Practical Magnetism VIII: reporting and visualization of magnetic anisotropy data

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1 Applications of magnetic anisotropy data
Magnetic fabrics are a powerful, rapid and non-destructive tool to characterize mineral alignment and dynamic processes, and as such are widely applied in structural geology and tectonics, volcanology, sedimentology, and pore fabric studies. Rock deformation, sediment transport and compaction, lava and pyroclastic flow directions and emplacement of igneous bodies are some examples of processes that can be studied using magnetic anisotropy. Furthermore, paleomagnetic remanence data may exhibit a dependence on the magnetic fabric and magnetic anisotropy can be used to correct those data. Different minerals, including the characteristic remanence carriers, contribute differently to the observed magnetic fabrics, and understanding their contributions to the anisotropy allows better interpretation of geodynamic processes and paleomagnetic data (e.g., Biedermann & Bilardello, 2021, IRM31-3).

Numerous anisotropy parameters and ways to visualize magnetic fabric data exist, and choice of parameters and plots to use is by no-means homogeneous across the different research communities. Choice of parameters is often adapted to the specific application: for example, certain parameters for lineation ($L$) and foliation ($F$), more below, are often used for structural geology because they relate to the equivalent parameters as derived from other strain markers and are often plotted against each other as conventionally done for strain ratios (e.g., see Tarling & Hrouda (1993) for a more extensive discussion). Likewise, in “applied” magnetic fabric studies, the anisotropy degree is commonly described by the parameters $P$ or $P'$; however, studies that focus on single crystals’ magnetocrystalline anisotropy more commonly use the $\Delta k$ or $k'$ parameters.

Describing non-ellipsoidal particles or quantities is even more complex, and requires additional parameters, such as roundness, angularity, sphericity, or irregularity (e.g., Blott & Pye, 2007). Similarly, complicated anisotropies arising from higher-order tensors need additional descriptors (e.g., Biedermann et al., 2020; Flanders &

Figure 1. Schematic representation of full vs deviatoric tensors. In full tensors, the eigenvalues correspond to the semi axes of the anisotropy ellipsoid, whereas in the deviatoric tensors, the eigenvalues are defined as the difference between the semi axes of the ellipsoid and the “isotropic radii” approximated by the mean susceptibility $k_m$. cont’d. on pg. 10...
Visiting Fellow Reports

Magnetofossils, with a side of siderite, in PETM sediments from Wilson Lake, New Jersey

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The Paleocene-Eocene Thermal Maximum (PETM, ~56 Ma ago) is the largest carbon cycle perturbation of the Cenozoic (Westerhold et al., 2020). Although the rate of warming during the PETM pales in comparison to the Anthropocene, it is one of the best natural analogs for understanding and predicting outcomes associated with modern climate change (McInerney & Wing, 2011; Zeebe et al., 2016). It is globally identified by a negative carbon isotope excursion (CIE) (Kennett & Stott, 1991; Thomas & Shackleton, 1996) and is best described by six stratigraphic intervals that reflect the structure of the CIE: the pre-CIE, CIE onset, CIE core, CIE recovery phase I, CIE recovery phase II, and post-CIE intervals (Westerhold et al., 2018).

The CIE is recorded at high temporal resolution within the Marlboro Clay that was deposited along the Paleogene New Jersey, Maryland, and Delaware coast (Bralower et al., 2018; John et al., 2008; Kent et al., 2003; Self-Trail et al., 2012, 2017; Stassen et al., 2012; Zachos et al., 2006). The magnitude of the CIE recorded within bulk carbonate from the CIE core interval varies by depth in the Marlboro Clay, with more negative δ13C values nearshore than offshore, although the magnitude of the CIE recorded in benthic foraminifera shows little to no deviation (Bralower et al., 2018; John et al., 2008; Kent et al., 2003; Self-Trail et al., 2012, 2017; Stassen et al., 2012; Zachos et al., 2006). This off-shore gradient has been variably attributed to differences in coastal marine productivity (Gibbs et al., 2006; John et al., 2008; Stassen et al., 2012), sediment reworking (John et al., 2008; Stassen et al., 2012), cometary debris (Kent et al., 2003), or as an artifact of authigenic siderite (Bralower et al., 2018; Self-Trail et al., 2017). This last interpretation is supported by clast counts and clast morphology of siderite in washed samples of PETM sediments from the Maryland and New Jersey coast (Bralower et al., 2018; Self-Trail et al., 2017), including sediments from the Wilson Lake-A core (Bralower et al., 2018) that have been the focus of several of our studies on magnetofossils (Lippert & Zachos, 2007; Wagner, Egli, et al., 2021; Wagner, Lascu, et al., 2021).

Siderite (FeCO₃) is a common diagenetic carbonate in iron and carbon-rich marine sediments (Ellwood et al., 1986, 1988; Vuillemin et al., 2019). Paleomagnetically, iron-rich carbonates can be problematic because they can quickly oxidize to magnetite, maghemite, or hematite with little or no heating and produce aberrant remanence directions and secondary nanoparticles of magnetite (Ellwood et al., 1986, 1988; Golden et al., 2004). Although siderite carries no remanence at room temperature (RT), it is antiferromagnetic below 37 K. Thus, it can be detected in bulk sediments using Field Cooled (FC) and Zero-Field Cooled (ZFC) remanence experiments (Bilandello & Jackson, 2013; Housen et al., 1996; Pan et al., 2002).

A principal goal for my fellowship was to understand the composition and remanence characteristics of the PETM magnetofossil assemblage from the Wilson Lake-A core. I also wanted to test if I could use non-destructive low temperature (LT) remanence measurements to assess the presence and relative abundance of siderite in these bulk samples. This approach allows me to directly compare the interpretations regarding siderite to our previous work distinguishing and quantifying morphologically distinct magnetofossil assemblages (Wagner, Lascu, et al., 2021).

Seventeen samples were selected from the Wilson Lake-A core that span four of the six PETM intervals: the pre-CIE, CIE onset, CIE core, and post-CIE intervals. The Wilson Lake-A core does not record either of the CIE recovery intervals. I measured the FC and ZFC LT saturation remanent magnetization (SIRM) and the LT demagnetization of RT SIRM of each of these samples using the Magnetic Properties Measurement System instruments at the Institute for Rock Magnetism (IRM). I then normalized the FC-ZFC-RT data for each specimen to the initial FC remanence. The derivatives of the FC and ZFC curves are shown here to better observe the LT remanence behavior in the samples, such as phase transitions like the Verwey transition (Tᵥ) for magnetite (e.g., Stacey & Banerjee, 1974), the double TV which distinguishes abiotic from biotic magnetite (Chang et al., 2016), and the Néel temperature for siderite (Jacobs, 1963).

The FC-ZFC-RT datasets from each of the four PETM intervals at Wilson Lake show little to no variation within each stratigraphic interval (i.e., pre-CIE, CIE onset, CIE core, and post-CIE intervals). Therefore, I focus on the variation, and similarities, observed between the intervals in the descriptions below using a representative sample from each of the four intervals (Figure). Verwey transitions (Tᵥ) are observed between ~90-120 K in each of the PETM intervals, indicating that magnetite is present within each of the samples (Figure). There is a weak double TV transition in the FC curves from the CIE onset interval, further suggesting mixtures of detrital and biogenic magnetite (Chang et al., 2016). This is consistent with our published RT measurements...
and electron microscopy (Lippert & Zachos, 2007; Wagner, Egli, et al., 2021; Wagner, Lascu, et al., 2021). A more thorough investigation of the different populations of magnetite as revealed by these LT measurements is discussed in a separate publication in preparation.

The LT remanence of specimens representing the CIE onset and core intervals show at least one other LT inflection between 25-30 K in the FC and ZFC curves (Figure). Although the FC and ZFC curves from both the pre- and post-CIE intervals do not show as clear an inflection over this temperature range, the remanence of both sets of curves decreases rapidly with warming over this range. Siderite was documented in smear slides from the Wilson Lake-A core (Bralower et al., 2018), so I conclude that this LT behavior is due to warming through the Néel temperature for siderite (Housen et al., 1996; Jacobs, 1963).

The LT data suggest that siderite is more abundant and well-mineralized within the CIE core interval at Wilson Lake: I interpret this from the muted magnetite signatures and sharper transitions at the Néel temperature within this interval (Figure). This interpreted increase in siderite content is corroborated by a separate study that reports a small, concomitant increase in siderite grains and more negative carbon isotope values from the low carbonate interval (approximately equivalent to the CIE onset interval) to the non-low carbonate interval (roughly equivalent to the CIE core interval) in the Wilson Lake-A core (Bralower et al., 2018). I note that the LT remanence data from the CIE core interval samples are very similar to FC, ZFC, and RT cycling datasets for modern samples known to contain siderite (Abdulkarim, 2020). A more thorough description and discussion of the implications of this siderite for magnetofossil and environmental interpretations is forthcoming in another publication.

Identification of different iron mineral phases is important for understanding the depositional, diagenetic, and authigenic processes in sediments that underly our interpretation of climatic and ecological events in the geologic record. The LT remanence measurements confirm the presence of siderite at Wilson Lake-A (Bralower et al., 2018), underscoring the capability of these measurements to detect iron minerals that otherwise elude RT measurements like first-order reversal curve measurements (Wagner, Lascu, et al., 2021).

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References

Figure. Low temperature remanence results from four representative stratigraphic intervals from the PETM at Wilson Lake-A. Rows: The four stratigraphic intervals are: the pre-CIE, CIE onset, CIE core, and post-CIE intervals. Columns: (a) Normalized FC and ZFC LT-SIRM curves (black and gray curves) with derivatives (red and blue curves). The three dotted vertical lines mark 30, 100, and 120 K. These temperatures correspond to the interpreted phase transitions for siderite, oxidized or biogenic magnetite, and unoxidized magnetite (Chang et al., 2016; Housen et al., 1996; Stacey & Banerjee, 1974), respectively. Low temperature features that correspond to these transitions are indicated by arrows (e.g., “Néel” for siderite, “T_N” for magnetite, and “Double T_N,” for both abiotic and biogenic magnetite transitions). (b) Normalized RT-SIRM cycling curves (blue is cooling, red is warming). Similar to (a), the two dotted vertical lines mark 100 and 120 K, or the two T_N’s for magnetite; features corresponding to these transitions are also indicated with arrows. I interpret the pronounced T_N’s from the post-CIE interval to physical grain movement during the measurement.

Current Articles

A list of current research articles dealing with various topics in the physics and chemistry of magnetism is a regular feature of the IRM Quarterly. Articles published in familiar geology and geophysics journals are included; special emphasis is given to current articles from physics, chemistry, and materials-science journals. Most are taken from ISI Web of Knowledge, after which they are subjected to Procrustean culling for this newsletter. An extensive reference list of articles (primarily about rock magnetism, the physics and chemistry of magnetism, and some paleomagnetism) is continually updated at the IRM. This list, with more than 10,000 references, is available free of charge. Your contributions both to the list and to the Current Articles section of the IRM Quarterly are always welcome.

Environmental Magnetism


Delusina, I., S. W. Starratt, and K. L. Verosub (2022), Environmental evolution of peat in the Sacramento - San Joaquin Delta (California) during the Middle and Late Holocene as deduced from pollen, diatoms and magnetism, Quaternary International, 621, 50-61, doi:10.1016/j.quaint.2020.05.012.


Sheikh, H. A., B. A. Maher, V. Karloukovski, G. I. Lam-


Extraterrestrial and Planetary Magnetism


Fundamental Rock Magnetism and applications


Geomagnetism, Paleointensity and Records of the Geomagnetic Field


Hyodo, M., et al. (2022), Intermittent non-axial dipolar-field dominance of twin Laschamp excursions, Communications Earth & Environment, 3(1), doi:10.1038/s43247-022-00401-0.


**Magnetic Fabrics and Anisotropy**


Magnetic Mineralogy and Petrology, Other


Mitra, K., E. L. Moreland, A. L. Knight, and J. G. Catalano (2022), Rates and Products of Iron Oxidation by Chlorate at Low Temperatures (0 to 25 degrees C) and Implications for Mars Geochemistry, AcS Earth and Space Chemistry, 6(2), 250-260, doi:10.1021/acsearthspacechem.1c00379.


Paleomagnetism


**Stratigraphy**


Marton, E., R. Pipik, D. Starek, E. Kovacs, M. Vidhya, A. Schwierzewska, A. K. Tokarski, R. Vojtko, and S. Schlogl (2022), Enhancing the reliability of the magnetostratigraphic age assignment of azimuthally nonoriented drill cores by the integrated application of palaeomagnetic analysis, field tests, anisotropy of magnetic susceptibility, and the evolution of the endemic fauna as documented on the upper Miocene limnic deposits of the Turiec Basin.
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This is an evolving Index, so expect changes (and additions) along the way.

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Schuele, 1964; Stacey, 1960). In this brief overview, we limit the discussion to the magnitude ellipsoids of second order tensors. While not describing every single parameter or plot (we do report many of the parameters proposed in Table 1), we share some general considerations on reporting and visualizing magnetic anisotropy data that apply to different areas of research employing magnetic fabrics, hoping this will be useful in navigating the anisotropy jungle. A similar article on the selection of anisotropy of magnetic susceptibility parameters was already published by Cañon-Tapia (1994), and we refer the readers to that article for completeness.

On a more general note, it is very common for “magnetic anisotropy” and/or “paleomagnetism” to be referred to as “tools”, as has also been done here. However, we take this opportunity to iterate that neither are tools in the sense that they can be applied blindly, but are in fact areas of research in their own right, and a-priori thought should be put into data acquisition and processing schemes, parameters to be used, and data visualization and interpretation. While for the most part we are certainly “preaching to the quire”, we hope that given the didactic nature of this article series we are able to make a broader group of users aware of the complexities behind applying paleomagnetism and, in this case, magnetic anisotropy.

2 The jungle of anisotropy parameters
2.1 Susceptibility tensors, eigenparameters and mean susceptibility

From here onwards we will adopt terminology specific to magnetic susceptibility for simplicity, but the reader should be aware that the same applies for determining anisotropy from magnetic remanences also. Anisotropy is normally described by either a second-order symmetric tensor or a combination of mean susceptibility ($k_{\text{mean}}$), principal susceptibilities ($k_1$, $k_2$, $k_3$) and their directions, and parameters describing the degree or shape of the anisotropy. Tensors can be reported as full or deviatoric tensors (Fig. 1), either describing directional susceptibilities or susceptibility differences (see the IRMQ31-1 companion article by Bilardello & Biedermann, 2021). The eigenvalues of these tensors are $k_1 \geq k_2 \geq k_3$ and can be normalized in different ways: for deviatoric tensors, $k_1+k_2+k_3=0$; while for full tensors, $k_1+k_2+k_3=3$, $k_1+k_2+k_3=1$, or $k_1+k_2+k_3=3k_{\text{mean}}$ are commonly used (more on $k_{\text{mean}}$ or $k_{\text{m}}$ for brevity, below). These provide equivalent information, and it is normally clear from the reported values what normalization was used. However, note that the ranges adopted by some parameters defining anisotropy degree, magnetic lineation and foliation depend on the definition and normalization of $k_{\text{m}}$ and the eigenvalues. Therefore, a clear description of how data are processed and presented will avoid confusion. Also note that for remanence anisotropy, a further normalization may be made to the applied field when determining anisotropy of anhysteretic susceptibility (McCabe et al., 1985). For a discussion on normalization and the different anisotropy types we refer to the IRMQ30-2 article.
by Bilardello (2020). Though $k_m$ is seemingly an easy parameter, several ways of calculating $k_m$ exist. The simplest calculation is that of an arithmetic mean, i.e., $k_m = \frac{1}{3}(k_1+k_2+k_3)$ or $k_1^++k_2^++k_3^+$. Because the susceptibility axes are often lognormally distributed, a logarithmic $k_m$ notation ($n$) is also used, where $n$ (or sometimes $n_i$) = $(n_i + n_i(n_i+n_i)^{1/3})$, and $n_i$ were the natural logarithms of the principal susceptibilities. Additional ways of calculating mean susceptibility include geometric means, $k_{geom} = (k_1k_2k_3)^{1/3}$, particularly useful where the anisotropy correlates with strain, and the average geometric logarithmic mean $k_m = \text{antilog } [\frac{1}{3} \sum \log k_i/n_i]$, where $n_i$ is the number of samples (we refer to Tarling & Hrouda (1993) for a discussion of these).

The definition of the eigenvalues is straightforward when these are all positive ($k_{i} \geq k_{j} \geq k_{k} > 0$), but different conventions exist when they are all negative or have mixed signs, for example in carbonate rocks (e.g., Hrouda, 2004). If all eigenvalues are negative, $k_3$ may be defined as either the smallest in absolute values, or the most negative. Using absolute values will yield anisotropy parameters that are most close to the known parameters for positive eigenvalues, and may therefore make the most sense. However, when a mix of positive and negative eigenvalues is (more rarely) observed, e.g., due to a superposition of diamagnetic and paramagnetic anisotropies (e.g., feldspar with impurities, Biedermann et al., 2016), then defining $k_3$ as the most negative value is more intuitive. In this case, however, many anisotropy parameters are not truly defined, and the best way

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<th>Parameter</th>
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<td><strong>Anisotropy degree</strong></td>
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<td>Anisotropy degree, $P$ or $P_2$</td>
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<td>Nagata (1961)</td>
<td>Percent anisotropy</td>
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<td>Khan (1962)</td>
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<td>Orientation strength, $C$ = $\ln(k_1/k_3)$</td>
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<td>Rees (1966)</td>
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<td>Percent anisotropy</td>
<td>100($k_2$-$k_3$)/$k_1$</td>
<td>Graham (1966)</td>
<td>Corrected anisotropy, $P^*$ or $P_2$ = $\exp[2(n_2-n_3)^2+(n_2-n_3)^2]^{1/2}$</td>
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<td>Jelinek (1981)</td>
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<td>Percent anisotropy</td>
<td>100($k_2$-$k_3$)/$k_m$</td>
<td>Taira (1989)</td>
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<td>Total anisotropy, $H$ = ($k_1$-$k_3$)/$k_m$</td>
<td>0 – 3</td>
<td>Owens (1974)</td>
<td>$\Delta KH = k_1-k_3$</td>
<td>0 – 1</td>
<td>Owens &amp; Rutter (1978)*</td>
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**Lineation**

| Lineation | Lineation*, $L$ = $k_1/k_2$ | 1 – ... | Balsley & Buddington (1960) | Lineation degree | $2k_1/(k_1+k_3)$ | 1 – ... | Hrouda et al. (1971) |
| Normalized lineation, $L$ = ($k_1$-$k_3$)/$k_m$ | 0 – 3 | Khan (1962) | $\Delta L$ = ($k_1$-$k_3$)/100/$k_m$ | 0 – 300 | Taira (1989) |
| Lineation, $L$ = ($k_1$+$k_3$)/$k_2$ | 1 – ... | Urrutia-Fucugauchi (1980) | $\Delta kL = k_1-k_2$ | 0 – 1 | Owens & Rutter (1978)* |

**Foliation**

| Foliation | $F$ = $k_2/k_3$ | 1 – ... | Stacey et al. (1960) | Foliation degree | $2k_2(k_1+k_3)$ | 0 – 2 | Urrutia-Fucugauchi (1980) |
| Normalized foliation, $F$ = ($k_2$-$k_3$)/$k_m$ | 0 – 1.5 | Khan (1962) | $\Delta F$ = ($k_2$-$k_3$)/100/$k_m$ | 0 – 150 | this article*** |
| Foliation, $F$ = ($k_1$+$k_2$)/$k_3$ | 500 – 500 | Balsley & Buddington (1960) | $\Delta kF = k_2-k_3$ | 0 – 0.5 | this article*** |

**Shape**

| Shape | Prolateness = ($k_1$-$k_3$)/($k_2$-$k_3$) | 0 – ... | Khan (1962) | $E$-factor, $P/L$ = $k_2^2/k_1k_3$ | 0 – ... | Hrouda et al. (1971) |
| Prolateness = ($2k_1-k_2-k_3$)/($k_2-k_3$) | 1 – ... | Urrutia-Fucugauchi (1980) | Difference shape factor, $U$ = | | |
| Oblateness = ($k_1$-$k_3$)/$k_1$ | 0 – ... | Khan (1962) | Shape indicator = | | |
| Oblateness = ($k_1$+$k_2-k_3$)/$k_3$ | 1 – ... | Urrutia-Fucugauchi (1980) | Shape parameter or angle, $V$ = $\sin^{-1}[(k_1-k_3)/(k_1+k_3)]$ | 0 – 90 | Graham (1966) |
| Shape gradient, $K$ = $\ln(k_1/k_3)/\ln(k_2/k_3)$ | 0 – ... | Woodcock (1977) | Shape parameter, $I$ = | | |
| $R$-factor, $L/F$ = $k_3/k_2$ | 1 – ... | Stacey et al. (1960)** | Ellipsoid shape = | | |
| $q$-factor, $L/F$ = ($k_1$+$k_2$)/($k_1$+$k_2$) | 0 – 2 | Granar (1958) | Shape indicator = | | |
| Strain indicator = ($k_1$+$k_2$)/($k_2$+$k_3$) | …... – 1 | Flinn (1962) | | | |

*Also, other parameters, e.g., $\Delta KH$, $\Delta kL$ (Owens & Rutter, 1978) or $\%L$ (Taira, 1989) were already used by others (e.g., Granar, 1958) or Khan (1962) nested in other parameters or simply normalized (e.g., by $k_m$ and/or expressed as percentage), so that their origin/original use is not obvious. For example, $\Delta kL$, un-normalized, had not been defined before, but its normalized counterpart was already defined by Khan (1962) as for $\Delta kL$.*

Note that Stacey et al. (1960) calculated $L$, and $F$ (and therefore $R$) using the differences of the demagnetizing factors ($N_L$, $N_F$, and $N_A$, which may be compared to $k_1/k_2$ and $k_1/k_3$, respectively). These parameters are yet unpublished to the best of our knowledge, and we are therefore jokingly “claiming” them here as a testament to the seemingly endless possibility of anisotropy parameters.
to proceed should be to describe each fabric component separately (e.g., Černý et al., 2020). In any case, stating which convention is used to define the eigenvalues is always recommended.

2.2 Anisotropy degree and shape
Parameters to describe anisotropy shape and degree, include $P$ or $P'$ (or $P$), $L$, $F$, $\Delta k$, $k'$ amongst many others (see Table 1). Cañon-Tapia (1994) notes that lineation and foliation parameters are necessarily inextricably linked to each other, and both incorporated in other shape parameters that will similarly allow the identification of a fabric with either a prominently developed magnetic foliation or lineation. Note that parameters describing AMS degree also depend on the AMS shape; e.g., an extremely prolate anisotropy with $k_1 = 0.01$ ($k_2 = 2.98$, $k_3 = 0.01$, using $k_1 + k_3 + k_2 = 3$) and an extremely oblate anisotropy with the same $k_1$ ($k_2 = 1.495$, $k_3 = 1.495$, $k_1 = 0.01$) would have $\Delta k = k_1 - k_3 = 2.97$ and 1.485, respectively, and $P = 298$ and 149.5, respectively. Thus, prolate anisotropies always appear larger compared to the oblate counterpart.

Of the many parameters, we will briefly discuss some of the simplest, and therefore straightforward, or popularly used. For most researchers, a combination of lineation ($L$), foliation ($F$), degree of anisotropy ($P$ or $P'$), corrected degree of anisotropy ($P_c$ or $P'$), and the shape parameter ($T$), or its lognormal-distributed equivalent ($J$), constitute the “bread and butter” of anisotropy parameters. Balsely and Buddington (1960) suggested defining $F$ as $(k_1 + k_3)/2k_2$, which intuitively makes sense for foliated rocks; however, Stacey et al. (1960) point out that a rock with a lineation but no foliation should by definition have a foliation of unity, which is not the case here. In fact, both $L$ and $F$ calculated this way will reach the same maximum value for two perfectly lineated and foliated samples with the same $k$, which is rather confusing (see Table 1), and we only recommend use of this parameter to compare samples with similar foliation-dominated fabrics with dispersed $k_1$ and $k_2$ axes, e.g., for sedimentary fabrics.

Parameters that are based on differences rather than ratios have also been proposed, e.g., the normalized lineation of Khan (1962), $L = (k_1 - k_3)/k_3$. Tarling & Hrouda (1993) recommend that use of this parameter should be abandoned, albeit merely based on the potential confusion with other lineation parameters also defined “$L$”, though many other similarly defined lineation parameters also existed at the time, including the magnetic lineation ($L = (k_1 - k_2)100/k_3$) of Taira (1989), which should ideally be termed %$L$, as we have done here (see Table 1). More to the point, Cañon-Tapia (1994) argues that given the small departure from isotropy typically observed in natural rocks, there is no practical difference between using ratios or difference-parameters. In fact, Ellwood et al. (1988) and Harling (1983) had already noted that the actual calculation of various parameters reduces to a simple arithmetical combination of the principal susceptibilities, provided that these can be uniquely determined, and consequently, they are all interdependent to some extent. However, it will be shown below that for mixtures of paramagnetic and ferromagnetic minerals with different anisotropies, difference-parameters can prove extremely useful.

Many commonly used parameters ($L$, $F$, $P$...) have the advantage that they are easily relatable to structural pa-
rameters, but the disadvantage that they are only valid for full tensors and cannot be applied to deviatoric tensors. Deviatoric tensors are often determined in single crystal studies, as they are more precisely defined because differences are measured directly (e.g., Jelinek, 1996), and because some methods only provide deviatoric tensors. Additionally, \( P \) and related anisotropy parameters need to be interpreted in conjunction with \( k_m \), when determining contributions to the overall fabric. For example, a rock consisting of 100% perfectly aligned olivine will have a \( P \) value of 1.07, which corresponds to the single crystal anisotropy of olivine (when the olivine is not perfectly aligned, the \( P \)-value will be lower) (Fig. 2). When isotropic secondary magnetite is added to this rock, the \( P \)-value rapidly decreases, and drops below 1.01 as the magnetite content becomes larger than 0.2 vol% (Fig. 2). Therefore, this rock’s \( P \)-value will appear nearly isotropic, leading to the misinterpretation that the rock is almost undeformed even though the olivine is perfectly aligned. When interpreting \( P \) in conjunction with \( k_m \), it becomes clear that even a \( P \)-value of 1.01 for a high \( k_m \) may indicate strong alignment of the paramagnetic minerals. On the other hand, difference-parameters such as \( \Delta k \) reflect the anisotropy of the olivine independently of the magnetite content, avoiding misinterpretation of complex fabrics (Fig. 2).

2.3 Units of anisotropy parameters
Eigenvalue normalization not only affects parameter ranges (cf. 2.1), but also bears effects on the units of the parameters. Without any normalization, the units of the eigenvalues and their mean will reflect the quantities measured, i.e., the susceptibility or remanence, and these will vary depending on whether they are mass- (e.g., m\(^3\)/kg, Am\(^3\)/kg) or volume-normalized (e.g., dimensionless, SI, or A/m), or additionally normalized by field to kg, Am\(^3\)). If measured, i.e., the susceptibility or remanence, and these parameters will all be dimensionless. It is of paramount importance, however, that some normalization is performed if the parameters are to be compared among different rocks, possibly even within the same lithology. Normalizing difference-parameters by \( k_m \) will generate a dimensionless parameter, which is inherently easier to compare among samples; however, one always needs to bear in mind that \( k_m \) and the difference parameters may not be carried by the same minerals, so that normalized parameters are only seemingly easier to compare, as demonstrated in the previous section.

2.4 Terminology for fabric shape
Finally, throughout the years a wide terminology has been used to refer to the different fabric shapes, regardless of how these are calculated. Terms include “flattened ellipsoid”, “disk-shaped”, “uniaxial girdles”, “oblate” fabrics (also, “oblateness”), “constricted ellipsoids”, “elongated ellipsoids”, “rod-shaped”, “uniaxial clusters”, “linedate” or “prolate” fabrics (“prolateness”) (e.g., Borradaile & Jackson, 2004, 2010; Woodcock, 1977). Borradaile & Jackson (2004) note that terms such as “flattened” and “constricted” used in structural geology are meaningless in the context of magnetic anisotropy, and geometrical descriptors are more appropriate. A flattening or constrictional strain field is expected to align all grains with their long axes in a plane or along a preferred direction, respectively. However, the single crystal principal magnetic susceptibility axes do not necessarily correspond to the mineral shape. For example, for most amphiboles, the maximum susceptibility is along the crystallographic [010] axis, but the longest axis is normally [001]. Similarly, the maximum susceptibility of most clinopyroxenes is at 45° to the [001] crystallographic axis, which is the longest axis (e.g., Biedermann, 2018). Other terms, such as “sedimentary fabrics” for oblate ellipsoids of sediments and sedimentary rocks, with scattered \( k_3 \) axes within the sedimentary bedding plane and subvertical \( k_1 \) axes, are also widely used and appropriate in the right context. However, we do recommend that defining fabrics by a generic “type”, for example “type A for oblate fabrics” and “type B for prolate”, is unnecessary and confusing.

3. Data visualization – finding the right plot and “more processed” parameters
3.1. Fabric strength and shape
As for the parameters, certain plots are more appropriate than others for different applications, exactly because they use the same, or comparable, derived quantities. For example, the Flinn diagram (Flinn, 1962) which plots \( L (k_L/k_L) \) versus \( F (k_F/k_F) \), with origin at \( F, L = 1, 1 \), is often used in structural geology to evaluate petrofabrics (Fig. 3a). Shape, on these diagrams, is represented by the slope of a line connecting the origin to the datapoint, and quantified by the confusingly named parameter \( k = (L-1)/(F-1) \), which has the further inconvenience of being asymmetric, so that oblate ellipsoids have \( k > 1 \) whereas prolate ellipsoids have \( k \leq 1 \). A disadvantage of the Flinn diagram is that weak anisotropies are clustered near the origin, which can make data hard to discern and evaluate their shapes. This effect is overcome on the Ramsay diagram (Ramsay, 1967), also derived for structural applications, where the logarithms of the eigenvalues are used, so that the origin is at 0, 0 and its shape parameter \( K = \ln(L)/\ln(F) \).

Unlike Flinn’s shape parameter \( k \), Jelinek’s (1981) \( U \) and \( T \) parameters range symmetrically between \( U \) or \( T = +1 \) for oblate ellipsoids and \( U \) or \( T = -1 \) for prolate ellipsoids, noting, however, that \( U \) is zero when \( k_1/k_2 = k_2/k_3 \), while \( T = 0 \) when \( k_1/k_3 = k_2/k_1 \).

As other parameters, these shape parameters are also interrelated, so that \( T \), which may also be expressed as \( \ln(L)/\ln(F) \), relates to Ramsay’s \( K \) as \( T = (1-K)/(1+K) \), whereas \( U \) may also be expressed as \( (F-L)/(F+L) \). Likewise, Jelinek’s (1981) \( P_j \) parameter \( P_j = \exp \{2[(n_1-n_2)\^2+(n_2-n_3)\^2+(n_3-n_1)\^2]\}^{1/2} \) relates to Nagata’s (1961) \( P \) parameter \( k_{L3} \) as \( \ln(P_j) = \ln(P) + 1 + [(1/E)/3]^{1/2} \).

The Jelinek diagram (Fig. 3b), which plots \( P_j \) versus the shape parameter \( T \), is popular in the rock-magnetic
community. Compared to the Flinn diagram, this plot has the advantage of expanding the region of low anisotropy; however, expanding this area of the plot necessarily increases the uncertainty around data, so that slight shape differences for weakly anisotropic fabrics appear just as significant as for larger degrees of anisotropy. In general, at low anisotropy degrees, shapes are poorly defined and subject to large uncertainty. Moreover, for perfect isotropy \((P_J = 1)\) the fabrics are spherical by definition, so any values of \(T\) at this point are not real, presenting a “non-possible” region, or axis, within the diagram itself.

To obviate for these issues, Borradaile & Jackson (2004) proposed a polar plot in which \(P_J\) is the radius and \(T\) is the arc-length, which extends from 0 to \(\pi/4\), representing increasingly oblate distributions, and from 0 to \(-\pi/4\), representing increasingly prolate ellipsoids (Fig. 3c). Borradaile & Jackson (2010) also extended the polar plot for negative values of \(P_J\), making the plot symmetrical about the origin (not shown here), so that the negative \(P_J\) oblate field is on the bottom-left, while the prolate negative \(P_J\) field on the top-left part of the plot, allowing representation of diamagnetic fabrics on the same diagram, if the convention of \(k_1 = \) largest negative susceptibility is used and substituting the terms in Jelinek’s (1981) formula with \(\ln(k_1/k_m)\) and so on.

When representing data from the same lithology and/or when there is not much variation among the fabrics of different specimens, another useful plot shows the cumulative distributions of the bootstrapped eigenvalues, introduced by Tauxe et al. (2018) (Fig. 4). Here, the bootstrapped individual eigenvalues are plotted with the intervals containing 95% of their means, versus their cumulative distribution. Strongly overlapping distributions of the three eigenvalues readily indicate more isotropic fabrics, whereas non-overlapping distributions indicate unique eigenvalues and thus triaxial (or neutral) fabrics. Likewise, overlapping \(k_1\) and \(k_3\), but distinct \(k_2\), distributions indicate fabrics that tend to oblate, while overlapping \(k_2\) and \(k_3\), but distinct \(k_1\), distributions indicate prolate fabrics. The larger the “spread” between \(k_1\) and \(k_3\), the stronger the anisotropy. Because the distributions are bootstrapped, including specimen-data or site-mean data that carry substantially different anisotropies will affect the cumulative distribution, so that the function becomes

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**Figure 3.** a) Flinn diagram of \(L\) versus \(F\), with superimposed curves for \(T(I)\) and \(P_J\) to demonstrate the relationship between these parameters in this space; b) Jelinek plot of \(T\) versus \(P_J\), which avoids the clustering of low anisotropy data towards the origin of the Flinn diagram, yet introduces distortion of the same data (see text for details); c) Polar plot of \(T(I)\) (diagram arc-length) and \(P_J\) (diagram radius) of Borradaile & Jackson (2004). Original figures by Borradaile & Jackson (2004 and 2010).

**Figure 4.** Cumulative distribution of the eigenvalues: a) triaxial-prolate data, note the similar magnitude of the minimum and intermediate eigenvalues that do not however fall within each other’s 95% confidence intervals, black solid lines for the minimum and blue dash-dot lines for the intermediate, yet they are both much smaller than the maximum eigenvalue distribution, red line, indicating a dominantly prolate fabric; b) triaxial data showing significant spacing and no overlap between the three eigenvalue distributions; c) perfectly oblate data, with indistinguishable maximum and intermediate eigenvalue distributions, much larger than the minimum. Data from Bilardello (2021), plotted using the PmagPy software package of Tauxe et al. (2016).
somewhat blocky. Such observation will alert the user that one or more fabric determinations may either be different from the others, or problematic. In this respect, however, to some extent both the sampling scheme and data do dictate which plots one should use, so that if there is a range in anisotropy degree and/or shape, the more isotropic data will be strongly clustered on a Flinn diagram, whereas the cumulative distribution plot will be somewhat off.

Triangular or ternary diagrams of the normalized eigenvectors have also been used for eigenvalues, with apices at $k_1 = 1$, $k_2 = 0.5$, and $k_3 = 0.33$ (Mark, 1974; Mark & Andrews, 1975), and are analogous to Harland and Bayly’s (1958) stress plot (Fig. 5). Such plots have a curved line separating the fields for clustered (lineated) and girdled (foliated) fields, and the overall lack of symmetry makes them somewhat difficult to interpret (Fig. 5a). Nevertheless, ternary diagrams are commonly used within the glaciology community, for example, where fabric elongation $(1 - k_3/k_1)$ is plotted against fabric isotropy $(k_2/k_1)$. In this diagram, the apices represent three end-member fabric shapes: $[1 - (k_3/k_2) = 0; k_2/k_1 = 1]$ for isotropic fabrics; $[1 - (k_2/k_1) = 0; k_2/k_3 = 0]$ for planar girdles; and $[1 - (k_3/k_1) = 1; k_2/k_3 = 0]$ for perfect axial clusters. (Benn, 1994).

As a final remark about different ways of plotting anisotropy parameters, one must bear in mind that for perfect fabrics, ranging from neutral to infinitely prolate and oblate, the magnitudes of the eigenvalues vary asymmetrically from being equal to one axis being unique and infinitely long $(k_i)$ and the other two equal and infinitely small $(k_i = k_j$) for the prolate case, or one being infinitely small $(k_i)$, and the other two equal and infinitely long $(k_i = k_j$) in the oblate case. Therefore, for prolate fabrics with finite, not infinite, principal axes, the maximum eigenvalue will always be twice the size of the maximum (and intermediate) eigenvalues of the oblate counterpart (see Fig. 5a). This condition necessarily generates an inherent asymmetry in any possible plot and shape parameter, as mentioned above for Flinn’s $k$, Ramsey’s $K$, or Jelinek’s $P$, for which prolate fabrics will always have more range. Moreover, incrementally reducing prolateness to neutrality and subsequently increasing oblatteness, the path will not follow a linear trajectory (cf. Fig. 1.6 of Tarling and Hrouda, 1993). For all the considerations made so far, every plot has its faults, so that choice of diagram is mostly a matter of preference, and to some extent dictated by the data at hand and/or sampling strategy.

3.2. Fabric orientation.

Plotting eigenvector orientation data on an equal area lower hemisphere stereonet is the most widely used and appropriate fabric orientation representation (Fig. 6a). However, to represent the confidence of a mean eigenvector of a group of samples varies among users. Granted that confidence circles (e.g., $\alpha_{95}$ circles) should never be used for anisotropy, Jelinek’s (1978) confidence ellipses are probably the most commonly used method of representing uncertainties (Fig. 6b). Alternatively, Con-
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Olivine single crystals (6-18 wt.% FeO) Hornblende single crystals

olivine and hornblende single crystals, in relation to their crystallographic axes. The figure demonstrates that the rainbow scale visually distorts data because the yellow is the brightest color, attracting the eye the most, yet it is not located at the center of the map, whereas the green shades form a wide band with low perceived color contrast, opposed to the narrow band with high contrast of the red tones, so that the maximum eigenvector location appears more clustered than the minimum counterpart (Crameri, 2018; Crameri et al., 2020): a bimodal scale with a neutral (white) color midway between the maximum and minimum eigenvector locations is more appropriate. In any case, adding contour lines to counter any distortion effect is recommended; similar considerations and plotting improvements have been made by Ramon Egli in the context of FORC diagrams, from which we have extensively “borrowed” here and whom we thank for the discussion.

To some degree the choice of plot will be defined by the data used, e.g., colormaps are great for single specimens, while bootstrapped data can only be applied when averaging a sufficient number of samples. Other than that, it is mainly the authors’ (or sometimes the reviewers’) personal preference that will define the type of plots used.

4. Finding the way through the jungle
To summarize, we put together a number of recommendations for processing and visualizing anisotropy data. One important topic that was left out, however, is that of data significance and uncertainties, which will be covered in a subsequent article.

The first step is to determine whether the data are from full or deviatoric tensors, which will likely correspond to the type of work one performed. These require different parameters, whereby only \( k' \), \( \Delta k \) and \( U \) work for deviatoric tensors, and \( P, P', T, L, F \), etc., are only applicable to full tensors.

Data should be normalized, so that they are comparable among specimens: choice of normalization will also dictate the ranges of calculated parameters, so care is needed if comparisons to other data are to be made. For difference-parameters, a normalization by \( k'_{\text{m}} \) is useful,
bearing in mind the caveats when several carrier minerals contribute to the magnetic anisotropy. Also note that the definition and normalization of $k_m$ controls the ranges adopted by other parameters.

Of the multitude of parameters proposed in the literature many different measures for $F$ exist, for example, and each bears some implications. What is the purpose of the study and target audience? For structural applications one should use parameters that are more easily correlated with similar data (e.g., $P$, $L$, $F$). For separating fabric components, $k'$ and $\Delta k$ can be interpreted without knowing $k_m$, avoiding the caveat referred to above. If data are to be compared to single-crystal data, then $k'$ and $\Delta k$ etc., are also the most appropriate, since most single crystal data are reported with these parameters. For palaeomagnetic applications, for example anisotropy corrections for deflection of remanences or paleointensity, one must use full remanent tensors that characterize the mineral sub-populations carrying the remanence. These include the appropriate type of anisotropy (of TRM, ARM or IRM) to correct a natural or laboratory remanence, in the correct units, and to isolate the magnetic carriers in the grain size range of interest.

One should next evaluate the amount of data, which dictates choice of plot and parameters: for many specimens, plots that involve bootstrap confidence bounds may be used, but for fewer data, plots that use analytical uncertainties are more appropriate.

Next, the range and distribution of the data will inform what plot is better-suited. Whether all data are comparable among each other, or whether they necessarily need to be grouped by sites or different lithologies, and whether site-means may be plotted together, will dictate which plots one can use. These considerations also fall within the scope of the study and will dictate what “shape and magnitude” plots one should use, particularly with linear versus logarithmic distributions, and whether cumulative distribution plots are appropriate.

Finally, all parameters and visualization options have their advantages and limitations (e.g., large uncertainty in anisotropy shape for low anisotropy degrees is amplified by the Jelinek plot). It is important to be aware that the choices made bear implications for each parameter and plot.

References
Geol. Mag., 95, 89–104.

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