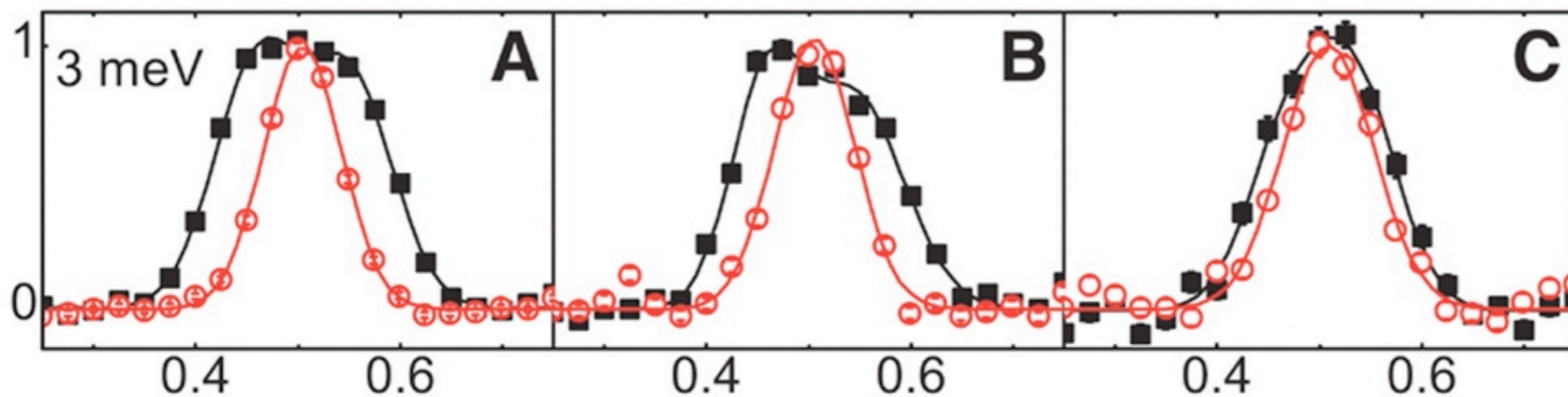


■ along  $a^*$     ○ along  $b^*$

5 K

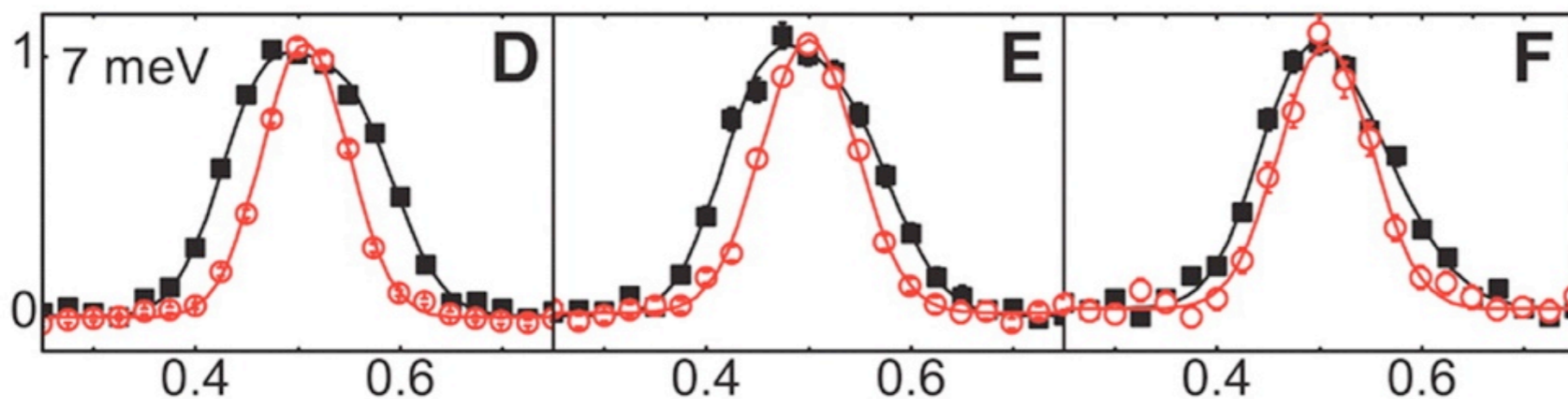
40 K

100 K

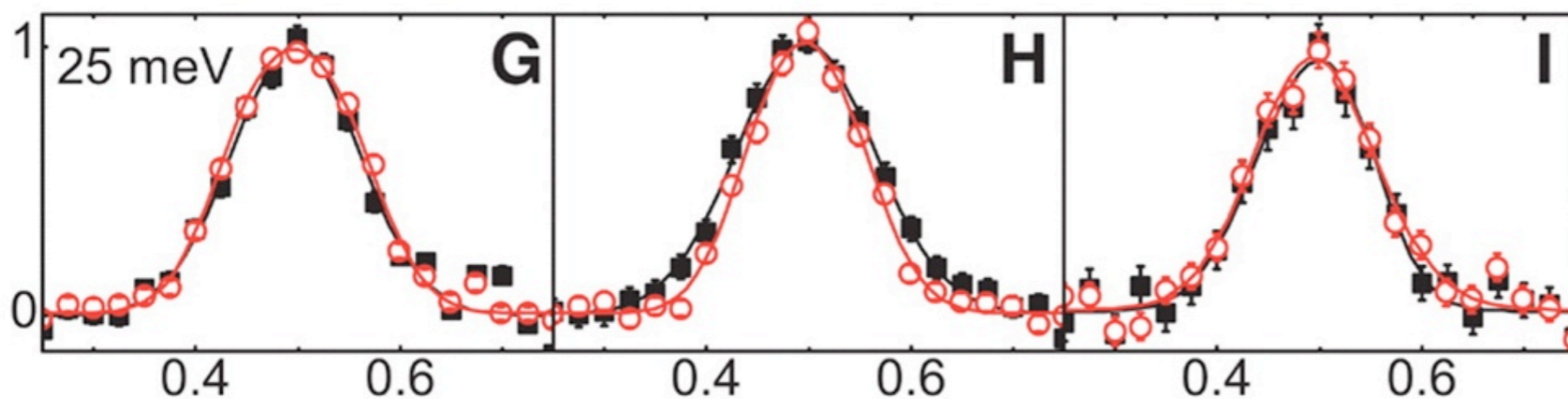


3 meV

Intensity (a.u.)



7 meV



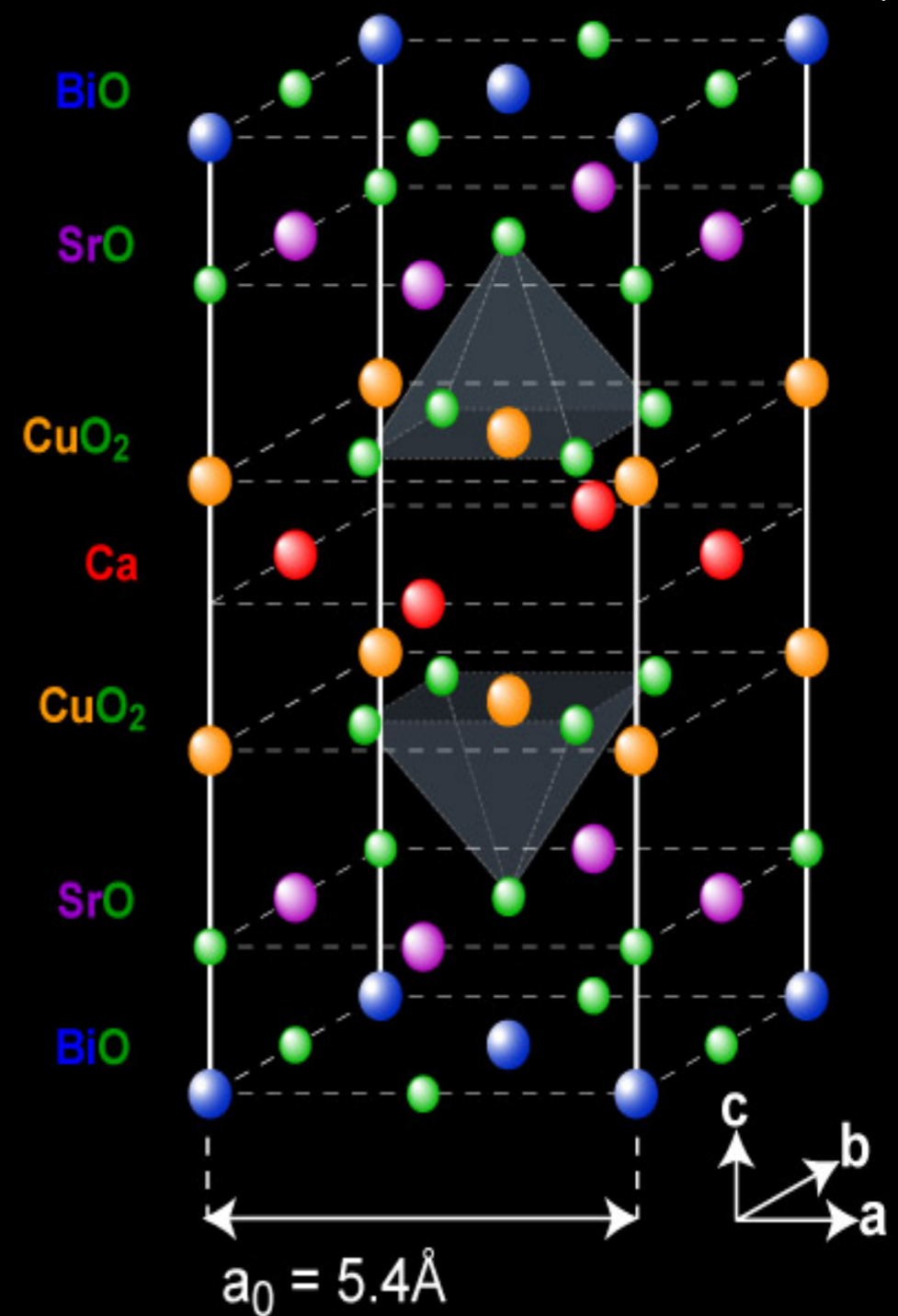
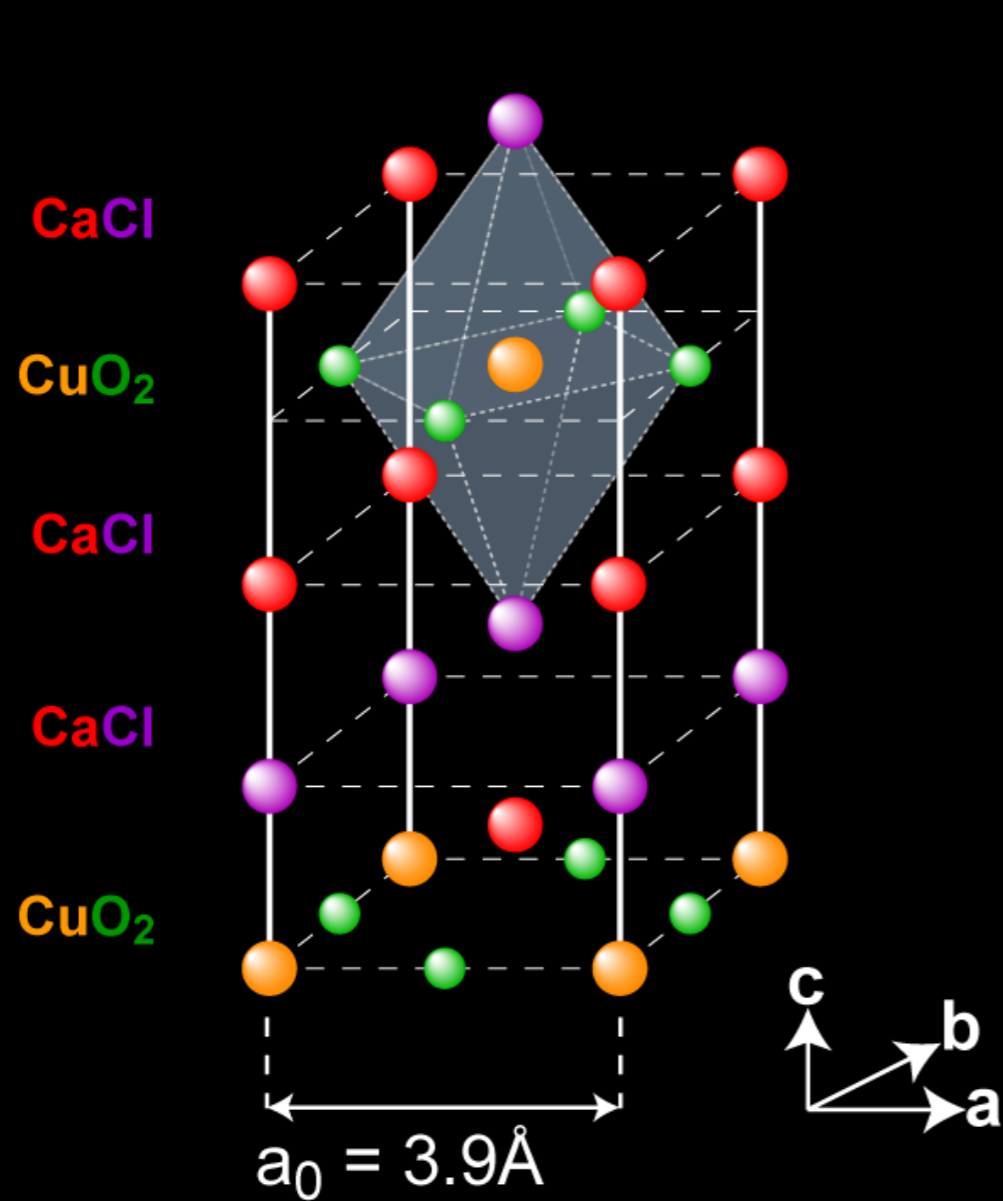
25 meV

Q (r.l.u.)

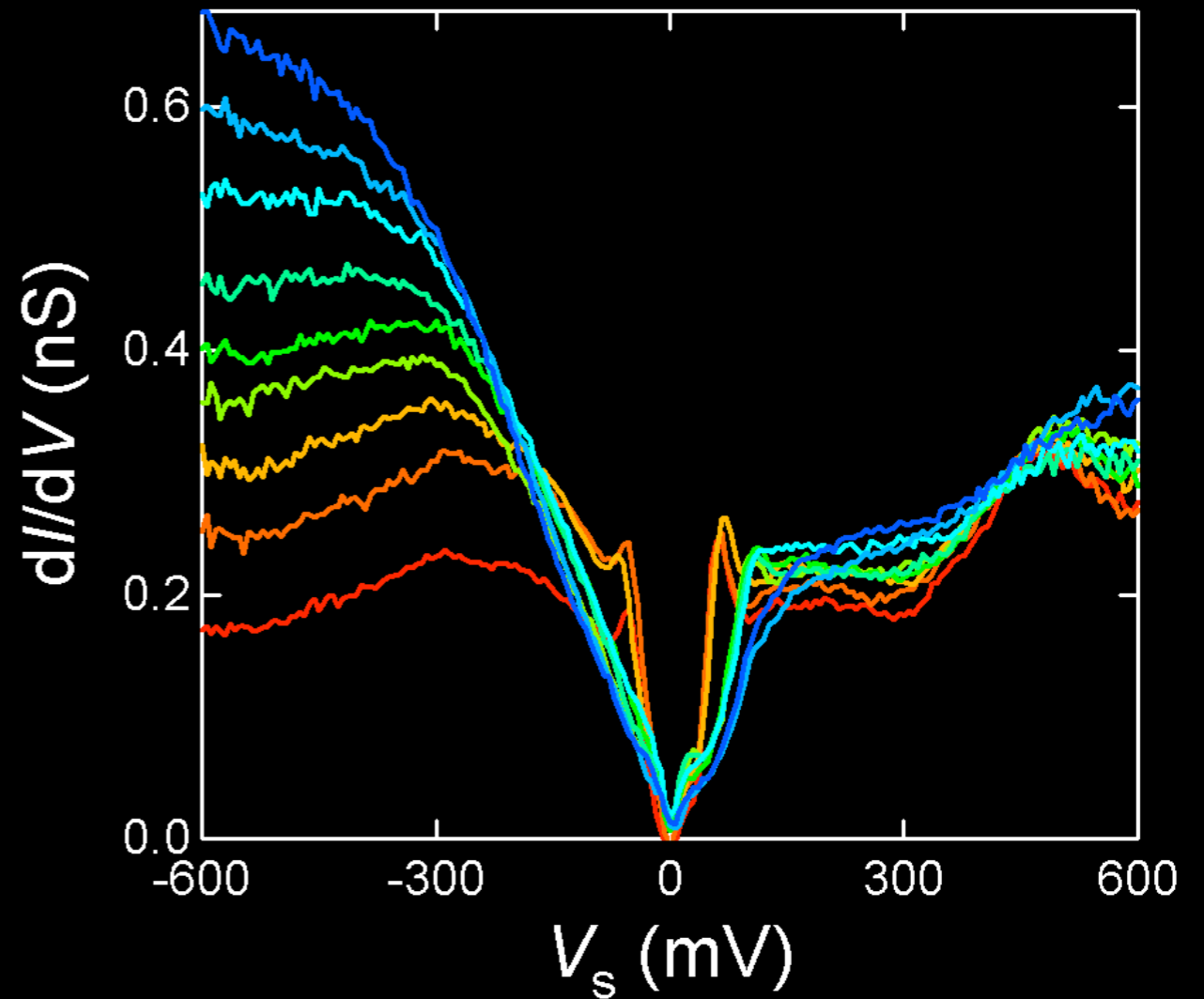
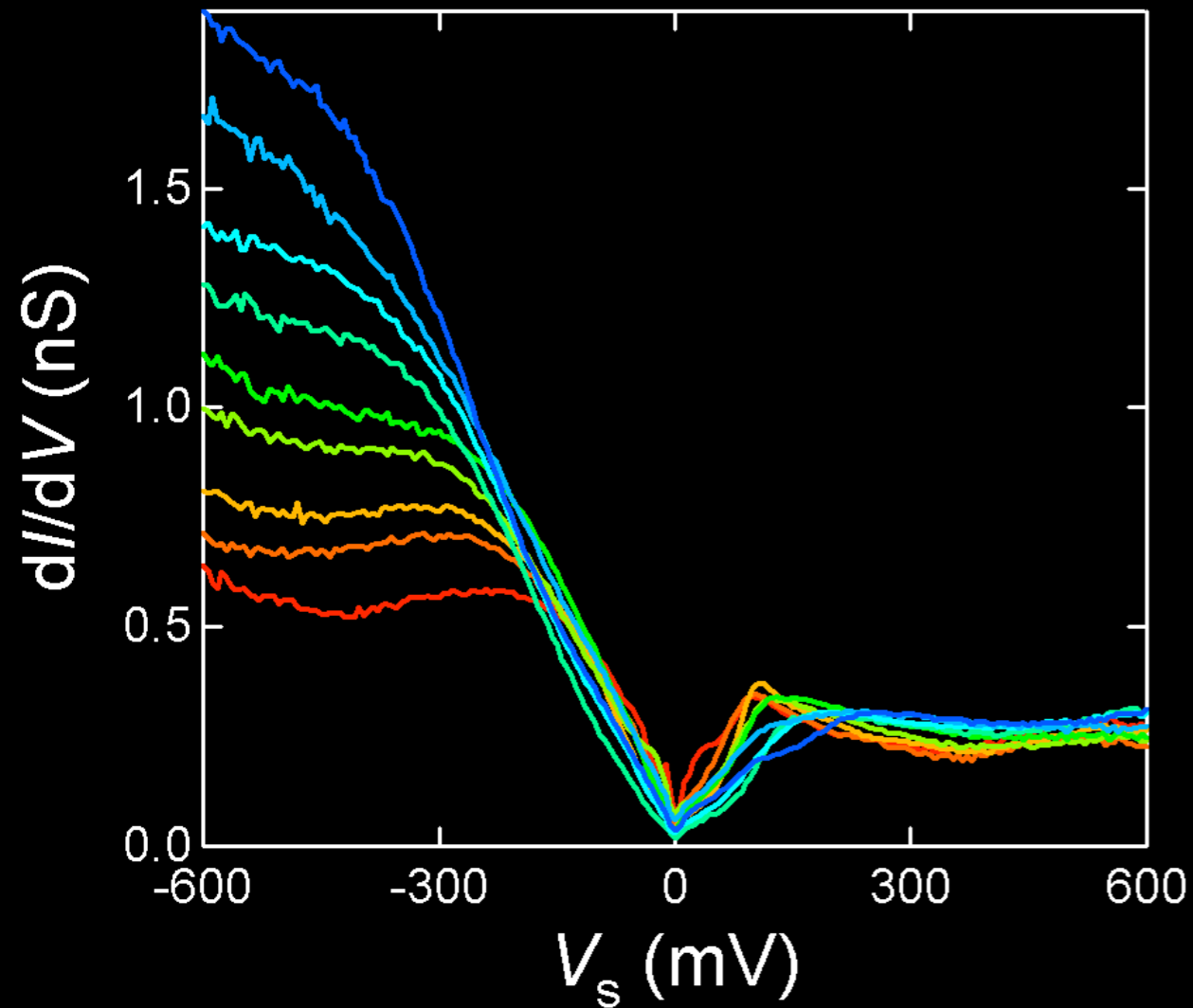
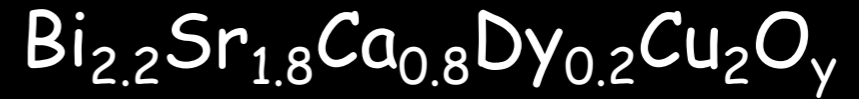
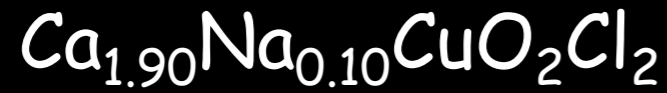
V. Hinkov, D. Haug,  
B. Fauqué, P. Bourges,  
Y. Sidis, A. Ivanov,  
C. Bernhard, C. T. Lin,  
and B. Keimer ,  
*Science* **319**, 597  
(2008)

# Nematic order in YBCO

# STM studies of the underdoped superconductor



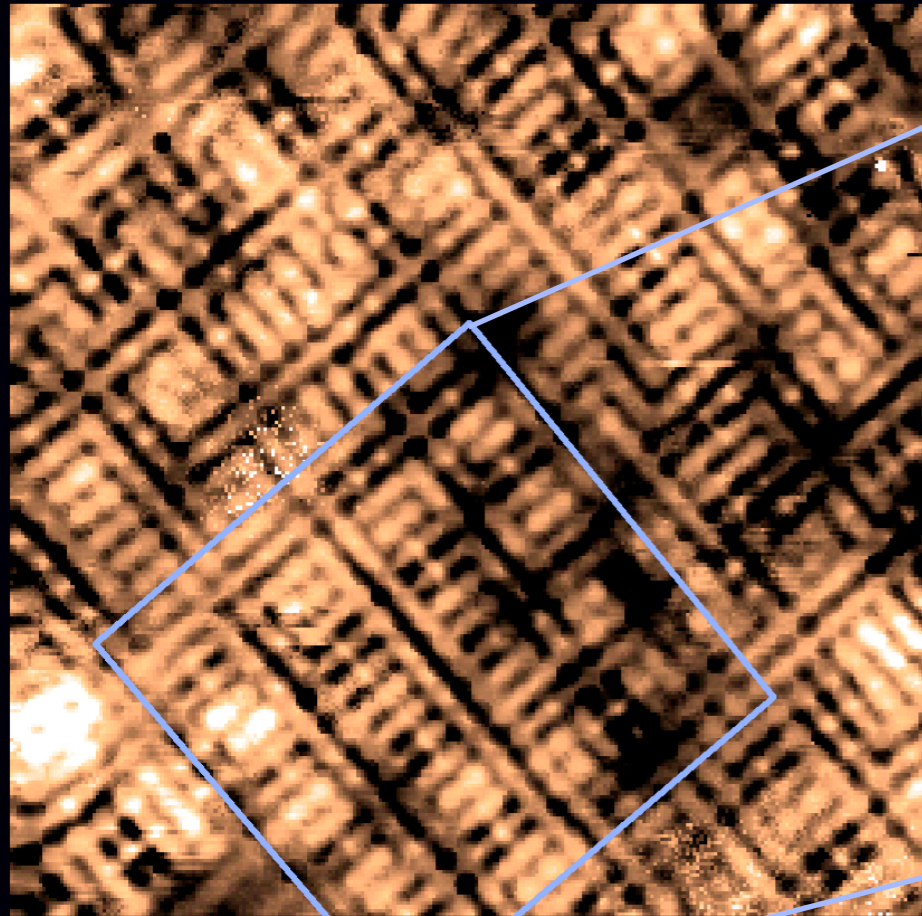
# dI/dV Spectra



Intense Tunneling-Asymmetry (TA)  
variation are highly similar

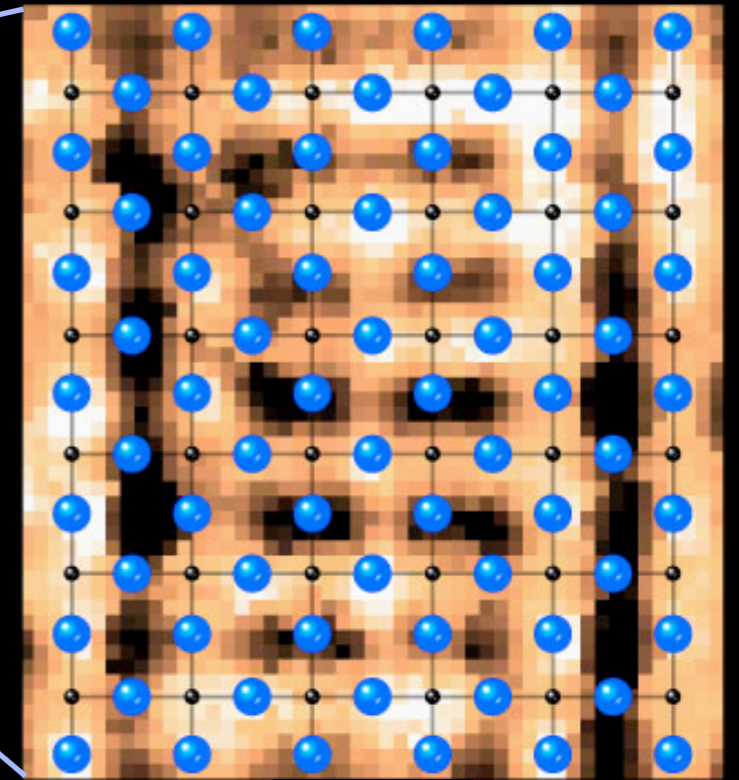
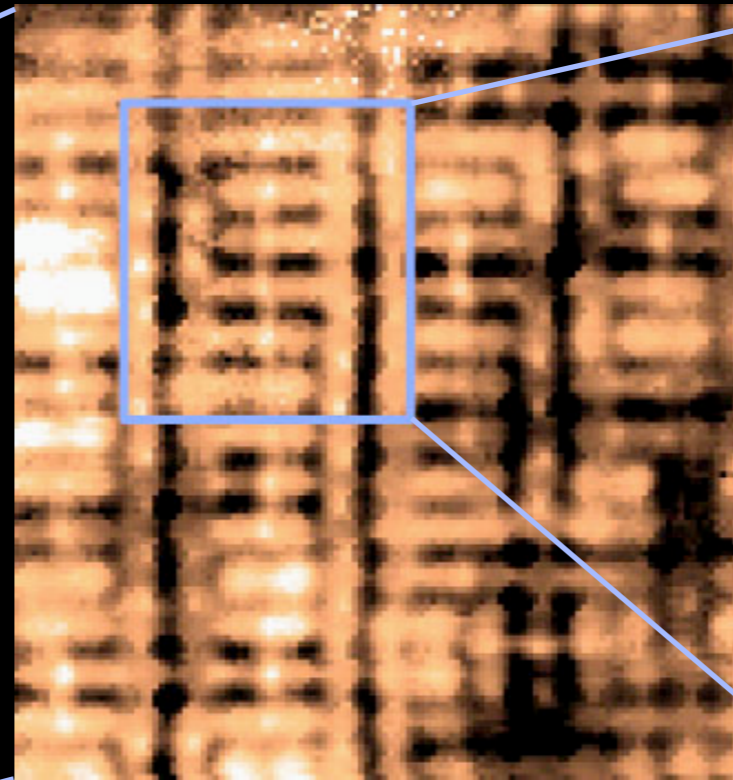
# TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

R map (150 mV)



← 12 nm →

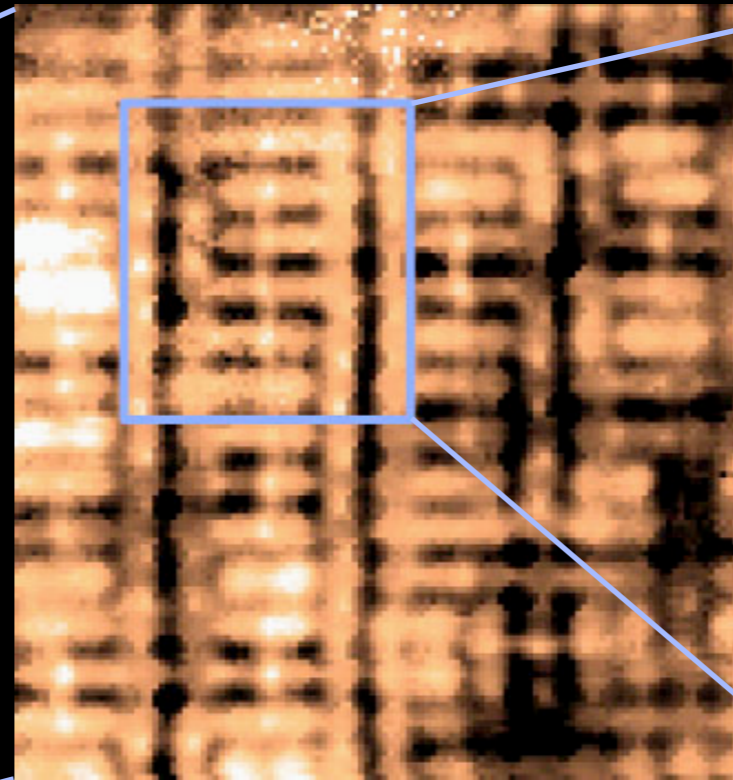
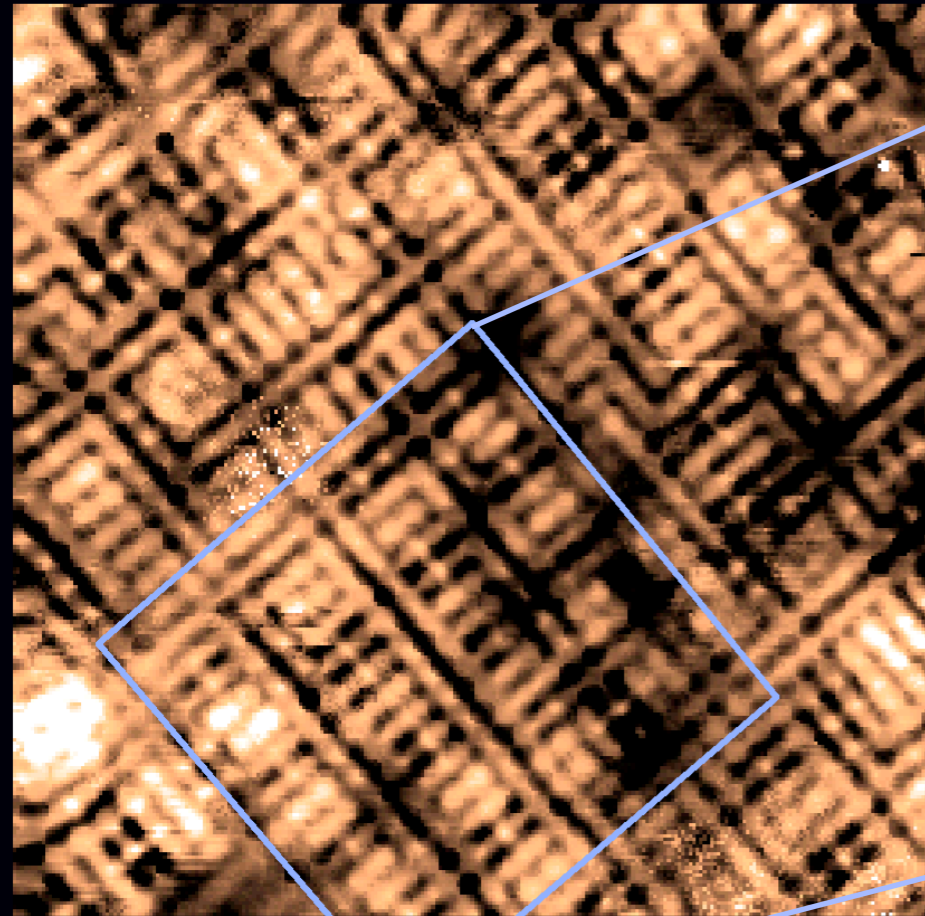
$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$ , 4 K



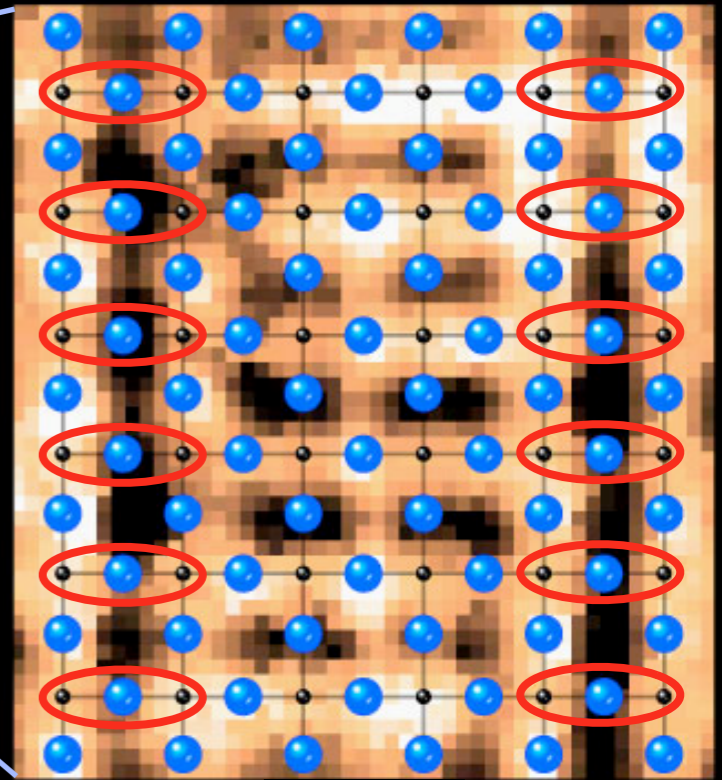
←  $4a_0$  →

# TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

R map (150 mV)



$\text{Ca}_{1.88}\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$ , 4 K



$4a_0$

12 nm

**Broken lattice translation and rotation symmetries:  
valence bond supersolid or supernematic**

S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991).

S.A. Kivelson, E. Fradkin, V.J. Emery, Nature 393, 550 (1998).

M. Vojta and S. Sachdev, Phys. Rev. Lett. 83, 3916 (1999).

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*Infinite anisotropy fixed point*

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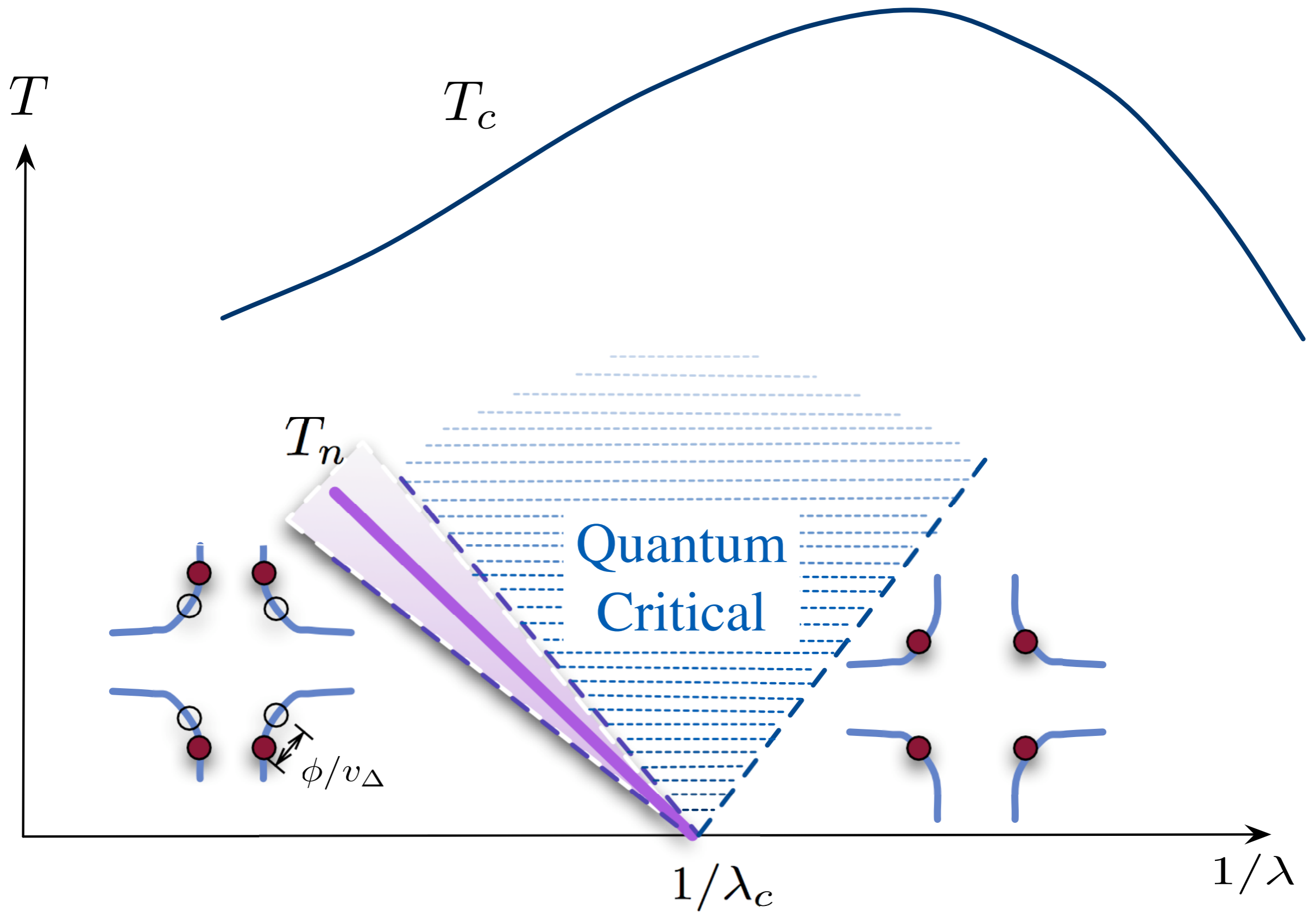
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M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000)

E.-A. Kim, M. J. Lawler, P. Oredo, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099

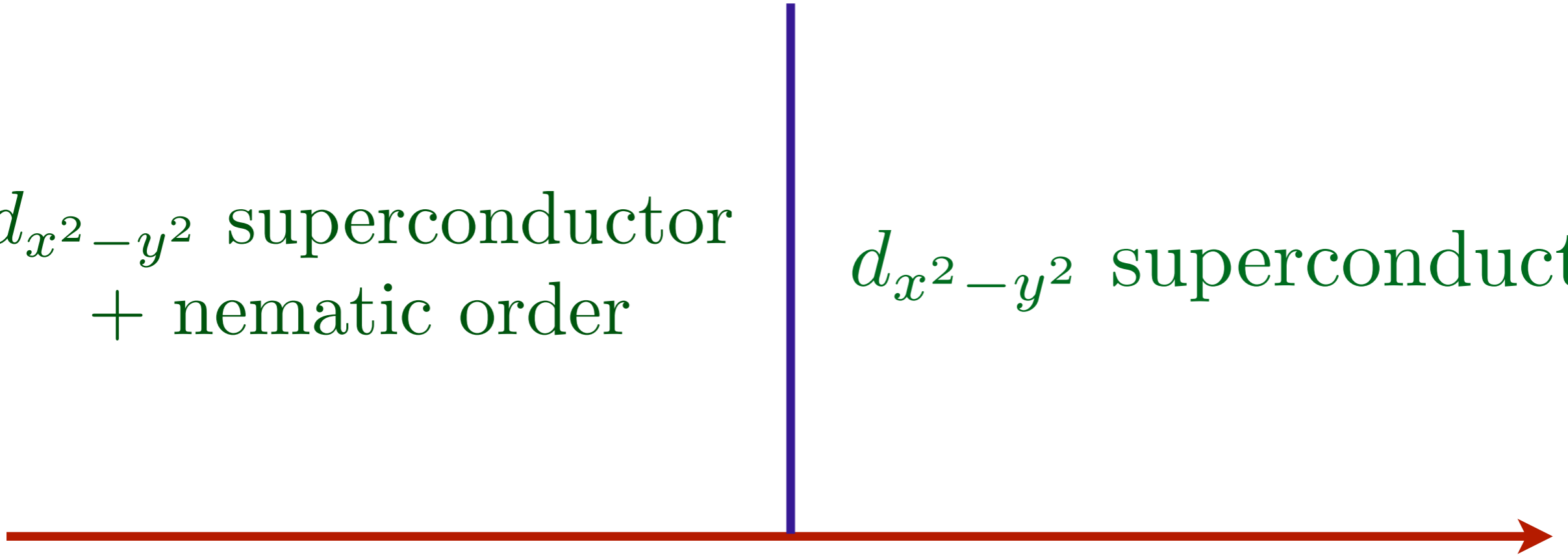


$d_{x^2-y^2}$  superconductor  
+ nematic order

$r_c$

$d_{x^2-y^2}$  superconductor

$r$



$d_{x^2-y^2} \pm s$   
superconductor

$d_{x^2-y^2}$  superconductor

$r_c$

$r$

Order parameter -  $s$  pairing amplitude  $\sim \phi$

Also  $\phi \sim |\Phi_1|^2 - |\Phi_2|^2$

# Field theory for dSC to dSC+nematic transition

$$S_{\phi}^0 = \int d^2x d\tau \left[ \frac{1}{2} (\partial_{\tau} \phi)^2 + \frac{c^2}{2} (\nabla \phi)^2 + \frac{r}{2} \phi^2 + \frac{u_0}{24} \phi^4 \right]$$

Ising theory for nematic ordering

$$S_{\Psi} = \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{1a}^{\dagger} (-i\omega_n + v_F k_x \tau^z + v_{\Delta} k_y \tau^x) \Psi_{1a} \\ + \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{2a}^{\dagger} (-i\omega_n + v_F k_y \tau^z + v_{\Delta} k_x \tau^x) \Psi_{2a}.$$

Free nodal quasiparticles

# Field theory for dSC to dSC+nematic transition

$$S_{\Psi\phi} = \int d^2x d\tau \left[ \lambda_0 \phi \left( \Psi_{1a}^\dagger \tau^x \Psi_{1a} + \Psi_{2a}^\dagger \tau^x \Psi_{2a} \right) \right],$$

Yukawa coupling is now permitted  
and is strongly relevant

RG analysis close to 3 dimensions  
yields runaway flow to strong coupling

## Expansion in number of fermion spin components $N_f$

Integrating out the fermions yields an effective action for the nematic order parameter

$$S_\phi = \frac{N_f}{v_\Delta v_F} \Gamma \left[ \lambda_0 \phi(x, \tau); \frac{v_\Delta}{v_F} \right] \\ + \frac{N_f}{2} \int d^2x d\tau \left( r \phi^2(x, \tau) \right) \\ + \text{irrelevant terms}$$

where  $\Gamma$  is a non-local and non-analytic functional of  $\phi$ .

The theory has only 2 couplings constants:  $r$  and  $v_\Delta/v_F$ .

## Expansion in number of fermion spin components $N_f$

Integrating out the fermions yields an effective action for the nematic order parameter

$$S_\phi = \frac{N_f}{2} \int_{k,\omega} |\phi(k, \omega)|^2 \left[ r + \frac{\lambda_0^2}{8v_F v_\Delta} \left( \frac{\omega^2 + v_F^2 k_x^2}{\sqrt{\omega^2 + v_F^2 k_x^2 + v_\Delta^2 k_y^2}} + (x \leftrightarrow y) \right) \right]$$

+higher order terms which cannot be neglected

# Renormalization group analysis

Couplings are local in the fermion action,  
so perform RG on fermion self energy

The  $1/N_f$  expansion has only one coupling constant  
at criticality:  $v_\Delta/v_F$ .

The RG has the structure:

$$\text{dynamic critical exponent : } z = 1 + \frac{1}{N_f} F_1(v_\Delta/v_F)$$

$$\text{fermion anomalous dimension : } \eta_f = \frac{1}{N_f} F_2(v_\Delta/v_F)$$

$$\text{RG flow equation : } \frac{d(v_\Delta/v_F)}{d\ell} = \frac{1}{N_f} F_3(v_\Delta/v_F)$$

where we have computed the functions  $F_{1,2,3}(v_\Delta/v_F)$ .

# Renormalization group analysis

The RG flow is to  $v_{\Delta}/v_F \rightarrow 0$  with

$$\frac{d(v_{\Delta}/v_F)}{d\ell} = -\frac{4}{\pi^2 N_f} (v_{\Delta}/v_F)^2 \ln \left( \frac{0.46987}{(v_{\Delta}/v_F)} \right)$$

This implies that at the critical point, as  $T \rightarrow 0$  we have the asymptotic result

$$\frac{v_{\Delta}}{v_F} = \frac{\pi^2 N_f}{4} \frac{1}{\ln \left( \frac{\Lambda}{T} \right) \ln \left[ \frac{0.1904}{N_f} \ln \left( \frac{\Lambda}{T} \right) \right]},$$

and a more precise result is obtained by numerically integrating the RG equation.



# Renormalization group analysis

In the limit  $v_{\Delta}/v_F \rightarrow 0$ , the effective action becomes

$$\begin{aligned} S_{\phi} &= \frac{N_f}{v_{\Delta} v_F} \Gamma \left[ \lambda_0 \phi(x, \tau); \frac{v_{\Delta}}{v_F} \right] \\ &\quad + \frac{N_f}{2} \int d^2 x d\tau \left( r \phi^2(x, \tau) \right) \\ &\quad + \text{irrelevant terms} \end{aligned}$$

# Renormalization group analysis

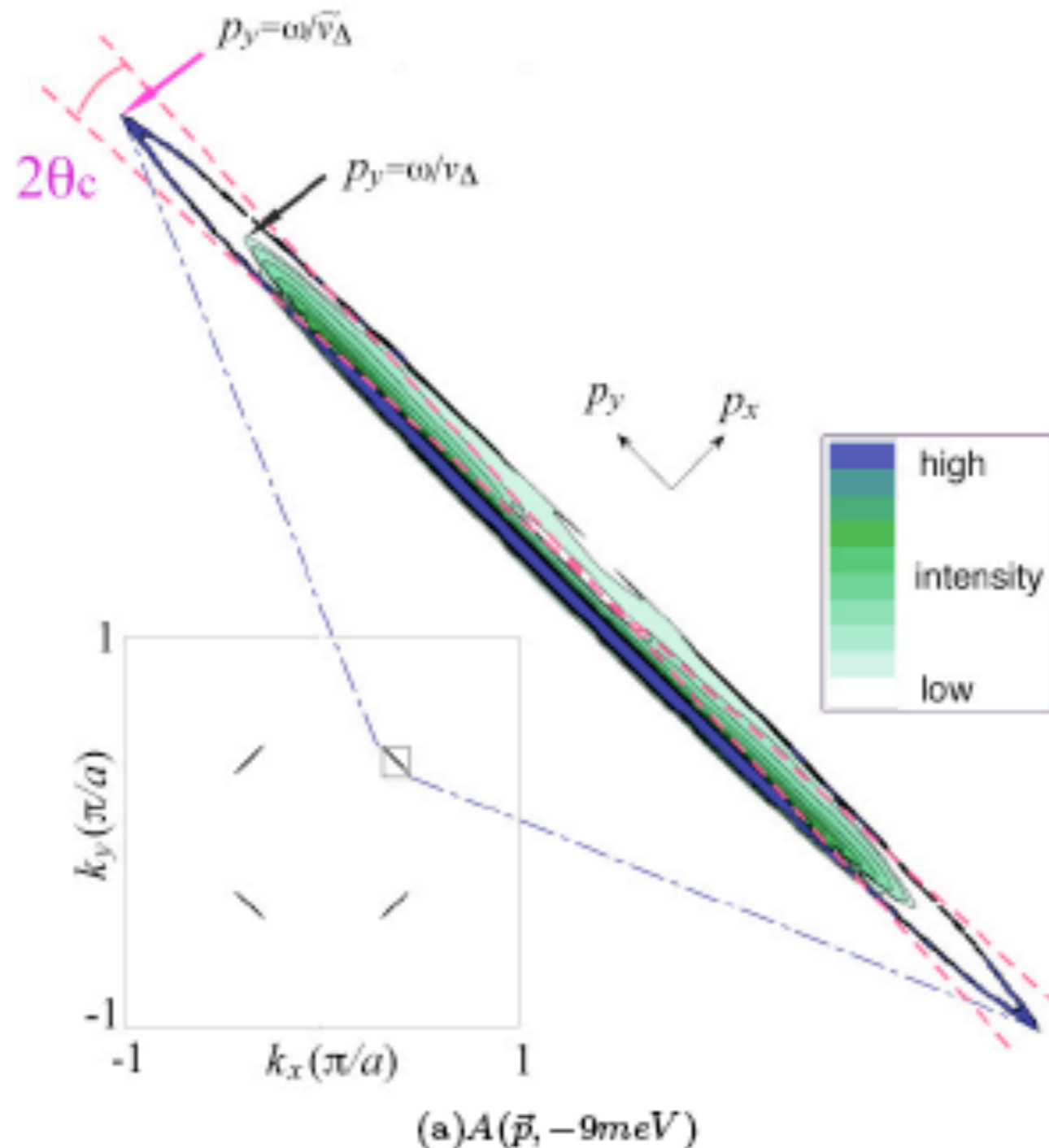
In the limit  $v_{\Delta}/v_F \rightarrow 0$ , the effective action becomes

$$S_{\phi} \sim \frac{N}{v_{\Delta}} \left[ \text{terms that have at most a divergence} \right. \\ \left. \text{which is a power of } \ln(v_F/v_{\Delta}) \right]$$

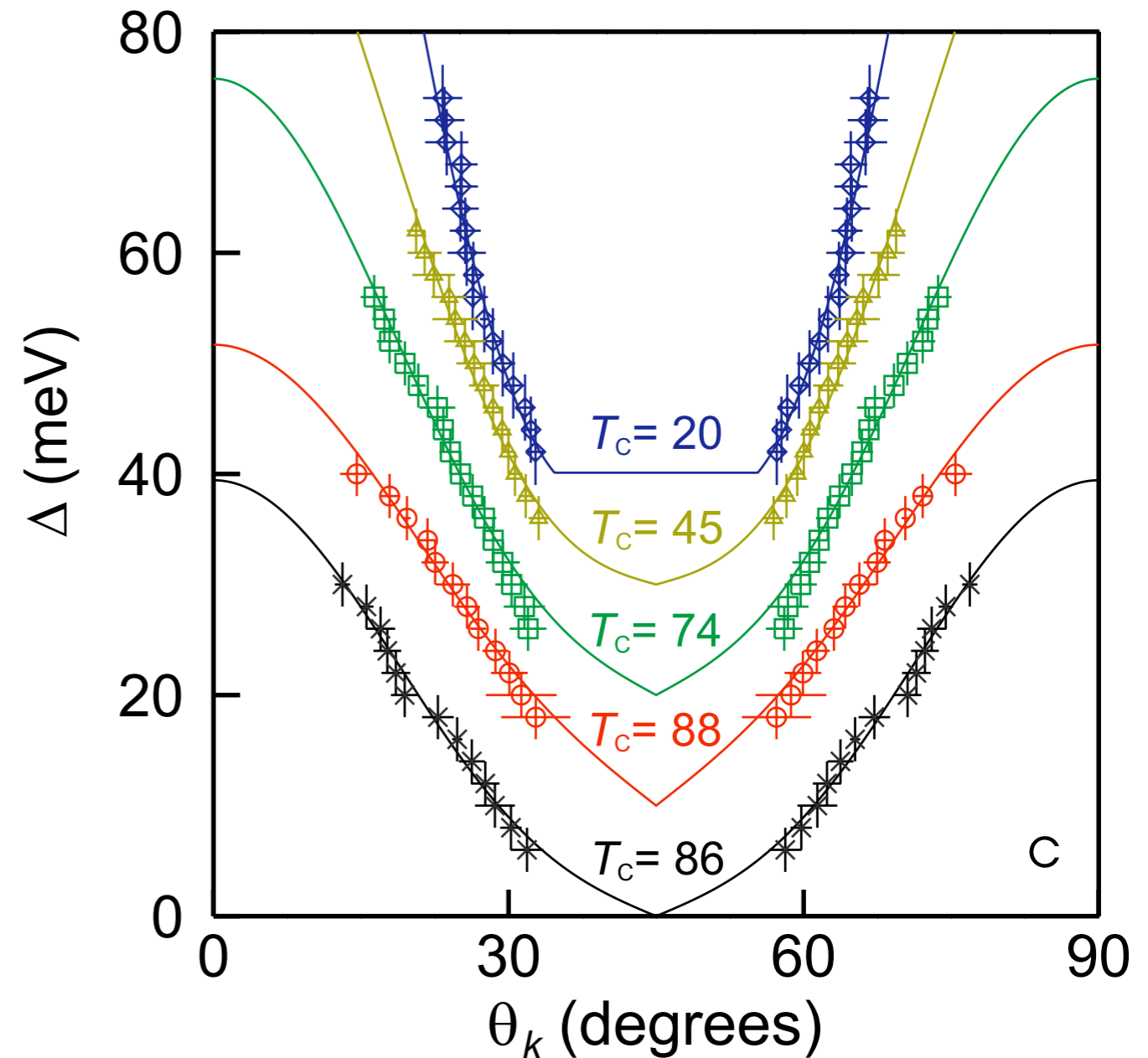
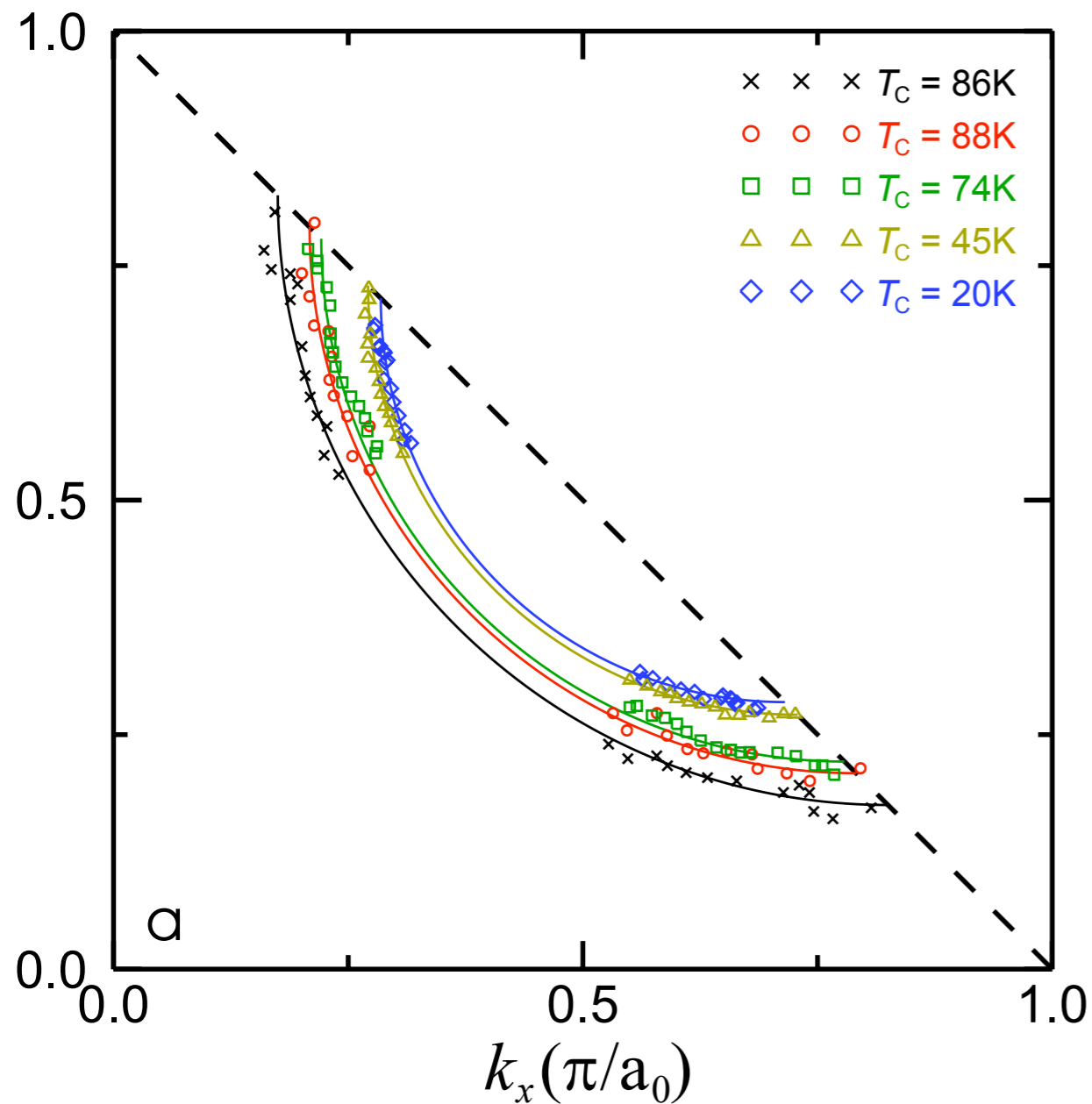
$\Rightarrow$  The theory is controlled by the expansion parameter  $v_{\Delta}/N_f$ , and so results are asymptotically exact even for  $N_f = 2$ .

# Fermion spectral functions

$\phi$  fluctuations broaden the fermion spectral functions except in a wedge near the nodal points



# Quasiparticle spectra from STM on BSCCO



Y. Kohsaka, C. Taylor, P. Wahl, A. Schmidt, Jinhwan Lee, K. Fujita, J. Alldredge, Jinho Lee, K. McElroy, H. Eisaki, S. Uchida, D.-H. Lee, & J.C. Davis, preprint

# Conclusions

1. Theories for damping of nodal quasiparticles in cuprates
2. SDW theory for LSCO yields (nearly) quantum critical spectral functions with arbitrary values of  $v_{\Delta}/v_F$
3. Nematic theory for YBCO/BSCCO has a fixed point with  $v_{\Delta}/v_F = 0$  which is approached logarithmically. The theory is expressed as an expansion in  $v_{\Delta}/v_F$
4. Theories yield “Fermi arc” spectra.