Masses and Boost-Invariant Wave Functions of Heavy Quarkonia from the Light-Front Hamiltonian of QCD

Stanisław D. Głazek

Institute of Theoretical Physics, Warsaw University

Method: Weak-coupling expansion for QCD in the LF Fock space

Result (in a crudest approximation)**:**

relativity + λ, α_λ, $m_\lambda \longrightarrow c\bar{c}$ or $b\bar{b}$ masses within few %

SDG, Jarosław Młynik - IFT/08/06

stglazek@fuw.edu.pl

No use of notions such as: scattering states for quarks or gluons, Feynman diagrams, path integral, euclidicity postulate, lattice, or vacuum expectation values.

Instead:

Renormalization Group Procedure for Effective Particles in QFT (RGPEP in LFQCD).

15 minutes?

stglazek@fuw.edu.pl

Two icebreakers

P. A. M. Dirac (1977):

"Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out."

K. G. Wilson (2004):

"...the time is ripe for a few accomplished theorists to switch into light-front theory and help build a growing research effort in this area."

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu} \, .$$

$$x^{\pm} = x^0 \pm x^3, \qquad t \to x^+, \qquad z \to x^-, \qquad x^{\perp} \, .$$

$$A^+ = 0 \, .$$

$$\begin{aligned} H_{can} = {} & H_{\psi^2} + H_{A^2} + H_{A^3} + H_{A^4} + H_{\psi A \psi} \\ & + H_{\psi A A \psi} + H_{[\partial A A]^2} + H_{[\partial A A](\psi\psi)} + H_{(\psi\psi)^2} \, . \end{aligned}$$

.

$$
\begin{aligned}
\mathcal{H}_{\psi^2} &= \frac{1}{2}\bar\psi\gamma^+\frac{-\partial^{\perp\,2}+m^2}{i\partial^+}\psi\,,\\
\mathcal{H}_{A^2} &= -\frac{1}{2}A^\perp(\partial^\perp)^2A^\perp\,,\\
\mathcal{H}_{\psi A\psi} &= g\,\bar\psi\not{A}\psi\,,\\
\mathcal{H}_{(\psi\psi)^2} &= \frac{1}{2}g^2\,\bar\psi\gamma^+t^a\psi\frac{1}{(i\partial^+)^2}\bar\psi\gamma^+t^a\psi\,.
\end{aligned}
$$

At $x^+ = 0$:

$$\psi = \sum_{\sigma c} \int [k] \left[\chi_c u_{k\sigma} b_{k\sigma c} e^{-ikx} + \chi_c v_{k\sigma} d^\dagger_{k\sigma c} e^{ikx} \right] ,$$

$$A^\mu = \sum_{\sigma c} \int [k] \left[t^c \varepsilon^\mu_{k\sigma} a_{k\sigma c} e^{-ikx} + t^c \varepsilon^{\mu *}_{k\sigma} a^\dagger_{k\sigma c} e^{ikx} \right] .$$

Regularization (boost invariant, 7 Poincaré symmetries).

Counterterms.

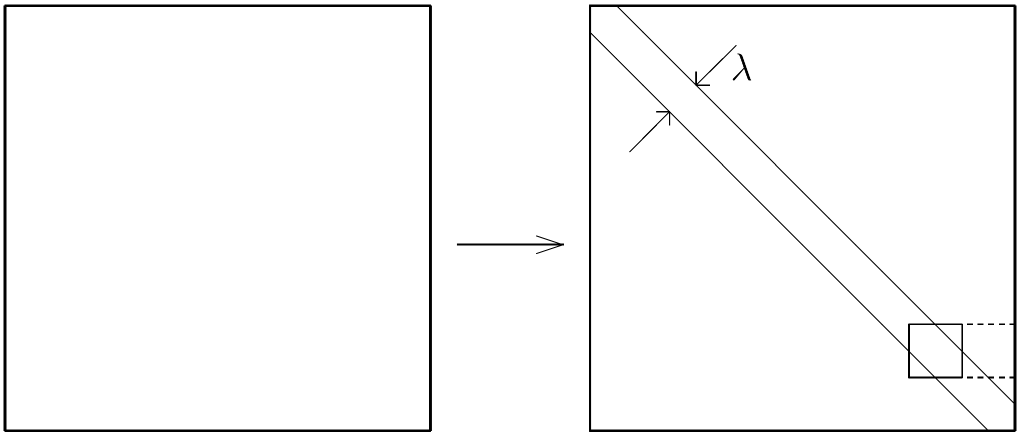
$$H = [H_{can} + H_{CT}]_{reg} \ .$$

RGPEP:

$$q_\lambda = U_\lambda \, q_{can} \, U_\lambda^\dagger \, ,$$

$$\hat{O}'_\lambda = [\mathcal{T}, \hat{O}_\lambda] \, , \quad \mathcal{T} = U'_\lambda U_\lambda^\dagger \, ,$$

$$\hat{O}_\infty = \hat{O}_{can(reg)} + \hat{O}_{CT} \quad \rightarrow \quad \hat{O}_\lambda \, .$$



RGPEP has roots in "similarity" RG procedure:

SDG and KGW ('93,'94,'98)

width λ in Hamiltonian matrices $\rightarrow$ form factors f_λ in RGPEP

$$H|P\rangle = M^2|P\rangle \quad \rightarrow \quad H_\lambda|P\rangle = M^2|P\rangle\,.$$

$$|P\rangle = |Q_\lambda \bar{Q}_\lambda\rangle + |Q_\lambda \bar{Q}_\lambda g_\lambda\rangle + \,\ldots\,.$$

$$[H_\lambda] = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & H_3 & Y \\ \cdot & Y^\dagger & H_2 \end{bmatrix} \rightarrow \begin{bmatrix} T_3 + \mu^2 & Y \\ Y^\dagger & T_2 + V_2 \end{bmatrix}.$$

$$H_{Q\bar{Q}\lambda} = T_{2\lambda} + V_{2\lambda} + Y_\lambda^\dagger \frac{1}{T_3 + \mu^2} Y_\lambda\,.$$

$$H_\lambda|P\rangle = M^2|P\rangle \rightarrow H_{Q\bar{Q}\lambda}|P\rangle = M^2|P\rangle\,.$$

$$\boxed{f_\lambda f_\lambda \frac{4m^2}{q_z^2} \frac{\mu^2}{q^2+\mu^2} \quad \longrightarrow \quad harmonic\ force\ in\ H_{Q\bar{Q}\lambda}\,.}$$

stglazek@fuw.edu.pl

$$|M, P^+, P^\perp\rangle = \int [ij]\, \tilde{\delta}\, P^+ \, \frac{\bar{u}_i \Psi_{ij} v_j}{-4m^2}\, |ij\rangle \, .$$

$$|ij\rangle = b^\dagger_{\lambda i} d^\dagger_{\lambda j}\, |0\rangle \, , \qquad \frac{\bar{u}_i \Psi_{ij} v_j}{-4m^2} \longrightarrow \chi_i^\dagger \, \phi(\vec{k}_{ij}) \chi_j \, .$$

$$0 = \left[\vec{p}^{\,2} - k_p \, \Delta_p - x\right] \phi(\vec{p}) - 2 \int \frac{d^3k}{(2\pi)^3} \, \mathcal{V} \, \phi(\vec{k}\,) \, ,$$

$$k_p = \frac{9}{128\sqrt{2\pi}} \left(\frac{\lambda^2}{\alpha m^2}\right)^3 ,$$

$$\mathcal{V} = f \, \frac{4\pi}{(\vec{p} - \vec{k}\,)^2} \, (1 + BF) \, ,$$

$$f = \exp\left\{ - \left[\frac{\mathcal{M}^2(p) - \mathcal{M}^2(k)}{\lambda^2} \right]^2 \right\} ,$$

$$M = 2m \sqrt{1 + x \left(\frac{2}{3}\alpha\right)^2} \, .$$

stglazek@fuw.edu.pl

Example of J/ψ or Υ:

$$\phi(\vec{k}\,) = \left[a(\vec{k}\,) + \vec{b}(\vec{k}\,)\vec{\sigma}\,\right]$$

$$a(\vec{k}\,) = 0\,, \quad b^m(\vec{k}\,) = \left[\delta^{mn}\,\frac{S(k)}{k} + \frac{1}{\sqrt{2}}\,\left(\delta^{mn} - 3\frac{k^m k^n}{k^2}\right)\,\frac{D(k)}{k}\right]\,s^n\,.$$

$$0 = \begin{bmatrix} p^2 - k_p\partial_p^2 - x\,, & 0 \\ 0\,, & p^2 - k_p\partial_p^2 + k_p\frac{6}{p^2} - x \end{bmatrix} \begin{bmatrix} S(p) \\ D(p) \end{bmatrix} - \frac{2}{\pi}\int_0^\infty dk\,f\,pk \begin{bmatrix} \mathcal{W}_{ss},\ \mathcal{W}_{sd} \\ \mathcal{W}_{ds},\ \mathcal{W}_{dd} \end{bmatrix}$$

$$\mathcal{W}_{ss} = J_0 + \frac{\alpha^2}{3}\left[(p^2+k^2)\,J_0 - 16/9\right]\,,$$

$$\mathcal{W}_{sd} = \frac{\alpha^2}{3}\left[p^2\,(J_2 - J_0) + 4/3\right]\frac{\sqrt{2}}{3}\,, \quad \mathcal{W}_{ds} = \frac{\alpha^2}{3}\left[k^2\,(J_2 - J_0) + 4/3\right]\frac{\sqrt{2}}{3}\,,$$

$$\mathcal{W}_{dd} = J_2 + (J_2 - J_0)/2 + \frac{\alpha^2}{3}\left\{(p^2+k^2)\,\left[J_0 - (J_2 - J_0)/6\right] - 20/9\right\}\,.$$

δ-functions with f_λ. J_0, J_2 are known functions.

stglazek@fuw.edu.pl

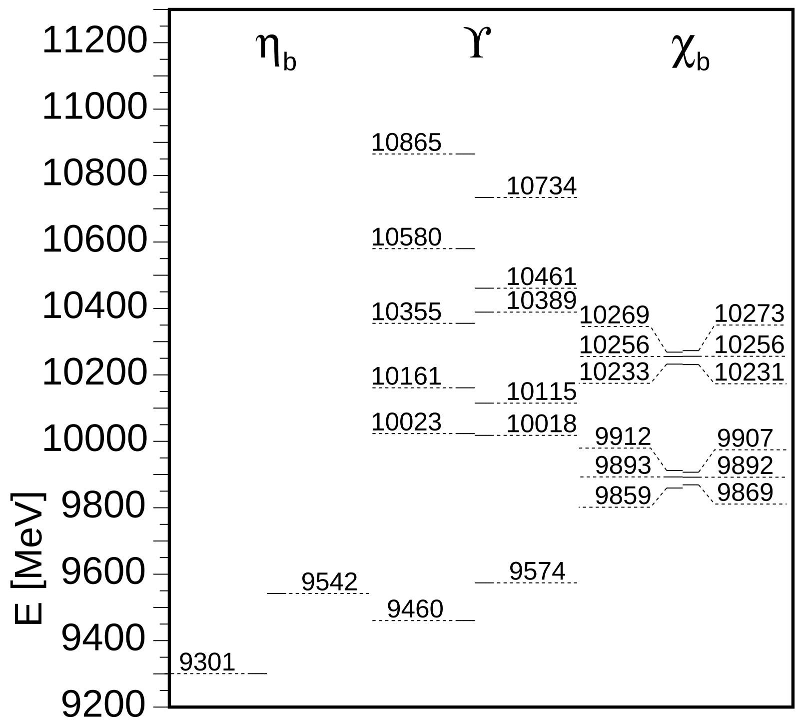


FIG. 1: 7 $b\bar{b}$ middle states: $\lambda = 3779.8$ MeV, $\alpha_\lambda = 0.28839$, $m = 4835.9$ MeV, ($\omega = 184.62$ MeV).

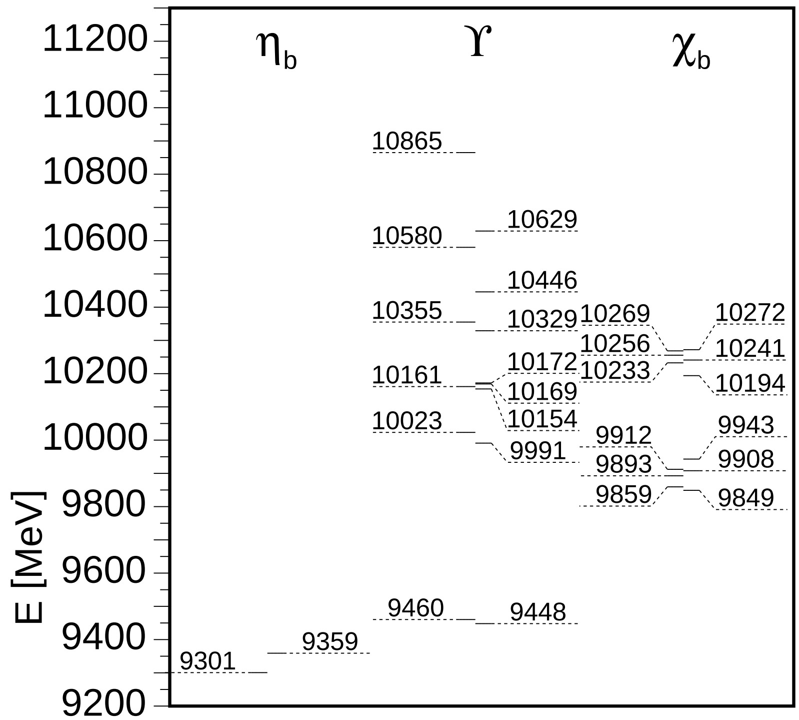


FIG. 2: 12 $b\bar{b}$ states: $\lambda = 3252.3$ MeV, $\alpha_\lambda = 0.50738$, $m = 4979.7$ MeV, ($\omega = 147.11$ MeV).

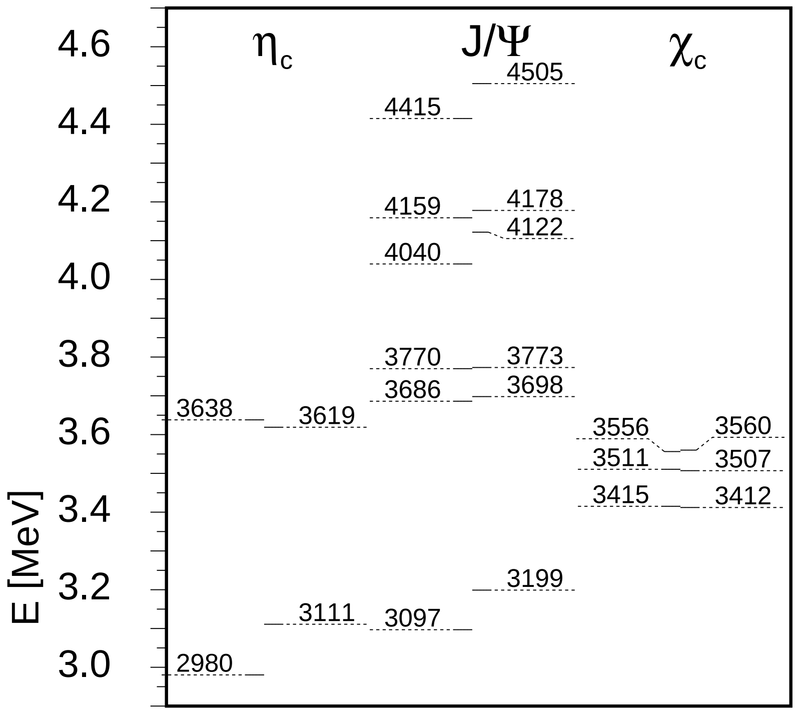


FIG. 3: 3 middle $c\bar{c}$ states: $\lambda = 1990.0$ MeV, $\alpha_\lambda = 0.34335$, $m = 1553.3$ MeV, ($\omega = 284.93$ MeV).

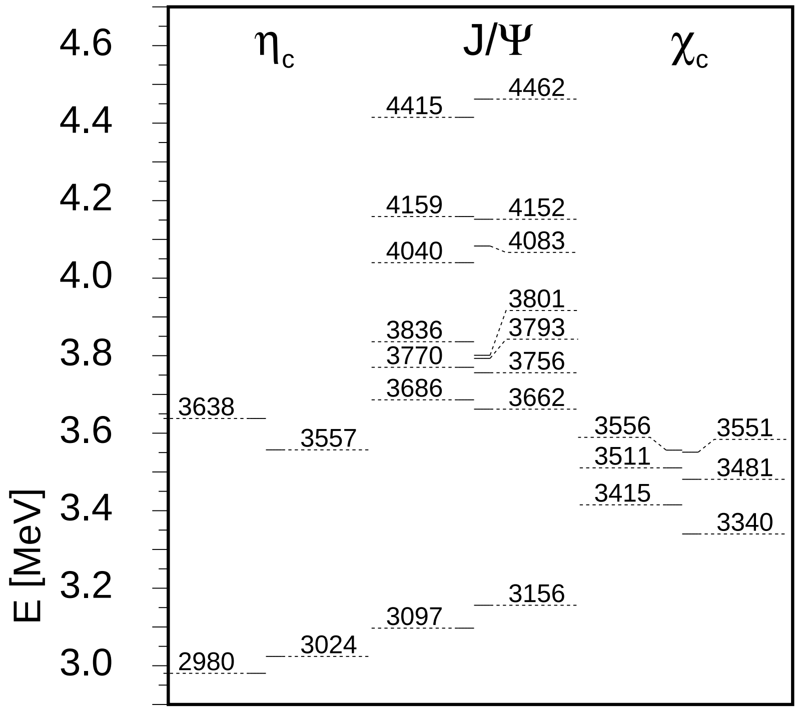
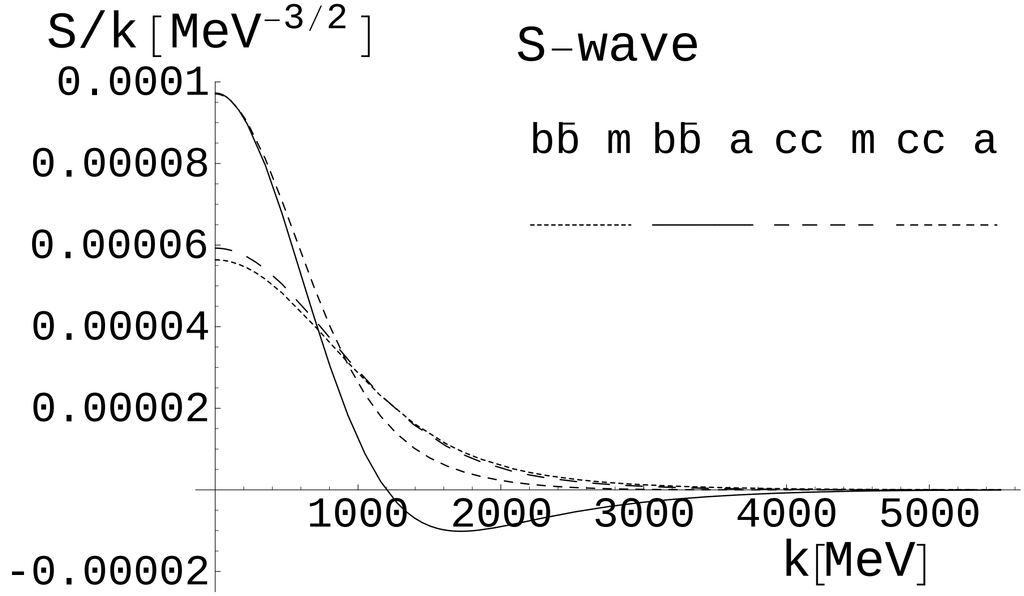
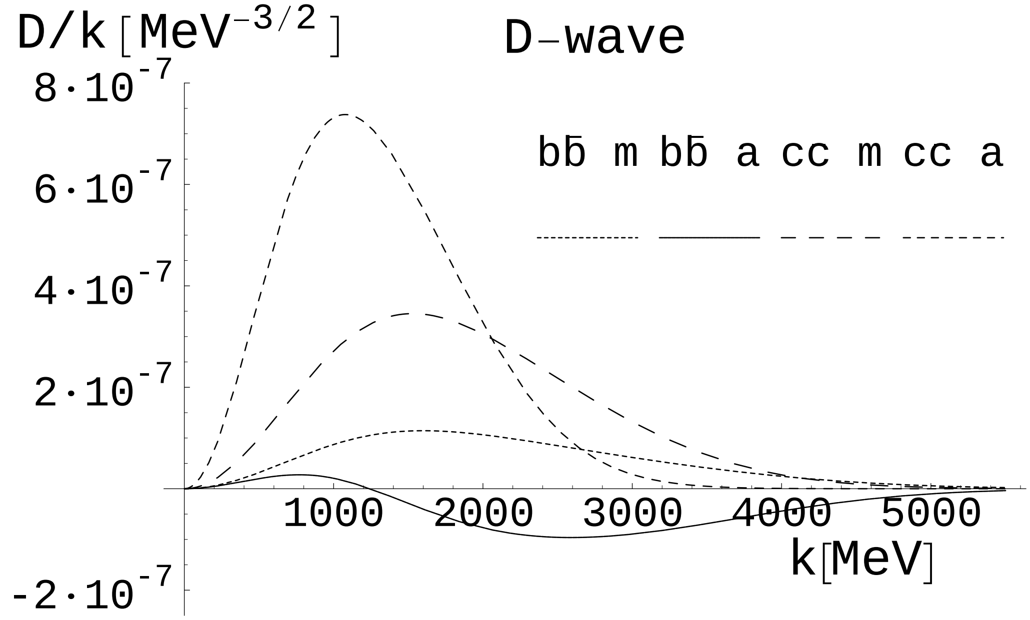


FIG. 4: 11 $c\bar{c}$ states: $\lambda = 1934.2$ MeV, $\alpha_\lambda = 0.41443$, $m = 1577.4$ MeV, ($\omega = 278.72$ MeV).



S/k [MeV^-3/2]
S-wave
b b̄ m
b b̄ a
cc m
cc a
0.0001
0.00008
0.00006
0.00004
0.00002
-0.00002
1000
2000
3000
4000
5000
k[MeV]


D/k [MeV$^{-3/2}$]
D−wave
8·10^{-7}
6·10^{-7}
4·10^{-7}
2·10^{-7}
-2·10^{-7}
bb̄ m bb̄ a cc m cc a
1000 2000 3000 4000 5000
k[MeV]

Conclusion:

- There exists an approximate constituent picture for heavy quarkonia in 1 flavor QCD: relativistic, simple, usable for fast mesons.
- Fits meson masses reasonably well for reasonable λ, α_λ, and m_λ.
- Provides very specific boost-invariant wave functions in the LF Fock space. Decays? Production? Unequal masses? Exclusive processes?
- Explicit bound states with J=0, J=1, J=2, waves S, P, D, F.
- Systematically improvable within RGPEP ($\alpha \sim 1/3$).

Will it survive further scrutiny?

stglazek@fuw.edu.pl

$$\frac{1}{2}\,\frac{m}{2}\,\omega^2\,r^2 + \mathcal{V}\,.$$

$$\omega = \sqrt{\frac{4}{3}\,\frac{\alpha}{\pi}}\;\lambda\;\left(\frac{\lambda}{m}\right)^2\;\left(\frac{\pi}{1152}\right)^{1/4}\;.$$

$$\mathcal{V} = f\;\left(\frac{4\pi}{p^2} + \alpha^2 R\right)(1+\alpha^2 S)(1+\alpha^2 M)\,.$$

.

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & H_4 & Y_1 & Y_2 \\ \cdot & Y_1^\dagger & H_3 & Y \\ \cdot & Y_2^\dagger & Y^\dagger & H_2 \end{bmatrix} \longrightarrow \begin{bmatrix} T_4 + \mu^2 & Y_1 & Y_2 \\ Y_1^\dagger & H_3 & Y \\ Y_2^\dagger & Y^\dagger & H_2 \end{bmatrix} .$$