

Precast, Prestressed Concrete Made with Alternative Supplementary Cementitious Materials

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Abstract

Supplementary cementitious materials (SCMs) can often be used as a partial replacement of portland cement to produce an environmentally friendly concrete. Concrete producers have commonly included fly ash in their concrete mixtures for the last 60 years. Unfortunately, the supply of fly ash is in decline as coal-fired power plants continue to shut down across the United States. Alternative SCMs being considered to partially replace portland cement include ground limestone, natural pozzolans, and ground glass. Most research investigating the mechanical properties of concrete that include SCMs has been conducted in a cast-in-place environment, but little research has been conducted in a precast environment. This research investigated the feasibility of including alternative supplementary cementitious materials in the production of precast, prestressed concrete. In addition to performance, material selection considerations also included cost, availability, particle size, and chemical composition. Initially, mortar mixtures were tested to ensure each alternative SCM could meet the requirements of the precast concrete plant, which included significant flow and a minimum compressive strength of 3500 psi after 16-18 hours of curing. The most suitable mortar mixture design, which contained Type III portland cement and 30% ground limestone, was scaled to produce concrete and material performance was assessed in the laboratory using fresh and mechanical properties such as slump, unit weight, air content, and compressive strength. After confirming the concrete mixture containing Type III portland cement and 30% ground limestone was suitable to be produced in a precast concrete plant, precast, prestressed beams

were fabricated, and their performance was assessed. Material and structural tests such as flow, compressive strength, modulus of elasticity, modulus of rupture, flexural strength, and bond strength were conducted. The goal was to replace 30% of the portland cement, but 27% was replaced when the concrete was mixed at the precast concrete plant. Structural performances of the concrete mixture containing Type III portland cement and 27.1% ground limestone performed as predicted by ACI 318-19 [1], and the 16-hour strength met all performance requirements of the precast concrete plant. Considerations and recommendations are also provided after presenting the conclusions.

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1 Introduction

1.1 Motivation

Concrete is one of the most versatile materials on Earth, and it is used to construct hospitals, schools, roads, homes, and many other parts of civil infrastructure. The main ingredients of concrete are fine aggregate, coarse aggregate, water, and portland cement. Portland cement is made by feeding raw materials including ground limestone and clay into a kiln that heats the materials inside to 1450°C. When limestone decomposes, carbon dioxide is produced. Carbon dioxide is also emitted when fuels are burned to keep the kiln at the required temperature. These carbon dioxide emissions negatively affect the environment.

Portland cement production accounts for an estimated 8% of global carbon dioxide emissions each year [2]. There is no indication that this number will decrease as the amount of portland cement produced has grown exponentially in the last 100 years [3] as shown in Figure 1.1.

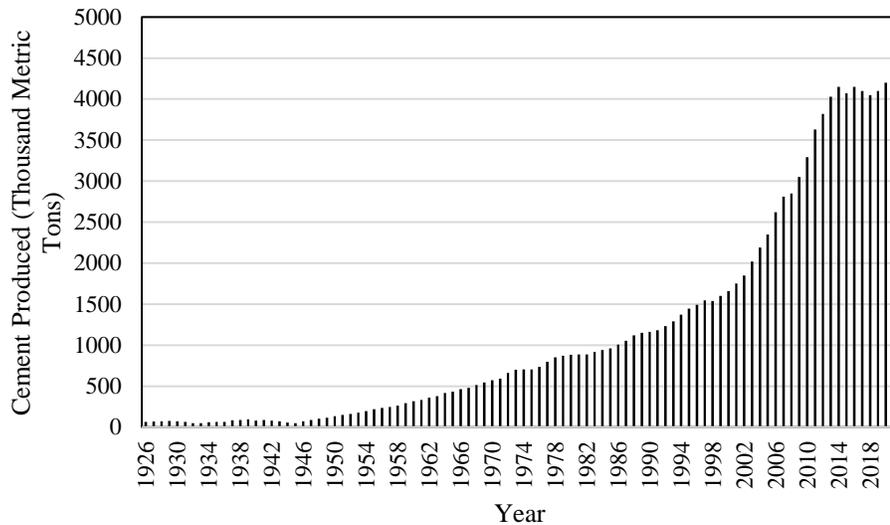


Figure 1.1 Global portland cement production each year since 1926 [4].

For decades, researchers have been studying ways to reduce carbon dioxide emissions during concrete production. One of the solutions to reduce the carbon footprint is the use of supplementary cementitious materials (SCMs) as a partial replacement of portland cement in concrete. Until recently, the most commonly used SCMs included fly ash, slag, and silica fume [5].

Fly ash is a byproduct of burning coal for electricity production and is a good replacement for portland cement because of its high silica alumina content, fine and spherically shaped particles, and previously low cost. Fly ash can provide many benefits to concrete when added as a partial replacement of portland cement. These benefits include increased strength, increased durability, increased workability, and reduced permeability [6]. Coal-powered energy, however, contributes 40% of global carbon dioxide emissions [7] and as the United States moves towards using cleaner energy sources in accordance with COP26 [8], coal power plants will be shut down and the supply of fly ash will dwindle. Figure 1.2

shows how the production of fly ash has decreased over the past decade.

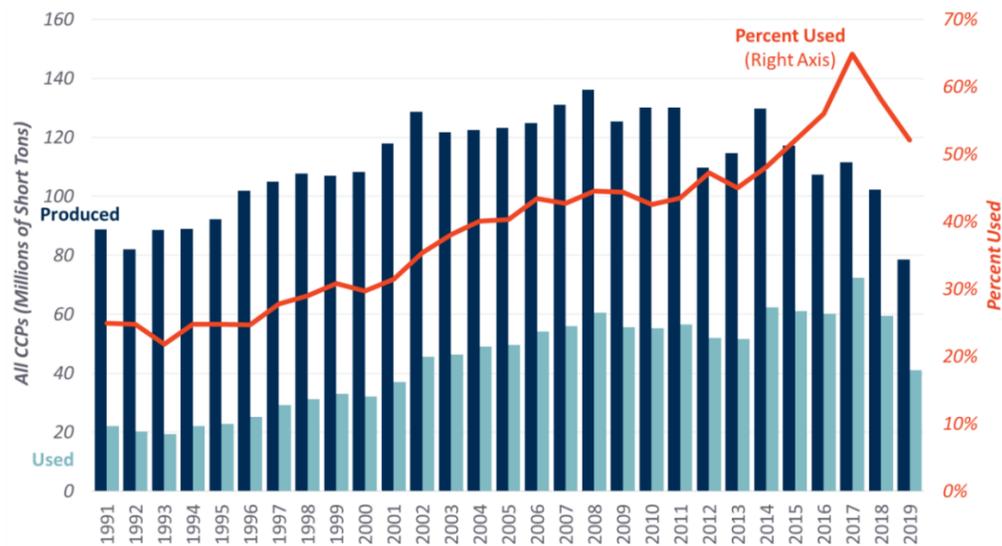


Figure 1.2 Fly ash production and use [9].

As the supply of fly ash declines, researchers have been looking for alternative materials to replace portland cement in concrete. Recent materials that researchers have been considering include calcined clay and shale, ground glass, natural pozzolans including metakaolin, diatomaceous earth, rice husk ash, and ground limestone.

Documented research has primarily been focused on using SCMs as a partial replacement of portland cement in cast-in-place concrete. An increasingly popular alternative to cast-in-place concrete building systems are precast concrete building systems. The components of precast concrete buildings are made in a factory-like manufacturing setting that is subject to strict quality assurance and quality control standards. Concrete specimens precast in a plant are shipped to jobsites for quick erection, saving time and money that is often associated with

formwork and curing cast-in-place concrete. To achieve this type of speed and efficiency, precast concrete components are removed from the formwork 16 to 18 hours after the concrete has been cast (daily). Precast concrete must meet a particular set of requirements that include a flow similar to that of self-consolidating concrete and a minimum compressive strength of 3500 psi when the concrete is removed from the molds. Therefore, properties such as cost, availability, particle size, and chemical composition of SCMs are important to consider when determining the feasibility of replacing portland cement with an SCM, other than fly ash, in a precast concrete environment. As the precast concrete industry continues to grow in popularity and the supply of fly ash continues to decrease, it is ever more important to create mixture designs with alternative SCMs that will meet the requirements set by precast concrete plants.

1.2 Project Goals

The primary goal of this research was to investigate the performance of concrete mixtures that included regionally-sourced alternative SCMs to partially replace portland cement in concrete. These mixture designs were created specifically to be used in a precast concrete plant. The major objectives of the project were to:

1. Develop a concrete mixture design using regionally-sourced alternative SCMs that is economically feasible and meets minimum workability and compressive strength requirements set forth by the precast concrete producer.

2. Compare the mechanical and structural performance of precast, prestressed concrete beams made with portland cement and fly ash to beams made with the portland cement and the best performing alternative SCM investigated.

1.3 Thesis Organization

This thesis is organized into five chapters. Chapter two provides a literature review that includes a general background on portland cement production, alternative SCMs, and a summary of previous literature on using alternative SCMs as partial replacements of portland cement. Chapter three presents information on the materials and methods used to fabricate mortar and concrete specimens, as well as to conduct tests on the fresh, mechanical, and structural properties of the specimens. Chapter four presents the data collected during mortar, concrete, and precast beam fabrication and testing and provides analysis and discussion of the results. Lastly, chapter five presents conclusions and provides recommendations for future work.

2 Literature Review

2.1 Portland Cement Concrete

Portland cement concrete is composed of portland cement, coarse aggregate, fine aggregate, and water. It is essential to building structures in communities and is one of the most durable construction materials. Concrete has many favorable attributes that include high strength and durability; it is also low in cost compared to other construction materials.

2.1.1 Portland Cement Production

Portland cement production commonly takes place near quarries filled with limestone and clay. The limestone, clay, and other materials are removed from the quarry and ground into a powder. The ground material is placed in a slowly rotating kiln that is built on a slight incline and heated to 1450°C. The kiln is powered by burning fossil fuels, which is one of the sources that contributes to the carbon footprint of portland cement. The other main source of greenhouse gas emissions is the calcination of the limestone. When limestone (CaCO_3) is heated above 800°C in the kiln, it begins to decompose and produces lime (C) and carbon dioxide ($\bar{\text{C}}$) as shown in Equation 2.1. Equation 2.1 is presented in cement chemistry notation; Table 2.1 presents formulas for commonly used oxides and main phases of portland cement production. The combination of burning fossil fuels and the decomposition of limestone is why portland cement production accounts for 8% of global carbon dioxide emissions each year.



Table 2.1 Cement chemist notation.

CCN	Formula	Name
C	CaO	Lime
\bar{C}	CO ₂	Carbon dioxide
C ₂ S	Ca ₂ SiO ₄	Dialcium silicate
C ₃ A	Ca ₃ Al ₂ O ₆	Tricalcium aluminate
C ₄ AF	Ca ₂ AlFeO ₅	Tetracalcium aluminoferrite
C ₃ S	Ca ₃ SiO ₅	Tricalcium silicate
CH	Ca(OH) ₂	Caesium hydroxide
C-S-H	CaH ₂ O ₄ Si	Calcium silicate hydrate
C-A-S-H	Ca ₂ Al ₂ O ₇ Si	Calcium aluminosilicate hydrate

As the lime continues to move through the kiln, it reacts with other raw elements to produce dialcium silicate (C₂S), tricalcium aluminate (C₃A), and tetracalcium aluminoferrite (C₄AF). As the materials continue to increase in temperature, C₂S reacts with lime to form tricalcium silicate (C₃S), as shown in Equation 2.2.

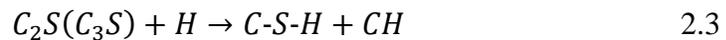


By the time the materials reach the end of the kiln, the resulting phases have formed small spherical balls called clinker. Clinker leaves the kiln at an extremely hot temperature and is cooled immediately. Once the clinker is cooled completely, it is interground with gypsum to create portland cement.

2.1.2 Hydration of Portland Cement in Concrete

Hydration is a series of chemical reactions that must occur for concrete to harden. There are a multitude of complex reactions that occur during hydration, however the scope of this explanation is limited to the hydration of the calcium

silicates. When water reacts with both C_2S and C_3S , calcium silicate hydrate (C-S-H) and calcium hydroxide (CH) are produced. C-S-H is a very desirable product and is the main contributor to the strength of the concrete. This reaction also produces an undesirable amount of calcium hydroxide (CH) as shown in Equation 2.3 [10], which does not provide any significant benefit to the performance of the concrete.



Tricalcium silicate (C_3S) reacts with water quickly, which contributes to the early strength of concrete. Alternatively, dicalcium silicate (C_2S) reacts with water more slowly, which contributes the later strength of the concrete.

2.2 Supplementary Cementitious Materials

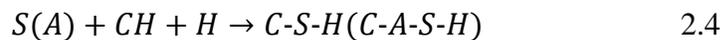
Supplementary cementitious materials (SCMs) are materials that replace a portion of portland cement in concrete and are cementitious, pozzolanic, or fillers. Cementitious materials chemically react with water to produce a hardened material, which also classifies these materials as hydraulic. Pozzolanic materials are not cementitious, rather they need to be in the presence of both water and calcium hydroxide to gain strength. This chemical reaction is slow, but ultimately produces a strong, hardened material. Fillers are SCMs that are not cementitious or pozzolanic. Common reasons to add SCMs to concrete include to decrease the total amount of portland cement in the concrete, improve the workability of the mixture, increase the durability, decrease the permeability, or to mitigate alkali silica reaction (ASR).

Important properties that set SCMs apart from one another include particle size, chemical composition, cost, and availability. An SCM with a finer particle size will have a larger total surface area of the particles, which will increase the reactivity. Accelerating the hydration of concrete will, in turn, accelerate the setting time and lead to a higher early strength [11]. Compositions of SCMs can include high amounts of calcium (C), silica (S), and alumina (A) that will react with CH to produce more C-S-H and calcium aluminate silicate hydrate (C-A-S-H). More C-S-H and C-A-S-H typically equate to increased concrete compressive strength.

It is also equally important to find a material that will be cost effective and locally available. When a material is not locally available, it can be costly to transport. Ideally, the cost of transportation of an SCM as well as the cost of the SCM itself is less than the cost of portland cement and its transportation. Finding a cost effective and locally available SCM would decrease the total cost of production of concrete as well as decrease the carbon footprint.

2.2.1 Pozzolans

Pozzolans are materials that are composed primarily of alumina and silica and are not cementitious but pozzolanic. This means that, unlike portland cement, when a pozzolan is hydrated, it will not harden due to chemical reactions. Rather, pozzolans need to be in the presence of water and calcium hydroxide to react, and when they do they form more C-S-H, as shown in the Equations 2.4 [12].



Across the globe, many pozzolans can be found naturally, while others are byproducts from industrial or commercial processes. The chemical and physical properties of pozzolans can differ significantly, which can affect the fresh, hardened, mechanical, and structural properties as well as the durability of concrete. However, in general, most pozzolans can provide a number of benefits to concrete, such as increased strength, the mitigation of chemical attacks, and a decrease in the carbon footprint of portland cement concrete [13].

2.2.1.1 Fly ash

Fly ash is a popular pozzolan that many concrete producers have used to partially replace portland cement in concrete for the last 60 years [14]. It is a byproduct from coal-fired electric plants. When pulverized coal burns, impurities within the coal fuse together and become suspended as spherical particles in the air above the combustion chamber. When the particles are cooled and solidified, they form spherical particles and are collected from the flue gas using electrostatic precipitators or other similar methods [15].

There are two different types of fly ash: Class C and Class F. Class C fly ash is a high calcium fly ash and Class F is a low calcium fly ash [16]. Fly ash is uniquely spherical in shape and often has a smaller particle size than portland cement. These are two of the main reasons why it is one of the most popular SCMs. Both the shape and size of fly ash are important to improve the workability and pozzolanic reactivity of portland cement concrete, as well as to decrease the water demand [17]–[22]. Bier and Rizwan [17] reported that compared to limestone and silica fume, fly ash required the least amount of high

range water reducer to reach the desired consistency of self-consolidating concrete. This is due to the small, spherical shape of the particles; the fly ash particles roll against each other like tiny ball bearings and can fill the spaces between the portland cement particles, decreasing the allowable space for water [23].

Additionally, fly ash provides benefits to hardened properties of concrete including increased ultimate strength, increased durability, reduced permeability, increased sulfate resistance, and decreased alkali silica reactivity [21], [24]–[28]. Although the addition of fly ash increases the ultimate strength of concrete, it is reported that, at larger replacement levels (50-60%), the seven-day strength is much lower than 100% portland cement concrete [24]–[26], [28]. When high early strength is a requirement, researchers have increased the concrete's early strength by adding different admixtures, such as water-reducers and accelerators [29], [30].

2.2.1.2 Natural Pozzolans

In 2022, the most common natural pozzolans that are used as a partial replacement of portland cement in concrete are calcined clay, calcined shale, and metakaolin [31]. These materials are sourced from natural mineral deposits, heat treated and then ground into pozzolans. Volcanic ash, volcanic glass, and rice husk ash are also categorized as natural pozzolans, but these materials are not used as frequently in concrete since they are not commonly found in every region of the world [31].

Natural pozzolans can differ in chemical composition, which means that the pozzolanic activity of each natural pozzolan is different. This affects the levels at which natural pozzolans can be used to replace portland cement and other properties such as the compressive strength of the concrete [32]–[34]. The level of replacement and compressive strength, as well as workability and water demand, also depends on the size and shape of the particles [35]. Compared to portland cement, natural pozzolans are generally ground to similar sizes, but can vary in shape [31]. Some natural pozzolans, like calcined clay and shale, have small, smooth particles which increases the early compressive strength and workability and decreases the water demand [31]. Other natural pozzolans, like pumicite are coarse and not finely divided, which would cause a decrease in workability and increase water demand [33]. Like fly ash, adding natural pozzolans to concrete generally increases the workability, decreases water demand, increases the late compressive strength, and can help mitigate chemical attack [31], [33], [36]–[38].

Tang et al. [39] found that when a natural pozzolan was used at 20% replacement levels, the workability of the fresh concrete increased by 16.7% compared to the portland cement concrete control. Unfortunately, this study reported lower compressive strengths for the concrete containing natural pozzolan compared to the portland cement concrete control. Compressive strengths of the concrete containing natural pozzolan at three days and 28 days were 2520 psi and 5480 psi, respectively; the portland cement concrete control had compressive strengths of 3520 psi and 6250 psi at three days and 28 days. The differences between the compressive strengths became smaller over time, decreasing from

1000 psi at three days to 770 psi at 28 days, which demonstrates that the natural pozzolan continued to contribute to the later strength of the concrete.

Similarly, Ghrici et al. [40] reported that the compressive strength of concrete containing 20% replacement of portland cement with natural pozzolan was less than the compressive strength of the portland cement concrete control. At two days, the compressive strengths of concrete containing natural pozzolan and portland cement concrete were 2310 psi and 2410 psi, respectively. In contrast to Tang et al. [39], the compressive strength of the concrete containing natural pozzolan exceeded the compressive strength of portland cement concrete at 28 days by 580 psi.

Mehta [41] and Massazza and Costa [42] reported similar results to Ghrici et al. [40]. They found that the early compressive strength of concrete containing natural pozzolans was less than that of the portland cement concrete. At lower replacement levels (5-20%), the late compressive strength of the concrete containing natural pozzolans exceeded that of the portland cement concrete. Mehta [41] reported that when the replacement level was increased to 30%, the late compressive strength did not exceed the portland cement concrete but was comparable.

2.2.1.3 Ground glass pozzolan

There are approximately 10 million metric tons of glass wasted in the United States each year. Only 33% of glass is recycled, while the remaining amount of glass ends up in landfills [43]. Although the rate of glass seems very

low, it is expected to increase as the market for SCMs becomes more competitive [43].

Research related to using glass in concrete has been conducted since approximately 1960 [44]. At first, recycled glass was considered as a coarse aggregate in portland cement concrete, but these studies were unsuccessful because the mechanical properties significantly decreased compared to portland cement concrete that contained natural coarse aggregates [45]. The decrease in strength was attributed to the fact that glass has a smooth, slippery surface, which weakened the bond between the glass coarse aggregate and portland cement paste [45].

More recently, glass has been considered as a supplementary cementitious material that can partially replace portland cement in concrete [46]. When ground into a fine powder and combined with portland cement and water, glass acts as a pozzolan. Glass is abundant in silica, which will react with CH to produce C-S-H. In early 2020, ASTM Subcommittee C09.24 published the specification “*Standard Specification for Ground-Glass Pozzolan for Use in Concrete*” [46]. Only three other specific specifications for SCMs exist for (1) slag; (2) silica fume; and (3) coal fly ash and natural pozzolans.

Many types of glass are produced for a variety of applications. The three most commonly produced types of glass are soda-lime container glass, soda-lime plate glass, and electrical glass (E-glass). Container glass is used to produce packaging, such as bottles or jars, and plate glass is used to produce windshields

and windows. E-glass is used to produce reinforcement in fiber-reinforced polymers which is commonly used in electrical insulation [46].

Research investigating if ground glass pozzolan can replace portland cement at similar levels to fly ash has been conducted. It was found that the addition of ground glass in concrete can increase the workability and decrease the water demand [36], [47]–[54]. Bueno et. al [54] reported that all mixtures that had portland cement replacement levels between 10% and 30% had a comparable or increased flow compared to the control. It was suspected that the workability increased due to the smooth surface of the ground glass particles, much like fly ash particles.

Since glass is composed primarily of amorphous silica, it can exhibit pozzolanic properties when ground fine enough. This has caused researchers to find low early compressive strengths in concrete containing ground glass and high late compressive strengths [54]–[59]. Krstic et al. [59] studied the effects of using post-consumer glass, such as recycled container glass, as a replacement for portland cement in concrete. At replacement levels of 20, 30, and 40%, the seven-day compressive strengths of the concrete containing ground glass were 4590, 4170, and 3860 psi, respectively. At 56 days, the compressive strengths were 6980, 7260, and 7610 psi. At 56 days, the compressive strength of concrete with 40% ground glass replacing portland cement was the highest, and it was concluded this was because of the increased pozzolanic reactions. No tests before seven days were performed. This study also used ground glass as a portland cement replacement in concrete that was placed as a sidewalk in Queens, New

York in 2016. The sidewalk compressive strength results met design specifications but were 13% lower than those obtained in the laboratory.

Bueno et. al [54] found that the addition of soda-lime glass at 10-15% replacement levels had comparable early and late compressive strength results to the portland cement concrete control, but any addition beyond 20% replacement had a decrease in compressive strength compared to the control.

2.2.2 Limestone

Limestone is a sedimentary rock that consists mostly of calcium carbonate (CaCO_3) and is readily available in quarries across the United States. These quarries can be several square miles and reach several hundred feet deep [60]. Although there are not limestone quarries in every region of the world, or every state in the U.S., other calcium carbonate sources are fairly common.

Researchers have been seeking ways to reduce the production cost of concrete using mineral additives such as pozzolans, nominally inert materials, and other cementitious materials to replace portland cement in concrete. Since 1820, researchers have studied the use of ground limestone as a partial replacement for portland cement in concrete [61]. Researchers have also studied the effects of ground limestone preblended with portland cement in concrete, which is different than adding limestone to the mixture during batching. It is important to note that some of these researchers report conflicting data on how ground limestone affects compressive strength.

Alone, ground limestone is not cementitious or pozzolanic, but it can be used as a filler in concrete. For limestone to act as a filler, particles must be

smaller than the portland cement particles. This would allow the limestone to tightly pack together between the portland cement particles in a way called particle packing [60]. Generally the size of portland cement particles are between one and 45 micrometers [62]. Since limestone is softer than clinker, it consequently takes less energy to grind limestone than clinker [61].

Good workability and adequate early strength are crucial when incorporating an inert material in concrete. Researchers, such as Suaiam et al. [63], Derabla et al. [64], and Kanazawa et al. [65], reported that including ground limestone improved the workability of concrete while Bier et al. [17] reported a decrease in workability in concrete when incorporating ground limestone. The workability of concrete can be affected by particle packing; when limestone fills the gaps in between portland cement particles, it decreases the water demand, which can reduce the workability of concrete. Impurities in the limestone could also cause a decrease in workability. Research has shown that when clay impurities are present in ground limestone, it will increase the water demand [66]. If an insufficient amount of water is added to the concrete mixtures, the impurities present will absorb the water, decreasing the water-to-cementitious material ratio and workability.

Many researchers have debated whether limestone is reactive when present in portland cement concrete [67]–[73]. Small additions of limestone can increase early strength due to particle packing [67]. This can be a cost-effective solution for producing concrete because ground limestone costs less than portland cement. Some researchers also argue that when limestone particles are ground to

less than one millimeter in diameter, they can act as a nucleation site for C-S-H growth. This could increase the rate of hydration, which in turn could increase the early compressive strength of concrete [61], [68], [69], [70].

Some researchers reported that the compressive strength of concrete is directly related to the level at which portland cement is replaced. Ghafoori et al. [74] found that limestone powder effectively reduced the required amount of portland cement in concrete as well as increased the flow and workability of the concrete. When replacement levels are increased from 5% to 30%, the 28-day compressive strengths of concrete containing limestone were reported to have increased from 8000 psi to 8700 psi, which is an increase of 9%. Ghafoori et al. [74] attributed the small increase in compressive strength to the particle packing of the limestone powder.

Meddah et al. [75] observed that at replacement levels of 15%, the addition of limestone had a reduction in compressive strength by 12%. Replacement levels above 15% resulted in a greater percentage of strength loss, significantly between 25-45% replacement. It was suspected that the loss of strength was because of the decrease in cementitious material in the concrete mixtures. Similarly, Schmidt et al. [76] and Sprung and Siebel [77] concluded that when ground limestone is added at levels no more than 10% and 15%, respectively, the compressive strength of the concrete increased.

3 Research Plan, Materials, and Methods

As fly ash increases in price and decreases in supply, use of alternative SCMs in structural concrete products need to be investigated. In this research, a series of experimental mortar mixtures were developed using three different types of portland cement and five different alternative SCMs. Mortar mixture development included mixing mortars to determine if the mixtures met the minimum strength and flow requirements of the precast concrete plant. Select mortar mixtures were used during concrete mixture development to ensure the concrete would be appropriate for fabricating precast, prestressed concrete beams at Molin Concrete Products Co. The structural properties of these beams were analyzed and compared to portland cement beams made with fly ash. Although a comparison to what is used in the industry now is important, it was crucial to compare the fresh and mechanical properties of the concrete to minimum fabrication requirements set by Molin.

3.1 Materials

3.1.1 Portland Cement

There were three different types of portland cement used during mortar mixture development: Type III, Type IL(10), and Type IIIL(10). Type III portland cement is ground finer than Type I, which produces concrete that has a higher early strength due to the increased reactivity of the cement. Molin typically uses Type III portland cement, produced by the Lehigh Cement Company. Type IL(10) is also produced by Lehigh Cement Company and is commonly referred to as EcoCem. Type IL(10) is an interground cement that includes 90% Type I portland

cement and 10% ground limestone [78]. Type IL(10) is a product that will likely be available on the market in the Midwest by 2023. It is predicted that an interground cement with Type III portland cement and ground limestone will soon follow. This prediction led to creating a Type IIIIL(10) in the lab at the University of Minnesota Duluth. Type IIIIL(10) was preblended immediately before being used in mortar mixtures rather than interground; it included 90% Type III portland cement and 10% ground limestone. The ground limestone in this study was produced by ILC Resources located in Savage, MN. Chemical compositions of Type III portland cement, EcoCem, and limestone are provided in Table 3.1. Results in Table 3.1 also report how much material is retained on the No. 325 sieve.

3.1.2 Fly Ash

Molin currently uses Class C fly ash produced by Boswell Energy Center. The chemical composition, as well as the amount retained on the No. 325 sieve, of the fly ash used is shown in Table 3.1.

3.1.3 Ground Limestone

The ground limestone used in this study was a mined from high calcium limestone that was processed at Lehigh Cement Company in Savage, MN. The chemical composition of the ground limestone, as well as the amount retained on the No. 325, is show in Table 3.1. The average particle size was 3.66 microns. This same ground limestone was also used to create the Type IIIIL(10) cement referred to in Section 3.2.

3.1.4 Natural Pozzolan

The natural pozzolan used in this research was removed from a quarry in Wassau, WI and produced by 3M. This SCM was advertised as appropriate for use in ternary concrete mixtures with fly ash. 3M also worked to produce a natural pozzolan that is consistent in size and similar in color to portland cement. The chemical composition of the natural pozzolan, as well as the amount retained on the No. 325, is show in Table 3.1.

3.1.5 Ground Glass Pozzolan

Two different ground glasses were used in this study: a commercially available ground glass pozzolan and a recycled soda-lime plate glass. The commercially available ground glass pozzolan is a ground soda-lime container glass pozzolan. These glass pozzolans are made from recycled post-consumer glass and are generally free from contaminants. The chemical compositions of both glass pozzolans, as well as the amount retained on the No. 325, are shown in Table 3.1.

Table 3.1 Chemical compositions of alternative supplementary cementitious materials used in this study. All values are reported as a percentage. Fineness of materials is also included.

Chemical Analysis	Boswell Fly Ash	3M Natural Pozzolan	Ground Plate Glass	Ground Glass Pozzolan	Type III	Type IL(10)	Ground Limestone
Silicon Oxide, SiO ₂	38.82	64.49	72.56	72.42	19.5	18.5	-
Aluminum Oxide, Al ₂ O ₃	19.39	17.67	0	1.83	4.8	5.1	-
Iron Oxide, Fe ₂ O ₃	5.58	5.5	0.4	0.3	2.2	2.2	-
Calcium Oxide, CaO	21.18	1.75	8.72	10.76	63.4	62.6	47
Magnesium Oxide, MgO	6.68	1.83	3.34	-	3.2	2.9	-
Sodium Oxide, Na ₂ O	2.33	3.62	12.55	13.15	0.05	0.06	-
Potassium Oxide, K ₂ O	0.8	3.73	0.09	-	0.45	0.6	-
Sulfur Trioxide, SO ₃	1.28	0.09	0.2	0.11	3.8	2.9	-
Loss on Ignition (LOI)	0.26	3.32	0.06	0.48	2.5	4.8	53
Amount Retained on No. 325	9.2	8	-	1.4	0.7	2.9	-

3.1.6 Aggregates

Both coarse and fine aggregates were sourced directly from Molin. The coarse aggregate used was 0.5625 in. crushed limestone and had an absorption of 1.9% and dry specific gravity of 2.66. There were two different fine aggregates used: 0.375 in. pea gravel and coarse sand. Each fine aggregate had an absorption of 1.5% and 0.6 and a dry specific gravity of 2.66 and 2.67, respectively.

3.1.7 Steel

The prestressed beams referred to in Section 3.4 contain both reinforcing steel and prestressed steel. The prestressed steel was placed on the bottom of the beam; it had a diameter of 0.5 in. and was Gr. 270. No. 5 rebar was placed on the top of the beam, No. 4 rebar was placed in the center of the beam, and both were Gr. 60. All prestressed beams contained Gr. 60, No. 4 stirrups.

3.2 Mortar Mixture Development

Preliminary mixture designs were investigated to determine which mixtures would meet minimum fabrication and design standards set by Molin. These minimum design standards include 3500 psi strength at strand release, according to ACI 318-19, and a flow near 150%. The average time that the strands are released at Molin is between 16 and 18 hours.

A total of 24 unique mortar mixtures were made and their compositions are summarized in Table 3.2. The control mixture composition was based on the concrete mixture that Molin currently uses, which contained Type III portland cement with 30% fly ash replacement by mass. The first batch of mortars emulated this mixture exactly, except the 30% fly ash by mass was replaced with

30% ground limestone, natural pozzolan, ground soda lime plate glass, or ground glass pozzolan by mass. The next set of mixtures was based on Type IL(10) portland cement and considered SCM replacement levels of both 20% and 30% by mass. The third set of mixtures contained Type III(10) portland cement and considered each SCM at a replacement level of 30% by mass.

Table 3.2 Mortar mixture compositions reported as percent by mass. Mortar mixtures were made with a water to cementitious materials (w/cm) ratio of 0.34 and included 0.25 ounces of superplasticizer. For each mixture, the portland cement and SCM content is recorded as a percentage of total cementitious material. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Mixtures	Type III	Ecocem Type IL(10)	Type IIIIL(10)	Fly ash	Limestone	Natural Pozzolan	Glass (Potters Plate)	Glass (Commercial)
TypeIL(10)	-	100	-	-	-	-	-	-
70TypeIL(10)30FA	-	70	-	30	-	-	-	-
70TypeIL(10)30L	-	70	-	-	30	-	-	-
70TypeIL(10)30NP	-	70	-	-	-	30	-	-
70TypeIL(10)30G	-	70	-	-	-	-	30	-
70TypeIL(10)30GP	-	70	-	-	-	-	-	30
80TypeIL(10)20FA	-	80	-	20	-	-	-	-
80TypeIL(10)20L	-	80	-	-	20	-	-	-
80TypeIL(10)20NP	-	80	-	-	-	20	-	-
80TypeIL(10)20G	-	80	-	-	-	-	20	-
80TypeIL(10)20GP	-	80	-	-	-	-	-	20

Table 3.2 (continued). Mortar mixture compositions reported as percent by mass. Mortar mixtures were made with a water to cementitious materials (w/cm) ratio of 0.34 and included 0.25 ounces of superplasticizer. For each mixture, the portland cement and SCM content is recorded as a percentage of total cementitious material. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

TypeIIIL(10)	-	-	100	-	-	-	-	-
70TypeIIIL(10)30FA	-	-	70	30	-	-	-	-
70TypeIIIL(10)30L	-	-	70	-	30	-	-	-
70TypeIIIL(10)30NP	-	-	70	-	-	30	-	-
70TypeIIIL(10)30G	-	-	70	-	-	-	30	-
70TypeIIIL(10)30GP	-	-	70	-	-	-	-	30
TypeIII	100	-	-	-	-	-	-	-
70TypeIII30FA (control)	70	-	-	30	-	-	-	-
70TypeIII30L	70	-	-	-	30	-	-	-
70TypeIII30NP	70	-	-	-	-	30	-	-
70TypeIII30G	70	-	-	-	-	-	30	-
70TypeIII30GP	70	-	-	-	-	-	-	30

All mortar mixtures were mixed in accordance with ASTM C305 – *Standard Practice for Mechanical Mixing of Hydraulic Cement Pastes and Mortars of Plastic Consistency* [79]. The mortars were mixed using a Hobart Model N50 tabletop mixer that had a capacity of 5 qt. Each mixture had a water-to-cementitious materials (w/cm) ratio of 0.34 and contained 0.25 oz of superplasticizer. The mixing process was 4 minutes long. Initially, water and superplasticizer were added to the mixing bowl, followed by the portland cement; these were mixed for 30 seconds at a slow speed. After 30 seconds, the fine aggregate was slowly added to the portland cement paste and the speed of the mixer was increased to a medium speed and allowed to mix for another 30 seconds. The mixture was then allowed to sit for 90 seconds before being mixed for a final time at a medium speed for another 60 seconds.

For each mixture, six 2 in. x 2 in. x 2 in. mortar cubes were cast and cured at room temperature for 16 hours, after which their compressive strength was measured. This was done to emulate the allowable curing time at the precast plant. The temperature of each mixture was recorded, along with the flow, which was determined in accordance with ASTM C1437 – *Standard Test Method for Flow of Hydraulic Cement Mortar* [80]. The compressive strength of the mortar cubes was measured in accordance with ASTM C109 – *Standard Test Method for Compressive Strength of Hydraulic Cement Mortars* [81]. After the initial 16 hours of curing, the mortar cubes were placed in a curing chamber set at 73°F and

95% humidity and the compressive strength of each mixture was tested at 16 hours, 7 days, and 28 days.

3.3 Concrete Mixture Development

A weighted decision chart, shown in Table 4.6, was used to determine which mixtures should be scaled up to concrete. Properties that were considered were cost, early strength, 28-day strength, workability, and availability. Each property was weighted in terms of importance on a scale of 1 to 5. The most highly weighted properties were cost and early strength, followed by workability, availability, and 28-day strength. Based on the weighted decision chart and discussion among the research team, three mortar mixtures were scaled to concrete mixtures in the lab to ensure adequate performance was achieved before casting precast, prestressed concrete beams at Molin.

The first mixture was the control mixture that is used at Molin. This mixture contained Type III portland cement with a 30% replacement of fly ash by mass. Two more concrete mixtures were made in the lab, which included: (1) Type III portland cement and 30% ground limestone and (2) Type IL(10) and a total of 30% ground limestone. Based on the weighted decision chart, the mixture containing ground limestone had the highest score. Factors that contributed to the high score of the mortar mixtures containing ground limestone were that the mixtures had a flow of 150%, had comparable early strengths to the control mixtures that contained Type III with 30% fly ash, and were the least expensive material.

All mixing procedures were followed in accordance with ASTM C192 – *Standard Practice for Making and Curing Concrete Test Specimens in the Laboratory* [82]. Concrete was mixed in a Husky Model GHM105890 mobile concrete mixer. Each mixture had a water-to-cementitious materials ratio (w/cm) of 0.34 and contained superplasticizer. The amount of superplasticizer varied in the concrete mixtures because it was evident the experimental mixture containing ground limestone needed more superplasticizer than the control mixture; using the same amount in both mixtures, the mixture containing Type III portland cement and 30% ground limestone had a significantly lower slump. The mixing procedure began by placing all coarse aggregate and half the water in the mixer; this was mixed until the aggregate was saturated surface dry. The portland cement, SCM, fine aggregate, and remaining water were then added to the mixer and mixed for a total of 3 minutes. The concrete mixture rested for 3 minutes and then was mixed for a final 2 minutes. As suggested by Molin, the superplasticizer was added to the concrete during the last 2 minutes of mixing.

Slump, unit weight, air content, and temperature were recorded as outlined in ASTM C143 – *Standard Test Method for Slump of Hydraulic-Cement Concrete*, ASTM C138 – *Standard Test Method for Density (Unit Weight), Yield, and Air Content (Gravimetric) of Concrete*, and ASTM C231 – *Standard Test Method for Air Content of Freshly Mixed Concrete by the Pressure Method*, respectively [83]–[85].

Cylindrical specimens that were 3 x 6 in. and 4 x 8 in. were cast and cured in an oven at 110°F for 16 hours to mimic internal curing temperatures of beams

produced at Molin. When the beams are cast, the internal temperature of the concrete increased from 64°F to 118 °F over 10 hours and then remained within 10°F of the maximum temperature before the beam was removed from the mold after 18 hours of curing. This increase in internal temperature is due to the exothermic hydration reaction of the cement and water. After 16 hours in the oven, the cylindrical specimens were placed in a curing chamber at 73°F and 95% humidity until it was time to measure the compressive strength and modulus of elasticity.

The compressive strength was measured at 16 hours, 7 days, 28 days, 90 days, and 180 days and tests were performed in accordance with ASTM C39 – *Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens* [86]. Tests for modulus of elasticity followed the procedure in ASTM C469 – *Standard Test Method for Static Modulus of Elasticity and Poisson's Ratio of Concrete in Compression* [87].

3.4 Precast, Prestressed Beam Fabrication

3.4.1 Precast, Prestressed Beam Material Properties

Based on the fresh and hardened properties of the concrete, one mixture was selected for use in precast, prestressed beams. The selected mixture was 70% Type III portland cement and 30% ground limestone by mass. The goal was to replace Type III portland cement by 30%, but only 27.1% ground limestone was placed in the mixture containing Type III portland cement and ground limestone. Before the concrete was mixed, a decision was made to increase the total amount of concrete made. An increased amount of cement was added to the mixer, but the

ground limestone had already been placed in the mixer by hand. In order for the two mixtures to remain comparable, a similar replacement rate of 27.5% was used in the control mixture containing Type III portland cement and fly ash. Beams were fabricated at Molin; two beams were made using Type III portland cement and 27.5% fly ash and four beams were made using Type III portland cement and 27.1% ground limestone. The mixture designs used are shown in Table 3.3.

Table 3.3 Mixture designs per batch. Batch size of control mixture was 2 yd and batch size of Type III with 30% ground limestone was 4 yd.

Material	Control Mixture	Type III & Ground Limestone
Type III Portland Cement (lb)	1046	2204
Fly Ash (lb)	398	0
Ground Limestone (lb)	0	821
9/16 in Limestone (lb)	2613	5309
Pea Gravel (lb)	764	1553
Coarse Sand (lb)	2315	4639
Total Water (lb)	534	1066
BASF 7700 (oz)	92	326

Both concrete mixtures were designed in the control room at Molin. The mixing sequence for the mixture containing Type III portland cement with 27.5% fly ash began by blending the 0.5625 in limestone, pea gravel, coarse sand, and water in the hopper. Then, the cement and fly ash were added and mixed. Lastly, the superplasticizer was added after all materials had thoroughly mixed and the mixture was given adequate time to chemically react. The mixture containing Type III portland cement with 27.1% ground limestone by mass followed the same mixing sequence as the control mixture except that the limestone was added by hand to the hopper after the aggregates and water were mixed. Once the portland cement and ground limestone were thoroughly mixed, the

superplasticizer was added in 50 oz increments until a desired consistency was observed.

3.4.2 Precast, Prestressed Beam Design Specifications

The beams were sized and designed to fail in flexure, but they were not to exceed a load capacity of 80 kips in order to stay within the 100-kip capacity of the load cell. All beams were designed to be 10 in. x 16 in. x 18.5 ft. For every beam, three Gr. 270, 0.5 in diameter prestressed steel strands were placed on the bottom 2 in. apart and were stressed to 70% of their ultimate strength. No more than three strands were used at the bottom of the beam to stay within the transfer stress limits at the top of the beam. Two Gr. 60, No. 5 rebar were placed 4 in. apart at the top of the cross section. All beams had stirrups placed 12 in. apart along their length since 12 in. was the maximum spacing allowed for the shear design of the beam. Typical beam dimensions are shown in Figure 3.1 and Figure 3.2. The predicted moment capacity of the beam containing fly as was 133.5 k-ft and the predicted moment capacity of the beam containing limestone was 133.3 k-ft; calculations of predicted moment capacity are shown in the Appendix.

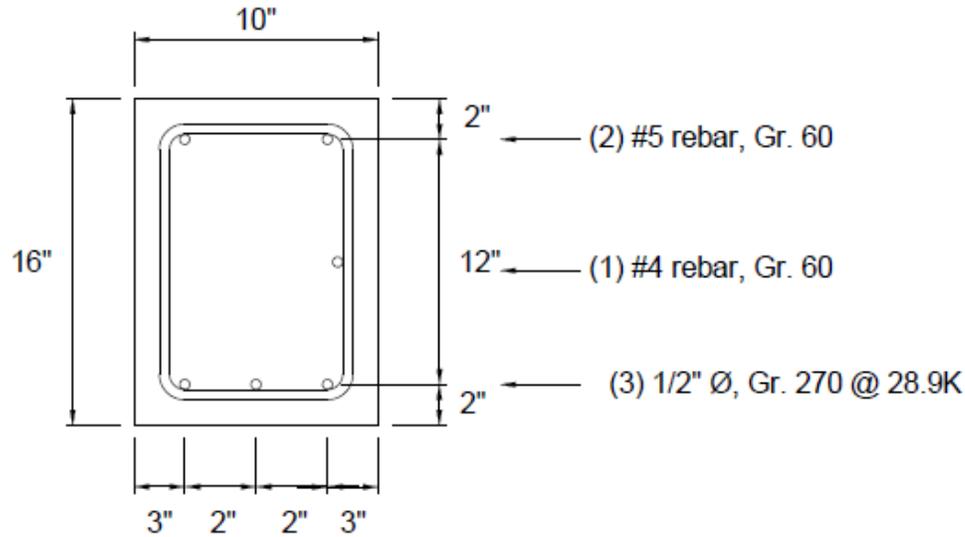


Figure 3.1 Typical cross section of prestressed, precast concrete beams.

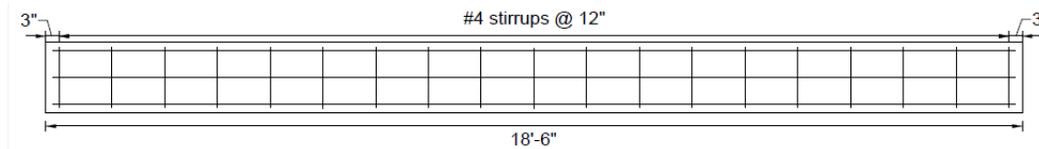


Figure 3.2 Typical elevation of prestressed, precast concrete beams

Various specimens were cast for characterizing the mechanical properties of concrete: 4 x 8 in. cylinders and 6 x 6 x 21 in. beams. All specimens were cast at Molin and then tested at the University of Minnesota Duluth. The companion specimens were tested to determine compressive strength, modulus of elasticity, and modulus of rupture. These tests are outlined in ASTM C39 – *Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens*, ASTM C469 – *Standard Test Method for Static Modulus of Elasticity of Poisson’s Ratio of Concrete in Compression*, and ASTM C78 – *Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)*, respectively [86]–[89]. The compressive strengths of both concrete mixtures were

tested at 16 hours, 7 days, 28 days, 90 days, and 180 days. Only one concrete cylinder that contained Type III with 27.1% ground limestone was placed in a SURE CURE cylinder mold. The SURE CURE cylinder mold mimics the same time-temperature curve of the concrete with the use of curing controllers and built-in heating capabilities.

3.4.3 Precast, Prestressed Beam Testing

Using a 110-kip Shore Western actuator and a 100-kip load cell, the precast beams were tested to determine flexural strength and bond strength between the longitudinal prestressing steel and concrete. The beams were tested between 15 and 24 days after they were cast. The beams were simply supported and were placed on a pin and roller that spanned 18 ft. Deflections were measured using a UniMeasure P510 Series string pot.

For flexural strength testing, a spreader beam was used to apply two point loads, 46 in. apart as shown in Figure 3.3. The loads were applied downward at a constant rate of 0.001 in./sec. The cracks were outlined in black marker when the load neared the calculated cracking moment of 81.9 k-ft. Calculations to determine the predicted cracking moment can be found in the Appendix. After the first cracks appeared, cracks were outlined every 5 kips. Failure of the beams was recorded once the concrete visually failed at the top of the cross section.

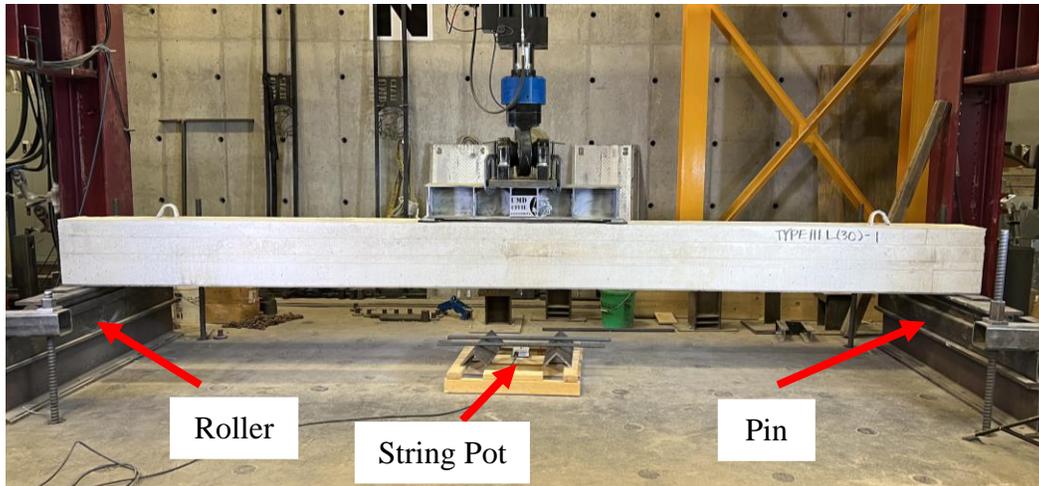


Figure 3.3 Flexure beam testing setup

For bond strength testing, a single point load was placed at 5.7 ft from the end of the beam, as shown in Figure 3.4, and was applied at a constant rate of 0.001 in/sec. The load was placed inside the calculated development length of 6.4 ft. Development length calculations are shown in the Appendix. Ideally, the load would have been placed at the end of the development length, but due to limitations of the mobility of the actuator the load was placed inside the development length. If the beams failed at predicted or higher loads, it indicated that the predicted development length was accurate or shorter than predicted. If the beams exhibited an unusual failure, it indicated that the development length was longer than predicted. The cracks were outlined when the load near the calculated cracking moment of 81.9 k-ft. Calculations to determine cracking moment are outlined in the Appendix. The cracks were then outlined every 5 kips after the initial cracks.

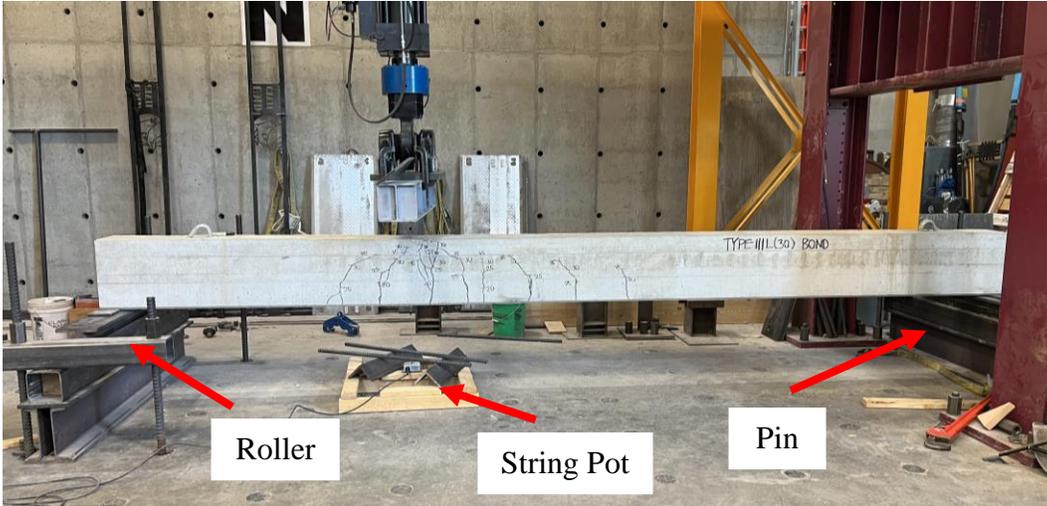


Figure 3.4 Bond strength testing setup

4 Results and Discussion

4.1 Mortar Mixture Development

The goal of mortar mixture development was to determine the workability and compressive strengths of mixtures made with each alternative SCM and to investigate how those properties compared to the requirements of a flow of 150% and a minimum compressive strength of 3500 psi at 16 hours. Fresh and hardened properties, such as flow, temperature, and 16-hour compressive strength, of each mixture were recorded and compared to the fresh and hardened properties of the control mixture that contained Type III portland cement and 30% fly ash by mass. Compressive strength at 7 and 28 days were reported but no measurements were of concern and were not discussed in detail in the following sections

4.1.1 Type III Portland Cement with 30% Alternative SCM Replacement

4.1.1.1 16-Hour Compressive Strength

The control mixture, containing Type III portland cement and 30% fly ash by mass, had a compressive strength of 3450 psi at 16 hours, which is less than the 3500 psi requirement. Data from Molin shows that the 16-hour compressive strength of the control mixture at strand release averages 4860 psi. The high heat of hydration causes the internal temperatures of the beams to get as high as 120°F, which ultimately leads to higher compressive strengths in the precast plant compared to the lab. All mortar mixtures that were mixed in the lab cured at room temperature. In hindsight, the lower compressive strength of the mortar mixtures recorded in lab were likely due to the lower temperature during curing.

Despite a lower heat of hydration, all remaining mixtures that contained Type III portland cement with 30% alternative SCM replacement by mass had 16-hour compressive strengths greater than 3500 psi but not more than 3800 psi. Compressive strength data for mortar mixtures containing Type III portland cement and 30% replacements are shown in Table 4.1 and Figure 4.1. Figure 4.1 illustrates which mortar mixes had a compressive strength of 3500 psi, the minimum 16-hour compressive strength required for prestressed concrete. The error bars shown in Figure 4.1 show the standard deviation between the 2 mortar samples broke on each date.

Table 4.1 Fresh and hardened properties of mortar mixtures containing Type III portland cement. Compressive strength data is an average of two samples. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Mixture	Temperature (°F)	Flow (%)	Compressive Strength (psi)		
			16 hr	7 day	28 day
TypeIII	78.2	147	5930	10130	11430
70TypeIII30FA	75.7	150	3450	8670	9490
70TypeIII30L	75.7	150	3770	7430	8480
70TypeIII30NP	74.3	150	3660	7170	9290
70TypeIII30G	74.6	141	3800	7890	9470
70TypeIII30GP	74.3	147	3670	7700	9330

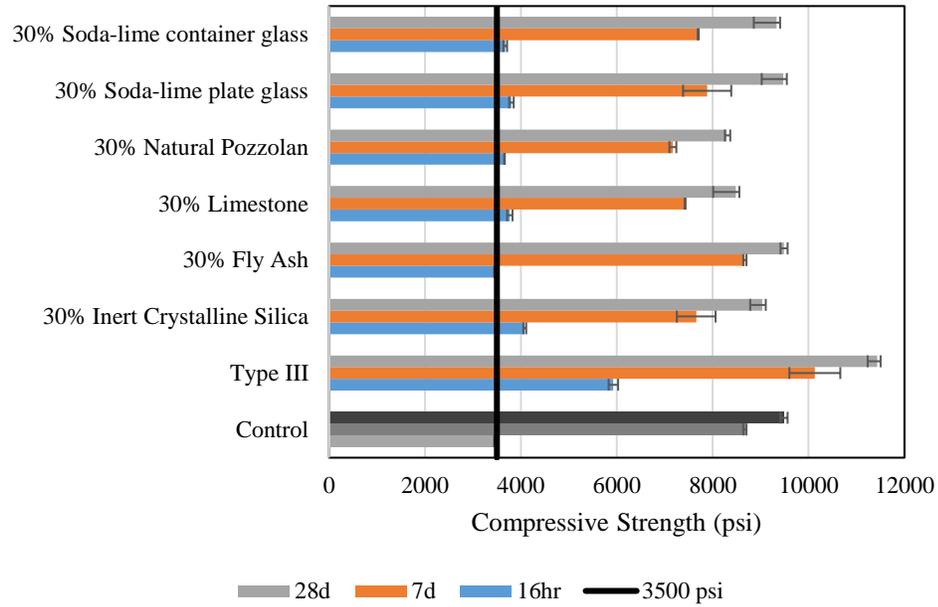


Figure 4.1 Compressive strength data of mortar mixtures containing Type III portland cement and 30% replacements. Compressive strength data is an average of two samples and the error bars illustrate the standard deviation. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

4.1.1.2 Flow

The control mixture, containing Type III portland cement and 30% fly ash by mass, had a flow of 150%. The results of the flow test were limited by the flow table, which measures 10 in in diameter. If the mortar began to spread beyond the 10 in table before the standard 25 taps, it was unfeasible to get an accurate measurement of how much further the mortar would have spread. Therefore, the highest flow any mortar mixture could have would have been 150%.

Mortar mixtures that contained Type III portland cement and 30% replacement with fly ash, ground limestone, and natural pozzolan had a flow of 150%. The mortar mixture that was composed of only Type III portland cement had a flow of 147%, which indicated that the fly ash, limestone, and natural pozzolan increased the workability. The exact comparison between the mortar mixtures containing fly

ash, limestone, and natural pozzolan was unattainable because of the limitation of the size of the flow table. The mortar mixture containing Type III portland cement and 30% ground glass pozzolan also had a flow of 147%, which demonstrated that the commercially available glass pozzolan did not hinder the workability of a mortar mixture containing Type III portland cement, but the mixture containing the commercially available glass pozzolan was not as workable as the control mixture. The mortar mixture that contained Type III portland cement and 30% ground plate glass had a flow of 141%, which is less than the control mixture and the mortar mixture containing only Type III portland cement, but this flow was still acceptable. Flow measurements of mortar mixtures containing Type III with 30% alternative SCM replacements by mass are recorded in Table 4.1.

4.1.2 Type IL(10) Cement with 30% Alternative SCM Replacements

4.1.2.1 16-Hour Compressive Strength

Early compressive strength results of mortars that used Type IL(10) cement with 30% replacement levels were all lower than 3500 psi and are presented in Table 4.2 and Figure 4.2; 7-day and 28-day compressive strength are also shown in Table 4.2 and Figure 4.2. The 16-hour compressive strengths of the mortar mixtures that contained Type IL(10) cement with 30% replacement levels were less than the control, but some were comparable. Mortar mixtures that contained Type IL(10) cement with 30% replacement of limestone and natural pozzolan had compressive strengths of 3100 and 3090 psi, respectively. The mortar mixtures that contained Type IL(10) cement and 30% fly ash, ground plate glass, and ground glass pozzolan had a 16-hour compressive strength of 2220,

2920, and 2900 psi, respectively, which were lower than the 16-hour compressive strength of the control mixture and the 3500 psi requirement. It is possible the lower 16-hour compressive strengths shown by the pozzolans are because of a combination between the slower pozzolanic reactions they exhibit as well as a higher total percent of portland cement replaced. The control mixture contained 30% replacement by mass, while mixtures containing Type IL(10) cement and 30% replacements had a total percent replacement higher than 30%. Compressive strength is directly correlated to the amount of portland cement in each mixture; a mixture that contains more portland cement by mass will have a higher compressive strength. Since Type IL(10) cement already had a 10% replacement by mass with ground limestone, the next set of mortar mixtures had 20% replacement levels with the expectation that the compressive strengths would increase. Figure 4.2 illustrates which mortar mixes reached a minimum compressive strength of 3500 psi. The error bars shown in Figure 4.2 show the standard deviation between the 2 mortar samples broke on each date.

Table 4.2 Fresh and hardened properties of mortar mixtures containing Type IL(10) cement at 30% replacement levels. Compressive strength data is an average of two samples. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Mixture	Temperature (°F)	Flow (%)	Compressive Strength (psi)		
			16 hr	7 day	28 day
TypeIL(10)	80.7	150	5310	9190	10340
70TypeIL(10)30FA	77.5	150	2220	7200	9280
70TypeIL(10)30L	75.3	145	3100	3880	8230
70TypeIL(10)30NP	74.3	150	2920	6780	7880
70TypeIL(10)30G	75.3	150	2900	7460	9260
70TypeIL(10)30GP	73.4	150	3090	7150	8620

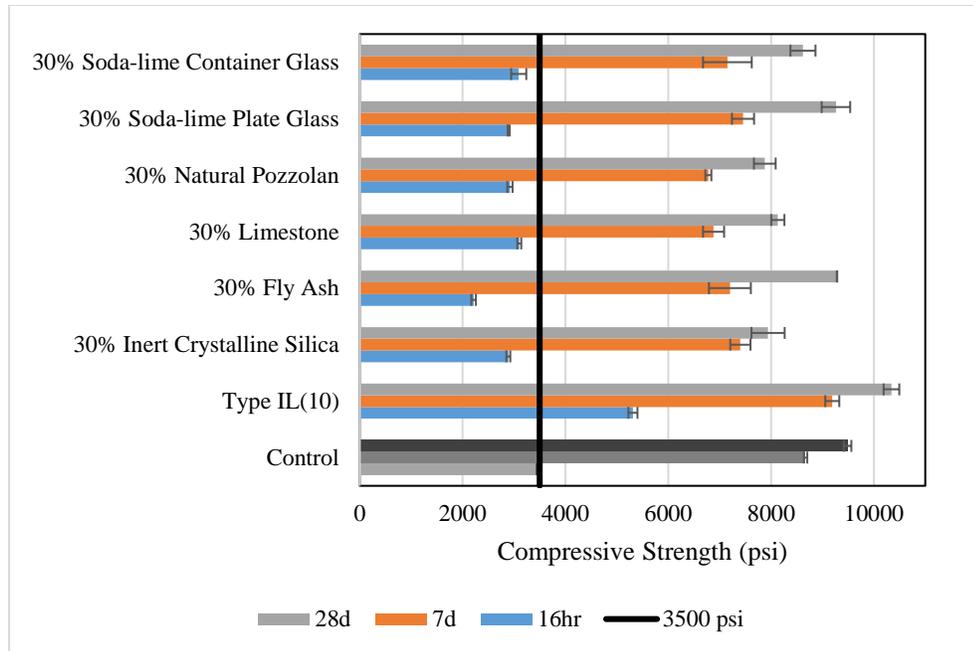


Figure 4.2 Compressive strength data of mortar mixtures containing Type IL(10) portland cement and 30% replacements. Compressive strength data is an average of two samples and the error bars illustrate the standard deviation. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

4.1.2.2 Flow

All mortar mixtures containing Type IL(10) cement with 30% replacement levels had a 150% flow, with the exception of the mortar mixture containing Type IL(10) cement with 30% ground limestone, which had a flow of 145%. The mortar mixture that contained Type IL(10) cement with 30% ground limestone by mass had a flow less than 150% because the total Type IL(10) cement replaced with limestone was 37% rather than 30%. Unlike fly ash, ground limestone particles are irregularly shaped which can lead to a decrease in flow, whereas the unique spherical shape of fly ash increases the flow of the mortar. The ground limestone in this study was ground finely, which can also lead to a decrease in flow since there is an increase in surface area of the particles. This causes the

mortar mixture to become sticky. All flow measurements can be found in Table 4.2.

4.1.3 Type IL(10) Cement with 20% Alternative SCM Replacement

4.1.3.1 16-Hour Compressive Strength

Since Type IL(10) cement already had a replacement of 10% limestone by mass, another set of mortar mixtures were created that contained Type IL(10) cement with 20% replacement levels instead of 30% with the anticipation that high compressive strengths would be observed. The compressive strength of all the mortar mixtures increased when the replacement level was decreased. Mortar mixtures containing Type IL(10) cement and 20% fly ash, ground limestone, and natural pozzolan increased the most. Particularly, the mortar mixture containing Type IL(10) cement and 20% ground limestone had a 16-hour compressive strength of 3530 psi, which is greater than the compressive strength of the control mixture and also exceeded the 3500-psi requirement. The mortar mixture containing Type IL(10) cement with 20% natural pozzolan had a 16-hour compressive strength of 3320 psi, which is comparable to the control mixture. This result was still less than the 16-hour compressive strength of the control mixture, but if the mortar cubes were heat cured, it was predicted that the 16-hour compressive strength would have likely increased. Although the 16-hour compressive strength of the mortar cubes that contained Type IL(10) cement with 20% fly ash increased from the mortar cubes that contained 30% replacement by mass, most 16-hour compressive strengths were still below the compressive strength of the control mixture and the 3500-psi requirement. Table 4.4 highlights

the increase in 16-hour compressive strengths that contained Type IL(10) cement with fly ash, ground limestone, and natural pozzolan when incorporated at 20% replacement rates and 30% replacement rates. Mortar mixtures that contained Type IL(10) cement with 20% replacement using ground plate glass and ground glass pozzolan had 16-hour compressive strengths of 2830 psi and 2950 psi, respectively. All compressive strengths for mortar mixtures containing Type IL(10) with 20% replacement levels are shown in Table 4.3 and Figure 4.3. Figure 4.3 illustrates which mortar mixes reached a minimum compressive strength of 3500 psi, which was only one mixture: the mixture containing Type IL(10) and 20% ground limestone. The error bars shown in Figure 4.3 show the standard deviation between the 2 mortar samples broke on each date.

Table 4.3 Fresh and hardened properties of mortar mixtures containing Type IL(10) cement at 20% replacement levels. Compressive strength data is an average of two samples. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Mixtures	Temperature (°F)	Flow (%)	Compressive Strength (psi)		
			16 hr	7 day	28 day
TypeIL(10)	80.7	150	5310	9190	10340
80TypeIL(10)20FA	73.9	150	2990	8520	9230
80TypeIL(10)20L	75.5	150	3530	7510	8850
80TypeIL(10)20NP	74.4	150	3320	7690	8770
80TypeIL(10)20G	75.3	150	2830	7480	8440
80TypeIL(10)20GP	73.9	150	2950	7200	8700

Table 4.4 Comparison between 16-hour compressive strengths of Type IL(10) cement with 20% replacement rates and 30% replacement rates. Compressive strength data is an average of two samples. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

SCM	TypeIL(10) with 30% replacement rate	TypeIL(10) with 20% replacement rate
	16-hour compressive strength (psi)	
Fly ash	2220	2990
Limestone	3100	3530
Natural Pozzolan	2920	3320
Plate glass	2900	2830
Glass pozzolan	3090	2950

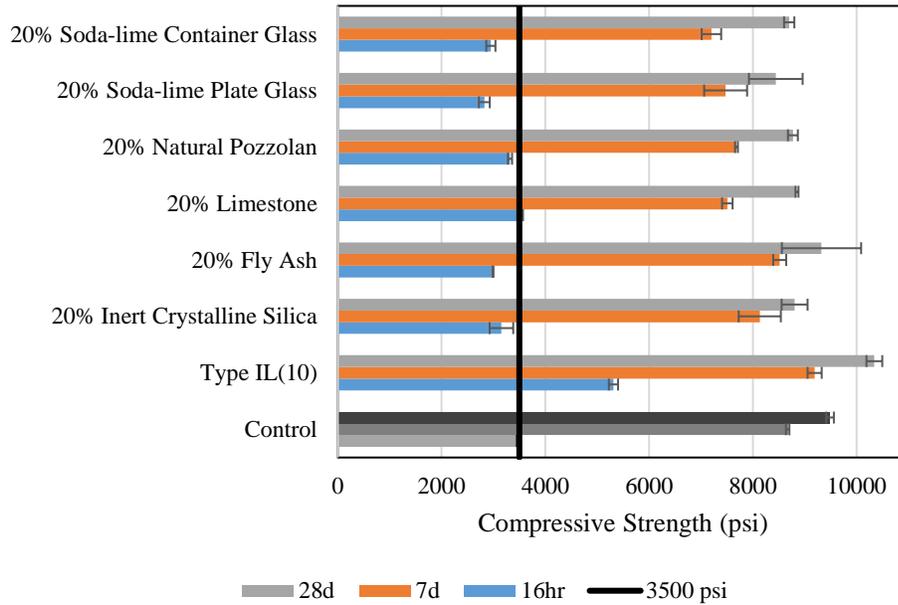


Figure 4.3 Compressive strength data of mortar mixtures containing Type IL(10) portland cement and 20% replacements. Compressive strength data is an average of two samples and the error bars illustrate the standard deviation. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

4.1.3.2 Flow

All mortar mixtures that contained Type IL(10) cement with 20% replacement levels had flow of 150% and were comparable to the control mixture.

The mixtures containing fly ash and ground limestone flowed over the edge of the table after five blows and the mixtures containing natural pozzolan, ground plate glass, and ground glass pozzolan flowed over the edge of the table after 12 to 13 blows. This indicated that the mortar mixtures containing limestone and fly ash had an increased workability compared to those mixtures containing natural pozzolan, ground plate glass, and ground glass pozzolan.

4.1.4 Type III-L(10) Cement with 30% Replacements

4.1.4.1 16-Hour Compressive Strength

Since Type II-L(10) cement is currently in production, it was predicted that Type III-L(10) will soon be produced as well. Mortar mixtures that used Type III-L(10) cement had 16-hour compressive strengths that were comparable to the control mixture. The 16-hour compressive strengths of mortar mixtures that contained Type III-L(10) cement are shown in Table 4.5 and Figure 4.4. One mortar mixture containing Type III-L(10) cement with 30% ground plate glass had a 16-hour compressive strength of 3560 psi, which was greater than the compressive strength of the control mixture and met the 3500-psi requirement. Other mortar mixtures that had a 30% replacement by mass had 16-hour compressive strengths between 3180 psi and 3330 psi. Type II-L(10) cement was interground in a production facility, rather than preblended in the lab, which could explain the lower early compressive strengths of mortar mixtures containing Type III-L(10). Furthermore, heat curing the mortar mixtures may have led to compressive strength results greater than the 3500-psi requirement.

Table 4.5 Fresh and hardened properties of mortar mixtures containing Type III L(10) cement. Compressive strength data is an average of two samples. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIII L(10)30FA contains 70% Type III L(10) and 30% fly ash.

	Temperature (°F)	Flow (%)	Compressive Strength (psi)		
			16 hr	7 day	28 day
Type III L(10)	N/A	150	3910	8630	10190
70Type III L(10)30FA	76.2	150	3280	7930	10080
70Type III L(10)30L	75.3	137	3330	6810	8010
70Type III L(10)30NP	75.5	150	3180	6920	7980
70Type III L(10)30G	76.6	150	3560	7800	8960
70Type III L(10)30GP	75.5	150	3240	7320	8670

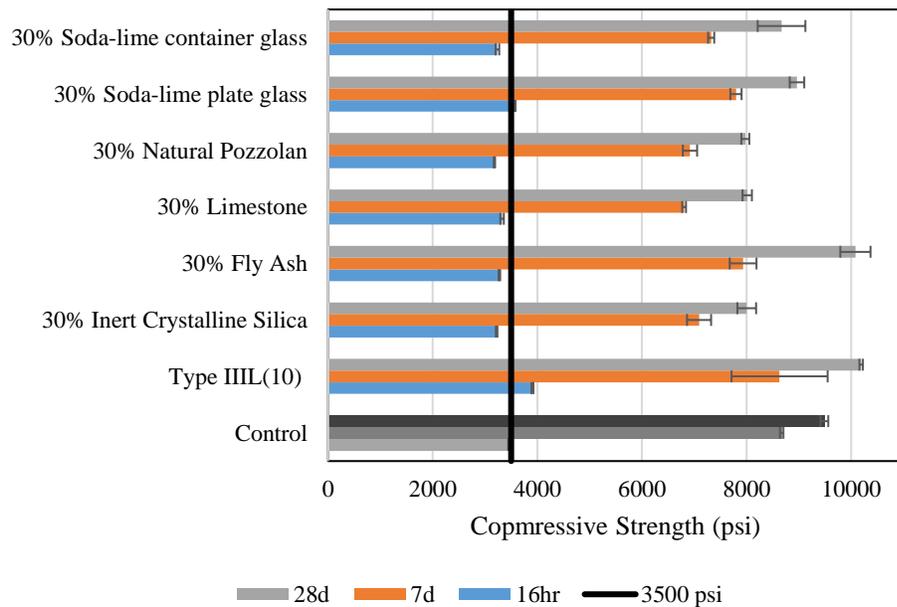


Figure 4.4 Compressive strength data of mortar mixtures containing Type III L(10) portland cement and 30% replacements. Compressive strength data is an average of two samples and the error bars illustrate the standard deviation. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIII L(10)30FA contains 70% Type III L(10) and 30% fly ash.

4.1.4.2 Flow

Every mortar mixture had a flow of 150% except the mortar mixture that contained Type III L(10) cement with 30% ground limestone, which had a flow of

137%. The mortar mixture that contained Type III(10) cement with 30% ground limestone by mass had a decreased flow since the total Type III(10) cement replaced with limestone is 37% rather than 30%. An increase in the amount of ground limestone decreases the flow of the mortar because of its irregularly shaped and fine particles. Table 4.5 reports the flow for each mortar mixture containing Type III(10) cement.

4.1.5 Summary of Mortar Mixtures

The weighted decision chart, discussed in Section 3.1 and shown in Table 4.6, was used to determine which mortar mixtures should be scaled to concrete. Although it was important that the mortar mixture had a flow of 150% and a minimum 16-hour compressive strength of 3500 psi, it was also important that the SCM that would be used in the concrete mixture was cost effective and a regionally available product. Ultimately, ground limestone was selected to be used as a partial replacement of portland cement in concrete mixture development since it had consistently higher early strengths, was the most cost-effective material, and is available around the world.

Ground limestone was used to replace Type III portland cement during concrete mixture development; Type III portland cement was used because it is currently used at Molin. Although Type II(10) will become available in the Midwest market soon, and it is anticipated it will become popular to use, it was outside the scope of this work to optimize a concrete mixture containing Type

IL(10) and a total of 30% ground limestone. Since Type III L(10) cement is not currently in production, it was not considered to be scaled to concrete.

Table 4.6 Weighted decision chart that was used to narrow the results found in the mortar mixture development. Each property was ranked on a scale from 1-5 and is highlighted in gray; one being the least important factor and five being the most important factor. Each material was then ranked on a scale from 1-5 based on its performance in the mortar mixtures.

Material	Cost	Early Strength	Later Strength	Workability	Availability	Score
Weight	5	5	2	3	3	
Limestone	5	4	3	1	4	66
Natural Pozzolan	4	3	3	4	3	62
Glass	1	2	4	4	4	47
Fly Ash	3	4	5	5	1	63

4.2 Concrete Mixture Development

One mortar mixture was scaled to concrete. Since Type III portland cement is currently used at Molin, a mixture containing Type III with 30% ground limestone was scaled to concrete. A control mixture containing Type III and 30% fly ash by mass was created to be used as a comparison to the experimental concrete mixture with ground limestone. Unlike the mortar mixtures, a set of concrete mixtures were cured in an oven at 110 °F for 16 hours to mimic internal curing temperatures of beams produced at Molin.

4.2.1 Type III Portland Cement with 30% Fly Ash Replacement

4.2.1.1 16-Hour Compressive Strength

Concrete mixtures containing Type III portland cement with 30% fly ash had an average 16-hour compressive strength of 4810 psi, which is similar to the average release strength reported to be 4860 psi by Molin. All compressive strength results are presented in Table 4.7.

Table 4.7 Optimized concrete mixture designs and properties. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = ground plate glass, GP = ground glass pozzolan. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

	70TypeIII30FA	70TypeIII30L
Portland Cement (lb)	17.6	17.6
Fly Ash (lb)	7.5	0
Limestone (lb)	0	7.5
Total Water (lb)	9.9	9.9
Superplasticizer (oz)	1.7	4.6
Unit Weight (pcf)	152.0	152.4
Air Content (%)	3.3	3.2
Slump (in.)	8	10
Temperature (°F)	72.7	N/A
Compressive Strength 16 hr (psi)	4810	5190
Compressive Strength 7 day (psi)	7050	7960
Compressive Strength 28 day (psi)	7620	7880
Modulus of Elasticity (GPa)	38.7	35.6

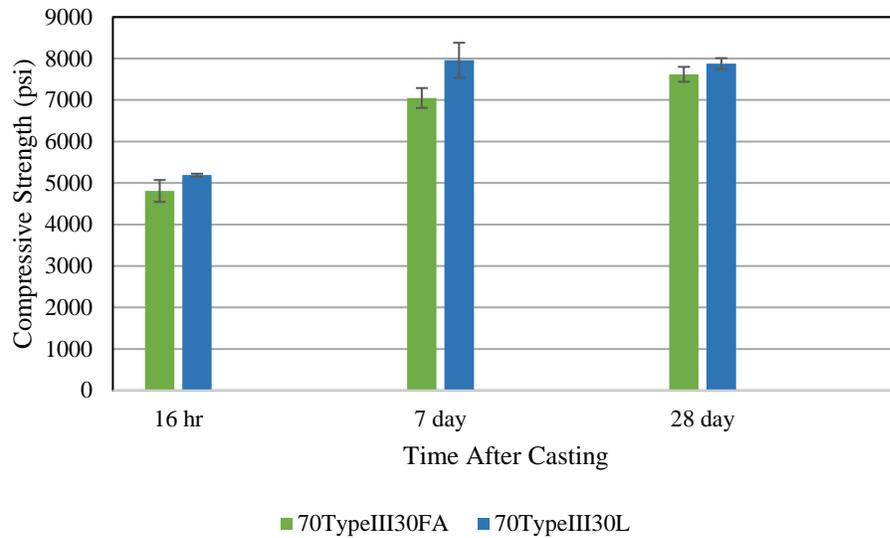


Figure 4.5 Compressive strengths of optimized concrete mixtures. The error bars on the graph are the standard deviation between 3 compressive strength cylinders. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

4.2.1.2 Slump

The concrete mixture contained 1.7 ounces of superplasticizer, which was scaled from the superplasticizer quantities used at Molin. Concrete mixtures containing Type III portland cement with 30% fly ash by mass had a slump of 8 in, shown in Table 4.7. The average spread at Molin is reported to be 26 inches. Molin typically measures the spread of the concrete since the concrete is self-consolidating. The slump of the concrete that contained Type III portland cement and ground limestone was unknown; to ensure the flow measurements were comparable, the slump was measured for all concrete mixtures made in the lab at UMD.

4.2.2 Type III Portland Cement with 28% Ground Limestone Replacement

4.2.2.1 16-Hour Compressive Strength

The average 16-hour compressive strength of the concrete mixture that contained Type III portland cement and 30% ground limestone was 5190 psi, which exceeded the 3500-psi requirement as well as the 16-hour compressive strength of the control mixture. This result ensured that the concrete was strong enough to withstand the force when the prestressed strands are released. Table 4.7 and Table 4.5 present the fresh and hardened properties of the concrete mixture containing Type III portland cement and 30% ground limestone.

4.2.2.2 Slump

Originally, 1.7 oz of superplasticizer was used in the concrete mixture containing Type III portland cement and 30% ground limestone and the slump was recorded as 1.5 in. While 1.7 oz as enough superplasticizer to produce a good slump for the control mixture, the mixture containing ground limestone required more to reach a comparable slump. Therefore, an additional set of mortar mixtures were created to optimize the amount of super plasticizer that was added in the concrete mixture containing ground limestone. Superplasticizer was added in increments of 0.02 oz, starting at 0.1 oz in the first mixture. A picture of the optimized mortar mixtures can be seen in Figure 4.6. It was found that the addition of 0.2 oz of superplasticizer in the mortar mixture that had Type III portland cement and 30% ground limestone produced a flow that was comparable to a control mortar mixture that contained Type III portland cement and 30% fly

ash; this amount of superplasticizer was equivalent to 4.6 oz when used in the concrete mixture.

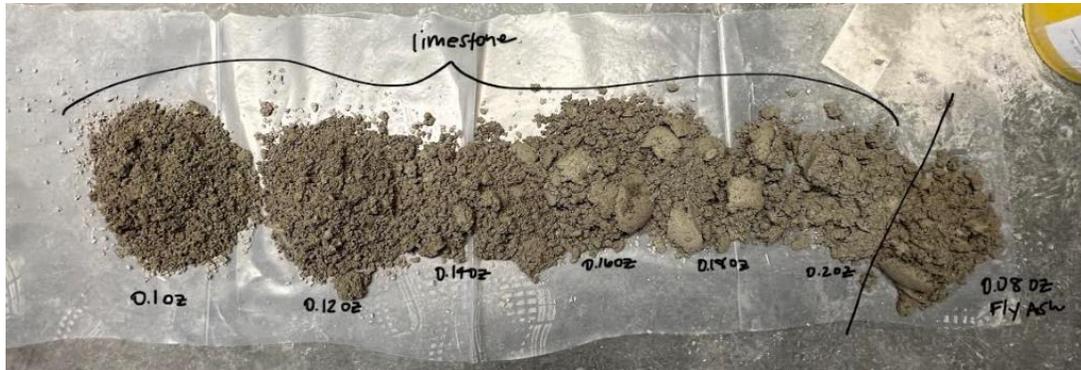


Figure 4.6 Mortar mixtures containing Type III with 30% ground limestone and increasing amounts of superplasticizer, starting with 0.01 oz. These were compared to the control mixture with 0.08 oz of superplasticizer that is shown on the right.

The additional superplasticizer produced a slump of 10 inches for the concrete mixture containing Type III portland cement and 30% ground limestone. Although a 10-inch slump is a greater slump than the slump of the control mixture, the set time decreased, and it became harder to work with over the span of 30 minutes. The slump of both the control mixture and the concrete mixture containing Type III portland cement and 30% ground limestone are presented in Table 4.7.

4.3 Precast, Prestressed Beam Material and Structural Properties

4.3.1 Material Properties

4.3.1.1 Compressive Strength

Compressive strength tests were performed at 16 hours, 7 days, and 28 days. Three cylinders composed of the control mixture had an average 16-hour strength of 4600 psi. Only one concrete cylinder that was composed of Type III portland cement and 27.1% ground limestone was placed in the SURE CURE

cylinder mold; at 16 hours, the specimen had a compressive strength of 5390 psi, which was greater than the required compressive strength of 3500 psi and the average compressive strength of the control mixture. Results from this cylinder were likely the most accurate since it was heated to the same temperature as the internal temperature of the beams while in the SURE CURE cylinder mold. Three other Type III portland cement and 27.1% ground limestone cylinders were cured under a tarp in the plant and had an average 16-hour compressive strength of 5700 psi. The difference in compressive strengths between the control cylinders and the cylinders composed Type III portland cement and 27.1% ground limestone was likely because of the slower pozzolanic reaction of the concrete containing fly ash, which corroborated results found in the UMD lab using smaller concrete batches. Results from cylinders made at Molin are shown in Table 4.8.

Table 4.8 Compressive strength results for cylinders fabricated at Molin. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Concrete Mixture	Compressive Strength (psi)		
	16 hr	7 days	28 days
70TypeIII27.5FA	4600	10640	11360
70TypeIII27.1L	5700	9650	11090

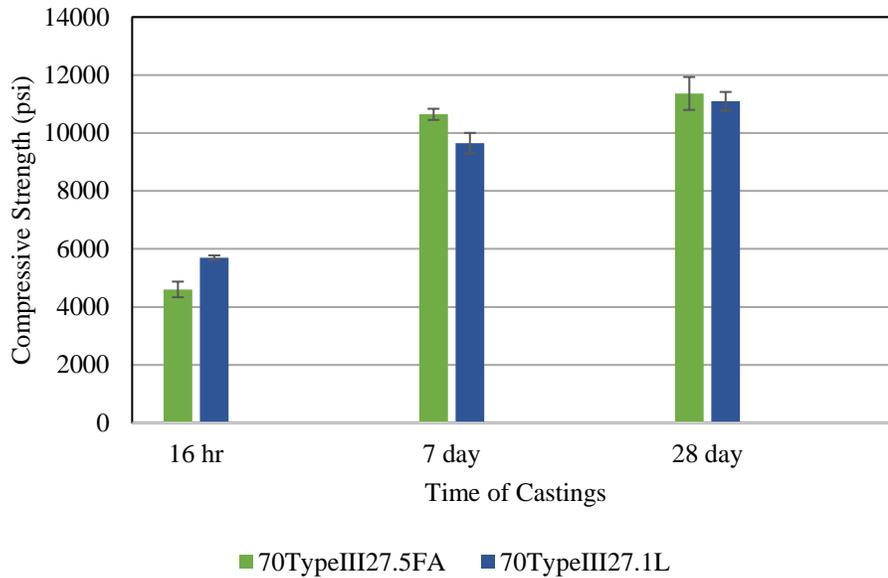


Figure 4.7 Compressive strength results for cylinders fabricated at Molin. The error bars on the graph are the standard deviation between 3 compressive strength cylinders. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Both concrete mixtures had an increase in compressive strength at 7 days; cylinders composed of Type III portland cement and 27.5% fly ash had an average compressive strength of 10640 psi and cylinders composed of Type III portland cement and 27.1% ground limestone had an average compressive strength of 9650 psi at 7 days. It was possible that the pozzolanic reaction from the fly ash had started contributing to the compressive strength of the cylinder with Type III portland cement and 27.5% fly ash. These results were slightly different than the compressive strengths results recorded in the lab at UMD, which indicated that cylinders composed of Type III portland cement and 27.1% ground limestone would still have a higher compressive strength at 7 days than cylinders composed of Type III portland cement and 27.5% fly ash. At 28 days, both concrete mixtures had a slight increase in compressive strength from the 7-day results, but the control mixture still remained stronger than the cylinder

composed of Type III portland cement and 27.1% ground limestone. At 28 days, the control mixture had a compressive strength of 11360 psi and the mixture composed of Type III portland cement and 27.1 % ground limestone had a compressive strength of 11090 psi. Compressive strength data from cylinders fabricated at Molin are shown in Table 4.8 and Figure 4.7.

4.3.1.2 Slump

The concrete mixture containing Type III portland cement and 27.5% fly ash had a spread of 22 in. Typically, concrete containing Type III portland cement and fly ash has an average spread of 26 in. The decrease in spread was because of the decreased amount of superplasticizer used in the control mixture. The standard mixture at Molin uses 52 oz of superplasticizer per 1 yd³, but the control mixture used 46 oz of superplasticizer. During casting of the beams for this research, the batching operator focused on the slump meter on the control panel, which measures the amp draw of the mixer motors, as well as the visual look of the concrete inside the mixer to determine how much superplasticizer to add. The control panel showed a similar reading to previous concrete mixtures when 46 oz of superplasticizer was added, but consequentially the spread had decreased.

The concrete mixture containing Type III portland cement and 27.1% ground limestone had a spread of 24 in. The mixture was easily flowing into the mold and additional consolidation was not needed. The spread was greater than that of the control mixture, and only 2 in. less than the typical spread of concrete mixed at Molin. The concrete mixture containing Type III portland cement and 27.1% ground limestone used 81.5 oz of superplasticizer per 1 yd³. This was

significantly more than what was used in the control mixture, which led to concern about a decrease in set time; however, no issues were encountered during fabrication of the four beams. Use of 81.5 oz of superplasticizer per 1 yd³ was less than the 136 oz of superplasticizer per 1 yd³ used to produce concrete that had a slump of 10 in. during concrete mixture development in the lab.

4.3.1.3 Modulus of Rupture and Modulus of Elasticity

Equation 4.1 from ACI 318-19 [1] was used to predict the modulus of elasticity for comparison to the value determined from testing a concrete cylinder at UMD.

$$E_c = 33 \cdot w_c^{1.5} \sqrt{f'_c} \quad 4.1$$

Table 4.9 shows predicted and observed modulus of elasticity results for both concrete mixtures produced at Molin. The predicted modulus of elasticity was calculated using the 28-day compressive strength results. The observed modulus of elasticity was measured using one cylinder specimen of each concrete mixture and was determined 28 days after casting. A stress versus strain graph is shown in Figure 4.8, the observed modulus of elasticity is the slope of the line between the two points. The observed modulus of elasticity for the concrete mixture composed of Type III portland cement and 27.5% fly ash was 5579 ksi and the predicted modulus of elasticity was 6462 ksi. The observed modulus of elasticity was 16% less than the modulus of elasticity predicted by ACI 318-19. Similarly, the concrete mixture containing Type III portland cement and 27.1% ground limestone had an observed modulus of elasticity of 5582 ksi, and a predicted modulus of elasticity of 6384 ksi. The observed modulus of elasticity

was 14% less than the modulus of elasticity predicted by ACI 318-19. However, the measured modulus of elasticity of both mixtures was approximately the same, with a difference of less than 1%.

Table 4.9 Predicted and observed modulus of elasticity at 28 days. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Concrete Mixture	Predicted Modulus of Elasticity (ksi)	Observed Modulus of Elasticity (ksi)
70TypeIII27.5FA	6462	5579
70TypeIII27.1L	6384	5582

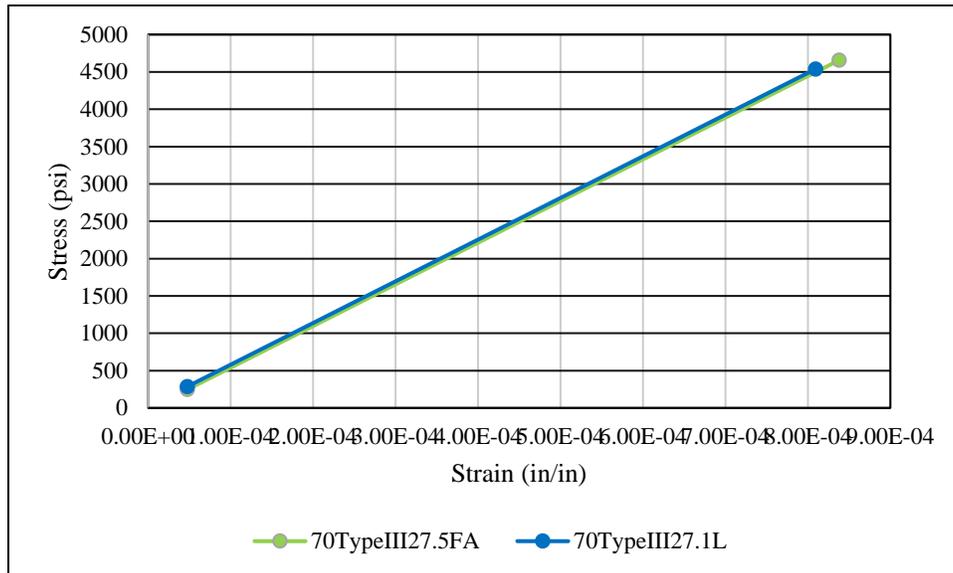


Figure 4.8 Stress versus strain data from both concrete mixtures. Modulus of elasticity is determined by calculating the slope of each line.

Equation 4.2 from ACI 318-19 [1] was used to predict the modulus of rupture for comparison to the value determined from testing concrete beams at UMD.

$$f_r = 7.5\sqrt{f'_c} \quad 4.2$$

Table 4.10 shows the predicted and observed modulus of rupture for both concrete mixtures produced at Molin. The predicted modulus of rupture was

calculated using the 28-day compressive strength results. The observed modulus of rupture was measured using one small beam for the concrete mixture containing Type III portland cement and 27.5% fly ash and the average of two small beams for the concrete mixture containing Type III portland cement and 27.1% ground limestone; both concrete mixtures were tested at 28 days. The observed moduli of elasticity were calculated by the software system used to break the small beams at UMD. The observed modulus of rupture of the control mixture was 781 ksi and the observed modulus of rupture of the concrete mixture containing Type III portland cement and ground limestone was 792 psi. Both concrete mixtures had a modulus of rupture that was comparable to the predicted modulus of rupture, indicating that both mixtures have similar direct tensile strength. This also indicates that the code is still applicable to concrete mixtures containing 30% ground limestone. The observed modulus of rupture of the concrete mixture composed of Type III portland cement and 27.5% fly ash was 2% less than the predicted modulus of rupture, which was 799 psi; the observed modulus of rupture of the concrete mixture composed of Type III portland cement and 27.1% ground limestone was less than 1% greater than the predicted modulus of rupture, which was 790 psi.

Table 4.10 Predicted and observed modulus of rupture at 28 days. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Concrete Mixture	Predicted Modulus of Rupture (psi)	Observed Modulus of Rupture (psi)
70TypeIII27.5FA	799	781
70TypeIII27.1L	790	792

4.3.2 Structural Properties

4.3.2.1 Flexural Capacity

The predicted flexural capacities were calculated using the 28-day compressive strength, the observed modulus of elasticity, and observed modulus of rupture results. Flexural tests were performed between 16 and 18 days after curing. The goal was to break additional concrete specimens at the time of breaking the concrete beams, but because of a limited number of concrete specimens, they were only tested at 28 days. All beams exhibited a flexural failure. Flexural failure was characterized by visually seeing a compressive failure of concrete at the top of the beam and flexural cracks along its length, as well as seeing a decrease in the applied load as the actuator was moving downward at a constant rate. A typical flexural failure is shown in Figure 4.9. All observed results, shown in Table 4.11, were greater than those predicted using ACI 318-19. It is important to note that all predicted flexural capacities were calculated with a phi-factor of 0.9, which would decrease the predicted flexural capacity compared to the actual capacity.

One beam composed of Type III portland cement and 27.5% fly ash was tested at midspan in flexure. The observed moment capacity of the control mixture was 150 k-ft, which was 13% greater than the predicted moment capacity of 133.5 k-ft.



Figure 4.9 Typical flexure test failure.

Two beams composed of Type III portland cement and 27.1% ground limestone were tested at midspan in flexure; both beams had similar observed moment capacities: 137.8 k-ft and 139 k-ft. These moment capacities were 3% and 4% higher than the predicted moment capacities. This indicates that the code is still applicable to concrete mixtures containing 30% replacement with ground limestone. While all mixtures had an observed moment capacity greater than predicted, the control mixture had a moment capacity almost 9% greater than the observed moment capacities of the concrete mixture containing ground limestone.

Table 4.11 Predicted and observed moment capacities for flexure beams. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone, NP = natural pozzolan, G = Potter plate glass, GP = Commercially available glass. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Concrete Mixture		Flexure	
		Predicted	Observed
		Moment (k-ft)	Moment (k-ft)
70TypeIII27.5FA	Beam 1	133.5	150
70TypeIII27.1L	Beam 1	133.3	139
	Beam 2		137.8

4.3.2.2 Bond Strength

All beams tested to evaluate the bond capacity exhibited normal, flexural failures. A typical bond strength test failure is shown in Figure 4.10. Bond strength tests were performed between 22 and 24 days after curing. All observed results shown in Table 4.12 were higher than those moment capacities predicted using ACI 318-19. One beam composed of Type III portland cement and 27.5% fly ash was tested to determine whether the predicted development length was accurate. The observed moment capacity of 136.6 k-ft was near 10% greater than the predicted moment capacity of 124.7 k-ft. Since the beam failed in flexure and had a higher than predicted moment capacity, it was concluded that the development length was shorter than predicted. If the observed moment capacity was less than predicted and exhibited an unusual failure, it would have indicated that the prestressed strands had not been fully developed.

Table 4.12 Predicted and observed moment capacities for bond strength beams. Nomenclature key for the SCMs is as follows: FA = fly ash, L = ground limestone. For example, 70TypeIL(10)30FA contains 70% Type IL(10) and 30% fly ash.

Concrete Mixture		Bond Strength	
		Predicted	Observed
		Moment (k-ft)	Moment (k-ft)
70TypeIII27.5FA	Beam 1	124.7	136.6
70TypeIII27.1L	Beam 1	124.8	134.1
	Beam 2		136.2

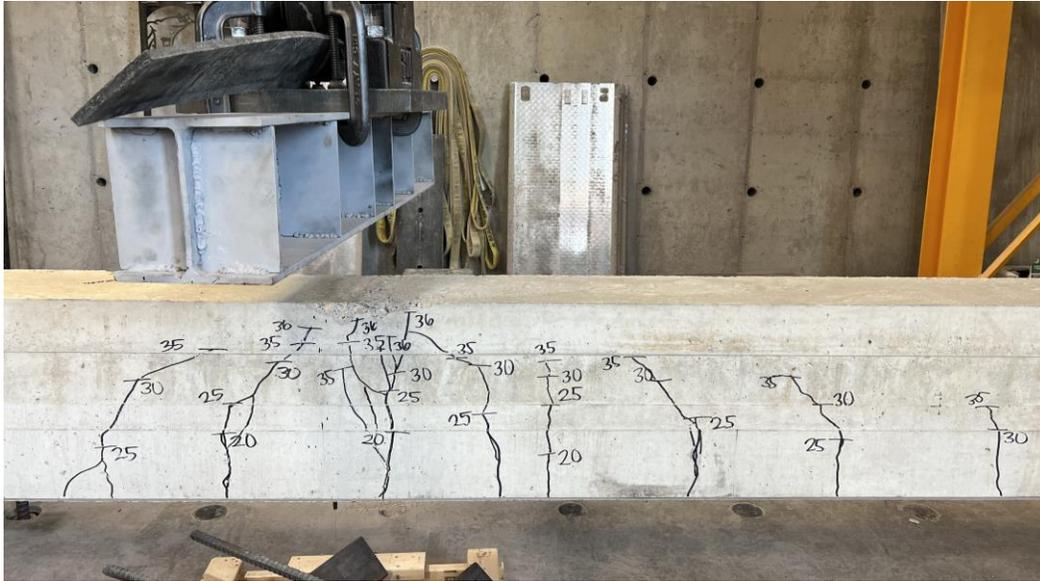


Figure 4.10 Typical bond strength test failure.

Similarly, the two beams composed of Type III portland cement and 27.1% ground limestone exhibited a flexural failure. The ground limestone mixture had an observed moment capacity of 134.1 k-ft and 136.2 k-ft. These were at least 7.5% greater than the predicted moment capacity of 124.8 k-ft. Again, this indicated that the actual development length was shorter than predicted. The predicted moment capacities for the bond strength beams were also greater than the predicted moment capacities for the flexure beams. This indicated that at the predicted development length, the prestressed strands are fully bonded and exhibit full moment capacity.

5 Conclusions and Recommendations

5.1 Project Summary

Portland cement production contributes 8% total global carbon dioxide emissions each year [2]. For decades, researchers have been studying ways to reduce these emissions. The partial replacement of portland cement with an SCM such as fly ash has proven to be successful; however, the supply of fly ash has already begun to decrease as the United States starts using cleaner energy sources. This project considered several alternative supplementary cementitious materials as a replacement for fly ash in precast, prestressed concrete.

5.2 Exploration of Alternative SCMs and New Cements

The alternative SCMs used in this project were selected based on availability and included natural pozzolan, ground limestone, and two different sources of ground glass. Using each of these materials at a 20 or 30% replacement level by mass, mortar mixtures were made using three different types of portland cement: Type III portland cement, Type IL(10) cement, and Type IIIL(10) cement. Each mortar mixture had to meet the workability (flow consistent with self-consolidating concrete) and compressive strength (3500-psi compressive strength requirement at 16 hours) requirements set forth by Molin. Type IL(10) cement is currently in production and will be available on the market in 2023, and Type IIIL(10) cement will likely soon follow. Using Type IL(10) or Type IIIL(10) cement instead of Type I portland cement or Type III portland cement will allow a larger total percent replacement of portland cement when used with another SCM.

All experimental mortar mixtures had a flow that was comparable to the control mixture that contained Type III portland cement and 30% fly ash. There was a larger contrast between the early compressive strength of the control mixture and experimental mixtures. Five out of 24 mixtures met the compressive strength requirements. This is possibly due to the slower pozzolanic reactions that occur in the mixtures that contain ground glass or natural pozzolan, but also due to the lack of heat during hydration. Despite the lack of curing heat, all mortar mixtures had a minimum 28-day compressive strength of 6700 psi which could be adequate for cast-in-place concrete, depending on the design requirements.

This research focused on using materials that produced concrete most suitable to be used in a precast, prestressed concrete plant, and based on the weighted decision chart, limestone was selected to be scaled up. Consistently, the mortar mixtures that contained ground limestone had a flow near 150% of the control and a higher early compressive strength than the other mixtures. Additionally, ground limestone is cost-effective, widely available, and met the minimum requirements to produce precast, prestressed concrete.

5.3 Implementing Limestone in Precast, Prestressed Concrete Plant

When the mortar mixture that contained Type III portland cement and 30% ground limestone was scaled to concrete, and heated to 110 °F for 16 hours, compressive strengths were higher than the control mixture. Initially, the fresh concrete had a decreased flow compared to the control mixture. A small amount of research was done to optimize the amount of superplasticizer to add to the concrete mixture, but more research should be conducted to optimize the use of

admixtures in the concrete mixture. Ultimately, the compressive strength and flow of the concrete mixture in the lab was adequate to be used to produce a set of precast, prestressed concrete beams at Molin.

Implementing limestone into precast concrete mixtures is manageable, but there will need to be some considerations. Portland cement production facilities already have the technology and resources to produce ground limestone, but not every precast concrete production facility may have the technology and capacity to store ground limestone. For example, a precast concrete plant may need an additional silo to store the ground limestone and implement additional methods to transport the material into the concrete mixer. For this research, the ground limestone was placed into the mixer by hand using 5-gallon buckets.

The concrete produced with ground limestone had a spread of 24 inches, which was comparable to the average spread of 26 in. for mixtures typically used at Molin. The amount of superplasticizer added to the mix was less than predicted amount needed, but it was still more than what was used in the control mixture, which had a spread of 22 in. The 16-hour compressive strength of the concrete containing Type III portland cement and 27.1% ground limestone was 5700 psi, which far exceeded the minimum 16-hour compressive strength requirement of 3500 psi.

5.4 Structural Performance

As the concrete aged, the control mixture had a higher compressive strength, which was possibly because of the slow, pozzolanic reactions that were occurring in the control mixture due to the abundance of alumina and silica in fly

ash as well as its unique spherical shape. While the modulus of elasticity and the modulus of rupture of both mixtures were less than predicted, the two concrete mixtures were comparable, indicating that both mixtures were less stiff than ACI predicted and had similar direct tensile strength.

All beams tested exhibited a flexural failure and failed at applied moments that were more than predicted flexural capacity. Although beams containing fly ash failed at higher applied moments, the code was still successfully used to predict the failure of beams containing ground limestone. Bond strength beams failed at higher applied moments than predicted when the loads were applied at midspan, indicating that not only was the development length shorter than predicted, but that at the predicted development length the beam still exhibited full moment capacity.

5.5 Future Research

5.5.1 Ternary blend investigation

Future research should investigate the use of the other SCMs to produce alternative cements and ternary blends. In this study, it was proven that ground limestone could be used at 30% replacement levels in precast, prestressed concrete. If alternative cements, such as Type IL(10), were used in combination with other SCMs, a higher percentage of portland cement could be replaced in concrete. It is predicted that Type IL(10) will become as commonly used as a replacement for portland cement. Researchers should optimize mixtures that contain Type IL(10) cement with an additional replacement with other SCMs; increasing the replacement level of cement is the most practical way to reduce

carbon dioxide emissions associated with concrete production [90]. Similarly, researchers should optimize mixtures containing Type III L(10) cement with higher replacement levels of the other SCMs. Mortar mixtures containing Type III L(10) cement in this study showed relatively higher compressive strengths compared to those using Type II L(10) cement, and if used in a ternary blend, compressive strength results could compare to those results of Type III portland cement with 30% ground limestone.

Additionally, if limestone was used in a ternary concrete mixture that included fly ash or another pozzolan, it could increase the flow of the concrete mixture. The ground limestone used in the study was finer than Type III portland cement and had a rigid microstructure, unlike fly ash which has a uniquely spherical shape. The addition of fly ash or another pozzolan in a concrete mixture with limestone could increase the flow of the concrete mixture without the addition of excess admixtures. Further, if ground limestone was used that was coarser, it could potentially produce a mixture that has an increase flow.

5.5.2 Additional structural performance

Testing the bond strength beams inside the development length proved that the development length was less than predicted. Additional research should be done to find the accurate development length; this would be done by moving the applied load closer to the end until a flexural failure no longer occurred, or the flexural capacity was less than predicted.

References

- [1] ACI Committee 318, *Building Code Requirements for Structural Concrete*. Farmington Hills, MI: American Concrete Institute, 2019.
- [2] International Energy Agency, “Technology Roadmap.”
- [3] A. Hatfield, “Cement,” 2020.
- [4] “Cement Statistics and Information | U.S. Geological Survey.”
<https://www.usgs.gov/centers/national-minerals-information-center/cement-statistics-and-information> (accessed Jun. 09, 2022).
- [5] “SUPPLEMENTARY CEMENTITIOUS MATERIALS.” Accessed: Jun. 08, 2021. [Online]. Available: www.silicafume.org.
- [6] “The Future of Fly Ash Use in Concrete - NPCA.”
<https://precast.org/2013/11/the-future-of-fly-ash-use-concrete/> (accessed Mar. 22, 2021).
- [7] M. Jakob, “The future of coal in a carbon-constrained climate,” *Nat. Clim. Chang.*, vol. 10, no. 8, pp. 704–707, 2020, doi: 10.1038/s41558-020-0866-1.
- [8] “COP26 Goals - UN Climate Change Conference (COP26) at the SEC – Glasgow 2021.” <https://ukcop26.org/cop26-goals/> (accessed Nov. 21, 2021).
- [9] T. H. Adams, “Fly Ash Use in Concrete Increases Slightly As Overall Coal Ash Recycling Rate Declines,” Accessed: Oct. 11, 2021. [Online]. Available: www.aaa-usa.org.
- [10] “National Pozzolan Association: The History of Natural Pozzolans.”

- <https://pozzolan.org/history-pozzolans.html> (accessed Jul. 28, 2021).
- [11] “Cement,” *Build. Mater. Civ. Eng.*, pp. 46–423, Jan. 2011, doi: 10.1533/9781845699567.46.
- [12] “What is a pozzolan?”
<https://www.concrete.org/tools/frequentlyaskedquestions.aspx?faqid=688>
(accessed Jul. 28, 2021).
- [13] “National Pozzolan Association: How Natural Pozzolans Improve Concrete.” <https://pozzolan.org/improve-concrete.html> (accessed Jul. 28, 2021).
- [14] “Coal Fly Ash - Material Description - User Guidelines for Waste and Byproduct Materials in Pavement Construction - FHWA-RD-97-148.”
<https://www.fhwa.dot.gov/publications/research/infrastructure/structures/97148/cfa51.cfm> (accessed Jul. 29, 2021).
- [15] “What is Fly Ash?| Concrete Construction Magazine.”
https://www.concreteconstruction.net/how-to/materials/what-is-fly-ash_o
(accessed Aug. 11, 2021).
- [16] “Chapter 1 - Fly Ash - An Engineering Material - Fly Ash Facts for Highway Engineers - Recycling - Sustainability - Pavements - Federal Highway Administration.”
<https://www.fhwa.dot.gov/pavement/recycling/fach01.cfm> (accessed Jul. 29, 2021).
- [17] S. A. R. and T. A. Bier, “Self-Consolidating Mortars Using Various Secondary Raw Materials,” *ACI Mater. J.*, vol. 106, no. 1, doi:

10.14359/56313.

- [18] I. Al-Jumaily, Q. Kareem, and N. Naji, "An overview on the Influence of Pozzolanic Materials on Properties of Concrete," *Int. J. Enhanc. Res. Sci. Technol. Eng.*, vol. 4, no. 3, pp. 81–92, 2015.
- [19] N. Bouzoubaâ and M. Lachemi, "Self-compacting concrete incorporating high volumes of class F fly ash: Preliminary results," *Cem. Concr. Res.*, vol. 31, no. 3, pp. 413–420, Mar. 2001, doi: 10.1016/S0008-8846(00)00504-4.
- [20] S. Sekino and T. Narita, "Study on mix design and properties of selfcompacting concrete containing high volume fly ash," *J. Taiheiyō Cem. Corp.*, vol. 145, pp. 18–35, 2003.
- [21] M. Sonebi, "Medium strength self-compacting concrete containing fly ash: Modelling using factorial experimental plans," *Cem. Concr. Res.*, vol. 34, no. 7, pp. 1199–1208, Jul. 2004, doi: 10.1016/J.CEMCONRES.2003.12.022.
- [22] R. Helmuth, *Fly Ash in Cement and Concrete*. 1987.
- [23] W. J. Halstead, "Use of Fly Ash in Concrete.," *Natl. Coop. Highw. Res. Program, Synth. Highw. Pract.*, vol. 96, no. Reapproved, pp. 1–34, 1986.
- [24] D. H. Alexandre Pavoine Thierry Chaussadent, Arezki Tagnit-Hamou, and Loïc Divet, "Impact of Alternative Cementitious Material on Mechanical and Transfer Properties of Concrete," *ACI Mater. J.*, vol. 111, no. 3, doi: 10.14359/51686828.
- [25] A. Bilodeau and V. Mohan Malhotra, "High-Volume Fly Ash System:

- Concrete Solution for Sustainable Development,” *Mater. J.*, vol. 97, no. 1, pp. 41–48, Jan. 2000, doi: 10.14359/804.
- [26] V. M. Malhotra, “Superplasticized Fly Ash Concrete for Structural Applications,” *Concr. Int.*, vol. 8, no. 12, pp. 28–31, Dec. 1986.
- [27] H. Yazici, “The effect of silica fume and high-volume Class C fly ash on mechanical properties, chloride penetration and freeze-thaw resistance of self-compacting concrete,” *Constr. Build. Mater.*, vol. 22, no. 4, pp. 456–462, Apr. 2008, doi: 10.1016/J.CONBUILDMAT.2007.01.002.
- [28] R. J. Detwiler, “SUBSTITUTION OF FLY ASH FOR CEMENT OR AGGREGATE IN CONCRETE: STRENGTH DEVELOPMENT AND SUPPRESSION OF ASR,” 2002, Accessed: Feb. 17, 2022. [Online]. Available: www.portcement.org.
- [29] R. N. Swamy, S. A. R. Ali, D. Theodorakopoulos, and D., “Early Strength Fly Ash Concrete for Structural Applications,” *ACI JOURNAL, Proc.*, vol. 80, no. 5, pp. 414–423, 1983.
- [30] M. C. Bhardwaj, V. S. Batra, and V. V Sastry, “Effect of Admixtures (Calcium Chloride) on Pozzalana Concrete,” *Indian Concr. J.*, vol. 54, no. 5, pp. 134–138, 1980.
- [31] “SCMs in Concrete: Natural Pozzolans.” <https://precast.org/2017/09/scms-concrete-natural-pozzolans/> (accessed Nov. 23, 2021).
- [32] “Use of Raw or Processed Natural Pozzolans in Concrete,” 2001.
- [33] B. Mather, “The Partial Replacement of Portland Cement in Concrete,” *Cem. Concr.*, pp. 37–73, 1958.

- [34] R. E. Philleo, "Freezing and Thawing Resistance of High-Strength Concrete," 1986.
- [35] V. M. Malhotra, *Fly ash, silica fume, slag, and natural pozzolans in concrete*. 1989.
- [36] R. T. Hemmings, "Process for Converting Waste Glass Fiber into Value Added Products, Final Report," Dec. 2005, doi: 10.2172/889402.
- [37] M. Kasaniya, M. D. A. Thomas, and E. G. Moffatt, "Efficiency of natural pozzolans, ground glasses and coal bottom ashes in mitigating sulfate attack and alkali-silica reaction," *Cem. Concr. Res.*, vol. 149, Nov. 2021, doi: 10.1016/J.CEMCONRES.2021.106551.
- [38] H. M. Owaid, R. B. Hamid, and M. Raihan Taha, "A Review of Sustainable Supplementary Cementitious Materials as an Alternative to All-Portland Cement Mortar and Concrete," *Aust. J. Basic Appl. Sci.*, vol. 6, no. 9, pp. 2887–303, 2012.
- [39] V. L. Tang, T. C. Nguyen, X. H. Ngo, V. P. Dang, and B. Bulgakov, "Effect of natural pozzolan on strength and temperature distribution of heavyweight concrete at early ages," vol. 193, no. MATEC Web Conferences, 2018, doi: <https://doi.org/10.1051/matecconf/201819303024>.
- [40] M. Ghrici, S. Kenai, and M. Said-Mansour, "Mechanical properties and durability of mortar and concrete containing natural pozzolana and limestone blended cements," *Cem. Concr. Compos.*, vol. 29, no. 7, pp. 542–549, Aug. 2007, doi: 10.1016/J.CEMCONCOMP.2007.04.009.
- [41] P. K. Mehta, *Natural Pozzolans: Supplementary Cementing Materials for*

- Concrete*. Ottawa, Canada: Canadian Government Publishing Center, 1987.
- [42] F. Massazza and V. Costa, "Aspects of the Pozzolanic Activity and Properties of Pozzolanic Cements," *II Cem.*, vol. 76, pp. 3–18, 1979.
- [43] Mitch Jacoby, "The state of glass recycling in the US," *C&EN Glob. Enterp.*, vol. 97, no. 6, pp. 28–32, Feb. 2019, doi: 10.1021/CEN-09706-COVER.
- [44] B. Taha and G. Nounu, "Utilizing Waste Recycled Glass as Sand/Cement Replacement in Concrete," *J. Mater. Civ. Eng.*, vol. 21, no. 12, pp. 709–721, Dec. 2009, doi: 10.1061/(ASCE)0899-1561(2009)21:12(709).
- [45] E. Harrison, A. Berenjian, and M. Seifan, "Recycling of waste glass as aggregate in cement-based materials," *Environ. Sci. Ecotechnology*, vol. 4, Oct. 2020, doi: 10.1016/J.ESE.2020.100064.
- [46] M. K. Amanda Kaminsky Prasad Rangaraju, Arezki Tagnit-Hamou, and Michael D.A. Thomas, "Ground-Glass Pozzolan for Use in Concrete," *Concr. Int.*, vol. 42, no. 11.
- [47] N. Schwarz, H. Cam, and N. Neithalath, "Influence of a fine glass powder on the durability characteristics of concrete and its comparison to fly ash," *Cem. Concr. Compos.*, vol. 30, no. 6, pp. 486–496, Jul. 2008, doi: 10.1016/J.CEMCONCOMP.2008.02.001.
- [48] A. Omran, N. Soliman, A. Zidol, and A. Tagnit-Hamou, "Performance of ground-glass pozzolan as a cementitious material-a review," *Adv. Civ. Eng. Mater.*, vol. 7, no. 1, pp. 237–270, 2018, doi: 10.1520/ACEM20170125.
- [49] A. Zidol, M. Tohoue-Tognonvi, and A. Tagnit-Hamou, "Advances in

- Durable Concrete Materials Applied to the African Context,” 2013.
- [50] A. Tagnit-Hamou, “Alternative Cementitious Materials-Effect of Glass Powder on Concrete Sustainability,” 2013.
- [51] A. T.-H. and A. Bengougam, “Glass Powder as a Supplementary Cementitious Material,” *Concr. Int.*, vol. 34, no. 3.
- [52] A. A. Aliabdo, A. E. M. Abd Elmoaty, and A. Y. Aboshama, “Utilization of waste glass powder in the production of cement and concrete,” *Constr. Build. Mater.*, vol. 124, pp. 866–877, Oct. 2016, doi: 10.1016/J.CONBUILDMAT.2016.08.016.
- [53] N. A. Soliman and A. Tagnit-Hamou, “Development of ultra-high-performance concrete using glass powder – Towards ecofriendly concrete,” *Constr. Build. Mater.*, vol. 125, pp. 600–612, Oct. 2016, doi: 10.1016/J.CONBUILDMAT.2016.08.073.
- [54] E. T. Bueno, J. M. Paris, K. A. Clavier, C. Spreadbury, C. C. Ferraro, and T. G. Townsend, “A review of ground waste glass as a supplementary cementitious material: A focus on alkali-silica reaction,” *J. Clean. Prod.*, vol. 257, p. 120180, Jun. 2020, doi: 10.1016/J.JCLEPRO.2020.120180.
- [55] A. Shayan and A. Xu, “Value-added utilisation of waste glass in concrete,” *Cem. Concr. Res.*, vol. 34, no. 1, pp. 81–89, Jan. 2004, doi: 10.1016/S0008-8846(03)00251-5.
- [56] C. Shi, Y. Wu, Y. Shao, and C. Riefler, “Alkali-Aggregate Reaction of Mortar Bars Ground Glass Powder,” 2004.
- [57] Y. Shao, T. Lefort, S. Moras, and D. Rodriguez, “Studies on concrete

- containing ground waste glass,” *Cem. Concr. Res.*, vol. 30, no. 1, pp. 91–100, Jan. 2000, doi: 10.1016/S0008-8846(99)00213-6.
- [58] A. Shayan and A. Xu, “Performance of glass powder as a pozzolanic material in concrete: A field trial on concrete slabs,” *Cem. Concr. Res.*, vol. 36, no. 3, pp. 457–468, Mar. 2006, doi: 10.1016/J.CEMCONRES.2005.12.012.
- [59] M. K. and J. F. Davalos, “Field Application of Recycled Glass Pozzolan for Concrete,” *ACI Mater. J.*, vol. 116, no. 4, doi: 10.14359/51716716.
- [60] “Portland-Limestone Cement U.S. Fact Sheet.”
- [61] “Portland-Limestone Cement - National Precast Concrete Association.” <https://precast.org/2014/06/portland-limestone-cement/> (accessed Aug. 13, 2021).
- [62] G. Schumacher and L. Juniper, “Coal utilisation in the cement and concrete industries,” *Coal Handb. Towar. Clean. Prod.*, vol. 2, pp. 387–426, Oct. 2013, doi: 10.1533/9781782421177.3.387.
- [63] G. Sua-iam and N. Makul, “Use of limestone powder during incorporation of Pb-containing cathode ray tube waste in self-compacting concrete,” *J. Environ. Manage.*, vol. 128, pp. 931–940, Oct. 2013, doi: 10.1016/J.JENVMAN.2013.06.031.
- [64] R. Derabla and M. L. Benmalek, “Characterization of heat-treated self-compacting concrete containing mineral admixtures at early age and in the long term,” *Constr. Build. Mater.*, vol. 66, pp. 787–794, Sep. 2014, doi: 10.1016/J.CONBUILDMAT.2014.06.029.

- [65] K. Kanazawa, K. Yamada, and S. Sogo, “Properties of Low-Heat Generating Concrete Containing Large Volumes of Blast-Furnace Slag and Fly Ash,” *Spec. Publ.*, vol. 132, pp. 97–118, May 1992, doi: 10.14359/1974.
- [66] L. Courard, F. Michel, and J. Pierard, “Influence of clay in limestone fillers for self-compacting cement based composites,” *Constr. Build. Mater.*, vol. 25, no. 3, pp. 1356–1361, Mar. 2011, doi: 10.1016/J.CONBUILDMAT.2010.09.009.
- [67] G. De Schutter, “Effect of limestone filler as mineral addition in self-compacting concrete,” *36th Conf. Our World Concr. Struct. Recent Adv. Technol. Fesh Concr. Proc.*, pp. 49–54, 2011, Accessed: Feb. 16, 2022. [Online]. Available: <http://hdl.handle.net/1854/LU-2939937>.
- [68] I. Soroka and N. Stern, “Calcareous fillers and the compressive strength of portland cement,” *Cem. Concr. Res.*, vol. 6, no. 3, pp. 367–376, 1976, doi: 10.1016/0008-8846(76)90099-5.
- [69] E. F. Irassar, “Sulfate attack on cementitious materials containing limestone filler — A review,” *Cem. Concr. Res.*, vol. 39, no. 3, pp. 241–254, Mar. 2009, doi: 10.1016/J.CEMCONRES.2008.11.007.
- [70] B. Craeye *et al.*, “Effect of mineral filler type on autogenous shrinkage of self-compacting concrete,” *Cem. Concr. Res.*, vol. 40, no. 6, pp. 908–913, Jun. 2010, doi: 10.1016/J.CEMCONRES.2010.01.014.
- [71] K. Vance, M. Aguayo, T. Oey, G. Sant, and N. Neithalath, “Hydration and strength development in ternary portland cement blends containing

- limestone and fly ash or metakaolin,” *Cem. Concr. Compos.*, vol. 39, pp. 93–103, May 2013, doi: 10.1016/J.CEMCONCOMP.2013.03.028.
- [72] D. P. Bentz *et al.*, “Multi-scale investigation of the performance of limestone in concrete,” *Constr. Build. Mater.*, vol. 75, pp. 1–10, Jan. 2015, doi: 10.1016/J.CONBUILDMAT.2014.10.042.
- [73] P. Thongsanitgarn, W. Wongkeo, A. Chaipanich, and C. S. Poon, “Heat of hydration of Portland high-calcium fly ash cement incorporating limestone powder: Effect of limestone particle size,” *Constr. Build. Mater.*, vol. 66, pp. 410–417, Sep. 2014, doi: 10.1016/J.CONBUILDMAT.2014.05.060.
- [74] R. S. Nader Ghafoori and Meysam Najimi, “Transport Properties of Limestone-Containing Self- Consolidating Concrete,” *ACI Mater. J.*, vol. 114, no. 4, doi: 10.14359/51689673.
- [75] M. S. Meddah, M. C. Lmbachiya, and R. K. Dhir, “Potential use of binary and composite limestone cements in concrete production,” *Constr. Build. Mater.*, vol. 58, pp. 193–205, Mar. 2009, doi: 10.1016/J.CONBUILDMAT.2013.12.012.
- [76] M. Schmidt, “Cement with interground additives. Capabilities and environmental relief, Part 1.,” *undefined*, 1992.
- [77] S. Sprung and E. Siebel, “Cement with interground additives - capabilities and environmental relief. part 1,” *Zem. Kalk. Gips*, vol. 45, no. 2, pp. 64–69, 1992.
- [78] “Sustainable Green Cement- EcoCem PLC® from Lehigh Hanson, Inc.” <https://www.lehighhanson.com/products/cement/EcoCemPLC> (accessed

Nov. 06, 2021).

- [79] “ASTM C305 - Standard Practice for Mechanical Mixing of Hydraulic Cement Pastes and Mortars of Plastic Consistency,” vol. 14, no. 2. ASTM International, West Conshohocken, PA, p. 147, 2020, doi: 10.1177/089033449801400218.
- [80] “ATM C1437 - Standard Test Method for Flow of Hydraulic Cement Mortar,” vol. 4, no. 1. ASTM International, West Conshohocken, PA, pp. 7–8, 2020, doi: 10.1520/C1437-20.2.
- [81] “ASTM C109 - Standard Test Method for Compressive Strength of Hydraulic Cement Mortars (Using 50mm Cube Specimens),” vol. 4, no. 1. ASTM International, West Conshohocken, PA, 2021, doi: 10.1520/C0109.
- [82] “ASTM C192 - Standard Practice for Making and Curing Concrete Test Specimens in the Laboratory.” ASTM International, West Conshohocken, PA, 2019, doi: 10.1520/C0192.
- [83] “ASTM C143 - Standard Test Method for Slump of Hydraulic-Cement Concrete,” vol. 04, no. 1. ASTM International, West Conshohocken, PA, pp. 1–4, 2020, doi: 10.1520/C0143.
- [84] “ASTM C138 - Standard Test Method for Density (Unit Weight), Yield, and Air Content (Gravimetric),” vol. 4. ASTM International, West Conshohocken, PA, pp. 23–26, 2017, doi: 10.1520/C0138.
- [85] “ASTM C231 - Standard Test Method for Air Content of Freshly Mixed Concrete by the Pressure Method.” ASTM International, West Conshohocken, PA, 2017, doi: 10.1520/mnl10913m.

- [86] “ASTM C39 - Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens,” vol. 4. ASTM International, West Conshohocken, PA, pp. 1–5, 2021, doi: 10.1520/C0039.
- [87] “ASTM C469 - Standard Test Method for Static Modulus of Elasticity and Poisson’s Ratio of Concrete in Compression,” vol. 04. ASTM International, West Conshohocken, PA, pp. 1–5, 2014, doi: 10.1520/C0469.
- [88] “ASTM C496 - Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens.” ASTM International, West Conshohocken, PA, 2017, doi: 10.1520/mnl10881m.
- [89] “ASTM C78 - Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading),” vol. 4. ASTM International, West Conshohocken, PA, pp. 1–4, 2022, doi: 10.1520/C0078.

Appendix: Sample Calculations of Precast, Prestressed Beams

Limestone Mixture

Flexure Beam Design

Beam Properties

$L_{bm} := 18.5 \text{ ft}$	Beam length @ transfer
$L := 18 \text{ ft}$	Span Length
$x := 0 \text{ ft}, 0.1 \text{ ft}..L$	Length of Beam as Function of x
$x_{ms} := 0.5 \cdot L = 9 \text{ ft}$	Midspan of Beam
$b := 10 \text{ in}$	Beam Width
$h := 16 \text{ in}$	Beam Height
$P_b := 2 \cdot b + 2 \cdot h = 52 \text{ in}$	Beam Perimeter
$A := b \cdot h = 160 \text{ in}^2$	Area of Cross-Section
$V := L_{bm} \cdot b \cdot h = 35520 \text{ in}^3$	Beam Volume
$SA := P_b \cdot L_{bm} = 11544 \text{ in}^2$	Surface Area of Beam
$I_g := b \cdot h^3 \div 12 = 3413.333 \text{ in}^4$	Gross Moment of Inertia
$y_t := h \div 2 = 8 \text{ in}$	Distance from Top of Beam to Neutral Axis
$y_b := h \div 2 = 8 \text{ in}$	Distance from Bottom of Beam to Neutral Axis
$S_t := I_g \div y_t = 426.667 \text{ in}^3$	Top Section Modulus
$S_b := I_g \div y_b = 426.667 \text{ in}^3$	Bottom Section Modulus

Concrete Properties

$f'_{ci} := 5700 \text{ psi}$	Compressive Strength at Release
$f'_c := 11090 \text{ psi}$	Compressive Strength at 28 Days
$w_c := 150 \text{ pcf}$	Unit Weight
$E_{ci} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_{ci} \cdot \text{psi}} = 4577 \text{ ksi}$	Initial Modulus of Elasticity of Concrete
$E_{epredicted} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_c \cdot \text{psi}} = 6384 \text{ ksi}$	Predicted Modulus of Elasticity of Concrete
$E_c := 5582 \text{ ksi}$	Observed Modulus of Elasticity of Concrete
$f_{rpredicted} := 7.5 \cdot \sqrt{f'_c \cdot \text{psi}} = 790 \text{ psi}$	Predicted Modulus of Rupture
$f_r := 792 \text{ psi}$	Observed Modulus of Rupture

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$$\beta_1 := \begin{cases} \text{if } 2500 \text{ psi} \leq f'_c \leq 4000 \text{ psi} & = 0.65 \\ \text{else if } 4000 \text{ psi} < f'_c \leq 8000 \text{ psi} & \left\| \begin{array}{l} 0.85 \\ 0.85 - \frac{0.05 \cdot (f'_c - 4000 \text{ psi})}{1000 \text{ psi}} \end{array} \right. \\ \text{else} & \left\| 0.65 \end{cases}$$

Stress Block Coefficient

Prestress Steel Properties

$$f_{pu} := 270 \text{ ksi}$$

Tensile Strength of Prestress

$$d_{bps} := 0.5 \text{ in}$$

Diameter of Prestress Steel

$$A_{ps12} := 0.153 \text{ in}^2$$

Area of Prestress Steel

$$E_{ps} := 28500 \text{ ksi}$$

Modulus of Elasticity of Prestress Steel

$$n_{ps} := 3$$

Number of Prestress Strands

$$A_{ps} := n_{ps} \cdot A_{ps12} = 0.459 \text{ in}^2$$

Total Area of Prestress Strands

$$d_p := h - 2 \text{ in} = 14 \text{ in}$$

Depth to Prestress Strands

$$e := d_p - y_t = 6 \text{ in}$$

Eccentricity of Prestress Steel

$$\rho_p := \frac{A_{ps}}{b \cdot d_p} = 0.003$$

Prestress Reinforcement Ratio

Compression Steel Properties

$$d_r := 0.625 \text{ in}$$

Diameter of Compression Steel

$$A_{r12} := 0.31 \text{ in}^2$$

Area of Compression Steel

$$n_r := 2$$

Number of Compressive Bars

$$A'_s := n_r \cdot A_{r12} = 0.62 \text{ in}^2$$

Total Area of Compressive Steel

$$d' := 2 \text{ in}$$

Depth to Compressive Steel

$$\rho'_p := \frac{A'_s}{b \cdot d'} = 0.031$$

Rebar Reinforcement Ratio

$$f_y := 60 \text{ ksi}$$

Yield Strength of Compressive Steel

$$E_s := 29000 \text{ ksi}$$

Modulus of Elasticity of Compressive Steel

Loads

$$w_{sw} := w_c \cdot A = 166.667 \text{ plf}$$

Distributed Self Weight of Beam

$$M_{sw}(x) := \frac{w_{sw} \cdot x}{2} \cdot (L - x)$$

Moment from Self Weight

$$M_{sw}(x_{ms}) = 6.75 \text{ kip} \cdot \text{ft}$$

Moment from Self Weight at Midspan

$$P := 20 \text{ kip}$$

Applied Point Loads, half of actuator load

$$2 \cdot P = 40 \text{ kip}$$

Total load applied by actuator

$$s_{point} := 46 \text{ in}$$

Distance Between Point Loads

$$aa := x_{ms} - \frac{s_{point}}{2} = 7.083 \text{ ft}$$

Distance of Point Loads to End of Beam

$$M_{LL}(x) := \begin{cases} P \cdot x & \text{if } x < aa \\ P \cdot aa & \text{else if } aa \leq x \leq L - aa \\ P \cdot (L - x) & \text{else if } L - aa < x \end{cases}$$

Moment from Applied Loads

$$M_u(x) := M_{sw}(x) + 0.9 \cdot M_{LL}(x)$$

Maximum Moment at Midspan

$$M_{sw}(x_{ms}) = 6.75 \text{ kip} \cdot \text{ft}$$

$$M_{LL}(x_{ms}) = 141.667 \text{ kip} \cdot \text{ft}$$

$$M_u(x_{ms}) = 134.3 \text{ kip} \cdot \text{ft}$$

Prestress Losses

Elastic Shortening

$$K_{cir} := 0.9$$

Prestress Coefficient

$$F_j := 0.7$$

Jacking Stress

$$P_j := F_j \cdot f_{pu} \cdot A_{ps} = 86.751 \text{ kip}$$

Initial Prestress Force

$$f_{cir} := K_{cir} \cdot \left(\frac{P_j}{A} + \frac{P_j \cdot e^2}{I_g} \right) - \frac{M_{sw}(x_{ms}) \cdot e}{I_g} = 1.169 \text{ ksi}$$

Net Compressive Strength

$$\Delta f_{es} := \frac{E_{ps}}{E_{ci}} \cdot f_{cir} = 7.279 \text{ ksi}$$

Loss from Elastic Shortening

$$f_{pi} := F_j \cdot f_{pu} - \Delta f_{es} = 181.721 \text{ ksi}$$

Initial Prestress Force After Short-Term Losses

$$P_i := f_{pi} \cdot A_{ps} = 83.41 \text{ kip}$$

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Creep

$$f_{cds} := 0$$

Stress in Concrete from Dead Load
(no additional DL applied)

$$K_{cr} := 2$$

Prestress Coefficient

$$\Delta f_{cr} := K_{cr} \cdot \frac{E_{ps}}{E_c} \cdot (f_{cir} - f_{cds}) = 11.938 \text{ ksi}$$

Loss from Creep

Shrinkage

$$RH := 72.5$$

Relative Humidity

$$K_{sh} := 1.0$$

Prestress Coefficient

$$VS := \frac{A}{P_b} = 3.077 \text{ in}$$

Volume to Surface Area Ratio

$$\Delta f_{sh} := 8.2 \cdot 10^{-6} \cdot K_{sh} \cdot E_{ps} \cdot \left(1 - (0.06) \cdot \frac{VS}{\text{in}}\right) \cdot (100 - RH) = 5.24 \text{ ksi} \quad \text{Loss from Shrinkage}$$

Relaxation

$$K_{re} := 5000 \text{ psi}$$

Prestress Coefficient

$$J := 0.04$$

Prestress Coefficient

$$R := \frac{f_{pi}}{f_{pu}} = 0.673$$

Ratio Needed For "C" Equation

$$C := \frac{R}{0.21} \left(\frac{R}{0.9} - 0.55 \right) = 0.634$$

Prestress Coefficient

$$\Delta f_{re} := (K_{re} - J \cdot (\Delta f_{sh} + \Delta f_{cr} + \Delta f_{es})) \cdot C = 2.55 \text{ ksi} \quad \text{Loss from Relaxation}$$

Total Long Term Losses

$$\Delta f_{total} := \Delta f_{re} + \Delta f_{sh} + \Delta f_{cr} + \Delta f_{es} = 27.007 \text{ ksi} \quad \text{Total Losses}$$

$$f_{se} := F_j \cdot f_{pu} - \Delta f_{total} = 161.993 \text{ ksi} \quad \text{Prestress Force After Losses}$$

$$P_e := f_{se} \cdot A_{ps} = 74.355 \text{ kip}$$

Transfer and Development Length

$$\gamma_p := 0.28$$

Prestress factor

$$f_{ps} := f_{pu} \cdot \left(1 - \frac{\gamma_p}{\beta_1} \cdot \left(\rho_p \cdot \frac{f_{pu}}{f'_c}\right)\right) = 260.716 \text{ ksi}$$

Prestress Force at Nominal Flexural Strength

$$l_t := \frac{f_{se}}{3000 \text{ psi}} \cdot d_{bps} = 2.25 \text{ ft}$$

Transfer Length

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$$l_d := l_t + \frac{(f_{ps} - f_{se})}{1000 \text{ psi}} \cdot d_{bps} = 6.363 \text{ ft}$$

Development Length

Flexural Capacity

$$a := \frac{A_{ps} \cdot f_{ps}}{0.85 \cdot f'_c \cdot b} = 1.269 \text{ in}$$

Depth of Stress Block

$$c := \frac{a}{\beta_1} = 1.953 \text{ in}$$

$$M_n := A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2} \right) = 133.284 \text{ kip} \cdot \text{ft}$$

Nominal Moment Capacity

$$M_n(x) := \text{if } x < l_t$$

$$\left\| \frac{x}{l_t} \cdot \frac{f_{se}}{f_{ps}} \cdot M_n \right.$$

else if $l_t \leq x \leq l_d$

$$\left\| \left(\frac{x - l_t}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) + \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right.$$

else if $l_d < x \leq L - l_d$

$$\left\| M_n \right.$$

else if $L - l_d < x \leq L - l_t$

$$\left\| \left(1 - \frac{x - L + l_d}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) \right) \cdot M_n \right.$$

else if $L - l_t < x \leq L$

$$\left\| \left(\frac{f_{se}}{f_{ps}} - \frac{x - L + l_t}{l_t} \cdot \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right.$$

Moment Capacity as a Function of x

$$M_{cr} := S_b \cdot \left(\frac{P_e}{A} + \frac{P_e \cdot e}{S_b} + f_r \right) = 81.861 \text{ kip} \cdot \text{ft}$$

Cracking Moment

$$1.2 \cdot M_{cr} = 98.233 \text{ kip} \cdot \text{ft}$$

Cracking Moment

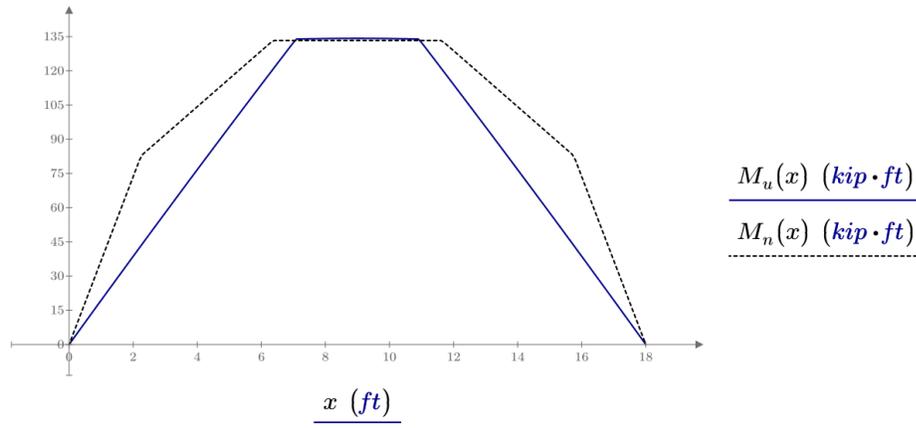
$$\text{if } M_n(x_{ms}) \geq 1.2 \cdot M_{cr} \mid = \text{"Okay"}$$

$\left\| \text{"Okay"}$

else

$\left\| \text{"Not Okay"}$

Check Cracking Moment is less than Moment Capacity



$$M_u(x_{ms}) = 134.25 \text{ kip} \cdot \text{ft}$$

Design Moment at Midspan

$$M_n(x_{ms}) = 133.284 \text{ kip} \cdot \text{ft}$$

Moment Capacity at Midspan

$$P_i = 83.41 \text{ kip}$$

Initial Prestress Force

$$P_i(x) := \text{if } x \leq l_t$$

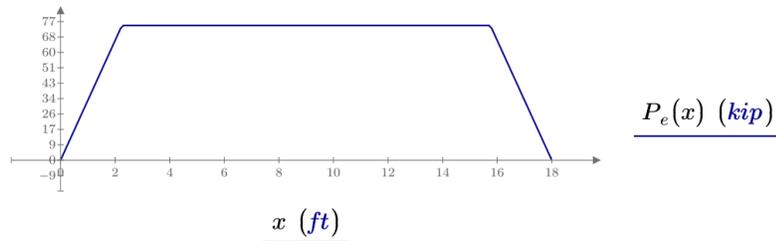
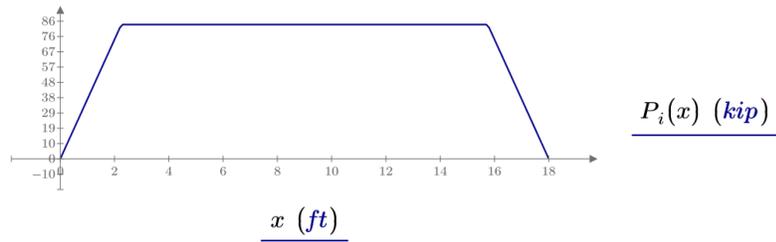
Initial Prestress Force as Function of x

$$\left\| \begin{array}{l} \frac{x}{l_t} \cdot P_i \\ \text{else if } l_t < x \leq L - l_t \\ P_i \\ \text{else if } x > L - l_t \\ P_i - \frac{P_i}{l_t} \cdot (x - (L - l_t)) \end{array} \right\|$$

$$P_e(x) := \text{if } x \leq l_t$$

Effective Prestress Force as Function of x

$$\left\| \begin{array}{l} \frac{x}{l_t} \cdot P_e \\ \text{else if } l_t < x \leq L - l_t \\ P_e \\ \text{else if } x > L - l_t \\ P_e - \frac{P_e}{l_t} \cdot (x - (L - l_t)) \end{array} \right\|$$



Transfer Stresses

$$f_{ttm}(x) := \frac{P_i(x)}{A} - \frac{P_i(x) \cdot e}{S_t} + \frac{M_{sw}(x)}{S_t} \quad \text{Transfer Stresses at Top of Beam}$$

$$f_{btm}(x) := \frac{P_i(x)}{A} + \frac{P_i(x) \cdot e}{S_b} - \frac{M_{sw}(x)}{S_b} \quad \text{Transfer Stresses at Bottom of Beam}$$

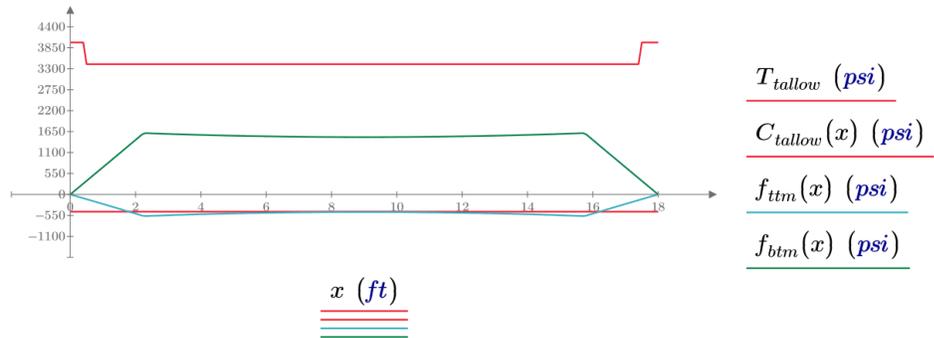
Limits

$$C_{tallow_end} := 0.7 \cdot f'_{ci} = 3990 \text{ psi} \quad \text{Compression Limits at Transfer (ACI Table 24.5.3.1)}$$

$$C_{tallow_ms} := 0.6 \cdot f'_{ci} = 3420 \text{ psi} \quad \text{Compression Limits at Transfer (ACI Table 24.5.3.1)}$$

$$T_{tallow} := -6 \cdot \sqrt{f'_{ci}} \cdot \text{psi} = -453 \text{ psi} \quad \text{Tension Limits at Transfer (PCI)}$$

$$C_{tallow}(x) := \begin{cases} \text{if } x < 0.5 \text{ ft} & C_{tallow_end} \\ \text{else if } x < L - 0.5 \text{ ft} & C_{tallow_ms} \\ \text{else} & C_{tallow_end} \end{cases} \quad \text{Compression Limits Along the Beam}$$



Tension Check

$$T := \frac{(f_{ttm}(x_{ms}) - T_{allow}) \cdot b \cdot c}{2} = -0.086 \text{ kip}$$

Tensile Force

$$f_s := 0.5 \cdot f_y = 30000 \text{ psi}$$

Permitted Stress in the Reinforcement

$$\left| \frac{T}{f_s} \right| = 0.003 \text{ in}^2$$

Required Area of Reinforcement

$$\left. \begin{array}{l} \text{if } A'_s \geq \frac{T}{f_s} \\ \quad \parallel \text{ "Okay"} \\ \text{else} \\ \quad \parallel \text{ "Not Okay"} \end{array} \right| = \text{"Okay"}$$

Service Stresses

$$f_{tsm}(x) := \frac{P_e(x)}{A} - \frac{P_e(x) \cdot e}{S_t} + \frac{M_u(x)}{S_t}$$

Service Stresses at Top of Beam

$$f_{bsm}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b} - \frac{M_u(x)}{S_b}$$

Service Stresses at Bottom of Beam

Limits

$$T_{sallow} := -7.5 \cdot \sqrt{f'_c \cdot \text{psi}} = -789.818 \text{ psi}$$

Tension Limits

$$C_{sallow} := 0.6 \cdot f'_c = 6654 \text{ psi}$$

Compression Limits

Shear Capacity and Demand

$$V_{sw}(x) := w_{sw} \cdot \left(\frac{L}{2} - x \right)$$

Shear from Self Weight

$$V_{LL}(x) := \begin{cases} P & \text{if } x \leq aa \\ 0 \text{ kip} & \text{else if } aa < x < L - aa \\ -P & \text{else if } L - aa \leq x \end{cases}$$

Shear from Applied Loads

$$V_u(x) := V_{sw}(x) + 1.2 \cdot V_{LL}(x)$$

Ultimate Shear

$$d_p := \max(d_p, 0.8 \cdot h) = 14 \text{ in}$$

Depth to Prestress

$$f_{pce}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b}$$

$$V_d(x) := V_{sw}(x)$$

Shear from Dead Loads

$$M_d(x) := M_{sw}(x)$$

Moment from Dead Loads

$$V_i(x) := V_u(x) - V_d(x)$$

$$M_{max}(x) := M_u(x) - M_d(x)$$

$$f_d(x) := \frac{M_d(x)}{S_b}$$

$$M_{cre}(x) := S_b \cdot (6 \cdot \sqrt{f'_c \cdot psi} + f_{pce}(x) - f_d(x))$$

$$V_{ci}(x) := \max\left(0.6 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p + |V_d(x)| + \frac{|V_i(x)| \cdot M_{cre}(x)}{M_{max}(x)}, 1.7 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p\right)$$

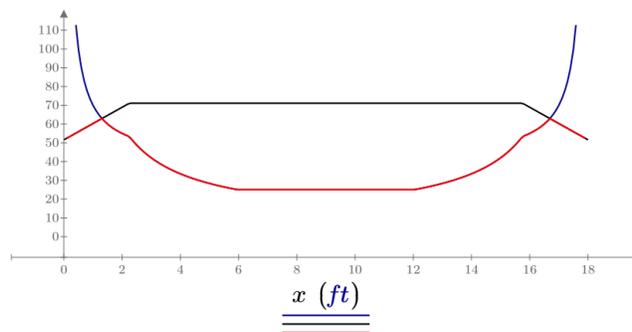
$$V_p := 0$$

$$f_{pc}(x) := \frac{P_e(x)}{A}$$

$$V_{cw}(x) := (3.5 \cdot \sqrt{f'_c \cdot psi} + 0.3 \cdot f_{pc}(x)) \cdot b \cdot d_p + V_p$$

$$V_{cw}(x_{ms}) = 71.1 \text{ kip}$$

$$V_c(x) := \min(V_{ci}(x), V_{cw}(x))$$



$$\underline{\underline{V_{ci}(x) \text{ (kip)}}}$$

$$\underline{\underline{V_{cw}(x) \text{ (kip)}}}$$

$$\underline{\underline{V_c(x) \text{ (kip)}}}$$

x (ft)

Non-Commercial Use Only

Stirrups

$$\phi_v := 0.75$$

$$V_s(x) := V_u(x) - V_c(x)$$

$$V_s(aa) = -0.744 \text{ kip}$$

$$A_v := 2 \cdot 0.20 \text{ in}^2 = 0.4 \text{ in}^2$$

Try double leg stirrups

$$s_{max} := \frac{A_v \cdot f_y \cdot d_p}{V_s(x_{ms})} = -13.406 \text{ in}$$

Maximum spacing allowed

$$s_{max} := \min(0.75 \cdot h, 24 \text{ in}) = 12 \text{ in}$$

$$s := s_{max}$$

$$A_{vmin} := \min \left(\max \left(0.75 \sqrt{f'_c \cdot \text{psi}} \cdot \frac{b \cdot s}{f_y}, 50 \text{ psi} \cdot \frac{b \cdot s}{f_y} \right), \frac{A_{ps} \cdot f_{pu} \cdot s}{80 \cdot f_y \cdot d_p} \cdot \sqrt{\frac{d_p}{b}} \right) = 0.026 \text{ in}^2$$

$$\begin{array}{l} \text{if } A_v \geq A_{vmin} \\ \quad \text{"Okay"} \\ \text{else} \\ \quad \text{"Not Okay"} \end{array} \quad \Bigg| = \text{"Okay"}$$

$$n_s := \frac{L}{s} + 1 = 19$$

Number of Stirrups Needed

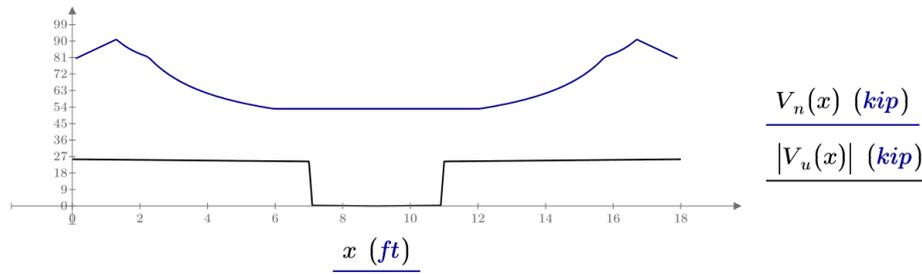
$$V_s := \frac{A_v \cdot f_y \cdot d_p}{s} = 28 \text{ kip}$$

Shear Strength of Stirrups

$$4 \cdot \sqrt{f'_c \cdot \text{psi}} \cdot b \cdot d_p = 58.973 \text{ kip}$$

$$V_n(x) := V_c(x) + V_s$$

Nominal Shear Strength



Non-Commercial Use Only

Bond Strength Beam Design

Beam Properties

$L_{bm} := 18.5 \text{ ft}$	Beam length
$L := 18 \text{ ft}$	Span Length
$x := 0 \text{ ft}, 0.1 \text{ ft}..L$	Length of Beam as Function of x
$x_{ms} := 0.5 \cdot L = 9 \text{ ft}$	Midspan of Beam
$b := 10 \text{ in}$	Beam Width
$h := 16 \text{ in}$	Beam Height
$P_b := 2 \cdot b + 2 \cdot h = 52 \text{ in}$	Beam Perimeter
$A := b \cdot h = 160 \text{ in}^2$	Area of Cross-Section
$V := L_{bm} \cdot b \cdot h = 35520 \text{ in}^3$	Beam Volume
$SA := P_b \cdot L_{bm} = 11544 \text{ in}^2$	Surface Area of Beam
$I_g := b \cdot h^3 \div 12 = 3413.333 \text{ in}^4$	Gross Moment of Inertia
$y_t := h \div 2 = 8 \text{ in}$	Distance from Top of Beam to Neutral Axis
$y_b := h \div 2 = 8 \text{ in}$	Distance from Bottom of Beam to Neutral Axis
$S_t := I_g \div y_t = 426.667 \text{ in}^3$	Top Section Modulus
$S_b := I_g \div y_b = 426.667 \text{ in}^3$	Bottom Section Modulus

Concrete Properties

$f'_{ci} := 5690 \text{ psi}$	Compressive Strength at Release
$f'_c := 11090 \text{ psi}$	Compressive Strength at 28 Days
$w_c := 150 \text{ pcf}$	Unit Weight
$E_{ci} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_{ci} \cdot \text{psi}} = 4573 \text{ ksi}$	Initial Modulus of Elasticity of Concrete
$E_{c\text{predicted}} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_c \cdot \text{psi}} = 6384 \text{ ksi}$	Predicted Modulus of Elasticity of Concrete
$E_c := 5582 \text{ ksi}$	Observed Modulus of Elasticity of Concrete
$f_{r\text{predicted}} := 7.5 \cdot \sqrt{f'_c \cdot \text{psi}} = 790 \text{ psi}$	Predicted Modulus of Rupture
$f_r := 792 \text{ psi}$	Observed Modulus of Rupture

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$$\beta_1 := \begin{cases} 0.85 & \text{if } 2500 \text{ psi} \leq f'_c \leq 4000 \text{ psi} \\ 0.85 - \frac{0.05 \cdot (f'_c - 4000 \text{ psi})}{1000 \text{ psi}} & \text{else if } 4000 \text{ psi} < f'_c < 8000 \text{ psi} \\ 0.65 & \text{else if } f'_c > 8000 \text{ psi} \end{cases} = 0.65$$

Stress Block Coefficient

Prestress Steel Properties

$$f_{pu} := 270 \text{ ksi}$$

Tensile Strength of Prestress

$$d_{bps} := 0.5 \text{ in}$$

Diameter of Prestress Steel

$$A_{ps12} := 0.153 \text{ in}^2$$

Area of Prestress Steel

$$E_{ps} := 28500 \text{ ksi}$$

Modulus of Elasticity of Prestress Steel

$$n_{ps} := 3$$

Number of Prestress Strands

$$A_{ps} := n_{ps} \cdot A_{ps12} = 0.459 \text{ in}^2$$

Total Area of Prestress Strands

$$d_p := h - 2 \text{ in} = 14 \text{ in}$$

Depth to Prestress Strands

$$e := d_p - y_t = 6 \text{ in}$$

Eccentricity of Prestress Steel

$$\rho_p := \frac{A_{ps}}{b \cdot d_p} = 0.003$$

Prestress Reinforcement Ratio

Compression Steel Properties

$$d_r := 0.625 \text{ in}$$

Diameter of Compression Steel

$$A_{r12} := 0.31 \text{ in}^2$$

Area of Compression Steel

$$n_r := 2$$

Number of Compressive Bars

$$A'_s := n_r \cdot A_{r12} = 0.62 \text{ in}^2$$

Total Area of Compressive Steel

$$d' := 2 \text{ in}$$

Depth to Compressive Steel

$$\rho'_p := \frac{A'_s}{b \cdot d'} = 0.031$$

Rebar Reinforcement Ratio

$$f_y := 60 \text{ ksi}$$

Yield Strength of Compressive Steel

$$E_s := 29000 \text{ ksi}$$

Modulus of Elasticity of Compressive Steel

Loads

$$w_{sw} := w_c \cdot A = 166.667 \text{ plf}$$

Distributed Self Weight of Beam

$$M_{sw}(x) := \frac{w_{sw} \cdot x}{2} \cdot (L - x)$$

Moment from Self Weight

$$M_{sw}(x_{ms}) = 6.75 \text{ kip} \cdot \text{ft}$$

Moment from Self Weight at Midspan

$$P := 35 \text{ kip}$$

Applied Point Load (1)

$$aa := 5.67 \text{ ft}$$

Distance of Point Loads to End of Beam

$$bb := L - aa = 12.33 \text{ ft}$$

$$aa \div d_p = 4.86$$

$$aa \div h = 4.253$$

$\text{if } \frac{aa}{d_p} \geq 2.5$	= "Okay"	$\text{if } \frac{aa}{h} \geq 2.5$	= "Okay"
$\parallel \text{"Okay"}$		$\parallel \text{"Okay"}$	
else		else	
$\parallel \text{"Not Okay"}$		$\parallel \text{"Not Okay"}$	

$$R_1 := P \cdot bb \div L = 23.975 \text{ kip}$$

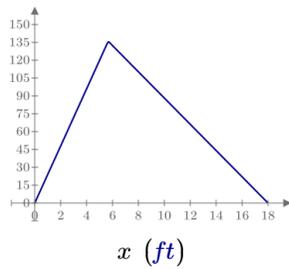
$$R_2 := P \cdot aa \div L = 11.025 \text{ kip}$$

$$R_1 + R_2 = 35 \text{ kip}$$

$$P = 35 \text{ kip}$$

$$M_{LL}(x) := \text{if } x < aa \left\{ \begin{array}{l} R_1 \cdot x \\ \text{else} \\ R_1 \cdot x - P \cdot (x - aa) \end{array} \right.$$

Moment from Applied Loads



$M_{LL}(x)$ (kip · ft)

$$M_u(x) := M_{sw}(x) + 0.9 \cdot M_{LL}(x)$$

Maximum Moment at Midspan

$$M_u(aa) = 128.17 \text{ kip} \cdot \text{ft}$$

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Prestress Losses

Elastic Shortening

$K_{cir} := 0.9$	Prestress Coefficient
$F_j := 0.7$	Jacking Stress
$P_i := F_j \cdot f_{pu} \cdot A_{ps} = 86.751 \text{ kip}$	Initial Prestress Force
$f_{cir} := K_{cir} \cdot \left(\frac{P_i}{A} + \frac{P_i \cdot e^2}{I_g} \right) - \frac{M_{sw}(x_{ms}) \cdot e}{I_g} = 1.169 \text{ ksi}$	Net Compressive Strength
$\Delta f_{es} := \frac{E_{ps}}{E_{ci}} \cdot f_{cir} = 7.286 \text{ ksi}$	Loss from Elastic Shortening
$f_{pi} := F_j \cdot f_{pu} - \Delta f_{es} = 181.714 \text{ ksi}$	Initial Prestress Force After Short-Term Losses
$P_i := f_{pi} \cdot A_{ps} = 83.407 \text{ kip}$	

Creep

$f_{cds} := 0$	Stress in Concrete from Dead Load (no additional DL applied)
$K_{cr} := 2$	Prestress Coefficient
$\Delta f_{cr} := K_{cr} \cdot \frac{E_{ps}}{E_c} \cdot (f_{cir} - f_{cds}) = 11.938 \text{ ksi}$	Loss from Creep

Shrinkage

$RH := 72.5$	Relative Humidity
$K_{sh} := 1.0$	Prestress Coefficient
$VS := \frac{A}{P_b} = 3.077 \text{ in}$	Volume to Surface Area Ratio
$\Delta f_{sh} := 8.2 \cdot 10^{-6} \cdot K_{sh} \cdot E_{ps} \cdot \left(1 - (0.06) \cdot \frac{VS}{\text{in}} \right) \cdot (100 - RH) = 5.24 \text{ ksi}$	Loss from Shrinkage

Relaxation

$K_{re} := 5000 \text{ psi}$	Prestress Coefficient
$J := 0.04$	Prestress Coefficient
$R := \frac{f_{pi}}{f_{pu}} = 0.673$	Ratio Needed For "C" Equation
$C := \frac{R}{0.21} \left(\frac{R}{0.9} - 0.55 \right) = 0.634$	Prestress Coefficient

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$$\Delta f_{re} := (K_{re} - J \cdot (\Delta f_{sh} + \Delta f_{cr} + \Delta f_{es})) \cdot C = 2.549 \text{ ksi} \quad \text{Loss from Relaxation}$$

Total Long Term Losses

$$\Delta f_{total} := \Delta f_{re} + \Delta f_{sh} + \Delta f_{cr} + \Delta f_{es} = 27.013 \text{ ksi} \quad \text{Total Losses}$$

$$f_{se} := F_j \cdot f_{pu} - \Delta f_{total} = 161.987 \text{ ksi} \quad \text{Prestress Force After Losses}$$

$$P_e := f_{se} \cdot A_{ps} = 74.352 \text{ kip} \quad \text{Effective Prestress Force}$$

Transfer and Development Length

$$\gamma_p := 0.28 \quad \text{Prestress factor}$$

$$f_{ps} := f_{pu} \cdot \left(1 - \frac{\gamma_p}{\beta_1} \cdot \left(\rho_p \cdot \frac{f_{pu}}{f'_c} \right) \right) = 260.716 \text{ ksi} \quad \text{Prestress Force at Nominal Flexural Strength}$$

$$l_t := \frac{f_{se}}{3000 \text{ psi}} \cdot d_{bps} = 2.25 \text{ ft} \quad \text{Transfer Length}$$

$$l_d := l_t + \frac{(f_{ps} - f_{se})}{1000 \text{ psi}} \cdot d_{bps} = 6.364 \text{ ft} \quad \text{Development Length}$$

Flexural Capacity

$$a := \frac{A_{ps} \cdot f_{ps}}{0.85 \cdot f'_c \cdot b} = 1.269 \text{ in} \quad \text{Depth of Stress Block}$$

$$c := \frac{a}{\beta_1} = 1.953 \text{ in}$$

$$M_n := A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2} \right) = 133.284 \text{ kip} \cdot \text{ft} \quad \text{Nominal Moment Capacity}$$

$$M_n(x) := \text{if } x < l_t \quad \text{Moment Capacity as a Function of } x$$

$$\left\| \frac{x}{l_t} \cdot \frac{f_{se}}{f_{ps}} \cdot M_n \right.$$

else if $l_t \leq x \leq l_d$

$$\left\| \left(\frac{x - l_t}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) + \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right.$$

else if $l_d < x \leq L - l_d$

$$\left\| M_n \right.$$

else if $L - l_d < x \leq L - l_t$

$$\left\| \left(1 - \frac{x - L + l_d}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) \right) \cdot M_n \right.$$

else if $L - l_t < x \leq L$

$$\left\| \left(\frac{f_{se}}{f_{ps}} - \frac{x - L + l_t}{l_t} \cdot \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right.$$

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$$P_e = 74.352 \text{ kip}$$

Effective Prestress Force

$$M_{cr} := S_b \cdot \left(\frac{P_e}{A} + \frac{P_e \cdot e}{S_b} + f_r \right) = 81.859 \text{ kip} \cdot \text{ft}$$

Cracking Moment

$$1.2 \cdot M_{cr} = 98.231 \text{ kip} \cdot \text{ft}$$

Factored Cracking Moment

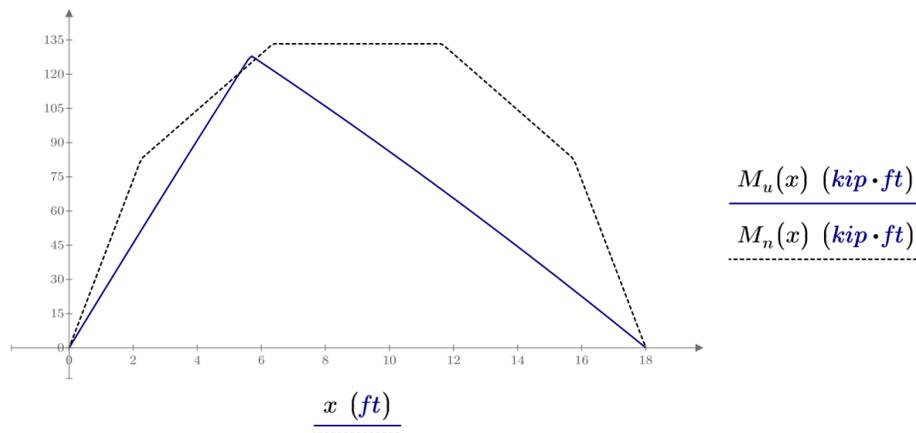
if $M_n(x_{ms}) \geq 1.2 \cdot M_{cr}$ = "Okay"

Check Cracking Moment is less than Factored Moment Capacity

|| "Okay"

else

|| "Not Okay"



$$M_u(aa) = 128.17 \text{ kip} \cdot \text{ft}$$

Design Moment at Critical Section

$$M_n(aa) = 124.774 \text{ kip} \cdot \text{ft}$$

Moment Capacity at Critical Section

$$P_i = 83.407 \text{ kip}$$

Initial Prestress Force

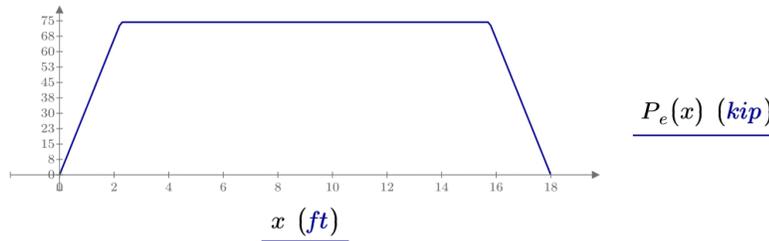
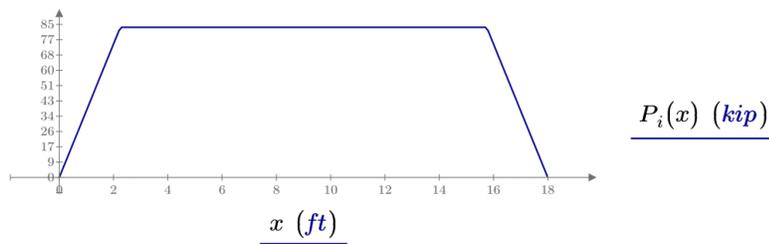
$$P_i(x) := \text{if } x \leq l_t$$

Initial Prestress Force as Function of x

$$\left\| \begin{array}{l} \frac{x}{l_t} \cdot P_i \\ \text{else if } l_t < x \leq L - l_t \\ P_i \\ \text{else if } x > L - l_t \\ P_i - \frac{P_i}{l_t} \cdot (x - (L - l_t)) \end{array} \right\|$$

$$P_e(x) := \begin{cases} \text{if } x \leq l_t \\ \left\| \frac{x}{l_t} \cdot P_e \right. \\ \text{else if } l_t < x \leq L - l_t \\ \left\| P_e \right. \\ \text{else if } x > L - l_t \\ \left\| P_e - \frac{P_e}{l_t} \cdot (x - (L - l_t)) \right. \end{cases}$$

Effective Prestress Force as Function of x



Transfer Stresses

$$f_{ttm}(x) := \frac{P_i(x)}{A} - \frac{P_i(x) \cdot e}{S_t} + \frac{M_{sw}(x)}{S_t}$$

Transfer Stresses at Top of Beam

$$f_{btm}(x) := \frac{P_i(x)}{A} + \frac{P_i(x) \cdot e}{S_b} - \frac{M_{sw}(x)}{S_b}$$

Transfer Stresses at Bottom of Beam

Limits

$$C_{tallow_end} := 0.7 \cdot f'_{ci} = 3983 \text{ psi}$$

Compression Limits at Transfer (ACI Table 24.5.3.1)

$$C_{tallow_ms} := 0.6 \cdot f'_{ci} = 3414 \text{ psi}$$

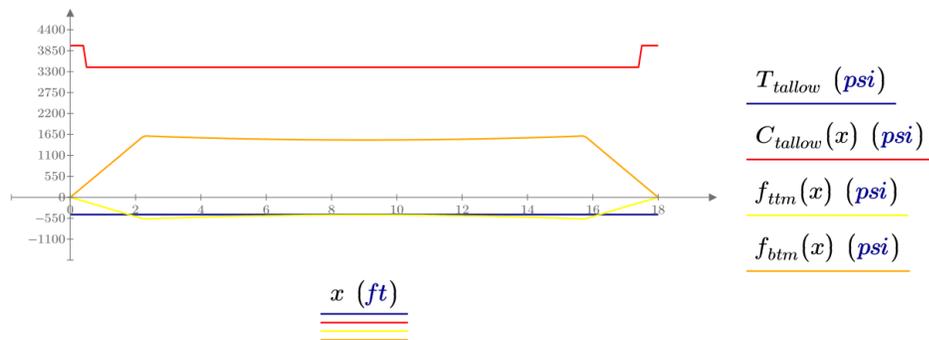
Compression Limits at Transfer (ACI Table 24.5.3.1)

$$T_{tallow} := -6 \cdot \sqrt{f'_{ci} \cdot \text{psi}} = -453 \text{ psi}$$

Tension Limits at Transfer (PCI)

$$C_{tallow}(x) := \begin{cases} \text{if } x < 0.5 \text{ ft} \\ \parallel C_{tallow_end} \\ \text{else if } x < L - 0.5 \text{ ft} \\ \parallel C_{tallow_ms} \\ \text{else} \\ \parallel C_{tallow_end} \end{cases}$$

Compression Limits Along the Beam



Tension Check

$$T := \frac{(f_{ttm}(x_{ms}) - T_{tallow}) \cdot b \cdot c}{2} = -0.09 \text{ kip}$$

Tensile Force

$$f_s := 0.5 \cdot f_y = 30000 \text{ psi}$$

Permitted Stress in the Reinforcement

$$\left| \frac{T}{f_s} \right| = 0.003 \text{ in}^2$$

Required Area of Reinforcement

$$\begin{cases} \text{if } A'_s \geq \frac{T}{f_s} \\ \parallel \text{"Okay"} \\ \text{else} \\ \parallel \text{"Not Okay"} \end{cases} = \text{"Okay"}$$

Service Stresses

$$f_{tsm}(x) := \frac{P_e(x)}{A} - \frac{P_e(x) \cdot e}{S_t} + \frac{M_u(x)}{S_t}$$

Service Stresses at Top of Beam

$$f_{bsm}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b} - \frac{M_u(x)}{S_b}$$

Service Stresses at Bottom of Beam

Limits

$$T_{sallow} := -7.5 \cdot \sqrt{f'_c} \cdot \text{psi} = -789.818 \text{ psi}$$

Tension Limits

$$C_{sallow} := 0.6 \cdot f'_c = 6654 \text{ psi}$$

Compression Limits

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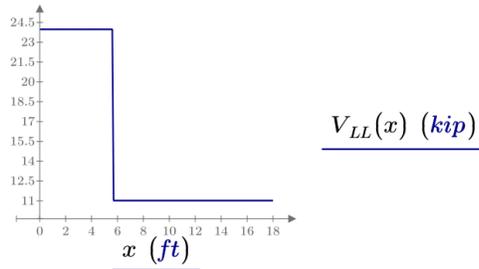
Shear Capacity and Demand

$$V_{sw}(x) := w_{sw} \cdot \left(\frac{L}{2} - x \right)$$

Shear from Self Weight

$$V_{LL}(x) := \begin{cases} \text{if } x < aa \\ \left\| P \cdot bb \div L \right\| \\ \text{else} \\ \left\| P \cdot aa \div L \right\| \end{cases}$$

Shear from Applied Loads



$$V_u(x) := V_{sw}(x) + 1.2 \cdot V_{LL}(x)$$

Ultimate Shear

$$d_p := \max(d_p, 0.8 \cdot h) = 14 \text{ in}$$

Depth to Prestress

$$f_{pce}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b}$$

$$V_d(x) := V_{sw}(x)$$

Shear from Dead Loads

$$M_d(x) := M_{sw}(x)$$

Moment from Dead Loads

$$V_i(x) := V_u(x) - V_d(x)$$

$$M_{max}(x) := M_u(x) - M_d(x)$$

$$f_d(x) := \frac{M_d(x)}{S_b}$$

$$M_{cre}(x) := S_b \cdot \left(6 \cdot \sqrt{f'_c \cdot psi} + f_{pce}(x) - f_d(x) \right)$$

$$V_{ci}(x) := \max \left(0.6 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p + |V_d(x)| + \frac{|V_i(x)| \cdot M_{cre}(x)}{M_{max}(x)}, 1.7 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p \right)$$

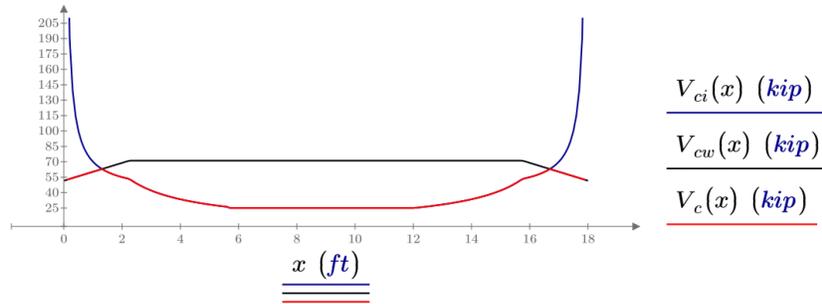
$$V_p := 0$$

$$f_{pc}(x) := \frac{P_e(x)}{A}$$

$$V_{cw}(x) := \left(3.5 \cdot \sqrt{f'_c \cdot psi} + 0.3 \cdot f_{pc}(x) \right) \cdot b \cdot d_p + V_p$$

$$V_c(x) := \min(V_{ci}(x), V_{cw}(x))$$

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Stirrups

$$s_{max} := \min(0.75 \cdot h, 24 \text{ in}) = 12 \text{ in}$$

$$s := s_{max}$$

$$A_{vmin} := \min \left(\max \left(0.75 \sqrt{f'_c} \cdot \text{psi} \cdot \frac{b \cdot s}{f_y}, 50 \text{ psi} \cdot \frac{b \cdot s}{f_y} \right), \frac{A_{ps} \cdot f_{pu} \cdot s}{80 \cdot f_y \cdot d_p} \cdot \sqrt{\frac{d_p}{b}} \right) = 0.026 \text{ in}^2$$

if $A_v \geq A_{vmin}$	= "Okay"	$A_v = 0.4 \text{ in}^2$
"Okay"		
else		
"Not Okay"		

$$n_s := \frac{L}{s} + 1 = 19$$

Number of Stirrups Needed

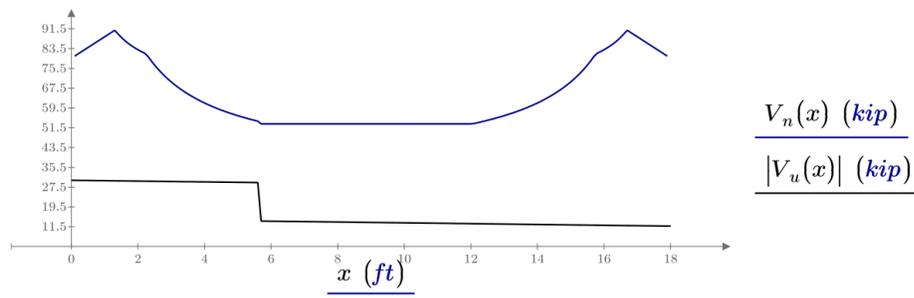
$$V_s := \frac{A_v \cdot f_y \cdot d_p}{s} = 28 \text{ kip}$$

Shear Strength of Stirrups

$$4 \cdot \sqrt{f'_c} \cdot \text{psi} \cdot b \cdot d_p = 58.973 \text{ kip}$$

$$V_n(x) := V_c(x) + V_s$$

Nominal Shear Strength



Non-Commercial Use Only

Fly ash mixture

Flexure Beam Design

Beam Properties

$$L_{bm} := 18.5 \text{ ft}$$

$$L := 18 \text{ ft}$$

$$x := 0 \text{ ft}, 0.1 \text{ ft}..L$$

$$x_{ms} := 0.5 \cdot L = 9 \text{ ft}$$

$$b := 10 \text{ in}$$

$$h := 16 \text{ in}$$

$$P_b := 2 \cdot b + 2 \cdot h = 52 \text{ in}$$

$$A := b \cdot h = 160 \text{ in}^2$$

$$V := L_{bm} \cdot b \cdot h = 35520 \text{ in}^3$$

$$SA := P_b \cdot L_{bm} = 11544 \text{ in}^2$$

$$I_g := b \cdot h^3 \div 12 = 3413.333 \text{ in}^4$$

$$y_t := h \div 2 = 8 \text{ in}$$

$$y_b := h \div 2 = 8 \text{ in}$$

$$S_t := I_g \div y_t = 426.667 \text{ in}^3$$

$$S_b := I_g \div y_b = 426.667 \text{ in}^3$$

Concrete Properties

$$f'_{ci} := 4600 \text{ psi}$$

$$f'_c := 11360 \text{ psi}$$

$$w_c := 150 \text{ pcf}$$

$$E_{ci} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_{ci} \cdot \text{psi}} = 4112 \text{ ksi}$$

$$E_{predicted} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_c \cdot \text{psi}} = 6462 \text{ ksi}$$

$$E_c := 5579 \text{ ksi}$$

Beam length @ transfer

Span Length

Length of Beam as Function of x

Midspan of Beam

Beam Width

Beam Height

Beam Perimeter

Area of Cross-Section

Beam Volume

Surface Area of Beam

Gross Moment of Inertia

Distance from Top of Beam to Neutral Axis

Distance from Bottom of Beam to Neutral Axis

Top Section Modulus

Bottom Section Modulus

Compressive Strength at Release

Compressive Strength at 28 Days

Unit Weight

Initial Modulus of Elasticity of Concrete

Observed Modulus of Elasticity of Concrete

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$$f_{r\text{predicted}} := 7.5 \cdot \sqrt{f'_c \cdot \text{psi}} = 799 \text{ psi}$$

$$f_r := 781 \text{ psi}$$

Observed Modulus of Rupture

$$\beta_1 := \begin{cases} \text{if } 2500 \text{ psi} \leq f'_c \leq 4000 \text{ psi} & 0.85 \\ \text{else if } 4000 \text{ psi} < f'_c \leq 8000 \text{ psi} & 0.85 - \frac{0.05 \cdot (f'_c - 4000 \text{ psi})}{1000 \text{ psi}} \\ \text{else} & 0.65 \end{cases} = 0.65$$

Stress Block Coefficient

Prestress Steel Properties

$$f_{pu} := 270 \text{ ksi}$$

Tensile Strength of Prestress

$$d_{bps} := 0.5 \text{ in}$$

Diameter of Prestress Steel

$$A_{ps12} := 0.153 \text{ in}^2$$

Area of Prestress Steel

$$E_{ps} := 28500 \text{ ksi}$$

Modulus of Elasticity of Prestress Steel

$$n_{ps} := 3$$

Number of Prestress Strands

$$A_{ps} := n_{ps} \cdot A_{ps12} = 0.459 \text{ in}^2$$

Total Area of Prestress Strands

$$d_p := h - 2 \text{ in} = 14 \text{ in}$$

Depth to Prestress Strands

$$e := d_p - y_t = 6 \text{ in}$$

Eccentricity of Prestress Steel

$$\rho_p := \frac{A_{ps}}{b \cdot d_p} = 0.003$$

Prestress Reinforcement Ratio

Compression Steel Properties

$$d_r := 0.625 \text{ in}$$

Diameter of Compression Steel

$$A_{r12} := 0.31 \text{ in}^2$$

Area of Compression Steel

$$n_r := 2$$

Number of Compressive Bars

$$A'_s := n_r \cdot A_{r12} = 0.62 \text{ in}^2$$

Total Area of Compressive Steel

$$d' := 2 \text{ in}$$

Depth to Compressive Steel

$$\rho'_p := \frac{A'_s}{b \cdot d'} = 0.031$$

$$f_y := 60 \text{ ksi}$$

$$E_s := 29000 \text{ ksi}$$

Loads

$$w_{sw} := w_c \cdot A = 166.667 \text{ plf}$$

$$M_{sw}(x) := \frac{w_{sw} \cdot x}{2} \cdot (L - x)$$

$$M_{sw}(x_{ms}) = 6.75 \text{ kip} \cdot \text{ft}$$

$$P := 20 \text{ kip}$$

$$2 \cdot P = 40 \text{ kip}$$

$$s_{point} := 46 \text{ in}$$

$$aa := x_{ms} - \frac{s_{point}}{2} = 7.083 \text{ ft}$$

$$M_{LL}(x) := \begin{cases} \text{if } x < aa & P \cdot x \\ \text{else if } aa \leq x \leq L - aa & P \cdot aa \\ \text{else if } L - aa < x & P \cdot (L - x) \end{cases}$$

$$M_u(x) := M_{sw}(x) + 0.9 \cdot M_{LL}(x)$$

Rebar Reinforcement Ratio

Yield Strength of Compressive Steel

Modulus of Elasticity of Compressive Steel

Distributed Self Weight of Beam

Moment from Self Weight

Moment from Self Weight at Midspan

Applied Point Loads, half of actuator load

Total load applied by actuator

Distance Between Point Loads

Distance of Point Loads to End of Beam

Moment from Applied Loads

Maximum Moment at Midspan

$$M_{sw}(x_{ms}) = 6.75 \text{ kip} \cdot \text{ft}$$

$$M_{LL}(x_{ms}) = 141.667 \text{ kip} \cdot \text{ft}$$

$$M_u(x_{ms}) = 134.3 \text{ kip} \cdot \text{ft}$$

Prestress Losses

Elastic Shortening

$$K_{cir} := 0.9$$

$$F_j := 0.7$$

Prestress Coefficient

Jacking Stress

$$P_j := F_j \cdot f_{pu} \cdot A_{ps} = 86.751 \text{ kip}$$

Initial Prestress Force

$$f_{cir} := K_{cir} \cdot \left(\frac{P_j}{A} + \frac{P_j \cdot e^2}{I_g} \right) - \frac{M_{sw}(x_{ms}) \cdot e}{I_g} = 1.169 \text{ ksi}$$

Net Compressive Strength

$$\Delta f_{es} := \frac{E_{ps}}{E_{ci}} \cdot f_{cir} = 8.103 \text{ ksi}$$

Loss from Elastic Shortening

$$f_{pi} := F_j \cdot f_{pu} - \Delta f_{es} = 180.897 \text{ ksi}$$

Initial Prestress Force After Short-Term Losses

$$P_i := f_{pi} \cdot A_{ps} = 83.032 \text{ kip}$$

Creep

$$f_{cds} := 0$$

Stress in Concrete from Dead Load
(no additional DL applied)

$$K_{cr} := 2$$

Prestress Coefficient

$$\Delta f_{cr} := K_{cr} \cdot \frac{E_{ps}}{E_c} \cdot (f_{cir} - f_{cds}) = 11.944 \text{ ksi}$$

Loss from Creep

Shrinkage

$$RH := 72.5$$

Relative Humidity

$$K_{sh} := 1.0$$

Prestress Coefficient

$$VS := \frac{A}{P_b} = 3.077 \text{ in}$$

Volume to Surface Area Ratio

$$\Delta f_{sh} := 8.2 \cdot 10^{-6} \cdot K_{sh} \cdot E_{ps} \cdot \left(1 - (0.06) \cdot \frac{VS}{\text{in}} \right) \cdot (100 - RH) = 5.24 \text{ ksi}$$

Loss from Shrinkage

Relaxation

$$K_{re} := 5000 \text{ psi}$$

Prestress Coefficient

$$J := 0.04$$

Prestress Coefficient

$$R := \frac{f_{pi}}{f_{pu}} = 0.67$$

Ratio Needed For "C" Equation

$$C := \frac{R}{0.21} \left(\frac{R}{0.9} - 0.55 \right) = 0.62$$

Prestress Coefficient

$$\Delta f_{re} := (K_{re} - J \cdot (\Delta f_{sh} + \Delta f_{cr} + \Delta f_{es})) \cdot C = 2.474 \text{ ksi}$$

Loss from Relaxation

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Total Long Term Losses

$$\Delta f_{total} := \Delta f_{re} + \Delta f_{sh} + \Delta f_{cr} + \Delta f_{es} = 27.761 \text{ ksi} \quad \text{Total Losses}$$

$$f_{se} := F_j \cdot f_{pu} - \Delta f_{total} = 161.239 \text{ ksi} \quad \text{Prestress Force After Losses}$$

$$P_e := f_{se} \cdot A_{ps} = 74.008 \text{ kip}$$

Transfer and Development Length

$$\gamma_p := 0.28 \quad \text{Prestress factor}$$

$$f_{ps} := f_{pu} \cdot \left(1 - \frac{\gamma_p}{\beta_1} \cdot \left(\rho_p \cdot \frac{f_{pu}}{f'_c} \right) \right) = 260.937 \text{ ksi} \quad \text{Prestress Force at Nominal Flexural Strength}$$

$$l_t := \frac{f_{se}}{3000 \text{ psi}} \cdot d_{bps} = 2.239 \text{ ft} \quad \text{Transfer Length}$$

$$l_d := l_t + \frac{(f_{ps} - f_{se})}{1000 \text{ psi}} \cdot d_{bps} = 1.949 \text{ m} \quad \text{Development Length}$$

Flexural Capacity

$$a := \frac{A_{ps} \cdot f_{ps}}{0.85 \cdot f'_c \cdot b} = 1.24 \text{ in} \quad \text{Depth of Stress Block}$$

$$c := \frac{a}{\beta_1} = 1.908 \text{ in}$$

$$M_n := A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2} \right) = 133.542 \text{ kip} \cdot \text{ft} \quad \text{Nominal Moment Capacity}$$

$$\begin{aligned}
 M_n(x) := & \text{if } x < l_t \\
 & \left\| \frac{x}{l_t} \cdot \frac{f_{se}}{f_{ps}} \cdot M_n \right. \\
 & \text{else if } l_t \leq x \leq l_d \\
 & \left\| \left(\frac{x-l_t}{l_d-l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) + \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right. \\
 & \text{else if } l_d < x \leq L-l_d \\
 & \left\| M_n \right. \\
 & \text{else if } L-l_d < x \leq L-l_t \\
 & \left\| \left(1 - \frac{x-L+l_d}{l_d-l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) \right) \cdot M_n \right. \\
 & \text{else if } L-l_t < x \leq L \\
 & \left\| \left(\frac{f_{se}}{f_{ps}} - \frac{x-L+l_t}{l_t} \cdot \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right.
 \end{aligned}$$

Moment Capacity as a Function of x

$$M_{cr} := S_b \cdot \left(\frac{P_e}{A} + \frac{P_e \cdot e}{S_b} + f_r \right) = 81.219 \text{ kip} \cdot \text{ft}$$

Cracking Moment

$$1.2 \cdot M_{cr} = 97.463 \text{ kip} \cdot \text{ft}$$

Cracking Moment

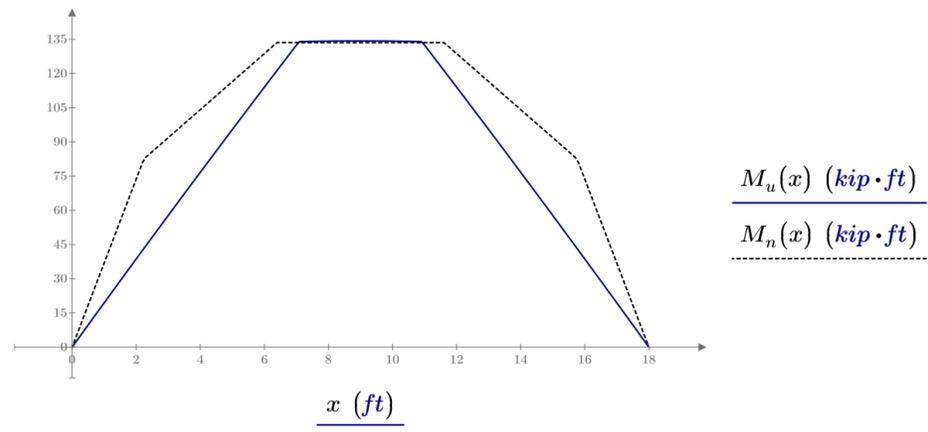
$$\text{if } M_n(x_{ms}) \geq 1.2 \cdot M_{cr} \text{ = "Okay"}$$

Check Cracking Moment is less than Moment Capacity

|| "Okay"

else

|| "Not Okay"



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$$M_u(x_{ms}) = 134.25 \text{ kip} \cdot \text{ft}$$

Design Moment at Midspan

$$M_n(x_{ms}) = 133.542 \text{ kip} \cdot \text{ft}$$

Moment Capacity at Midspan

$$P_i = 83.032 \text{ kip}$$

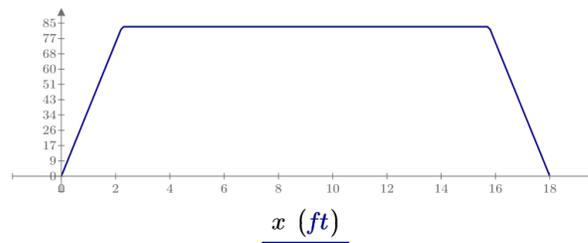
Initial Prestress Force

$$P_i(x) := \begin{cases} \text{if } x \leq l_t & \left\| \frac{x}{l_t} \cdot P_i \right. \\ \text{else if } l_t < x \leq L - l_t & \left\| P_i \right. \\ \text{else if } x > L - l_t & \left\| P_i - \frac{P_i}{l_t} \cdot (x - (L - l_t)) \right. \end{cases}$$

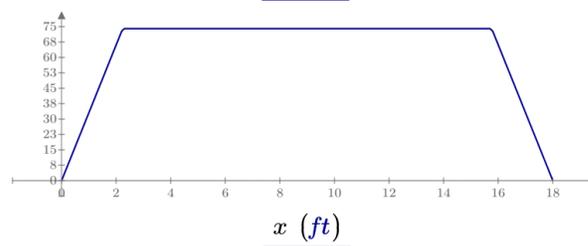
Initial Prestress Force as Function of x

$$P_e(x) := \begin{cases} \text{if } x \leq l_t & \left\| \frac{x}{l_t} \cdot P_e \right. \\ \text{else if } l_t < x \leq L - l_t & \left\| P_e \right. \\ \text{else if } x > L - l_t & \left\| P_e - \frac{P_e}{l_t} \cdot (x - (L - l_t)) \right. \end{cases}$$

Effective Prestress Force as Function of x



$$\underline{P_i(x) \text{ (kip)}}$$



$$\underline{P_e(x) \text{ (kip)}}$$

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Transfer Stresses

$$f_{ttm}(x) := \frac{P_i(x)}{A} - \frac{P_i(x) \cdot e}{S_t} + \frac{M_{sw}(x)}{S_t}$$

Transfer Stresses at Top of Beam

$$f_{btm}(x) := \frac{P_i(x)}{A} + \frac{P_i(x) \cdot e}{S_b} - \frac{M_{sw}(x)}{S_b}$$

Transfer Stresses at Bottom of Beam

Limits

$$C_{tallow_end} := 0.7 \cdot f'_{ci} = 22201118 \text{ Pa}$$

Compression Limits at Transfer (ACI Table 24.5.3.1)

$$C_{tallow_ms} := 0.6 \cdot f'_{ci} = 19029530.129 \text{ Pa}$$

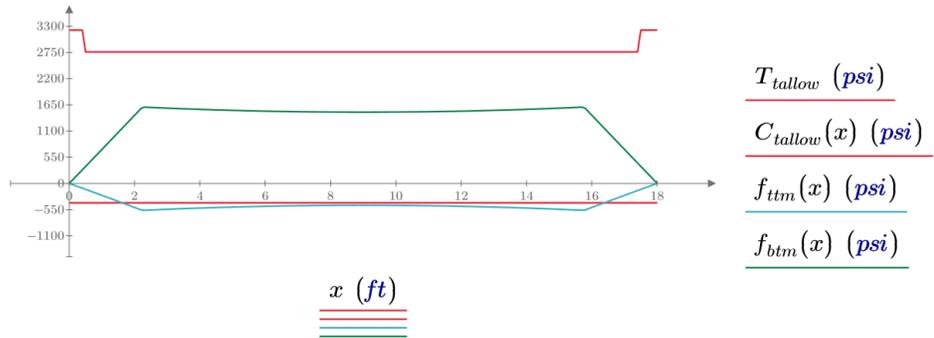
Compression Limits at Transfer (ACI Table 24.5.3.1)

$$T_{tallow} := -6 \cdot \sqrt{f'_{ci} \cdot psi} = -3 \cdot 10^6 \text{ Pa}$$

Tension Limits at Transfer (PCI)

$$C_{tallow}(x) := \begin{cases} \text{if } x < 0.5 \text{ ft} \\ \parallel C_{tallow_end} \\ \text{else if } x < L - 0.5 \text{ ft} \\ \parallel C_{tallow_ms} \\ \text{else} \\ \parallel C_{tallow_end} \end{cases}$$

Compression Limits Along the Beam



Tension Check

$$T := \frac{(f_{ttm}(x_{ms}) - T_{tallow}) \cdot b \cdot c}{2} = -0.495 \text{ kip}$$

Tensile Force

$$f_s := 0.5 \cdot f_y = 206842718.795 \text{ Pa}$$

Permitted Stress in the Reinforcement

$$\left| \frac{T}{f_s} \right| = 0.017 \text{ in}^2$$

Required Area of Reinforcement

$$\left. \begin{array}{l} \text{if } A'_s \geq \frac{T}{f_s} \\ \quad \parallel \text{ "Okay"} \\ \text{else} \\ \quad \parallel \text{ "Not Okay"} \end{array} \right| = \text{"Okay"}$$

Service Stresses

$$f_{tsm}(x) := \frac{P_e(x)}{A} - \frac{P_e(x) \cdot e}{S_t} + \frac{M_u(x)}{S_t}$$

Service Stresses at Top of Beam

$$f_{bsm}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b} - \frac{M_u(x)}{S_b}$$

Service Stresses at Bottom of Beam

Limits

$$T_{\text{allow}} := -7.5 \cdot \sqrt{f'_c \cdot \text{psi}} = -5.511 \cdot 10^6 \text{ Pa}$$

Tension Limits

$$C_{\text{allow}} := 0.6 \cdot f'_c = 46994665.71 \text{ Pa}$$

Compression Limits

Shear Capacity and Demand

$$V_{sw}(x) := w_{sw} \cdot \left(\frac{L}{2} - x \right)$$

Shear from Self Weight

$$\left. \begin{array}{l} V_{LL}(x) := \text{if } x \leq aa \\ \quad \parallel P \\ \quad \text{else if } aa < x < L - aa \\ \quad \parallel 0 \text{ kip} \\ \quad \text{else if } L - aa \leq x \\ \quad \parallel -P \end{array} \right|$$

Shear from Applied Loads

$$V_u(x) := V_{sw}(x) + 1.2 \cdot V_{LL}(x)$$

Ultimate Shear

$$d_p := \max(d_p, 0.8 \cdot h) = 14 \text{ in}$$

Depth to Prestress

$$f_{pce}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b}$$

$$V_d(x) := V_{sw}(x)$$

Shear from Dead Loads

$$M_d(x) := M_{sw}(x)$$

Moment from Dead Loads

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$$V_i(x) := V_u(x) - V_d(x)$$

$$M_{max}(x) := M_u(x) - M_d(x)$$

$$f_d(x) := \frac{M_d(x)}{S_b}$$

$$M_{cre}(x) := S_b \cdot (6 \cdot \sqrt{f'_c \cdot psi} + f_{pce}(x) - f_d(x))$$

$$V_{ci}(x) := \max \left(0.6 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p + |V_d(x)| + \frac{|V_i(x)| \cdot M_{cre}(x)}{M_{max}(x)}, 1.7 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p \right)$$

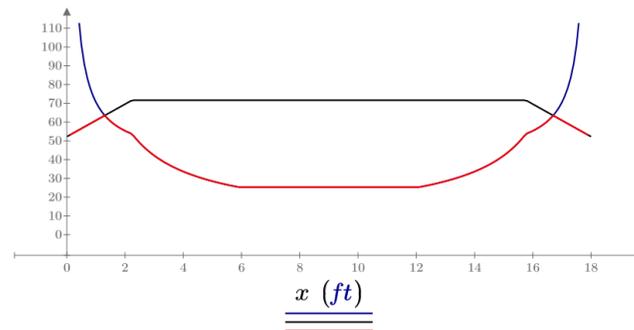
$$V_p := 0$$

$$f_{pc}(x) := \frac{P_e(x)}{A}$$

$$V_{cw}(x) := (3.5 \cdot \sqrt{f'_c \cdot psi} + 0.3 \cdot f_{pc}(x)) \cdot b \cdot d_p + V_p$$

$$V_{cw}(x_{ms}) = 71.7 \text{ kip}$$

$$V_c(x) := \min(V_{ci}(x), V_{cw}(x))$$



Stirrups

$$\phi_v := 0.75$$

$$V_s(x) := V_u(x) - V_c(x)$$

$$V_s(aa) = -1.047 \text{ kip}$$

$$A_v := 2 \cdot 0.20 \text{ in}^2 = 0.4 \text{ in}^2$$

Try double leg stirrups

$$s_{max} := \frac{A_v \cdot f_y \cdot d_p}{V_s(x_{ms})} = -13.246 \text{ in}$$

Maximum spacing allowed

$$s_{max} := \min(0.75 \cdot h, 24 \text{ in}) = 12 \text{ in}$$

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$$s := s_{max}$$

$$A_{vmin} := \min \left(\max \left(0.75 \sqrt{f'_c \cdot \text{psi}} \cdot \frac{b \cdot s}{f_y}, 50 \text{ psi} \cdot \frac{b \cdot s}{f_y} \right), \frac{A_{ps} \cdot f_{pu} \cdot s}{80 \cdot f_y \cdot d_p} \cdot \sqrt{\frac{d_p}{b}} \right) = 0.026 \text{ in}^2$$

if $A_v \geq A_{vmin}$ | = "Okay"
 || "Okay"
 else
 || "Not Okay"

$$n_s := \frac{L}{s} + 1 = 19$$

Number of Stirrups Needed

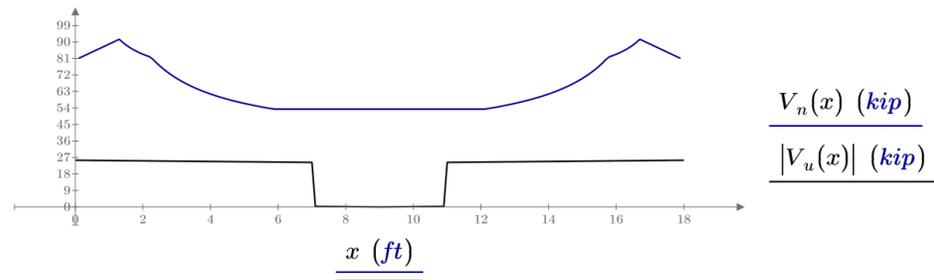
$$V_s := \frac{A_v \cdot f_y \cdot d_p}{s} = 28 \text{ kip}$$

Shear Strength of Stirrups

$$4 \cdot \sqrt{f'_c \cdot \text{psi}} \cdot b \cdot d_p = 59.687 \text{ kip}$$

$$V_n(x) := V_c(x) + V_s$$

Nominal Shear Strength



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Bond Strength Beam Design

Beam Properties

$L_{bm} := 18.5 \text{ ft}$	Beam length
$L := 18 \text{ ft}$	Span Length
$x := 0 \text{ ft}, 0.1 \text{ ft}..L$	Length of Beam as Function of x
$x_{ms} := 0.5 \cdot L = 9 \text{ ft}$	Midspan of Beam
$b := 10 \text{ in}$	Beam Width
$h := 16 \text{ in}$	Beam Height
$P_b := 2 \cdot b + 2 \cdot h = 52 \text{ in}$	Beam Perimeter
$A := b \cdot h = 160 \text{ in}^2$	Area of Cross-Section
$V := L_{bm} \cdot b \cdot h = 35520 \text{ in}^3$	Beam Volume
$SA := P_b \cdot L_{bm} = 11544 \text{ in}^2$	Surface Area of Beam
$I_g := b \cdot h^3 \div 12 = 3413.333 \text{ in}^4$	Gross Moment of Inertia
$y_t := h \div 2 = 8 \text{ in}$	Distance from Top of Beam to Neutral Axis
$y_b := h \div 2 = 8 \text{ in}$	Distance from Bottom of Beam to Neutral Axis
$S_t := I_g \div y_t = 426.667 \text{ in}^3$	Top Section Modulus
$S_b := I_g \div y_b = 426.667 \text{ in}^3$	Bottom Section Modulus

Concrete Properties

$f'_{ci} := 4600 \text{ psi}$	Compressive Strength at Release
$f'_c := 11360 \text{ psi}$	Compressive Strength at 28 Days
$w_c := 150 \text{ pcf}$	Unit Weight
$E_{ci} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_{ci} \cdot \text{psi}} = 4112 \text{ ksi}$	Initial Modulus of Elasticity of Concrete
$E_{ci} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_{ci} \cdot \text{psi}} = 4112 \text{ ksi}$	
$E_{c\text{predicted}} := 33 \cdot (w_c \div \text{pcf})^{1.5} \cdot \sqrt{f'_c \cdot \text{psi}} = 6462 \text{ ksi}$	

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$$E_c := 5579 \text{ ksi}$$

Observed Modulus of Elasticity of Concrete

$$f_{r\text{predicted}} := 7.5 \cdot \sqrt{f'_c} \cdot \text{psi} = 799 \text{ psi}$$

Observed Modulus of Rupture

$$f_r := 781 \text{ psi}$$

Stress Block Coefficient

$$\beta_1 := \begin{cases} \text{if } 2500 \text{ psi} \leq f'_c \leq 4000 \text{ psi} & = 0.65 \\ \parallel 0.85 \\ \text{else if } 4000 \text{ psi} < f'_c < 8000 \text{ psi} & \\ \parallel 0.85 - \frac{0.05 \cdot (f'_c - 4000 \text{ psi})}{1000 \text{ psi}} \\ \text{else if } f'_c > 8000 \text{ psi} & \\ \parallel 0.65 \end{cases}$$

Prestress Steel Properties

$$f_{pu} := 270 \text{ ksi}$$

Tensile Strength of Prestress

$$d_{bps} := 0.5 \text{ in}$$

Diameter of Prestress Steel

$$A_{ps12} := 0.153 \text{ in}^2$$

Area of Prestress Steel

$$E_{ps} := 28500 \text{ ksi}$$

Modulus of Elasticity of Prestress Steel

$$n_{ps} := 3$$

Number of Prestress Strands

$$A_{ps} := n_{ps} \cdot A_{ps12} = 0.459 \text{ in}^2$$

Total Area of Prestress Strands

$$d_p := h - 2 \text{ in} = 14 \text{ in}$$

Depth to Prestress Strands

$$e := d_p - y_t = 6 \text{ in}$$

Eccentricity of Prestress Steel

$$\rho_p := \frac{A_{ps}}{b \cdot d_p} = 0.003$$

Prestress Reinforcement Ratio

Compression Steel Properties

$$d_r := 0.625 \text{ in}$$

Diameter of Compression Steel

$$A_{r12} := 0.31 \text{ in}^2$$

Area of Compression Steel

$$n_r := 2$$

Number of Compressive Bars

$$A'_s := n_r \cdot A_{r12} = 0.62 \text{ in}^2$$

Total Area of Compressive Steel

$$d' := 2 \text{ in}$$

$$\rho'_p := \frac{A'_s}{b \cdot d'} = 0.031$$

$$f_y := 60 \text{ ksi}$$

$$E_s := 29000 \text{ ksi}$$

Depth to Compressive Steel

Rebar Reinforcement Ratio

Yield Strength of Compressive Steel

Modulus of Elasticity of Compressive Steel

Loads

$$w_{sw} := w_c \cdot A = 166.667 \text{ plf}$$

Distributed Self Weight of Beam

$$M_{sw}(x) := \frac{w_{sw} \cdot x}{2} \cdot (L - x)$$

Moment from Self Weight

$$M_{sw}(x_{ms}) = 6.75 \text{ kip} \cdot \text{ft}$$

Moment from Self Weight at Midspan

$$P := 35 \text{ kip}$$

Applied Point Load (1)

$$aa := 5.67 \text{ ft}$$

Distance of Point Loads to End of Beam

$$bb := L - aa = 3.758 \text{ m}$$

$$aa \div d_p = 4.86$$

$$aa \div h = 4.253$$

if $\frac{aa}{d_p} \geq 2.5$ "Okay" else "Not Okay"	= "Okay"	if $\frac{aa}{h} \geq 2.5$ "Okay" else "Not Okay"	= "Okay"
--	----------	--	----------

$$R_1 := P \cdot bb \div L = 23.975 \text{ kip}$$

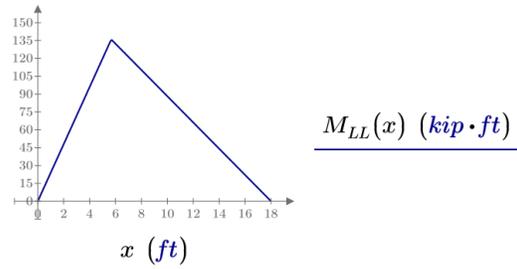
$$R_2 := P \cdot aa \div L = 11.025 \text{ kip}$$

$$R_1 + R_2 = 35 \text{ kip}$$

$$P = 35 \text{ kip}$$

$$M_{LL}(x) := \begin{cases} \text{if } x < aa \\ \quad || R_1 \cdot x \\ \text{else} \\ \quad || R_1 \cdot x - P \cdot (x - aa) \end{cases}$$

Moment from Applied Loads



$$M_u(x) := M_{sw}(x) + 0.9 \cdot M_{LL}(x)$$

Maximum Moment at Midspan

$$M_u(aa) = 128.17 \text{ kip} \cdot \text{ft}$$

Prestress Losses

Elastic Shortening

$$K_{cir} := 0.9$$

Prestress Coefficient

$$F_j := 0.7$$

Jacking Stress

$$P_i := F_j \cdot f_{pu} \cdot A_{ps} = 86.751 \text{ kip}$$

Initial Prestress Force

$$f_{cir} := K_{cir} \cdot \left(\frac{P_i}{A} + \frac{P_i \cdot e^2}{I_g} \right) - \frac{M_{sw}(x_{ms}) \cdot e}{I_g} = 1.169 \text{ ksi}$$

Net Compressive Strength

$$\Delta f_{es} := \frac{E_{ps}}{E_{ci}} \cdot f_{cir} = 8.103 \text{ ksi}$$

Loss from Elastic Shortening

$$f_{pi} := F_j \cdot f_{pu} - \Delta f_{es} = 180.897 \text{ ksi}$$

Initial Prestress Force After Short-Term Losses

$$P_i := f_{pi} \cdot A_{ps} = 83.032 \text{ kip}$$

Creep

$$f_{cds} := 0$$

Stress in Concrete from Dead Load
(no additional DL applied)

$$K_{cr} := 2$$

Prestress Coefficient

$$\Delta f_{cr} := K_{cr} \cdot \frac{E_{ps}}{E_c} \cdot (f_{cir} - f_{cds}) = 11.944 \text{ ksi}$$

Loss from Creep

Shrinkage

$$RH := 72.5$$

Relative Humidity

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$$K_{sh} := 1.0 \quad \text{Prestress Coefficient}$$

$$VS := \frac{A}{P_b} = 3.077 \text{ in} \quad \text{Volume to Surface Area Ratio}$$

$$\Delta f_{sh} := 8.2 \cdot 10^{-6} \cdot K_{sh} \cdot E_{ps} \cdot \left(1 - (0.06) \cdot \frac{VS}{\text{in}}\right) \cdot (100 - RH) = 5.24 \text{ ksi} \quad \text{Loss from Shrinkage}$$

Relaxation

$$K_{re} := 5000 \text{ psi} \quad \text{Prestress Coefficient}$$

$$J := 0.04 \quad \text{Prestress Coefficient}$$

$$R := \frac{f_{pi}}{f_{pu}} = 0.67 \quad \text{Ratio Needed For "C" Equation}$$

$$C := \frac{R}{0.21} \left(\frac{R}{0.9} - 0.55 \right) = 0.62 \quad \text{Prestress Coefficient}$$

$$\Delta f_{re} := (K_{re} - J \cdot (\Delta f_{sh} + \Delta f_{cr} + \Delta f_{es})) \cdot C = 2.474 \text{ ksi} \quad \text{Loss from Relaxation}$$

Total Long Term Losses

$$\Delta f_{total} := \Delta f_{re} + \Delta f_{sh} + \Delta f_{cr} + \Delta f_{es} = 27.761 \text{ ksi} \quad \text{Total Losses}$$

$$f_{se} := F_j \cdot f_{pu} - \Delta f_{total} = 161.239 \text{ ksi} \quad \text{Prestress Force After Losses}$$

$$P_e := f_{se} \cdot A_{ps} = 74.008 \text{ kip} \quad \text{Effective Prestress Force}$$

Transfer and Development Length

$$\gamma_p := 0.28 \quad \text{Prestress factor}$$

$$f_{ps} := f_{pu} \cdot \left(1 - \frac{\gamma_p}{\beta_1} \cdot \left(\rho_p \cdot \frac{f_{pu}}{f'_c}\right)\right) = 260.937 \text{ ksi} \quad \text{Prestress Force at Nominal Flexural Strength}$$

$$l_t := \frac{f_{se}}{3000 \text{ psi}} \cdot d_{bps} = 2.239 \text{ ft} \quad \text{Transfer Length}$$

$$l_d := l_t + \frac{(f_{ps} - f_{se})}{1000 \text{ psi}} \cdot d_{bps} = 1.949 \text{ m} \quad \text{Development Length}$$

Flexural Capacity

$$a := \frac{A_{ps} \cdot f_{ps}}{0.85 \cdot f'_c \cdot b} = 1.24 \text{ in} \quad \text{Depth of Stress Block}$$

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$$c := \frac{a}{\beta_1} = 1.908 \text{ in}$$

$$M_n := A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2} \right) = 133.542 \text{ kip} \cdot \text{ft}$$

Nominal Moment Capacity

$$M_n(x) := \text{if } x < l_t$$

$$\left\| \frac{x}{l_t} \cdot \frac{f_{se}}{f_{ps}} \cdot M_n \right.$$

else if $l_t \leq x \leq l_d$

$$\left\| \left(\frac{x - l_t}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) + \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right.$$

else if $l_d < x \leq L - l_d$

$$\left\| M_n \right.$$

else if $L - l_d < x \leq L - l_t$

$$\left\| \left(1 - \frac{x - L + l_d}{l_d - l_t} \cdot \left(1 - \frac{f_{se}}{f_{ps}} \right) \right) \cdot M_n \right.$$

else if $L - l_t < x \leq L$

$$\left\| \left(\frac{f_{se}}{f_{ps}} - \frac{x - L + l_t}{l_t} \cdot \frac{f_{se}}{f_{ps}} \right) \cdot M_n \right.$$

Moment Capacity as a Function of x

$$P_e = 74.008 \text{ kip}$$

Effective Prestress Force

$$M_{cr} := S_b \cdot \left(\frac{P_e}{A} + \frac{P_e \cdot e}{S_b} + f_r \right) = 81.219 \text{ kip} \cdot \text{ft}$$

Cracking Moment

$$1.2 \cdot M_{cr} = 97.463 \text{ kip} \cdot \text{ft}$$

Factored Cracking Moment

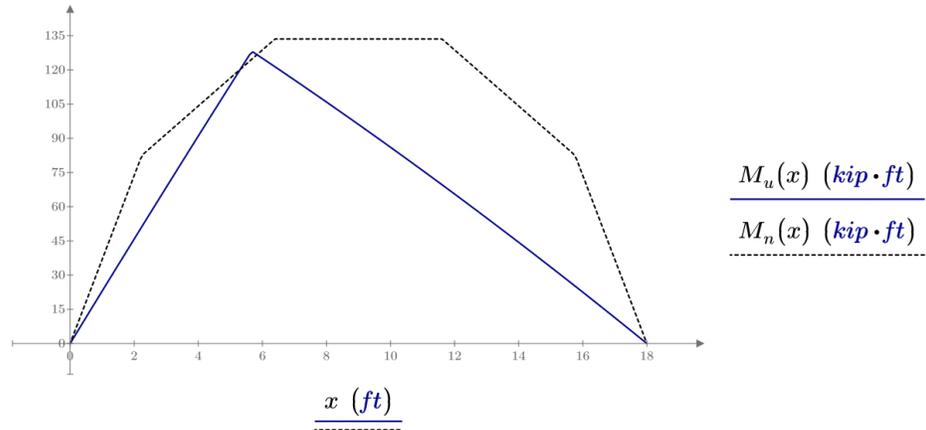
$$\text{if } M_n(x_{ms}) \geq 1.2 \cdot M_{cr} \mid = \text{“Okay”}$$

$\left\| \text{“Okay”} \right.$

else

$\left\| \text{“Not Okay”} \right.$

Check Cracking Moment is less than Factored Moment Capacity



$$M_u(aa) = 128.17 \text{ kip} \cdot \text{ft}$$

Design Moment at Critical Section

$$M_n(aa) = 124.655 \text{ kip} \cdot \text{ft}$$

Moment Capacity at Critical Section

$$P_i = 83.032 \text{ kip}$$

Initial Prestress Force

$$P_i(x) := \text{if } x \leq l_t$$

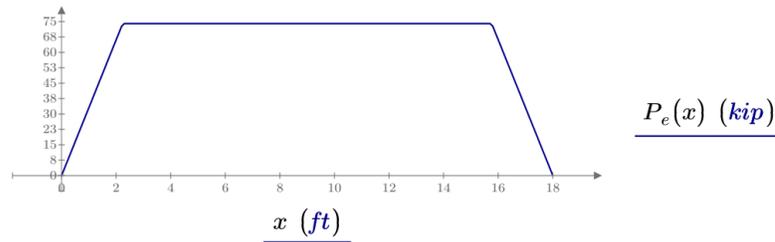
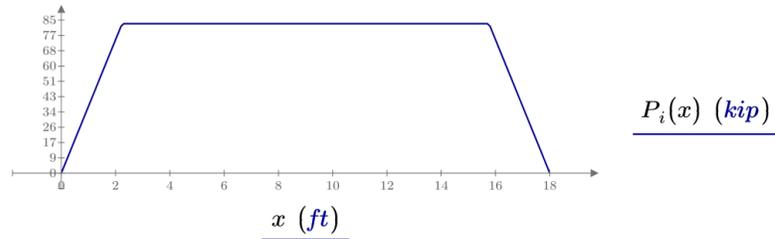
Initial Prestress Force as Function of x

$$\begin{cases} \frac{x}{l_t} \cdot P_i \\ \text{else if } l_t < x \leq L - l_t \\ P_i \\ \text{else if } x > L - l_t \\ P_i - \frac{P_i}{l_t} \cdot (x - (L - l_t)) \end{cases}$$

$$P_e(x) := \text{if } x \leq l_t$$

Effective Prestress Force as Function of x

$$\begin{cases} \frac{x}{l_t} \cdot P_e \\ \text{else if } l_t < x \leq L - l_t \\ P_e \\ \text{else if } x > L - l_t \\ P_e - \frac{P_e}{l_t} \cdot (x - (L - l_t)) \end{cases}$$



Transfer Stresses

$$f_{ttm}(x) := \frac{P_i(x)}{A} - \frac{P_i(x) \cdot e}{S_t} + \frac{M_{sw}(x)}{S_t}$$

Transfer Stresses at Top of Beam

$$f_{btm}(x) := \frac{P_i(x)}{A} + \frac{P_i(x) \cdot e}{S_b} - \frac{M_{sw}(x)}{S_b}$$

Transfer Stresses at Bottom of Beam

Limits

$$C_{tallow_end} := 0.7 \cdot f'_{ci} = 22201118 \text{ Pa}$$

Compression Limits at Transfer (ACI Table 24.5.3.1)

$$C_{tallow_ms} := 0.6 \cdot f'_{ci} = 19029530.129 \text{ Pa}$$

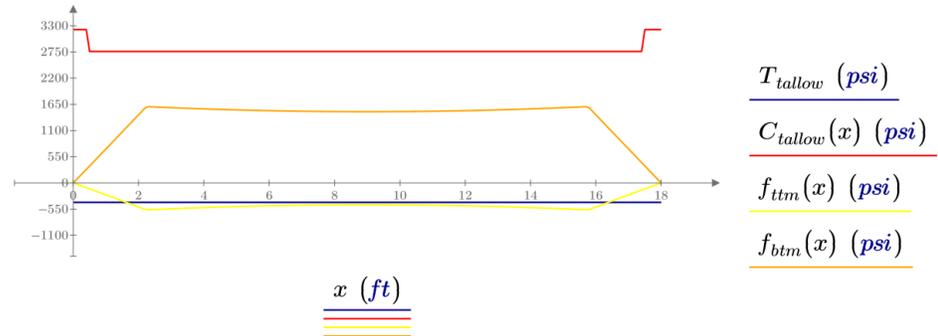
Compression Limits at Transfer (ACI Table 24.5.3.1)

$$T_{tallow} := -6 \cdot \sqrt{f'_{ci} \cdot psi} = -3 \cdot 10^6 \text{ Pa}$$

Tension Limits at Transfer (PCI)

$$C_{tallow}(x) := \begin{cases} \text{if } x < 0.5 \text{ ft} & C_{tallow_end} \\ \text{else if } x < L - 0.5 \text{ ft} & C_{tallow_ms} \\ \text{else} & C_{tallow_end} \end{cases}$$

Compression Limits Along the Beam



Tension Check

$$T := \frac{(f_{ttm}(x_{ms}) - T_{tallow}) \cdot b \cdot c}{2} = -0.495 \text{ kip}$$

Tensile Force

$$f_s := 0.5 \cdot f_y = 206842718.795 \text{ Pa}$$

Permitted Stress in the Reinforcement

$$\left| \frac{T}{f_s} \right| = 0.017 \text{ in}^2$$

Required Area of Reinforcement

$$\begin{aligned} &\text{if } A'_s \geq \frac{T}{f_s} && \text{“Okay”} \\ &\quad \parallel && \text{“Okay”} \\ &\text{else} && \\ &\quad \parallel && \text{“Not Okay”} \end{aligned}$$

Service Stresses

$$f_{tsm}(x) := \frac{P_e(x)}{A} - \frac{P_e(x) \cdot e}{S_t} + \frac{M_u(x)}{S_t}$$

Service Stresses at Top of Beam

$$f_{bsm}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b} - \frac{M_u(x)}{S_b}$$

Service Stresses at Bottom of Beam

Limits

$$T_{sallow} := -7.5 \cdot \sqrt{f'_c \cdot \text{psi}} = -5.511 \cdot 10^6 \text{ Pa}$$

Tension Limits

$$C_{sallow} := 0.6 \cdot f'_c = 46994665.71 \text{ Pa}$$

Compression Limits

Shear Capacity and Demand

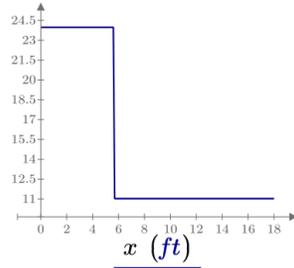
$$V_{sw}(x) := w_{sw} \cdot \left(\frac{L}{2} - x \right)$$

Shear from Self Weight

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$$V_{LL}(x) := \begin{cases} \text{if } x < aa \\ \quad \parallel P \cdot bb \div L \\ \text{else} \\ \quad \parallel P \cdot aa \div L \end{cases}$$

Shear from Applied Loads



$V_{LL}(x)$ (kip)

$$V_u(x) := V_{sw}(x) + 1.2 \cdot V_{LL}(x)$$

Ultimate Shear

$$d_p := \max(d_p, 0.8 \cdot h) = 14 \text{ in}$$

Depth to Prestress

$$f_{pce}(x) := \frac{P_e(x)}{A} + \frac{P_e(x) \cdot e}{S_b}$$

$$V_d(x) := V_{sw}(x)$$

Shear from Dead Loads

$$M_d(x) := M_{sw}(x)$$

Moment from Dead Loads

$$V_i(x) := V_u(x) - V_d(x)$$

$$M_{max}(x) := M_u(x) - M_d(x)$$

$$f_d(x) := \frac{M_d(x)}{S_b}$$

$$M_{cre}(x) := S_b \cdot (6 \cdot \sqrt{f'_c \cdot psi} + f_{pce}(x) - f_d(x))$$

$$V_{ci}(x) := \max\left(0.6 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p + |V_d(x)| + \frac{|V_i(x)| \cdot M_{cre}(x)}{M_{max}(x)}, 1.7 \cdot \sqrt{f'_c \cdot psi} \cdot b \cdot d_p\right)$$

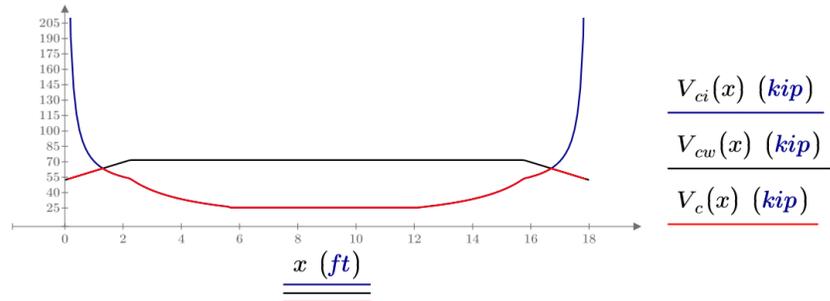
$$V_p := 0$$

$$f_{pc}(x) := \frac{P_e(x)}{A}$$

$$V_{cw}(x) := (3.5 \cdot \sqrt{f'_c \cdot psi} + 0.3 \cdot f_{pc}(x)) \cdot b \cdot d_p + V_p$$

$$V_c(x) := \min(V_{ci}(x), V_{cw}(x))$$

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Stirrups

$$s_{max} := \min(0.75 \cdot h, 24 \text{ in}) = 12 \text{ in}$$

$$s := s_{max}$$

$$A_{vmin} := \min\left(\max\left(0.75 \sqrt{f'_c} \cdot \text{psi} \cdot \frac{b \cdot s}{f_y}, 50 \text{ psi} \cdot \frac{b \cdot s}{f_y}\right), \frac{A_{ps} \cdot f_{pu} \cdot s}{80 \cdot f_y \cdot d_p} \cdot \sqrt{\frac{d_p}{b}}\right) = 0.026 \text{ in}^2$$

if $A_v \geq A_{vmin}$ "Okay" else "Not Okay"	$=$ "Okay" $A_v = 0.4 \text{ in}^2$
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$$n_s := \frac{L}{s} + 1 = 19$$

Number of Stirrups Needed

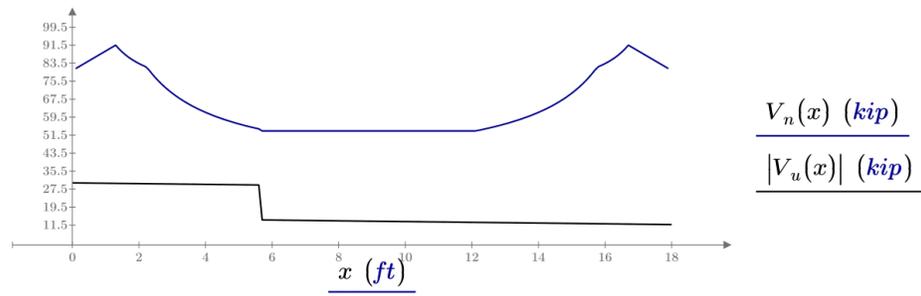
$$V_s := \frac{A_v \cdot f_y \cdot d_p}{s} = 28 \text{ kip}$$

Shear Strength of Stirrups

$$4 \cdot \sqrt{f'_c} \cdot \text{psi} \cdot b \cdot d_p = 59.687 \text{ kip}$$

$$V_n(x) := V_c(x) + V_s$$

Nominal Shear Strength



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