

GREEN'S FORMULAS FOR LINEARIZED PROBLEMS WITH LIVE LOADS

BY

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IMA Preprint Series # 121

December 1984

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

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GREEN'S FORMULAS FOR LINEARIZED PROBLEMS WITH LIVE LOADS

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Let \mathcal{B} be an elastic body, that is, a pair (Ω, T) where $\Omega \subset \mathbb{R}^3$, the reference placement, is a bounded set with smooth or piecewise smooth boundary $\partial\Omega$ and exterior normal n , and T is a response function:

$$T(x, F) : \Omega \times \text{Lin}^+ \rightarrow \text{Lin}$$

by Lin (respectively, Lin^+) we denote, as usual, the space of second-order tensors (with positive determinant) .

We may specify the choice of T in order to satisfy the axioms of frame indifference and balance of angular momentum and, possibly, to reflect certain material symmetries, but we don't need to consider such issues.

If we denote by (b^f, s^f) the loading pair, where b^f is the body force in $x \in \Omega$ and s^f is the surface traction in $x \in \partial\Omega$ exerted by the environment on \mathcal{B} in the deformation f , the traction problem in finite elastostatics can be stated as follows:

Find a smooth, orientation preserving ($\det \nabla f > 0$) diffeomorphism $f: \bar{\Omega} \longleftrightarrow f(\bar{\Omega})$, such that

$$-\partial_j T_{ij}(x, \nabla f) - b_i^f = 0 \quad \text{in } \Omega$$

$$T_{ij}(x, \nabla f) n_j - s_i^f = 0 \quad \text{in } \partial\Omega .$$

If the loads b^f and s^f don't depend on the deformation f , they are said to be dead and we have the classical traction problem; otherwise, the loading is live.

An interesting class of live loadings has been considered by Spector [6, 7]. These are the simple loadings, defined by constitutive equations of the form:

$$b^f(x) = b(x, f(x), \nabla f(x)) \quad x \in \Omega$$

$$s^f(x) = s(x, f(x), D_t f(x)) \quad x \in \partial\Omega ,$$

where D_t denotes the tangential gradient operator. For example, a hydrostatic environment, practically the only well understood example of live loading, is a simple loading in the sense of Spector.

We will consider more general live loadings, replacing the condition on surface traction by the condition

$$s^f(x) = s(x, f(x), \nabla f(x)) \quad x \in \partial\Omega ,$$

that is, allowing the surface traction to depend on the whole gradient and not only on the tangential derivatives [3].

With this choice of loading, the traction problem linearized about a reference deformation f_0 with gradient F_0 is the following boundary-value problem:

Find a vector field $u(x)$ such that

$$L[u]_i = -(S_{ijhk} u_{h,k})_{,j} - b_{ihk} u_{h,k} - B_{ih} u_h = g_i \quad \text{in } \Omega$$

$$M[u]_i = S_{ijhk} n_j u_{h,k} - s_{ihk} u_{h,k} - S_{ih} u_h = d_i \quad \text{on } \partial\Omega$$

where the comma indicates differentiation with respect to the corresponding space variable,

$$S_{ijhk} = \frac{\partial T_{ij}(x, F_0)}{\partial F_{hk}} \quad \text{is the linearized elasticity tensor}$$

$$s_{ihk} = \frac{\partial s_i(x, f_0, F_0)}{\partial F_{hk}} \quad \text{is the linearized environment tensor,}$$

and

$$b_{ihk} = \frac{\partial b_i}{\partial F_{hk}} , \quad B_{ih} = \frac{\partial b_i}{\partial f_h} , \quad S_{ih} = \frac{\partial s_i}{\partial f_h} .$$

The study of the linearized problem may be a useful tool to attempt to solve the nonlinear problem, for example, by the inverse function theorem and the implicit function theorem in suitable Banach spaces. This has been done by Valent [8] for the nonlinear displacement problem, following an idea of Gurtin and Spector.

The linearized problem is also useful when we try to study, in finite elasticity, the stability of a solution by Signorini's perturbation method (see, for example, [1]).

Moreover, the linearized problem coincides formally with the corresponding problem in the theory of small deformations superimposed on large. In this context, the vector $b_{ihk} u_{h,k} + B_{ih} u_h$ specifies the body forces exerted by the environment in the small deformation u , $s_{ihk} u_{h,k} + S_{ih} u_h$ is the surface traction exerted by the environment at $x \in \partial\Omega$, and g, d are small incremental loadings, independent of u .

We do not attempt to justify the linearization of the traction problem; in fact, Stoppelli has shown that the solutions of the linearized theory do not always approximate the solutions of the nonlinear problem even when the applied traction is small.

Now, for the pair of operators (L, M) we look for a Green's formula generalizing the Betti's reciprocity theorem. Finding a Green's formula is a crucial point in order to apply the theory of existence of linear boundary value problems for elliptic operators. More precisely, if we denote by L^* the operator formally adjoint to L , we wish to find an operator M^* , depending on L, M , such that

$$(G) \quad \int_{\Omega} L[u] \cdot v \, dx + \int_{\partial\Omega} M[u] \cdot v \, d\sigma = \int_{\Omega} L^*[v] \cdot u \, dx + \int_{\partial\Omega} M^*[v] \cdot u \, d\sigma$$

for every $u, v \in C^2(\bar{\Omega})$.

Capriz and Podio Guidugli [2] demonstrate this Green's formula in the particular case of Spector's simple loadings, supposing $\partial\Omega$ smooth. Their method of proof is inspired by that used by Fiehera to deal with the problem of regular oblique derivatives for a scalar equation. They construct a bilinear form

$$a(u, v) = \int_{\Omega} (\alpha_{ijhk} u_{h,k} v_{i,j} + \beta_{ihk} u_{h,k} v_i + \gamma_{ih} u_h v_i) dx - \int_{\partial\Omega} S_{ih} u_h v_i d\sigma$$

such that

$$a(u, v) = \int_{\Omega} L[u] \cdot v dx + \int_{\partial\Omega} M[u] \cdot v dx \quad \text{for every } u, v \in \mathcal{C}^2(\bar{\Omega}),$$

and then, by an integration by parts, they obtain (G).

In order to construct the bilinear form, they make use of smooth extensions in Ω of both the vector field n and the tensor field s defined on $\partial\Omega$, and hence they need the regularity of the boundary (for example, $\partial\Omega \in \mathcal{C}^3$). Moreover, they show that a Green's formula like (G) can be obtained if the linearized environment tensor is tangential, that is, $s_{ihk} n_k = 0$. To see that the last condition is also necessary, we begin by setting

$$t_{ihk} = S_{ijhk} n_j - s_{ihk}, \quad E_{ih}^{(n)} = S_{ijhk} n_j n_k$$

$$A_{ih}^{(n)} = t_{ihk} n_k = E_{ih}^{(n)} - s_{ihk} n_k.$$

Then we consider the classical Green-Betti formula

$$\begin{aligned} \text{(G.B.)} \quad \int_{\Omega} L[u] \cdot v dx + \int_{\partial\Omega} S_{ijhk} n_j u_{h,k} v_i d\sigma &= \\ &= \int_{\Omega} L^*[v] u dx + \int_{\partial\Omega} (S_{ijhk} n_k v_{i,j} - b_{ihk} n_k v_i) u_h d\sigma. \end{aligned}$$

Since we can split the gradient of a function u into its normal and tangential parts and since the value on $\partial\Omega$ of a function u and its normal derivatives are independent, we can choose in formulas (G) and (G.B.), $u = 0$ and $\partial u / \partial n$ arbitrary on $\partial\Omega$. Subtracting the formulas so obtained from each other, we get the condition

$$A(n) = E(n) ,$$

that is,

$$s_{ihk} n_k = 0 .$$

It follows that in order to deal with nontangential loadings, one must first generalize the Green's formula (G), for example, in this way:

Find a matrix V and an operator M^* , depending on L and M such that

$$(G) \quad \int_{\Omega} L[u] \cdot v \, dx + \int_{\partial\Omega} M[u] \cdot Vv \, d\sigma = \int_{\Omega} L^*[v] \cdot u \, dx + \int_{\partial\Omega} M^*[v] \cdot u \, d\sigma .$$

The previous argument immediately gives us, for the matrix V , the condition

$$V^T A(n) = E(n) .$$

Hence a sufficient condition in order to find V is $\det A(n) \neq 0$: we may call this the normality condition [5].

If n is not characteristic for L on $\partial\Omega$, that is, if $\det E(n) \neq 0$, then the normality condition becomes a necessary condition also and, moreover, $\det V \neq 0$. This situation occurs when L is elliptic up to the boundary.

To construct M^* under the normality condition, we introduce the tensor

$$c_{ihk} = S_{ijhk} n_j - V_{pi} t_{phk} .$$

We call this tensor the tangential correction, since $c_{ihk} n_k = 0$ by definition of V . When the loading is simple,

$$c_{ihk} = s_{ihk} .$$

Looking for M^* , we multiply the tangential correction by $u_{h,k} v_k$ and substitute in (G. B.) to get

$$\begin{aligned}
& \int_{\Omega} L[u] \cdot v \, dx + \int_{\partial\Omega} M[u] \cdot Vv \, d\sigma + \int_{\partial\Omega} c_{ihk} u_{h,k} v_i \, d\sigma = \\
& = \int_{\Omega} L^*[v] \cdot u \, dx + \int_{\partial\Omega} (S_{ijhk} n_k v_{ij} - b_{ihk} n_k v_i - S_{jh} V_{ji}) u_h \, d\sigma .
\end{aligned}$$

The integral $\int_{\partial\Omega} c_{ihk} u_{h,k} v_i \, d\sigma$ can be evaluated by Stokes' formula in the case of $\partial\Omega$ a single regular surface or else, for example, in the case $\partial\Omega = \Sigma_1 \cup \Sigma_2$ with $\partial\Sigma_1 = \partial\Sigma_2 = \Gamma$ and $\Sigma_1, \Sigma_2, \Gamma$ regular.

Indeed, set $w_k = c_{ihk} v_i u_h$ and consider $z_\ell = N_{\ell k} w_k$, where N is the skew tensor associated to n and $P - N^2 = I$, with $P = n \otimes n$. For any smooth extension of $z = Nw$ to a neighborhood of Σ_1 , since $w \cdot n = 0$ and $n \cdot n = 1$ on Σ_1 , we have

$$n \cdot \text{rot} z = N_{\ell s} z_{\ell, s} = N_{\ell s} N_{\ell k, s} w_k + N_{\ell s} N_{\ell k} w_{k, s} = -N_{ks}^2 w_{k, s} \quad \text{on } \Sigma_1 .$$

Then by applying the Stokes formula, we obtain

$$\int_{\Sigma_1} -N_{sk}^2 (c_{ihs} u_{h,i} v_k) \, d\sigma = \int_{\Gamma} c_{ihk} u_{h,i} v_k N_{\ell k} t_\ell^{(1)} \, ds$$

where $t^{(1)}$ is the tangential vector to Γ oriented coherently with the normal n on Σ_1 . The same argument on Σ_2 leads to an analogous formula with $t^{(2)} = -t^{(1)}$. If the surfaces Σ_1 and Σ_2 have the same normal on Γ , by adding these two formulas the line integral vanishes and, since $c_{ihk} = -c_{ihs} N_{sk}^2$, we get:

$$-\int_{\partial\Omega} c_{ihk} u_{h,k} v_i \, d\sigma = \int_{\partial\Omega} (c_{ihk} v_{i,k} - N_{ks}^2 c_{ihk, s} v_i) u_h \, d\sigma .$$

From this, we can easily derive that:

$$M^* [v]_h = (S_{ijhk} n_k + c_{ihj}) v_{i,j} - (b_{ihk} n_k + S_{\ell h} V_{\ell i} + N_{ks}^2 c_{ihk, s}) v_i ,$$

hence the Green's formula is constructed in the case Ω regular.

If n is discontinuous across Γ , that is, if on Γ the normal $n^{(1)}$ of Σ_1 is different from the normal $n^{(2)}$ of Σ_2 , the line integral

$$\oint_{\Gamma} u_h v_i \left[c_{ihk} N_{kl} \right]_2^1 t_{\ell}^{(1)} ds ,$$

does not vanish, and we don't have a Green's formula.

However, consider for simplicity the case of a hydrostatic environment in which $s_{ihk} = c_{ihk} = -\pi(n_i \delta_{hk} - n_h \delta_{ik})$; in this case the environment tensor depends on the normal and the line integral disappears. More generally, for any tangential correction such that $c_{ihk} = c_{ihk}(x, n)$, the condition

$$c_{ihk}(x, n^{(1)}) n_k^{(2)} + c_{ihk}(x, n^{(2)}) n_k^{(1)} = 0 \quad \text{on } \Gamma$$

guarantees the vanishing of the line integral.

In summary, if $\partial\Omega$ is smooth, or if $\partial\Omega$ is piecewise smooth but the environment tensor satisfies suitable conditions, we can obtain the Green's formula (G) under the hypothesis that the boundary operator M is normal.

From the Green's formula we can easily derive a necessary condition for the existence of solutions of the linearized traction problem. In fact, if we consider (L^*, M^*) as the adjoint of (L, M) , we can state the following compatibility condition for the data (g, d) :

$$\int_{\Omega} g \cdot v \, dx + \int_{\partial\Omega} d \cdot Vv \, d\sigma = 0$$

for every solution v of the homogeneous adjoint problem

$$\begin{aligned} L^*[v] &= 0 && \text{in } \Omega \\ M^*[v] &= 0 && \text{on } \partial\Omega . \end{aligned}$$

This condition is analogous to the compatibility condition well known in linear elasticity except for the presence of the matrix V .

Moreover, if we suppose L elliptic in the sense of Petrowskii, M normal, L and M satisfying the complementing condition, $\partial\Omega$ and the coefficients smooth, then, by the usual techniques for the elliptic boundary value problems, we can prove the existence and the regularity of the solution under the compatibility condition for the data.

To conclude, we wish now to give an example of the ambiguities that may accompany the failure of the normality condition [4]. Let S_{ijkl} be the classical elasticity tensor with L the associated differential operator, and let $P = n \otimes n$. If we choose $s_{ihk} = P_{il} S_{ljhk} n_j$ and $S_{ih} = -P_{ih}$ we obtain a boundary operator which is not normal. Precisely, if we denote by $t_i[u] = S_{ijkl} n_j u_{h,k}$ the traction vector on $\partial\Omega$, the boundary operator becomes

$$M[u] \equiv (1 - P)t[u] + Pu$$

and the resulting boundary condition can be equally well classified as a live-boundary condition of traction or as a dead-boundary condition of contact.

In addition, a Green's formula for this problem follows directly from the Betti's formula

$$(B) \quad \int_{\Omega} L[u] \cdot v \, dx + \int_{\partial\Omega} t[u] \cdot v \, d\sigma = \int_{\Omega} L^*[v] \cdot u \, dx + \int_{\partial\Omega} t^*[v] \cdot u \, d\sigma$$

using the identity

$$\begin{aligned} t[u] \cdot v - t^*[v] \cdot u &= ((1 - P)t[u] + Pu) \cdot (-Pt^*[v] + (1 - P)v) - \\ &\quad - ((1 - P)t^*[v] - Pv) \cdot (Pt[u] + (1 - P)u) . \end{aligned}$$

In fact, if we set

$$\begin{aligned}
M[u] &= (1-P)t[u] + Pu \quad , & M^*[v] &= (1-P)t^*[v] - Pv \quad , \\
V[v] &= -Pt^*[v] + (1-P)v \quad , & V^*[u] &= Pt[u] + (1-P)u \quad ,
\end{aligned}$$

we can rewrite (B) as

$$\int_{\Omega} L[u] \cdot v \, dx + \int_{\partial\Omega} M[u] \cdot V[v] \, d\sigma = \int_{\Omega} L^*[v] \cdot u \, dx + \int_{\partial\Omega} M^*[v] \cdot V^*[u] \, d\sigma \quad .$$

But this Green's formula differs from the one we have obtained under the normality assumption, since V is a differential operator of order 1 and not a matrix.

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