An Analysis of the Impact of Algae Harvesting on a Lake Eutrophication Model

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May 9, 2022

1 Introduction

Eutrophication is a biological process where the phytoplankton population increases significantly in an aquatic environment due to the presence of excess nutrients, especially phosphorus [1]. It is highly detrimental to the environment and has several negative consequences, such as increased fish deaths due to anoxic events and excessive plant and algae growth [2]. There have been multiple models developed to study this phenomenon. A number of these focus on Lake Mendota, WI, which is considered by some "the most studied lake in the world" [3]. This paper will focus on the 2005 Carpenter model, which uses data from Lake Mendota [2, 4]. This model is composed of a system of three differential equations that model the phosphorus concentrations in different lake elements: soil, water, and sediment. The current model does not account for lake restoration methods, so it will be modified to include algae harvesting, one standard lake restoration method.

2 Literature Review

2.1 Eutrophication

The growth of algal populations leads to higher photosynthesis rates, which decreases carbon concentrations and increases the pH of the water [7]. Eutrophication occurs due to excess nutrients in the water, especially phosphorus. These nutrients usually come from agricultural runoff, sewage, and industrial discharges, and the restoration of an eutrophic lake can have high associated costs, depending on its current state [4]. The reversal of eutrophication varies depending on their responses to the reduction of the phosphorus input. Certain lakes, called "reversible," are able to leave the eutrophic state solely by decreasing the phosphorus input. Others require a combination of decreasing it and some outside intervention, such as chemical treatment, and are called hysteretic [4].

2.2 Lake Restoration Methods

Currently, it is estimated that over 30% of lakes worldwide are classified as eutrophic [5]. The restoration process takes a long time and has high associated costs, and currently, one of the main factors when considering lake management is climate change. Climate change affects patterns of rainfall and temperature, both of which contribute to the aquatic environment of the lakes [6]. The lake restoration strategies can be divided based on their focus: external and internal measures. The main external measure is the reduction of external nutrient loading, which happens by modifying land use and agricultural practices, and also sewage load reductions [6]. When considering internal measures, one important method is biomanipulation, where increased grazing rates deter the phytoplankton growth. Like most lake restoration methods, this method works best when combined with the reduction of external nutrient loading [5, 6].

However, even with the decrease in external nutrient loading, internal nutrient loading from accumulated phosphorus in sediments may prevent any changes in the algae population in the lakes [7]. Further in the analysis of the eutrophication model, this information will be used to explain certain observed behaviors.

2.2.1 Algae Harvesting

Another important method of lake restoration, and the focus of this research, is algae harvesting. Removing algae, fish, or any other living organism from the lake environment will cause a decrease in nutrient availability, decreasing the species growth rates [5]. Different phytoplankton species are able to absorb varying concentrations of phosphorus, but this process depends on the concentrations of other chemical species, such as nitrogen [8]. They are able to remove phosphorus from the lake water through a mixture of adsorption and chemical precipitation, and, in certain lakes, are able to remove between 60% and 70% of the phosphorus present [8, 9].

The harvested algae can be reused as biofuels, fertilizers, and animal feed, among others [7, 10]. The main issue with the algae harvesting restoration method is the harvesting process itself, which can add up to 20% and 30% of the total cost of algal biomass production for reusing [5, 9]. This is a large discouragement for algae harvesting in natural lakes as the sole restoration method since it would be much more expensive and energy-intensive than other methods [11]. However, the combination of algae harvesting with additional restoration techniques can be effective [9, 8].

3 Original Model

Essentially, lake eutrophication depends on the phosphorus input, the rate of loss, and the recycling of phosphorus in the lake water [4]. Basic models deal with only the effects of phosphorus concentration in water, but a more complex model was developed to include the effect of phosphorus in the soil and sediments in the process of eutrophication [2]. The model is composed of the three differential equations below.

$$
\begin{aligned}
\frac{dU}{dt} &= W + F - H - cU \\
\frac{dP}{dt} &= cU - (s+h)P + rMf(P) \\
\frac{dM}{dt} &= sP - bM - rMf(P) \\
f(P) &= \frac{P^q}{m^q + P^q}
\end{aligned}
$$

where U is the phosphorus density in the soil, P is this density in lake water, and M is the phosphorus density in surface sediments, all in units of g/m^2 [2].

Symbol	Definition	Nominal Value
b	Permanent burial rate of sediment P	$0.001y^{-1}$
c	P runoff coefficient	$0.00115y^{-1}$
F	Annual agricultural import of P to the watershed	$31.6g/m^2y$
h	Outflow rate of P	$0.15\ y^{-1}$
H	Annual export of P from the watershed in farm products	$18.6g/m^2y$
m	P density in the lake when recycling is 0.5 r	$2.4g/m^2$
r	Maximum recycling rate of P	$0.019y^{-1}$
q	Parameter for steepness of $f(P)$ near m	8
s	Sedimentation rate of P	$0.7y^{-1}$
W	Nonagricultural inputs of P to the watershed	$1.55\ g/m^2y$

Table 1: Parameter definitions and nominal values [2]

Using the values determined by Carpenter in table 1, we first fix the value of $W + F - H$. Doing this, we get that $\frac{dU}{dt} = 0$ when $cU = W + F - H$, which we then plug into the second and third equations to find the equilibrium solutions, so we only have to worry about the last two equations. We can see that for values of $W + F - H$ between around $0.38g/m^2y$ and $0.85g/m^2y$ there are multiple equilibria [2]. The two stable equilibria represent the eutrophic and the non-eutrophic states of the lake. In figure 1, we are considering the system at $W + F - H = 0.6g/m^2y$, and have the first equilibrium, represented by A, at $(0.70651, 494.02425)$. This point represents the non-eutrophic state of the system. The other stable equilibrium is represented by B, and is at $(3.18391, 122.41418)$, which is the eutrophic state of the system. The non-eutrophic state has low water phosphorus concentrations but high sediment phosphorus concentrations, while the eutrophic state has low sediment phosphorus but higher water phosphorus. There is also an unstable equilibrium at $(2.0115, 298.299)$, which is a threshold state.

When we consider the system with $W+F-H = 1.55g/m^2y$, we have that the state only has one equilibrium solution at $(8.37831, 293.25346)$, which represent

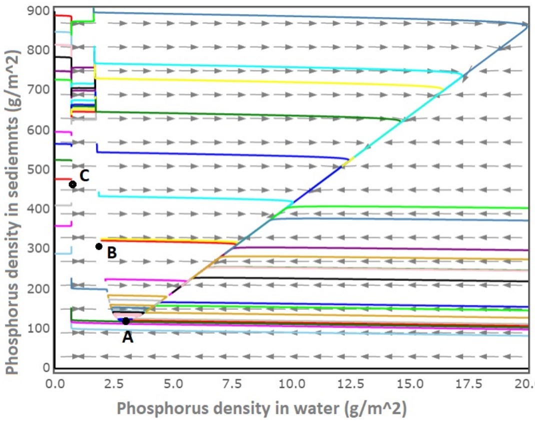

Figure 1: Phase diagram at $W + F - H = 0.6g/m^2y$, with two stable equilibria, A and C, and one unstable equilibrium at B.

the eutrophic state. As seen in figure 2, the equilibrium solution happens in a high water phosphorus density and a low sediments phosphorus density.

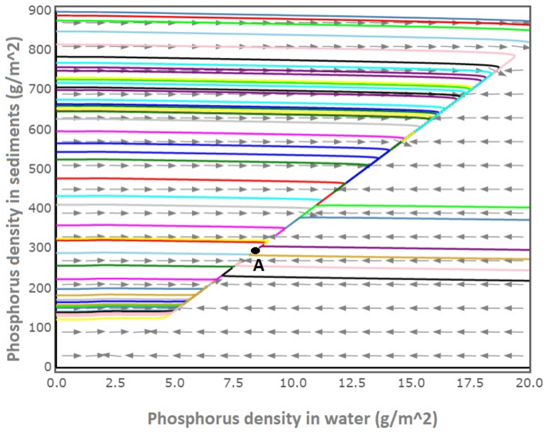

Figure 2: Phase diagram at $W+F-H = 1.55g/m^2y$, with one stable equilibrium A.

To determine the resilience of the three-equation model, the value of $W + F - H$ is varied while the others remain constant. Starting with $W + F - H = 0.6g/m^2y$, the initial coordinates for the system were chosen to be $P = 0.1\frac{g}{m^2}$ and $M = 100\frac{g}{m^2}$, and then were let freely flow for 100 years. Then, the system was switched to $W + F - H = 1g/m^2y$ for another 100 years. Lastly, the system was switched to the new parameter value of $W + F - H$ for a certain period of time. The coordinate of where the new system was after this time was then returned to the original equation where $W + F - H = 0.6g/m^2y$ and was let freely flow to see is the system would return to the non-eutrophic equilibrium. This process was repeated to determine the maximum time length the system can remain with the new parameter values before it does not return to the non-eutrophic state once the system is switched back to $W + F - H = 0.6g/m^2y$, instead tending towards the eutrophic equilibrium. This process was repeated for several values of $W + F - H$, and the results are plotted in Figure 3. The results match with what was expected from Cessi since the resilience is in the form of a power function [12]. This result shows that there is very little time to reverse the effects of eutrophication. Some values of $W+F-H$ have only months before reducing the phosphorus input is not enough to prevent eutrophication.

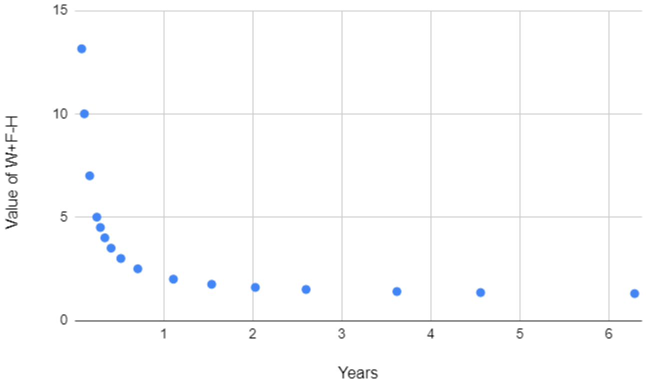

Figure 3: The minimum time length of a perturbation required to switch the system to the eutrophic equilibrium.

4 Model Modifications

While Carpenter's model is already a good way to estimate how the phosphorus concentration changes for different lake components, we can modify it to include other parameters. In this case, the only change is adding a parameter μ that represents the rate of algae harvesting happening in the lake.

$$\frac{dU}{dt} = W + F - H - cU$$

$$\frac{dP}{dt} = cU - (s + h + \mu)P + rMf(P)$$

$$\frac{dM}{dt} = sP - bM - rMf(P)$$

$$f(P) = \frac{P^q}{m^q + P^q}$$

Once again, the values determined by Carpenter are used for the parameters c, s, h, r, b, q, m, which can be found on table 1. To analyse how μ behaves, we fix the value of $W + F - H$ at $1.55g/m^2y$. Doing this, we get that $du/dt = 0$ when $cU = 1.55$, which we then plug into the second and third equations to find the equilibrium solutions. As we increase the value of μ, the system exhibits different behaviors. The case where $\mu = 0$ is the original system, and the discussion of this scenario can be found above. Between $\mu = 0$ and $\mu = 0.36$, there is only one equilibrium solution representing the eutrophic state. These solutions are all stable equilibria, and the system is similar to the one seen on figure 2.

Once we get at $\mu = 0.40$, we get an interesting behavior arising. The system now has three equilibrium solutions. Figure 3 displays the phase diagram for this scenario. The first, at $(2.567, 138.286)$, represents the lake in its eutrophic state and is seen as point A in the figure, with high phosphorus density in the water and low phosphorus density in the sediments. The analysis of the stability of this solution indicates that it is an unstable equilibrium. We also have an equilibrium at $(1.387, 787.055)$, which represents the non-eutrophic lake state, and the stability analysis indicates that this is a stable node. This point is seen as point C in the figure, with high sediment phosphorus and low water phosphorus There is an additional equilibrium at $(1.545, 700.133)$, seen in point B, whose stability analysis shows it is a saddle point. The three equilibrium solutions exist for values between $\mu = 0.40$ and $\mu = 0.47$.

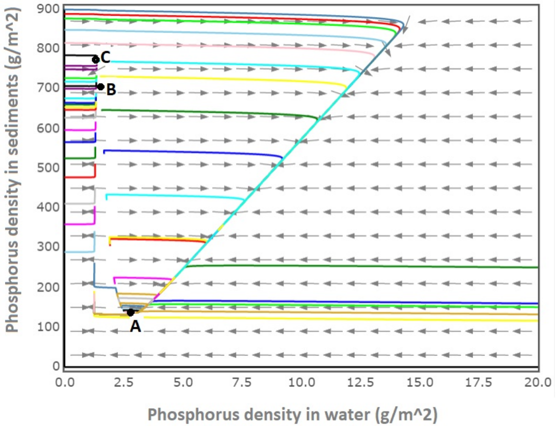

Figure 4: Phase diagram at $W + F - H = 1.55g/m^2y$ and $\mu = 0.40$, with one stable equilibrium at C and two unstable equilibria at A and B.

For values of μ above $\mu = 0.48$ or above, there is only one equilibrium solution representing the non-eutrophic state of the lake. The physical meaning of this is that the algae harvesting at these levels will lead to the reversal of the lake eutrophication. Figure 4 shows the phase diagram for the system with $\mu = 0.48$, and there is only one equilibrium solution, A, which happens in a low water phosphorus and high sediment phosphorus environment.

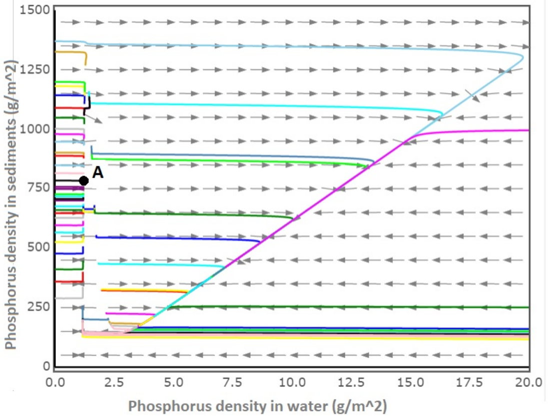

Figure 5: Phase diagram at $W + F - H = 1.55 g/m^2 y$ and $\mu = 0.48$, with one stable equilibrium A.

5 Periodic Solution

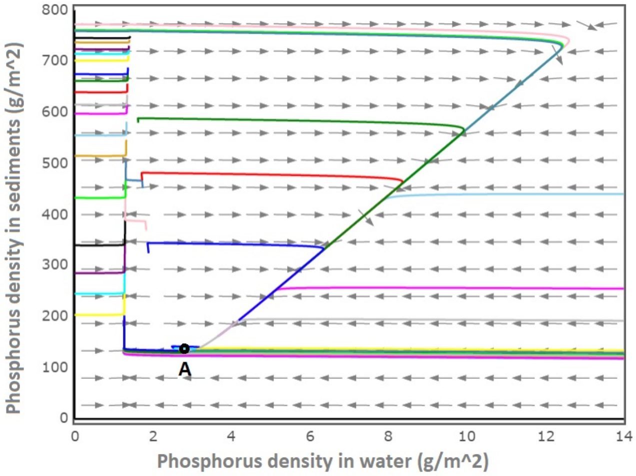

Figure 6: Phase diagram at $W + F - H = 1.55 g/m^2 y$ and $\mu = 0.39$, with one unstable equilibrium A.

On the interval of $\mu = 0.37$ to $\mu = 0.39$, the system behaves differently than any other values. Firstly, we have one equilibrium solution, which is unstable. When we focus on the $\mu = 0.39$ case, the unstable equilibrium is at $(2.6229, 133.629)$ represented in Figure 5 by point A. When looking at the phase diagram, it is possible to notice some interesting behavior. We can observe the presence of a loop. Suppose we start on the top right of the graph, with a high phosphorus density in water and in sediments. In that case, the amount of phosphorus decreases in both the sediments and water until we reach a value close to the unstable equilibrium. Then, there is a short period where the phosphorus in the sediments does not change much, but the density of it in water is still decreasing. After this period, when the phosphorus density in water is close to $1.4g/m^2$, this value stabilizes and the phosphorus quantity in the sediments starts to rise very slowly, from around $130g/m^2$ to about $750g/m^2$. This forms an almost perfectly vertical line seen on the left of the graph. Once we reach those values, the phosphorus density in sediments stabilizes and the density in water increases very fast. This returns us to the original position, of high phosphorus in both sediments and water. The period necessary to complete this cycle is of around 2880 years. From the literature, we have that without algae harvesting and in a noneutrophied lake, the turnover time for sediment phosphorus is about 1000 years [2]. Adding the algae to the lake makes sense for the phosphorus cycling process to take longer than expected.

6 Conclusion

Paleolimnological analyses of other lakes have shown that the amount of algae in lakes has changes over time, even when there are no additional inputs [14, 15]. This indicates that lakes have internal processes that cause phosphorus concentrations to change over time, since the amount of algae depends on the amount of nutrients available. Thus, even without algae harvesting, it is possible to have a periodic cycle about the phosphorus concentrations. In essence, the periodic behavior of the phosphorus concentrations when algae harvesting is taking place could be replicated by increasing the value of the outflow rate h. The results imply that, over thousands of years, lakes could go through different states regarding phosphorus.

Algae harvesting is a viable way of restoring a lake from its eutrophic state, but it would take hundreds of years for any visible change to happen. For any short-term response, algae harvesting needs to be combined with additional restoration techniques.

In this paper, we have explored the effects of adding an algae harvesting parameter to a model of lake eutrophication. At a sufficiently large input, we have observed that a eutrophied lake will tend towards a noneutrophied state. We also saw that, for certain values of the new parameters, there is the appearance of a periodic cycle, which we were able to quantify the length of this period. The results illustrate the urgency of mitigating the amount of phosphorus added to bodies of water to prevent eutrophication, for even with algae harvesting, it will be long before results start being seen.

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