A Cautionary Note on the Use of LISREL's Automatic Start Values in Confirmatory Factor Analysis Studies

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The accuracy of parameter estimates provided by the major computer programs for confirmatory factor analysis studies is questioned. This note demonstrates an inconsistency in parameter estimates across two of the major programs (LISREL and EQS), with the inconsistency attributed to the use of LISREL VI's automatic start values for the estimation of generalized least squares models.

Recently the social science literature has abounded with studies attempting to validate or confirm various factor analytic-based models. These studies have been termed confirmatory factor analysis (CFA) studies; this term simply refers to a test of the validity of a proposed model which is assumed to account for a body of data in terms of relatively few parameters, with the model supposedly based on a priori information (theory or hypothesis).

The confirmatory factor analytic model can be given as

$$\mathbf{X} = \Lambda \xi + \delta \quad , \tag{1}$$

where $\mathbf{X}$ is a vector of k observed measures,
ξ is a vector of m underlying or latent factors, such that $m < k$,
Λ is a $k \times m$ matrix of factor loadings relating the observed $\mathbf{X}$ to the latent factors ξ, and
δ is a vector of k variables representing random measurement error.

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The variance-covariance matrix (hereafter denoted as the covariance matrix) for $\mathbf{X}$ may be defined as

$$\Sigma = \Lambda \Phi \Lambda' + \Theta_\delta \quad , \tag{2}$$

where Φ is the $m \times m$ covariance matrix of ξ, and Θ_δ is the diagonal $k \times k$ covariance matrix of δ. The structure of a factor analytic model is then defined a priori, and the measurements for parameter estimation on the matrices are submitted, usually in a covariance matrix format. The appropriateness of the factor structure is then tested by attempting to replicate the covariance matrix. Thus, an initial hypothesis is established for test of fit (H_0: $\mathbb{S} = \hat{\Sigma}$), where $\mathbb{S}$ is the original covariance matrix and $\hat{\Sigma}$ is the recovered covariance matrix.

One probable reason for the increase in CFA studies is the availability of computer programs. The major programs available are Jöreskog and Sörbom's (1976, 1978, 1982, 1984) LISREL, Browne and Cudeck's (1983) BENWEE, Schoenberg's (1982) MILS, and most recently Bentler's (1985) EQS. By far the most cited program used for CFA is the Jöreskog and Sörbom (1984) LISREL program. However, the literature fails to provide any comparison of parameter estimates across the various software packages, forcing the user to trust the accuracy of the estimates.

This brief note compares three estimation procedures across two of the more popular programs, LISREL VI Version 6.6 (Jöreskog & Sörbom, 1984) and EQS Version 2.0 (Bentler, 1985). The estimation procedures used were (1) ordinary least squares (OLS; Equation 3), (2) generalized least squares (GLS; Equation 4), and (3) maximum likelihood (ML; Equation 5). Both programs attempt to minimize the following fit functions for the estimates (see Jöreskog, 1983, and Bentler, 1985, for more details):

$$\text{OLS} = 2^{-1}\text{tr}(\mathbf{S} - \Sigma)^2 \quad (3)$$

$$\text{GLS} = 2^{-1}\text{tr}[(\mathbf{S} - \Sigma)\mathbf{S}^{-1}]^2 \quad (4)$$

$$\text{ML} = \text{tr}(\Sigma^{-1}\mathbf{S}) - \log_e|\Sigma^{-1}\mathbf{S}| - k \quad (5)$$

where $\mathbf{S}$ is the observed moment matrix (covariance),
Σ is the theoretical moment matrix (covariance), and
k is the number of observed variables.

The data used in this comparison were obtained from Long (1983) on Wheaton's (1978) study concerning the psychological disorders of patients over two time periods (1967 and 1971). The variables are (1) psychological disorders in 1967, (2) psychophysiological disorders in 1967, (3) psychological disorders in 1971, and (4) psychophysiological disorders in 1971.[1] The correlations, variances, and covariances are reported in Table 1.

The initial CFA model proposed by Long (1983) was used as the comparison device to obtain parameter estimates from both programs (see Figure 1). The model was not further refined. Parameter estimates, standard errors and tests of significance (where applicable) for OLS, GLS, and ML procedures from both the LISREL and EQS programs are in Table 2; the estimates obtained by Long (1983) using LISREL V (Jöreskog & Sörbom, 1982) and MILS (Schoenberg, 1982) are in Table 3. Also indicated are the degrees of freedom, estimated likelihood ratio test (chi-square value), probability estimates, and various indices of fit.

One very interesting finding concerns the estimates given by the LISREL VI GLS procedure. Because these data were previously analyzed by Long (1983) using LISREL V and Schoenberg's (1982) MILS programs, initial parameter estimates are available for comparison, if Long's estimates are correct. Because the EQS program essentially recovered all of Long's parameter estimates (with minor differences, most likely due to rounding errors) and the LISREL VI OLS and ML did the same, it is safe to say that Long's estimates are appropriate. This leaves the problem of why the LISREL VI GLS procedure did not recover the estimates.

The LISREL VI GLS procedure produced confusing results. The most obvious inconsistency was produced by the likelihood ratio (L^2), providing a value of .03 with one degree of freedom. This ratio was very different from the ratio reported by Long (1983) and the finding provided by Bentler's EQS. A recalculation of the L^2 ratio using Equation 6 indicated a value of 29.58 with one degree of freedom:

$$L^2_{\text{GLS}} = [(N-1)/2]\ \text{tr}[(\mathbf{S}-\hat{\Sigma})\mathbf{S}^{-1}]^2 \quad , \quad (6)$$

where df $= .5k(k + 1) - t$,
k is the number of observed variables in the analysis, and
t is the number of independent parameters to be estimated.

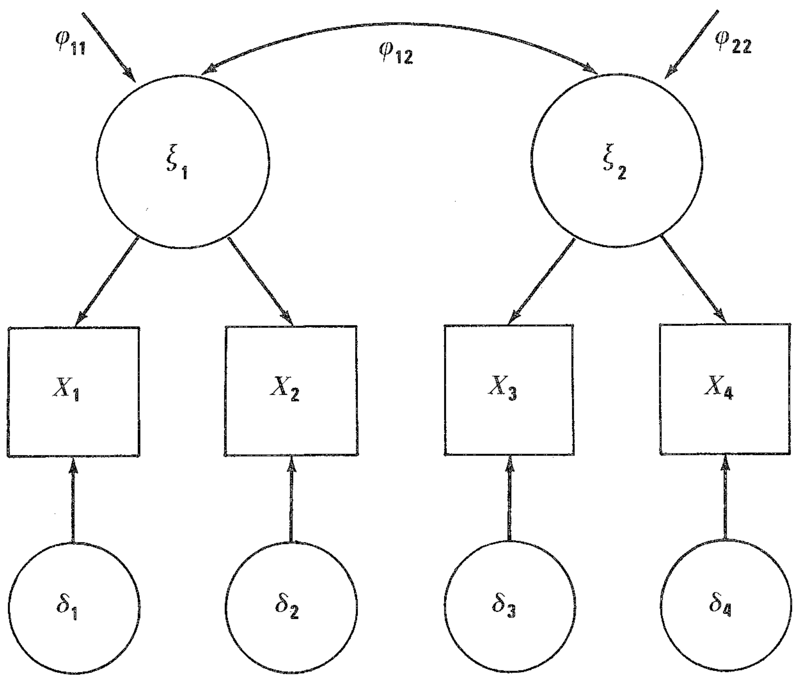
[1]It should be noted that the number of patients reported in Long's (1983) appendix is a misprint and should read 630 patients, not 63 (Long, personal communication, March 6, 1986).

Table 1
Correlations, Covariances and Variances (N=630)
(Correlations in upper triangle)

Variable	1	2	3	4
1	2.102	0.454	0.526	0.377
2	0.365	0.308	0.247	0.309
3	1.053	0.189	1.904	0.549
4	0.275	0.086	0.381	0.253

Note. Adapted from Wheaton (1978).

Figure 1
Model 1: A Model for the
Measurement of Psychological Disorders



This initial finding indicates a LISREL VI problem in the calculation of L^2. Because L^2 is contingent upon the parameter estimates, the accuracy of these estimates is also questionable, because the L^2 value provided by LISREL VI did not equal the values given by Long (1983) or by using Bentler's (1985) EQS program. Recalculation of Jöreskog and Sörbom's (1984) goodness-of-fit index (GFI) and adjusted goodness-of-fit index (AGFI) also provides evidence of a problem with the estimates, with a GFI of .9836 (versus 1.000) and an AGFI of .8362 (versus 1.000), using Equations 7 and 8 respectively.

$$\text{GFI} = 1 - \frac{\operatorname{tr}(\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{S} - \mathbf{I})^2}{\operatorname{tr}(\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{S})^2} \quad . \tag{7}$$

$$\text{AGFI} = 1 - [K(K + 1)/2d](1 - \text{GFI}) \quad , \tag{8}$$

where d is the degrees of freedom.

An examination of the GLS estimates provided by LISREL VI (Table 2), along with their standard errors, reveals problems (especially with the $\boldsymbol{\Theta}_\delta$ matrix) as compared with the other programs' estimates. Also apparent, though not expected, is striking similarity between the GLS and the OLS estimates.

A second model was also estimated using ML and GLS estimates from both programs. Data were obtained from Kenny (1979, p. 146). The inconsistency in GLS estimates between LISREL VI Version 6.6 and EQS was again detected (results are not reported here).

One possible source of the problem was hypothesized to be the use of automatic start values in obtaining the parameter estimates. To test this hypothesis, data from Long's (1983) initial model were submitted to the LISREL and EQS programs for both ML and GLS estimates, using automatic start

Table 2
Parameter Estimates from LISREL VI and EQS

Method and Parameter	OLS	GLS	$\hat{\sigma}$	t^b	ML	$\hat{\sigma}$	t^b
LISREL VI Version 6.6							
λ_{11}	1.000[a]	1.000[a]	0.000	0.000	1.000[a]	0.000	0.000
λ_{21}	0.188	0.187	0.065	2.858	0.205	0.205	8.057
λ_{32}	1.000[a]	1.000[a]	0.000	0.000	1.000[a]	0.000	0.000
λ_{42}	0.268	0.268	0.077	3.466	0.271	0.024	11.389
ϕ_{11}	1.938	1.953	0.941	2.075	1.781	0.221	8.059
ϕ_{22}	1.424	1.414	0.633	2.233	1.408	0.148	9.502
ϕ_{12}	1.051	1.051	0.035	29.671	1.045	0.090	11.615
θ_{11}	0.164	0.150	0.961	0.156	0.322	0.188	1.709
θ_{22}	0.239	0.250	0.240	1.042	0.233	0.015	15.207
θ_{33}	0.480	0.491	0.662	0.742	0.497	0.110	4.534
θ_{44}	0.151	0.165	0.347	0.476	0.150	0.011	13.065
COR(ξ_1,ξ_2)	-	-	-	-	-	-	-
REL(ξ_1,x_1)	0.922	0.929	-	-	0.847	-	-
REL(ξ_1,x_2)	0.224	0.188	-	-	0.243	-	-
REL(ξ_2,x_3)	0.748	0.742	-	-	0.739	-	-
REL(ξ_2,x_4)	0.403	0.347	-	-	0.408	-	-
GFI	1.000	1.000	-	-	0.985	-	-
AGFI	0.998	1.000	-	-	0.852	-	-
df	1	1	-	-	1	-	-
L^2	-	0.030	-	-	19.130	-	-
p	-	0.867	-	-	0.000	-	-
EQS Version 2.0							
λ_{11}	1.000[a]	1.000[a]	0.000	0.000	1.000[a]	0.000	0.000
λ_{21}	0.188	0.205	0.025	8.054	0.205	0.025	8.056
λ_{32}	1.000[a]	1.000[a]	0.000	0.000	1.000[a]	0.000	0.000
λ_{42}	0.268	0.271	0.024	11.391	0.271	0.024	11.391
ϕ_{11}	1.938	1.799	0.223	8.085	1.781	0.221	8.066
ϕ_{22}	1.424	1.431	0.149	9.573	1.408	0.148	9.499
ϕ_{12}	1.051	1.045	0.090	11.598	1.045	0.090	11.615
θ_{11}	0.164	0.303	0.190	1.593	0.322	0.188	1.710
θ_{22}	0.239	0.220	0.015	14.602	0.233	0.015	15.209
θ_{33}	0.480	0.468	0.111	4.218	0.497	0.110	4.535
θ_{44}	0.151	0.141	0.011	12.424	0.150	0.011	13.064
COR(ξ_1,ξ_2)	0.633	0.652	-	-	0.660	-	-
REL(ξ_1,x_1)	-	-	-	-	-	-	-
REL(ξ_1,x_2)	-	-	-	-	-	-	-
REL(ξ_2,x_3)	-	-	-	-	-	-	-
REL(ξ_2,x_4)	-	-	-	-	-	-	-
GFI	-	-	-	-	-	-	-
AGFI	-	-	-	-	-	-	-
df	1	1	-	-	1	-	-
L^2	-	18.296	-	-	19.130	-	-
p	-	0.000	-	-	0.000	-	-

[a] Parameters that were constrained to equal 1.000.
[b] $t=Z=(\hat{\omega}-\omega)/\hat{\sigma}$; where $\hat{\omega}$ = parameter estimate, and ω = hypothesized value.

Table 3
LISREL V and MILS Estimates Obtained by Long(1983)

Method and Parameter	OLS	GLS	ML
λ_{11}	1.000[a]	1.000[a]	1.000[a]
λ_{21}	0.188	0.205	0.205
λ_{32}	1.000[a]	1.000[a]	1.000[a]
λ_{42}	0.268	0.270	0.271
ϕ_{11}	1.938	1.800	1.781
ϕ_{22}	1.424	1.431	1.408
ϕ_{12}	1.051	1.046	1.045
θ_{11}	0.164	0.300	0.322
θ_{22}	0.239	0.220	0.233
θ_{33}	0.480	0.467	0.497
θ_{44}	0.150	0.141	0.150
COR(ξ_1,ξ_2)	0.633	0.652	0.660
REL(ξ_1,x_1)	0.922	0.857	0.847
REL(ξ_1,x_2)	0.222	0.286	0.244
REL(ξ_2,x_3)	0.748	0.755	0.739
REL(ξ_2,x_4)	0.407	0.443	0.407
GFI	-	-	-
AGFI	-	-	-
df	1	1	1
L^2	-	17.320	18.310
p	-	0.000	0.000

[a]Parameters that were constrained to equal 1.000.

values and user-specified start values. Table 4 shows the results. No difference across programs could be detected between the ML estimates using either automatic or user-specified start values, while an inconsistency was discovered between the LISREL VI GLS estimates. It may be concluded that the use of automatic start values provides inappropriate estimates when using the GLS procedure. This has been verified by Jöreskog (personal communication, May 7, 1986). The problem was identified as a programming error in LISREL VI Version 6.6, with intermediate calculations being placed in the S^{-1} matrix location of the program. This subsequently leads to inaccurate results when the S^{-1} matrix is used to complete the estimates.

This problem may be easily avoided by specifying NS (not to compute initial estimates and start iteration by the steepest descent from a given start value) on the output line, and then specifying user start values. Use of the two-stage least squares solution as starting values is also suggested for optimal performance in larger models.

Summary

Most university computer centers maintain an array of software that will provide parameter estimates in confirmatory factor analysis. Because new statistical techniques make researchers dependent on computer programs, the accuracy of which can rarely be verified independently, it is suggested that replication of parameter estimates using two independent programs may not be inappropriate.

Table 4
Parameter Estimates from LISREL VI and EQS Using Automatic Start Values and User-Specified Start Values

Method and Parameter	Automatic Start Values		User-Specified Start Values	
	ML	GLS	ML	GLS
LISREL VI Version 6.6				
λ_{11}	1.000[a]	1.000[a]	1.000[a]	1.000[a]
λ_{21}	0.205	0.187	0.205	0.205
λ_{32}	1.000[a]	1.000[a]	1.000[a]	1.000[a]
λ_{42}	0.271	0.268	0.271	0.271
ϕ_{11}	1.781	1.953	1.781	1.789
ϕ_{22}	1.408	1.414	1.408	1.431
ϕ_{12}	1.045	1.051	1.045	1.045
θ_{11}	0.322	0.150	0.322	0.303
θ_{22}	0.233	0.250	0.233	0.220
θ_{33}	0.497	0.491	0.497	0.468
θ_{44}	0.150	0.165	0.150	0.141
df	1	1	1	1
L^2	19.130	0.030	19.130	18.300
p	0.000	0.867	0.000	0.000
EQS Version 2.0				
λ_{11}	1.000[a]	1.000[a]	1.000[a]	1.000[a]
λ_{21}	0.205	0.205	0.205	0.205
λ_{32}	1.000[a]	1.000[a]	1.000[a]	1.000[a]
λ_{42}	0.271	0.271	0.271	0.271
ϕ_{11}	1.781	1.799	1.781	1.799
ϕ_{22}	1.408	1.431	1.408	1.431
ϕ_{12}	1.045	1.045	1.045	1.045
θ_{11}	0.322	0.303	0.322	0.303
θ_{22}	0.233	0.220	0.233	0.220
θ_{33}	0.497	0.468	0.497	0.468
θ_{44}	0.150	0.141	0.150	0.141
df	1	1	1	1
L^2	19.130	18.296	19.130	18.296
p	0.000	0.000	0.000	0.000

[a]Parameters that were constrained to equal 1.000.

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