The Partonic Nature of Instantons

David Tong

UNIVERSITY OF CAMBRIDGE

Work in progress with Ben Collie

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Happy Birthday Misha!!

What are Instantons Made of?

- Instantons have a number of collective coordinates:

- SU(N) Yang-Mills
  - 4 translations, 1 scale size, 4N-5 orientation
  - 4N in total

- CP^N Sigma-Model
  - 2 translations, 1 scale size, 2N-3 orientations
  - 2N in total

- An old idea: Instantons are composed of N partons

Belavin et al '79

The Partonic Nature of Instantons

- The conjecture of partons usually framed for
  - d=3+1 dimensions (for Yang-Mills)
  - d=1+1 dimensions (for sigma-models)

- Here we revisit this idea in context of
  - d=4+1 dimensions (for Yang-Mills)
  - d=2+1 dimensions (for sigma-models)

- Theories are non-renormalizable (effective theories)
- Instantons are particle-like excitations
- Is the single instanton a multi-particle state?
  - …and how can we tell?

5d Yang-Mills

- There exists a UV completion of gauge theories in d=4+1 dimensions — Seiberg '96
  - (at least with supersymmetry)
  - …but we don't know much about it
- With N=2 supersymmetry, the UV completion is the (2,0) theory in d=5+1 dimensions
- The UV completion has N^3 degrees of freedom. — Klebanov and Tseytlin '96

The Instanton

- The instanton is the KK mode from the sixth dimension

$$M_{\mathrm{inst}} = \frac{8\pi^2}{e^2} = \frac{1}{R}$$

Rozali, '97

- Proposal: <u>The instanton is an N-particle state</u>

- The N partons inside the instanton are the remnant of the UV degrees of freedom which comprise the (2,0) theory.

Circumstantial Evidence

- Turn up the heat and look at cross-over in free-energy

Itzhaki et al ‘98

$$F \sim N^2 T^5 \longrightarrow RN^3 T^6$$

- The transition happens at the temperature

$$T \sim \frac{1}{NR} \sim \frac{1}{e^2 N}$$

- This had to be the case: this is where the 5d theory is strongly coupled, so this is where we need new UV degrees of freedom.

More Circumstantial Evidence

- Anomaly coefficient for G = ADE is conjectured to be

Intriligator, '00

$$c_2(G) \times |G|$$

dual coxeter number
= number of instanton moduli

dimension of group

- Are the partons in the adjoint of G?
  - What is the mechanism that confines them?

- Quantizing the scale size of the instanton gives a continuous spectrum…natural for a multi-particle state.
  - But the moduli space is *not* that of N free objects…does this contain clues about confining mechanism?

A Toy Model

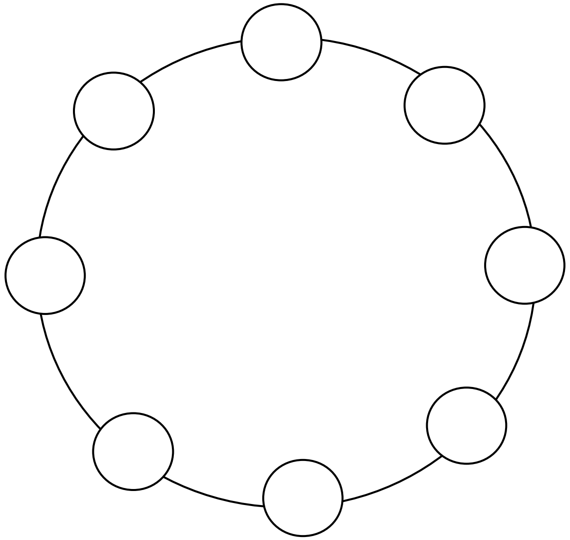
- These questions are difficult to answer in case of Yang-Mills
- The CP^N sigma-model provides (as always!) a nice toy-model where we can see how some of these issues are resolved.

A Toy Model

Intriligator and Seiberg, '96

- Our toy will be in d=2+1 dimensions
- It is a gauge theory with N=4 supersymmetry
  - Vector multiplet: $V = (A_\mu, \phi_i, \text{fermions}) \qquad i = 1, 2, 3$
  - Hypermultiplet: $Q = (q, \tilde{q}, \text{fermions})$

- $U(1)^N$ + N hypermultiplets
  - gauge coupling = g^2
  - mass of hypers = m



The Low-Energy Dynamics

- The hypermultiplets have physical mass

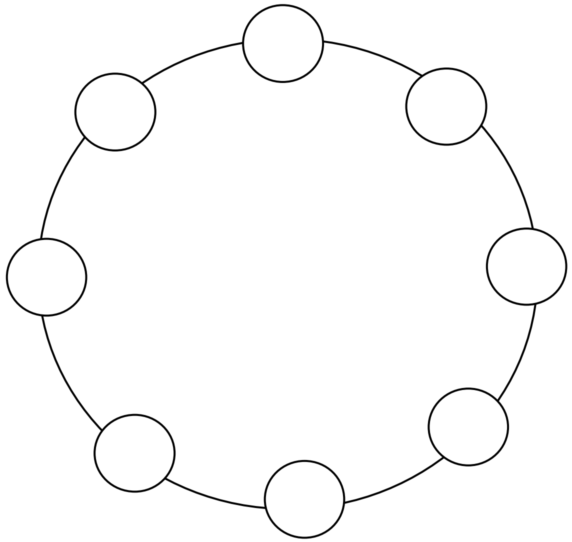
"bare" mass

$$M^a = (\phi^a - \phi^{a+1} + m)$$

vevs of vector multiplets

- At low energies, we want an effective action for the massless vector multiplets

ϕ_i^a and $F_{\mu\nu}^a = g^2 \epsilon_{\mu\nu\rho} \partial_\rho \sigma^a$



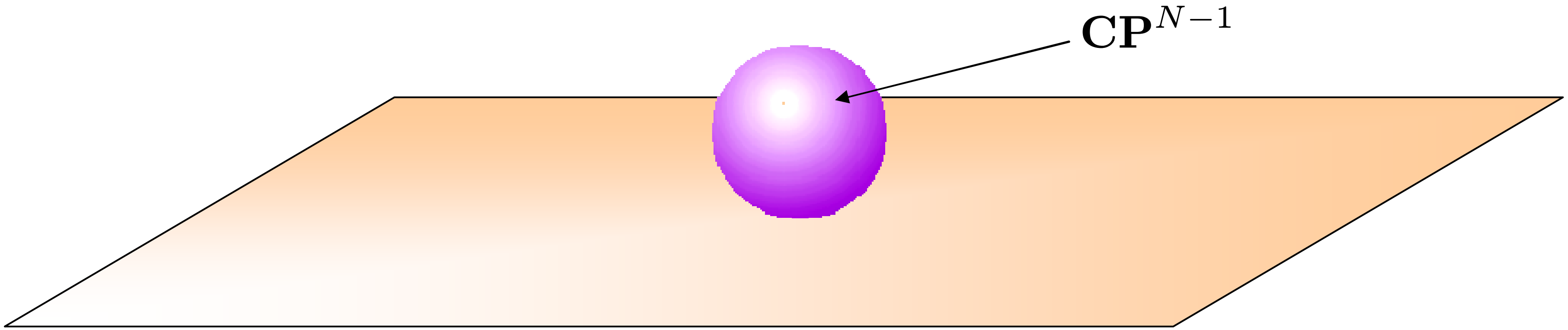
Low-Energy Dynamics

Intriligator and Seiberg, '96

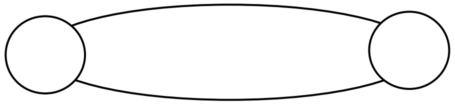
- We integrate out the hypermultiplets
- This induces interactions for vector multiplets

- The low-energy dynamics is a sigma-model with target space

$$\mathbf{R}^4 \times T^\star \mathbf{CP}^{N-1}$$



An Example: **CP^1**



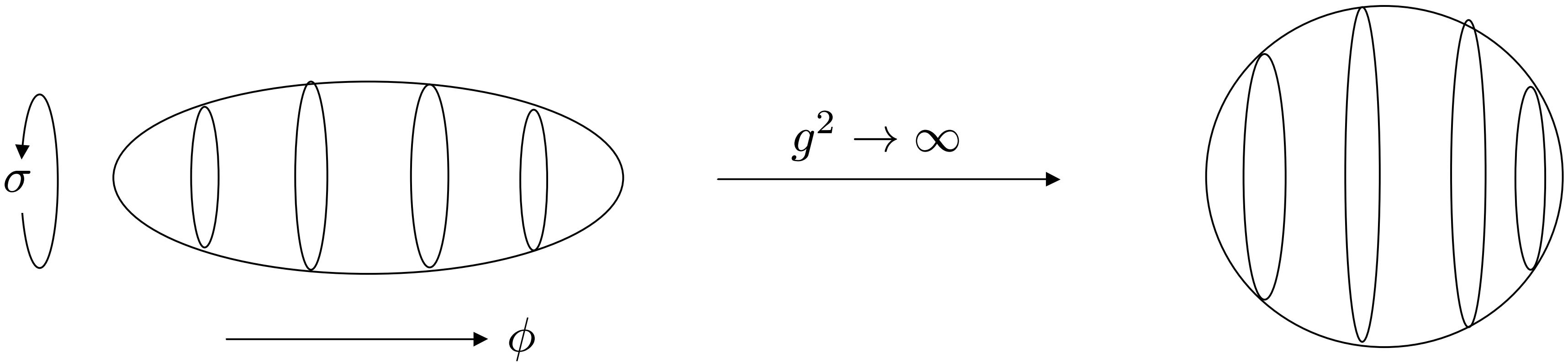
Diagonal U(1) decouples

Axial U(1) has two hypers, with charge +1 and -1

Integrate out hypers:

$$\mathcal{L}_{\rm eff} = \frac{1}{g_{\rm eff}^2}(\partial\phi)^2 + g_{\rm eff}^2(\partial\sigma)^2$$

$$\frac{1}{g_{\rm eff}^2} = \frac{1}{g^2} + \frac{1}{m-\phi} + \frac{1}{m+\phi}$$

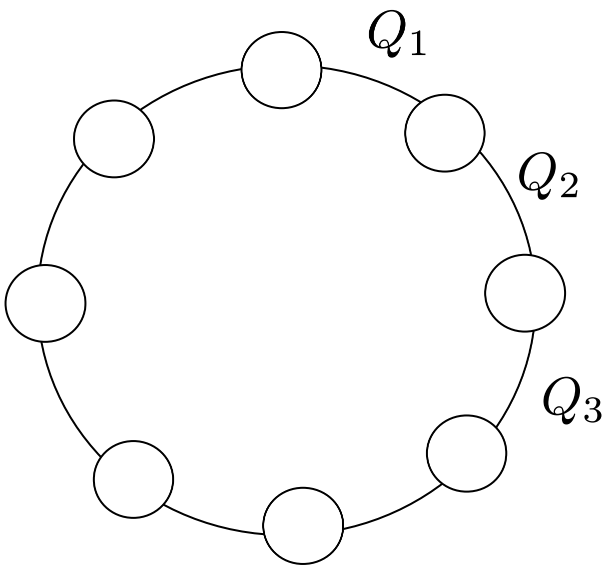


The Soliton

- The low-energy theory has a soliton: it is a sigma-model lump
  - 2 translation modes
  - 1 scale size
  - 2N-3 orientation modes

$$\partial_\mu \phi = g_{\rm eff}^2(\phi)\epsilon_{\mu\nu}\partial^\nu \sigma$$

- What is this soliton in the microsopic theory?
  - It is BPS
  - Mass = Nm



- It is an N-particle state.
  - The soliton is made of the objects that we thought we'd integrated out!

$$Q_1 Q_2 Q_3 \ldots Q_N$$

Aharony, Hanany, Intriligator, Seiberg and Strassler, '97

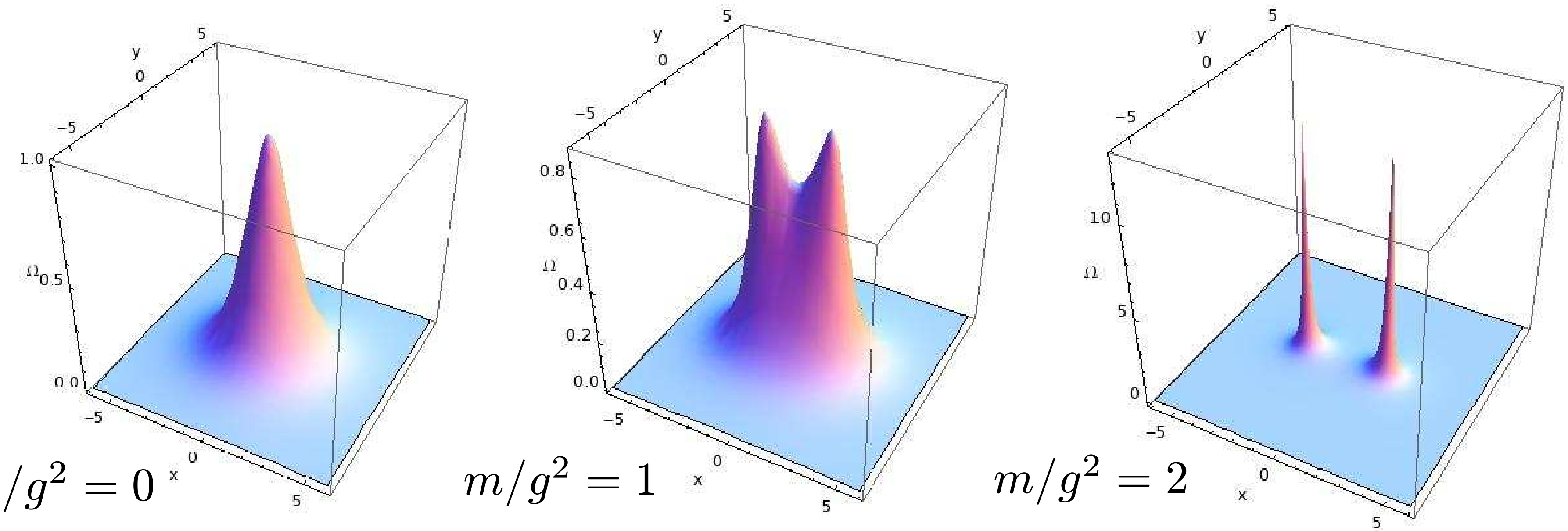
Reconstructing the UV Physics

- Question: Suppose we just have access to the IR physics on the Coulomb branch. What does the soliton tell us about the UV completion?

- Answer: Pretty much everything!

Seeing the Partons

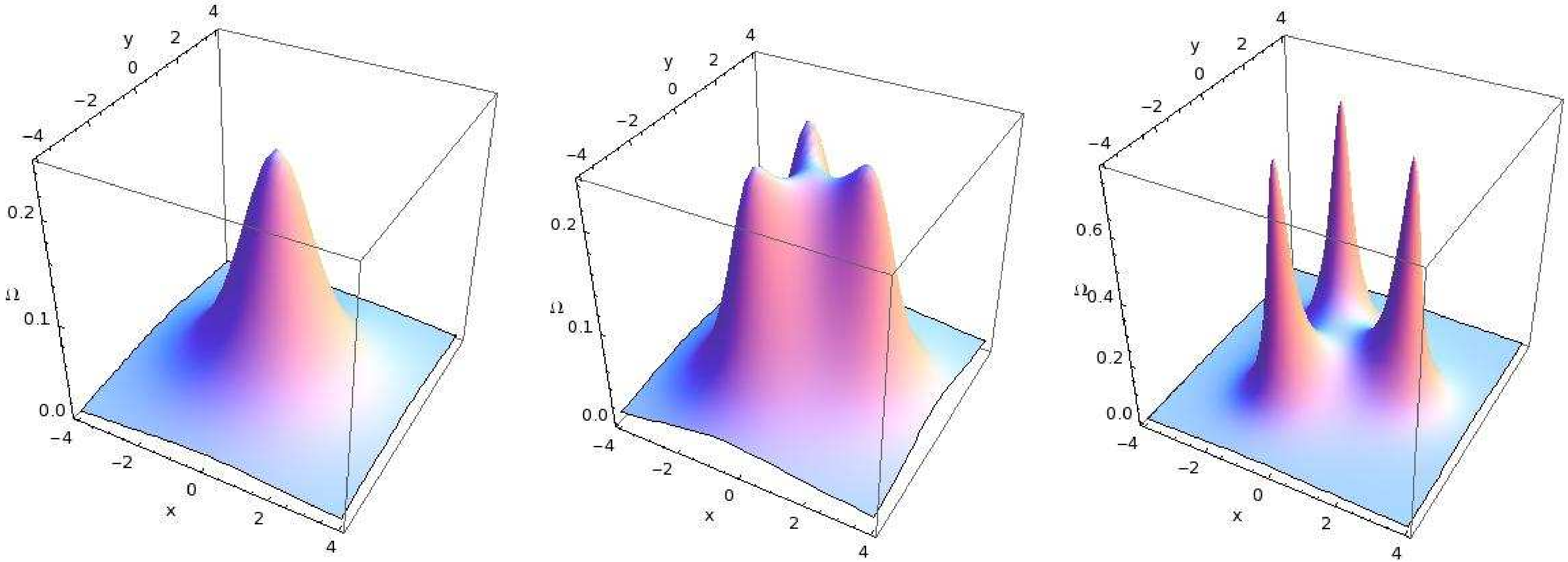
- We get the round metric on $\mathbf{CP}^{N-1}$ only in the limit $g^2 \to \infty$
- If we study solitons on the squashed target space, the solitons dramatically reveal themselves. Here's a soliton on $\mathbf{CP}^1$



$m/g^2 = 0$ $\quad$ $m/g^2 = 1$ $\quad$ $m/g^2 = 2$

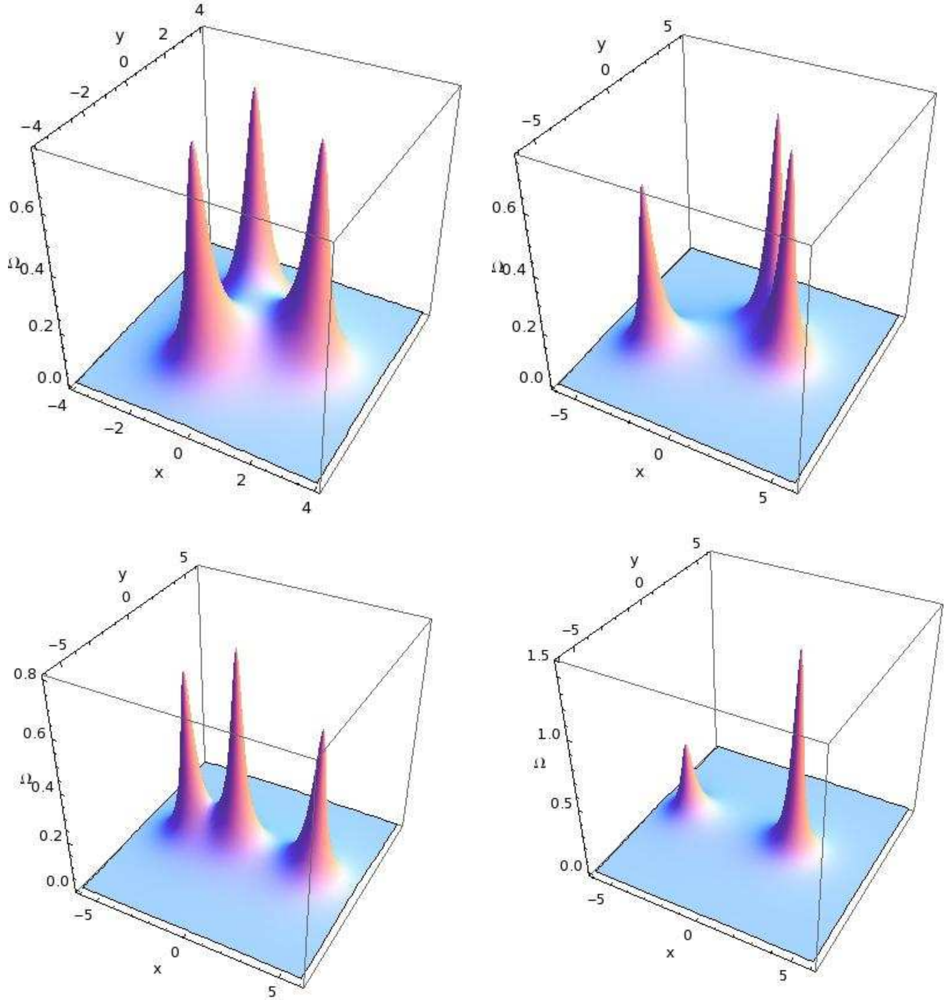
Seeing the Partons

- Here's a single soliton on a squashed $\mathbf{CP}^2$



Changing the Orientation

- Keep the “scale size” of the lump fixed
- Change the “orientation modes”
- Watch the partons move



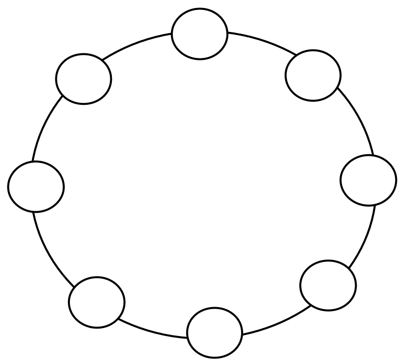
Confinement of Partons

- Why does the low-energy theory only include the N-particle state?

$$Q_1 Q_2 \ldots Q_N$$

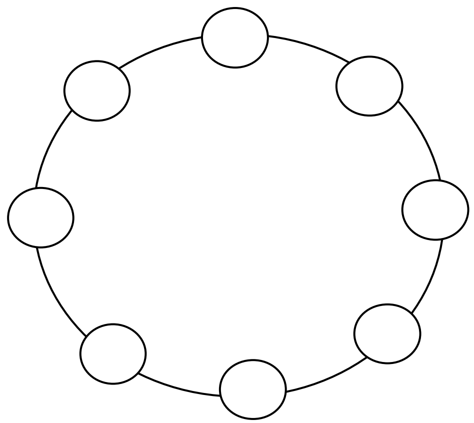
- Answer from microscopic theory: logarithmic confinement
  - In d=2+1 dimensions, electric charges have $E \sim \frac{1}{r}$
  - This gives log divergent mass
  - Only gauge singlet states have finite mass

- This log divergence re-appears when partons move.
  - Seen in IR theory as a log divergence in moduli space metric. Only modes which don't change dipole moments have finite norm



Parton Quantum Numbers

- Can we reconstruct the quantum numbers of the partons?
  - In other words, can we reconstruct the full UV theory?



Dual Bogomolnyi Equations

- The Bogomonyi equations for **CP^1** are

$$\partial_\mu \phi = g_{\mathrm{eff}}^2(\phi)\epsilon_{0\mu\nu}\partial^\nu \sigma$$

- They have a moduli space of solutions: dimension 2kN for k soliton sector

- Can rewrite this equation in dual variables $F_{\mu\nu} = g_{\mathrm{eff}}^2(\phi)\epsilon_{\mu\nu\rho}\partial_\rho \sigma$

$$F_{0\mu} = \partial_\nu \phi$$

Dual Bogomolnyi Equations

$$F_{0\mu} = \partial_\nu \phi$$

- The dual Bogomolnyi equations have *no* smooth solutions

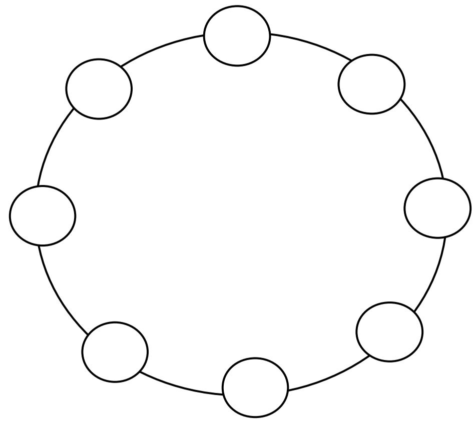
- But we can reproduce the exact soliton solution if we introduce electric sources

$$\partial_\mu \left(\frac{1}{g_{\text{eff}}^2} F_{0\mu} \right) = \sum_{n=1}^{k} \delta(z - z_n^+) - \delta(z - z_n^-)$$

In soliton description, these are collective coordinates. Here, they are sources.

Dual Bogomolnyi Equation

- The dual formulation of the Bogomolnyi equation provides an explicit map between a soliton and fundamental fields
- It also works for $\mathbf{CP}^{N-1}$
- Can reconstruct quantum numbers of partons which determines the UV microscopic theory

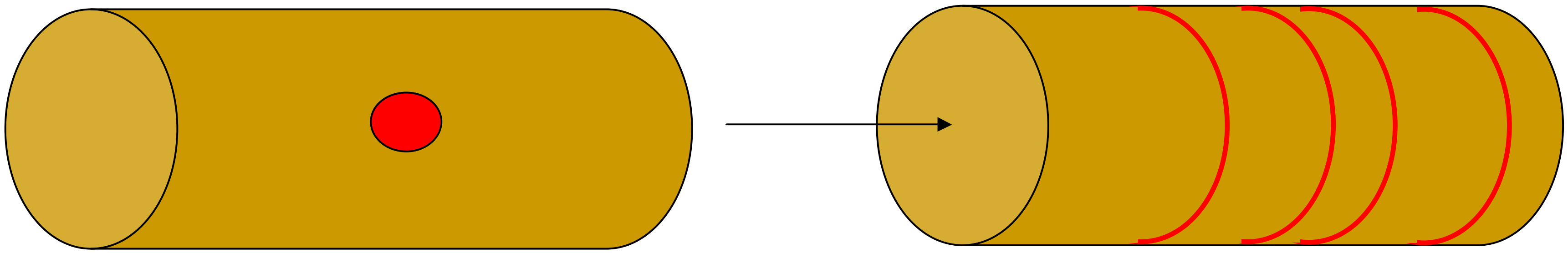


Calorons: A Red Herring?

- There is one way that instantons are known to split into N partons
  - Calorons

Lee and Yi, Van Baal

- Put d=4+1 theory on circle. Add a Wilson line.
  - Instantons ⟶ N monopole strings



- Doesn't seem possible that this can happen in non-compact space
- Also happens for lumps in the toy model
  - But the calorons have nothing to do with the true partons

Summary: Questions, not Answers

- Toy model in d=2+1
  - Explicit demonstration that solitons can be thought of as multi-particle states
  - A study of the soliton allows us to reconstruct the UV behaviour
- Real Interest: d=4+1 Yang-Mills
  - Does the instanton solution hold clues about the constituents of the (2,0) theory?
  - What is the confinement mechanism?
    - No hint of log confinement…partons are probably not merons

Happy Birthday Misha!!