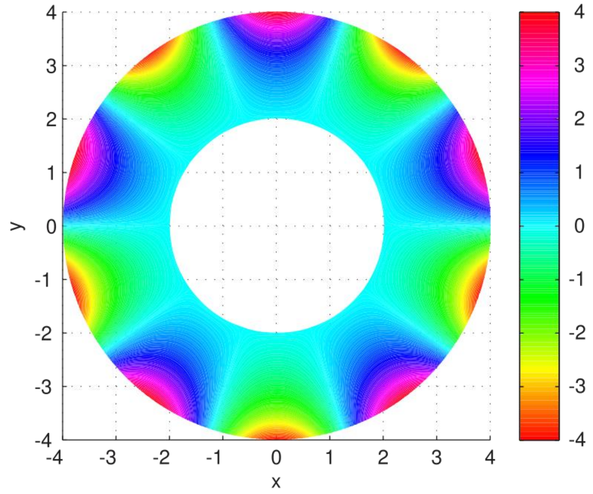
Quantifying the Relationship Between Robin Parameter and Solutions to the Discrete Laplace's Equation

Nathan Elango & Dr. Max Engelstein

University of Minnesota, UROP Project, Spring 2025

Introduction:

- What is a partial differential equation?
  - Relates derivatives of multivariable function in a region (called a domain)
  - Relates function values and derivatives at boundary
  - We solve for the function
- What is Laplace's Equation?
  - Relevant to systems in equilibrium. "No peaks".



- What is a Robin boundary condition?

$$\frac{1}{a}\frac{\partial u}{\partial \vec{n}} + u = f$$

Robin parameter

Slope of function in direction perpendicular to boundary

Function value

Known, depends on position

Motivation:

- Want to quantify how solutions change as Robin parameter changes!

Methods:

- Solutions to Laplace's Equation closely related to Green's functions

$$G(x,y) = Pr(\text{go from } x \text{ to } y \text{ without absorption into boundary})$$

$$a = \frac{p}{1-p}$$

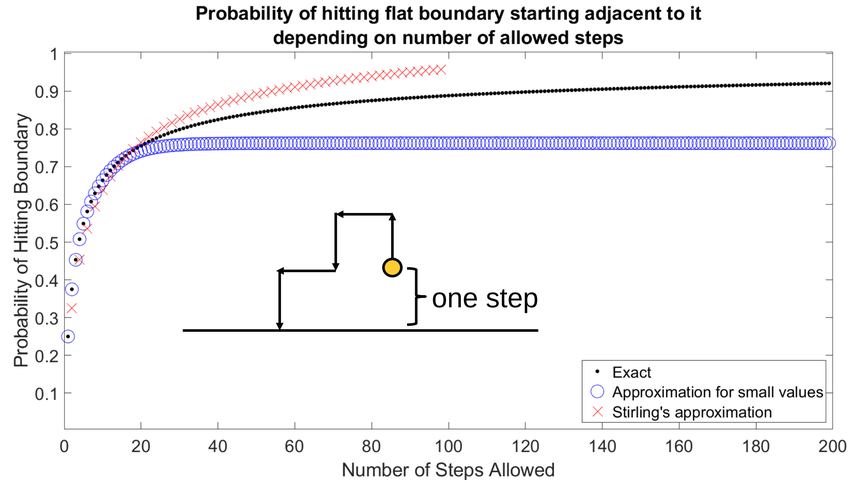
p: Probability of reflection

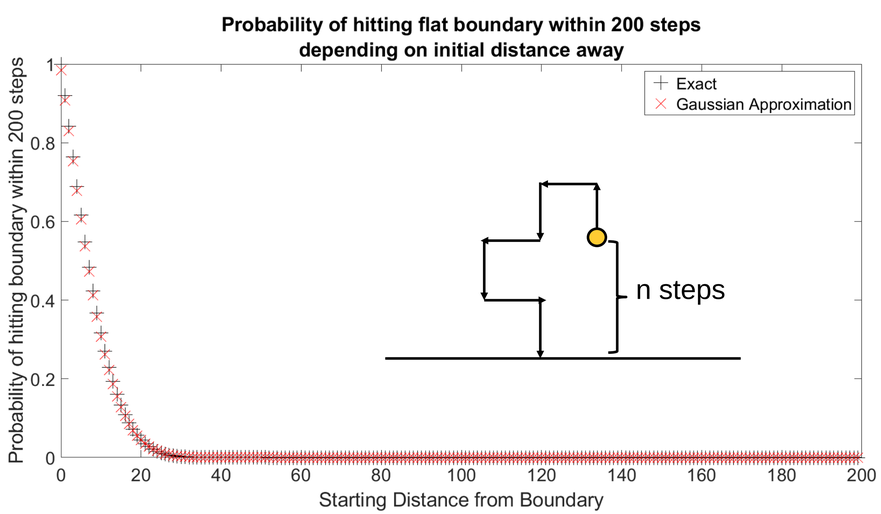
- Only effect of Robin parameter is at boundary, so we care what happens there
  - How long it takes to hit boundary?
    - Starting near vs far
  - How far away particle gets before hitting boundary?
    - Exact solution known, difficult to compute

Next Steps:

- Approximate distance traveled before hit
- Study different boundary geometries
- Find representation formula

Results





Acknowledgements

We thank Dr. Engelstein for wonderful mentoring and the Undergraduate Research Opportunities Program for funding.