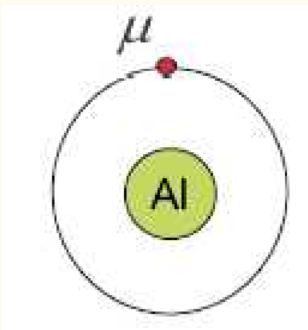
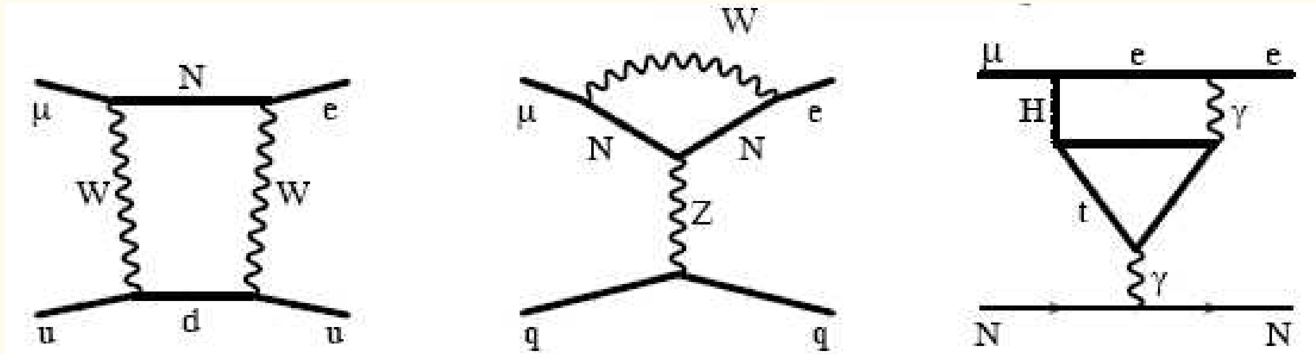
Decays of Muons: Free and Bound





Continuing Advances in QCD
Minneapolis, May 16, 2013

Andrzej Czarnecki   University of Alberta

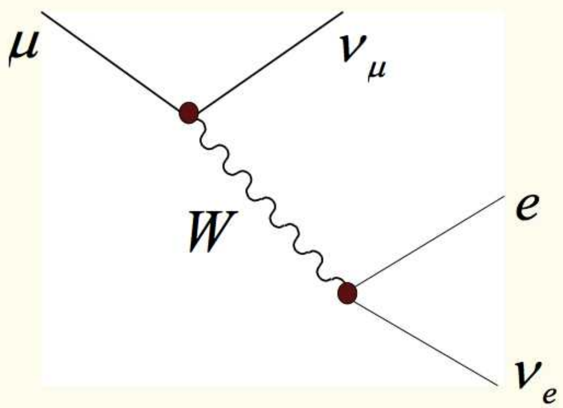
Outline

Free muon decay
- Theoretical vs experimental accuracy
- How to improve the theory

Lepton-flavor violation: searches

Background: bound muon decay
- Modification of the electron spectrum by binding
- Precise description of the spectrum

Free muon decay



A model process in particle physics
(tools for quark decays)

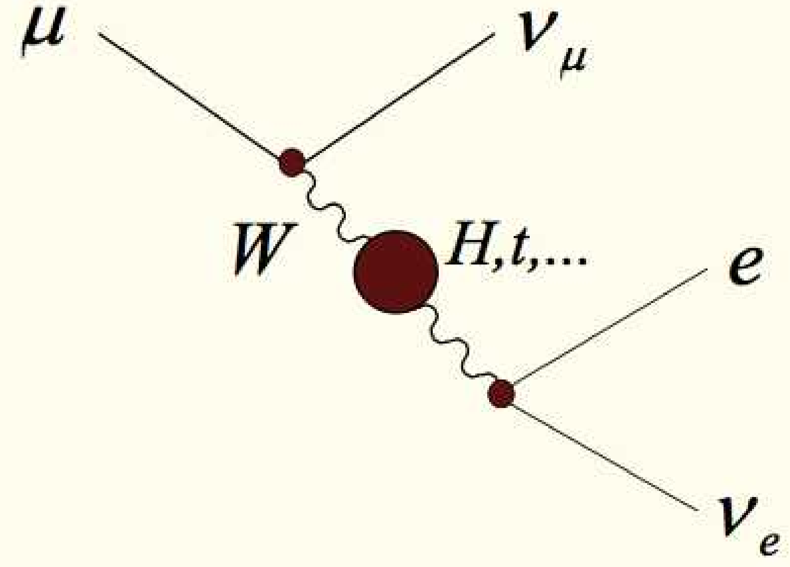
The first decay process known with one- and two-loop QED effects.

Anastasiou, Melnikov, Petriello, JHEP 0709 (2007) 014
van Ritbergen + Stuart, PRL 82 (1999) 488
Pak + Czarnecki, PRL 100 (2008) 241807

Also very thoroughly studied experimentally; most recently
* decay distributions ("Michel parameters") TWIST PRD 85 (2012) 092013
* total rate (1 ppm!) MuLan PRL 106 (2011) 041803

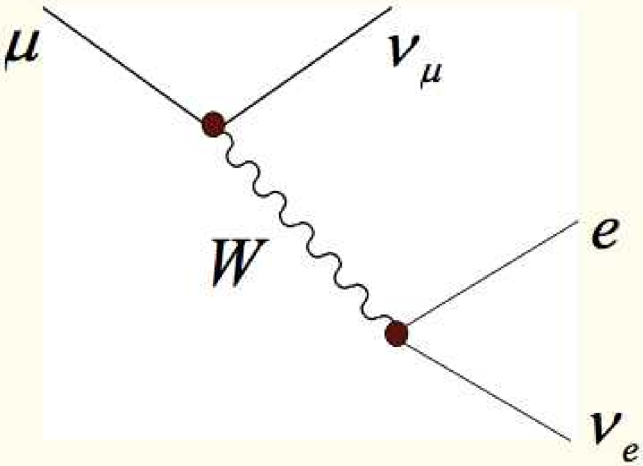
Fermi constant and tests of the SM

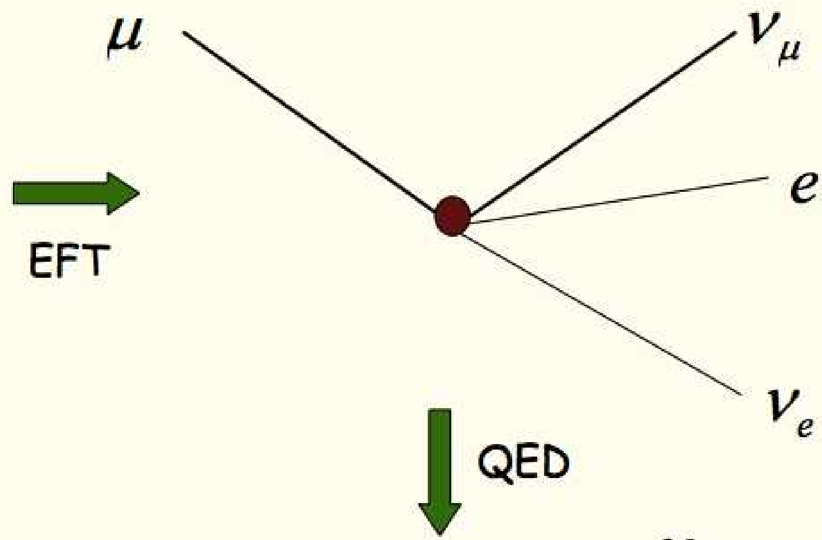
$$G_\mu \sim \frac{\alpha}{M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}\left(1+\Delta r\right)$$

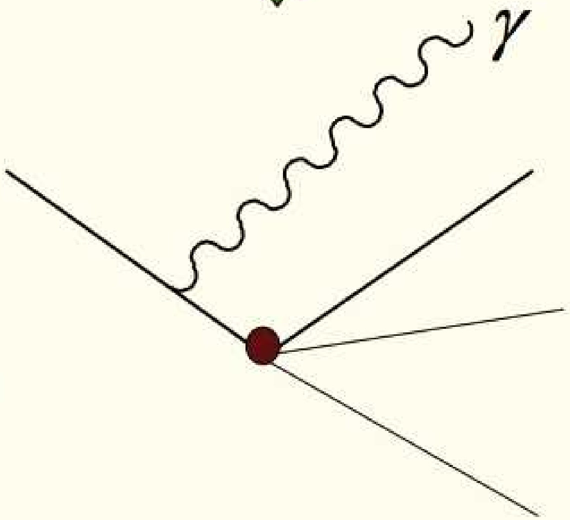


$$\Delta\alpha_{\rm had}^5 - c\, m_t^2 + c' \ln M_H^2 + \ldots$$

Determination of the Fermi constant (convention)







$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[1 + \Delta q\right]$$

Finite m_e and QED corrections in the four-fermion EFT

QED radiative corrections in Fermi theory

1956: one-photon, with m_e — Behrends, Finkelstein, Sirlin

1999: two-photon, $m_e=0$ — van Ritbergen and Stuart

2008: two-photon, with m_e — Pak, AC

Related work:
Numerical tests of the $O(\alpha^2)$ result (not able to determine the m_e effect):
Chetyrkin, Harlander, Seidensticker, Steinhauser (1999);
Blokland, AC, Ślusarczyk, Tkachov (2004)

2005, Anastasiou, Melnikov, Petriello: $O(\alpha^2)$ electron spectrum

Muon lifetime in Fermi theory, with QED

$$\Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left[X_0 + \frac{\alpha}{\pi}X_1 + \left(\frac{\alpha}{\pi}\right)^2 X_2 + \dots\right]$$

$$X_0 = 1 - 8\rho^2 - 24\rho^4 \ln\rho + 8\rho^6 - \rho^8$$

$$\rho \equiv \frac{m_e}{m_\mu}$$

$$X_1 = \frac{25}{8} - \frac{\pi^2}{2} - (34 + 24\ln\rho)\rho^2 + 16\pi^2\rho^3 - \left(\frac{273}{2} - 36\ln\rho + 72\ln^2\rho + 8\pi^2\right)\rho^4 + \dots.$$

$$X_2 = X_2(\rho = 0) - \frac{5}{4}\pi^2\rho + \dots$$

Pak, AC

Example: electron vacuum polarization

$$X_C = -\frac{1009}{288} + \frac{8\zeta_3}{3} + \frac{77\pi^2}{216}$$

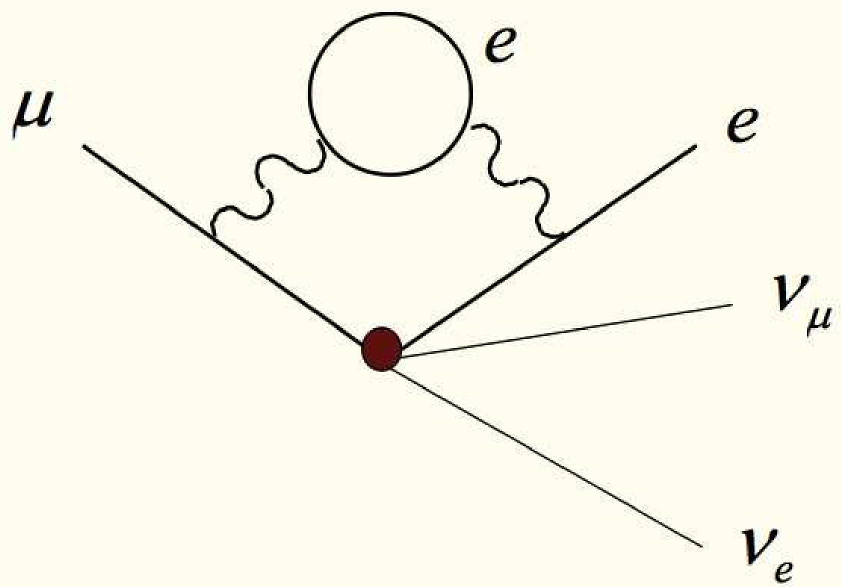
$$-\frac{5}{4}\pi^2\rho$$

$$+\left[\frac{145}{3} + \frac{52}{3}\ln\rho - 8\ln^2\rho + \frac{16\pi^2}{3}\right]\rho^2$$

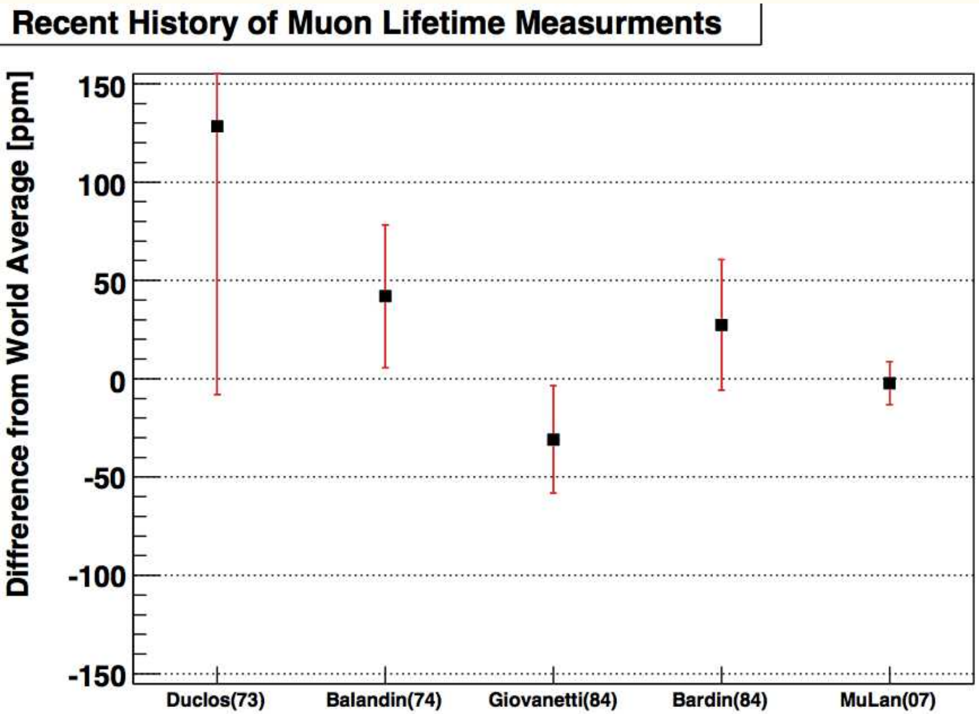
$$+\left[\frac{569}{36} + \frac{64}{3}\ln\rho\right]\pi^2\rho^3 + \ldots$$

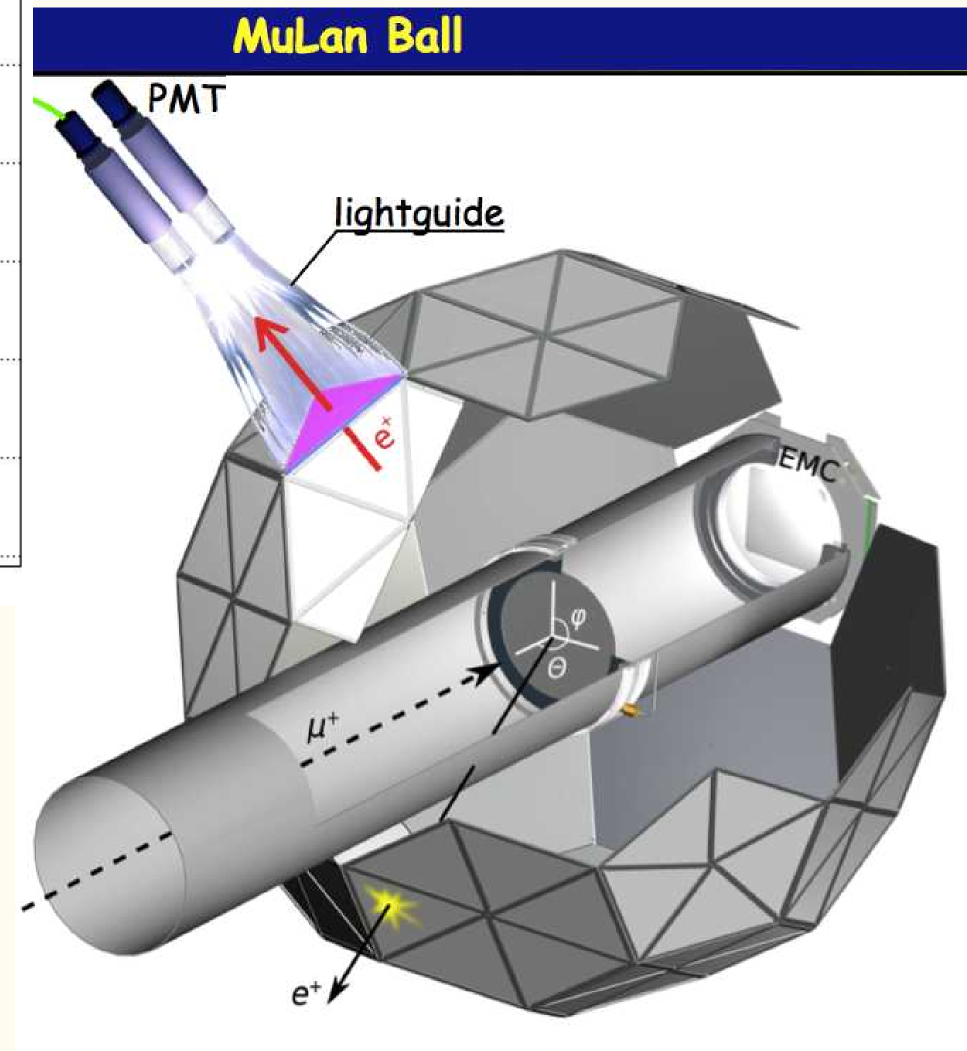
Series in powers and logs of the mass ratio, $\rho = m_e / m_\mu$

$$\Delta q(m_e) \simeq -0.43 \cdot 10^{-6}$$



How well can the muon lifetime be measured?





Theoretical accuracy of the muon decay rate

$$\Gamma = \Gamma_0 \left[1 + \frac{\alpha}{\pi} x_1 + \left(\frac{\alpha}{\pi}\right)^2 \left(x_2 - \frac{x_1}{3} \ln \rho^2\right)\right],$$

$$x_1 = \frac{25}{8} - \frac{\pi^2}{2} \simeq -1.81,$$

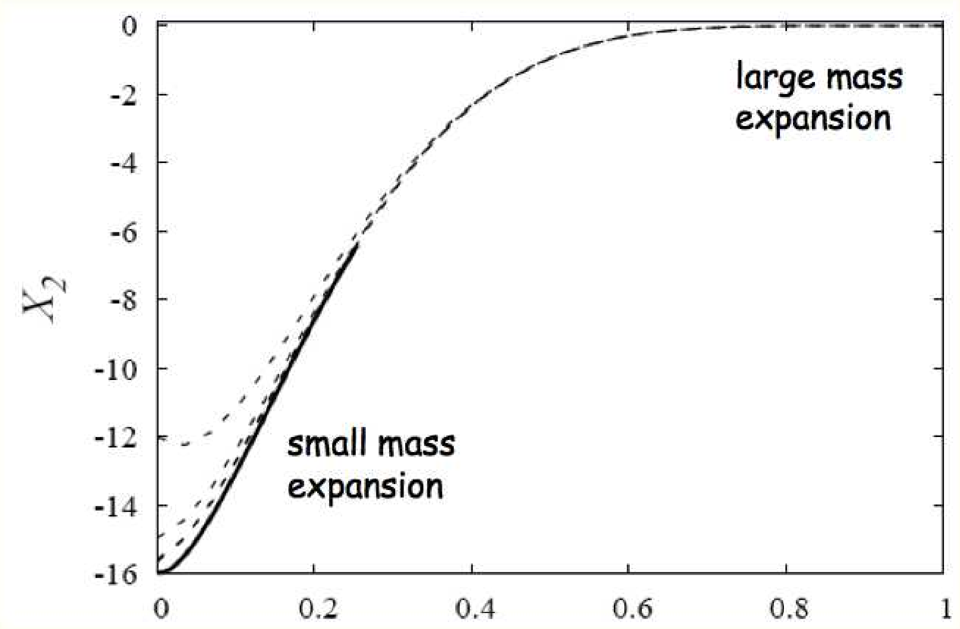
$$x_2 = \frac{156815}{5184} - \frac{1036}{27}\zeta_2 - \frac{895}{36}\zeta_3 + \frac{67}{8}\zeta_4 + 53\zeta_2 \ln 2 \simeq 6.74.$$

If x_3 is ~40, it gives rise to Δq~0.5 ppm.

Other sources of uncertainty:
m_μ: 0.2 ppm
hadronic loops: 0.02 ppm

Can the three loop effect be determined?

We have found an interesting way while checking the two-loop result: the calculation would be easier if the electron was very heavy, almost as heavy as the decaying muon.



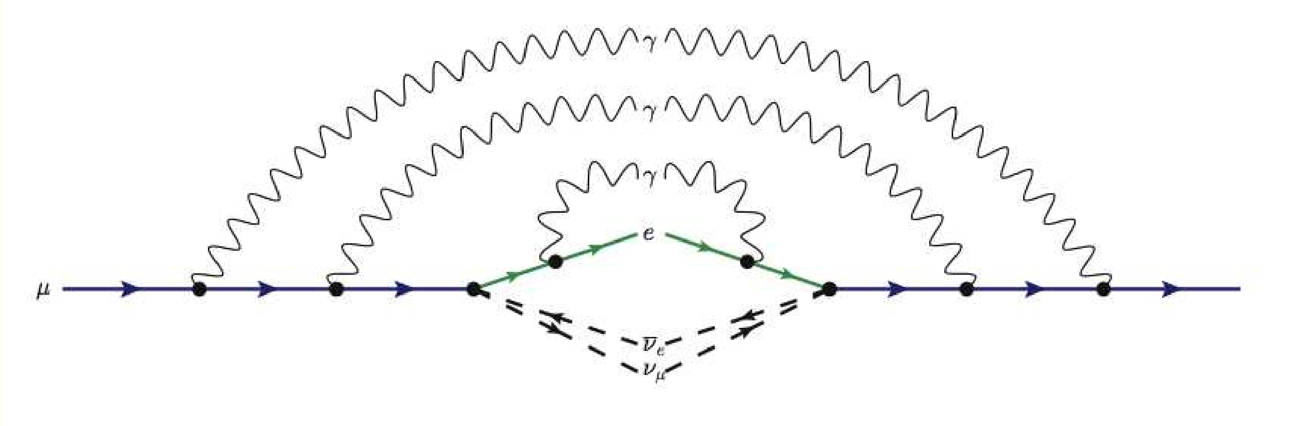
"Electron to muon" mass ratio

From Dowling, Piclum, AC

Note: the plot actually for QCD.
QED given by a subset of QCD results.

Expansion around the equal mass limit

with M. Dowling
and M. Czakon



Electron and muon masses: the only characteristic scales.

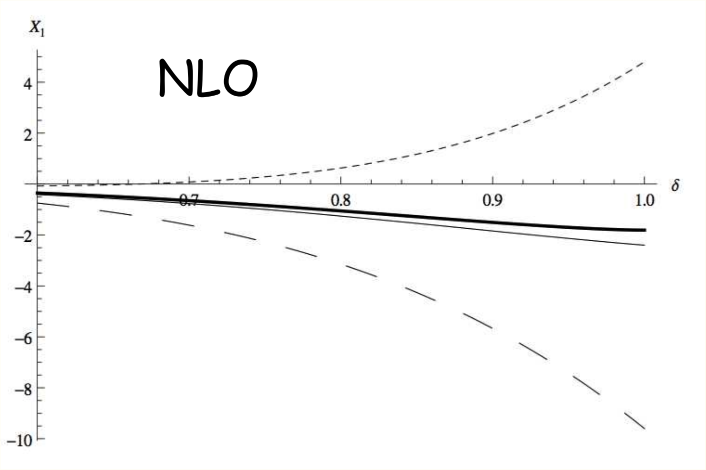
We are interested in the limit $m_\mu - m_e \ll m_\mu$

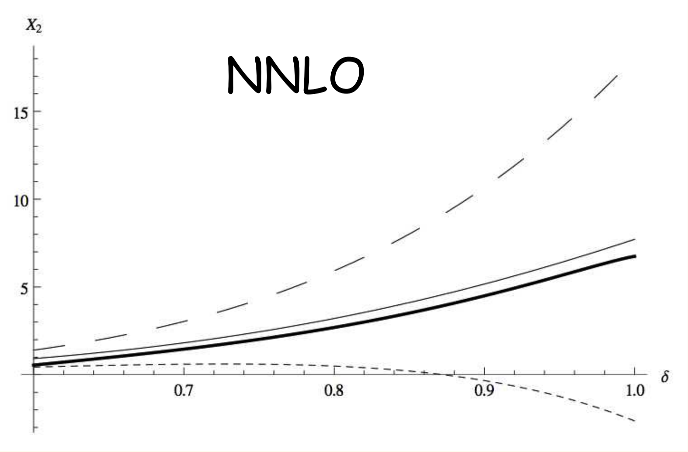
The imaginary part of five-loop integrals needed;
no contribution when all loop momenta "hard" $\sim m_\mu$
(this simplifies the task – a lot).

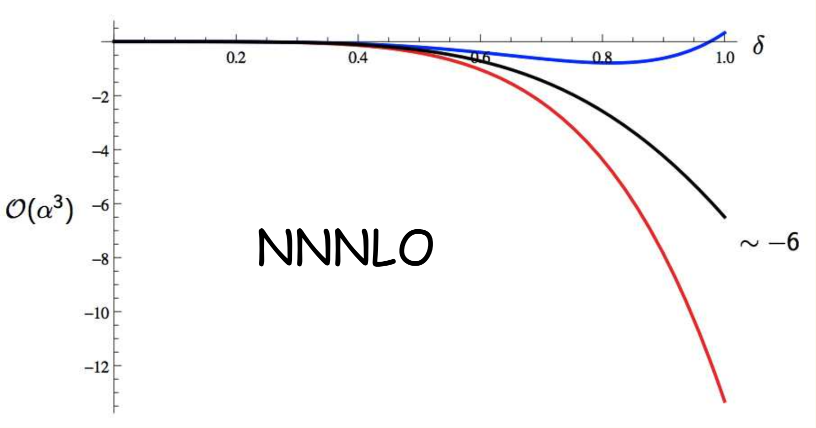
Estimate of the three-loop coefficient

The NNNLO axial formfactor is known in the zero-recoil limit.

It determines the first two orders in the mass expansion
(real radiation more strongly suppressed).

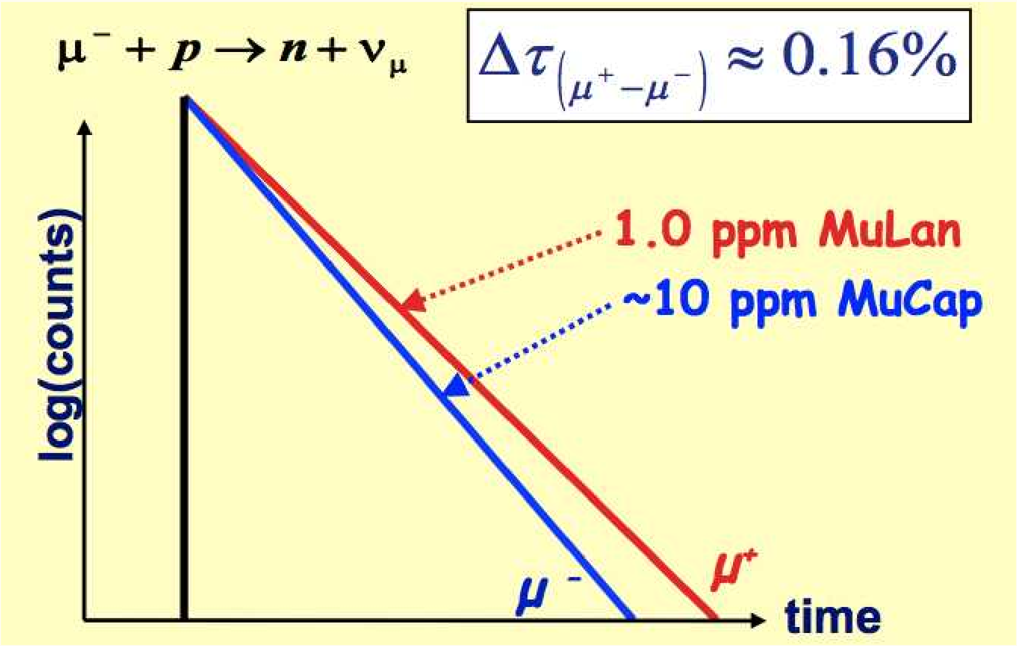






Another application of the muon lifetime

Muons and antimuons behave differently in matter.
Negatively-charged muons form muonic atoms and can be captured by nuclei.



V.Tishchenko

The rate of capture, found from the lifetime difference, determines nucleon formfactors.

Lepton-flavor violating processes and the muon decay in orbit

Muon g-2: ~3.6σ discrepancy

Encouragement for lepton flavor violation searches:

$$a_\mu^{\mathrm{NP}} \frac{e}{2m} \bar{\mu} \sigma \cdot F \mu \;\rightarrow\; \frac{e}{2m} \bar{e} \left(f_M + f_E \gamma_5 \right) \sigma \cdot F \mu$$

$$f_{M,E} \sim a_\mu^{\mathrm{NP}} \cdot \delta$$

$$BR(\mu \rightarrow e\gamma) \sim 10^{-3} \delta^2$$

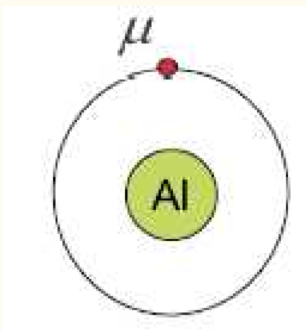
New bound (MEG @ Paul Scherrer Institute) $\mathrm{BR}\,(\mu \to e\gamma) < 5.7 \cdot 10^{-13}$ (2013)

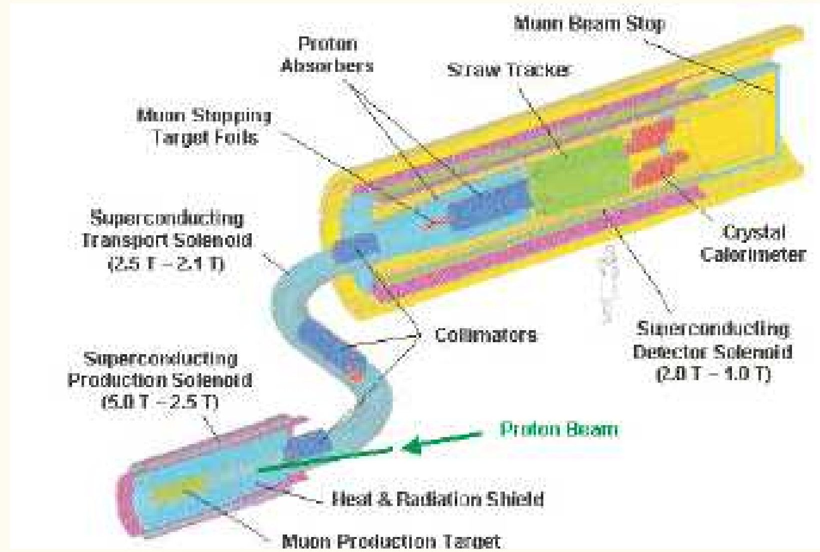
probes $\delta \lesssim 10^{-5}$

(Also lots of theoretical encouragement from "new physics" models.)

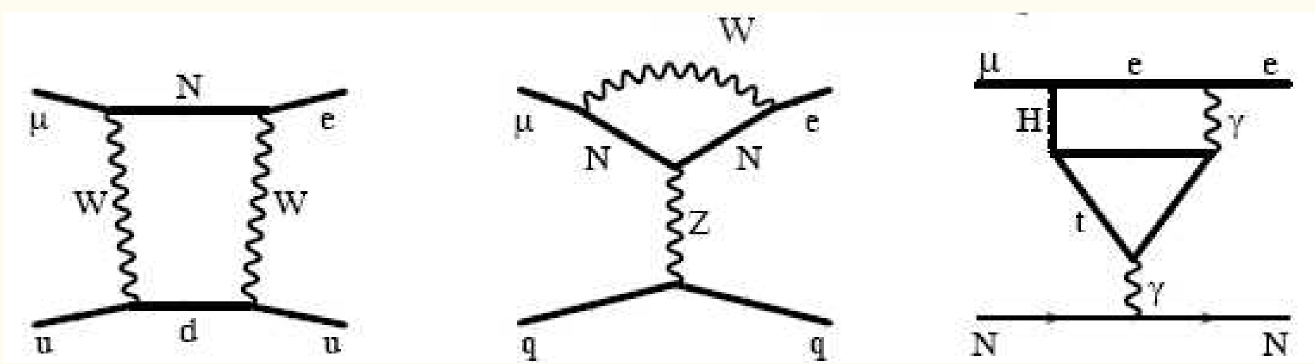
Muon-electron conversion

"The best rare process"
No accidental bkgd
(single monochromatic e^-);
10^{-17} sensitivity envisioned





Variety of mechanisms:

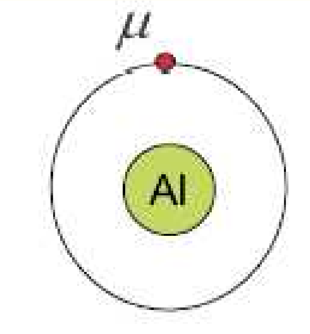


Comparison with scattering experiments

Highest luminosity in fixed-target experiments

$$\sim 10^{37\ldots38} / \left(\mathrm{cm}^2 \cdot \mathrm{s}\right)$$

In a single muonic atom



$$= \text{density} \times \text{velocity}$$

$$= \left|\psi(0)\right|^2 \cdot Z\alpha = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \sim Z^4 \cdot 4 \cdot 10^{39} / \left(\mathrm{cm}^2 \cdot \mathrm{s}\right)$$

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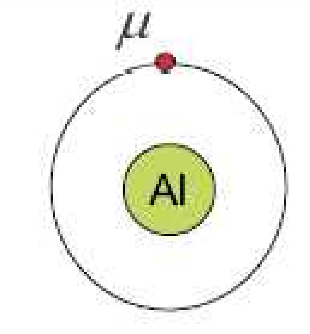

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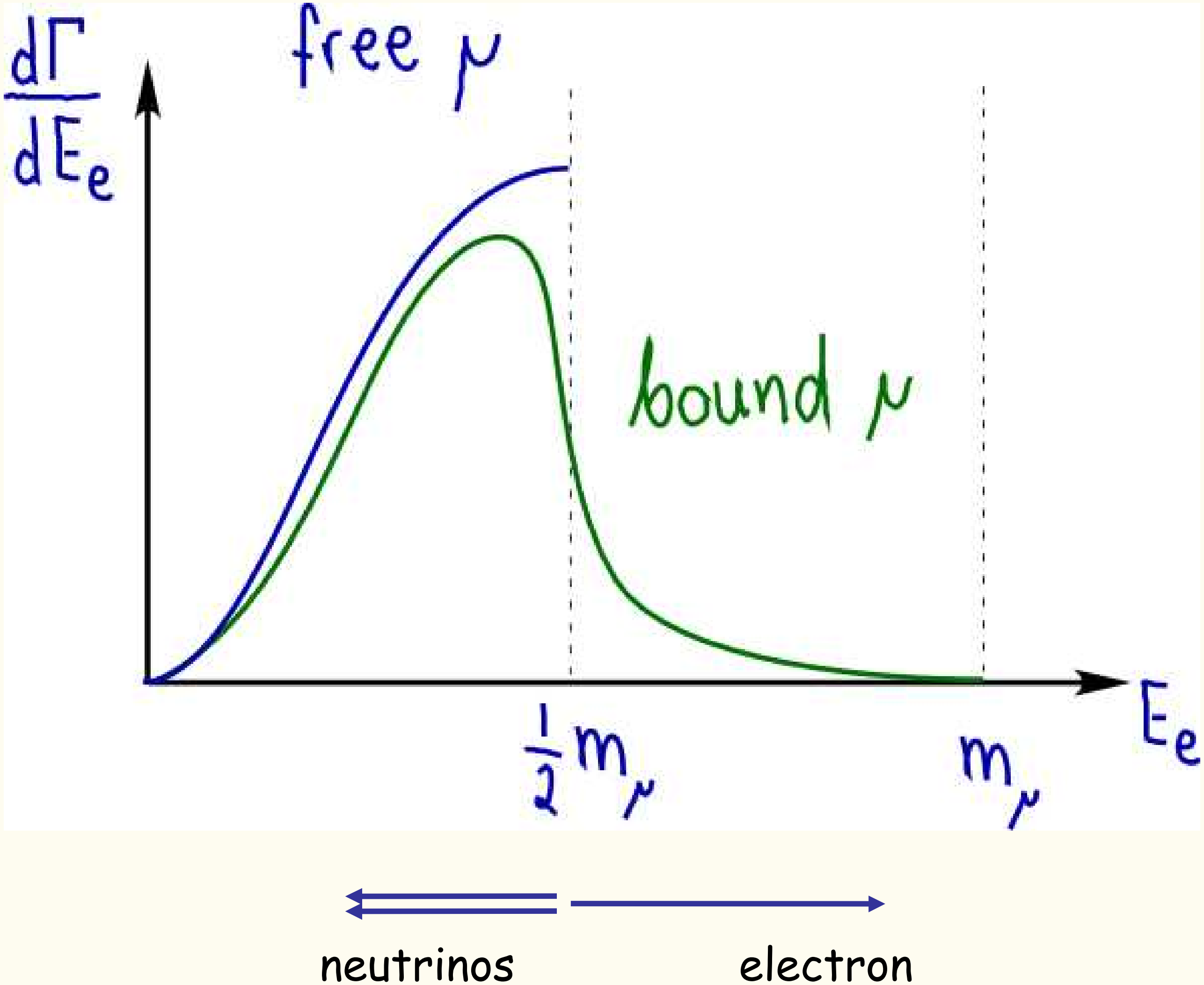
Many atoms are studied in parallel: ~10^{11} muons stopped per second, each lives about 10^{-6} seconds: 10^5 atoms present:

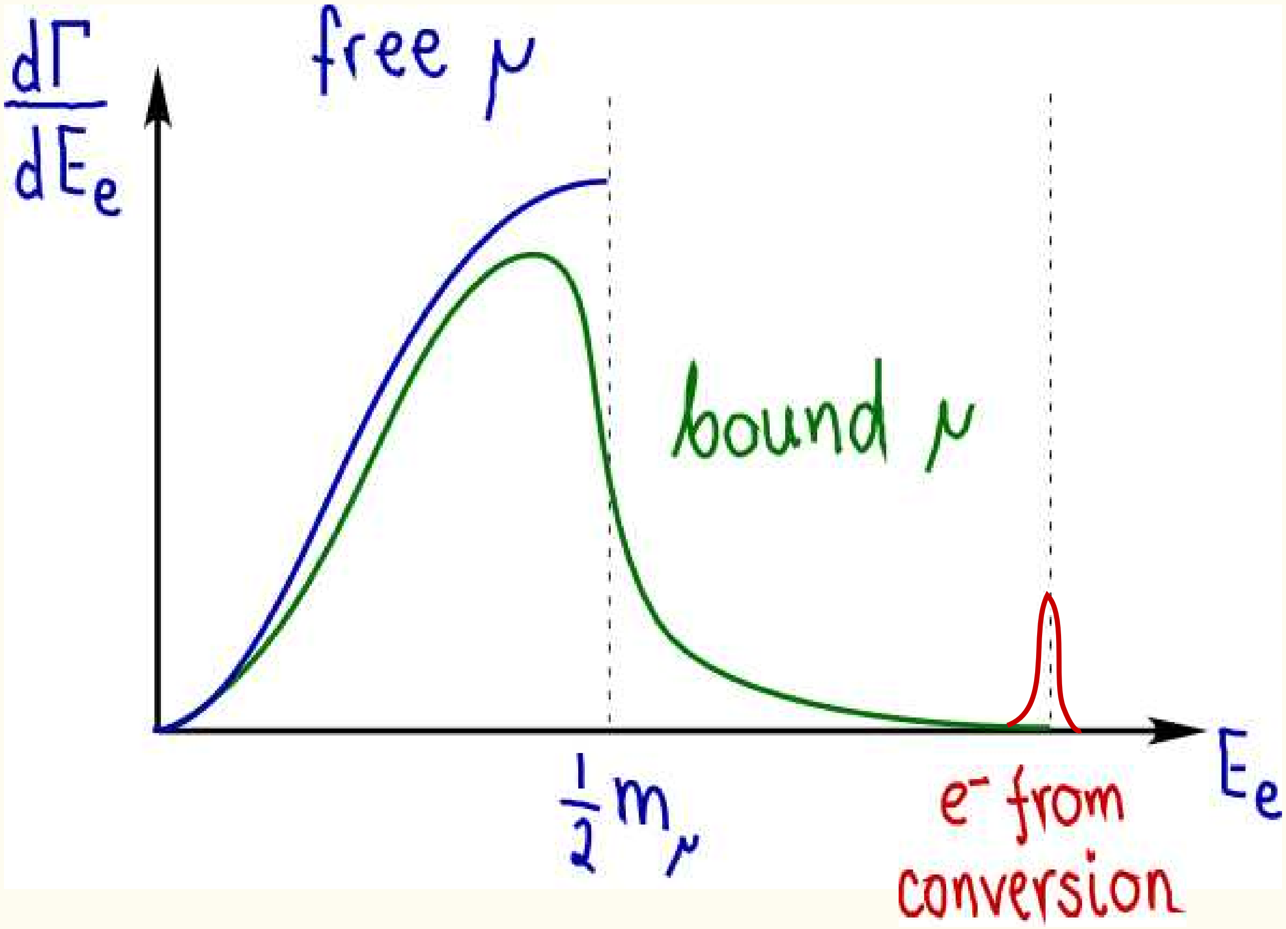
$$\sim 10^{49} / \left(\mathrm{cm}^2 \cdot \mathrm{s}\right)$$

Muon decay in orbit (DIO)

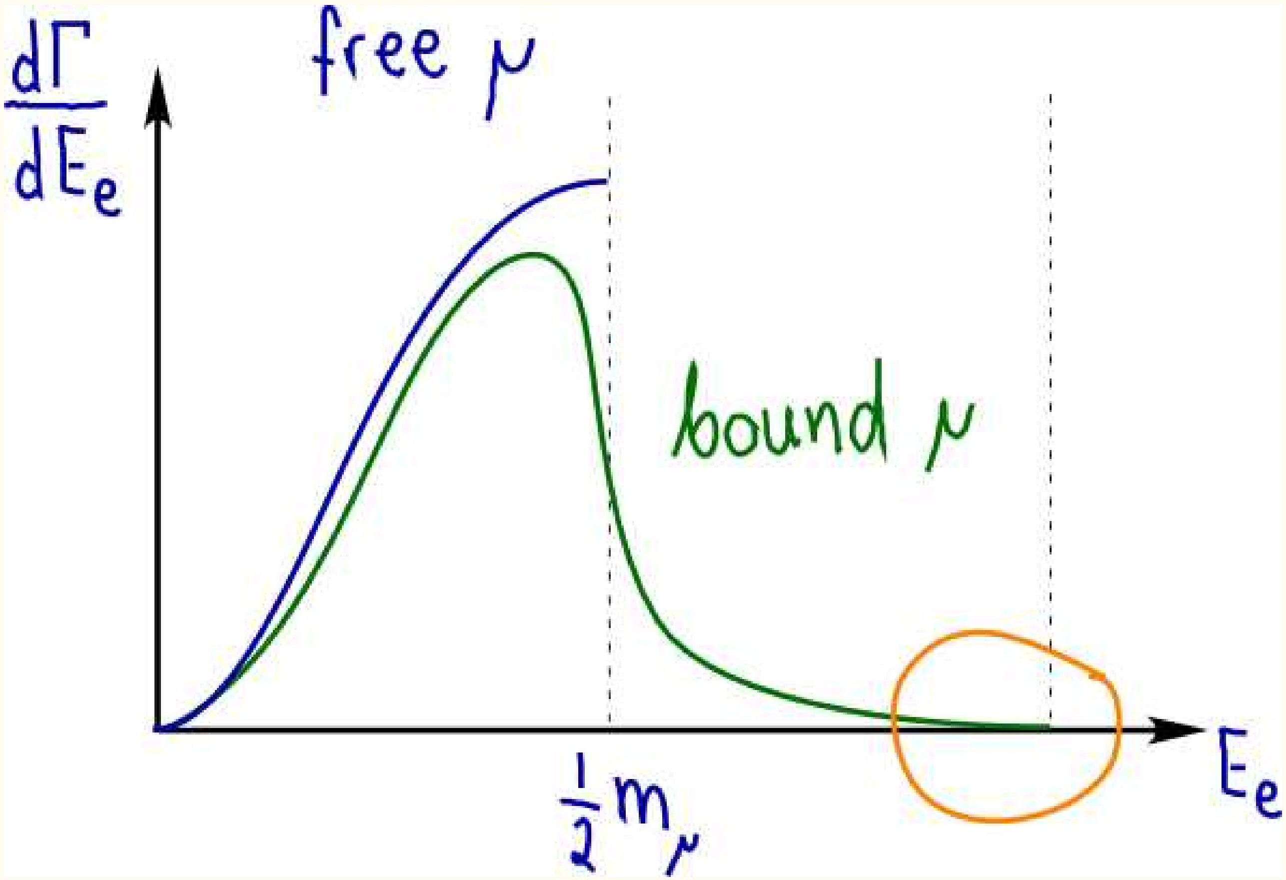
Background from the standard muon decay



Background from the standard muon decay



End point spectrum must be well understood



$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 (E_{max} - E)^5$$

End point spectrum

Previous studies: Shanker & Roy, Hänggi et al., Herzog & Alder

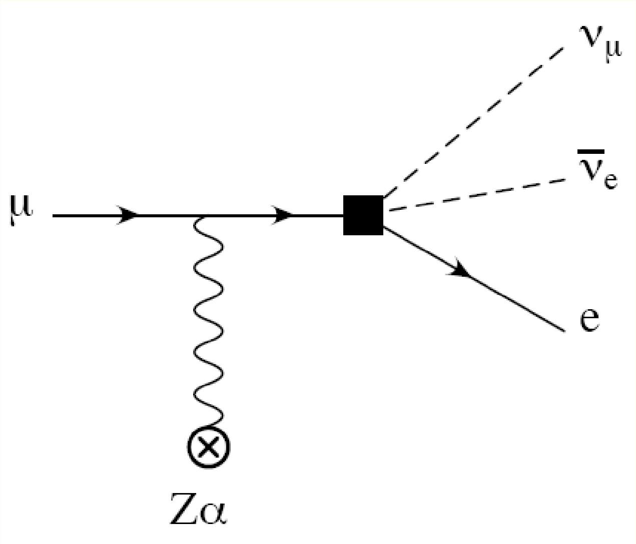
Relativistic muon wave function, nuclear size and recoil, electron final state interactions: all taken into account.

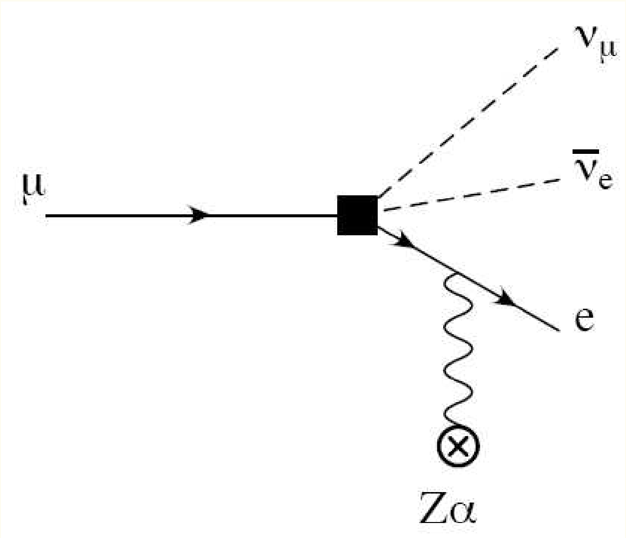
$$N(E_e)dE_e \simeq 0.4 \cdot 10^{-21} \left(1 - \frac{E_e}{E_{\max}}\right)^5 dE_e$$

New evaluation: AC, X. Garcia i Tormo, W. J. Marciano PRD84,013006,2011

Planned energy resolution in Mu2e: ~250 keV → 0.22 background events.

How can the electron get muon's whole energy?





Neutrinos get no energy;
The nucleus balances electron's momentum, takes no energy.
Near the end point:

$$
\begin{aligned}
\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e} &\sim |\psi(0)|^2 (Z\alpha)^2 \frac{\mathrm{d}^3\nu_e}{\nu_e}\frac{\mathrm{d}^3\nu_\mu}{\nu_\mu}\delta\left(E_{\mathrm{max}} - E_e - \nu_e - \nu_\mu\right) \mathrm{Tr} \ldots \not{\nu}_e \ldots \not{\nu}_\mu \\
&\sim (Z\alpha)^5 \left(E_{\mathrm{max}} - E_e\right)^5
\end{aligned}
$$

μ-e conversion may be caused by a majoron

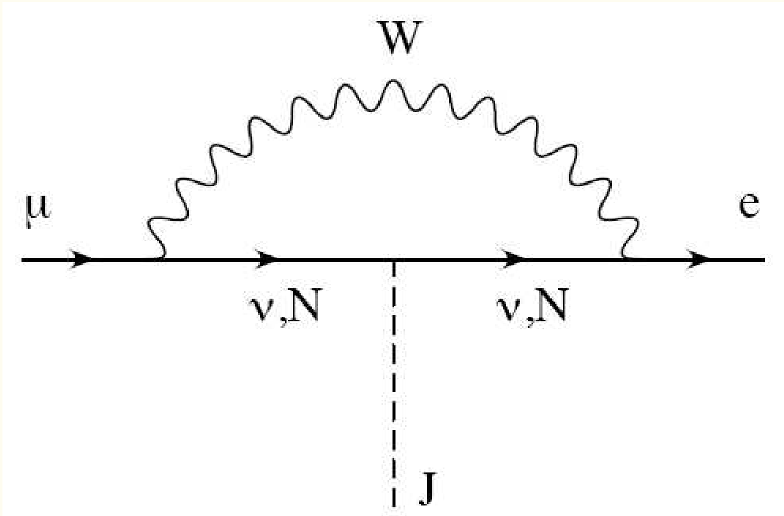
What is the majoron?

If neutrinos have Majorana masses: lepton number is not conserved.

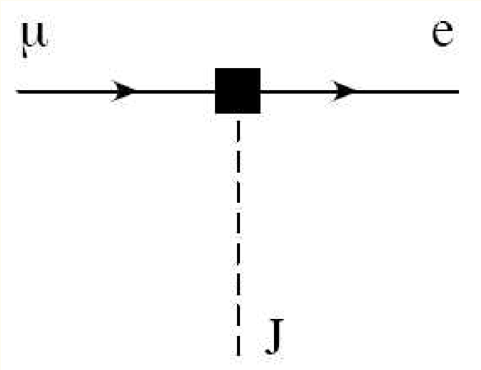
How can lepton conservation be broken?

* explicitly by the Majorana mass term;
* spontaneously, locally; or
* spontaneously, globally → Goldstone boson.

Majoron can violate lepton flavor number



$$\bar{e}\,(g_L L + g_R R)\,\mu$$



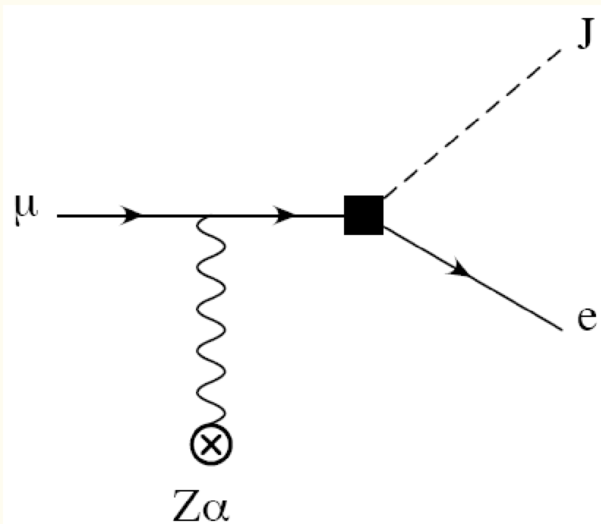
The resulting extra muon decay rate:

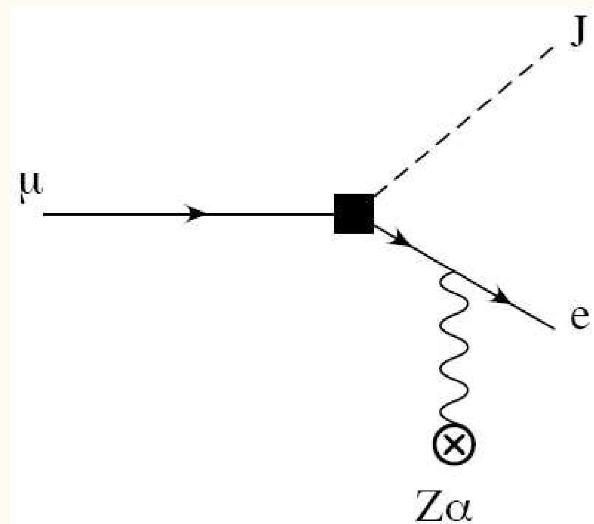
$$\Gamma\,(\mu \rightarrow eJ) = \frac{m_\mu}{32\pi}\left(g_L^2 + g_R^2\right)$$

What is the electron spectrum in $\mu \to e + J$?

Free muon: monoenergetic electron, $E_e = m_\mu/2$

Muon bound in an atom: spread out up to m_μ





$$\frac{d\Gamma}{dE_e} \sim |\psi(0)|^2 (Z\alpha)^2 \frac{d^3 J}{J} \delta\left(E_{\max} - E_e - J\right) |\mathcal{M}|^2$$

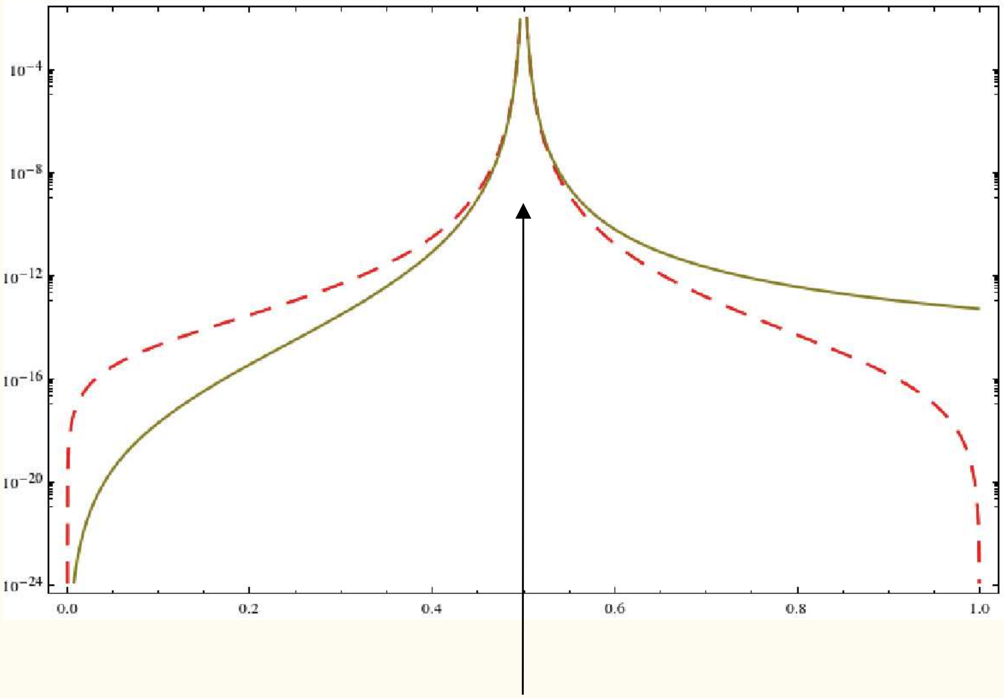
$$\sim (Z\alpha)^5 \left(E_{\max} - E_e\right)^3$$

Vanishes at end point

With D. Bryman, M. Dowling, X. Garcia i Tormo

Results: electron spectrum in $\mu \rightarrow e + J$

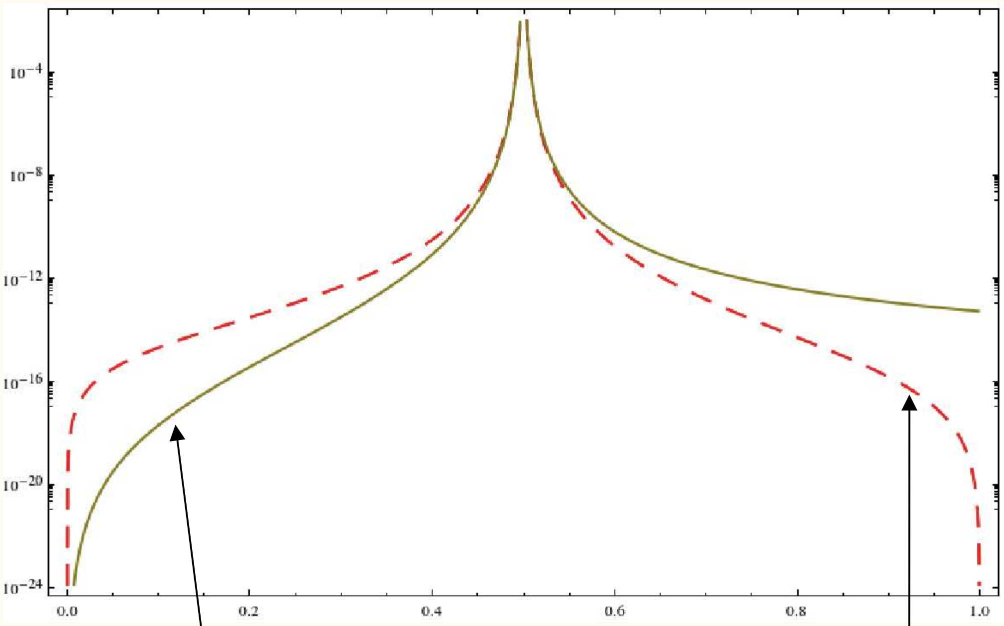
Robert Szafron

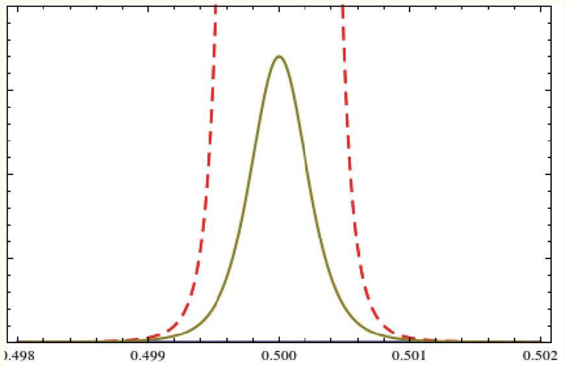


without binding effects,
the electron spectrum is
monochromatic,
concentrated here
at half muon mass

Results: electron spectrum in $\mu \rightarrow e + J$

Robert Szafron





smearing due to
muon's motion.
Dominates in the center.

expansion
in Z*alpha
Correct far
from the center

Summary

We have determined spectra of daughter electrons in decays of bound muons.

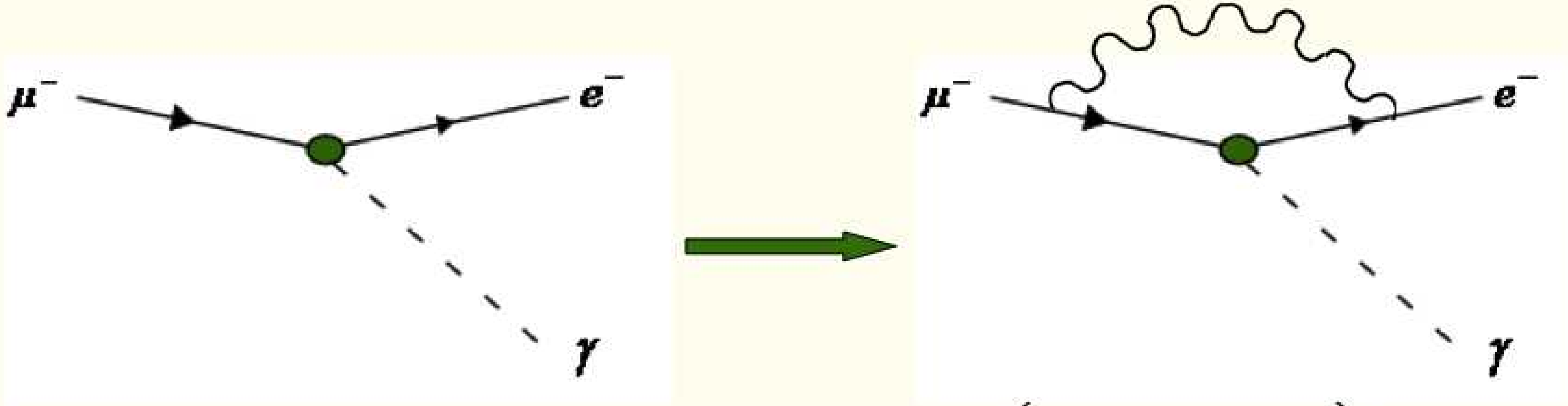
Simple interpretation of the high-energy tail: hard photon exchange with the nucleus.

The signal of possible decays into majorons is enhanced by two powers of (E_{max} - E_e) but not by four powers.

Ongoing work:

better understanding of the muon decay in orbit: an effective theory approach to various regions of the spectrum (with R. Szafron).

QED suppression of the decay $\mu \to e\gamma$



$$\sigma_{\alpha\beta} q^{\beta} \left(E - M\gamma_5 \right) A^{\alpha}$$

$$\times \left(1 - \frac{4\alpha}{\pi} \ln \frac{\Lambda}{m_\mu} \right)$$

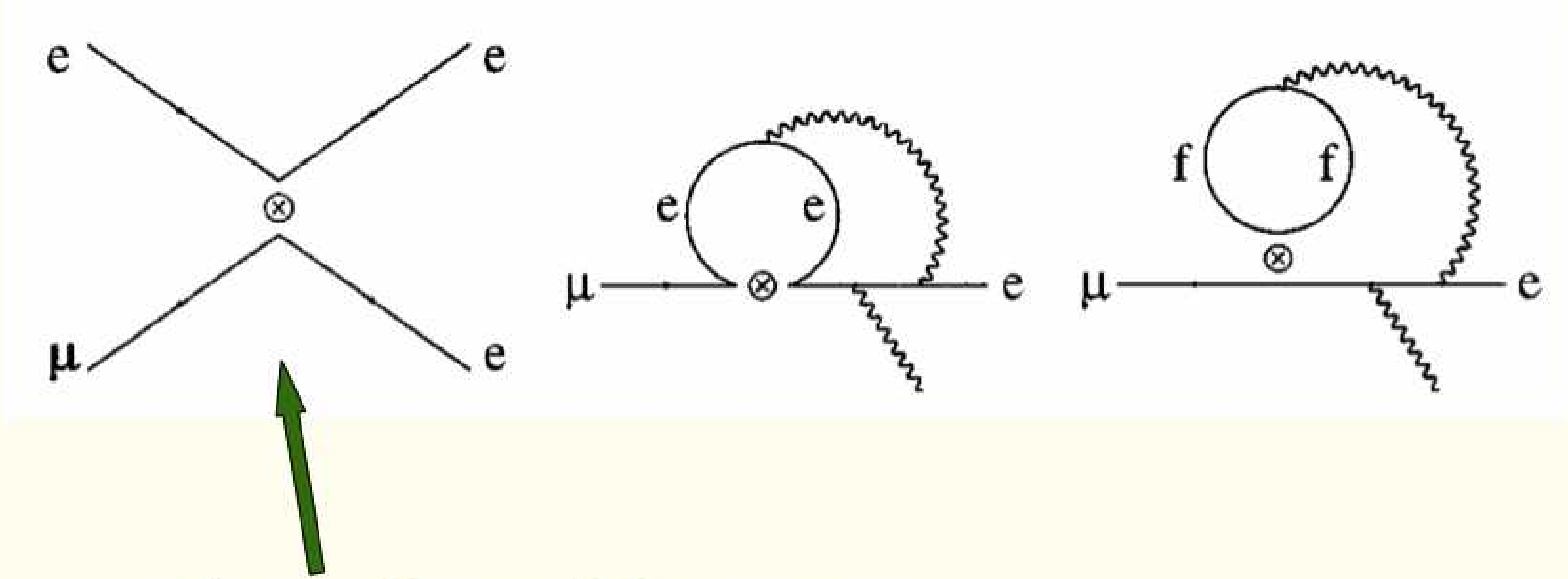
AC and Jankowski,
PRD 65, 113004 (2002)

This is the largest known QED correction to a decay rate;
15 percent for $\Lambda \sim 250$ GeV. In general, $\sim 2\ln(\Lambda/m_\mu)$ percent.

For comparison, correction for the normal muon decay is 0.4 percent.

The rate suppression $\times\left(1-\frac{8\alpha}{\pi}\ln\frac{\Lambda}{m_\mu}\right)$ is universal (independent of the mechanism of LFV)

This is because the non-dipole operators (four-fermion) which would have a different scaling, contribute little:



Wilson coefficient of this operator
is constrained by direct searches (SINDRUM),

$$\frac{\Gamma(\mu\rightarrow eee)}{\Gamma(\mu\rightarrow e\nu\nu)}<10^{-12}$$

Lepton-flavor violating processes and the muon decay in orbit

Why such a large correction?

Difference between the normal muon decay and the eγ channel

$$\bar{\mu}\gamma^{\mu}Le \cdot W_{\mu}$$

dimension=4, renormalizable

$$\bar{\mu}\sigma^{\mu\nu}e \cdot F_{\mu\nu}$$

dimension=5, non-renormalizable:
large logs in the photon loops

Other processes, $\mu \to eee$ and the conversion,

have a mixture of both structures. Large logs can be present.