CONSTRAINED LOCALIZATION IN SENSOR NETWORKS

By

Mona Mahmoudi

and

Guillermo Sapiro

IMA Preprint Series # 2072

(October 2005)

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

UNIVERSITY OF MINNESOTA

400 Lind Hall

207 Church Street S.E.

Minneapolis, Minnesota 55455–0436

Phone: 612/624-6066 Fax: 612/626-7370

URL: http://www.ima.umn.edu

Constrained Localization in Sensor Networks

Mona Mahmoudi and Guillermo Sapiro

Abstract— **In this note we show how to introduce physical constraints in the localization problem in sensor networks. This is based on extending the classical STRESS function from distance geometry and multidimensional scaling. We present the underlying framework and demonstrate it with three examples: penalizing the sensors for being in high elevation areas, removing sensors from forbidden areas, and forcing the sensors to be on pre-described curves.**

Index Terms - **Sensor localization, sensor networks, STRESS function, multidimensional scaling, distance geometry, physical constraints.**

I. Introduction

Automatic sensor localization is one of the most fundamental problems in the area of sensor networks. Sensor data needs to be registered to its physical location to be of use in the major applications of sensor networks. For large scale and inexpensive networks, it is not possible to include GPS capability on every device. Therefore, automatic sensor localization based on pairwise (local) information has received a lot of attention in recent years. The basic idea is to use information such as signal strength, time-of-arrival, or angle-of-arrival, between a sensor and some of its local neighbors (often denoted as pairwise *dissimilarities*), to compute the physical coordinates of the sensors. See for example [3] for some literature on the subject and details on the basic requirements of sensor localization algorithms.

Computing point (sensor) coordinates from pairwise dissimilarities is a classical problem in distance geometry [2]. Such problems arise for example in molecular biology, where protein structures are to be determined from a few noisy measurements of pairwise distances obtained from X-ray crystallography or NMR. The same task is the fundamental problem in multidimensional scaling [1], [4], where the primary goal is to represent and visualize in low dimensional Euclidean space a set of pairwise dissimilarities obtained, for example, from psychophysical experiments. The sensor localization problem is nothing else than another application/extension of these theories.

Often, in addition to the pairwise dissimilarities, some prior or learned information about the physical environment is also available. For example, if we are localizing active cellular phones in the heart of the winter in Minnesota, it is very unlikely that they will be located in the middle of a lake. Similarly, it is unlikely that sensors for ocean waves activity are located far inland. The same physical prior knowledge is valid for chemical and control sensors for example. This physical information is very common and imposes an additional constraint in the sensor localization problem. Although multidimensional scaling has studied the constrained scenario [1], [4], this important problem has not been part of the sensor localization techniques developed in the literature. It is the goal of this paper to present a simple framework for sensor localization with physical constraints.

Next, Section II, describes the framework for sensor localization with constraints. Experimental results for particular physical scenarios are presented in Section III, while concluding remarks are given in Section IV.

We thank Alejandro Ribeiro for comments on the work here described. The work here presented is partially supported by the Office of Naval Research, the National Science Foundation, DARPA, and the National Geospatial-Intelligence Agency.

Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455, {mahmo022,guille}@umn.edu

II. Constrained sensor localization

We now introduce the proposed framework for constrained node localization in sensor networks. Consider a network with N nodes in a D dimensional space (usually $D = 2, 3$). Let $x_i \in \Re^D$, $i = 1..N$, be the coordinates for each one of the sensors. We assume that we measure pairwise dissimilarities δ_{ij} between sensors i and j at positions x_i and x_j as Euclidean distances:[1]

$$\delta_{ij} = \| x_i - x_j \| = \sqrt{(x_i - x_j)^T (x_i - x_j)}.$$

These pairwise distances can be obtained for example via received signal strength or time of arrival, and are often noisy. Also, not all the pairwise distances need to be available.

We propose to find the constrained sensor positions from the available set δ_{ij} via the minimization of the following global cost function:

$$S = \sum_{ij} w_{ij} (\delta_{ij} - d_{ij}(X))^2 + \lambda \sum_i f(x_i). \tag{1}$$

Here, w_{ij} represents the accuracy of the measurements δ_{ij} (e.g., $w_{ij} = 0$ is a measurement between sensors i and j is not available), X stands for the $N \times D$ matrix of unknown coordinates, $d_{ij}(X)$ is the Euclidean distance between the searched coordinates for the sensors i and j, $f(x_i)$ provides the penalty (constraint) for positioning sensor i at coordinate x_i, and λ is a scalar parameter that controls the tradeoff between the accuracy to the provided dissimilarities δ_{ij} and the constraints given by $f(\cdot)$. The first sum is over all the pairs, while the second one if over each sensor.

The first term in the energy (1) is the classical STRESS in multidimensional scaling [1], [4], and has been previously used for sensor localization, see [3] and references therein. The novelty in our approach is in the introduction of the second term, $\sum_i f(x_i)$. The penalty function f can represent

[1] The framework here introduced, as well as the general theory of multidimensional scaling, is applicable to other measures of dissimilarities, including geodesic distances for example [6].

probability of finding sensor i at a given location, or can penalize for locating the sensors in forbidden or unreasonable areas. In other words, it provides constraints on the sensor localization that come from prior or learned knowledge about the sensors network physical environment.[2]

Note that all practical algorithms for sensor localization that follow energy functions of the form of the STRESS cost are prune to local minima. As a consequence, it is likely they will locate the sensors in low probability or even forbidden areas. This is also due to the noise in the available dissimilarities δ_{ij}. Thereby, using the cost function (1), even as a second refinement step, is crucial.

III. Experimental results

We now present examples of our proposed constrained sensor localization framework. In particular, three different selections for the penalty function f in Equation (1) are presented. First, in Figure 1 we localize the sensors following a topographic map. We define f to act as a probability function, with probability proportional to the local elevation, the higher the point the lower the probability to find a sensor there. In Figure 2, we penalize for sensors located inside the two large lake areas. This is done by designing f to be strictly positive inside the forbidden areas (lake) and zero otherwise. Moreover, f increases with the distance to the border of the lake. Finally, in Figure 3, we constrain the sensors to be on a given curve (the yellow line). This is done by defining f to be the unsigned distance to the curve.

In all the examples we start from a given configuration, blue squares for Figure 1 and green squares for figures 2 and 3, that can be obtained from any of the current state-of-the-art and unconstrained sensor localization techniques, e.g., [3]. Then, we minimize the energy given by (1), obtaining the red dots as the new and now constrained sensors position. We used standard optimization techniques from the Matlab Optimization Toolbox, e.g., *fminsearch* [5], while more sophisticated ones could be used as well. In particular, the local cost decomposition for the STRESS developed in [3] is also valid for the constrained STRESS here introduce, thereby leading to the development of distributed optimization algorithms if desired.

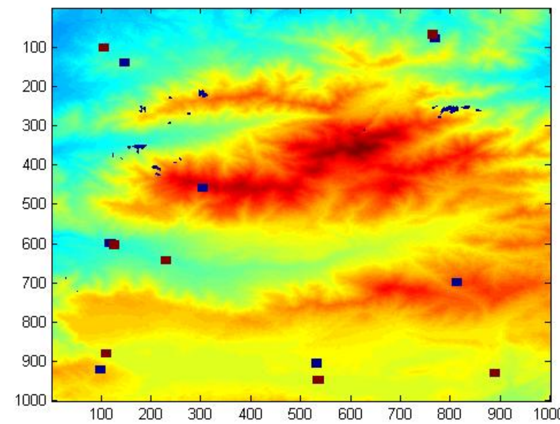

Fig. 1. *The elevation data is used to define the penalty function f. The higher the area, the lower the probability of finding a sensor there. Note how the blue squares, marking the original sensor locations, move to the red squares located at lower elevation regions. The elevation map goes from dark blue, representing low elevations, to dark red, representing high elevations. The blue spots are zones of holes in the elevation information.*

IV. Conclusions

In this note we have addressed for the first time the problem of sensor localization with physical constraints. We have extended the classical STRESS function from distance geometry and multidimensional scaling theories, which is frequently used in the sensor networks arena, to include an extra term that represents the available prior information about the physical environment. We exemplified the ideas by constraining the sensors to be in low elevation areas, outside of forbidden zones, and to be located on predefined curves.

The proposed algorithm can be used as a second step, after constraint-free and efficient sensor localization techniques have been applied to the available pairwise dissimilarity measures. To efficiently use our proposed energy directly, distributed minimization algorithms have to be developed. For some constraints, these are already available in the multidimensional and distance geometry literature, while for others, such as those coming from distance functions, they need to be developed (based on these available algorithms). Extending the current work to angle-of-arrival information is of great significance as well. These directions, together with the use of our technique for large real sensor networks, are the subject of future research.

[2]Costa *et al.*, [3], proposed an extra quadratic term in their modified STRESS function that penalizes for individual sensors i to be located far from pre-established positions $\bar{x}_i$: $\sum_i r_i \parallel x_i - \bar{x}_i \parallel$, for confidence values r_i. This is a particular case of the physically motivated constrained framework here proposed.

References

[1] I. Borg and P. Gorenen, *Modern Multidimensional Scaling: Theory and Applications*, Springer, 1996.

[2] L. M. Blumenthal, *Theory and Applications of Distance Geometry*, Chelsea House Pub, 1970.

[3] J. Costa, N. Patwari, and A. O. Hero III, "Distrbuted multidimensional scaling with adaptive weighting for node localization in sensor netwroks," *ACM Trans. on Sensor Networks*, to appear, 2005.

[4] T. Cox and M. Cox, *Multidimensional Scaling*, Chapman & and Hall, London, 1994.

[5] *Matlab Optimization Toolbox*, http://www.mathworks.com/products/optimization/

[6] J. B. Tenenbaum, V. DeSilva, and J. C. Langford, "A global geometric framework for nonlinear dimensionality reduction," *Science* **290**, pp. 2319-2323, 2000.

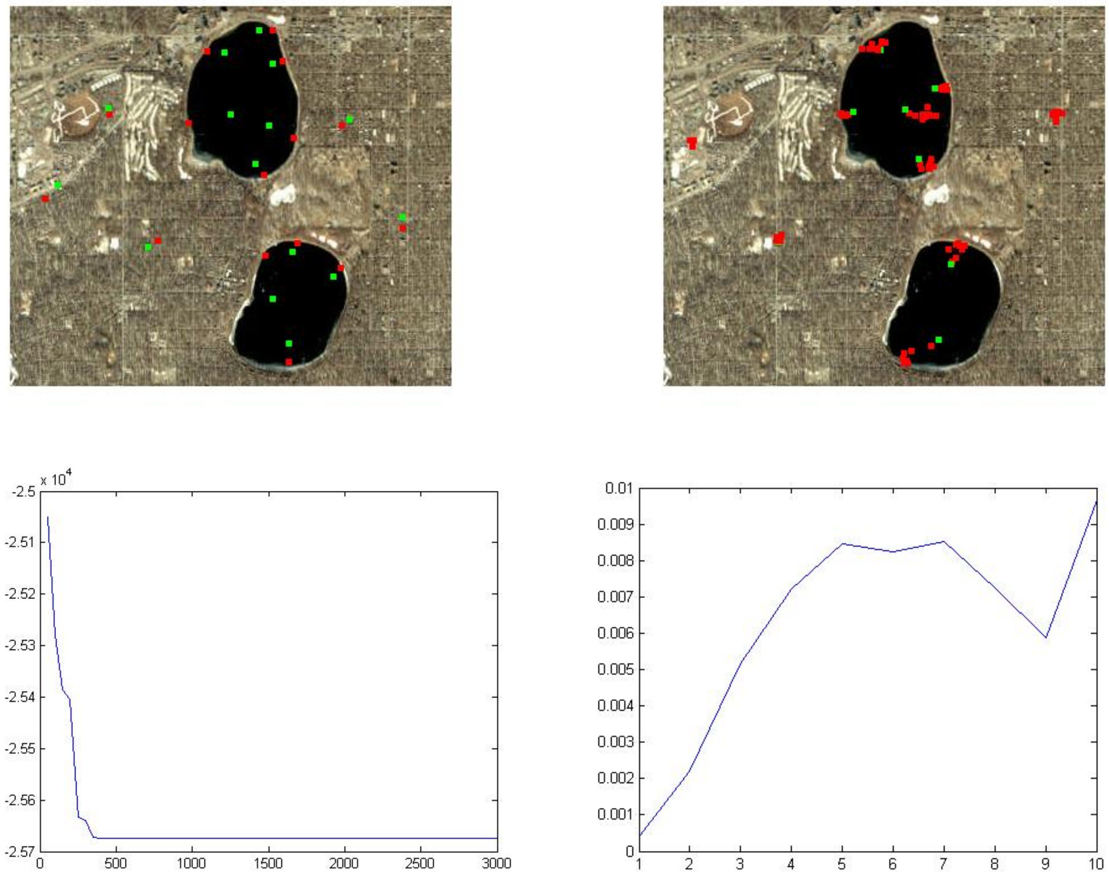

Fig. 2. *Examples of placing the sensors outside of the lake areas. On the top left we see the results, where all the sensors have been moved out of the lake. On the right we show the sensor positions for different values of the parameter λ in Equation (1). On the second row, left, we plot the energy (1) as a function of the minimization iteration. On the second row, right, we plot the average percentage deviation of the pairwise distance function, with respect to the initial condition of the optimization, as a function of λ (this corresponds to the figure on the top right).*

Fig. 3. *Examples of placing the sensors in a curve, the yellow line. Results for two different values of λ are shown in the first row, with a higher value on the right (note how all the sensors are located on the yellow line). A different distances configuration is shown in the second row, left. In the second row, right, an additional example is shown, where some of the pairwise distances between the sensors are not available ($w_{ij} = 0$ in Equation (1)).)*