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REGIONAL RESOURCE DEPLETION AND EXPLORATION

by

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I. INTRODUCTION

Most work considering the depletion of a nonrenewable natural resource stock has been conducted in the context of optimal global depletion, exploration, and price paths (from Hotelling [3] to the more recent contributions of Dasgupta and Heal [1], Fisher [2], Schulze [6], Weinstein and Zeckhauser [9], and Pindyck [5] to name but a few). Firms are viewed as either operating under perfectly competitive or monopolistic conditions (Stiglitz [8] and Lewis [4] providing the best examples of the latter). This approach, however, has little value when considering the behavior of a regional resource firm which controls an often insignificant proportion of the world's total proven reserves of a particular natural resource in a market structure often dominated by a few reserve-rich firms. This paper will investigate the depletion and exploration strategies of this type of regional firm, as well as compare its behavior to that of the traditional economic firm. Section II describes the market situation envisioned for the firm. Section III considers the firm's depletion path over time while Section IV expands this to include the possibility of exploration. Section V concludes.

II. THE ORGANIZATION OF THE INDUSTRY

A given nonrenewable natural resource often occurs naturally in many regions throughout the world although some regions are blessed with much greater concentrations than others.[1] Because of this uneven distribution (and often ownership) of the resource reserves, it is typical in natural resource industries that a few firms with substantial reserve holdings have considerable control over the market for the resource.[2] Other producers typically are little more than price-takers. Regional resource firms in a global market are often in this category.

To investigate how such a market structure effects such firms the following assumptions will be made. First, the regional firm views the market price of the extracted resource as given and beyond its control. Clearly this is likely to be the case since the regional extraction of one firm would be negligible when compared to global production. Further, since the extracted resource of firms in the industry are basically homogeneous, there is no reason to expect that the consumer (e.g., a refinery) cares from which firm the resource comes. Secondly, the regional resource firm is assumed to initially hold the rights over a known quantity of reserves, R_0, and these may be augmented only through costly exploratory operations.

Finally, it will be assumed that the firm views the current real price of the extracted resource as that which will prevail in all future periods. This appears to "fly-in-the-face" of the classic Hotelling result which requires in general that the economic rent earned on units of a nonrenewable resource stock rise at the rate of discount. Typically this is achieved via an increasing price for the resource over time. This pattern, however, is seldom observed due to a myriad of real-world violations of the underlying assumptions from changing extraction costs and demand conditions over time to unforeseen discoveries and less than pure market structures.[3] These uncertainties basically leave the regional producer at best guessing what the future price might be. An expectation of a constant real price in the future is just one reasonable possibility. Other possible expectations, however, may also be considered within the framework presented below.

In summary, the regional resource firm is assumed to be a price-taker in all periods whose production (extraction) is ultimately constrained by input availability (known reserves).

III. REGIONAL RESOURCE DEPLETION

In order to maximize the present value of profits over time an unconstrained price-taker (unconstrained with respect to input availability over time) produces, as is well-known, at that output level, q**, where price equals the marginal cost of production and does so in all periods. The resource firm, however, is constrained by R_0 in the absence of exploration. Consider its problem:

$$\max_{q} \ \pi = \int_0^T [pq - C(q)]e^{-\delta t} \, dt \qquad (1)$$

subject to

$$\dot{R} = -q \qquad (2)$$

$$R \geq 0, \ q \geq 0, \ R(0) = R_0 > 0 \qquad (3)$$

where p is the market price of the extracted resource, q the quantity of the resource extracted and sold, R the amount of known unextracted reserves controlled by the firm, and C represents the costs of extracting a given amount of the resource with $C(0) > 0$, $C' > 0$, and $C'' > 0$.

The solution a la Pontrayagin's maximum principle is straightforward given the Hamiltonian,

$$H = pqe^{-\delta t} - C(q)e^{-\delta t} + \lambda(-q). \qquad (4)$$

Differentiating H with respect to q and assuming an interior solution ($q > 0 \ \forall \ t$) yields the following necessary condition:

$$p - C' = \lambda e^{\delta t}. \qquad (5)$$

Differentiating H with respect to R gives the dynamic equation for λ which is simply $\dot{\lambda} = 0$. Thus, it is clear from (5) that the difference between price and the marginal cost of extraction must grow over time at a rate of δ -- the basic Hotelling result. However, since the firm assumes $\dot{p} = 0$, this implies a falling output level over time. In particular, differentiating (5) with respect to time and remembering $\dot{p} = 0$ yields

$$\dot{q} = \frac{-\delta(p-C')}{C''} < 0. \quad (6)$$

The path of q can be described more precisely using the necessary condition that the Hamiltonian evaluated at time T, the terminal time, must equal zero. Setting (4) equal to zero and rearranging gives

$$p - \frac{C(q(T))}{q(T)} = \lambda e^{\delta T}. \quad (7)$$

This with (5) evaluated at T implies that at the terminal time q must be that output level at which average and marginal extraction costs are equal or that output level where per unit extraction costs are minimized. Thus, the time path of q appears as is shown in Figure 1 by path A where q** is that output level where p = C' and q* is that where per unit extraction costs are minimized.[4]

Since the area under this path must equal R_0, the value of λ will depend on R_0 and be inversely related to it. Therefore, path B in Figure 1 would represent the path of q if initial reserves were greater. Note that the less binding the initial reserve constraint (the larger R_0), the closer the resource firm behaves like its unconstrained counterpart (output levels close to q** as one would expect). However, note that if $\delta = 0$, the resource firm will produce in all periods at q* where per unit profits are maximized regardless of the level of R_0.

Finally, Figure 2 illustrates how the absolute level of p effects the depletion path. Path B represents the case of a lower p. Clearly, this results in a lower value for q**, q_B^{**}, which in turn leads to a flatter time profile with more extraction delayed to later periods and a later exhaustion date.

IV. EXPLORATION

The introduction of the possibility of exploration to a large degree reduces the intertemporalness of the resource firm's optimization problem since

FIGURE 1.

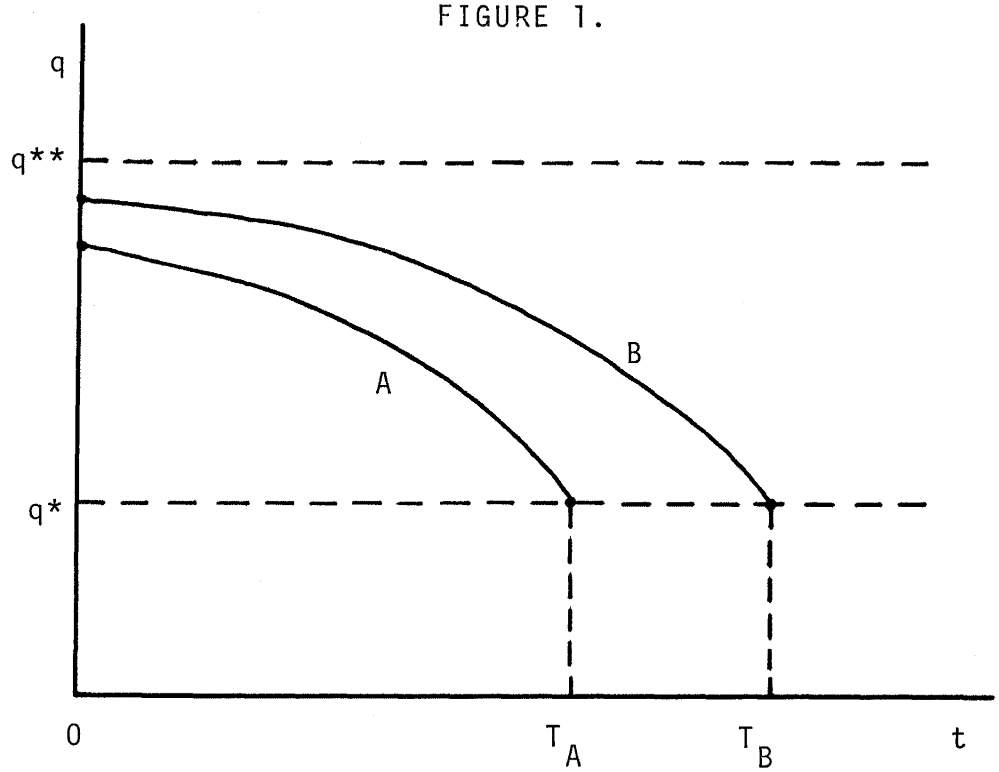
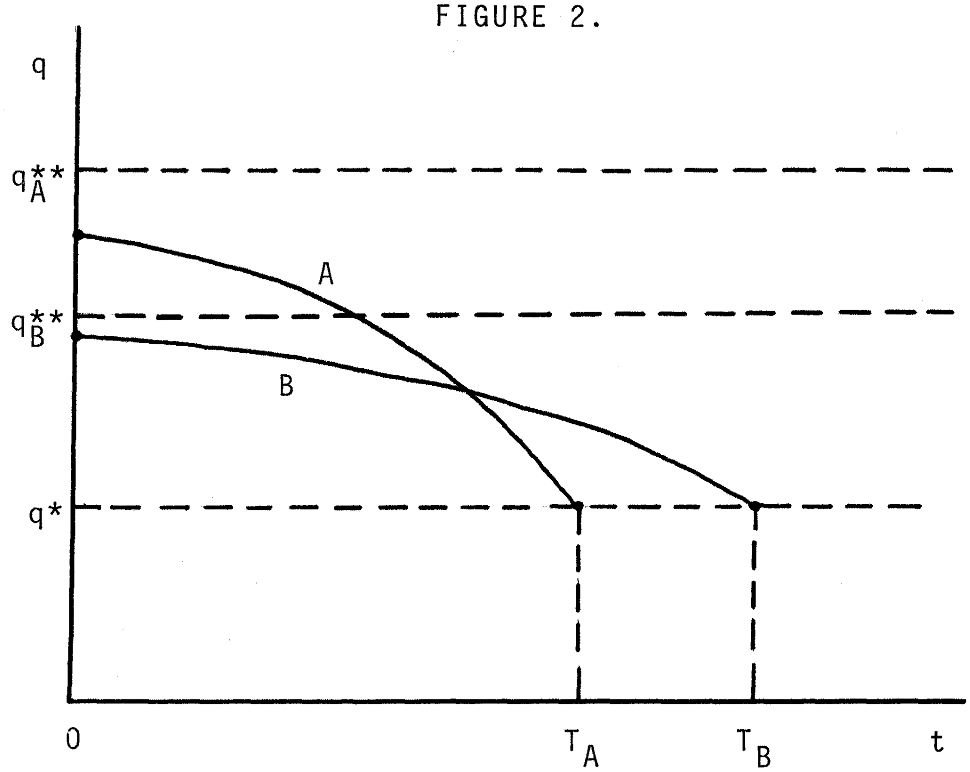


FIGURE 2.



input availability (again the input being reserves) will be less constrained. To highlight some of the basic principles, the nature of the exploration process will be somewhat simplified. In particular it will be assumed that additional units of reserves can be explored for and discovered at a cost of w per unit within the firm's region. This obviously ignores the tremendous uncertainty and "lumpiness" associated with exploratory activities, as well as the likelihood that the cost of exploration rises and the probability of discovery falls as the region is explored (Pindyck [5]). However, w may best be interpreted as the expected cost of exploring for and discovering an additional unit of reserves. Given this the resource firm's problem now appears as follows:

$$\underset{q,d}{\text{Max}}\ \pi = \int_0^T [pq - C(q) - wd]e^{-\delta t}dt \tag{8}$$

subject to

$$\dot{R} = d - q \tag{9}$$

and the other constraints given in (3) plus $d \geq 0$ where d is the amount of additional reserves discovered through exploration.

To analyze the behavior of the firm, consider the case where $R_0 = 0$. In the first period it is clear that $q \leq d$ must be true since the firm can't extract and sell more units than it has discovered. Further, however, it would not be optimal for q to be less than d since the present value of the exploration cost of the extra units, d - q, would be less if the search for them and their discovery were delayed until they were also extracted and sold. Thus, the firm would choose to have q = d in the initial period. A similar argument may be made for all subsequent periods so that q = d in all periods. The problem above, in fact, becomes identical to that of a firm whose processing costs are represented by C(q) and whose ingredient or component costs are w per unit (in essence, a unit of reserves is required to "produce" a unit of the extracted resource). The problem, therefore, reduces to maximizing the expression pq - C(q) - wq with respect to q which results in the following necessary condition:

$$p - C' - w = 0. \qquad (10)$$

The problem is no longer intertemporal and the firm would produce at that output level where the above is true, q_w, indefinitely (as long as $q_w > q^*$). Of course, the firm realistically cannot produce indefinitely because at some time the total resource stock will all be discovered and extracted. As this time is approached, w is likely to begin rising very rapidly stopping all exploration and extraction. However, if we assume that this time is beyond the reasonable planning horizon of the resource firm (as it would be for most resources at this time), the above scenario provides a reasonable approximation of the firm's behavior.

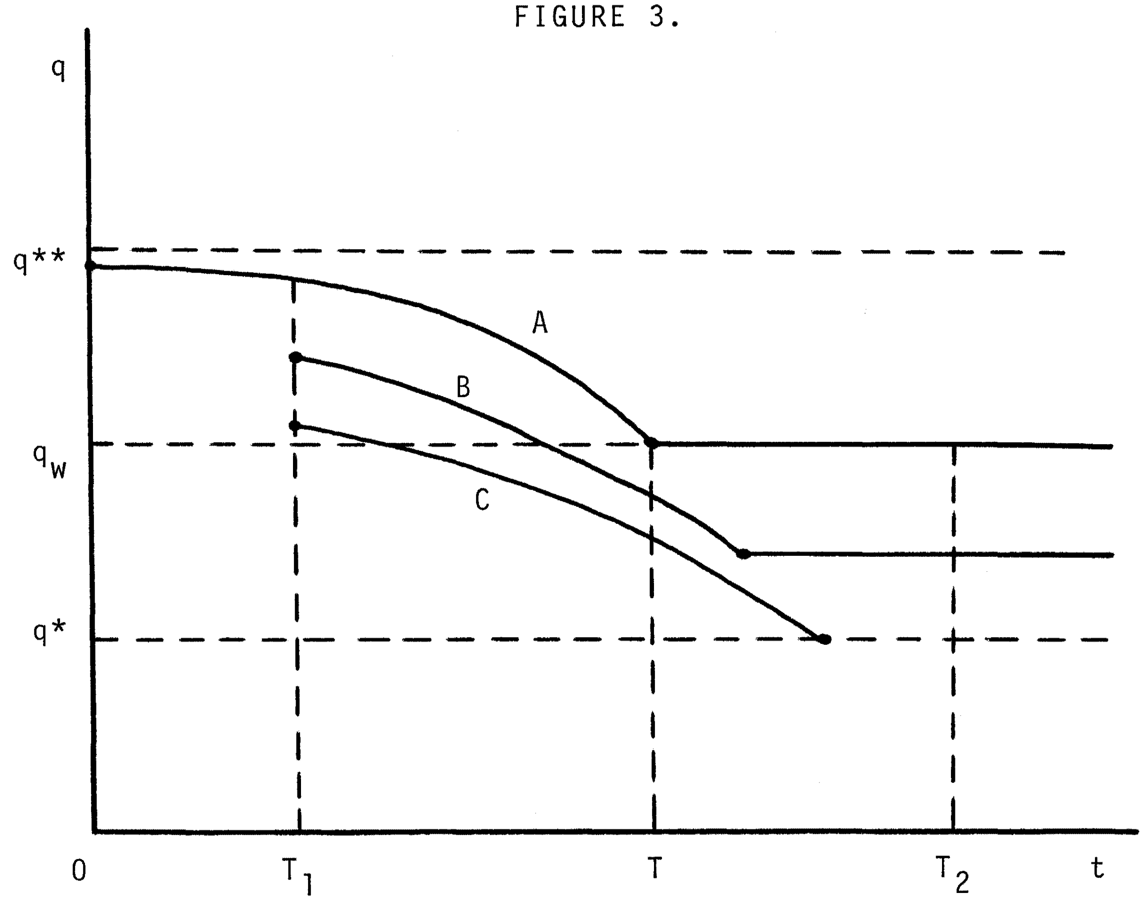
If $R_0 > 0$, exploration will be put off until R = 0 so that exploration costs may be discounted as much as possible. Thus, until R = 0 the behavior of the resource firm would be much like that discussed already in Section III except that at the time the initial reserves are exhausted, T, the following must now be true:

$$p - C' = \lambda e^{\delta T} = w. \qquad (11)$$

This can be seen by noting $\lambda e^{\delta T}$ represents the shadow price of an additional unit of reserves while w represents the additional cost of acquiring one.[5] This, in turn, implies a lower λ and higher output levels in all periods resulting in a quicker depletion of R_0 than before. Once exhaustion occurs production then continues at q_w as described above. The extraction path is illustrated in Figure 3 by path A.

Note that path A is drawn assuming p - w is greater than C*, the minimum of per unit extraction costs.[6] If this is not the case q_w will be less than q* and exploration would not be profitable. In which case the path of q would appear

FIGURE 3.



as it did in Figure 1. Clearly then, the level of exploratory activity and actual production are highly dependent on the level of p - w relative to C*.

Recently dramatic falls in the price of crude oil on the world market have had severe impacts on those regions dependent on the oil industry (e.g., Oklahoma, Texas, Wyoming). The model presented above may be applied in the case of firms in these regions to investigate the impact of a sudden price drop that is anticipated to be somewhat permanent. Returning to Figure 3, suppose the firm is moving along path A and at time T_1 the price falls to a new lower level, p_n, which the firm expects to prevail at least for the near future. If $p_n - w > C^*$, the firm will jump to a path like B resulting in lower production levels and lower future exploratory activity. If, however, the price drop is severe enough such that $p_n - w < C^*$, the firm will drop to a path like C and forego any future exploration. Should the drop in price occur at T_2 again exploration and production will fall and possibly even to zero.

Clearly, the severity of the effect of a price drop on a regional resource firm depends on the magnitude of the price change and the cost of exploration in the region. For those regions where the exploration costs are high so that the margin of profit is low, almost any price drop will bring to a halt any exploration and reduce extraction levels. Those regions where exploration costs are lower will suffer drops in activity levels but not likely complete cessations.

V. CONCLUDING REMARKS

Exploration by its nature is typically a lumpy process in the sense that discoveries much larger than could be extracted in a single period are possible (as well as no discoveries at all despite heavy exploration expenditures). Thus, as opposed to the smooth paths for exploration and extraction presented here, there is likely to be more than just one period of extraction without exploration (following a major discovery) and possibly even periods where no extraction occurs because exploration has turned up no new reserves. Further, due to the uncertainty associated with exploration it is likely that firms would begin exploration activity before their reserves were depleted. How long before would clearly depend on the actual riskiness of exploration and the firm's attitude towards risk-bearing. While inclusion of these factors would add a degree of realism to the model above, the basic behavior of the firms would not drastically differ from that presented.

ENDNOTES

1 While crude oil is a prime example of the scenario to be presented here, other natural resource industries (aluminum, copper, nickel) also fit the general framework.

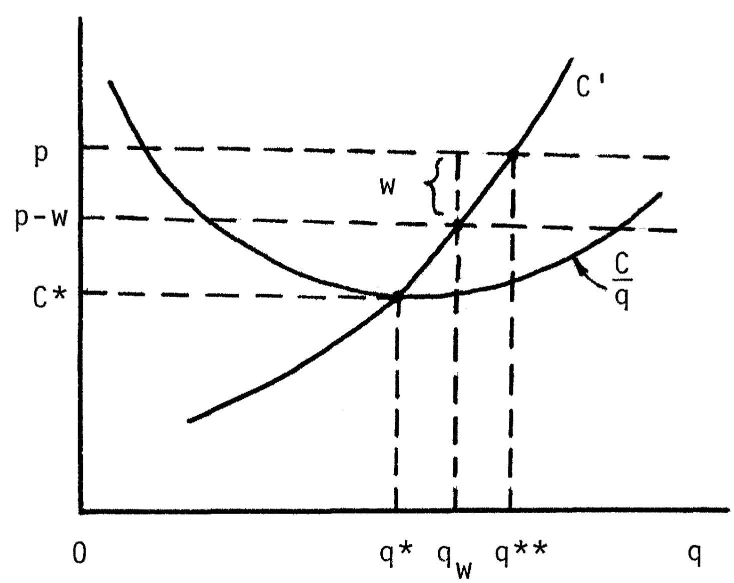
2 For example, OPEC and British Petroleum in crude oil, Alcoa in aluminum, and Kennecott in copper.

3 See Slade [7]. The actual relative-price movements of several nonrenewable natural resources are investigated and U-shaped price paths over time are suggested.

4 The curvature of the path comes from noting that $\ddot{q} < 0$ (assuming $C''' = 0$).

5 This result is very similar to that achieved in depletion models which consider a "backstop" technology. Fisher [2], pgs. 17-20, provides a simple discussion of this.

6 The figure below provides the relationship between q^*, q^{**}, q_W, p, C^*, and w envisioned.



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