THE COST OF CAPITAL AND THE DEMAND FOR MONEY BY FIRMS

by

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ABSTRACT

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The paper presents a simple model which incorporates the financial measure of cost of capital in the firms' demand for money. The model is tested using cross-section data of firms. The empirical results are consistent with the prediction and show the potential usefulness of the Modigliani-Miller cost of capital approach to resources allocation by firms.

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Several cross-section estimates for the firms' demand for cash were reported in the recent literature. Among these are Meltzer [4], Whalen [10], and Vogel and Madala [9].[1]

The two main difficulties with the above studies are:

(a) They did not use a cost of capital variable in their cross-section analysis, and thus assuming implicitly that all the firms in a given cross-section have the same cost of capital. This assumption, however, is inconsistent with the theory of finance, which suggests that cost of capital of a firm depends on its appropriate risk class;[2]

(b) They use aggregate data of firms in different industries rather than data of individual firms.

In this study we use data of individual firms and suggest a measure of cost of capital,[3] which is derived from the theory of firm valuation and is based on the approach suggested by Modigliani and Miller ([5], [8]).

In the first section of the paper, we derive a simple extension of Baumol's model [1] to the case where money also enters into the

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production function of the firm.[4] Section II presents the empirical formulation of the model, in which Modigliani-Miller's measure of the cost of capital is applied to the firm demand for money. Section III presents the results, and section IV summarizes the conclusions and the possible implications of this approach.

I. A MODEL OF THE FIRM DEMAND FOR CASH

Let $S(X,M)$ be the firm production function where S is the firm's sales (in value terms), X is a measure of the firm cost of production (excluding only the cost of holding cash), and M is the average amount of money held by the firm. We assume that S is a linear homogeneous production function, in which money enters as a factor of production.[5] The total costs are given by:

$$T(X,M) = X + r\cdot M + \frac{a\cdot X}{2M} \quad . \tag{1}$$

The firm profits $\Pi(X,M)$ are given by

$$\Pi(X,M) = S(X,M) - X - rM - \frac{a\cdot X}{2M} \tag{2}$$

The equilibrium conditions are:

$$MP_X = \frac{\partial S(X,M)}{\partial X} = \left(1 + \frac{a}{2M}\right) \tag{3}$$

$$MP_M = \frac{\partial S(X,M)}{\partial M} = r - \frac{a\cdot X}{2M^2} \tag{4}$$

where MP_X and MP_M are the marginal products (in value terms) of one unit of cost and cash, respectively.

To illustrate the nature of our solution, let us assume for simplicity that the firm costs X are given, and the production function S is a Cobb-Douglas Production function.

$$S = A \cdot M^{\alpha} X^{1-\alpha} \quad . \tag{5}$$

In this case the equilibrium condition (4) is read as:

$$\frac{\alpha S}{M} = r - \frac{a \cdot X}{2M^2} \quad . \tag{6}$$

The average amount M will be given by the solution of the quadratic equation

$$rM^2 - \alpha SM - \frac{aX}{2} = 0 \quad . \tag{7}$$

The optimal solution is thus:

$$M = \frac{\alpha \cdot S}{2r} + \frac{\sqrt{\alpha^2 S^2 + 2aXr}}{2r} \quad . \tag{8}$$

The solution in (8) has two interesting special cases:

(a) $\alpha = 0$.

In this case money does not enter into the production and (7) reduces to

$$M = \sqrt{\frac{aX}{2r}} \tag{8'}$$

which is the classical Baumol's solution. The demand elasticities in this case are:

$$\eta_{MX} = 1/2 \tag{9'}$$

$$\eta_{Mr} = -1/2 \quad .$$

(b) $a = 0$.

In this case there is no transaction cost of cashing an interest-bearing asset. The solution in this case is

$$M = \frac{\alpha \cdot S}{r} \tag{8''}$$

which implies unit elasticity with respect to size.

$$\eta_{MS} = 1 \tag{9''}$$

$$\eta_{Mr} = -1 \quad .$$

These are basically the results obtained by Meltzer [4]. In general, however, we can expect both $\alpha > 0$ and $a > 0$ and that the general solution will fall between the solutions of the two extreme cases.

The theoretical predictions of the model are:[6]

$$0.5 < \eta_{MS} < 1 \tag{10}$$

$$-1 < \eta_{Mr} < -0.5 \quad .$$

It should be also noted that in (8′) we have used cost as a measure of the firm's size while in (8′′) we have used sales. Since the model is applied to a cross-section study, it seems that the two variables are very related measures of the firm's size.

II. A CROSS-SECTION TEST OF FIRMS DEMAND FOR CASH -- THE EMPIRICAL MODEL

The classical theory of corporate valuation has derived a simple equation to a valuation of a stock price.

$$P = \frac{E}{\rho - \lambda} \tag{11}$$

where

P is the price of a corporate share

E is the earnings per share

ρ is the cost of capital which takes into account the risk of the firm

λ is the long-run growth of earnings per share.

From (11) we can get

$$\rho = E/P + \lambda \quad . \tag{12}$$

The variable λ -- the expected increase in earnings per share, is not directly measured from the data. We assume, however, that investors use data of the past rate of growth of the firm to predict its future rate of growth. As a growth variable, therefore, we have used the rate of growth of sales of the firm in the last five years.

The cost of capital of the firm is thus defined as a function

$$\rho = f(E/P, G) \tag{13}$$

where G -- is the average rate of growth of the firm sales in the period of five years.[7] For simplicity we assume:

$$Lg(\rho) = Lg(E/P) + \gamma \cdot G \quad . \tag{14}$$

The demand for cash equation which we test is

$$Lg\ M = \beta_0 + \beta_1\ Lg\ S + \beta_2\ Lg\ \rho \tag{15}$$

where S is the size of the firm which is measured by its level of sales. Substituting (14) in (15) we get:

$$\text{Lg } M = \beta_0 + \beta_1 \text{ Lg } S + \beta_2 \cdot \text{Lg } (E/P) + \beta_3 \cdot G \tag{16}$$

where

$$\beta_3 = \gamma \cdot \beta_2 \quad .$$

The predictions of our model as derived in (10) are:

$$0.5 < \beta_1 < 1 \tag{17}$$

$$-1 < \beta_2 < -0.5$$

$$0 < \gamma = \frac{\beta_3}{\beta_2} \leq 1 \quad .$$

III. THE CROSS-SECTION TEST OF THE MODEL -- THE RESULTS

The sample consists of 546 firms from Compustat tape with fiscal year ending in December. We have selected all the firms which have relevant data for the years 1964 and 1965. Few firms with negative profits were excluded.

The variables are:

- M_{65} - cash and equivalent of the firm in 1965.
- M_{64} - cash and equivalent of the firm in 1964.
- S_{64} - sales of the firm in 1964.
- S_{59} - sales of the firm in 1959.
- $(E/P)_{64}$ - earnings/price ratio in 1964.
- G_{64} - growth in sales. We used the log of the ratio $\frac{\text{sales } 64}{\text{sales } 59}$.

In the empirical work, we have estimated two versions of the demand for cash equation.

$$\text{(I)} \quad LgM_{64} = \beta_0 + \beta_1\, LgS_{64} + \beta_2\, Lg(E/P)_{64} + \beta_3\, G_{64} + V$$

$$\text{(II)} \quad LgM_{65} = \beta_0 + \beta_1\, LgS_{64} + \beta_2\, Lg(E/P)_{64} + \beta_3\, G_{64} + V$$

The use of (II) in addition to (I) is based on two considerations: (a) The independent variable S_{64} and $(E/P)_{64}$ were not known exactly to the firm in the period where the decision on cash holding in December 1964 was made. (b) The independent variables may be endogenous in equation (I) and thus will bias the regression coefficients. From these considerations, the use of lagged independent variables may be preferred.

The results of the estimation are reported in Table 1.

Table 1

REGRESSION RESULTS OF THE FIRM DEMAND FOR CASH
USING CROSS-SECTION SAMPLE FOR A GIVEN YEAR
(t values are given in parentheses)

Dependent Variable	Coefficients of the Independent Variables and R^2				
	Constant	LgS_{64}	$Lg(E/P)_{64}$	G_{64}	R^2
LgM_{64}	-1.884 (11.6)	0.882 (28.7)			0.603
LgM_{64}	-0.9292 (3.4)	0.871 (28.7)	-0.471 (4.3)		0.616
LgM_{64}	-0.694 (2.5)	0.866 (28.8)	-0.506 (4.7)	-0.397 (3.5)	0.625
LgM_{65}	-1.880 (11.6)	0.889 (29.1)			0.610
LgM_{65}	-0.667 (2.5)	0.874 (29.3)	-0.598 (5.6)		0.631
LgM_{65}	-0.513 (1.8)	0.871 (29.3)	-0.621 (5.8)	-0.261 (2.4)	0.634

The coefficient β_1 seems to be above 1/2 which was the predicted value of Baumol's model [1]; it is, however, below one which suggests some economies of scale in holding money. This is consistent with the prediction in (17).

The coefficient β_2 is somewhat above 1/2 (in absolute value) and is consistent with our prediction as well.

The coefficient β_3 suggests that the investors use past growth data in their prediction of future growth. The weight attached to the "growth of sales" factor in equation (14) is less than one ($\gamma = 0.8$ for 1964 and $\gamma = 0.4$ for 1965).[8]

The finding that both β_2 and β_3 have the correct signs and order of magnitude is an important empirical evidence that the concept of the cost of capital which was derived by Miller and Modigliani [5] is a useful variable in explaining the firm's decision regarding the allocation of its scarce resources.

IV. CONCLUSIONS AND IMPLICATIONS

The paper shows that the cost of capital variable for the firm which was derived from finance theory seems to be very useful in explaining the firm demand for money.

The paper may have implications in two other fields. First, in the theory of inventory, it suggests an approach in which inventory is a productive factor, and that the optimal decision of inventory holding will have to take this into account. Second, the empirical finding on the usefulness of the cost of capital measure may have some implications to the study of investment, in particular, in the direction of "integration of uncertainty into the theory and econometrics of investment" which Jorgenson [3] views as the most important open question in the study of investment.

FOOTNOTES

1. See Vogel and Madala [9] for more detailed reference to the literature.

2. See Modigliani and Miller [8].

3. This idea of applying a long-term "cost of capital" measure to the analysis of the firm demand for money was emphasized by Miller and Orr ([6], [7]).

4. This possibility is suggested by Friedman [2] and Meltzer [4].

5. For simplification, we use as an argument the costs of factors of production X rather than the detailed inputs in the production function. This is justified by our assumption of a linear homogeneous production function and by the implicit assumption of given and fixed prices of all the factors of production.

6. For simplicity we view both η_{Ms} and η_{Mx} as the same measure of the elasticity of the demand for cash with regard to the firm size.

7. We assume that the rate of growth of sales is a better proxy for the long-run growth in earnings.

8. The difference between Version I (1964) and Version II (1965) may also reflect the fact that in the first regression the independent variables are endogenous variables which may bias the estimates.

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