SCALAR-PSEUDOSCALAR SPLITTING IN HOT QCD.

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"DELAYED FOLLOWUP" OF DUNNE-KOGAN-KOVNER-TEKIN (2000-2001) ON HOT 2+1 GAUGE THEORIES

with Gerald Dunne

HOW IMPORTANT IS ANOMALY AT HIGH T?

$$U_A(1): \qquad \psi_L(x) \to e^{i\alpha}\psi_L(x); \;\; \psi_R(x) = e^{-i\alpha}\psi_R(x)$$

ANOMALY EQUATION

$$\partial_\mu J_A^\mu = -N_F \frac{g^2}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

OPERATOR EQUATION - AS SUCH OF COURSE IT HOLDS AT ANY TEMPERATURE

THE "VEHICLES" OF ANOMALY ARE INSTANTONS

$$S_{INST} = \frac{2\pi}{\alpha_s}$$

INSTANTONS COME IN DIFFERENT SIZES - DILATATIONAL INVARIANCE OF THE CLASSICAL QCD ACTION.

QUANTUM CORRECTIONS:

$$S_{INST}(\rho) = \frac{2\pi}{\alpha_s(\rho)} \propto b \log \frac{1}{\rho \Lambda_{QCD}}$$

AT ZERO TEMPERATURE INSTANTON CONTRIBUTIONS ARE DOMINATED BY LARGE SIZES

INSTANTON SIZE IS STABILIZED BY INTERINSTANTON INTERACTIONS, STILL INSTANTON CONTRIBUTION IS PATENTLY NONPERTURBATIVE AND LARGE

AT HIGH TEMPERATURE INSTANTONS WITH $\rho \geq \frac{1}{T}$ ARE SUPPRESSED - THEY "'DON'T FIT"' INTO SMALL IMAGINARY TIME DIRECTION. NO PROBLEM WITH LARGE INSTANTONS.

BUT INSTANTONS AT HIGH T ARE "CONFINED"

EVERY INSTANTON AND ANTIINSTANTON HAS N_F FERMIONIC ZERO MODES

$$\not{D}\psi = 0$$

INSTANTON: ψ_L; ANTIINSTANTON: ψ_R FOR EVERY FLAVOR

PATH INTEGRAL IN AN INSTANTON SECTOR VANISHES DUE TO THE ZERO MODE. FOR WIDELY SEPARATED A-I PAIR THIS INDUCES INTERACTION POTENTIAL

$$\int D\psi d\bar{\psi} e^{\bar{\psi}\not{D}[A_I + A_A]\psi} = Det[\not{D}]$$

THE DETERMINANT IS SMALL BECAUSE OF THE SMALL OVERLAP BETWEEN THE ZERO MODES OF I AND A.

AT ZERO TEMPERATURE THE ZERO MODE WAVE FUNCTION (I at x_I)

$$\psi(x) \propto \frac{1}{|x - x_I|^3}$$

THE OVERLAP IS A POWER

$$\bar{\psi}_R^I \not{D} \psi_L^A \propto |x_I - x_A|^{-3}$$

SO "POTENTIAL" BETWEEN THE INSTANTONS IS LOGARITHMIC:

$$S[A_I + A_A] = S_I + S_A + \# N_F \log |x_I - x_A|$$

AT HIGH TEMPERATURE: EXPONENTIAL $\psi(x) \propto e^{-\pi T |x - x_I|}$

THE "'POTENTIAL"' IS LINEAR

$$S[A_I + A_A] = S_I + S_A + 2 N_F \pi T |x_I - x_A|$$

INSTANTONS ARE "CONFINED" WITH "STRING TENSION" $\sim T$ - APPEAR IN THE VACUUM ENSEMBLE ONLY IN CLOSELY BOUND PAIRS.

HENCE THE IDEA THAT INSTANTONS ARE IRRELEVANT AT HIGH T.

ANOMALY AND THE SCALAR - PSEUDOSCALAR MIXING

BUT WHAT DOES IT MEAN, IRRELEVANT?

NO INSTANTONS $\rightarrow$ NO TOPOLOGICAL CHARGE $\rightarrow$ NO ANOMALY.

E.G.: NO ANOMALY $\rightarrow$ SCALAR AND PSEUDOSCALAR MESONS ARE DEGENERATE.

FLAVOR SINGLETS (FOR SIMPLICITY)

$$S(x) = \bar{\psi}_i \psi_i = \bar{\psi}_i^L \psi_i^R + \bar{\psi}_i^R \psi_i^L \quad ; \quad P(x) = \bar{\psi}_i \, i\gamma_5 \psi_i = i \left(\bar{\psi}_i^L \psi_i^R - \bar{\psi}_i^R \psi_i^L \right)$$

IF THE AXIAL ANOMALY IS ABSENT (AND NO SPONTANEOUS BREAKING OF CHIRAL SYMMETRY)

$$\langle \bar{\psi}_i^L(x) \psi_i^R(x) \; \bar{\psi}_j^L(y) \psi_j^R(y) \rangle = 0 = \langle \bar{\psi}_i^R(x) \psi_i^L(x) \; \bar{\psi}_j^R(y) \psi_j^L(y) \rangle$$

AND THEREFORE

$$\langle S(x) S(y) \rangle = \langle P(x) P(y) \rangle$$

WHAT DO WE KNOW ABOUT THE DIFFERENCE OF CORRELATION LENGTHS OF S AND P?

NOT MUCH UNTIL RECENTLY.

LATTICE CALCULATIONS - AT NOT VERY HIGH TEMPERATURES (BUT ABOVE CHIRAL SYMMETRY RESTORATION).

SUGGEST THAT THE DEGENERACY IS NOT THERE, BUT THE RESULTS ARE NOT CLEAN ENOUGH TO MAKE A DEFINITIVE STATEMENT.

RECENTLY KARSH ET.AL. (2010) - AT $T_c < T < 1.3T_C$ THE SPLITTING IS DEFINITELY THERE.

SINCE AT HIGH TEMPERATURE INSTANTON CALCULATION IS UNDER CONTROL LET'S DO IT AND SEE WHAT HAPPENS.

FINITE TEMPERATURE - A GEDANKEN CALCULATION

WE CONSIDER ONLY TWO FLAVOR CASE $N_F = 2$

WE ARE AT HIGH TEMPERATURE $T \gg T_C$ - DIMENSIONALLY REDUCED SETUP.

ALL THE ACTION HAPPENS FAR FROM THE INFRARED SCALE "'MAGNETIC MASS"', THUS WE DO NOT WORRY ABOUT GENUINELY NONPERTUBATIVE QCD EFFECTS

$$G_{LL}(x-y) = \langle \bar{\psi}_i^R(x)\psi_i^L(x)\ \bar{\psi}_j^R(y)\psi_j^L(y)\rangle$$

ONE INSTANTON: EXACTLY RIGHT NUMBER OF ZERO MODE TO BE SATURATED BY THE FERMIONIC OPERATORS IN THE CORRELATOR

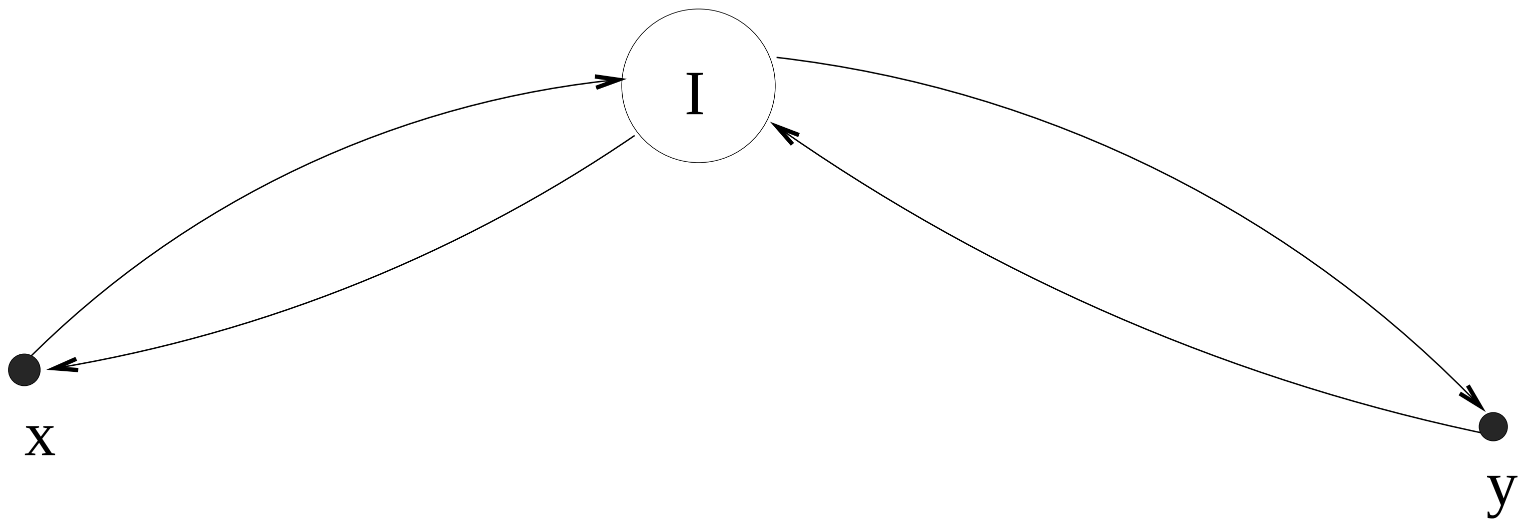

Figure 1: Single Instanton contribution to G_{LL}.

$$\langle \bar{\psi}_i^L(x)\psi_i^R(x)\ \bar{\psi}_j^L(y)\psi_j^R(y)\rangle_I \approx a\,T^6 e^{-S_I} e^{-2\pi T\{|\vec{x}-\vec{a}|+|\vec{y}-\vec{a}|\}}$$

INSTANTON SHOULD SIT ALONG THE LINE BETWEEN x AND y. INTEGRATING OVER a WITH Tdz:

$$\langle \bar{\psi}_i^R(x)\psi_i^L(x)\ \bar{\psi}_j^R(y)\psi_j^L(y)\rangle_I \approx (aT^6)(cT)|\vec{x}-\vec{y}|e^{-S_I(\beta)}e^{-2\pi T|\vec{x}-\vec{y}|}$$

WE CAN ALSO HAVE TWO INSTANTONS AND AN ANTI INSTANTON.

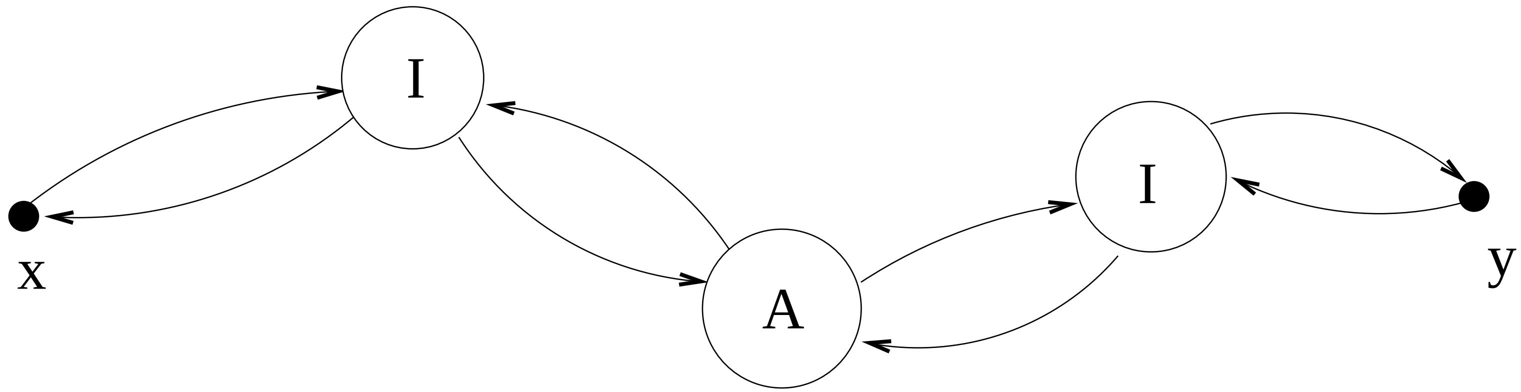

Figure 2: I-A-I contribution to G_{LL}.

$$\langle \bar{\psi}_i^R(x)\psi_i^L(x)\ \bar{\psi}_j^R(y)\psi_j^L(y)\rangle_{IAI} \approx (aT^6)\frac{1}{3!}c^3T^3|\vec{x}-\vec{y}|^3 e^{-3S_I(\beta)}e^{-2\pi T|\vec{x}-\vec{y}|}$$

AND n I's AND $n-1$ A's

$$\langle \bar{\psi}_i^R(x)\psi_i^L(x)\ \bar{\psi}_j^R(y)\psi_j^L(y)\rangle_{I^nA^{n-1}} \approx$$

$$(aT^6)(cTe^{-S_I(\beta)})^{2n-1}e^{-2\pi T|\vec{x}-\vec{y}|}\int_x^y dy_n \int_x^{y_n} dx_{n-1}\int_x^{x_{n-1}} dy_{n-1}\ldots \int_x^{x_1} dy_1$$

SUMMING OVER n

$$\langle \bar\psi_i^R(x)\psi_i^L(x)\ \bar\psi_j^R(y)\psi_j^L(y)\rangle \approx (aT^6)e^{-2\pi T|\vec{x}-\vec{y}|}\,\sinh\left(cTe^{-S_I(\beta)}|\vec{x}-\vec{y}|\right)$$

THE AXIALLY SYMMETRIC PART PERTURBATIVELY IS

$$G_{LR}(x_y) = \langle \bar\psi_i^L(x)\psi_i^R(x)\ \bar\psi_j^R(y)\psi_j^L(y)\rangle_{\text{perturbative}} \approx (aT^6)\, e^{-2\pi T|\vec{x}-\vec{y}|}$$

BUT IT ALSO GETS CONTRIBUTION FROM IA CHAINS. THIS TIME THE NUMBER OF I's IS EQUAL TO THE NUMBER OF A's. SUMMING OVER n JUST LIKE BEFORE GIVES:

$$\langle \bar\psi_i^L(x)\psi_i^R(x)\ \bar\psi_j^R(y)\psi_j^L(y)\rangle \approx (aT^6)e^{-2\pi T|\vec{x}-\vec{y}|}\,\cosh\left(cTe^{-S_I}|\vec{x}-\vec{y}|\right)$$

NOW WITH SOME JUSTIFICATION

PATH INTEGRAL FOR n I's AND $n-1$ A's.

KEEP ONLY THE ZERO MODES IN THE FERMI FIELDS - THE REST HAVE MUCH SHORTER RANGE.

$\phi_a(x)$ THE WAVE FUNCTION OF THE ZERO MODE OF a-TH INSTANTON, $\phi_b(x)$ - OF THE b-TH ANTIINSTANTON.

DECOMPOSE FIELDS AS

$$\psi_i^L(x) = \Sigma_{a=1}^n \phi_a(x)\psi_i^{La} \quad ; \qquad \psi_i^R(x) = \Sigma_{b=1}^{n-1} \phi_b(x)\psi_i^{Rb}$$

THEN

$$G_{LL}(x,y) = \int \left[\Pi_{iab} d\psi_i^{La} d\psi_i^{\dagger Ra} d\psi_i^{Rb} d\psi_i^{\dagger Lb}\right]$$
$$\left(\Sigma_{b=1}^n \phi_b^*(x)\psi_i^{\dagger Rb}\right)\left(\Sigma_{a=1}^n \phi_a(x)\psi_i^{La}\right)\left(\Sigma_{b=1}^n \phi_b^*(y)\psi_j^{\dagger Rb}\right)\left(\Sigma_{a=1}^n \phi_a(y)\psi_j^{La}\right) e^{-S[\psi]}$$

FERMIONIC ACTION

$$S[\psi] = (2n-1)S_I + \Sigma_{ab}\left[\bar\psi_i^{Ra} T_{ab}\psi_i^{Rb} + \bar\psi_i^{Lb} T_{ba}^* \psi_i^{La}\right]$$

WITH THE OVERLAP MATRIX

$$T_{ab} = \int dx \phi_a^*(x) \not{D} \phi_b(x)$$

ITS ALL EXPONENTIAL

$$\phi_{a(b)}(x) \propto e^{-\pi T |x - x_{a(b)}|} \; ; \quad T_{ab} \propto e^{-\pi T |x_a - x_b|}$$

NOW EXPAND THE EXPONENTIAL AND CALCULATE PATH INTEGRAL OVER ψ.

THE RESULT HAS EXACTLY THE SAME FORM AS DESCRIBED - THE ZERO MODES AT x CONTRACT INTO THE ZERO MODES OF SOME INSTANTON AND SO ON. THE "'LINKS"' BETWEEN THE INSTANTONS ARE THE SQUARES OF THE OVERLAP MATRIX ELEMENTS FOR EACH FLAVOR

THE LEADING CONTRIBUTION IS CLEARLY THE ALTERNATING I-A CHAIN.

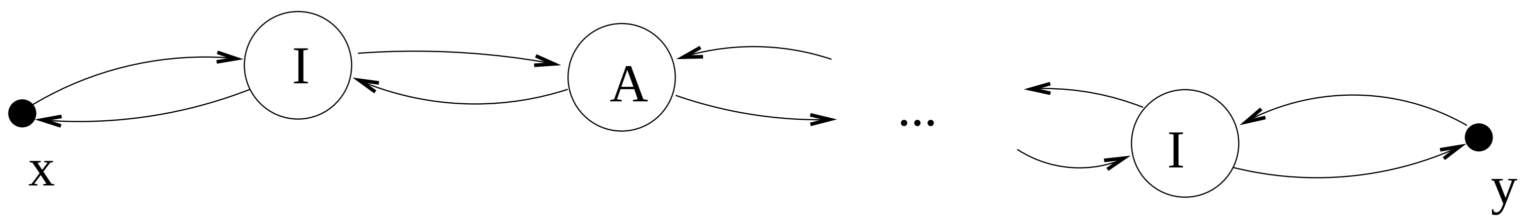

Figure 3: The Instanton - Antiinstanton chain that gives the leading contribution to the correlator.

TWO COMMENTS:

1. THE INSTANTONS IN SU(3) HAVE DIFFERENT COLOR ORIENTATIONS, THUS THE ZERO MODE WAVE FUNCTIONS DEPEND ON COLOR. HOWEVER GIVEN A FIXED I-A CONFIGURATION ONE CAN INTEGRATE OVER THE COLOR ORIENTATION, AND SINCE THIS AVERAGING COMMUTES WITH THE INTEGRATION OVER THE POSITIONS, THE RESULT SIMPLY DETERMINES THE CONSTANT c.

2. THE CHAIN ALONG THE STRAIGHT LINE IS LEADING BUT FLUCTUATIONS OF COURSE ARE ALLOWED. THOSE WILL LEAD TO PREEXPONENTIAL CORRECTION (ENTROPY) OF THE POWER FORM $|x-y|^l$.

TAKING LINEAR COMBINATIONS OF G_{LL} AND G_{LR} WE FIND

$$\langle S(x)S(y)\rangle \propto e^{-M_S|x-y|} \quad ; \quad \langle P(x)P(y)\rangle \propto e^{-M_P|x-y|} \quad ; \quad \langle S(x)P(y)\rangle = 0$$

WITH

$$M_S = 2\pi T - c\,T e^{-S_I(\beta)}; \qquad M_P = 2\pi T + c\,T e^{-S_I(\beta)}$$

THUS THE SPLITTING DOES NOT VANISH

$$\frac{\Delta M}{M} \propto e^{-S_I(\beta)} \propto \left(\frac{\Lambda_{QCD}}{T}\right)^b$$

$b = (11N_c - 2N_f)/3$

THIS RESULT IN CHIRALLY SYMMETRIC PHASE IS VALID NOT ONLY FOR ISOSINGLETS SINCE $\bar{\psi}_i \psi_i$ and $\bar{\psi}_i \tau^a_{ij} \psi_j$ FORM AN IRREDUCIBLE REPRESENTATION OF $SU(2) \times SU(2)$. THUS WE HAVE THE SAME RELATION FOR THE CORELATION LENGTH IN THE π AND a_0 CHANNELS

$$\frac{M_{a_0} - M_\pi}{M_\pi} \propto \left(\frac{\Lambda_{QCD}}{T}\right)^b$$

THIS IS IN PRINCIPLE EASIER TO VERIFY ON THE LATTICE, BUT STILL TOUGH - THE POWER IS VERY BIG.

IS THIS SEMICLASSICS IN "'ELECTRIC"' THEORY (HTL)?

IMAGINE "INTEGRATING OUT" FERMIONS - EFFECTIVE THEORY OF POLYAKOV LOOP P.

INSTANTONS $\to$ SKYRMIONS OF P (Atyah-Manton construction)

FERMIONIC BILINEARS $\to$ GLUONIC OPERATORS IN EFFECTIVE THEORY - "HALF A SKYRMION".

MASS SCALE IN EFFECTIVE THEORY - DEBYE MASS $m_D \propto gT$

INSTANTON "STRING TENSION" - $\sigma \sim T = m_D^2/g^2$ - VERY SEMICLASSICAL.

WHAT KIND OF "SKYRME MODEL" LEADS TO SEMICLASSICAL CONFINEMENT OF SKYRMIONS?