Essays on Inequality and Macroeconomics

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Dedication

To my dad, Manuel, for his unconditional love, and to my wife, Claudia, for her support through this journey and for sacrificing so much to help me pursue my dreams. Finally, a special mention to the loving memory of my mother, Montserrat Munté Corresa (1961-2015),

"Sense tu, res d'això hagués estat possible".

Abstract

This dissertation consists of three chapters. In chapter 1, I study and document a positive and strong relationship between households' permanent income and their tendency to accumulate a disproportionate amount of wealth. I do so by estimating a structural income process using detailed household survey panel data from the Bank of Spain. In chapter 2, I investigate why high permanent income households tend to accumulate and save relatively more. To accomplish this I build a standard partial equilibrium life-cycle model of consumption and savings, augmented with preference heterogeneity, a realistic public pension system, transmission of wealth from parents to children in the form of bequests and also transmission of ability. I show all these features generate non-homothetic behavior. Next, I calibrate the model for Spain and use it to decompose the quantitative contribution of each non-homothetic source. I find that the pension system and the introduction of bequests bring empirical and model moments very close. In particular, the pension system accounts for at least 60% of heterogeneity in wealth-to-income ratios. The intuition is that the public pension system generates income replacement ratios that are low and motivates high income households to complement their pension by saving more over the life cycle. In chapter 3, I study the role of incomplete information as a possible explanation to rationalize the lack of fiscal adjustment that triggered a debt accumulation process during and after the Great Recession in many European countries. In particular, I build a simple two-period model of government debt issuance and default in which i) the decision maker does not observe the shocks hitting the economy, ii) policy is assumed to depend on aggregate labor, and iii) policy and labor are endogenous and determined simultaneously. In this setup, I show that optimal fiscal reaction features endogenous little fiscal adjustment as well as larger debt accumulation and default probabilities.

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Chapter 1

Heterogeneous Saving Behavior and Permanent Income: An Empirical Investigation

1.1 Introduction

A large body of the literature argues the existence of a positive and strong relationship between different measures of lifetime or permanent income, saving rates, and wealth accumulation over the life cycle (see Dynan et al. (2004), Straub (2019), and Feiveson and Sabelhaus (2019)), where permanent income captures returns to individual fixed skills or abilities. In particular, some assert that consumption and saving behavior are non-homothetic with respect to permanent income and that households with persistent high income tend to save more out of it[1].

In this paper I document and quantify differences in wealth accumulation across the distribution of permanent income among Spanish households. I claim this quantification is important at least for three reasons. First, many studies use only U.S data to understand household behavior. Some important sources are the Survey of Consumer Finances (SCF), the Panel Study of Income Dynamics (PSID), the Consumer Expenditure Survey (CEX), and Social Security administrative data. This is typically the trend due to data availability

[1]This idea dates back to Fisher (1930) and Keynes (1936).

and quality concerns. However, it remains uncertain whether or not their conclusions are U.S.-specific or similar inferences can be drawn from other countries. In the past two decades many countries made an effort to build comprehensive and rich data sets. To this end, I use the Survey of Household Finances (SHF), arguably the most important data set at household level in Spain, to understand wealth accumulation behavior among Spanish households.

Second, the existence of non-homothetic behavior matters for macroeconomic aggregates[2]. Some recent examples in this direction are Straub (2019) and Kaplan et al. (2018). The former studies the secular decline of real interest rates since the 1970s. Specifically, he relates the increase in permanent labor income inequality, in conjunction with non-homothetic consumption and saving behavior, to conclude that it caused a decline in real interest rates of 1%. The latter assesses the role of monetary policy in a New Keynesian heterogeneous agent model with two assets, one illiquid and another liquid, which give rise to the so-called wealthy hand-to-mouth households. In this economy, the channels by which monetary policy affects real economy are very different from standard heterogeneous agents New Keynesian models. In both, non-homothetic behavior is a *sine qua non* condition to obtain their results[3].

Third, generating additional empirical moments can help economists discipline economic models and quantify the importance of different sources of non-homothetic behavior such as preference heterogeneity or bequest motives.

In this paper I complement the literature that mostly uses U.S. data to find evidence of non-homothetic behavior. To do so, I use Spanish panel data from the Survey of Household Finances (SHF) from 2002 to 2017 to estimate permanent income at household-level and document its relationship with wealth accumulation relative to income. My findings suggest this might be a wide phenomenon that it is not specific to the institutional or cultural characteristics of the U.S. and that is potentially present in other countries as well.

[2]Macroeconomic models with individual heterogeneity from the 1990s and 2000s feature *approximate* neutrality, where macroeconomics matters for inequality but inequality does not affect much macroeconomic outcomes.

[3]Straub (2019) assumes non-homotheticity *directly* in preferences and abstracts from other sources that may create it. Kaplan et al. (2018) assumes the existence of liquid and illiquid assets with fixed adjustment costs. The latter represents the main source of non-homotheticity in the model.

The analysis of Spanish survey data suggests the existence of a strong and positive relationship between *estimated* permanent income level and wealth accumulation relative to income, in line with Dynan et al. (2004) and Hubbard et al. (1995) for the U.S. In particular, for households between 20 and 65 years old, I find that the median household in the 8th, 9th, and 10th decile of the permanent income distribution tends to accumulate at least 50% more wealth relative to its income than the median household in the 2nd decile. A similar picture emerges when I split the panel between households younger than 45 years of age and older, suggesting that non-homothetic saving behavior is present over the life cycle, not only when old. In Chapter 2, I combine the empirical moments obtained in this chapter with other moments to quantify the importance of different sources of this non-homothetic behavior in wealth accumulation.

I also estimate the income process at household level. This will serve as an important input in the quantitative analysis of Chapter 2.

Related literature. This chapter relates to several strands of a broad literature at the intersection of measurement of non-homothetic saving behavior in the data, theoretical explanations of the latter and wealth inequality, and the implications for public policy programs design.

First, economists have studied empirically whether households with high permanent income tend to save more than those with low permanent income. In his groundbreaking work on the permanent income hypothesis, Friedman (1957) makes the case that the positive *cross-sectional* correlation between income and saving rates reflects the idea that individuals respond to positive (negative) transitory income shocks by running up (down) assets and smoothing consumption. Over time, as more and better quality data was available, some economists challenged this view and concluded in favor of the existence of differential saving behavior (e.g., Mayer (1972)).

In the 90's, Hubbard et al. (1995) revisits the question and claims, at least for the U.S., wealth levels are disproportionally higher among households with high lifetime income. A main drawback of the analysis is the use of cross-sectional data where lifetime income is approximated by education level. More recently, using better quality and newer micro data from the PSID, SCF, and CEX, Dynan et al. (2004) finds a strong positive relationship between saving rates and lifetime income. Furthermore, Straub (2019) uses PSID data from

1999 to 2013 to estimate a linear relationship between log consumption and log permanent income. He finds a coefficient significantly smaller than one ($\phi \approx 0.7$) and argues that this is evidence of non-homothetic consumption-saving behavior along the permanent income distribution. Also, for the U.S., Feiveson and Sabelhaus (2019) finds clear evidence in favor of the highest permanent income households being the ones that save relatively the most.

While most of research focuses on the U.S., my paper contributes to the literature by studying the question in another country. This is particularly interesting because one may have concerns that the results are driven by specific cultural or idiosyncratic institutional factors. However, this paper shows that a similar behavior is present in another country, very different in many regards, like Spain. Even though I do not measure saving rates directly, I do measure a consequence of it - wealth accumulation.

Finally, there is an important tradition of using *changes* to the joint dynamics of income and consumption-choice data to measure how much consumption insurance is in the economy, and also to infer important features of the income process. Relevant examples of the former are Blundell et al. (2008) and Kaplan and Violante (2010). For the latter, Guvenen and Smith (2014) is a leading example. Relative to these papers, I focus on the long-run relationship between the *level* of lifetime income and the *level* of wealth relative to income over the life cycle. The reason is that I am interested in wealth accumulation rather than consumption smoothing and insurance.

Roadmap. The remainder of the chapter is organized as follows. Section 1.2 describes the data set. Section 1.3 describes and implements the estimation strategy. Section 1.4 shows the empirical evidence on non-homothetic behavior across the permanent income distribution for Spain. Section 1.5 presents and discusses estimated income parameter values. Section 1.6 presents a discussion of some limitations of the empirical analysis. Finally, Section 1.7 concludes. Appendix A contains all additional empirical and quantitative results as well as details of the numerical implementation.

1.2 Data

The main data set is composed of six waves that run from 2002 to 2017 of the Survey of Household Finances. The SHF, sponsored by Spain's central bank, collects data on

investment, consumption, and financial decisions of Spanish households. It is the sole source of disaggregated data of its kind in Spain and forms an important source of data for research and public policy[4]. The SHF has a panel dimension in which a household might be included at most during four consecutive waves (some less and some only one). For households interviewed in four waves, they are followed for a period of 10 years. Table 1.1 shows the number of households and observations after imposing a series of restrictions. In the analysis, I restrict the set of households to those that are interviewed four times and that exhibit positive income. In principle, households that are interviewed three times could be included as well. However, this generates a tradeoff between extending the dimension in the cross section and adding more noise in the estimation of permanent income at household level, which crucially depends on the longitudinal dimension. Given that the amount of households when the restriction of four years is imposed (2,848 individuals) is comparable to other studies, I choose to avoid including households with less than four observations.

Table 1.1: Data Sample Restrictions

Restriction	**Households**	**Observations**
Full sample	35,941	35,941
4 waves	2,848	11,392
a) 02-05-08-11	1,497	5,868
b) 05-08-11-14	876	3,504
c) 08-11-14-17	475	1,900
Positive income	2,768	11,072

Note: SHF data from 2002 to 2017.

[4]Based in part on the success of this project, all European Union member states are now mandated by the European Central Bank to conduct a similar nationwide survey in their home countries, known as the "Household Finance and Consumption Survey" (HFCS).

For each household, I construct measures of income, assets, debts, spending commitments such as monthly mortgage payments, and a measure of non-durable consumption (see Appendix A for details). Income includes labor income, self-employment income, unemployment benefits, retirement and permanently disable pensions, public transfers, and private transfers such as child support and alimony. I differentiate between this source of income, which I call *main*, from other sources such as rents from real estate and returns on financial assets. I defined wealth as assets (cash, savings accounts, listed shares, bonds, real estate, unlisted shares, among others) minus debts (credit card debt, mortgages, personal loans, leasing, and others). In the following section I estimate a model for income which corresponds to main. However, when constructing wealth-to-income ratios, income measure includes main income and real estate and financial assets income.

1.3 Estimation Strategy

The statistical model. In line with the standard practice, I assume log income for household i at age t can be represented by the following parametric specification[5],

$$\begin{aligned} \log y_{it} &= \overbrace{\mathbf{x}_{it}'\beta}^{\text{Controls}} + \gamma_{it} \\ \gamma_{it} &= \overbrace{\alpha_i}^{\text{Permanent}} + \overbrace{z_{it}}^{\text{Persistent}} + \overbrace{\nu_{it}}^{\text{Transitory}} \\ z_{it} &= \rho_z z_{it-1} + \epsilon_{z,it}, \end{aligned} \tag{1.1}$$

where $\mathbf{x}_{it}$ is a set of controls and γ_{it} is log income net of predictable individual components. The latter consists of a *deterministic* permanent component α_i, which is the focus in this paper, a persistent component z_{it}, and a pure transitory component ν_{it}. This way of residualizing income by projecting it onto household's observable characteristics follows a

[5]Due to sample size and in particular to the small number of observations for each household (4), the set of specifications I can estimate is highly restrictive. This motivates a simple enough process that is widely used in the literature (see Straub (2019) and others). See Guvenen et al. (2019) for an exposition on more elaborate labor income processes, which may include features like: i) unemployment risk, ii) idiosyncratic persistent labor income, both in levels and in growth, and many others. In this analysis I stick to the simple "standard" approach.

long tradition (some recents examples are Arellano and Blundell (2017) and Blundell et al. (2008)).

The following set of assumptions is made: 1) all random variables are iid across households and time, 2) permanent income α_i and controls $\mathbf{x}_{it}$ are uncorrelated with income shocks $(\nu_{it}, \epsilon_{it})$. Also denote standard deviation of shocks by $(\sigma_\nu, \sigma_\epsilon)$.

The vector of controls includes a cubic polynomial in household head's age, household size, number of earners, number of members below age 14, and year dummies[6]. I include household size, number of kids, and numbers of earners to equivalize household earnings. For instance, suppose two households in the panel have the same controls and income observations but they differ in family size and number of kids. In particular, take a family with two earners and two kids and another family with just two earners. If I do not include number of kids, estimated income residuals $\hat{\gamma}_{jt}$ are the same for both households and I might conclude they have the same permanent income α. However, it is natural to think that the household with two kids will be less capable of saving as much as the household with no kids. Without controlling for family composition, this approach would deliver that these two households have the same estimated permanent income but hold very different wealth-to-income ratios across times. In other words, it would generate too much unexplained variance within permanent income deciles.

I do not include employment status because of two reasons: i) unemployment risks should be contained in the stochastic part of income since it is one of the most important risks households face, and ii) in Spain unemployment risk is a widespread risk faced by many households. Through the lens of this income process, unemployment shocks are captured either by ϵ_{it} or ν_{it}, depending on the nature of the risk.

Estimation of permanent income. The combination of the process described in (1.1) and the SHF data set produces a linear panel-data model with cross-sectional dimension $I = 2,768$ and time dimension $T = 4$[7]. The goal is to obtain estimates for common parameters $(\hat{\rho}_z, \hat{\sigma}_\epsilon, \hat{\sigma}_\nu)$ and individual-household permanent income estimates $\{\hat{\alpha}_i\}_{i \in I}$. The latter are the parameters I am mostly interested in. The remaining parameters will be used

[6] In light of sample size, I do not attempt a richer conditioning.

[7] Remember this is an unbalanced panel because each household appears four times during the six waves that conform the SHF.

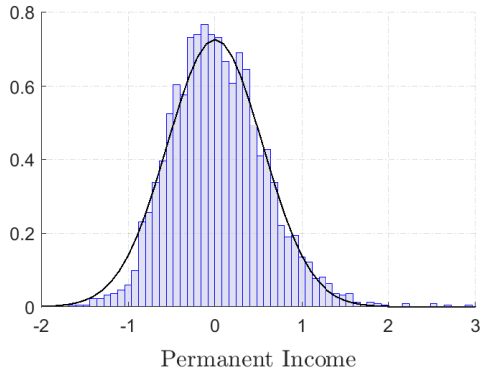
as inputs in the structural model of Chapter 2. By regressing log income on controls, I obtain estimates of the unpredictable component of income $\{\hat{\gamma}_{it}\}$. Following a similar approach to Straub (2019), I approximate permanent income by averaging $\hat{\gamma}_{it}$ across time for each household i,

$$\hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{it}.$$

For long averages, $T \to \infty$, under an appropriate law of large numbers for income process γ_{it}, average of log income residuals $\hat{\alpha}_i$ measures α_i without noise because $\mathbb{E}[z_{it} + \nu_{it}] = 0$.

Two main reasons might cause that the difference between estimated and true permanent income to be non-negligible. First, the fact that $T < \infty$ makes difficult to distinguish between pure persistent income shocks and permanent income. This is a problem hard to overcome due to panel dimension limitations. Second, the closer is ρ_z to one the more indistinguishable persistent income shocks are from permanent income. Importantly, each problem makes the other more severe. To address this issue, in Section 1.6, I run several robustness exercises to assess the severity of these problems. Finally, Figure 1.1 plots the distribution of estimated permanent income levels for each individual household in the SHF. Since it is well approximated by a normal distribution, I will assume in Chapter 2 that α follows a normal distribution whose standard deviation σ_α will be estimated.

Figure 1.1: Permanent Income Distribution



Note: This figure plots the distribution of estimated permanent income (histogram) for each individual household in the SHF sample against a normal distribution (solid).

1.4 Results

In this section I study how wealth accumulation relative to income varies across the permanent income distribution. The goal is to assess whether or not high permanent income households tend to accumulate a disproportionately large amount of wealth compared to low permanent income households.

To do so, I use household-level estimates of permanent income $\{\hat{\alpha}_i\}_{i\in I}$, obtained in the previous Section 1.3, to classify each household i within a permanent income decile[8]. Clearly, $\hat{\alpha}_i$ is an estimate and is used to rank households according to their level of "average" income once controlled by a set of observables. Next, I construct a meaningful measure of the stock of wealth relative to income, where income includes labor income, income from real estate (rents) and income from financial assets. For each household, I compute the wealth-to-income ratio each time it is interviewed, giving a sequence of wealth-to-income ratios $\{\frac{b_{it}}{y_{it}}\}_{i,t=1,2,3,4}$. Further, denote by $\frac{b}{y}(i)$ the median wealth-to-income ratio over a period of 10 years (four observations). The latter gives a sense of how much wealth household i tends to accumulate relative to income. Finally, for each permanent income decile, I compute the median of $\left\{\frac{b}{y}(i)\right\}$ for all i whose permanent income belongs to that decile. This gives an idea of how wealth relative to income varies with permanent income.

Figure 1.2 shows the results for the cross section of households between 20 and 65 years of age. For an easy comparison, I normalize these values to the minimum wealth-to-income value that corresponds to the 2nd decile. A clear pattern stands out, wealth relative to income is positive and strongly correlated with permanent income. For instance, households above the 8th decile exhibit at least ratios 80% larger than households in the second permanent income decile[9]. Also, this analysis shows that wealth-to-income ratios are almost monotonically increasing in the permanent component of income. It means that non-homothetic saving behavior is not only a phenomenon for top earners but also across the distribution.

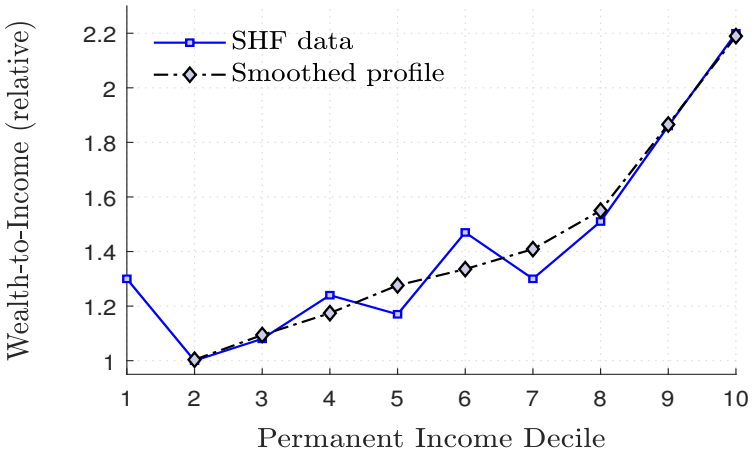
In order to check the robustness of this result, Figure A.1 in Appendix A contains a similar exercise but for different measures of wealth relative to income. Despite some

[8] Motivated by samples size I choose to use deciles instead of percentiles.

[9] As will become clear, the model used in this paper will bot be able to capture behavior for the first decile.

differences, a similar pattern arises, confirming the existence of a positive relationship between permanent income levels and accumulation of wealth relative to income.

Figure 1.2: Wealth to Income across the Permanent Income Distribution - Cross Section

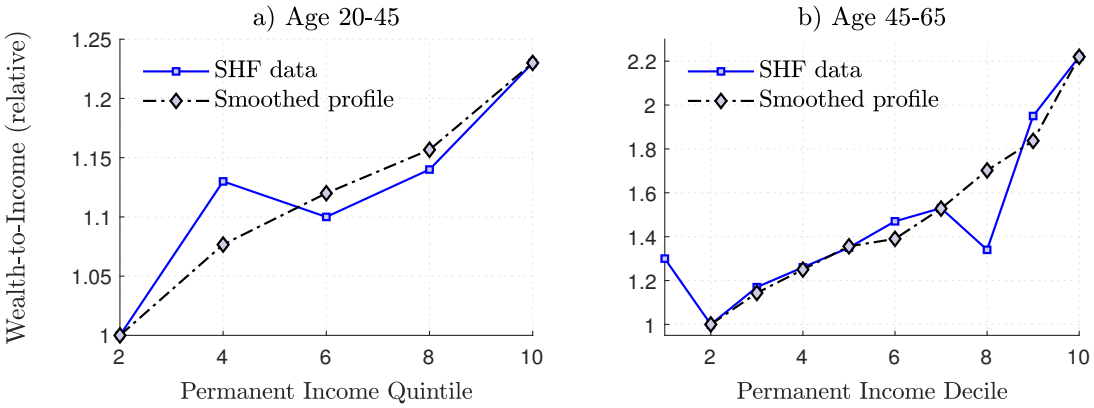


Note: Cross section refers to households with head's age between 20 and 65 years old during the entire period the household is observed. For comparability reasons, median wealth-to-income ratios are shown relative to the the second decile. The smoothed profile (black dashed line) is calculated using LOWESS regression, with bandwidth set to 0.5, omitting the value for the first decile.

Figure 1.2 shows the cross-section of households from age 20 to age 65. However, it is important to condition by age to see whether these differences are always present or get amplified over time. Figure 1.3 splits households below and above 45 years of age and plots relative wealth-to-income ratios across the permanent income distribution. As it can be observed, 1) differences exist for the young and for the old, and 2) these differences tend to be exacerbated over time. While between ages 20 and 45 the top quintile holds median wealth-to-income ratios that are round of 25% larger than the bottom quintile, for households between 45 and 65 years old this difference increase up to more than 100%. Intuitively, if what give rise to these differences is that high permanent income households save relatively more, then it makes sense that wealth-to-income differences get magnified

over time.

Figure 1.3: Wealth to Income across the Permanent Income Distribution - Age



Note: Age refers to household head's age during the entire period the household is observed. Due to sample size, the plot for age 20-45 is restricted to quintiles while for age 45-65 deciles are used. For comparability reasons, median wealth-to-income ratios are shown relative to the quintile (decile) group with a lower value. The smoothed profile (black dashed line) is calculated using LOWESS regression, with bandwidth set to 0.5. On the right panel I omit the value for the first decile.

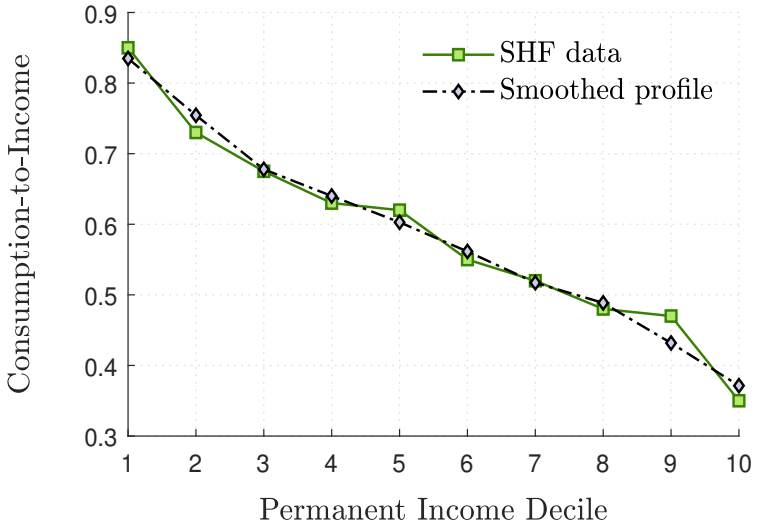
Another piece of information that is informative about differential saving rates across the permanent income distribution is consumption reported in the SHF. If heterogeneous saving rates are important, they should be reflected, at least partially, in consumption to income ratios. That is, households that tend to save a larger share out of income should exhibit lower consumption-to-income ratios. Figure 1.4 shows constructed consumption-to-income ratios across the permanent income distribution using SHF data.

Specifically, consumption includes non-durable consumption reported in the SHF and also spending commitments[10]. The latter consist of all type of loan payments as well as rent payments. This gives a real sense of what is "left" to be saved. Clearly, Figure 1.4 is suggestive evidence that low permanent income households tend to consume a larger fraction of their income compared to high ones. This fact seems to point out that differences

[10]See Appendix A for details on how these variables are constructed.

in active saving behavior out of income might be an important reason. If that is the case, why do high permanent income households tend to accumulate more wealth over time relative to their income? Do they save more? Do they inherit relatively more assets? In Chapter 2 I will use a structural life-cycle consumption-savings model to answer these questions.

Figure 1.4: Consumption to Income across the Permanent Income Distribution



Note: Cross section refers to households with head's age between 20 and 65 years old during the entire period the household is observed. Within each decile, I compute the median consumption-to-income ratio. The smoothed profile (black dashed line) is calculated using LOWESS regression, with bandwidth set to 0.5.

Finally, moments from Figure 1.2 are very valuable because they provide information about the strength of non-homothetic behavior in wealth accumulation for different permanent income groups. I collect them in Table 1.2 to be used in Chapter 2.

Table 1.2: Table of Relevant Empirical Moments

	Permanent Income Decile								
	2	3	4	5	6	7	8	9	10
Age $[20-65)$	1	1.09	1.17	1.28	1.34	1.41	1.55	1.86	2.19

Note: These moments are obtained directly from the smoothed profile using LOWESS regressions, with bandwidth set to 0.5, from Figure 1.2.

1.5 Income Process Parameters

Log income residuals at household level, $\{\hat{\gamma}_{it}\}_{i\in I,t=1,...,4}$, are used to estimate the *stochastic* component of log income, $\alpha_i + z_{it} + \nu_{it}$, in equation 1.1. I denote the set of parameters governing it by $\theta = \{\sigma_\alpha, \rho_z, \sigma_\epsilon, \sigma_{z_0}, \sigma_\nu\}$, where σ_α controls the variance of the distribution from which each household draws its permanent income level, ρ_z controls the autocorrelation coefficient of the persistent component of income, σ_ϵ controls variance of innovations to the latter, σ_{z0} governs variance of initial draw of z, and σ_ν determines variance of transitory iid shocks.

Many alternative methods are used to obtain these estimates. A widely used method by now is the minimum distance estimation (MDE), which finds $\hat{\theta}$ that minimizes the distance between theoretical and sample covariance moments (see details in Appendix A.3). Specifically, I estimate an *annual* income process using triennial longitudinal household-level income data, which is the frequency available in the SHF[11]. Results from MDE are reported in Table 1.3.

The autoregressive coefficient for the persistent income component is 0.8. This value is in line with Garcia-Cabo and Madera (2021) that uses panel data for social security

[11]The idea is to estimate parameters for an annual income process whose *triennial* covariance structure matches the *triennial* empirical structure obtained using SHF data. Differently from Klein and Telyukova (2013), there exists an exact analytical formulas for covariance moments. In their model measured annual income is the sum of 12 monthly income realizations (time aggregation) and Jensen's inequality prevents from exchanging the log of the sum with the sum of logs. However, here I do not face that problem.

labor income[12]. This value is considerably smaller than estimates for the U.S., which typically are larger than 0.9. One possible explanation is that my income measure is very broad, including income from self-employment, which is very volatile, and even from family members outside the household. The standard deviation of the permanent component of income, σ_α, and the standard deviation of shocks to the persistent component of income, σ_ϵ, absorb the larger fraction of variation in the data. Surprisingly, the standard deviation of the i.i.d. component, σ_ν, is very low[13]. Finally, σ_{z0} represents the estimated initial variance households face when entering the job market.

Table 1.3: Income Process Parameters

Parameter	**Description**	**Value**	**Standard Errors**
ρ_z	Persistence of persistent component	0.80	(0.075)
σ_α	Std. dev. permanent component	0.27	(0.08)
σ_ϵ	Std. dev. shocks persistent component	0.39	(0.11)
σ_ν	Std. dev. of iid component	0.013	(0.005)
σ_{z_0}	Initial std. dev. of persistent component	0.05	(0.0023)

Note: This table shows the estimated parameters of the process for log income residuals. Standard errors are block-bootstrapped with 500 iterations and clustered at household level.

[12]They are able to estimate an age-dependent autoregressive coefficient, with values that range from 0.6 for early age to 0.9 for older age.

[13]Given this fact and to reduce the number of states, I will assume in the calibrated model for Spain that $\sigma_\nu = 0$.

1.6 Discussion

In this section I discuss some relevant concerns affecting the validity of the results drawn in Section 1.4.

Performance of the estimation approach. A major issue is that the estimation approach might not very precise because the SHF surveys the same household only fours times in a decade and then drops that household from the panel. This represents a relatively short period of time compared, for instance, to the Panel Study of Income Dynamics in the U.S. This short panel dimension might cause permanent income estimates $\{\hat{\alpha}_i\}_{i\in I}$ to be far away from their true values $\{\alpha_i\}_{i\in I}$. To assess to what extent this problem might lead to wrong conclusions, I perform the following exercise. I test my estimation approach on artificial data generated from a *true homothetic* life-cycle model calibrated for Spain[14]. The estimation approach should be understood as a filter applied to the data. A good filter would be one that is able to pick the homothetic behavior of agents in an economy in which agents are truly homothetic.

Figure 1.5 combines the empirical moments of Figure 1.2 (in blue) and the result of applying the filter to a panel of 50,000 individuals using homothetic model as the true data generating process (in red). Because permanent income is approximated, the line from the full information case[15] (black dashed line) does not fully coincide with the one that comes out of the filter (red line).

Two main conclusions can be drawn. First, the full information case and the filter are very close one to each other. This means that, while noisy, the filter picks the homothetic behavior. A second conclusion is that the filter gives results that are very different when applied to data from the SHF compared to artificial data generated from a true homothetic model. I interpret these differences to be large enough that make it difficult to think that data from the SHF comes from a homothetic model. In other words, I see this as clear evidence of non-homothetic behavior.

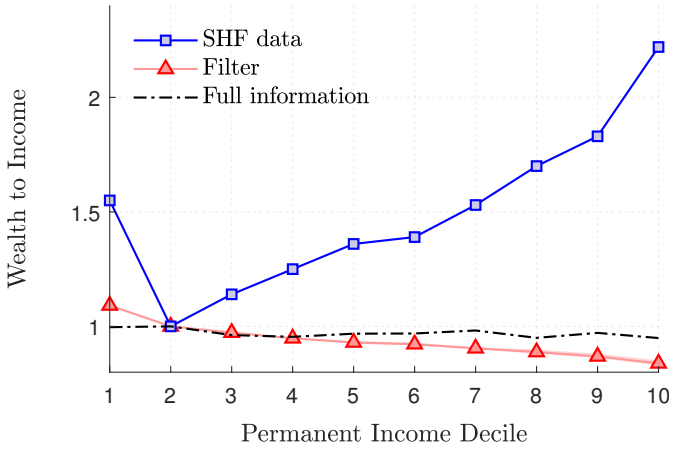
Furthermore, it seems the filter tends to underestimate the wealth-to-income ratios

[14] See Chaper 2 for a details of the model and the parameterization strategy.

[15] Full information case is the case where I observe permanent income type for each household without noise.

for high permanent income households and overestimate it for the low permanent income ones. Intuitively, the reason is that at the bottom of the distribution of estimated α's, I pick transitory shocks to income. Since wealth is much less volatile than income, the ratio goes up. The same logic applies at the top of the permanent income distribution.

Figure 1.5: Performance of the Estimation Approach - Wealth



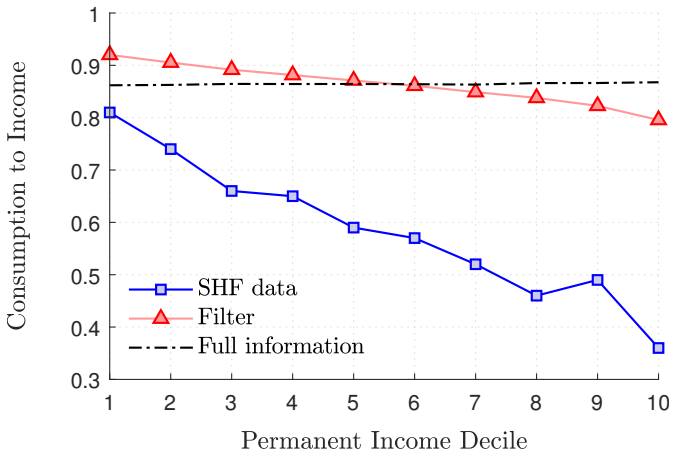
Note: The estimation is applied to households with age between 20 and 65 years old. For comparability reasons, median wealth-to-income ratios are shown relative to the second decile as in the empirical analysis. Within each decile the value corresponds to the median wealth-to-income ratio.

Similarly, I perform the same exercise for consumption-to-income ratios, shown in Figure 1.6. Clearly, the filter I use tend to tilde down a line that is horizontal[16]. Notwithstanding, the magnitude observed when the filter is applied to true data is too large to come from a homothetic model.

I also check how robust the filter (estimation approach) is to sample size. To do so, I restrict the sample to be of the same size as in the the empirical analysis. Figure A.2 in Appendix A.2 shows that the results presented are robust to small sample issues.

[16]The reasoning is that negative income shocks are accompanied by relative small consumption changes (consumption smoothing), pushing measured ratios down (up) at the top (bottom) of the distribution of permanent income.

Figure 1.6: Performance of the Estimation Approach - Consumption



Note: The estimation is applied to households with age between 20 and 65 years old. Within each decile the value corresponds to the median consumption-to-income ratio.

Issues when ρ_z is high. When income is highly persistent the task of differentiating permanent income and highly persistent shocks is arduous. To investigate this problem I replicate main Figure 1.2 when ρ_z is close to one. Figure A.3 in Appendix A.2 shows that the results are robust to this case as well.

Permanent income classification accuracy. Finally, in Table A.1 of Appendix A.4 I show how well the filter classifies households within their true permanent income decile for different values of the persistent component ρ_z. The conclusion is that it does a good job classifying households' permanent income level even when the estimate $\hat{\rho}_z$ is a bit off. For a more detailed explanation see Appendix A.4.

1.7 Conclusion

In this chapter I study the empirical relationship between permanent income and wealth accumulation relative to income for a panel of Spanish households using the Survey of Households Finances run by the Bank of Spain. The analysis finds a strong and positive

relationship between permanent income and different measures of wealth-to-income for working-age households. I find that the median household in the top three deciles of the permanent income distribution tends to exhibit wealth-to-income ratios at least 50% larger than the median household in the 2nd decile. This large variation points out the need to understand why high permanent income households tend to accumulate such a disproportionate amount of wealth.

Chapter 2

Disentangling Key Determinants of Heterogeneous Saving Behavior

2.1 Introduction

For Spain, Chapter 1 provides evidence of a strong and positive relationship between household permanent income and wealth accumulation over the life-cycle relative to income[1]. In other words, that high income households tend to accumulate a disproportionately large amount. This behavior is also referred as non-homothetic saving behavior with respect to permanent income[2].

Previous work has uncovered the empirical importance of the public pension system, transmission of wealth and ability across generations, and preference heterogeneity to understand wealth accumulation and, when introduced in standard life-cycle models, to help improve the fit of the wealth distribution (see De Nardi and Fella (2017)). This paper shows that previous elements are also important to understand non-homothetic saving behavior and, when studied in combination, to make it easier to quantify the relative importance of each.

[1]For the U.S., similar evidence can be found in Dynan et al. (2004), Straub (2019), and Feiveson and Sabelhaus (2019).

[2]See Chapter 1 for empirical evidence for Spain.

I argue that quantifying the contribution of these sources to heterogeneity in wealth-to-income ratios, wealth inequality, and saving behavior is important for at least three reasons.

First, the existence of non-homothetic behavior matters for macroeconomic aggregates[3]. Some recent examples in this direction are Straub (2019) and Kaplan et al. (2018). The former studies the secular decline of real interest rates since the 1970s. Specifically, he relates the increase in permanent labor income inequality, in conjunction with non-homothetic consumption and saving behavior, to conclude that it caused a decline in real interest rates of 1%. The latter assesses the role of monetary policy in a New Keynesian heterogeneous agent model with two assets, one illiquid and another liquid, which give rise to the so-called wealthy hand-to-mouth households. In this economy, the channels by which monetary policy affects real economy are very different from standard heterogeneous agents New Keynesian models. In both, non-homothetic behavior is a *sine qua non* condition to obtain their results[4].

Second, for those that are concerned about wealth inequality, how much of it is a direct consequence of the incentive to save generated by the pension system, wealth accumulated across generations, and preference heterogeneity? In other words, it is important to quantify how much observed inequality stems from income inequality and how much is created and amplified by other factors.

Third, if one is interested in public policy changes (e.g, a delay in retirement age, an increase in inheritance tax rates, and many others), it is important to be aware which of these changes affect the most and how much. Understanding the role of public policies currently in place is important to propose reforms. As Heckman et al. (1998) writes and De Nardi and Yang (2014) remarks, "It is potentially very dangerous to 'solve' problems whose origins are not well understood".

In this paper I decompose the quantitative contribution of different mechanisms the

[3]Macroeconomic models with individual heterogeneity from the 1990s and 2000s feature an *approximate* neutrality, where macroeconomics matters for inequality but inequality does not affect much macroeconomic outcomes.

[4]Straub (2019) assumes non-homotheticity *directly* in preferences and abstracts from other sources that may create it. Kaplan et al. (2018) assumes the existence of a liquid asset and an illiquid asset with fixed adjustment costs, which introduces the main source of non-homotheticity.

literature seems to agree that generate relevant non-homothetic behavior when operating in isolation. For Spain, I develop a quantitative partial equilibrium life-cycle model of consumption and wealth accumulation. The main three sources of non-homothetic behavior are: i) a detailed retirement pension system that captures the main features of the Spanish system, ii) intergenerational links in the form of transmission of permanent income or ability, and wealth between parents and children, and iii) preference heterogeneity in discount factors for different households in the permanent income distribution.

It is important to introduce intergenerational linkages for two reasons. First, previous work in the literature (see De Nardi (2004)) argues that wealth accumulated over generations can partially explain why high income households hold relatively more wealth. The latter is true as long as parents and children permanent income is sufficiently correlated. Second, limitations in the SHF do not allow to distinguish between *accumulated* wealth and *inherited* wealth. To do so, I explicitly include this mechanism in the model.

Some recent papers show heterogeneous returns and preference heterogeneity are quantitatively important to understand wealth inequality (see Xavier (2021)) and differential saving behavior (see Aguiar et al. (2020)). To account for this possibility, I introduce preference heterogeneity in discount factors and use empirical moments inferred from the SHF to quantify how much heterogeneity the model needs to match these moments perfectly. As a need argued by De Nardi and Fella (2017), this paper fills the gap of measuring preference heterogeneity in conjunction with other explanations generating wealth inequality to better understand them and to avoid overstating the role of other individual factors. The empirical and model approach established in this project is an explicit first step towards that goal.

I calibrate the model to the Spanish economy, paying special attention to empirical moments that are informative about parameters governing non-homotheticities. The model allows to turn off one source of non-homothetic behavior at a time, serving as a laboratory to perform the main decomposition exercise in which I quantify the relative contribution of each source of non-homothetic behavior for households along the permanent income distribution.

I find that the pension system and the existence of bequest motives bring empirical and model wealth-to-income ratio moments very close. The intuition is simple, high permanent

income households tend to save more to complement their public pension and also receive more wealth from their parents. However, these two sources are not able to account for the large amount of wealth - relative to income - the top 20% of households accumulate. To this end, the model calibration approach requires a sizable amount of preference heterogeneity in order to match wealth-to-income ratios at the top. That is, it is required that households at the top of the distribution to be relatively more patient to justify the excessive amount of wealth they accumulate. The decomposition approach also allows to assess the role of the pension system, transmission of wealth and ability, and preference heterogeneity as channels that amplify inequality that stems from permanent income differences. The analysis shows that around 63% of total wealth inequality can be explained by precautionary reasons[5]. Around 50% of the remaining inequality can be explained by the pension system, 30% by bequest, and 20% by preference heterogeneity.

Related literature. This paper relates to several strands of a broad literature at the intersection of measurement of non-homothetic saving behavior in the data, theoretical explanations of the latter and wealth inequality, and the implications for public policy programs design.

First, economists have studied empirically whether households with high permanent income tend to save more than those with low permanent income. In his groundbreaking work on the permanent income hypothesis, Friedman (1957) makes the case that the positive *cross-sectional* correlation between income and saving rates reflects the idea that individuals respond to positive (negative) transitory income shocks by running up (down) assets and smoothing consumption. Over time, as more and better quality data was available, some economists challenged this view and concluded in favor of the existence of differential saving behavior (e.g., Mayer (1972)).

In the 90's, Hubbard et al. (1995) revisits the question and claims, at least for the U.S., wealth levels are disproportionally higher among households with high lifetime income. A main drawback of the analysis is the use of cross-sectional data where lifetime income is approximated by education level. More recently, using better quality and newer micro data from the PSID, SCF, and CEX, Dynan et al. (2004) finds a strong positive relationship

[5]Households with good luck tend to accumulate more and households with bad luck smooth shocks by disaving.

between saving rates and lifetime income. Furthermore, Straub (2019) uses PSID data from 1999 to 2013 to estimate a linear relationship between log consumption and log permanent income. He finds a coefficient significantly smaller than one ($\phi \approx 0.7$) and argues that this is evidence of non-homothetic consumption-saving behavior along the permanent income distribution. Also, for the U.S., Feiveson and Sabelhaus (2019) finds clear evidence in favor of the highest permanent income households being the ones that save relatively the most.

This paper is not the first one to study the role of non-homothetic household behavior to try to understand the large amount of wealth differences observed in the data. Hubbard et al. (1995) studies the role of means-tested social insurance programs in causing low permanent income households to save disproportionately less. It does so by introducing a government guaranteed minimum consumption floor in a standard life-cycle model of consumption and savings. De Nardi (2004) constructs a quantitative overlapping-generations model in which parents and children are linked by voluntary bequests and by earning ability. It shows that both ingredients are important to understand empirical wealth concentration in the U.S. Huggett and Ventura (1999) studies the role of the social security pension system in the U.S. and the incentives it generates for high income households to save a disproportionally larger share of their income.

I complement these papers by including in a standard consumption-saving model not just one but many sources of non-homothetic behavior[6]. This allows me to decompose the relative quantitative contribution of each of them. The goal is not only to construct a model that is able to generate non-homothetic behavior, but also to let the model speak, in combination with data, of the importance of each mechanism.

Furthermore, Hubbard et al. (1995) and De Nardi and Fella (2017) point out the potential importance of discount factor heterogeneity in accounting for non-homothetic behavior and wealth concentration in the data, respectively. In principle, it is difficult to measure how much discount factor heterogeneity is in the data because 1) it is not clear which set of moments are useful for identification, and 2) any model with not enough sources of non-homothetic behavior will overestimate its quantitative importance. This paper addresses

[6]The current version of the paper includes voluntary bequests and a social security pension system, but omits means-tested social insurance programs.

this issue because it includes different sources of non-homothetic behavior previous literature argued as relevant, and also because it constructs a set of empirical moments (i.e., median wealth-to-income differences across the permanent income distribution[7]) that, in conjunction with a structural model, are informative about discount factor heterogeneity. This way, I develop a strategy to discipline it.

I build on a partial equilibrium version of the model used in De Nardi and Yang (2014), which documents that, for the U.S., households with similar lifetime earnings tend to exhibit very different wealth levels close to retirement. While De Nardi and Yang (2014) focuses on heterogeneity within a lifetime earnings group, I focus on heterogeneity between lifetime earnings groups.

Finally, this work does not study all possible sources of non-homothetic behavior. For instance, it omits entrepreneurship, health expenses, non-homothetic utility with respect to consumption, and heterogeneous income uncertainty. It also omits behavioral explanations of why low income households tend to save less. For instance, Olivi (2019) proposes a theoretical and identification approach that finds that low income households tend to be overconfident and optimistic, leading them to consume relatively more and to save relatively less.

Roadmap. The remainder of the paper is organized as follows. Section 2.2 introduces the set of non-homotheticities studied in this paper. Section 2.3 introduces the model economy. Section 2.4 provides a homothetic parameterization that will serve as a benchmark. Some results are also discussed. Section 2.5 introduces preference heterogeneity and shows a model calibration in which it is the only source of non-homothetic behavior. Section 2.6 discusses the identification strategy of the full model and shows the main decomposition exercise of the paper. Finally, Section 2.7 concludes.

2.2 Non-Homotheticities

As shown in Chapter 1, high permanent income households tend to accumulate more wealth relative to income, which is true when young but also when old and close to retirement. This section introduces the different sources generating non-homothetic behavior

[7] See Section 1.4.

studied in this paper that will be introduced formally later. Previous work has uncovered that these sources are empirically important to understand wealth accumulation, and that, when introduced in standard life-cycle models, help improve the fit of the wealth distribution. Therefore, they have the potential to be relevant to understand non-homothetic behavior in savings and wealth accumulation behavior. First of all, it is important to formally define what I mean by non-homothetic behavior in savings or consumption. Definition 1 precisely establishes what homotheticity is.

Definition 1 *Let $\mathcal{C}_t(b, \alpha, z, \nu)$ be the consumption function that is a solution of household's individual problem (see model of Section 2.3) where b is net wealth. Households are said to exhibit homothetic behavior if $\mathcal{C}_t$ is homogeneous of degree 1 in assets and permanent income,*

$$\mathcal{C}_t(kb, k\alpha, z, \nu) \;=\; k\mathcal{C}_t(b, \alpha, z, \nu) \qquad \forall\; b, \alpha, z, \nu$$

for $k \geq 0$. The linearity result holds if and only if all households start with zero assets in the first period, or the initial asset position is proportional to permanent income. Another implication is that the saving rate s_t is the same for each permanent income type, where s_t is defined as

$$s_t \;=\; \frac{b_{t+1} - b_t}{y_t + rb_t} \;=\; 1 \;-\; \frac{c_t}{y_t + rb_t}$$

where y_t is income at time t and r net wealth return.

Non-homothetic behavior exists when the properties established in definition 1 are not met.

Preference heterogeneity. The literature finds heterogeneity in preferences to be quantitatively important for describing empirical consumption and saving decisions of households. This heterogeneity might take different forms ranging form differences in discount factors or risk-aversion to differences in inter-temporal elasticity of substitution. In this paper, I choose heterogeneity in discount factors (β) for two reasons. First, Aguiar et al. (2020) uses survey data aggregate moments to identify the amount of heterogeneity in preferences for U.S. households. It infers differences in discount factors play a prominent role in wealth accumulation and differential marginal propensities to consume across consumers. Second,

there is a direct connection between discount factors and wealth returns. Therefore, allowing for variation in β is a way of introducing other sources of heterogeneity as well in a parsimonious way. Xavier (2021) uses data from the Survey of Consumer Finances to document that high wealth households tend to gain a return for their wealth much larger than low wealth households. This feature of the data is also captured in a reduced form way by introducing heterogeneity in β.

Finally, I argue that allowing for variation in discount factors is convenient for two motives. First, I use it to match moments in Table 1.2 and ask how much heterogeneity is needed in the model when it is the only source of non-homothetic behavior. This gives provides an implicit *upper-bound* on the amount of heterogeneity a calibrated version of the model for Spain needs to match important moments. Second, once others sources of non-homothetic behavior such as the pension system are explicitly incorporated in the model, heterogeneity can also be understood as the amount of unexplained variation the model is not able to capture explicitly.

Retirement public pension. As in many countries, in Spain the minimum and maximum pension contributions and pension perceived after retirement from the public pension system are limited from below and above. These limits introduce non-homothetic saving behavior with respect to permanent income. Intuitively, households with different permanent income levels tend to hit lower and upper bounds with different frequencies, which generates a correlation between the median household within each permanent income group, the savings rates, and the amount of accumulated wealth relative to income over the life cycle. In conclusion, the pension system generates incentives for high income households to save relatively more to complement their public pension. Unlike many papers such as De Nardi (2004), I do assume a pension system that is non-homothetic by construction.

Inter-generational links. The literature argues that inter-generational links in the form of bequests and transmission of ability between parents and children are important mechanisms to understand various dimensions of wealth inequality. For the U.S., De Nardi (2004) shows that voluntary bequests can explain the emergence of large states, which are often accumulated over many generations. Bequest motives are assumed to incorporate

two forces, one that controls their strength and that is symmetric to all households (homothetic component) and another that controls to what extent bequests are a *luxury* good (non-homothetic component). The latter implies that only sufficiently wealthy households will decide to voluntary leave bequests to the next generation. Since wealth and permanent income are positively correlated, intergenerational links have the ability to potentially explain part of the moments in Table 1.2. Finally, because permanent income is correlated across generations, high permanent income households also receive more bequests, which amplifies the role of inheritance.

2.3 Benchmark Economy

This section presents a partial equilibrium life-cycle consumption-savings model in the tradition of Bewley (1977), which I refer to as the benchmark model. The main features of this model economy are i) it includes relevant sources of non-homothetic behavior, and ii) it nests an *exact* homothetic behavior model under a certain parameterization, which I call *homothetic parameterization* and explain in detail in Section 2.4. Under the homothetic parameterization, saving rates out of income and wealth accumulation with respect to income are the same across all agents, conditional on a given sequence of shocks and for all permanent income types. This property makes the benchmark economy an attractive laboratory to study and compare other non-homothetic parameterizations and the contribution of each source. I complete this section by explaining the parameterization strategy and some interesting results.

Demographics. The stationary economy is populated by a continuum of dynasties, indexed by d. Each dynasty is composed by an infinite sequence of households, where the dying household is replaced by a new one. Within each dynasty, households are indexed by i. Therefore, each household is indexed by di. Age is indexed by $t = 1, 2, ..., T$. Each household enters the labor market at age 20 ($t = 1$ in the model) and deterministically retires at age 65 ($T^R = 46$ in the model). Households face life-span risk and die with certainty after age 90 ($T = 71$).

Preferences. For household i in dynasty d, preferences are represented by the expected

lifetime utility function[8]

$$\mathbb{E}_0\left[\sum_{t=1}^{T}\Delta_t\beta^{t-1}\Big\{u(c_{it},s_{it}) \;+\; \delta_{t+1}B(\omega_{it+1})\Big\}\right],$$

where $c_{it}\geq 0$ is consumption of nondurables, $s_{it}\geq 0$ is the service flow from housing, β is the discount factor, $B(\cdot)$ is a bequest motive, and ω_{it+1} is wealth. The death probability from age $t-1$ to age t is δ_t and $\Delta_t=\prod_{l=1}^{t}(1-\delta_l)$ is the probability that a household survives to age t. In section 2.4, I discuss the role of housing services in the model.

Income. Household i's gross log income at age t follows the process

$$\log y_{it} \;=\; \begin{cases} \overbrace{g(t)}^{\text{Life-cycle}} + \overbrace{\alpha_i}^{\text{Permanent}} + \overbrace{z_{it}}^{\text{Persistent}} + \overbrace{\nu_{it}}^{\text{Transitory}} & t<T^R \\ \log y_i^R & t\geq T^R \end{cases}$$

where $g(t)$ is the deterministic life-cycle component, α_i is the permanent component, z_{it} is a stochastic persistent shock and ν_{it} is a stochastic transitory idd shock. Following the empirical analysis of Chapter 1, I model the persistent component as the AR(1) process $z_{it}=\rho_z z_{it-1}+\epsilon_{z,it}$, where $\epsilon_{z,it}\sim\mathcal{N}(0,\sigma^2_{\epsilon_z})$. Finally, y^R denotes public retirement pension, which will be explained later.

Intergenerational transmission of wealth and ability. For the US, De Nardi and Yang (2014) shows that introducing voluntary bequests, and realistically calibrating them, matters for the empirical correlation between lifetime earnings and wealth inequality at retirement. There is a large body of evidence that suggests i) households leave bequests, and ii) idiosyncratic labor ability is correlated between parents and children. The goal of this paper is to asses the role of different mechanisms in shaping the relationship between permanent income and wealth accumulation. I assume that children know the amount they will receive from their parents in the form of bequest. However, they are unsure at what age they will receive it (there is uncertainty about parent's death date). I assume all bequests are received between age 30 and 60 within the same dynasty (parents die between age 67 and 91). This assumption avoids having parents's state variables as extra variables

[8]Except when it is needed, I omit subscript d.

but retains the main features of inheritance[9]. Finally, I assume an imperfect transmission of permanent income or ability between households of the same dynasty, governed by the process

$$\alpha_{d,i} \;=\; \rho_\alpha \alpha_{d,i-1} \;+\; \epsilon^T_{d,i}, \qquad \text{where} \quad \epsilon^T_{d,i} \sim \mathcal{N}(0, \sigma^2_{\epsilon^T}).$$

Constraints. Household i starts with inherited initial assets $b_{i1} = 0$. Budget and borrowing constraints at age t are

$$\begin{aligned} c_{it} \;+\; b_{it+1} \;&=\; y^{net}_{it} \;+\; Rb_{it} \;+\; tr^g_{it} \\ b_{it+1} \;&\geq\; \underline{b}(t, \alpha_i) \\ b_{i1} \;&\text{ given} \end{aligned}$$

where y^{net}_{it} is after tax income, b_{it} is assets, and $\underline{b}(t, \alpha_i)$ is a borrowing limit that depends on age and agent type, α_i. Due to the prevalence of housing among households' assets[10], I follow Kaplan and Violante (2014) and assume households potentially obtain a utility flow with proportionality parameter $\xi > 0$ from the entire stock of assets (i.e., $s_{it} = \xi b_{it}$). This utility flow enters the utility function as a service flow from housing[11].

Social security pension system. In Spain, the government runs a *pay-as-you-go* social security system. Working-age households pay a flat payroll tax rate τ_{ss} for income above minimum contribution base $\underline{y}^C$ and below maximum contribution base $\overline{y}^C$. Thus net income can be expressed as

$$y^{net}_{it} \;=\; \begin{cases} y_{it} & y_{it} < \underline{y}^C \\ \underline{y}^C + (1 - \tau_{ss})(y_{it} - \underline{y}^C) & y_{it} \in [\underline{y}^C, \overline{y}^C] \\ y_{it} - \tau_{ss}(\overline{y}^C - \underline{y}^C) & y_{it} \geq \overline{y}^C \end{cases}$$

[9] De Nardi (2004) assumes children form expectations about the size and timing of bequests and this distribution is an endogenous object that needs to be solved for in equilibrium. This approach makes the numerical computation more burdensome.

[10] The median value is 100% and the mean value is 93%.

[11] It is unrealistic that households accumulate such a large amount of assets relative to income just to self-insure against idiosyncratic income shocks and to smooth consumption after retirement. It is important to include as many saving motives as possible.

When retired, households do not pay social security taxes and $y_{it}^{net} = y_i^R$. This system works similarly to FICA in the U.S., where contributions made by workers are used to finance current pensions paid to each retiree. I follow a similar approach to Díaz-Giménez and Díaz-Saavedra (2009) and assume a system that replicates the main characteristics of the Spanish pension scheme. When a household retires, its pension y_i^R is computed using an average of *realized* gross labor income during the previous T^P years to retirement. Concretely, pension is computed as follows

$$\widetilde{y}_i^R = \phi \frac{1}{T^P} \sum_{j=1}^{T^P} \max\left\{\underline{y}^R \ , \ \min\left\{\overline{y}^R \ , \ y_{iT^R-j}\right\}\right\} \tag{2.1}$$

$$y_i^R = \max\left\{\underline{y}^P \ , \ \min\left\{\widetilde{y}_i^R \ , \ \overline{y}^P\right\}\right\} \tag{2.2}$$

where $\underline{y}^R$ and $\overline{y}^R$ are the minimum and maximum covered income, respectively, and ϕ is the contribution rate for income between $\underline{y}^R \leq y_{it} \leq \overline{y}^R$. Public pension is bounded above and below by a minimum and a maximum pension any household can get from the system. I denote them as $\underline{y}^P$ and $\overline{y}^P$, respectively. Observe that parameters $(\underline{y}^C, \overline{y}^C, \underline{y}^R, \overline{y}^R, \underline{y}^P, \overline{y}^P)$ make household's problem non-homothetic while ϕ does not.

For instance, suppose households j and i are equal up to age $t-1$ with the same accumulated amount of pensions rights[12]. Suppose at age t they are hit by different income shocks such that incomes satisfy $y_{it} > \underline{y}^R > y_{jt} > \underline{y}^R$. Using pension formula (2.1), household j's accumulated pension up to age t increases by $\phi \frac{1}{T^P} y_{jt}$, a constant fraction of current earnings. However, for household j it increases by $\phi \frac{1}{T^P} \overline{y}^R$, which is independent of current income. In other words, for income realizations above maximum covered income, accumulated pension does not change. This implies that households in this situation have an incentive to save a larger fraction of current income the larger is the difference $y_{it} - \overline{y}^R$. On the other hand, there is no difference in saving behavior of households j and i if both $y_{it}, y_{jt} \in [\underline{y}^R, \overline{y}^R]$, even though $y_{jt} > y_{it}$. That is, their behavior is just a scaled version one of each other and they save the same fraction of income.

Government and aggregate constraints. The government taxes income from working-age households and uses this income to pay retirees promised pension income. In this paper

[12]They started with the same initial amount of wealth and experienced the same sequence of income shocks up to $t-1$

I perform a partial equilibrium analysis and thus I abstract from imposing any government budget constraint or market clearing condition in the market for bonds.

2.4 Homothetic Parameterization

This section presents the benchmark parameterization that shuts down all sources of non-homothetic household behavior. As previously argued, without non-homotheticities there are no systematic differences in saving behavior across permanent income types. In the calibration approach some parameter values are assigned using external information, others are taken as standard from the literature, and another subset is calibrated inside the model to match certain relevant moments in the data. The last subset is obtained using simulated method of moments.

Preferences and demographics. In the spirit of Carroll (2000), I assume per period utility function is a standard CRRA that additively combines nondurable consumption and housing services as follows

$$\frac{c^{1-\gamma}-1}{1-\gamma} \; + \; \omega \frac{(s+\underline{s})^{1-\gamma}}{1-\gamma},$$

where ω measures the weight of nondurables relative to housing services in per period-utility, γ determines risk aversion ($1/\gamma$ is the *inter-temporal* elasticity of substitution), and $\underline{s}$ represents minimum housing services. Because housing represents 96% of wealth for the median household, I assume households derive utility from the full stock of wealth, $s = \xi b$, where the parameter ξ measures the service flow. Following Kaplan and Violante (2014), I set equal to 1% ($\xi = 0.01$). In line with De Nardi (2004), the bequest motive is assumed to be $B(\omega) = \frac{B_0}{1-\gamma}\big(\omega + B_1\big)^{1-\gamma}$, where B_0 captures the strength of the warm-glow bequest motive, and B_1 captures non-homotheticity in bequest motives[13]. Non-homotheticities in bequest motives are shut down by assuming $B_1 = 0$. Agents live up to $T = 90$ years and each period is a year. They retire at age $T^R = 65$ and enter the economy at each 20.

[13]In particular, it captures the degree to which a bequest is a luxury good. The larger is B_1 the less wealth a household leaves as a bequest. Also, it might happen that for sufficiently low levels of wealth at T, the household decides to leave no bequest and actually wants to borrow. Meaning, that it hits the borrowing constraint.

Mortality probabilities are taken directly from the *Spanish Statistics Institue* (INE).

Income. I use estimates obtained in the empirical section 1.5 to set parameters for the income process. In particular, the life-cycle component of income, $g(t)$, where t denotes age, is modeled as a 3rd order polynomial, $g(t) = g_1(t) + g_2(t)^2 + g_3(t)^3$, where estimated parameter values (g_1, g_2, g_3) are shown in Table 2.1. The AR(1) process autoregressive coefficient is set to $\rho_z = 0.8$ and the standard deviation of innovations to $\sigma_{\epsilon_z} = 0.39$. This implies that $\sigma_z = 0.65$. The estimated standard deviation of i.i.d. shocks to income is 0.03%. This allows to set $\sigma_\nu = 0$, saving a significant amount of computing time. To ensure that the model is homothetic I assume permanent income is perfectly correlated across individuals of the same dynasty, that is, $\alpha_{d,i+1} = \alpha_{d,i}$ for all $d, i \in \mathbb{N}$. Given that the estimated standard deviation for permanent income is $\sigma_\alpha = 0.273$, I assume $\epsilon^T_{d,1} \sim N(0, 0.273^2)$ and then $\epsilon^T_{d,i} = 0$ for all $i = 2, 3, ...$ This assumption will be relaxed in Section 2.6 when I estimate the full non-homothetic model.

Social security. One potential contributor to the non-homothetic empirical evidence presented in Chapter 1 is the specifics of the public pension system. I set parameter values of social security taxes and retirement pension to shut down any non-homotheticity induced by it. In particular, I make sure tax rate is proportional to income as well as the pension received by setting $\underline{y}^C = \underline{y}^R = \phi \underline{y}^P = 0$ and $\overline{y}^C = \overline{y}^R = \phi \overline{y}^P = N$, where N is sufficiently high such that household income is lower than N always. Social security tax rate is set to $\tau^{ss} = 0.08$. In Spain, the replacement rate is a function of the number of months worked. The maximum replacement rate is 0.86 and it can only be achieved if the retiree worked at least 35 years without any unemployment spell. In the pooled SHF data set, the average replacement rate from last salary is approximately 0.41[14]. Therefore, I set $\phi = 0.41$. This parameter is important because it crucially shapes incentives to save near retirement. The lower the replacement rate, the stronger the incentive to accumulate assets the years before retirement. This is common to all households and does not introduce any

[14]The Spanish law penalizes a lot agents with unemployment spells during the 15 years before retirement. This explains the low realized replacement rate observed in the survey.

type of non-homotheticity.

Other. For household with permanent income α_i, the borrowing limit at time t, $\underline{b}(t, \alpha_i)$, takes the form

$$\underline{b}(t, \alpha_i) \;=\; -\kappa \, \frac{\underline{y}(\alpha_i, t+1)}{R} \;>\; \overbrace{-\frac{1}{R}\left(\sum_{s=0}^{T-1-t} \frac{\underline{y}(\alpha_i, t+1+s)}{R^s} \right)}^{\text{Natural borrowing limit}}$$

where $\kappa \in [0, 1]$ and $\underline{y}(\alpha_i, t+1)$ is next period minimum income realization for the household. I assume no borrowing occurs by setting $\kappa = 0$. Since this is a partial equilibrium exercise, I set net asset return to 2%, that is, $R = 1.02$.

Estimation. I use Simulated Method of Moments to jointly estimate the set of parameters $\{\beta, \gamma, B_0\}$. I use the cross-sectional median wealth-to-annual-income ratio from the SHF to inform a the value of the discount factor[15]. In the SHF, the values is 4.6. Similarly, I use a median wealth-to-annual-income ratio of 0.95 for households between 20 and 30 years of age to inform risk-aversion. This moment is relevant because in the model the main saving motive for the young is self-insurance against income fluctuations. Finally, I use a ratio of median wealth at age 80 to age 65 of 0.68, which is an *imperfect* indicator of the importance of bequests as a saving motive. For all moments I use the pooled sample containing all six waves.

[15] As Kaplan and Violante (2014) points out, in the literature on quantitative macroeconomic models with heterogeneous households and incomplete markets, there are two approaches to estimating the discount factor. The first is to match median wealth (e.g., Carroll (1992) and Carroll (1997)). The second is to match average wealth (e.g., Aiyagari (1994), Ríos-Rull (1995), and Krusell and Smith (1998)). There is a trade off in this choice. Matching median wealth allows to reproduce the wealth distribution more closely for the vast majority of households, with the exception of the upper tail that holds a large fraction of total assets. Matching average (and aggregate) wealth allows to fully incorporate equilibrium effects on prices at the cost of overstating wealth holdings for a large fraction of the population. I choose the former approach because for the question at hand, a realistic distribution of wealth across the population if more important that aggregate effects.

Table 2.1: Parameters - Homothetic Economy

Parameter	Description	Value	Target
Preferences			
R	Asset return	1.02	–
β	Discount factor^i	0.963	Wealth-to-income age [20,64]*
γ	CRRA (IES of –)^i	2.466	Wealth-to-income age [20,30]*
ω	Relative weight of housing services in utility	0	–
ξ	Service flow from housing	0.01	–
$\underline{s}$	Minimum housing service	0.05	–
B_0	Bequest motive^i	15.59	Ratio wealth age 80 to 65*
B_1	Non-homotheticity in bequests	0	Assumption (see text)
T	Life-cycle periods	71	–
Social Security			
T^P	Periods used to compute retirement pension^o	15	Data
$\overline{y}^C$	Max. contribution base ($= \overline{y}^R = \overline{y}^P$)	100	Assumption (see text)
$\underline{y}^C$	Min. contribution base ($= \underline{y}^R = \underline{y}^P$)	0	Assumption (see text)
τ^{ss}	Payroll social security tax rate^o	0.08	Data
ϕ	Pension replacement rate^o	0.41	SHF
Income			
g_1	Linear term^o	$1/38.01$	SHF
g_2	Quadratic term^o	$-1/10,482$	SHF
g_3	Cubic term^o	$-1/478,469$	SHF
ρ_z	Persistency of persistent component^o	0.8	SHF
σ_z	Std. of persistent component (%)^o	65	SHF
σ_{ϵ_z}	Std. of innovations to persistent component (%)^o	39	SHF
σ_ν	Std. of transitory component (%)^o	0	SHF (see text)
κ	Borrowing limit	0	Assumption (see text)

*__Note__: Wealth moments correspond to median values. Superscript o indicates parameters estimated outside the model (directly from the SHF). Superscript i indicates parameters estimated inside the model. Parameters of stochastic component of income refer to annual values.

2.4.1 Results

Model and data. Table 2.2 shows targeted and untargeted model and data moments. The model matches targeted data moments almost perfectly. Regarding untargeted moments, the model is not able to capture the high skewness in assets data, a well known problem with the standard formulation. In other words, the model and the parameterization are not designed to capture the upper end of the distribution of wealth-to-income ratios. This can be seen because the model generates a wealth-to-income ratio of 7.72 for the 75th percentile, while in the data is 8.75.

Table 2.2: Key Moments - Model and Data

	Model	Data
Targeted Moments		
Wealth-to-income of non-retirees*	4.6	4.6
Wealth-to-income of age [20,30)*	0.95	0.95
Ratio wealth ages 80/65	0.61	0.68
Selected Untargeted Moments		
Std. log income at age [20,60]	0.72	0.74
Wealth-to-income (all)*	7.33	6.25
Wealth-to-income non-retirees (p75)	7.72	8.75
Median pension to median income	0.46	0.43

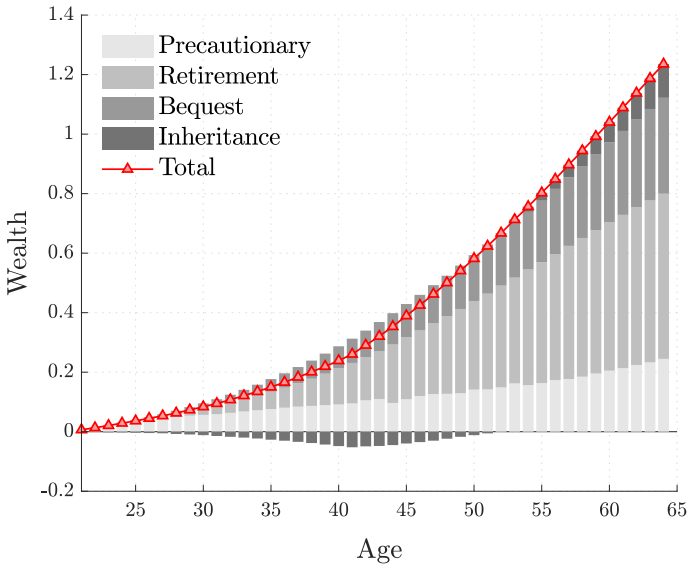
*__Note__: Median values. Shares and ratios are in percentage terms. Income is annual.

Figure B.1 in Appendix B shows that the model-implied cross-sectional distribution of income for working households aligns very well with its empirical counterpart. I use administrative data from Arellano et al. (2021) to construct the empirical cross-sectional

distribution of labor income for Spanish individuals for the year 2018.

Wealth and saving behavior over the life cycle. Figure 2.1 decomposes the contribution of each saving motive on the accumulation of wealth over the life cycle. Precautionary reasons dominate savings in early ages, while retirement is the main motive at the age of 40 and above. Bequest motives, while important, remain relatively small (less than 18 %).

Figure 2.1: Wealth Decomposition - Benchmark Economy



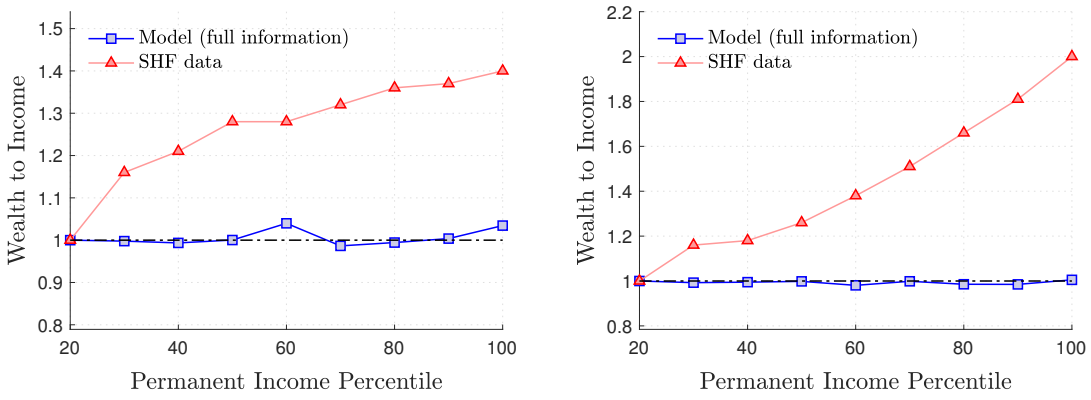
Note: Bequest refers to wealth left to children and inheritance refers to wealth received from parents.

Finally, Figure 2.2 illustrates the main implication of the benchmark model. It plots median wealth-to-income ratios across the distribution of permanent income. By construction, the model implies a flat distribution up to numerical error. In contrast, the data clearly implies a positive and strong relationship. This is true for young age households (left panel) and for older ones (right panel). The inability of the standard model under the homothetic parameterization to account for moments of interest implies that it is an appropriate benchmark to use when studying the implications of each source of non-homothetic

behavior.

Figure 2.2: Wealth across the Permanent Income Distribution: Benchmark Economy

a) Young (between 20 and 44) b) Closer to Retirement (between 44 and 64)



Note: Shares are constructed using the full sample and cross-sectional survey weights.

2.5 Introducing Preference Heterogeneity

In this section I investigate how much heterogeneity in preferences is required so that the model explains observed differences in wealth accumulation relative to income across the permanent income distribution (see moments in Table 1.2). By construction, under the homothetic parameterization of Section 2.4, the benchmark model cannot account for the type of non-homothetic behavior documented so far. To overcome this, I allow agents' discount factor to correlate with permanent income. Within the model, this correlation is the only mechanism that generates non-homothetic behavior.

Methodologically, I proceed as follows. First, I take as given the homothetic parameterization of Section 2.4. Second, I allow for the possibility that (α, β) have non-zero correlation. Third, I calibrate this correlation so that the model perfectly matches moments in Table 1.2. This approach allows to identify the amount of heterogeneity inferred from the data when viewed through the lens of the model.

Parameterization and identification. I set all parameters equal to the ones in the

homothetic parameterization (see Table 2.1) except those calibrated within the model ,$\{\beta, \gamma, B_0\}$. The latter three will be calibrated again within the context of this model. As in many situations, given the limited amount of longitudinal data I have, it is very difficult to measure any form of heterogeneity in preferences. In this paper I take a stand and assume heterogeneity only in discount factors[16]. This choice is motivated by recent empirical evidence on return heterogeneity documented in Xavier (2021) and preference heterogeneity in discount factors found in Aguiar et al. (2020). Agent i has $R_i\beta_i$ as the *effective* discount factor between marginal utility today and tomorrow. Therefore, it is difficult to isolate the effect of idiosyncratic gross returns to wealth, R_i, and patience, β_i. For this reason, I fixed R for all households and load all heterogeneity in β. In particular, I assume the discount factor can take three values, $\beta_i \in \{\beta^L, \beta, \beta^H\}$ such that

$$\beta^L \;=\; (1 - m_\beta)\beta, \qquad \beta^H \;=\; (1 + m_\beta)\beta,$$

where m_β is some scaling parameter and β will be calibrated. Importantly, in order to introduce heterogeneity, I allow the probability of having a specific discount factor to be a function of the permanent income type.

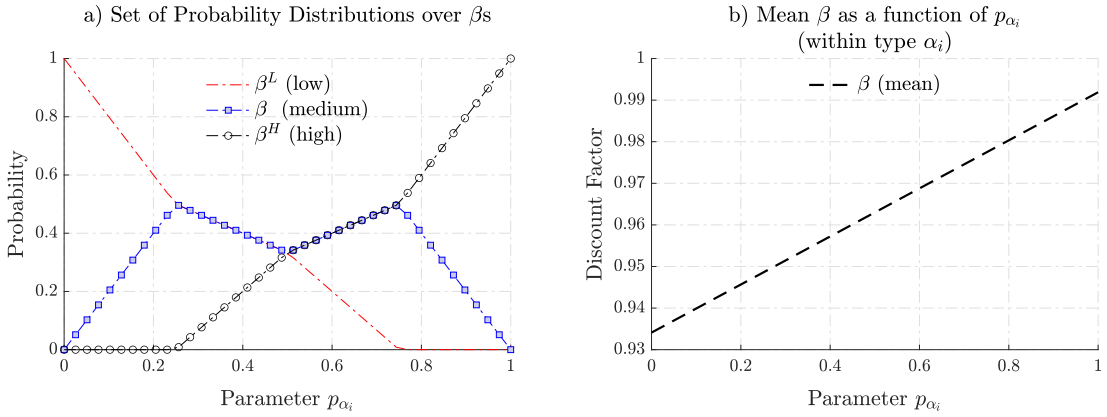
Suppose there are n_α different permanent income types. For each type $\alpha_i, i = 1, 2, ..., n_\alpha$, the probability of getting one of the three discount factors is

$$P\big(\beta = \widetilde{\beta} \mid \alpha_i\big) \;=\; \begin{cases} p_1(\alpha_i) & \text{if} \quad \widetilde{\beta} = \beta^L \\ p_2(\alpha_i) & \text{if} \quad \widetilde{\beta} = \beta \\ p_3(\alpha_i) & \text{if} \quad \widetilde{\beta} = \beta^H \end{cases}$$

where $p_j(\alpha_i)$ is clearly a function of permanent income. Note that this implies there are $3 \times n_\alpha$ parameter values to be estimated. Given the information I have from the SHF, it is very hard to jointly identify all these parameters. For this reason, I parameterize the probability distribution over β's for each permanent income type with just one parameter denoted by p_{α_i}. This way, I reduce the set of parameter values to just n_α, one per permanent income type. Figure 2.3 shows the set of admissible probability distributions parameterized by p_{α_i}.

[16]The SHF contains four observations for each household.

Figure 2.3: Main Features of β's Parameterization



Note: Panel a) shows how the probability of each β_i-type depends on parameter p_{α_i}. As p_{α_i} moves from 0 to 0.5, the probability β^L falls in favor of the other two. At $p_{\alpha_i} = 0.5$, the probability of each discount factor is 1/3. As p_{α_i} approaches 1, the probability of β^H increases. Panel b) shows how the average discount factor within a permanent income type changes as the parameter p_{α_i} moves.

This function assumption restricts the set of convex combinations of the following parameters $\{p_1(\alpha_i), p_2(\alpha_i), p_3(\alpha_i)\}$. In particular, it selects a subset within the 2-simplex. The restriction generates smooth transitions between discount factors' probabilities as p_{α_i} moves. For instance, I do not allow a scenario in which $p_1(\alpha_i) = 0.5$, $p_2(\alpha_i) = 0$, and $p_3(\alpha_i) = 0.5$.

Interestingly, this particular specification of preferences allows me to calibrate preference heterogeneity with the minimal set of parameters. Another problem arises if the number of permanent income types is larger than 10 in the numerical solution because this implies I have more parameters than moments. To solve this problem, I use linear interpolation if $n_\alpha > 10$. That is, if the cumulative density for a specific permanent income type lies between two deciles (say 2 and 3), I interpolate using $(p_{\alpha_2}, p_{\alpha_3})$ to obtain its corresponding parameter.

Table 2.3 shows targeted data moments and model moments. Clearly, the parameter governing the distribution for the second decile cannot be identified given this information. This means I am left with 9 moments and 10 parameters. Furthermore, I reduce the set

of moments to 8 by focusing on moments for the 3rd decile and above. This implies I have 8 moments and 10 parameters There are two options to solve this issue, either I impose further restrictions or I add extra moments that are informative about these two parameters $\{p_{\alpha_1}, p_{\alpha_2}\}$. I choose to set $p_{\alpha_1} = p_{\alpha_2} = 0.5$, that is, the bottom two deciles are composed by an equal share of each discount factor type. This allows me to have 8 parameters to be calibrated using the 8 moments in blue from Table 2.3.

Table 2.3: Table of Cross-Sectional Moments - Data and Model

	Permanent Income Decile									
	1	2	3	4	5	6	7	8	9	10
Data	1.33	1	1.09	1.17	1.28	1.34	1.41	1.55	1.86	2.19
Model	–	1	1.09	1.17	1.28	1.34	1.41	1.55	1.86	2.19

Note: These numbers express the ratio of median wealth-to-income between each decile and the 2nd decile for households between 20 and 65 years of age during the entire period the household is observed. It represents a relative measure capturing long-run non-homotheticities in the data. In blue are the data moments used in the calibration exercise.

The calibration strategy allows the model to perfectly match all data moments. Intuitively, the calibration finds parameters governing the probability distribution over discount factors for each α-type, $\{p_{\alpha_3}, p_{\alpha_4}, ..., p_{\alpha_10}\}$, so that the median household in each decile accumulates as much wealth (relative to income) as in the data. Intuitively, high permanent income households exhibit larger wealth-to-income ratios because 1) they are more patient or have higher returns on their wealth, creating incentive to save relatively more, and 2) since permanent income types are perfectly correlated across generations, they also inherit disproportionately larger amounts of wealth.

How much heterogeneity is needed? In the calibrated version of the model the inferred correlation between permanent income and discount factor is 0.53. This implies

that the model requires a large correlation in order to explain the heterogeneity in wealth accumulation across permanent income types. This must be interpreted as the maximum amount of admissible heterogeneity. The reason is that all other sources of non-homothetic behavior are shut down. Once I turn them off in the following section, this correlation should be much smaller. The question is how much?

Finally, Table 2.4 displays, within each permanent income type, the share of households with discount factors β^L, β, or β^H. Clearly, average discount factor across α-types is positively correlated with α. Mean discount factor is 0.961 with a standard deviation in the cross section of 2.28%. This implies that little ex-ante heterogeneity in discount factors can imply profound ex-post heterogeneity in wealth-to-income ratios.

Table 2.4: Inferred Relationship between Permanent Income and Discount Factor

	Permanent Income Decile (α)							
Discount Factor	3	4	5	6	7	8	9	10
β^L	0.49	0.42	0.4	0.33	0.3	0.11	0.04	0
β	0.49	0.42	0.4	0.33	0.35	0.45	0.48	0.18
β^H	0.02	0.16	0.2	0.33	0.35	0.45	0.48	0.82
Average β	0.947	0.953	0.956	0.961	0.963	0.971	0.974	0.985

Note: This tables shows the inferred share of each preference type and average discount factor within each permanent income decile.

2.6 Full Model: Identification and Results

In this section I use a calibrated version of the benchmark economy for Spain (see Section 2.4) to assess the quantitative importance of the public pension system, bequest motives and intergenerational transmission of wealth, and preference heterogeneity in explaining non-homothetic saving behavior (see empirical moments in Table 1.2). I also use the calibrated model to study the role of the previous mechanisms as amplification channels

of income inequality. To do so, I decompose the quantitative importance of each mechanism in explaining observed wealth inequality.

Identification. The identification approach distinguishes between two sets of parameters. The first one consists of all parameters that govern all sources of non-homothetic behavior, and the second one are the remaining parameters of the model. In particular, the first set is $\{\underline{y}^C, \underline{y}^R, \underline{y}^P, \overline{y}^C, \overline{y}^R, \overline{y}^P, B_0, B_1, \rho_\alpha, p_{\alpha_3}, p_{\alpha_4}, ..., p_{\alpha_{10}}\}$, which govern the public pension system, bequest motives, transmission of ability, and preference heterogeneity[17].

In Spain, the minimum and maximum contribution bases, $\{\underline{y}^R, \overline{y}^R\}$, are also used to define social minimum and maximum security taxable bases, $\{\underline{y}^C, \overline{y}^C\}$. Therefore, it is natural to set $\underline{y}^C = \underline{y}^R$ and $\overline{y}^C = \overline{y}^R$. Furthermore, in Spain the minimum and maximum pension levels, $\{\underline{y}^P, \overline{y}^P\}$ are such that $\overline{y}^P \approx \phi \overline{y}^R$ and $\underline{y}^P \approx \phi \underline{y}^R$. Given this, two parameters pin down the entire pension system, so let's focus on $\{\underline{y}^R, \overline{y}^R\}$. There are two different ways to assign values to these parameters. The first is to assign their corresponding statutory values. However, I think this is a bad idea due to the fact that the system is very complex, with many exceptions and changes over time. For this reason, I follow the second approach, which consists of assigning values to match a key feature of the system that is relevant for non-homothetic behavior. Using social security data, I find that, approximately, 18.7% of individuals receive the minimum retirement pension and that 1.9% receive the maximum. These two moments are very informative about non-homotheticities generated by the pension system. For this reason I calibrate $\{\underline{y}^R, \overline{y}^R\}$ such that the model hits these two moments[18].

Bequest motives are governed by $\{B_0, B_1\}$. There are a wide set of moments used in the literature two pin down these two parameters. Some good examples are De Nardi (2004) and Kaplan at al. (2020). In general, the strength parameter B_0 is used to match the fraction of wealth of households around 60 years old to the amount of wealth of older households. The idea is that this slope is a moment that informs how important bequests are. Regarding the intercept, B_1 is pinned down to match the share of households that

[17] Preference heterogeneity is parameterized and calibrated using the same approach explained in Section 2.5.

[18] The data does not include information at the household level. For this reason, I make the assumption that within a household all agents are equal.

leave no or small bequests. In the SHF the first moment is 0.68[19]. According to the OECD (Balestra and Tonkin (2018)), approximately 35% of Spanish households receive/give estates or donations. I use estimates by Cervini-Plá (2014) for Spain and set the value for intergenerational transmission coefficient to $\rho_\alpha = 0.45$. Given an estimated standard deviation for permanent income of $\sigma_\alpha = 0.273$, the remaining parameter which is unobserved is σ_{ϵ^T}, which takes a value of $\sigma_{\epsilon^T} = \sqrt{\sigma_\alpha^2(1-\rho_\alpha^2)} = 0.2438$.

Finally, as in the previous section, I re-estimate remaining preference parameters $\{\beta, \gamma\}$ to match the same set of moments[20]. The remaining set of parameters are kept fixed and equal to the parameterization in Table 2.1. Table 2.5 shows parameter values, and model and data moments used.

Table 2.5: Selected Parameters - Full Model

Parameter	Description	Value	Target Moment	Model	Data
β	Discount factor	0.969	Wealth-to-income of non-retirees*	4.6	4.6
γ	RRA (IES $\approx$ –)	2.47	Wealth-to-income of age [20,30)*	0.98	0.95
B_0	Bequest motive (strength)	51.7	Ratio wealth ages 80/65*	0.61	0.68
B_1	Bequest motive (intercept)	0.49	Share of HH receiving bequest > 0	0.35	0.35
$\overline{y}^R$	Max. contribution	0.191	Share min. pension (%)	18.6	18.7
$\underline{y}^R$	Min. contribution	0.088	Share max. pension (%)	1.9	1.9

*__Note:__ Data moments are obtained using median values in the pooled sample of the SHF.

Decomposition. In order to understand the role of each source of non-homotheticity in generating the patterns in the data, I re-compute different versions of the benchmark model in which I shut down one mechanism at a time and keep others operating. All parameters, except those involving directly the mechanism, are kept the same as in the calibration of Table 2.5.

[19]This is a large number and the main reason is that this model does not include health care and other type of expenditures important for old people. These expenditures generate an extra motive to save and motives not to run down wealth very fast. In this model these motives are "absorbed" by bequest motives.

[20]I apply the same filter (estimation approach) of Section 1.3 to model-generated data.

Panel a) in Figure 2.4 shows median wealth-to-income ratios across the permanent income distribution in a model with non-homothetic pension system, inter-generational linkages, but no preference heterogeneity. Even though I do not target this set of moments directly, the model is able to capture the shape and magnitude pretty well. However, the model runs short in accounting for the amount of wealth relative to income accumulated by the top 2 deciles of the permanent income distribution. This is where preference heterogeneity kicks in to fill the gap between empirical and model moments. This way preference heterogeneity can be interpreted as a residual or unexplained variation.

In panel b) of Figure 2.4 I show the contribution of the pension system, the recipiency of bequests and bequests left to children in the non-homothetic behavior. Given my calibration, the pension system plays the major role, specifically for the bottom of the permanent income distribution. This result is driven by the minimum pension contribution, which insures households against negative income shocks. Contrary, for the top half of the distribution the reason is that these households receive income whose contribution towards pension is capped, generating incentive to accumulate a disproportionate amount of wealth.

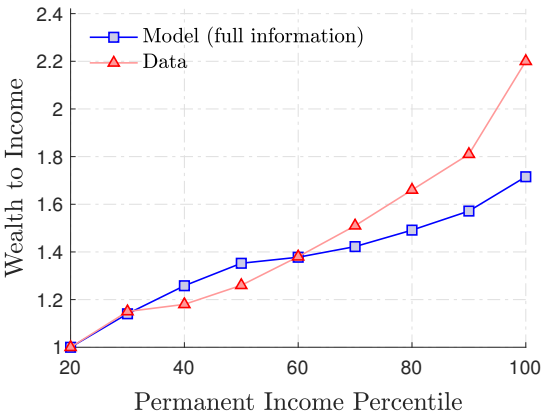
At first sight, it seems striking that despite the fact that only 18% of households perceive the minimum pension and 2% receive the maximum pension, it matters a lot for the median household. The intuition is that they do experience periods of high income in which it does not contribute fully in the pension calculation because the existence also of a maximum contribution. Optimally, they save more to compensate that. The pension system affects households's incentives across the board.

Bequest motives are relevant across the distribution as well. For low permanent income households, the recipiency of bequest from parents is not much relevant but the will to leave bequests to children is. For households on the top 20th percentile, recipiency matters as

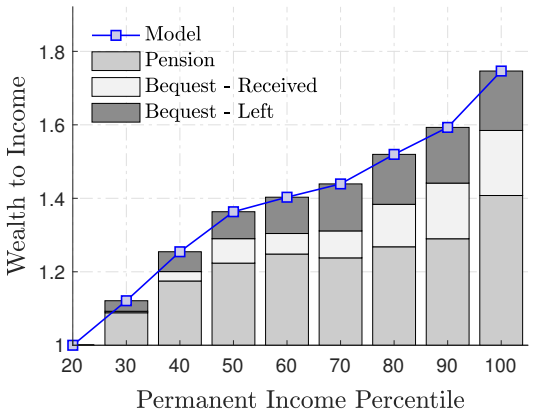
much as giving.

Figure 2.4: Wealth to Income across Permanent Income - Decomposition

a) Model vs Data



b) Decomposition



Note: Plots a) and b) refer to households with head's age between 20 and 65 years old during the entire period the household is observed. For comparability reasons, median wealth-to-income ratios are shown relative to the the second decile.

Table 2.6 quantifies the decomposition shown in Figure 2.4, including preference heterogeneity. Clearly, the public pension system and the incentives generated by it is the primary driver of non-homothetic behavior in the model, accounting for at least 65% of heterogeneity in wealth-to-income ratios. Furthermore, bequest motives to accumulate wealth (either to give or receive) are relatively more important for households at the top of the permanent income distribution, accounting for at least 30% of their non-homothetic behavior.

Interestingly, the contribution of preference heterogeneity for the top decile of the distribution is big. Approximately, 34% of differences observed with respect to households in the second decile can be attributed to differential discount factors. As already pointed out, this should be interpreted with caution. Preference heterogeneity is a residual in the model and should be interpreted as such. However, it is useful because it represent an upper-bound to the amount of heterogeneity in discount factors required to match the data. Clearly, if

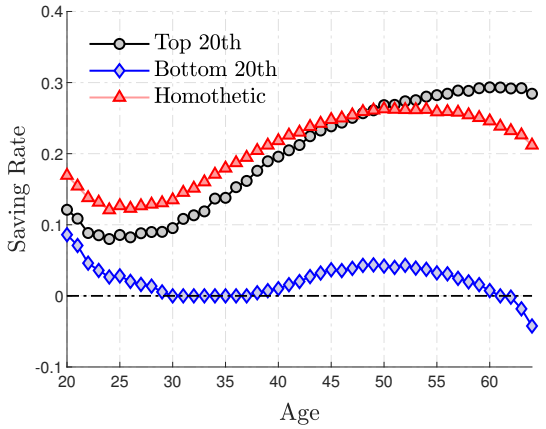
other sources of non-homothetic behavior were added, the 34% contribution would fall.

Table 2.6: Decomposition of Non-Homotheticities

	Pension		Bequest		Preferences
Decile	Min.	Max.	Received	Left	
3rd	73%	0%	3%	20%	4%
6th	53%	9%	13%	25%	0%
10th	0%	35%	17%	14%	34%

Implications for saving rates. Saving rates are very difficult to measure in the data due to limitations in the number of periods households are observed, measurement error, and others. For this reason it is very convenient to use the model to infer saving rates out of income over the life cycle for different permanent income groups. Figure 2.5 shows saving rates for the median household in the bottom and top 20% of the permanent income distribution.

Figure 2.5: Saving Rates



Note: Saving rate is computed as the share of income, excluding returns on wealth, households saves and do not consume.

In both, homothetic and non-homothetic versions of the model, saving rate patters are clear. Early, the median household saving rates is relatively high in order to build a buffer stock of wealth and self-insure against adverse income shocks. Then, saving rate falls to start increasing again to save toward retirement, bequests, etc. Interestingly, the calibrated version of the model with non-homotheticities implies very different saving rate patterns for the median household at the top and bottom 20% of the permanent income distribution. Saving rate difference are big over the life cycle, and compound over time, which explains the large difference of median wealth-to-income ratios for households at the top and bottom of the permanent income distribution. For instance, close to retirement, households at the top save almost 30% of income, while household at the bottom exhibit negative rates. As shown in Table 2.6, the pension system plays a huge role in these inferred patterns.

Implications for wealth inequality. A well known result in the inequality literature is that labor income differences are not enough to explain the large wealth concentration observed in the data, (see De Nardi and Fella (2017)). In a certain class of models, there is the sharp prediction that wealth cannot be more concentrated than earnings which is at odds with the global empirical evidence. In this section, I use the model to study the role of the pension system, bequests, and preference heterogeneity as amplification mechanisms of differences stemming from income. Table 2.7 shows Gini coefficients for income consumption, and wealth. Importantly, even though these moments are not targeted, the full model does a good job at capturing all three Gini coefficients and it can be interpreted as the model doing a good job in that regard. The decomposition approach also allows to assess the role of the pension system, transmission of wealth and ability, and preference heterogeneity as channels that amplify inequality that stems from permanent income differences. The analysis shows that around 63% of total wealth inequality can be explained by precautionary reasons. Around 50% of the remaining inequality can be explained by the pension system, 30% by bequest, and 20% by preference heterogeneity. In conclusion, a standard consumption and savings model calibrated to match income inequality would not be able to match wealth inequality by far. However, the addition of different sources of non-homothetic behavior helps understand why there exists such large wealth inequality in the data.

Table 2.7: Wealth Inequality - Amplification Channels

	Gini Coefficient			
	Permanent Income	Income	Consumption	Wealth
Data (SHF)	–	0.35	0.3	0.63
Full model	0.33	0.39	0.31	0.65
- Precautionary			–	0.41
- Pension			–	0.12
- Bequests			–	0.07
- Preferences			–	0.05

Note: The decomposition of the Gini coefficient for wealth is performed by comparing the difference between the coefficient in the full model and the coefficient in a model in which each source of non-homothetic behavior is turned off.

2.7 Conclusion

This paper studies the role of the public pension system, transmission of wealth and ability, and preference heterogeneity as drivers of non-homothetic wealth accumulation behavior and as amplification channels of income inequality to wealth inequality.

To understand the role of the non-homotheticities proposed in this paper, I build a partial equilibrium model of consumption and saving behavior that nests, under a homothetic parameterization, an economy that cannot account, by construction, for the type of documented non-homothetic behavior. Next, I calibrate for Spain the model and use it to decompose the quantitative contribution of each non-homothetic source. I find that the pension system and the introduction of bequests bring empirical and model moments very close. In particular, the pension system accounts for at least 60% of heterogeneity in wealth accumulation relative to income. The intuition is that the public pension system generates income replacement ratios that are low and motivates high income households

to complement their pension by saving more over the life cycle.

However, these two sources are not able to account for the large amount of wealth - relative to income - the top 20% of households accumulate. To this end, the model calibration approach requires of a sizable amount of heterogeneity in discount factors in order to hit wealth-to-income ratios at the top. This differences should be interpreted as differences in preferences but also in wealth returns. For instance, Xavier (2021) and Fagereng et al. (2020) find these are quantitatively important for the U.S. and for Norway, respectively.

The decomposition approach also allows to assess the role of the pension system, transmission of wealth and ability, and preference heterogeneity as channels that amplify inequality that stems from income differences. Interestingly, the public pension system generates substantial wealth inequality, followed by bequests received and left.

Finally, the analysis in this paper present a series of limitations that will be interesting and fruitful to address in future research. I group these in two main categories. First, I omit other sources of non-homothetic behavior that, once included, may reduce the role of preference heterogeneity. For instance, the role of business ownership, which is specially important for the wealthy, or portfolio choice and capital gains. Since housing represent 95% of net wealth for the median household, incorporating it might have an important impact. Further research will need to introduce explicitly housing in the model as an illiquid asset households derive utility from. This can help disentangle bequest motives from just simply holding housing to enjoy it. In the current model bequest motives and housing motives are confounding factors. Finally, in order to study the impact of policy changes, the model needs to consider general equilibrium to take into account changes in prices and aggregation seriously.

Chapter 3

Optimal Fiscal Policy and Sovereign Default under Incomplete Information

3.1 Introduction

The economic crisis starting in of 2008 in Spain was characterized by a dramatic fall in tax revenues and an increase in government spending. However, statutory labor income tax rates (the largest source of revenue) remained unchanged until 2010 when the government increased rates on all tax brackets. For instance, for the middle income bracket tax rates jumped from 37% to 40% and for the high income bracket from 45% to 52%. A similar picture arises with valued added tax rates, that climbed from 16% to 21%. The initial lack of fiscal adjustment triggered a process of debt accumulation and eventually a rise in sovereign bond spreads.

In this paper I study the role if incomplete information as a possible explanation to rationalize the lack of government fiscal adjustment when the crisis hit and the subsequent debt accumulation episode. In particular, taking advantage of recent theory developed in Hauk et al. (2021), I consider a fully microfounded two-period optimal policy model of sovereign debt and default in which the government issues debt, and decides whether or not to default. I assume the government does not observe the state of the economy and

importantly, it sets tax policy based on a signal -aggregate labor- that is endogenous to the policy itself.

The model provides interesting implications for fiscal policy, debt issuance and pricing. First, the interaction of incomplete information with optimal policy and endogenous signal can generate tax smoothing. That is, optimal tax policy calls for a smaller reaction to shocks compared to the complete information case. This lack of fiscal adjustment can lead to accumulation of large debt over time and an increase in default probabilities. Second, optimal policy implies highly non-linear response of taxes to shocks[1]. The government might change very quickly from reacting very little when labor and output fall to increasing taxes strongly as employment and output further falls. This second implication is relevant in light of what happened to several European countries during the so called Great Recession. It was observed that many countries did little fiscal adjustment at the beginning, which led to rapid accumulation of debt and eventually to a large fiscal adjustment.

Finally, it is important to keep in mind this paper abstracts from other valid explanations of the late observed fiscal adjustment and debt accumulation that took place during and after the 2008 recession. It is important to acknowledge that other explanations related to political economy issues had first-order importance. However, in this paper I solely focus on one explanation.

Related literature. This paper is in the intersection of the literature that studies sovereign debt dynamics and default and the literature that focuses on optimal policy when signals are *endogenous* and information is *incomplete*.

This paper builds on a simple defaultable debt version of the Lucas and Stokey (1983) model of optimal dynamic taxation under uncertainty. Defaultable debt is modeled in the tradition of Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2006), and Aguiar et al. (2019), where default is determined by a strategic government comparing the value of repayment to the value achieved by default.

Signal extraction with an *exogenous* signal is well understood and it goes as far back

[1]Optimal policies that are non-linear have been studied in the literature but for other reasons. As pointed out in Hauk et al. (2021), Swanson (2006) obtains a non-linear policy when the normality assumption with separable shocks is relaxed. The non-linearity arises entirely from the Bayesian updating on prior non-Gaussian shocks.

as Muth (1960). However, fewer papers have studied optimal policy when signals are endogenous to the policy itself. Some instances are Pearlman (1992), Svensson and Woodford (2003), and Svensson and Woodford (2004). Unlike these examples the model in this paper does not feature either the "separation principle" or the "certainty equivalence" property that makes the problem of solving for the optimal policy - when the signal is endogenous- much simpler. To deal with this technical difficulties, I make extensive use of theory develop in Hauk et al. (2021).

Roadmap. The remainder of the chapter is organized as follows. Section 3.2 describes the model in detail. Section 3.3 presents the Ramsey problem and characterizes the solution. Section 3.4 explains the solution algorithm. Section 3.5 presents and discusses the outcome of this economy. Finally, Section 3.6 concludes.

3.2 Model

In this section I present a simple defaultable debt version of the Lucas and Stokey (1983) model of optimal dynamic taxation under uncertainty. Defaultable debt is modeled in the tradition of Eaton and Gersovitz (1981), Arellano (2008), Aguiar and Gopinath (2006), and Aguiar et al. (2019), where default is determined by a strategic government comparing the value of repayment to the value achieved by default. I relegate details on information structure and associated policy instruments to Section 3.3.

Preferences and technology. The economy lasts two periods $t = 1, 2$ and it is populated by a continuum of households of measure one, indexed by $i \in [0, 1]$. I assume households are all identical and I drop the subscript i for notational convenience when needed. Each household receives in period one a realized value of labor productivity θ and consumption preference parameter γ for that period. Let $\delta = (\theta, \gamma)$. The random variable δ is drawn from probability distribution $\mathcal{P}$ over support Δ. Each household has expected utility function

$$\gamma \frac{c_1^{1-\eta} - 1}{1-\eta} - \phi \frac{l_1^{1+\sigma}}{1+\sigma} + \beta \mathbb{E}_\kappa \left[\frac{c_2^{1-\eta} - 1}{1-\eta} - \phi \frac{l_2^{1+\sigma}}{1+\sigma} \right], \tag{3.1}$$

where c_t and l_t are consumption and hours worked in period t, respectively. I refer

to random variable γ as demand shock with support $[\underline{\gamma}, \overline{\gamma}]$. High realizations of γ imply i) consumption becomes relatively more valuable and households work more and consume more, and ii) consumption in the first period becomes relatively more important, which means that households are less willing to save towards second period consumption. Observe expectation operator for continuation utility is with respect to random default cost variable κ. In this economy production is a linear function of labor. Thus output takes the form

$$y_t = \theta_t l_t - \kappa d,$$

for $t = 1, 2$. Variable d takes value of one if the government defaults on its debt or 0, otherwise. Variable κ is an exogenous aggregate cost the economy incurs if government decides to default. Productivity in the second period, θ_2, is deterministic but productivity in the first period, $\theta_1 = \theta$, is a random variable with support $[\underline{\theta}, \overline{\theta}]$. Period budget constraints of the representative agent are

$$\begin{aligned} c_1 + qb &= \theta l_1(1 - \tau_1) \\ c_2 &= \theta_2 l_2(1 - \tau_2) + b(1 - d) \end{aligned}$$

where τ_t is labor income tax rate, b are bonds used to transfer resources between the first and the second period, and q is bond price. In the first period households pay income taxes, consume and saver or borrow by buying or selling an amount of bonds qb. In the second period, they pay income taxes and receive the face value of bonds if government does not default and zero otherwise. They use all available resources to consume.

The government. The government needs to finance a deterministic stream of spending g_1 and g_2. To do so it can levy distortionary labor income taxes (τ_1, τ_2) and also it can issue defaultable bonds b^g in the first period at price q. In the second period the government decides whether or not to default and then income tax rate τ_2. Default decision is strategic and it is done by comparing the value of paying back debt against the value of not paying back. Default has an associated stochastic cost in terms of aggregate resources of $\kappa \sim U[0, \overline{\kappa}]$[2]. This cost is realized at the beginning of second period. Government is

[2]Observe that absent this default cost government would always default in the second period, which would make positive amounts of debt unsustainable in the first period.

subject to the following budget constraints

$$g_1 = \theta l_1 \tau_1 + q b^g \tag{3.2}$$

$$g_2 + b^g(1-d) = \theta_2 l_2 \tau_2 \tag{3.3}$$

Household problem. The problem of the representative household consists of choosing functions $\{l_1(\delta), c_1(\delta), b(\delta)\} : \Delta \to \mathbb{R}$ and $\{l_2(\delta,\kappa), c_2(\delta,\kappa)\} : \Delta \times U[0,\overline{\kappa}] \to \mathbb{R}$ mapping exogenous states into actions to attain

$$\max \int_{\delta\in\Delta} \left\{ \gamma(\delta) \frac{c_1(\delta)^{1-\eta}-1}{1-\eta} - \phi \frac{l_1(\delta)^{1+\sigma}}{1+\sigma} + \beta \mathbb{E}_\kappa \left[\frac{c_2(\delta,\kappa)^{1-\eta}-1}{1-\eta} - \phi \frac{l_2(\delta,\kappa)^{1+\sigma}}{1+\sigma} \right] \right\} d\mathcal{P}(\delta) \tag{3.4}$$

subject to budget constraints

$$c_1(\delta) + q b(\delta) \leq \theta l_1(\delta)(1-\tau_1) \qquad \forall\ \delta \tag{3.5}$$

$$c_2 \leq \theta_2 l_2 (1 - \widetilde{\tau}_2(\kappa) + b\big[1 - \widetilde{d}(\kappa)\big] \qquad \forall\ (\delta,\kappa) \tag{3.6}$$

$$c_1, c_2 \geq 0\ , \quad l_1, l_2 \in [0,1] \tag{3.7}$$

Importantly, households treat fiscal functions mapping exogenous states into actual policies, $\{\tau_1(\delta), q(\delta), \tau_2\big(\delta,\kappa\big), \tau_2\big(\delta,\kappa\big)\}$, as exogenously given. Because households have rational expectations, these are actual equilibrium values of taxes and government bonds for each δ and κ. Next, I provide the standard definition of competitive equilibrium in this economy, which applies to complete and incomplete information economies.

Definition 2 (Competitive Equilibrium) *Given spending levels (g_1, g_2), a competitive equilibrium is defined by fiscal policy functions $g = \{\tau_1(\delta), \tau_2(\delta,\kappa), b^g(\delta), d(\delta,\kappa)\}$, bond price function $q(\delta)$ and allocation functions $x = \{c_1(\delta), c_2(\delta,\kappa), l_1(\delta), l_2(\delta,\kappa)\}$ such that when agents take as given g, allocation x solves households' problem. Also, bond market and goods markets clear, so that*

$$\begin{aligned} b^g &= b, \\ \theta_1 l_1 &= c_1 + g_1, \\ \theta_2 l_2 &= c_2 + g_2 + \kappa d. \end{aligned}$$

Government's budget constraints hold in both periods due to Walras' law.

Equilibrium characterization. The following set of equations characterize any competitive equilibrium for this economy:

$$\gamma u'(c_1)\theta(1-\tau_1) = \phi v'(l_1) \tag{3.8}$$

$$\gamma u'(c_2)\theta_2(1-\tau_2) = \phi v'(l_2) \tag{3.9}$$

$$q = \frac{\beta}{\gamma}\mathbb{E}_\kappa\left[\frac{u'(c_2)}{u'(c_1)}(1-d)\right] \tag{3.10}$$

$$\theta l_1 = c_1 + g_1 \tag{3.11}$$

$$\theta_2 l_2 = c_2 + g_2 + d\kappa \tag{3.12}$$

$$c_1 + qb = \theta l_1(1-\tau_1) \tag{3.13}$$

$$c_2 = \theta_2 l_2(1-\tau_2) + b(1-d) \tag{3.14}$$

Equations (3.8) and (3.9) are intratemporal optimality conditions between leisure and consumption. Equation (3.10) is the pricing condition for bonds and represents intertemporal optimality of households. Equations (3.11) and (3.12) are market clearing conditions and finally, equations (3.13) and (3.14) are household's budget constraints[3].

3.3 Ramsey Equilibrium with Incomplete Information

This section presents assumptions about information structure and timing in this economy, the definition of a Ramsey equilibrium with incomplete information, the problem government faces, and the set of first order conditions that characterize the Ramsey solution.

Beyond the primal approach. Following Hauk et al. (2021) I deviate from the "primal approach" as a standard way of finding the Ramsey solution of this type of economies. I proceed backwards from period two. State variables in the second period are stock of bonds b and default cost κ. For each pair (b, κ) and given fiscal decisions $\{\tau_2, d\}$, intratemporal optimality condition (3.9), market clearing condition (3.12), and household

[3]Government's budget constraints hold in both periods due to Walras' law.

budget constraint (3.14) can be combined to obtain equilibrium allocations (c_2, l_2). I use these to obtain the continuation value $\mathbb{E}_\kappa\big[V(b,\kappa)\big]$. This represents the expected utility obtained by households when the economy enters the second period with an outstanding amount b of debt.

In the first period, I combine intratemporal condition (3.8) and goods market condition (3.11) to obtain households' labor optimal choice given the state of the economy

$$l_1 \;=\; h(\theta, \gamma, \tau_1) \tag{3.15}$$

One should think of function $h(\cdot)$ as households' best response to fiscal policy τ_1 given state of nature (θ, γ). Next, I combine goods market condition (3.11), household's budget constraint (3.13), and bond pricing equation (3.10) to define an implicit bond function

$$b \;=\; b_{\text{imp}}(\theta, \gamma, l_1, \tau_1) \tag{3.16}$$

Unlike to the primal approach, this way of proceeding allows to map any fiscal policy $g = \big\{\tau_1(\delta), \tau_2(\delta,\kappa), b^g(\delta), d(\delta,\kappa)\big\}$ into a competitive equilibrium[4]. The main difference is that it generates a direct mapping between taxes, bond and default decisions into equilibrium quantities. This will prove to be a useful way to study optimal policy under incomplete information.

Information structure and timing. First, nature draws state $\delta = (\theta, \gamma)$ using $\mathcal{P}$, which only households can observe. Next, government announces and commits to a labor-contingent tax policy function $T : \mathcal{S} \to \mathcal{T}$ such that $\tau_1 = \tau = T(l)$. That is, tax function maps the set of signals $\mathcal{S}$ (realized aggregate labor in this case) into the set of first-period tax rate, $\mathcal{T}$. In other words, the government cannot condition its actions to the state of the economy. It has to commit to a tax policy that maps equilibrium labor to tax rates. In real life policy making, a government decides policy based on real time aggregate outcomes that are actually endogenous to that policy. When this is the case, one can say that the signal is endogenous to the policy.

[4]In the primal approach the goal is to get rid off tax instruments and to find allocations that maximize the objective function subject to a set of constraints as a function of allocations only and to a set of implementability constraints any competitive equilibrium has to satisfy.

Household i, given belief about aggregate behavior l, chooses labor $l_i = h\big(\delta, T(l)\big)$. Because households are assumed to have rational expectations, it must be the case that[5]

$$\int_i l_i \;=\; h\big(\delta, T(l)\big) \;=\; l.$$

Once households decide how much labor to supply, government tax rate is set by the fiscal policy such that $\tau = T(l)$ and debt issuance by $b^g = b_{\text{imp}}\big(\delta, l, T(l)\big)$. I assume the second period is trivial and it features complete information. That is, given state (κ, b^g), the government decides whether or not to default and the tax rate. For high levels of default cost κ, the government will choose not to default. However, for low levels of κ default becomes a great tool. The reason is that default allows to transfer resources from creditors (the households) to the government in a non-distortionary way. The government can default and simultaneously reduce the labor tax rate, which becomes an optimal strategy as long as κ is small enough. Finally, households choose how much to work and consume.

Definition 3 (Ramsey Equilibrium with Incomplete Information) *In this economy, a Ramsey equilibrium with incomplete information is characterized by a signal-contingent policy function*

$$\tau_1 \;=\; T(l) \quad \textit{for all} \quad \delta \in \Delta$$

such that T maximizes households' utility, T is a measurable function from $\mathbb{R}$ into $\mathbb{R}$, and allocations are determined in a competitive equilibrium.

Government problem. The Ramsey planner chooses signal-contingent policy function T to maximize households's utility subject to the constraints any competitive equilibrium must satisfy. A compact version of the problem is[6]

$$\max_{T} \int_{\delta \in D_\delta} \left\{ \gamma \frac{(\theta l - g_1)^{1-\eta} - 1}{1-\eta} - \phi \frac{l^{1+\sigma}}{1+\sigma} \;+\; \beta \mathbb{E}_\kappa \Big[V(b, \kappa) \Big] \right\} d\mathcal{P}(\delta)$$

[5] Since agents are assumed to be identical, aggregate and individual choices coincide in equilibrium. However, it is important to understand that the policy *only* depends on aggregate outcomes, justifying the assumption that households take the policy action τ_1 as given.

[6] See that I plug into the objective function households' consumption from aggregate feasibility and I use continuation value as a function of bonds. Continuation function is the result of imposing a competitive equilibrium in the second period.

subject to:

$$l = h(\delta, \tau) \tag{3.17}$$

$$b = b_{\text{imp}}(\delta, l, \tau) \tag{3.18}$$

$$\tau = T(l) \tag{3.19}$$

where (3.17) is individual reaction function, (3.18) is the implicit bond function , and (3.19) is the tax policy function. In order to map this problem to the canonical problem in Hauk et al. (2021), I plug the implicit bond function (3.18) in the continuation value to get the following problem,

$$\max_{T} \int_{\delta \in D_\delta} \underbrace{\left\{ \gamma_1 \frac{(\theta l - g_1)^{1-\eta} - 1}{1-\eta} - \phi \frac{l^{1+\sigma}}{1+\sigma} + \beta \mathbb{E}_k \Big[V\big(b_{\text{imp}}(\delta, l, \tau), \kappa\big) \Big] \right\}}_{\equiv W(\tau, l, \delta)} d\mathcal{P}(\delta) \tag{3.20}$$

subject to (3.17) and (3.19), where $W(\tau, l, \delta)$ is the utility achieved by households given the labor they supply, the state of the world, and the fiscal policy in period one.

Characterization of the Solution. I follow theory developed in Hauk et al. (2021) to obtain a FOC that characterizes the Ramsey equilibrium with incomplete information. First, I combine reaction function (3.17) and policy function (3.19) to get

$$l = h(\theta, \gamma, T(l)). \tag{3.21}$$

Given T, equation (3.21) implies that for an observed labor signal, the government is always confused because he does not know which combination of (θ, γ) gives rise to equilibrium l. Next, denote $\mathcal{S}(\delta, T)$ as the set of equilibrium labor for a specific exogenous state δ and for a given policy function T. Formally, It is defined as

$$\mathcal{S}(\delta, T) \equiv \Big\{ l \in \mathbb{R} \mid l = h(\delta, T(l)) \Big\}. \tag{3.22}$$

In other words, this represents the set of fixed points. Under certain conditions $\mathcal{S}(\delta, T)$ is nonempty and a singleton[7]. In this paper I assume conditions to ensure $\mathcal{S}$ is a singleton

[7]Mainly, $|h_\tau T'| < 1$ and compactness of $\mathcal{S}$. Intuitively, condition $|h_\tau T'| < 1$ ensures what is known in the literature by *weak strategic complementary* between agents, which ensures it is a contraction and a unique fixed point exists. For a more detailed explanation see Hauk et al. (2021).

are met. Also, define $\mathcal{T}(\delta, T) \equiv T(\mathcal{S}(\delta, T))$, which is the realized tax rate for a given exogenous variable δ and tax scheme T. Now, I am ready to state government's problem in a compact and general form,

$$\max_{T:[0,1]\to[0,1]} \int_{\delta\in\Delta} W\big(\mathcal{T}(\delta, T), \mathcal{S}(\delta, T), \delta\big)\, d\mathcal{P}(\delta). \tag{3.23}$$

That is, basically the Ramsey planner needs to choose and commits to a policy function T that maps equilibrium labor to tax rates and this is the instrument it uses to maximize welfare. Finally, Hauk et al. (2021) shows that the optimality condition for this canonical problem (3.23) is

$$\mathbb{E}_\delta\left[\frac{W_\tau^* + W_l^* h_\tau^*}{1 - h_\tau^* T^{*\prime}} \mid \mathcal{S}(T^*, \delta)\right] \;=\; 0 \tag{3.24}$$

Observe that the numerator in (3.24) is the standard first order condition for a Ramsey planner under complete information. Specifically, the government understands the direct effect of tax rate changes in welfare (W_τ^*), but also internalizes that tax rates do affect labor supply and subsequently welfare ($W_l^* h_\tau^*$). Furthermore, the Ramsey panner under incomplete information gives more weight to realizations such that a small change in the policy has large effects on equilibrium labor, represented by the scaling factor $\frac{1}{1-h_\tau^* T^{*\prime}}$, which acts as a kernel or measure change[8].

3.4 Solution Algorithm

Because the condition (3.24) is hard to deal with[9], Hauk et al. (2021) shows that a necessary and sufficient condition optimal policy T^* also has to satisfy is

$$F(T^*, l) \;\equiv\; \int_{\gamma\in\Gamma(l,T(l))} \frac{W_\tau + W_l h_\tau}{\mid h_\theta \mid} \cdot f_{\theta|\gamma}(\Theta(l,\gamma),\gamma) \cdot f_\gamma(\gamma) \cdot d\gamma \;=\; 0 \tag{3.25}$$

for all l with positive density, where

- $\Gamma(l, T^*(l)) \;=\; \Big\{\gamma \;:\; l = h(\theta, \gamma, T^*(l)) \text{ for some } \;(\theta, \gamma)\Big\}$. In words, this is the set of γ's that are consistent with the reaction function for some θ.

[8] For a more detailed discussion see Hauk et al. (2021).

[9] Observe it contains the derivative of the equilibrium policy function $T^{*\prime}$.

- $\Theta(l, \gamma) = \theta^*$ where it satisfies $l = h(\theta^*, \gamma, T^*(l))$

The algorithm I use to solve this problem works as follows:

1. **Discretization**

   (a) Discretize the set of possible values of l. Be smart when finding l_{min} and l_{max}, because in the extremes there is full revelation of the state. $I_l = \{l_1, l_2, ..., l_{n_l}\}$.

   (b) For θ and γ: Find $I_\theta \approx \{(\theta_1, w_1), ..., (\theta_{n_\theta}, w_{n_\theta})\}$, and $I_\gamma \approx \{(\gamma_1, w_1), ..., (\gamma_{n_\gamma}, w_{n_\gamma})\}$. If variables are Gaussian I use *Gauss-Hermite Quadrature* to select grid nodes and corresponding weights. One may use uniform distributions or Gaussian. In the analysis of Section 3.5 I use independent uniform distributions.

2. **Newton's**

   (a) Initial guess $\tau = T_n(l)$. Represent it using a piece-wise linear function. In particular,

$$T_n(l) = \sum_{i=1}^{n_l} \lambda_i \psi(l)$$

   (b) For each $l_i \in I_l$

      i. Construct the set $\Gamma(l_i, T_n(l_i))$

      ii. For each $\gamma_i \in \Gamma(l_i, T_n(l_i))$, find $\theta_j \in \Theta(l_i, \gamma_i)$

      iii. Compute $f_{\theta|\gamma}(\theta_j, \gamma_i) = f_\theta(\theta_j)$, which is true because shocks are assumed to be independently distributed.

      iv. With all of this at hand, I can compute

$$F(T_n, l_i) \equiv \int_{\Gamma(l_i, T(l_i))} (\cdot)$$

   (c) Then, I have $\left\{F(T_n, l_1), ..., F(T_n, l_{n_l})\right\}$

   (d) The goal is to find $\{\lambda_1, ..., \lambda_{n_l}\}$ such that

$$\underset{(n_l \times 1)}{\begin{bmatrix} F(\mathcal{T}, l_1) \\ F(\mathcal{T}, l_2) \\ ... \\ F(\mathcal{T}, l_{n_l}) \end{bmatrix}} = \overrightarrow{0}$$

(e) Update as follows until the vector is sufficiently close to zero:

$$\underset{(n_l \times 1)}{\begin{bmatrix} \lambda_{n+1,1} \\ \lambda_{n+1,2} \\ \dots \\ \lambda_{n+1,n_l} \end{bmatrix}} = \underset{(n_l \times 1)}{\begin{bmatrix} \lambda_{n,1} \\ \lambda_{n,2} \\ \dots \\ \lambda_{n,n_l} \end{bmatrix}} - \underset{(n_l \times 1)}{\begin{bmatrix} \lambda_{n+1,1} \\ \lambda_{n+1,2} \\ \dots \\ \lambda_{n+1,n_l} \end{bmatrix}}$$

3.5 Results

In this section I show optimal policy and equilibrium variables for the model in Section 3.2 and information structure and equilibrium concept introduced in Section 3.3.

Parameterization. I follow a simple parameterization to illustrate the qualitative properties of this economy. Table 3.1 shows such parameter values. I set government spending in the first period to be bigger than in the second period. This assumption is important to make sure that in equilibrium the government issues debt and has incentive to smooth distortionary taxes over time. I set disutility of labor jointly with other parameters to ensure average equilibrium labor is one third and average output is one.

Table 3.1: Model Parameters

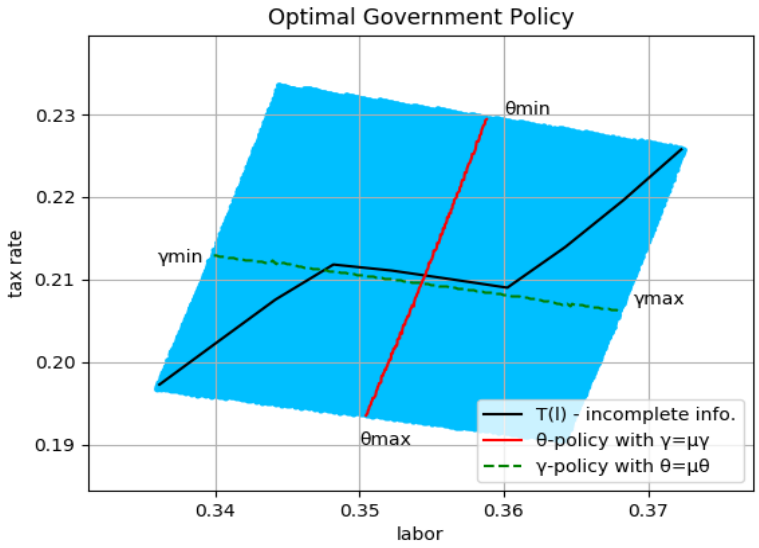
Parameter	Description	Value
β	Discount factor	0.96
η	Coefficient of relative risk aversion	1.1
σ	Inverse of Frisch elasticity	1.1
ϕ	Disutility of labor	10
$\overline{\kappa}$	Maximum default cost	0.3
g_1	First period gov. spending	$\in \{0.2, 0.3\}$
g_2	Second period gov. spending	0.1

To make results clear and neat, I assume random variables are independently distributed and follow a uniform distribution. The support for productivity variable is defined as $[0.9*$

μ_θ , $1.1*\mu_\theta$], where μ_θ is mean productivity level. I allow positive and negative productivity shocks of up to 10%. The support for preference variable is defined as $[0.9 * \mu_\gamma$, $1.1 * \mu_\gamma]$, where μ_γ is the mean level for the utility of consumption parameter. I set $\mu_\theta = 3$ and $\mu_\gamma = 1$. The support for default cost is $[0,\ \overline{\kappa}]$.

Optimal policy function. Figure 3.1 illustrates optimal policy when government spending in the first period is not very large ($g_1 = 0.2$). The zig-zag horizontal solid line represents optimal policy function $\tau = T(l)$, while the shaded region is the set of all equilibrium pairs (l, τ) that could have been realized under complete information.

Figure 3.1: Optimal Policy - Low Gov. Spending



Note: Government spending is low, $g_1 = 0.2$. Zig-zag horizontal solid line: optimal tax rate as a function of equilibrium labor under incomplete information; vertical solid line: optimal tax rates under complete information for different values of productivity θ when preference random variable is set to its mean (i.e., $\gamma = \mu_\gamma$); horizontal dashed line: optimal tax rates under complete information for different values of preference γ when productivity random variable is set to its mean (i.e., $\theta = \mu_\theta$); shaded region: set of complete information pairs (l, τ) for all possible realizations of exogenous random variables (θ, γ).

To better understand the intuition behind incomplete information, it is crucial to understand model mechanisms with complete information. There are two reasons that can push equilibrium labor to low levels. Either productivity θ is high or consumption preference γ is low. Although both situations imply similar outcomes to labor, they imply very different dynamics of revenues, debt, and default. When θ is high, the government optimally decides to lower the tax rate and issue less bonds. The goal is to use the associated increase in government revenues to smooth tax distortions over time. This scenario is represented in Figure 3.1 with the vertical solid line. The higher productivity is, the lower equilibrium labor and optimal tax rate are.

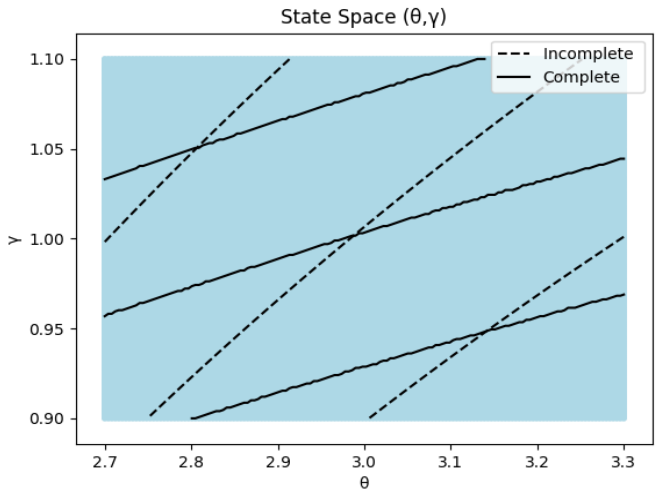
Contrary, when consumption preference fall, agents decided to consume less and work less, which reduces government tax revenues and pushes optimal tax rate to increase (i.e., see downward-sloping dashed line).

This two examples show that in an ideal situation, if the government sees an increase in aggregate labor, it would want to react in opposite directions depending on the type of shock hitting the economy. However, when incomplete information is assumed, the above implementation of optimal policy is not feasible, generating interesting dynamics of taxes, debt, and default.

Figure 3.1 shows that the limits of l are easy to compute. For extreme values of the signal (labor), there is full revelation of the state of the economy because there is only one combination of (θ, γ) that can give rise to the outcome. That explains why optimal policy under incomplete information mimics optimal policy under complete information at the extremes. However, in between these two extreme values the government is uncertain of what combination of (θ, γ) generates equilibrium tax rate and labor. As argued above, when productivity is maximum and preference for consumption is minimum (i.e., $(\theta_{max}, \gamma_{min})$), households work very little and full revelation occurs. The government knows it and sets a low tax rate. Then, as equilibrium labor increases, tax rate increases: higher labor signal calls for lower expected productivity and also tax revenues. This happens monotonically up to a point in which any realization for productivity is possible and higher l signals higher γ, which increases tax revenue. Because of this, tax rate falls. This happens again up to the point the highest θ is ruled out and the tax rate increases again up to the full revelation point (i.e., $(\theta_{min}, \gamma_{max})$).

Figure 3.2 partitions the state space of (θ, γ) for three different values of the signal, low, medium, and high. The right lower corner (low γ and high θ) corresponds to the case of low labor. Notice that under complete information (solid line) almost any productivity level is possible (from 2.8 until 3.3). Contrary, under incomplete information (dashed line) only a restricted subset of productivity levels is feasible. For middle level of the signal almost any productivity level is consistent with it. Finally, by comparing optimal partitions, under incomplete information it is tilted upwards, shrinking the set of feasible productivities consistent with a given signal. This gives an idea how important is tax revenue for smoothing of distortions and welfare.

Figure 3.2: State-Space Partition



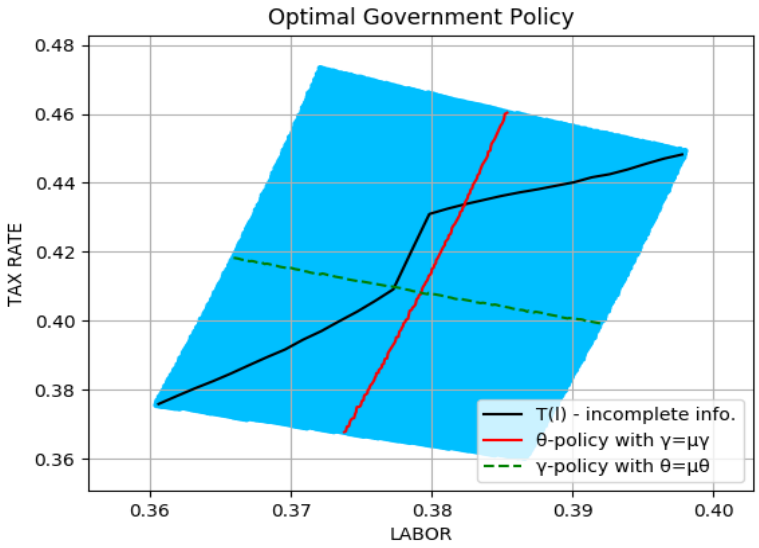
Note: Set of admissible combinations of (θ, γ) for three different levels of the signal -labor- under complete information (solid line) and incomplete information (dashed line). Upper left corner corresponds to $l = l^{\text{high}}$, the middle of the region to $l = l^{\text{middle}}$, and the lower right corner to $l = l^{\text{low}}$.

Consequently, optimal policy is highly non-linear. Observe that a marginal change in the value of either productivity or preference changes the slope of the tax rate as a function of aggregate labor.

Finally, Figure 3.3 shows optimal policy when government spending is relatively high ($g_1 = 0.3$). Unlike the case with low g_1, here optimal tax policy is monotonically increasing

in labor. When g_1 is high, the government is very concerned about reaching the top of the Laffer curve, where taxes are most distortionary. If a low productivity realization occurs, a large amount of debt needs to be issued, which if default does not occur in the second period, calls for high distortionary taxes. Like the case of low g_1, optimal policy is highly non-linear. When g_1 is sufficiently high, for the middle region, the government is sufficiently confident about low realizations of θ, which makes optimal policy under incomplete information (zig-zag horizontal solid line) to mimic optimal policy under complete information when only productivity varies (vertical solid line).

Figure 3.3: Optimal Policy - High Gov. Spending

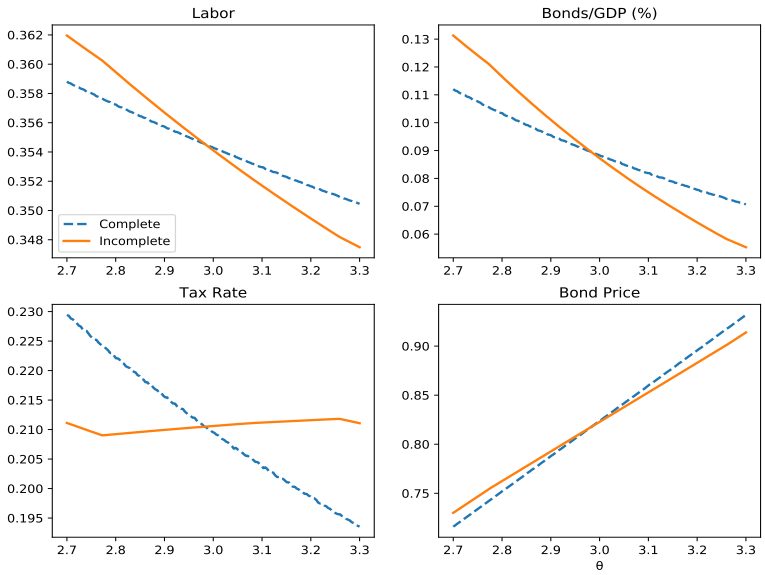


Note: Government spending is high, $g_1 = 0.3$. Zig-zag horizontal solid line: optimal tax rate as a function of equilibrium labor under incomplete information; vertical solid line: optimal tax rates under complete information for different values of productivity θ when preference random variable is set to its mean (i.e., $\gamma = \mu_\gamma$); horizontal dashed line: optimal tax rates under complete information for different values of preference γ when productivity random variable is set to its mean (i.e., $\theta = \mu_\theta$); shaded region: set of complete information pairs (l, τ) for all possible realizations of exogenous random variables (θ, γ).

Productivity shocks. It is important to show some comparative statics for equilibrium

variables of interest in this model. To do so, in Figure 3.4 I plot labor, tax rate, the ratio of bond to GDP, and bond price when only productivity varies in this economy.

Figure 3.4: Equilibrium Variables under Productivity Shock - Low Gov. Spending



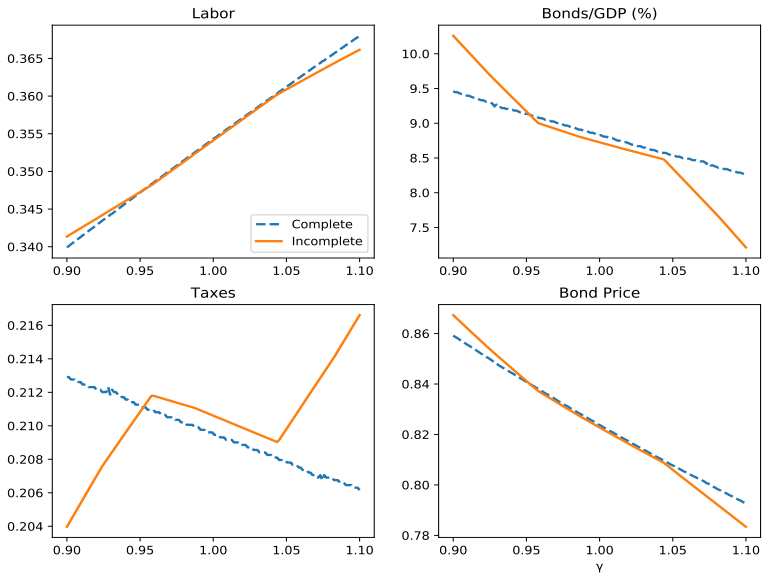
Note: Government spending is low, $g_1 = 0.2$. Solid orange line represents equilibrium variable as a function of productivity θ under incomplete information. Dashed blue line represents equilibrium variable as a function of productivity θ under complete information. Left panels: top is labor and bottom is effective tax rate. Right panels: top is bond-to-GDP ratio and bottom is bond price. Preference level is set fixed to its mean ($\gamma = \mu_\gamma$).

Under complete (dashed blue line) and incomplete information (solid orange line) a negative shock to productivity causes labor to increase (wealth effect dominates), government revenue falls and bonds issued by the government increase, as well as bond-to-GDP ratio, and bond price falls. The latter is true since more bonds increases the probability of default next period. Interestingly, optimal tax becomes highly irresponsive to productivity shocks (either positive or negative). This causes that after a negative productivity shock occurs, optimal policy implies more debt accumulation under incomplete that complete information. Also hours become more volatile in response to these shocks and taxes become

smoother and change sign.

Preference shocks. Figure 3.5 shows equilibrium outcome when preference shocks hit the economy and productivity is set to its mean value, ie., $\theta = \mu_\theta$. Similarly to the case of only productivity shocks, labor becomes more volatile and, under negative preference shocks, the economy accumulates a larger ratio of bonds to GDP with incomplete information.

Figure 3.5: Equilibrium Variables under Preference Shock - Low Gov. Spending



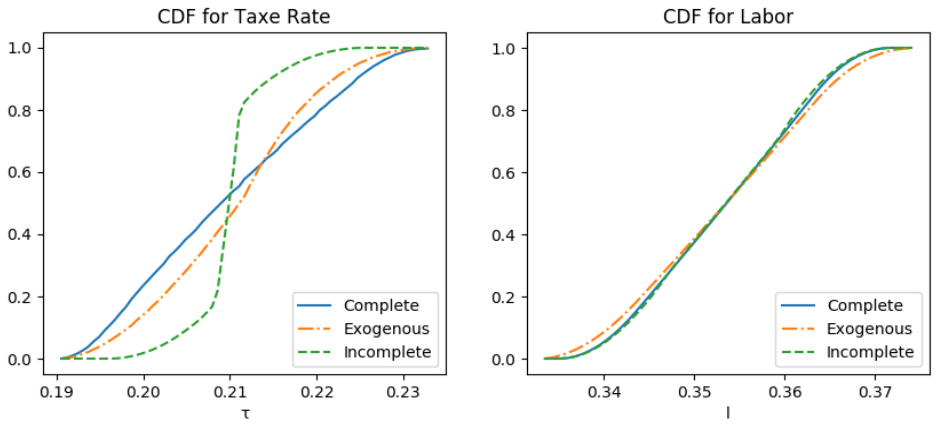
Note: Government spending is low, $g_1 = 0.2$. Solid orange line represents equilibrium variable as a function of productivity θ under incomplete information. Dashed blue line represents equilibrium variable as a function of productivity θ under complete information. Left panels: top is labor and bottom is effective tax rate. Right panels: top is bond-to-GDP ratio and bottom is bond price. Productivity level is set fixed to its mean ($\theta = \mu_\theta$).

Effects of the optimal filter on taxes[10]. As shown, optimal policy under incomplete information calls for a substantial amount of tax rigidity across exogenous states. To

[10]The density function $f_{\theta|\gamma}(\Theta(l,\gamma),\gamma) \cdot f_\gamma(\gamma)$ in optimality condition (3.25) is an endogenous object and it can be interpreted as the optimal filter

confirm this, Figure 3.6 shows the equilibrium cumulative distribution function for taxes under complete information, incomplete information and endogenous signal, and also under exogenous signals. Given the results and intuition presented so far, the outcome shown in Figure 3.6 is very intuitive. Basically optimal fiscal policy under incomplete information features some sort of endogenous rigidity: when the Ramsey planner is unsure of what type of shocks are hitting the economy, it makes sense to chose a fiscal policy that is not very aggressive. Clearly, second period taxes, absent a default, will become more volatile since they have to adjust to meet the budget.

Figure 3.6: Cumulative Density Function - Optimal Policy



Note: Cumulative distribution function for tax rate (left panel) and labor (right panel) under complete information (solid blue line), incomplete information with endogenous signal (dashed green line), and incomplete information with exogenous signal (dashed dotted orange line).

Default probabilities. The limited fiscal adjustment endogenously generated by assuming incomplete information has relevant implications for debt accumulation and probability of default. Table 3.2 summarizes this statistic. Incomplete information, coupled with endogenous policy, generates higher default probabilities compared to the complete information case. The intuition is that under incomplete information taxes are less volatile and less responsive to shocks, causing the government to issue more debt and to increase

default incentive in the future, which pushes up probability of default.

Table 3.2: Default Probabilites

Shock	Complete Info.	Incomplete Info.
θ	3.5%	4.5%
γ	2.8%	3.2%

3.6 Conclusion

This paper presents a simple two-period model of optimal fiscal policy, debt issuance, and default under incomplete information. In this setup the government does not observe the state of the economy, as it happens in real life, but nonetheless it has to design a policy contingent on endogenous objects in the economy. In the model I assume it conditions tax rate on equilibrium labor. I show that optimal policy is highly non-linear and that endogeneity between the signal and the policy interact in a non-trivial way.

The optimal design of fiscal policy calls for relative tax rigidity when compared to the case of complete information (i.e., when the government observes the state of the economy). Consequently, debt accumulation as a a percentage of GDP increases relatively more as well as default probability. This paper shows that this is a promising mechanism to try to understand the reaction of many European governments during the Great recession and, in particular, the Spanish government.

Finally, it would be interesting to extend this framework to an economy with infinite horizon where many signals are observed simultaneously. Also it would be important to further study the implications for some interesting aspects in the debt literature such as debt dynamics, spreads, and maturity.

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Appendix A

Appendix to Chapter 1

A.1 Data

In this section, I explain in detail how each variable is constructed using information from the SHF for each household.

Income. Gross income is differentiated depending on its source, which I denote by main, real estate, and financial assets: 1) *Main* - it include wages and salaries, self-employment income[1], unemployment benefits, retirement and permanently disabled pensions, regular private transfers such as child support and alimony, and regular public transfers. In many papers, this is the common measured used, which is referred as labor income; 2) *Real Estate* - it includes income from renting out rooms of the main residence, as well as, monthly rents obtained from other real estate assets. For some households, mostly with large amounts of illiquid wealth, this represents a sizable fraction of their total monthly income; 3) *Financial Assets* - it includes income derived from financial assets such as options, futures or swaps, mutual funds, fixed-income assets, and others such as managed accounts. One may worry about the role of the state in redistribution and shaping risk. To this end, Arellano et al. (2021) finds that, in Spain, labor earnings measures of risk before and after taxes are

[1]For independent professional, sole proprietors of business or self-employed worker, I use previous year pre-tax profits as income. I divide by 12 to obtain an approximation to gross monthly income. For owners or partners in family business or non-family partnership with a role in the management, I use the fixed amount they are paid each year of their work, but not their stake in distributed dividends.

almost identical.

Assets. It includes *liquid* assets such as cash, checking and savings accounts, term deposits, listed shares, mutual funds, public fixed-income securities (treasury bills and gov. bonds), private fixed-income securities (commercial paper, bonds issued by private companies, etc), and house-purchase savings accounts. Also, I include *illiquid* assets such as the main residence and other real estate, means of transport such as cars and motorcycles[2], unlisted shares, loans to family members, and occupational and voluntary pension plans. A potential problem arises because income measures are before taxes[3].

Debt. I include outstanding amounts of debit balances on credits cards from financial institutions and non-financial corporations[4], loans not used to purchase real estate such as mortgages, personal loans, credit lines, leasing and renting, and others. Finally, I include mortgages and unsecured debt obtained to purchase real estate assets.

Spending Commitments. Monthly loan payments (capital and interest) associated to the main residence and other real estate assets. I also include payments of other type of loans, as well as, monthly rent when the household does not own the main residence.

Consumption. It includes family's total average spending on consumer goods, including food but excluding consumer durables (such as cars, electrical appliances), housing rent, mortgage payments, insurance policies, renovations, etc.

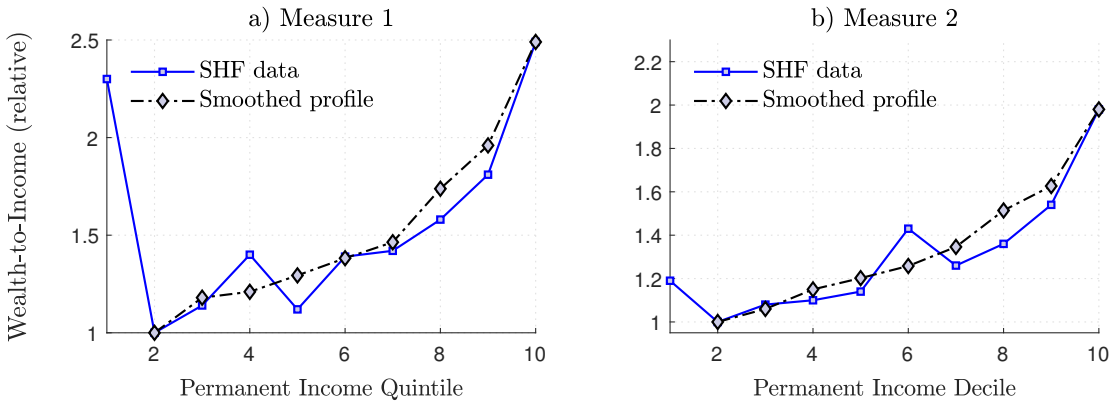
[2]Jewelry, works of art, antiques, furnishing, fittings and appliances are not included either on liquid or illiquid assets category. Life insurance and other forms of insurance such as health-care, home and vehicle policies are not included as well.

[3]Due to measurement problems, I decide not to include self-reported value of the own business.

[4]This is asked from the 2005 wave onwards. Credit card limits were not asked before 2005.

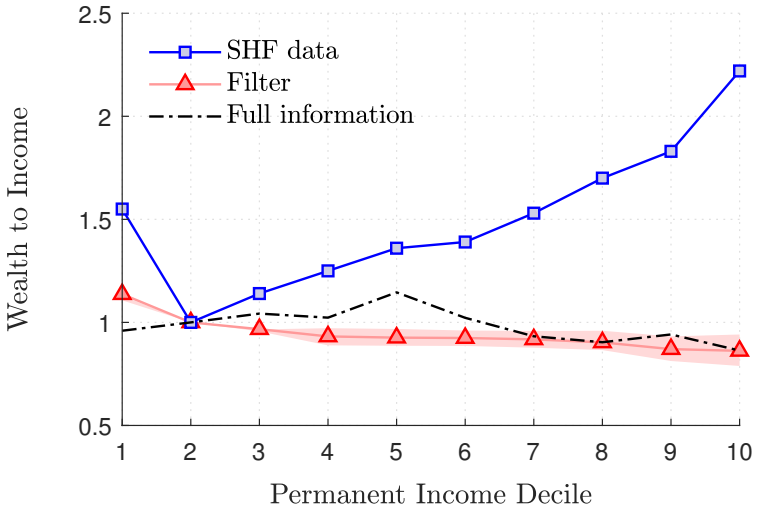
A.2 Figures and Tables

Figure A.1: Alternative Measures



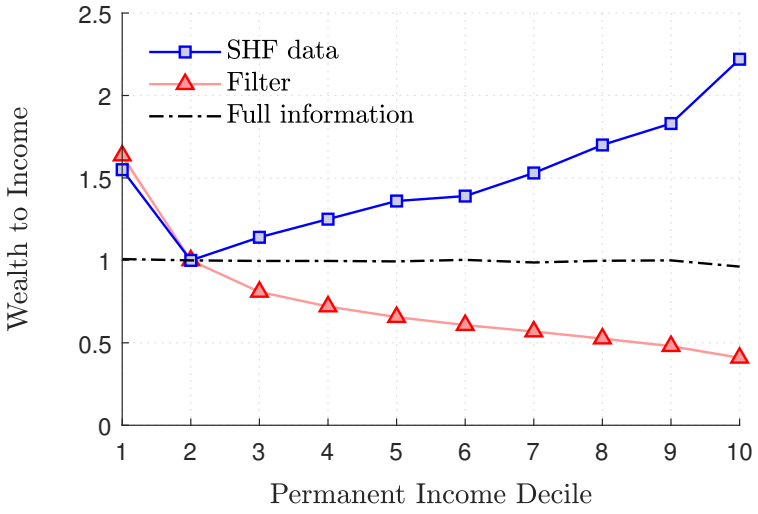
Note: *Measure 1* corresponds to the same measure as in Figure 1.2 in the main text but within each decile the mean is taken instead of the median. *Measure 2* computes for each household the median wealth-to-income ratio where income is the mean over the 10 years it is observed. Then, within each decile the median ratio is taken. Since income is more volatile than wealth, it allows to obtain a measure that is less sensitive to transitory income movements. The qualitative implications remain unchanged regardless of the measure used. Households included are those with head's age between 20 and 65 years old during the entire period the household is observed. For comparability reasons, median wealth-to-income ratios are shown relative to the the second decile. The smoothed profile (black dashed line) is calculated using LOWESS regression, with bandwidth set to 0.5, omitting the value for the first decile.

Figure A.2: Performance of the Filter - Small Sample Size



Note: Sample size is 3,000 individual households as in the panel obtained from the Survey of Household Finances. The estimation is applied to households with age between 20 and 65 years old. For comparability reasons, median wealth-to-income ratios are shown relative to the the second decile as in the empirical analysis. Within each decile the value corresponds to the median wealth-to-income ratio.

Figure A.3: Performance of the Filter - High Persistency ($\rho_z = 0.95$)



Note: Persistency of income process is set to $\rho_z = 0.95$. Sample size is 50,000 individual households as in the panel obtained from the Survey of Household Finances. The estimation is applied to households with age between 20 and 65 years old. For comparability reasons, median wealth-to-income ratios are shown relative to the the second decile as in the empirical analysis. Within each decile the value corresponds to the median wealth-to-income ratio.

A.3 Estimation of the Income Process

Consider income process from section 1.3 where controls have been removed,

$$\gamma_{it} = \alpha_i + z_{it} + \nu_{it} \tag{A.1}$$

$$z_{it} = \rho_z z_{it-1} + \epsilon_{it} \tag{A.2}$$

where $t = 1, ..., T$ represents age and goes from 1 (30 years of age) to $T = 26$ (55 years of age). This restriction leaves me with a panel of 1,318 households. I follow the standard estimation strategy in the literature and use a Minimum Distance Estimator. The goal is to find parameters $(\rho_z, \sigma_\alpha, \sigma_\epsilon, \sigma_\nu, \sigma_{z_0})$ that minimize the distance between empirical and theoretical covariance moments from the data generating process described by (A.1) and (A.2).

Theoretical moments. If the panel was balanced, there would be $\frac{T(T+1)}{2}$ moments. However, I only observe households four times since they enter the panel, which implies I have $T \times T = 144$ moments. Clearly, the model is overidentified. Let the vector of moments be

$$M(\theta) = vec\begin{pmatrix} \mathbb{E}[\gamma_{i1}\gamma_{i1}] & - & - & \mathbb{E}[\gamma_{i1}\gamma_{i4}] & - & - & \mathbb{E}[\gamma_{i1}\gamma_{i7}] & ... & - \\ - & \mathbb{E}[\gamma_{i2}\gamma_{i2}] & - & - & \mathbb{E}[\gamma_{i2}\gamma_{i5}] & - & - & ... & - \\ - & - & \mathbb{E}[\gamma_{i3}\gamma_{i3}] & - & - & \mathbb{E}[\gamma_{i3}\gamma_{i6}] & - & ... & - \\ - & - & - & \mathbb{E}[\gamma_{i4}\gamma_{i4}] & - & - & \mathbb{E}[\gamma_{i4}\gamma_{i7}] & ... & - \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ - & - & - & - & - & - & - & - & \mathbb{E}[\gamma_{iT}\gamma_{iT}] \end{pmatrix}$$

where $\theta = (\rho_z, \sigma_\alpha, \sigma_\epsilon, \sigma_\nu, \sigma_{z_0})$. Note that I have already imposed that expectations of innovations are zero. Clearly, the matrix of moments is sparse. Applying some algebra and imposing assumptions one can derive theoretical moments:

$$\mathbb{E}[\gamma_{it}\gamma_{it+h}] = \sigma_\alpha^2 + 1_{\{h=0\}}\sigma_\nu^2 + \rho^h \mathbb{E}[z_{it}^2]$$

where

$$\mathbb{E}[z_{it}^2] = \rho_z^{2t}\sigma_{z0}^2 + \sum_{j=1}^{t} \rho_z^{2(t-j)}\sigma_\epsilon^2$$

Sample moments. Let $\hat{m}_{t,h}$ be the empirical counterpart of each moment $\mathbb{E}[\gamma_{it}\gamma_{it+h}]$, which is constructed as follows

$$\hat{m}_{t,h} = \frac{1}{|I_t|}\sum_{i\in I_t}\hat{\gamma}_{it}\hat{\gamma}_{it+h}$$

where I_t is the set of households with an initial observed age t in the panel data. Finally, let $\hat{M}$ be the column vector with all moments stacked up.

Estimation. Estimated set of parameters, $\hat{\theta}$, is the solution to

$$\min_{\theta}\ \big[M(\theta)-\hat{M}\big]'W\big[M(\theta)-\hat{M}\big]$$

where W is a weighting matrix. In the implementation, to minimize the possibility of falling in local minima, I start the minimization problem at different initial guesses for all parameters and pick the one that attains the lowest value.

Standard errors. I compute standard errors by bootstrap.

A.4 Permanent Income Classification

One important concern has to do with how good permanent income estimates $\{\hat{\alpha}_i\}_{i\in I}$ are obtained using the proposed estimation strategy. To assess goodness of fit, I apply Monte Carlo simulations using income process (1.1) as the true data generating process. Then I estimate the artificial data using the sam estimation approach I apply to survey data. I do so for different values of the persistence componenet of income ρ_z fro which I include the one I estimated for the SHF.

Table A.1 shows how many α's the procedure classifies within the same permanent income decile where the true values lies on, in adjacent deciles up to a maximum of two deciles. Clearly, the share of well classified households is decreasing the more persistent is the stochastic component z_{it}. However, even when $\alpha_z = 0.95$, the estimation classifies almost 80% of households within two deciles of distance. For instance, α_i lie on the 1st decile of the distribution and I classify it at most in the 3rd decile.

Table A.1: Goodness of Fit of the Estimation Approach

Criterion	Persistence Parameter Value			
	$\rho_z = 0.70$	$\rho_z = 0.80^*$	$\rho_z = 0.9$	$\rho_z = 0.95$
Same decile	0.43	0.47	0.33	0.26
Adjacent deciles	0.80	0.85	0.66	0.56
Max. 2 deciles of distance	0.97	0.98	0.88	0.79

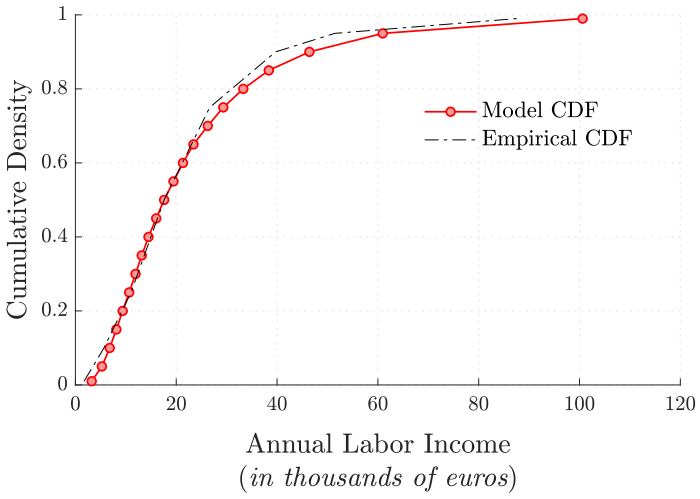
Note: First column corresponds to the parameter estimate obtained using SHF data.

Appendix B

Appendix to Chapter 2

B.1 Figures and Tables

Figure B.1: Distribution of Labor Income



Note: Data source is the Continuous Work History Sample, which is a 4% non-stratified random sample from the Spanish population registered with the social security administration. Income includes both women and men and it is reported in 2018 euros. Decile numbers are obtained from Arellano et al. (2021).