Subjective Feeling

A Thesis
SUBMITTED TO THE FACULTY OF THE UNIVERSITY OF MINNESOTA
BY

Orges Ormanidhi

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Advisor: Professor Jay S. Coggins
Co-Advisor: Professor C. Ford Runge

2023

Acknowledgment

I want to thank my advisors Professors Jay Coggins, C. Ford Runge, Tian-Jun Li, and Elton Mykerezi. I am grateful to Professor Coggins for his constant support of my work. I am grateful to Professor Runge for his unlimited insights to my research. I am grateful to Professor Li for helping me in my mathematical education. I am grateful to Professor Mykerezi for welcoming me in the department from the very beginning. I cannot see how I could have achieved my results during my studies without my advisors. Thank You All.

Abstract

In the 19th century, Jevons wished for a way to measure the quantity of feeling, which he envisioned as the integral of the intensity of feeling from an activity on which a subject was spending time. I have derived the measure of intensity of feeling from both the engaged activity and non-engaged activities with a theory founded on subjective feeling as the primitive of human behavior. The integral of my measure of intensity of feeling from the engaged activity measures the quantity of feeling, which I call integrated experience in my research.

I have developed my theory of choice with weaker assumptions than those in neoclassical economic theory because subjective feeling is a more basic primitive than the preference relation. The application of my subjective feeling theory is in two parts. In the first part, I explain with a descriptive framework any optimal or non-optimal choice of time-allocations to activities without the observer's assumption of optimization for the subject. In the second part, I predict with a normative framework the optimal choice of time-allocations to activities, as well as the sequence of engaged activities, with the observer's assumption of optimization for the subject who maximizes his overall experience.

In the descriptive framework, the subject chooses a sequence of engaged activities in a schedule based on the switch-time determination through a matching process between the engaged activity and non-engaged activities. In the normative framework,

the conditionally rational subject chooses the sequence of engaged activities in the optimal schedule through a sorting process of all matchings. The choices that the observer is able to analyze with my intensity of feeling functions extend those that she is able to analyze with utility functions.

Table of Contents

Chapter 1

Introduction: The Research

1.1 The Primitive

The very foundation of neoclassical economic theory is a human characteristic believed to be the drive, the force, or similar determining influence that compels us to make a choice at any moment in time. This characteristic, treated as "the primitive characteristic of the individual" (Mas-Colell et al., 1995, p. 5), is the *preference relation*. The preference relation is defined as a mathematical binary relation over a choice set X. In order to be well-defined, a preference relation (denoted by $\succcurlyeq$) should be complete (Mas-Colell et al., 1995, p. 6): for any pairs x and y in X, x is at least as good as y ($x \succcurlyeq y$), y is at least as good as x ($y \succcurlyeq x$), or both $x \succcurlyeq y$ and $y \succcurlyeq x$ ($x \sim y$, in which case an individual is indifferent between x and y).[1] I refer to the individual studied by neoclassical economic theory as *the agent*.

Neoclassical economic theory is the standard theory in modern economics. Its utilization extends well beyond household economic decisions ranging from addiction (Schelling, 1978; Becker and Murphy, 1988; Gruber and Köszegi, 2001; Allcott et al.,

[1]Another term for $x \succcurlyeq y$ is x is preferred to y, and $x \succ y$ usually means x is strictly preferred to y.

2022) and marriage (Becker, 1973; Becker, 1974; Lundberg and Pollak, 1993; Lafortune and Low, 2023) to pollution and carbon dioxide levels (Hotlz-Eakin and Selden, 1995; Grossman and Krueger, 1995; Stern et al., 1996; Copeland and Taylor, 2004; Muller et al., 2011; Acemoglu et al., 2012; Anderson, 2019). Predicated on the preference relation as the primitive characteristic of the individual, the reliance on the capability of neoclassical economic theory to explain a large variety of choices goes hand in hand with the belief in the preference relation as a biological force that determines these choices. Perhaps such belief may undergird the claim that "economics is everywhere" (Hamermesh, 2020), because if an *observer* of human behavior utilizes a theory founded on an individual characteristic possessing the power to determine any type of choice, then economics would be found in many places (if not everywhere).

However, the observation of choices made every day could cause to doubt that economics is actually everywhere. Economists are aware that neoclassical economic theory is unable to predict[2] a variety of choices. Rather than considering a new theory predicated on another individual characteristic, possibly with a greater potential than the preference relation to explain[3] and/or predict a larger variety of choices, the current view of the emerging field of behavioral economics is to treat choices that fall outside predictions of the standard theory as "mistakes." Here is a description of the origin of behavioral economics at Berkeley and its development as a new field:

> In 1987, a cross-disciplinary course at the University of California, Berkeley marked a milestone for a new field of study. Two professors, an economist and a psychologist, joined forces to teach a PhD class that used the analytical tools of social and cognitive psychology to investigate economic problems. Their teaching was a foundation for what is now known as behavioral economics. . . . The two professors, George Akerlof and Daniel

[2]By predict, I mean that the theory utilized to study choice is normative. Neoclassical economic theory is a normative theory because an agent *must* make an optimal choice among alternatives.

[3]By explain, I mean that the theory utilized to study choice is descriptive. A descriptive theory does not require from the individual to behave optimally and hence he may or may not make an optimal choice.

> Kahneman, went on to win Nobel prizes. ... Berkeley's philosophy is that behavioral economics enriches rather than contradicts standard economic thought. When the tools of psychology are joined with the proven analytical methods of economics, the results are more powerful and scholars get closer to underlying truths about human behavior. ... People make mistakes, they have biases, they can't always control their impulses, they react to social pressure. (About the Initiative, Initiative for Behavioral Economics & Finance, behavioral.berkeley.edu/about.html, Accessed 9/24/2023.)[4]

This view holds that these so-called mistakes could be understood and corrected with methods developed by this field of study. In trying to correct these so-called mistakes, some researchers (Gilboa et al., 2019) suggest that a more effective mathematical rhetoric might convince people to make the choices that are recommended by neoclassical economic theory.

They claim that a mathematical theorem stating that utility maximization is equivalent to the two axioms of completeness and transitivity[5] possesses rhetorical power (Gilboa et al., 2019). Supposedly, such rhetoric could then help correct behavior: "[A mathematical theorem] can also convince some decision makers to make the extra step and attempt to assess their own utility functions in the hope of guaranteeing that their choices are consistent" (Gilboa et al., 2019, pp. 343-344). Taking their efforts to correct human behavior even further, they suggest that the axioms of standard economic theory might teach people *how* to decide: "... axiomatic systems such as von Neumann and Morgenstern (1944) or Savage (1954) might convince people that the way *they would like to make decisions* in the face of risk or of uncertainty is by maximization of expected utility" (Gilboa et al., 2019, p. 344, emphasis in original).

[4]The Initiative for Behavioral Economics & Finance (IBEF) has recently become part of the Robert G. and Sue O'Donnell Center for Behavioral Economics and "errors" are among its research areas: "For reference some of the current research by members in IBEF include: ... How do errors in beliefs and judgements such overconfidence and the sunk-cost fallacy hamper entrepreneurs, and which organizational fixes might help to overcome these hurdles (About the Robert G. and Sue O'Donnell Center for Behavioral Economics, haas.berkeley.edu/behavioral-economics/about/, Accessed 10/27/2023, grammatical errors in original.)

[5]A preference relation is transitive when $x \succcurlyeq y$ and $y \succcurlyeq z$ imply $x \succcurlyeq z$.

I study human behavior differently. The observer of human behavior in my research does not make normative judgments regarding choices of agents. Although the individual whose behavior I study might make judgments about his own choice, the observer of someone else's behavior does not judge. Therefore, I do not aim to correct behavior or choice[6] because no choice is considered to be a mistake (or error) for the observer.

In my research, I study the behavior of an individual whose state allows him to do several things such as eating, studying or even doing nothing either at the same time or at different times during a given period. If the individual does some things simultaneously, I call these things together an *activity*. I refer to the individual studied by my theory as *the subject*.

Definition. ***An activity*** *is a group of things that the subject does simultaneously. The activity on which the subject is spending time is the engaged activity and activities on which he is considering spending time are non-engaged activities.*

Simultaneity in the definition of activity implies that activities are distinct. For example, if an individual walks and chews gum for 1.5 hours and then reads and chews gum for another 1.5 hours, he has spent a total of three hours on two different activities doing three things: walking, reading and chewing gum. Although the individual chewed gum for three hours, he did not walk and read simultaneously at any moment in time.

If his choice is studied by my theory, the observer assumes that the subject considers spending time on two distinct activities such as "walk and chew gum" and "read and chew gum." Similarly, if his choice is studied by neoclassical economic theory, the observer assumes that the agent (same individual) considers spending time on same alternatives. In both my research and neoclassical economic settings, "walk and chew gum" and "read and chew gum" are distinct categories under consideration.

Not all activities are part of my research. On one hand, things that are associated

[6] I use behavior and choice interchangeably because behavior in my research is represented by any choice.

with activities create mental images that can be perceived if the subject is in a conscious state: "[The conscious state of mind] ... is awake rather than asleep. It is alert and focused rather than drowsy or confused or distracted. It is oriented to time and place. The images in mind—sounds, visual images, feelings, you name it—are properly formed, exhibited with clarity, and inspectable. They would not be if you were under the action of "psychoactive" molecules, from alcohol to psychedelic drugs" (Damasio, 2018, p. 143). So some activities such as sleeping or being under the influence of substances are not part of my research.

On the other hand, recent research has found a small degree of consciousness called cognitive motor dissociation even in patients appearing in an unconscious state of mind due to acute brain injury (Franzova et al., 2023). Because of the difficulty of defining conscious versus unconscious states, Nilsen et al. (2022) suggest the use of a pragmatic approach of characterizing such states in both scientific settings and clinical practices. Without joining in the long debate on consciousness or lack thereof (Chalmers, 1995; Melloni et al., 2021; Yaron et al., 2022; Lenharo, 2023), I adopt a pragmatic approach in studying the behavior of a subject who is conscious, and thus able to consider spending time on his activities.

I have created a new primitive characteristic, which I call *subjective feeling*, by combining activities and time with individual experience.

Definition. ***Subjective feeling*** *is the instantaneous experience of a subject's mind and body from the engaged activity as well as non-engaged activities.*

Its advantage as "the primitive" is that it is more basic or more primitive than the preference relation. A more basic primitive needs weaker assumptions or requirements to found a theory. The weaker the assumptions, the wider the applicability of the theory.[7]

Before elaborating on this new primitive, I use the term "subjective" for two purposes. First, by subjective I mean two distinctions: 1) different subjects might experience things differently even if they are engaged in the same activity; and 2) unless one analyzes her own behavior, the subject and the observer are different individuals.[8] Second, by subjective I also mean the deep connection between the subject's own subjectivity and his feelings: "Feelings provide the qualia element included in subjectivity. In turn, subjectivity permits feelings to be scrutinized as specific objects in conscious experience" (Damasio 2018, p. 147).

Subjective feeling represents the role of feelings in the functioning of human organism as described by Damasio (2021, p. 83):

> The sensory maps and images that are part of elaborate feelings incorporate in the ongoing mental flow facts regarding the state of the organism's interior. This informational role is a primary contribution of feelings, but feelings have another role to play: they provide the urge and the incentive to behave according to the information they carry and do what is most appropriate for the current situation, be it running for cover or hugging the person you have missed.

Summarizing this quote by Damasio (2021), feelings have:

1. an *informational* role, by which they inform the organism about its interior state; and
2. a *behavioral* role, by which they tell the organism what to do.

The part of feelings consisting of sensory maps and images that inform the interior of an organism about its state is outside the scope of my current research. I take the interior state of the organism at every moment in time t as given by a positive constant

[7] Weaker assumptions mean that less is assumed and so more cases can be explained and/or predicted. A similar point is made by Özgur and Ok (2011) to note the wide applicability of their theorems of multi-utility representation of incomplete preference relations thanks to weak requirements of such representations.

[8] I will use pronouns "she/her" for the observer and "he/him" for the subject.

function $\alpha(t)$ that changes in a stepwise fashion over time, that is $\alpha(t)$ is a piecewise positive constant.

Definition. $\alpha(t) > 0$ *is a piecewise constant during intervals of time that measures the interior state of the living organism at every moment in time.*

The values of $\alpha(t)$ come from measurements of the feeling of different states of the organism or self-assessments. Here is a description of such measurements in an interview of ScienceNews (SN) with the neuroscientist Antonio Damasio discussing his book “Knowing & Feeling” (2021):

> SN: How do feelings help an organism manage life?
> Damasio: Feelings are representations of the state of your body. To have a feeling of pain, pleasure, well being, sickness, thirst, hunger or desire is to generate a picture of some parts of your organs. For example, the feeling of well-being is related to parameters that you can locate and measure. This is something that we can analyze; we can actually study it in the laboratory. A lot of what would be described as [the feeling of] well being is related to the simple musculature that is around blood vessels in organs, like the stomach, the gut and so forth. And even muscular skeletal components of our body, how they are: Are they contracted? Are they distended, a large part, or not so much? What we are feeling from well-being is, in fact, describing states of our body; that's what feelings are about. So the root of feeling in the state of the organism is unquestionable. ('Feeling & Knowing' explores the origin and evolution of consciousness: Neuroscientist Antonio Damasio discusses his latest book, ScienceNews, sciencenews.org/article/feeling-knowing-book-consciousness-origin-evolution,
> Accessed 10/10/2023.)

With the informational role of feelings represented by $\alpha(t)$ measuring the subject's being (well, sick, exited, and so on) depending on interior state of the organism, subjective feeling represents the behavioral role of feelings. The non-zero value of $\alpha(t)$ means that feelings are able to communicate to a living organism about its state. Its positive value ensures that the dual role of feelings, as a combination of their information role represented by $\alpha(t)$ and their behavioral role represented by functions measuring subjective feeling, is correct. For example, suppose that at some point in time $t = \tau$, a function $f(t)$ measuring subjective feeling is negative. Then the product $\alpha(\tau)f(\tau) < 0$ ensures that the dual role of feelings is correct since it would be contradictory to the organism to receive a positive value from feelings if both their informational role ($\alpha(t)$) and behavioral role ($f(t)$) were communicating to this organism negative values. Also, $\alpha(t)$ is piecewise because although the state of the organism can change, it needs to be stable or constant during intervals of time in order for the subject to experience feelings, which are measured by $f(t)$.[9]

As seen from their definitions, the preference relation and subjective feeling are biologically different primitives. When the preference relation is the primitive, the agent creates a mathematical relation that determines choice. Neoclassical economic theory does not make clear which biological processes, if any, would support the use of a mathematical notion like the preference relation to be the drive that compels us to make a choice.[10] Nonetheless, Mas-Colell (1995, p. 6) recognize the considerable amount of work required by the agent in neoclassical economic setting to create a preference relation: "It takes work and serious reflection to find out one's own preferences."

Although presumably unintentionally, it would be misleading to use the term preferences for the preference relation, as Mas-Colell (1995) in this quote and others elsewhere[11] do, because preferences differ from the preference relation biologically (rather

[9] Although my analysis is the same for both constant or non-constant $\alpha(t)$, it is unreasonable for $\alpha(t)$ to change at every moment in time while $f(t)$ may do so. If $\alpha(t)$ were not stable during some uninterrupted time, then both the informational role and the behavioral role would be interfering with each others' roles.

[10] Neoclassical economic theory (Samuelson, 1938; Samuelson, 1948; Houthakker, 1950; Debreu, 1954; Arrow, 1959; Debreu, 1959; Rader, 1963; Matzkin, 1991; Mas-Colell et al., 1995; McFadden, 2005; Jehle and Reny, 2011; Kreps, 2013; Chambers and Echenique, 2016) does not report any findings from neuroscience or other sciences to support the use of the preference relation as the primitive.

[11] With a few exceptions (Schmeidler, 1971; Gilboa and Schmeidler, 1989; Ok, 2002; Evren znd Ok, 2011), it is not unusual to see the use of the term preferences for the preference relation (Cox et al., 1985; Barsky et al., 1997; Becker and Mulligan, 1997; Laibson, 1997; Andreoni and Miller, 2002; Charness and Rabin, 2002; Frederick et al., 2002; Levitt and List, 2007; Fischbacher and Gachter, 2010; Polisson et al., 2020).

than semantically). Preferences exist in basic form in the organism: "... the organism has a basic set of preferences—or criteria, biases, or values. Under their influence and the agency of experience, the repertoire of things categorized as good or bad grows rapidly, and the ability to detect new good and bad things grows exponentially" (Damasio, 1994, p. 116). In contrast, the preference relation does not exist in an equally basic form because from comparisons of areas in the brain that are responsible for mathematical/logical and non-logical reasoning (Kroger et al., 2008; Amalric and Dehaene, 2018), creating a mathematical binary relation such as the preference relation requires considerable mental effort and cognition.

Subjective feeling is more basic than the preference relation because, rather than the required mathematical ability to create the preference relation, the experience—a defining ingredient—in subjective feeling represents preferences (or biases, or values) in their basic form in the human organism, as Damasio (1994) noted that it is through the agency of experience that the subject has the ability to categorize things as good or bad and that his ability grows exponentially when new things are considered. Furthermore, if the ability to categorize new things as good or bad grows so rapidly, then the ability to evaluate how good and bad they are in (the preference) relation to each other must grow even more rapidly. When the alternatives are infinite, such as when allocating how much time to spend on each activity or alternative, the agent deciding with the preference relation as the primitive looks like a superhuman being because of his ability to order infinitely many bundles of every single moment spent on every alternative.

The difficulty in evaluating alternatives in the preference relation was noted by Camerer et al. (2005, p. 10): "The variables that enter into the formulation of the decision problem—the preferences,[12] information, and constraints—are precisely the variables that should affect the decision, if the person had unlimited time and comput-

[12]Notwithstanding the use of preferences for the preference relation once again.

ing ability." Such difficulty was also noted in connection with the individual experience by Mas-Colell et al. (1995, p. 6): "Introspection quickly reveals how hard it is to evaluate alternatives that are far from the realm of common experience."

Individual experience has been considered in other contexts as well, but its role in my research is different because it is a defining ingredient of the primitive. The role of experience in other contexts is supplemental or external because it is not part of the primitive, i.e. the preference relation. I note here two main contexts where experience is considered: prospect theory (Kahneman and Tversky, 1979) where past experience is considered among other elements to predict choice under uncertainty; and experienced utility (Kahneman et al., 1997)[13] instantaneous and past experiences were considered for their experienced utility representations of the hedonic quality of utility as introduced by Bentham (1789). In both contexts, experience has a secondary role in choice because it is not a defining ingredient of the preference relation as the primitive. In my research, experience has a primary role in choice because it is a defining ingredient of subjective feeling as the primitive. There are other differences on the role of experience in the theories of these contexts and my theory. I discuss further differences in the next chapter.

Subjective feeling is also more basic than the preference relation because of where feelings originate and how easily they function in the human organism. Rather than the brain areas responsible for creating a mathematical notion like the preference relation, feelings originate from the visceral interior of the organism: "Feelings arise in the interior of organisms, in the depth of viscera and fluids where the chemistry responsible for life in all its aspects reigns supreme" (Damasio, 2021, p. 95). When the subject makes a choice with subjective feeling as the primitive, he does not spend much mental

[13] Kahneman et al. (1997) use the term experienced utility to distinguish it from decision utility commonly known as utility, or ordinal utility, in modern economics. Utility/decision utility is a function that represents the ordering of a preference relation without preserving differences that may exist between any pairs.

effort because feelings "move with ease from mind to body and back again" (Damasio, 2021, p. 109). As they move easily between his mind and body, feelings execute their behavioral role "effortlessly" by telling the agent what to do: "[Feelings] provide the mind with facts on the basis of which we know, effortlessly, that whatever else is in mind, at the moment, also belongs to us, is happening in us. Feelings allow us to experience and become conscious, to unify our mental holdings around our singular being" (Damasio, 2021, p.148).

In subjective feeling, the way in which feelings allow the subject to experience what is happening to him is by expressing the interconnection between feelings and experience as a process that involves both his mind and body at every instant in time, from the engaged activity as well as non-engaged activities. Feelings and experience are interconnected because they cannot exist without each other: "When the [mental] images are properly placed in the perspective of the organism *and* are suitably accompanied by feelings, a *mental experience* ensues" (Damasio, 2018, p. 152). The subject's experience occurs in both his mind and body because feelings are both mental and physical: "...feelings are not purely mental; ...they are hybrids of mind and body" (Damasio, 2021, p. 109).

Experience in subjective feeling is instantaneous because mental experiences happen over time: "Mental experiences are not "instant pictures" but processes in time ..." (Damasio, 2018, p. 121). *Time* is intrinsically linked to human experience through what are called time cells, which are human hippocampal neurons that were first identified in humans by Umbach et al. (2021) in their study of 27 human epilepsy patients performing an episodic memory task.[14] The significance of these time cells for human experience was further elaborated by Reddy et al. (2022), who tracked neuronal activity in 9 epilepsy patients and concluded that: "Our results provide further evidence

[14] "Episodic memory describes our ability to weave temporally contiguous elements into rich and coherent experiences" (Umbach et al., 2021, p. 28463).

that human hippocampal neurons represent the flow of time in an experience" (p. 6724). In my research, the experience is instantaneous and it happens over a period during which the subjects consider spending time on a finite number of activities.

Activities constitute the final ingredient of subjective feeling. While the state of the organism, measured by $\alpha(t)$, represents the interior of the subject's being, in my research, activities represent the exterior of his being. It is through the interaction of $\alpha(t)$ and activities on which the subject is considering spending time that I obtain the measure of subjective feeling. As noted in the definition of subjective feeling, experience occurs at every instant from the engaged activity as well as non-engaged activities. Hence, experience is an instantaneous process that occurs in both the mind and body because it is created from things that he is doing while spending time on the engaged activity as well as things that he is considering doing when, and if, he is spending time on non-engaged activities.

As noted earlier, a mental experience ensues when mental images are accompanied by feelings (Damasio, 2018). The subject's mental images in my research are created from events associated with the engaged activity as well as non-engaged activities. Events associated with the engaged activity are real in the sense that they unfold while the subject interacts with the exterior environment in which such activity takes place, such as when walking the dog or even doing nothing but sitting comfortably in a sofa. Events associated with non-engaged activities are imagined simultaneously as the subject is spending time in the engaged activity.

Lee et al. (2021) identified which specific areas of the brain are responsible for imagining the future by investigating the hypothesis that imagination consists of: 1) a constructive process (the vividness of imagined events) to create a new future event as proposed by Addis et al. (2007), Hassabis et al. (2007) and Schacter et al. (2007): and 2) an evaluative process (the valence of imagined events) to judge the imagined event as positive or negative as proposed by D'Argembeau and van der Linden (2004), Gilbert and Wilson (2007) and Sharot et al. (2007). They found that "The vividness of imagined events modulates the ventral [Default Mode Network] DMN, but not the dorsal DMN, while the valence of imagined events modulates the dorsal DMN, but not the ventral DMN" (Lee et al, 2021, p. 5243). As the observer of human behavior, I organize all events, real or imagined, into activities.

Before I present in the next section my model of behavior, a note on the difference between feelings and emotions. Although they sometimes may be considered equivalent in everyday language, in my research, they are different. Here is how Damasio (1994, p. 138) described his understanding of their difference: "In conclusion, emotion is the combination of *a mental evaluative process*, simple or complex, with *dispositional responses to that process*, mostly *toward the body proper*, resulting in an emotional body state, but also *toward the brain itself* (neurotransmitter nuclei in brain stem), resulting in additional mental changes. ... I reserve the term *feeling* for the experience of those changes." In my research, the primitive that determines behavior is subjective feeling defined by three ingredients—experience, time and activities. Therefore, the subject in my research acts on his feelings as defined in the primitive rather than his emotions.

Although some behaviors or choices might reflect the subject's emotions, in my research, his choice need not coincide with his emotions because not all feelings are experienced as emotions: " ... all emotions generate feelings if you are awake and alert, but not feelings originate in emotions" (Damasio, 1994, p. 143). An illustration of the difference between feelings and emotions is given by the description that "pain is a feeling but not an emotion" in Psychology Today[15] by the psychiatrist Neel Burton presenting his book "Heaven & Hell" (2020). In my research, if the subject were in pain,

[15] "What's the Difference Between a Feeling and an Emotion? Pain is a feeling but not an emotion," Psychology Today, first posted in 2014 and revised in 2020, psychologytoday.com/us/blog/hide-and-seek/201412/whats-the-difference-between-feeling-and-emotion, Accessed 10/28/2023.

his choice would be determined by his painful experience—the ingredient of subjective feeling—rather than his emotion.

My goal in choosing subjective feeling as the primitive is twofold. First, to use a primitive that finds support in neuroscience, notably by Damasio (1994, 2018, 2021). Second, to create a theory that is able to explain and predict a wider variety of choices than neoclassical economic theory. At the same time, an unintended consequence in founding a theory of human behavior with a different primitive from the preference relation is that it may be viewed as a radical change in studying choice: "The radical approach involves turning back the hands of time and asking how economics might have evolved differently if it had been informed from the start by insights and findings now available from neuroscience" (Camerer et al., 2005, p. 10).

Rather than a radical change in neoclassical economic theory, choosing subjective feeling as the primitive for my theory is my answer to this question: "What if we studied choice with a primitive that is more basic than the preference relation?" After choosing subjective feeling as a more basic primitive than the preference relation, I explain how I study choice in the next section where I present the model I have developed to study the behavior of the subject who is considering spending time on a finite set of activities during a given period.

1.2 The Model of Behavior

In my research, one can view my role as an observer sitting in a bench in a park and observing a subject going about his daily business by switching from one activity to another. While observing the subject, I then ask myself such questions as: 1) What motivates this subject to choose an activity?; 2) When would he switch to another activity?; 3) Is the subject optimizing anything with his behavior?

My answer to the first question is that subjective feeling motivates the subject which activity to choose and how much time to spend on it. In the previous section I explained why subjective feeling is a powerful force guiding our daily business, which I organize into activities. To emphasize their power, feelings come first and cognition comes next (Damasio, 1994, pp. 159-160): "... because of their inextricable ties to the body, [feelings] come first in development and retain a primacy that subtly pervades our mental life. ...feelings are winners among equals. And since what comes first constitutes a frame of reference for what comes after, feelings have a say on how the rest of the brain and cognition go about their business." I answer the other two questions by applying the model of behavior I have developed to analyze the choice of the subject considering spending time t on a finite set of n activities over a given period of time.

The model of behavior includes a theory (explained in chapter 2) of the measure of subjective feeling and the analysis choice includes its application (explained chapter 3) in two steps: a) a descriptive framework where the subject is not required to make an optimal choice; and b) a normative framework where he is required to make an optimal choice.

At any moment in time t, given a finite set of n activities, the subject is spending time on the engaged activity and considering spending time on non-engaged activities. I call the instantaneous experience of the subject's mind and body *the intensity of feeling*. The differentiable function $f_i(t)$ represents the intensity of feeling from the engaged activity i, $i = 1, 2, \dots n$. Given i, the sum $\sum f_{j|i}(t)$ of differentiable functions $f_{j|i}(t)$ represents the intensity of feeling from non-engaged activities $j = 1, 2, \dots, i-1, i+1, \dots, n$ while the subject is spending time on the engaged activity i. Since the sum of $f_i(t)$ and $\sum f_{j|i}(t)$ depends on the engaged activity i, I call it the subject's subjective feeling given i, $f_i^*(t)$. It is the measure of the intensity of feeling from all activities given the engaged activity i.

Definition. ***The intensity of feeling*** *given the engaged activity* i*,* $f_i^*(t) = f_i(t) + \sum f_{j|i}(t)$*, measures subjective feeling, where* $f_i(t)$ *is the instantaneous experience from* i *and, given* i*,* $\sum f_{j|i}(t)$ *is the instantaneous experience from* j*,* $j \neq i$*.*

Using rates of change $\dot{f}_i$ and $\dot{f}_{j|i}$, I obtain $f_i(t)$ and $f_{j|i}(t)$ with three assumptions.

Assumption I The subject is considering spending time on a finite set of activities.

Assumption II At any moment in time, the subject is engaged in an activity.

Assumption III Given $\alpha(t) > 0$ as the common coefficient of proportionality, the rate of change in intensity of feeling from an activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities.

These assumptions give rise to a model with two systems of differential equations working simultaneously. *The engaged system* for the intensity of feeling from each engaged activity and *the non-engaged system* for the intensity of feeling from each non-engaged activity given the engaged activity, which work together at every moment in time.

The engaged system. The intensity of feeling from each engaged activity is given by

$$\dot{f}_i = \alpha(t) \left(f_i(t) - \sum f_k(t) \right), \quad \forall i = 1, 2, \cdots n; \; k \neq i$$

The non-engaged system. The intensity of feeling from each non-engaged activity is

$$\dot{f}_{j|i} = \alpha(t) \left[f_{j|i}(t) - \left(\sum f_{l|i}(t) + f_i(t) \right) \right], \quad \forall j = 1, 2, \cdots, i{-}1, i{+}1, \cdots, n; \; l \neq j$$

Changes in the organism's intensities of feeling from different activities occur through the interaction of the informational role of feelings represented by $\alpha(t)$ and their behavioral role represented by intensities of feeling. This interaction is expressed by the right-hand sides of the engaged and non-engaged systems. These changes in the organism's intensities of feeling from activities are traced by their rates of change in the left-hand sides of both systems.

Although $\alpha(t)$ may change piecewise, it represents the common state of the organism for all activities. For example, if the subject does not feel good today compared to yesterday, the rate of change in the intensity of feeling from drinking coffee today might be different from yesterday, but not because of how he likes coffee but because his organism feels different.

In this model, the rate of change in the intensity of feeling depends on the state of the organism $\alpha(t)$ and the number of activities competing for time. Without loss of generality, suppose that $\alpha(t) = 0$ represents a state of the organism either dead or completely unresponsive, and $\alpha(t) = 1$ represents a normal state. Then $0 < \alpha(t) < 1$ represents states that are not so good and $\alpha(t) > 1$ states that are really good. The larger the value of $\alpha(t)$ the better the organism and the greater the change in the intensity of feeling from each activity.

With n activities, each activity competes for time with the other $n-1$ activities.[16] From the definition of an activity, the most suitable form to express such competition is by the difference between the intensity of feeling from an activity and the sum of intensities of feeling from the other activities.[17] Then the greater the force from each activity to pull the subject in spending time on that activity as opposed to the

[16] In the engaged system, each activity competes with the other $n-1$ activities to engage in any of them. In the non-engaged system, given the engaged activity i, each non-engaged activity competes with the other $n-2$ non-engaged activities to engage in any of them as well as with the engaged activity i, so each non-engaged activity also competes with $n-1$ activities.

[17] If the model was formulated in a different form, say intensities of feeling from two activities were multiplied, then their force would act simultaneously, hence they would not be two different activities.

combined force of the remaining $n-1$ activities, the greater the (absolute) change in its intensity of feeling.

With my two-system model, I obtain unique functions $f_i(t)$ and $f_{j|i}(t)$ to derive the measure of subjective feeling $f_i^*(t)$. By extending the model, I also obtain intensity of feeling functions that include discounting and uncertainty in the subject's choice.

I illustrate my assumptions with an example of a hypothetical subject who has planets in the universe of his mind. These mind-planets are a metaphor for activities. By Assumption I, the number n of mind-planets is finite. Imagine that the mind-planets move with what the 19th century economist William Jevons (1888) called "incessant variation" when describing our minds: "Incessant variation characterises our states of mind, and this is the source of the main difficulties of the subject. Nevertheless, if these variations can be traced out at all, or any approach to method and law can be detected, it will be possible to form a conception of the resulting quantity of feeling." The notion of the quantity of feeling introduced by Jevons (1888) has inspired me to choose subjective feeling as the primitive in my research. I trace such incessant variation of our minds by rates of change in the intensity of feeling functions.

Now suppose that each mind-planet is pulling the subject to jump on its surface with a force of gravity. The force of gravity is a metaphor for the intensity of feeling. Also, the subject can jump effortlessly to a mind-planet. By Assumption II, the subject is on a single mind-planet at any moment in time. While spending time on a mind-planet, the other mind-planets are pulling the subject with a combined force equal to the sum of their individual forces. By Assumption III, the rate of change of the force of each mind-planet is proportional the difference between its own force and the combined force of the other mind-planets.

When the three assumptions work together, each mind-planet competes with the other $n-1$ planets in two ways. First, as in the engaged system, the force of each mind-planet i pulls the subject on its surface by withstanding the combined force of the other $n-1$ mind-planets that push the subject away from i onto another surface. Second, as in the non-engaged system, while the subject is on mind-planet i, each mind-planet j pulls the subject toward its surface by withstanding the combined force of the remaining $n-2$ mind-planets he is not on and the force of the mind-planet i he is on. But the subject is human and the rate of change of the force of each mind-planet also depends on the state of his organism.

Models of rates of change proportional to suitable combinations of unknown functions have been used widely. For example, proportional rates of change are used in natural or logistic models to find the growth function for a given population as well as in the predator-prey system, also known as Lotka-Volterra equations,[18] to find the growth functions for populations of two species that co-exist in the same habitat with one species as the predator and the other species as the prey (Wangersky, 1978). In neuroscientfic studies, rates of change are used to study neuronal activity of short-term brain plasticity to model the rate of change of an unknown probability function proportional to the difference between this (unknown) probability and a maximum probability limit (similar to my model) by McElvain et al. (2015) and to model neural population activity in order to understand (the unresolved problem) of decision-related neural activity by Boucher et al. (2023).

My model has similarities with the Lotka-Volterra system of equations because activities compete for time, but there are differences. Activities would be both "predator" and "prey" because each activity can be the engaged activity and my model has two systems working simultaneously. The coefficient of proportionality $\alpha(t)$ is subject-specific rather than activity-specific and it is not necessarily constant. If $\alpha(t)$ is combined with activity-specific coefficients accounting for discounting and uncertainty, the new coef-

[18] The name comes from the differential equations that were initially developed by Lotka (1910) and then further utilized by Volterra (1926).

ficients of proportionality become activity-specific, but they more involved and they also are not necessarily constant.

Assumptions impose requirements that are needed to develop a theory. Since subjective feeling drives the subject to make a choice, the assumptions in my theory are weaker than the assumptions in standard economic theory because the two dynamical systems of my model do not require that the subject spend mental effort to create a mathematical relation. From the observer's point of view, weaker assumptions are advantageous because they provide her with a stronger analytical tool to study choice. In my theory, the analytical tool to study choice is the measure of subjective feeling $f_i^*(t)$. In standard economic theory, the analytical tool is utility (or ordinal utility).

One of the main advantages of my theory is that $f_i^*(t)$ is independent from rationality,[19] which is defined as the agent's ability to create a complete and transitive preference relation (Mas-Colell et al., 1995). Rationality is a strong requirement because completeness and transitivity are strong assumptions: "The strength of the completeness assumption should not be underestimated. . . . Transitivity is also a strong assumption, and it goes to the heart of the concept of rationality" (Mas-Colell et al., 1995, pp. 6-7).[20] In turn, a utility function can represent a preference relation "only if

[19]The independence of my theory from rationality does not have anything to do with anyone being rational or not. All it means is that the observer utilizing a theory that does not depend on the strong assumption of rationality has more opportunities to apply the theory and to explain and/or predict a wider variety of choices, including those that from the observer's point view are considered non-rational.

[20]Efforts to predict choices that are not predicted by the standard model include theories that include individual experience such as prospect theory (Kahneman and Tversky, 1979) and experienced utility (Kahneman et al., 1997) and those that can be figuratively described as "Mas-Colell et al. (1995) +" (because they add to the standard model) such as the "dual-self" models (Bernheim and Rangel, 2004; Fudenberg and Levine, 2006). I compare later my results with theories that involve individual experience because they are more directly related to my research. However, I focus on comparing my model with the standard model because although "dual-self" models have interesting features, they impose considerable restrictions on the agent's behavior: "It is important that we do not allow the long-run self to precommit for the entire dynamic game (Fudenberg and Levine, 2006, p. 1450)." Moreover, they aim, as in Thaler and Shefrin (1981), to extend orthodox models in order to characterize the observed choice as rational (Fudenberg and Levine, 2006, p. 1451). In contrast, I do not impose restrictions on the subject's behavior or require that he is rational.

it is rational" (Mas-Colell et al., 1995, p.9).

Note however that what I, as well as Mas-Colell et al. (1995), call assumptions are presented as axioms elsewhere.[21] Although again presumably unintentionally, the use of the term axioms rather than assumptions might be misleading for two reasons. I illustrate these reasons with the use of the term axioms I noted earlier about the claim that the observer's rhetoric has convincing power to influence behavior: "Simple axioms such as completeness and transitivity, by contrast, are likely to be accepted by most listeners. A mathematical theorem stating that these two axioms are equivalent to behavior that can be described by utility maximization is then a powerful rhetorical device" (Gilboa et al., 2019, p. 343).

The first reason why using axioms for assumptions could be misleading is technical. If rationality and utility were in fact equivalent, technically, it would be correct to call them axioms because you can derive them from one another.[22]. As noted, rationality is an "only if" condition for the existence of utility. Therefore, completeness and transitivity as the two defining components of rationality are not equivalent to utility.

The second reason why using axioms for assumptions could be misleading is behavioral in the sense that axioms hide implicit requirements that the observer imposes on the behavior of the agent being studied. Note that Gilboa et al. (2019) claim that the behavior described by utility maximization is equivalent to the axioms of completeness and transitivity (or rationality). This equivalence does not reveal the implicit requirement on behavior that the agent choose the most preferred bundle. Even if the agent is able to create the preference relation, it is not guaranteed that he will optimize (or

[21]It is not unusual to find the use of the term axioms for assumptions in formulations of economic theories of choice (von Neumann and Morgenstern, 1944; Arrow, 1950; Tversky et al., 1990; Loewenstein and Prelec, 1992; Machina and Schmeidler, 1992; Stoye, 2011; Harrison et al., 2015: Chambers and Echenique, 2018).

[22]The physicist Richard Feynman discussed axioms in a now YouTube video when explaining the difference between what he called Greek and Babylonian mathematics (Feynman: 'Greek' versus 'Babylonian' mathematics, YouTube, youtube.com/watch?v=YaUlqXRPMmY, Accessed 10/21/2023.)

maximize) with his choice, as reflected by the revealed preference notion introduced by Samuelson (1948, p. 243): "...the individual guinea-pig, by his market behavior, reveals his pattern—if there is such a consistent pattern."

The distinction I am making between axioms and assumptions [23] points to the earlier distinction I made between the subject and the observer, because while axioms could be simple facts about human behavior of the subject, assumptions are requirements that the observer imposes on the subject to create a theory so that she can analyze his behavior. Indeed, the need for optimizing an ordinal utility is for the observer to analyze the agent's choice. There is no reason for the agent, who knows his preference relation and presumably chooses his most preferred bundle, to spend extra mental efforts in figuring out an ordinal utility in order to choose the most preferred bundle that he already knew what is was. There is every reason for the observer to choose an ordinal utility function and optimize it because the agent's preference relation is unobservable for the observer and it needs to be consistently revealed by his market choice (Samuelson, 1948) in order for the observer to be able to analyze the agent's choice. Even if the agent and the observer were the same person, in contrast with the characterization of completeness and transitivity as "simple" axioms by Gilboa et al. (2019), as noted in the previous section, they are not simple because they are based on the preference relation. In the next section, I make optimizing behavior an explicit assumption where I apply my theory to analyze choice, but in its descriptive framework the theory works without an assumption on optimization.

The difficulty, or what I would call the inaccuracy, of analyzing choice based on the preference relation was also noted by Camerer et al. (2005, p. 10): "Although

[23] An axiom is "a statement or principle that is generally accepted to be true, but need not be so" (dictionary.cambridge.org/us/dictionary/english/axiom) and an assumption is "something that you accept as true without question or proof," (dictionary.cambridge.org/us/dictionary/english/assumption), Cambridge Dictionary, Accessed 10/15/2023.

economists may privately acknowledge that actual flash-and-blood human beings often choose without much deliberation, the economic models as written invariably represent decisions in a "deliberative equilibrium," i.e., that are at a stage where further deliberation, computation, reflection, etc. would not by itself alter the agent's choice." Maintaining the distinction between the subject and the observer, I present next how I apply my theory to analyze choice.

1.3 The Analysis of Choice

Neoclassical economic theory consists of the three intrinsic concepts: the preference relation, rationality; and optimization, which are taken as given abilities of the agent. Meanwhile, the observer cannot analyze the agent's choice with any of these three concepts because they are unobservable. Instead she uses utility, also known as ordinal utility. Since the observer chooses which function to use when analyzing the agent's choice, she has basically unchecked freedom to use any function she sees fit. Once the observer has decided which utility function to use, the agent's choice is predicted by the choice that maximizes such function. The implication of the process I am describing here was also noted by Becker (1962, p. 1): "...now everyone more or less agrees that rational behavior simply implies consistent maximization of a well-ordered function, such as a utility or profit function."

In this process, if the predicted choice is not supported by the data, there are a few options. One option is to look for utility functions that are believed to represent the agent's preference relation and that are consistent with the observed behavior.[24] Another option is to try and correct behavior as is done in the field of behavioral

[24] For example, Schmeidler (1989) replaced the axiom of independence in von Neumann-Morgenstern (1994) by the weaker axiom of comonotonic independence to derive expected utility without additivity and Campbell and Cochrane (1999) included habit in their utility function to explain asset pricing.

economics noted earlier.[25] Yet another option is to analyze behavior with a more basic primitive than the preference relation, as I do by applying a theory founded on subjective feeling as the primitive of behavior to explain and predict a wider variety of choices than those in neoclassical economic theory. I illustrate these choices with Figure 1.1, where I call my theory *subjective feeling theory*.

In Figure 1.1, P_0 is the set of choices studied by neoclassical economic theory and P_1 is the set of choices studied by my subjective feeling theory. Their union is $P_0 \cup P_1$ and their intersection is $P_0 \cap P_1$. $P_0 \setminus P_1$ is the set of choices from subjects with impaired brain damage affecting their ability to experience feelings and resulting in poor choices.[26] $P_1 \setminus P_0$ is the set of choices from subjects who: a) have not created or are unable to create a preference relation; b) are not rational (either because their preference is not complete, or it is not transitive, or both); c) or do not optimize with their behavior.

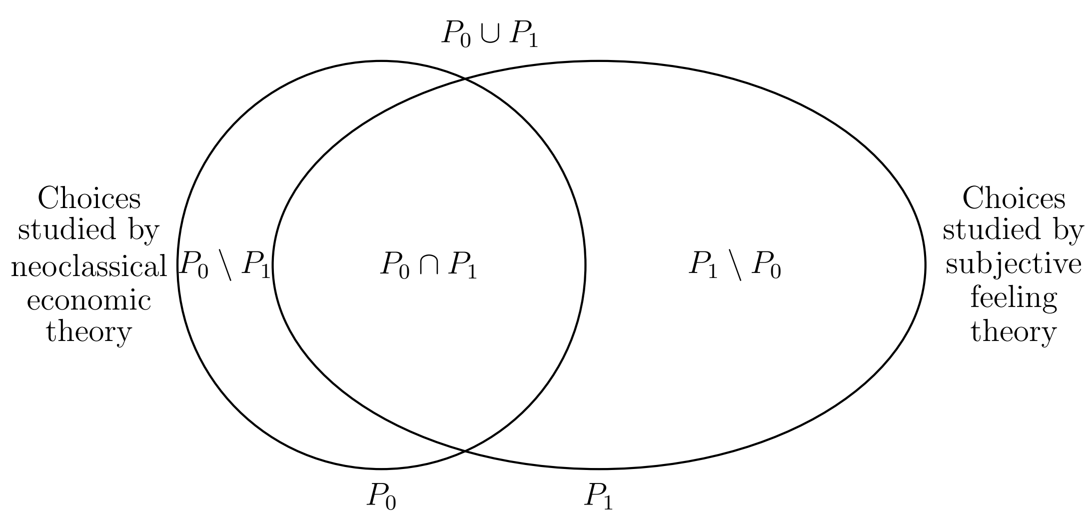

Figure 1.1: Choices studied in neoclassical economic theory and subjective feeling theory

[25]For example, incorporating psychology and sociology, Akerlof and Kranton (2000) propose a utility function that shows how people in different social categories should behave.

[26]Patients with damaged ventromedical prefrontal cortex but otherwise normal intellectual functions make poor real-life decisions (Bechara et al., 1994; Bechara et al., 2000; Moretti et al, 2009).

The larger size of P_1 relative to P_0 reflects the additional number of choices that can be analyzed in subjective feeling theory compared to neoclassical economic theory. This is more clear in the much bigger size of $P_1 \setminus P_0$ relative to $P_0 \setminus P_1$. In Figure 1.1, area $P_0 \setminus P_1$, I exclude choices that can be explained by subjective feeling theory because it includes patients with impaired ability to experience feelings. It was the existence of poor choices by such patients that led to the conclusion that: "It is not enough to "know" what should be done; it is also necessary to "feel" it" (Camerer et al., 2005, 29).

In my theory, feelings are center stage and the whole set of choices represented by P_1 is based solely on subjective feeling as the primitive. The set P_1 is such a large set that it can include choices from subjects who do not even have a preference relation. The inability, or also unwillingness, of subjects to create a preference relation was noted by Sen (1973, p. 248): "If a person chooses x rather than y, it is presumed that he regards x to be at least as good as y, and not that may be[27] he has no clue about what to choose and has chosen x because he had to choose something." And yet, the observer in neoclassical economic theory has no choice but to interpret the observed choice (not a pun) as the most preferred choice because her analytical tool to study choice is the maximization of her chosen utility. It is through the maximization of her chosen utility function to study the agent's behavior that she concludes that the utility-maximizing bundle is the most preferred choice from the agent.

As noted, the preference relation, rationality and optimization are presumed abilities of the agent and utility is the observer's analytical tool that binds them together in order to study choice in neoclassical economic theory. As also noted, the preference relation is not a basic concept and rationality is a strong assumption. If the agent is rational as assumed in neoclassical economic theory, unless utility represents his cardi-

[27] Grammatical error in original.

nal utility, there will be no benefit for this rational agent to construct an ordinal utility function in order to find the most preferred bundle because he already knows which bundle is his most preferred choice.

However, in its early stages, economic theory did not comprise the preference relation or rationality, and the meaning of utility was different from its meaning today. In the 19th century, classical economic theory consisted of two main concepts: utility; and optimization, used jointly. Utility was both a concept believed to be the guiding force of human behavior and an analytical tool used to explain an observed choice as an optimization. Although not as well-defined as the preference relation, utility in classical economic theory acted as the primitive of behavior, on the belief that it represented individual "pleasure/displeasure." For example, Bentham (1781) talked about utility/disutility, Edgeworth (1881) imagined a hedonimeter, and as noted earlier Jevons (1888) introduced the quantity of feeling.

In today's literature, when a need to distinguish between the two meanings of utility arises, utility in classical economic theory is usually referred to as cardinal utility and utility in neoclassical economic theory as ordinal utility. The preference relation and rationality, as well as the change in the meaning of utility, are products of further developments in economic theory. By the middle of the 20th century, the classical concept of (cardinal) utility as the guiding force of human behavior had been replaced by the preference relation.

Nowadays cardinal utility is rarely mentioned because it has been largely replaced by ordinal utility that is commonly referred to simply as utility.[28] Although modern/ordinal utility is a convenient analytical tool to study *rational* behavior, its use has come at the cost of requirements imposed on the agent by presuming that he has a rational preference relation that guides him to make a choice—and not any choice

[28]It is convenient to simply use the term utility if everybody uses ordinal utility.

but the optimal choice as prescribed by the observer's chosen utility function to study his behavior.

In my research, I analyze choice differently. I explain any choice with a descriptive framework where the subject is not assumed to optimize or to be rational. Then I also predict his choice with a normative framework where the subject is assumed to optimize and to be rational to some extent, by which I mean that the subject is not presumed to order as many alternatives as the agent in neoclassical economic theory. In fact, in the normative framework the subject is presumed to make only a few orderings.

Neoclassical economic theory does not have an assumption on optimization. The reason is because it is a normative theory and optimization is an inherent feature of any normative theory. In my research, I include an extra assumption on optimization when the theory of subjective feeling is applied with its normative framework where the subject's choice must be optimal, still based on a weaker version of rationality than rationality in neoclassical economic theory. An assumption on optimization is not needed when the theory of subjective feeling is applied in the descriptive framework, where the subject's choice need not be optimal.

The choice in my current research is the subject's choice of time spent on each activity. The explanation and prediction of such choice constitutes the immediate application of subjective feeling theory because time is an ingredient of its primitive. Once the choice of time spent on each activity is explained and predicted, the observer can explain and predict the subject's economic, health, social, and other choices associated with the time he spends on his activities. Both the descriptive and normative frameworks provide vast opportunities to develop new consumer and producer theories. These opportunities arise because subjective feeling theory has weaker assumptions than neoclassical economic theory as the result of using subjective feeling as a more basic primitive than the preference relation.

The descriptive framework relies on the concepts of *zero-state*, *switch-time*, *time-allocation*, and *schedule*. Given the engaged activity i, the zero-state is the instant of time t^* when intensities of feeling from non-engaged activities $f_{j|i}(t)$ are all 0, that is the instant when no intensity of feeling function from any of non-engaged activities affects affects the subject.

Definition. ***The zero-state*** *t^* is when $f_{j|i}(t^*) = 0 \; \forall \; j = 1, 2, \ldots, i-1, i+1, \ldots, n$.*

The switch-time is the instant when the subject switches the current engaged activity with another engaged activity from one of non-engaged activities. That is, the switch-time is the moment when the subject stops spending time on the current engaged activity i and starts spending time on another engaged activity $j \neq i$. The switch-time can only happen at the zero-state t^*, otherwise at least one of $f_{j|i}(t)$ would have affected the subject, in which case activity i would have been the engaged activity.

Definition. ***The switch-time*** *is the instant t^* when the engaged activity becomes a non-engaged activity and one of non-engaged activities becomes the engaged activity.*

It is important for the switch-time to happen at an instant of time (rather than an interval). A switch-time happening during an interval of time, however small it might be, would mean that a subject is engaged in more than one activity at a time, a contradiction of Assumption II. I prove that the switch-time can only happen at an instant of time.

Given the zero-states during a period starting at $t = 0$ and ending at $t = T$, a time-allocation to an engaged activity is the length of interval between any two switch-times, the initial time $t = 0$ and the first switch-time, or the last switch-time and the end of period $t = T$. The time-allocation to an activity that is not the engaged activity during an interval is 0, and if an activity is not the engaged activity over $[0, T]$ its time-allocation is then 0.

Definition. ***A time-allocation*** *to an activity* i*,* T_i*, is the length of interval between: two consecutive switch-times* t_i^* *and* t_h^**,* $t = 0$ *and first switch-time, or last switch-time and* $t = T$*. The time-allocation to an activity that is not the engaged activity is* 0*, and the time-allocation to an activity that remains non-engaged over* $[0, T]$ *is* 0*.*

Given the time-allocations, a schedule is a sequence of engaged activities over $[0, T]$.

Definition. ***A schedule*** *is a sequence of engaged activities.*

With these concepts, I explain the subject's choice of any schedule, that is any time-allocations to different activities for any sequence of engaged activities. The ability to create a sequence has been found to be unique in humans in the first empirical study that tested bonobos and humans for the stimulus sequences-hypothesis (Lind et al., 2023). The choice of the subject's schedule of time-allocations might or might not be optimal because it does not depend on the assumption of optimization. Also, the explanation of the subject's choice of a schedule is by the switch-times, which do not depend on the assumption of rationality.

In terms of what the subject needs to know in the descriptive framework, he does not need to know anything expect to feel—as given by subjective feeling—because when intensities of feeling from non-engaged activities are all 0 he will switch from the current engaged activity to another activity, for example, when he stops reading and starts eating because he feels hungry. Therefore, compared with Camerer et al. (2005), for the subject in my descriptive framework, it is enough to "feel"; it is not necessary to "know" what should be done. With my explanation of the subject's choice, I answer the second question raised in the beginning of the previous section by noting that the subject will switch from one activity to another when the intensities of feeling from non-engaged activities are all 0.

The normative framework includes an assumption on optimization, and it relies on the concepts of *integrated experience* from the engaged activity i denoted by E_i, *overall experience* from schedule s denoted by E_s, and *conditional rationality.*

E_i denotes the accumulation of the subject's intensity of feeling from the engaged activity, that is during the time-allocation T_i. If activity i becomes the engaged activity at the switch-time t_i^* and activity h becomes the engaged activity at the next switch-time t_h^*, I measure the accumulation of the subject's intensity of feeling from the engaged activity during the time-allocation T_i to the engaged activity i as the integral of $f_i(t)$ from t_i^* to $t_h^* - t_i^*$. Since T_i is allocated to the engaged activity i, I take integrated experience to be the accumulation of the intensity of feeling only from the engaged activity but not non-engaged activities.

Definition. ***Integrated experience*** $E_i = \int_{t_i^*}^{t_h^*} f_i(t)dt$ *is the subject's accumulation of the intensity of feeling from the engaged activity during interval* $t_h^* - t_i^*$*, that is during* T_i*.*

The next concept, overall experience E_s, represents the subject's integrated experience over $[0, T]$ from a schedule s, that is during all the time-allocations to the engaged activities in the sequence of s. For a finite number $M-1$ of switch-times during $(0, T)$, there are M (including the last activity) engaged activities in a sequence s denoted by M_s. Note that M_s does not necessarily equal n because the subject might be spending time only on some of his activities or he might repeat spending time on some activities over $[0, T]$. For example, if the subject engages once in only $n-1$ activities, then $M_s = n - 1 < n$, and if the subject engages twice in an activity and once in the other $n-1$ activities, then $M_s = n + 1 > n$. Given M_s, E_s is the sum of E_i over $[0, T]$.

Definition. ***Overall experience*** $E_s = \sum_{i=1}^{M_s} E_i$ *is the subject's integrated experience from schedule* s *over* $[0, T]$*.*

I predict the subject's choice with an additional fourth assumption on optimization that involves a comparison of overall experiences of different schedules over $[0, T]$, and the concept of conditional rationality, which requires that the subject order n initial conditions at the initial time $t = 0$ for each intensity of feeling function $f_i(t)$, $i = 1, 2, \dots n$. From the uniqueness of engaged activity functions $f_i(t)$, if the subject knows how he will feel if he starts to engage in each activity, then he knows the (finite) number N of all overall experiences from his possible schedules over $[0, T]$.

Note that N is not necessarily equal to $n!$. If the subject engages once in every activity in any possible schedule, then $N = n!$ because there are $n!$ possible schedules. However, M_s might be greater than n, as explained earlier. If M_s^* is the maximum number of engaged activities for some schedule s over $[0, T]$, then $N \leq M_s^*!$. In any case, given that switch-times are determined when intensities of feeling from non-engaged activities are all 0 and the intensity of feeling from each engaged activity is (uniquely) determined by the initial conditions at $t = 0$, the only requirement on the subject's cognitive ability is to order n initial conditions for intensities of feeling from engaged activities.

Definition. *Given a maximum number of switch-times $M_s^* - 1$, **conditional rationality** is the cognitive ability of the subject to know n initial conditions for his intensities of feeling from engaged activities.*

With this (weak) requirement of rationality, I present the fourth assumption where I assume that a subject who satisfies conditional rationality maximizes overall experience.

Assumption IV The conditionally rational subject maximizes overall experience.

With Assumptions I-IV, I predict the subject's choice of time-allocations to activities. In addition, I also predict the order of engaged activities, or the order of time-allocations because of the predicted schedule with the maximum overall experience.

In terms of what the subject needs to know in the normative framework, he only needs to know how feels when he starts to engage in each activity. Therefore, compared with Camerer et al. (2005), for the subject in my normative framework, it is not enough to "feel"; it is necessary to "know" only initial conditions for his intensities of feeling from engaged activities. With my prediction of the subject's choice, I answer the third question raised in the beginning of the previous section by noting that the subject chooses the schedule that optimizes his overall experience. Also, note that my theory explains how the subject optimizes his overall experience by the optimizing mechanism consisting of the switch-times and the (finite) ordering of initial conditions for intensities of feeling from engaged activities. In contrast, neoclassical economic theory does not explain how the agent optimizes utility.

Thanks to the weaker assumption of conditional rationality as opposed to the stronger assumption of rationality, the choice predicted by subjective feeling theory is possibly more accurate than the choice predicted by neoclassical economic theory. Thanks to these weaker assumptions, by predicting time-allocations of the optimal schedule of activities, I also predict the order of engaged activities, something that is not possible to achieve when applying neoclassical economic theory to predict time-allocations.

But knowing the sequence of engaged activities has many advantages. Imagine how useful the order of activities is for an agricultural expert to learn when a farmer will switch from harvesting the crop to starting a degree, an environmental researcher to learn when an horticulturalist will switch from cultivating the garden to driving the truck, or a financial analyst to learn when an investor will switch from holding stocks to investing in real estate. By defining these descriptions of what the subject does as activities on which he is spending time, subjective feeling theory provides the observer with the opportunity to explain the choice of time-allocation to engaged activities and to predict the order of these activities.

Time-use is important and it has been studied in neoclassical economic theory as well (Becker, 1965). Although economic theory can predict time-allocations, it does so with stronger assumptions than mine because rationality in neoclassical economic theory requires from the agent to order infinitely many bundles whereas conditional rationality in subjective feeling theory requires from the subject to order a finite number of initial conditions. The larger the number of activities n the greater the cognitive ability required to order their initial conditions for intensities of feeling from engaged activities. Regardless how big n is, it is always finite by Assumption I. So the subject in my theory is not required to order infinitely many bundles, as is the case when the agent in neoclassical economic theory orders infinitely many bundles of time allocations even though the number of alternatives is finite.

To illustrate the practical benefits of building a theory with weaker assumptions, imagine an app where you can enter the number of activities n, a constant for your $\alpha(t)$, the time period $[0, T]$, $f_i(0)$ and a representative $f_{j|i}(0)$ for each of your activities. This mind-app is a metaphor for the brain, but it can actually be programmed quite easily with the unique functions obtained by my theory. Then when you wake up in the morning and simply enter these values, your mind-app will give you your optimal schedule. You can add options to your mind-app to give you your optimal schedule even if you want to enter a function rather than a constant for your $\alpha(t)$, or discounting and uncertainty for each activity.

As an example, suppose that the subject is deciding in the morning how to spend the next two hours on three (distinct) activities: walking the dog; reading his newspaper; and drinking coffee. Let's call these activities: Walk; Read; and Coffee. Suppose that today he feels like enjoying walking the dog and drinking coffee, but not so much reading any news (although he does want to stay in touch with what is going). In this scenario, the decision for the subject is to decide how much time to spend on Walk, Read, and Coffee. If the subject does not care about optimizing his overall experience (he just wants to go with the flow), he follows a random schedule among his possible schedules given by the switch-times that tell him how much time to spend on each activity today. If his decides to optimize, then he makes an extra effort to make three evaluations on how he feels when he starts: walking the dog; reading his newspaper, and drinking coffee, and he follows the schedule given by his mind-app that tell him how much time to spend on Walk, Read, and Coffee today.

As noted earlier, the explanation and prediction of time-allocations in my research is the immediate application of my theory because time is the defining ingredient of subjective feeling. Now suppose that when spending time on these activities, the subject can also eat (consume), work (produce), or maybe both (hence consume and produce simultaneously). Then with the descriptive and normative frameworks, choices of goods consumed and/or produced could also be explained by their association with non-optimal and/or optimal choices of time spent on each activity. I will analyze such choices in my future research.

Before I present in the next chapter my theory of subjective feeling, a note on a recent finding about human behavior that I discovered from "The Heart of Mathematics" book by Burger and Starbird (2013) where they quote Galileo Galilei saying that: "...where the senses fail us, reason must step in" (p.3). To the extent that my descriptive framework represents the role of senses and my normative framework represents the role of reason, the great Galileo understood human behavior long before I was able to understand it with my subjective feeling theory, which I discuss in the next chapter.

Chapter 2

The Theory

The preference relation is represented by a (continuous) utility function if it is rational and continuous[1] on a choice set X (Mas-Colell et al., 1995, p. 47). Subjective feeling is measured by the intensity of feeling $f_i^*(t)$ if the rate of change in intensity of feeling from an activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities. Utility is ordinal, and it is subject to the observer's choice of which function to use; hence it *represents* the preference relation.

In contrast, intensity of feeling is unique, and it is not subject to the observer's choice; hence it only represents but *measures* subjective feeling. Since the concepts of utility and intensity of feeling are tools at the disposal of the observer to study human behavior, I have chosen to present my theory through a comparative discussion of these concepts.

Differences between utility and intensity of feeling are theoretical and applicable. By theoretical, I mean differences in terms of requirements to obtain a function that the observer can use to analyze choice. By applicable, I mean differences in terms

[1]The preference relation is continuous if it is preserved under limits (Mas-Colell et al., 1995, p. 46).

of requirements to use this function in applied work and/or further theoretical developments. I discuss theoretical differences in this chapter and applicable differences in the next chapter. The discussion of theoretical differences is presented in three parts: 1) primal, by which I mean differences in terms of primitives; 2) existential, by which I mean differences in terms of assumptions; and 3) functional, by which I mean differences in terms of specifications.

The mathematical presentation of functional differences in this chapter includes only intensities of feeling from engaged activities because they are the most comparable with utility. As seen in the previous chapter, the measure of subjective feeling includes both the intensity of feeling from the engaged activity and intensities of feeling from non-engaged activities. There is no comparable representation of intensities of feeling from non-engaged activities in neoclassical economic theory because bundles that are not chosen have no impact on utility. In contrast, activities in which the subject is not currently engaged while spending time on the engaged activity continue to have impact on his choice of time-allocations to activities. The role intensities of feeling from non-engaged activities is discussed in the next chapter where I apply my theory to explain and predict the choice of time-allocations.

2.1 Primal Differences

Utility and intensity of feelings are both functions that the observer can use to analyze choice, but they represent different primitives. Utility is an ordinal representation of the preference relation, which has been the primitive of human behavior for economists for about 100 years, based on an early axiomatic presentation by Frisch (1926)[2] formalizing

[2] It is not easy to pinpoint the exact date when the preference relation was formalized as a mathematical relation. The earlier formalization by Frisch in 1926 was republished later in Metroeconomica in 1957.

it as a mathematical relation.

In contrast, intensity of feeling is a unique representation of subjective feeling, which I propose as the primitive to study human behavior by combining experience, time and activities. Although it is a new primitive, elements of subjective feeling can be traced back to classical economic theory. For example, aspects of experience over time are in the notion of utility as a property of an object to produce benefit, pleasure, etc. evaluated by the pleasure's (or pain's) intensity, duration, uncertainty and remoteness by Bentham (1781), and the idea of a hedonimeter as a psycho-physical machine to continuously measure the pleasure that is experienced and determined by the individual's consciousness by Edgeworth (1881). More directly, subjective feeling can be traced to the notions of intensity of feeling and quantity of feeling introduced by Jevons (1888), illustrated in Figure 2.1 below.

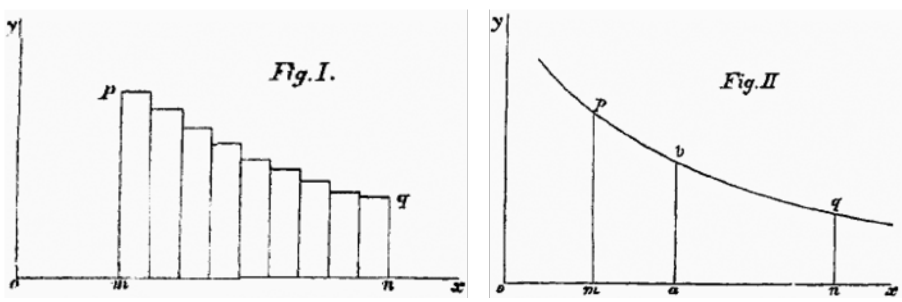

Figure 2.1: Jevons' Quantity of Feeling

In his *Fig. I*, Jevons noted that the quantity of feeling during each minute may be represented by a rectangle with base one minute and height proportional to intensity of feeling during that minute. While pointing out that it is artificial to assume that the intensity of feeling varies by such sudden steps at regular time intervals, he noted that the error becomes smaller and smaller as time intervals become shorter shorter, and the error is avoided when the interval becomes infinitely short. According to Jevons, the proper representation of the variation of feeling is by a curve, in his *Fig. II*, where each point on the curve indicates the intensity of feeling at a moment in time and the whole quantity of feeling during a given time interval is measured by the area under the curve during this interval. For example, the quantity of feeling during interval *mn* is measured by *pmnq* and during *ma* by *pmab*.

Intensity of feeling and quantity of feeling, as envisioned by Jevons, correspond to intensity of feeling $f_i(t)$ and integrated experience E_i, respectively. However, intensity of feeling in my theory includes intensities of feeling from both the engaged activity i and non-engaged activities $j|i$.

Aspects of experience over time in subjective feeling can also be found in the concept of experienced utility by Kahneman et al. (1997). However, their theory is fundamentally different from my theory. Their theory extends decision (or ordinal) utility: "We propose a formal normative theory of the total experienced utility of temporally extended outcomes [TEOs]. … *Decision utility* is a measure on TEOs which is inferred from choices, either by direct comparisons of similar objects or by indirect methods, such as elicited willingness to pay " (Kahneman et al., 1997, pp. 375-377, emphasis in original).

While experienced utility theory in Kahneman et al. (1997) is based on the preference relation of outcomes extended by the time component as the primitive, my theory is based on subjective feeling as the primitive. Therefore, unlike experienced utility in Kahneman et al. (1997), the intensity of feeling in my theory is independent from utility (or decision utility). Furthermore, my theory is applied in both descriptive and normative frameworks, and is thus not confined to being only a normative theory.[3]

[3]Given that experienced utility by Kahneman et al. (1997) is a known representation of the concepts by Bentham (1781), Edgeworth (1881), and Jevons (1888), in the beginning of my research I used the term experience utility for my primitive, which was limited to the hedonic experience from the engaged activity and hence does not represent in full the role of feelings. Hedonic experience represents experience only from the engaged activity whereas subjective feeling represents the behavioral role of feelings through instantaneous experience of the subject's mind and body from the engaged activity as well as non-engaged activities.

Although the agent in a neoclassical economic setting might derive individual experience from alternatives under consideration, utility leaves out his experience for two reasons: the preference relation is not defined in terms of time and utility cannot explain how outcomes are formed. The first reason why utility cannot represent experience is because the preference relation is not defined in terms of time, that is time is not its defining ingredient. It might be tempting to think of analyses of choices that involve time with utility as counterexamples, but they only make the inability of utility to represent individual experience more evident. I present next two such cases, choice of time allocated to different alternatives and inter-temporal choice.

In the case of analyzing choices of time allocated to alternatives with utility, the agent with the preference relation as the primitive orders bundles of intervals of time that he allocates to different alternatives rather than instances of time. In contrast, the subject with subjective feeling as the primitive accumulates intensity of feeling at each instant to create his individual experience, measured by the integrated experience E_i. A utility function can only represent alternatives as ordered elements of a mathematical relation that are usually called outcomes (Kahneman et al., 1997; Dolan and Kahneman, 2008; Berridge and O'Doherty, 2014). These outcomes are bundles of times allotted rather than processes over time. As seen in the previous chapter, experience is not an "instant picture" but a process over time (Damasio, 2018). Utility represents the ordering of outcomes as instant pictures and not as temporal processes leading to these outcomes, because outcomes are bundles of time allocated as a mathematical relation and not as sources of individual experience.

In the case of analyzing choices of goods over time in inter-temporal one-shot or recursive models with utility, these choices do not depend on time. In one-shot models, time serves as an index to distinguish between goods in two different periods. A good x at period t is considered different from the same good x at period $t + 1$, denoted by two different variables x_t and x_{t+1}. In these models, choice comes from maximizing a utility function of these variables. The solution of models with these variables does not include experience because x_t and x_{t+1} do not measure any interval of time during which individual experience occurs.

In contrast with one-shot models, goods in recursive models— with finite or infinite periods—are the same goods in all periods even though they are still indexed by period. However, the assumptions that are needed to solve these models make time irrelevant and the inter-temporal choice of goods does not depend on time at all. To illustrate that inter-temporal choice makes time irrelevant take the classic model by Stokey et al. (1989):

$$\max_{(c_t,k_{t+1})_{t=0}^{\infty}} \sum_{t=0}^{\infty} \eta^t U(c_t)$$

$$\text{s.t.} \quad c_t + k_{t+1} \leq f(k_t),$$

$$c_t, k_{t+1} \geq 0, \quad t = 0, 1, \cdots,$$

$$\text{given } k_0 > 0,$$

where c is consumption, η a discounting factor,[4] k the capital stock, $f(k)$ total supply of goods available per worker and $U(c_t)$ is a utility function that represents the preference relation over inter-temporal consumption bundles for a representative household.

In this sequential problem, period-to-period utility remains constant and it is equal to the discounting factor η (that is $U(c_{t+1})/U(c_t) = \eta$). So the inter-temporal choice does not depend on time. Further, when the sequential problem leads to the so-called functional equation form, the choice does not depend on time either. Taking v as

[4] In their work Stokey et al. (1989) use β for the discounting factor but because I use β when extending my model to include discounting and uncertainty, I denote here their discounting factor by η.

equal to the maximum value of the sequential problem above, Stokey et al. (1989) show that there exists a unique function v called a value function that depends only on the capital stock k, $v(k) = \max_{c,y}[U(c) + \eta v(y)]$ ($c + y \leq f(k)$, $0 \leq y \leq f(k)$, $c, y \geq 0$), providing the solution to the recursive model. Note how the variables in this solution are no longer indexed by time, because time is irrelevant, and hence the agent's inter-temporal choice does not depend on time.

The second reason why utility does not represent experience is because it cannot explain how outcomes are formed. I illustrate why this is the case by comparing lotteries of amounts of money for uncertain events. Suppose that the observer uses the classic model to evaluate these lotteries, the expected utility from von Neumann and Morgenstern (1944).

While it is clear that outcomes for lotteries are (uncertain) amounts of money, it is not clear what outcomes are for the expected utility, or utility in general. If the observer uses the expected utility, the question is: Are outcomes (uncertain) amounts of money, or utilities from these amounts? If we take outcomes to be amounts of money and the preference relation defined on lotteries of amounts of money as in Mas-Colell (1995, p. 173), then the choice set consists of lotteries but not amounts of money directly. Since the observer cannot observe the preference relation on lotteries, she uses the expected utility.

However, the expected utility is obtained from utilities from amounts of money directly rather than lotteries. So if the observer uses the expected utility, the choice set must consist of utilities but not lotteries because utilities are not defined from lotteries but from amounts of money directly. If the choice set is utilities from amounts of money directly but the preference relation is on lotteries rather than these amounts, utilities do not represent this preference relation because they do not maintain the ordering of lotteries. If utilities represent amounts of money but not the preference relation on lotteries, then outcomes cannot be amounts of money directly as in Mas-Colell et al. (1995) because the expected utility represents utilities from amounts of money but not amounts directly.

So when using the expected utility, outcomes are utilities from amounts of money. Then the next question is: Are utilities from amounts of money ordinal or cardinal? It makes no sense for utilities from amounts of money to be ordinal for the agent. Amounts of money are already ordered when expressed with a given currency, say in US dollar. Using ordinal utilities for amounts expressed in US dollars is equivalent to expressing these amounts in another currency, say the British pound, which makes no sense for the agent.

It remains that utilities must be cardinal for the agent. If utilities were cardinal, they would give the agent individual experience, which the preference relation on lotteries does not explain how it is formed because it orders lotteries but not cardinal utilities. Furthermore, since utility in neoclassical economic theory is ordinal but not cardinal, the expected utility would also be an ordinal representation of the preference relation on lotteries to predict the agent's choice of the optimal lottery. But if the expected utility is ordinal, then utilities from amounts of money cannot be cardinal. Hence utilities must be ordinal, and ordinal utility is unable to explain how outcomes are formed because it is not cardinal.

However outcomes are defined, utility cannot explain how they are formed, but concluding that utilities from amounts of money in a lottery are ordinal[5] points to the distinction I make between the observer and the agent. It is the observer who chooses ordinal utilities for the agent to be able to predict his optimal lottery. It may come as no surprise that utility only maintains the ordering of the preference relation but cannot explain how outcomes are formed because the preference relation is void of

[5]The same conclusion follows from the same reasoning if lotteries did not have money for uncertain events but other outcomes.

individual experience.

In contrast, the intensity of feeling explains how the outcome of individual experience is formed. In my research, the outcome is measured by the overall experience from a schedule E_s formed by the sequence of integrated experiences E_i as integrals of intensity of feeling functions over time-allocations in that schedule. The observer who chooses utility is limited because she can only look at outcomes and not where they come from. The source of outcomes is "blocked from the observer's view" when using utility because the preference relation does not explain from neuroscience or other findings how outcomes are formed. However, the observer who chooses the intensity of feeling is not limited because she can look at both outcomes and where they come from. The source of outcomes is open when using the intensity of feeling because subjective feeling explains how outcomes are formed.

I illustrate the advantage of knowing how outcomes are formed with a hypothetical example of river flows by Professor C. Ford Runge. Suppose that the observer wants to analyze the behavior of a grand-source where a given number of rivers originate from. Suppose that these rivers form cascades as they flow. The grand-source is a metaphor for the agent or the subject and rivers are a metaphor for alternatives or activities. Let's assume that each river's rate of flow is proportional to the difference between the force of its own current and sum of forces of other rivers' currents (as in my Assumption III). A popular tool called utility allows the observer to measure the volume from every river by standing at the bottom of cascades where she can see the water coming down but not the streams flowing. Imagine now if there is a new tool called the intensity of feeling allowing her to measure the volume from every river by standing at the top of cascades where she can see both the water coming down and the streams flowing. I would choose the intensity of feeling.

2.2 Existential Differences

Given that the preference relation is represented by a (continuous) utility function if it is rational and continuous (Mas-Colell et al., 1995), the conditions for the existence of utility can be grouped into three requirements or assumptions: I^u) Finite number of goods; II^u) Choice of a single bundle; and III^u) Rational and continuous preference relation, where the superscript u denotes the assumptions needed to derive a utility function. The conditions for the existence of intensity of feeling are also grouped into three assumptions, which I write in abbreviated form as: I^f) Finite number of activities; II^f) Choice of the engaged activity; and III^f) Proportionality in the rate of change, where the subscript f denotes the assumptions needed to derive intensity of feeling functions.

Assumption I^f is similar to I^u. Assumptions II^f and II^u are similar only to the extent of the single choice, but they are substantially different. A utility function takes its value from every bundle of alternatives whereas intensity of feeling functions from the engaged activity and non-engaged activities take their values from every instant of time. Practically, II^u has no impact on I^u because choosing a bundle does not change the number of alternatives under consideration. If there are n alternatives, for every choice, the number of alternatives remains n. Although the number of activities remains the same, II^f works together with I^f because choosing to spend time on an activity changes the number of activities an individual has under consideration. If there are n activities, for every activity the subject engages in, there are $n-1$ other activities to engage in, as in the non-engaged system.

In my theory, non-chosen activities continue to influence the subject when he is engaged in an activity. But these non-engaged activities cannot give intensity of feeling in the same way as the engaged activity because from II^f the subject spends time on a single activity. For this reason, I use the term non-engaged activities for the other activities the subject has under consideration while engaging in an activity.

There are other differences. Assumption II^u implies that for each bundle of alternatives there is a a utility function *value* that represents this bundle, and because of the ordinal nature of utility there are many such values to represent the same (fixed) bundle. However, assumption II^f implies that there are intensity of feeling *functions* that represent intensities of feeling from the engaged activity as well as non-engaged activities. Given initial conditions, these intensities of feeling functions are unique, so there is only one (cardinal) intensity of feeling function for the engaged activity as well as each of non-engaged activities while the subject is spending time on the chosen engaged activity. Therefore, intensity of feeling functions measure subjective feeling.

Assumptions III^f and III^u are similar only to the extent of a technicality because my theory assumes differentiability (with the rates of change) and neoclassical economic theory assumes continuity of the preference relation. Also, $\alpha(t)$ in my theory can be continuous or have a finite number of jump discontinuities during $[0, T]$.

If the subject in the earlier example is considering spending time on walking the dog, drinking coffee, and reading the news while in his cabin, and suddenly notices a bear outside the window, then $\alpha(t)$ could change. If he perceives a threat, the activities have changed, and hence my assumption I has changed. If he does not perceive a threat, and the activities remain the same, nevertheless it could be a new value for $\alpha(t)$ due to a jump discontinuity that might have occurred at the moment when the subject noticed the bear.

However, assumptions III^f and III^u are intrinsically different because assumption III^f is independent of rationality, that is a complete and transitive preference relation (Mas-Colell et al., 1995). Any usage of utility automatically assumes rationality because utility is an only-if condition for rationality (Mas-Colell et al., 1995). The widespread usage of utility makes the assumption of a rational agent the predominant assumption in economic and other analyses. However, rationality has been repeatedly challenged (May, 1954; Tversky, 1969; Elster, 1979; Kahneman and Tversky, 1984; Schelling, 2006).

Assumption III^f is weaker than III^u because assuming that the rate of change in the intensity of feeling from each activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities up to the coefficient $\alpha(t)$ is practical. Presumably, values of the coefficient of proportionality $\alpha(t)$ can come from clinical measurements of the organism, but they need not. The values of $\alpha(t)$ do not have to be precise as long as they are sufficient—and not perfect—to represent the state in which the subject feels his organism is when making a choice. All that is needed from $\alpha(t)$ for the observer is a value (or values if it changes over period $[0, T]$) for a working assessment of how the subject feels[6] when planning on spending time on a given number of activities.

For all practical purposes, without loss of generality, I normalize $\alpha(t)$ by three states:

$$\alpha(t) = \begin{cases} (0,1) & \textbf{State 0}.\ \text{Feels not so good: lower value, worse one feels} \\ 1 & \textbf{State 1}.\ \text{Feels good: normal, average, or usual way one feels} \\ (1,\infty) & \textbf{State 2}.\ \text{Feels really good: higher value, better one feels} \end{cases}$$

Making State 2 unbounded allows for values associated with certain states such as ecstatic ones. With the range of values for $\alpha(t)$ by these three states, all the observer needs to know is a self-assessment by the subject of the state he is in and an associated value for that state. For example, the observer can ask the subject "How do you feel?"

[6] It is striking how often the question "How do you feel?" can be heard in medical settings. Perhaps how the patient answers this question gives the healthcare provider useful information.

and then ask him to put a value if he is in State 0 or 2 (because State 1 already has 1 assigned to it).

Evaluations of states from self-assessments are not uncommon. For example, Bradley and Lang (1994) used the so called self-assessment manikin (SAM)[7] to evaluate pleasure, arousal, and dominance experienced by 78 subjects on an 18-point scale, Redelmeier and Kahneman (1996)[8] used self-assessment to evaluate the intensity of pain experienced by two colonoscopy patients on a 10-point scale, and Lee et al. (2021) used self-assessment to evaluate the intensity of arousal, current emotion, future emotion, etc. experienced by 32-34 participants on a 7-point scale.

Given that $\alpha(t)$ represents the informational role of feelings, as in the previous chapter, its value (either from self-assessments or clinical measurements) cannot be 0. If $\alpha(t) = 0$, then feelings have no role. In my research, individuals with $\alpha(t) = 0$ are individuals who do not experience feelings. I described these individuals in the previous chapter as patients with impaired ability to experience their feelings and their choices fall in the set $P_0 \setminus P_1$ (Figure 1.1), which does not include choices that can be explained by my subjective feeling theory.

Likewise, intensity of feeling from the engaged activity i and intensities of feeling from non-engaged activities $j|i$ cannot be all 0 at the same time because they represent the behavioral role of feelings, that is either $f_i(t) \neq 0$ or $f_{j|i}(t) \neq 0$ for at least one j. As Damasio (2018, p. 120) noted: "A neutral, plain account of feelings as perceptual maps/images misses these critical ingredients: their valence and their power to capture one's attention." When an intensity of feeling (from either engaged or non-

[7] "The Self-Assessment Manikin (SAM) is a non-verbal pictorial assessment technique that directly measures the pleasure, arousal, and dominance associated with a person's affective reaction to a wide variety of stimuli" (Bradley and Lang, 1994, p. 49).

[8] Reported in Kahneman et al. (1997).

engaged activity) is 0, its valence[9] is neutral. If $f_i(t) = 0$ *and* $f_{j|i}(t) = 0, \forall j = 1, 2, \ldots, i-1, i+1, \ldots n$, that is intensities of feeling from the engaged activity and non-engaged activities are all 0, feelings have no role to play. If all intensities of feeling are 0, subjective feeling has no power as the primitive to capture the subject's attention by urging or guiding him to spend time on any activities.[10]

If the valence of feeling from an (engaged or non-engaged) activity is positive, I represent it with a positive value of intensity of feeling, and if is is negative, I represent it with a negative value of intensity of feeling. As noted, if the value of intensity of feeling from an activity is 0, it has neutral valence and that activity has no power or influence on the subject.

Definition. ***The Valence*** *of feeling from an activity is positive if* $f_i(t) > 0$ *or* $f_{j|i} > 0$ *and negative if* $f_i(t) < 0$ *or* $f_{j|i} < 0$ *for at least one* $j = 1, 2, \cdots, i-1, i+1, \cdots, n$. *The valence of feeling is neutral if* $f_i(t) = 0$ *or* $f_{j|i} = 0$ *for any activity.*

Assumption III^f is weaker than III^u because assuming that the rate of change in the intensity of feeling from an activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities up to $\alpha(t)$ is not only more practical but also more realistic than rationality. As noted, in practice, the subject only needs to how he feels for $\alpha(t)$ to have a value. In reality, since $\alpha(t) > 0$, as long as the valence from an activity acts on the subject with a greater force than the combined valences from other activities, the rate of change in the intensity of feeling from that activity is positive, otherwise it is negative or zero. This dynamics for each system is as follows:

The engaged system. $\dot{f}_i > 0$ if $f_i(t) > \sum f_k(t)$, otherwise $\dot{f}_i \leq 0$

[9] Valence is "1. in the field theory of Kurt Lewin, the subjective value of an event, object, person, or other entity in the life space of the individual. An entity that attracts the individual has **positive valence**, whereas one that repels has **negative valence**," dictionary.apa.org/valence, American Psychological Association, boldface in original, Accessed 11/6/2023.

[10] If at some τ, $f_i^*(\tau) = 0$, but $f_i(\tau) \neq 0$ or $f_{j|i}(\tau) \neq 0$ for at least one j, feelings still have a role to play.

The non-engaged system. $\dot{f}_{j|i} > 0$ if $f_{j|i}(t) > \left(\sum f_{l|i}(t) + f_i(t)\right)$,

otherwise $\dot{f}_{j|i} \leq 0$

As in the example of the previous chapter, suppose that the subject is deciding how to spend the next two hours on Walk; Read; and Coffee, and at the initial time $t = 0$ (in the morning when deciding) he feels like enjoying walking the dog (Walk) and drinking coffee (Coffee), but does not feel anything positive or negative about the news (Read). At $t = 0$ when he starts to engage in any of these activities, let intensities of feelings be $f_W(0) = 2$ for Walk, $f_C(0) = 1$ for Coffee, and $f_R(0) = 0$. Walk and Coffee have positive valences, but walking the dog is more enjoyable than drinking coffee, and Read has neutral valence. Since $\alpha(t) > 0$, as long as $f_C(t) > f_W(t) + f_R(t)$, $\dot{f}_C > 0$, which means that he would enjoy drinking coffee, otherwise he will not. Similarly, for $\dot{f}_W, \dot{f}_R$. Suppose the subject starts the day drinking coffee (and thinking what to do next). While drinking coffee, as long as $f_{W|C}(t) > f_{R|C}(t) + f_C(t)$, $\dot{f}_{W|C} > 0$, which means that he would would enjoy walking the dog (while drinking the coffee), otherwise he would not. Similarly, for $\dot{f}_{R|C}$. In the next section, I explain how intensities of feeling functions from engaged activities are obtained.

2.3 Functional Differences

Utility is an unspecified function that the observer chooses to predict the agent's choice, and its specifications depend on the observer's goals. For example, Epstein and Stanley (1991) generalized conventional time-additive expected utility, Fuhrer (2000) included current consumption relative to past consumption, Andersen et al. (2008) used a statistical specification involving latent trade-off between long-run optimization and short-run temptation, and Miettinen et al. (2020) investigated six functional forms to study behavior in a sequential prisoner's dilemma setting.

In contrast, intensity of feeling functions for engaged activities as well as non-engaged activities are unique, and their specifications do not depend upon the observer's goals. Intensities of feeling functions are unique because they belong to the sole family of functions that solve the engaged system and, as seen in the next chapter, non-engaged system as well. Given initial conditions, the specifications of functional forms for intensities of feeling are the unique (the only) specifications representing subjective feeling.[11] Hence, intensities of feeling measure the primitive in my research by $f_i^*(t)$ for any i. If $\mathbf{f}(t) = (f_1(t), f_2(t), \ldots, f_n(t))$ is a vector of intensities of feeling from engaged activities, then $\mathbf{f}(t)$ is a vector-function from $\mathbb{R} \to \mathbb{R}^n$ in general. For a given period of time, $\mathbf{f}(t)$ is a function from $[0, T] \to \mathbb{R}^n$. In contrast, utility is a function from $\mathbb{R}^n \to \mathbb{R}$ in general, or $\mathbb{R}_+^n \to \mathbb{R}$ if alternatives are non-negative such as consumption goods. The engaged system is:

The engaged system. $\dot{f}_i = \alpha(t)\left(f_i(t) - \sum f_k(t)\right), \quad \forall i = 1, 2, \cdots n;\ k \neq i$

As seen from this system, conceptually my theory starts from a position where we do not know the subject's intensities of feeling from engaged activities. However, I can see that he spends time on an activity and as time goes by he switches to another activity. It is in this conceptual setting where the intensity of feeling from each activity i pulls the subject to engage in i and intensities of feeling from other activities push him away from i.

Earlier I noted that positive values of intensities of feeling measure positive valence and their negative values measure negative valence. If all intensities of feeling are positive, the subject enjoys or finds pleasure from engaging in any of his activities, and values of intensities of feeling show how pleasurable activities are. However, if the

[11] A note on my use of the term "unique." By unique functions, or unique family of functions, I mean the same functional form up to constants or linear combinations. For example $y = C^* e^{0t} = C^*, y = C^0 e^{2t}, y = C^* + C^0 e^{2t}$ are unique functions for any C^*, C^0. By unique specification, I mean the only functional form. For example, $y = 2, y = 3e^{2t}, y = 2 + 3e^{2t}$ are all different specifications.

intensity of feeling from any activity is negative, the subject actually finds displeasure from that activity, and values of the intensity of feeling show how displeasurable the activity is.[12] There are displeasurable activities, and yet the subject spends time on them such as a job that he hates but he does anyway.

In matrix notation, the engaged system can be written as the product of $\alpha(t)$ and a circulant matrix A with 1 in the main diagonal and -1 for other elements, and $\mathcal{A}(t) = \alpha(t)A$.

The engaged system in matrix notation.

$$\begin{bmatrix} \dot{f}_1 \\ \dot{f}_2 \\ \vdots \\ \dot{f}_n \end{bmatrix} = \alpha(t) \begin{bmatrix} 1 & -1 & \cdots & -1 \\ -1 & 1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix} = \begin{bmatrix} \alpha(t) & -\alpha(t) & \cdots & -\alpha(t) \\ -\alpha(t) & \alpha(t) & \cdots & -\alpha(t) \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha(t) & -\alpha(t) & \cdots & \alpha(t) \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix}$$

$$\dot{\mathbf{f}} = \alpha(t)A\mathbf{f}(t) = \mathcal{A}(t)\mathbf{f}(t)$$

Since A is a real symmetric matrix, it has a complete set of (orthogonal) eigenvectors and corresponding eigenvalues (some of which may be repeated). If $K = \begin{bmatrix} \mathbf{k}^1 & \mathbf{k}^2 & \ldots & \mathbf{k}^n \end{bmatrix}$ denotes the matrix with eigenvectors of A, $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$ the diagonal matrix with its corresponding eigenvalues, $\tilde{\alpha}(t) = \int \alpha(t)dt$, and $\tilde{\Lambda} = \tilde{\alpha}\Lambda$,

[12] If values of intensities of feeling turn from positive to negative, they show that the activity turns from pleasurable to displeasurable, and vice versa, such as when the stomach hurts from eating too many sweets.

then in matrix notation:

$$K = \begin{bmatrix} k_1^1 & k_1^2 & \cdots & k_1^n \\ k_2^1 & k_2^2 & \cdots & k_2^n \\ \vdots & \vdots & \ddots & \vdots \\ k_n^1 & k_n^2 & \cdots & k_n^n \end{bmatrix}, \text{ and } \tilde{\Lambda} = \begin{bmatrix} \lambda_1\tilde{\alpha} & 0 & \cdots & 0 \\ 0 & \lambda_2\tilde{\alpha} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n\tilde{\alpha} \end{bmatrix}$$

Let $\mathbf{c} = (c_1, c_2, \ldots c_n)$ denote a vector of constants and e the exponential function. Then my Assumptions I-III guarantee the existence of unique intensities of feeling from engaged activities that are: expressed explicitly; real-valued; and linearly independent. Given initial conditions $\mathbf{f}(0)$, specifications of intensities of feeling from engaged activities are unique.

Theorem 1 (The Family of Intensities of Feeling from Engaged Activities)**.** *Given n activities, one engaged activity at a time, and the common coefficient of proportionality $\alpha(t) > 0$ that is piecewise constant with a finite number of jump discontinuities, if the rate of change in intensity of feeling from an activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities, there exists a unique family of intensities of feeling functions from engaged activities expressed explicitly:*

$$\mathbf{f}(t) = Ke^{\tilde{\Lambda}}\mathbf{c}$$

Given $c_1, c_2, c_3, \ldots, c_n$ from the initial conditions $\mathbf{f}(0)$, intensities of feeling from en-

gaged activities have the unique specifications:

$$
\begin{aligned}
f_1(t) &= c_1 e^{(2-n)\tilde{\alpha}t} + \sum_{i=2}^{n} c_i e^{2\tilde{\alpha}t} \\
f_2(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_2 e^{2\tilde{\alpha}t} \\
f_3(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_3 e^{2\tilde{\alpha}t} \\
&\vdots \\
f_n(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_n e^{2\tilde{\alpha}t}
\end{aligned}
$$

Proof. See Appendix A. □

The uniqueness property of intensities of feeling from engaged activities gives the observer more analytical power to study choice than utility. Unlike using unspecified functions for utility, given initial conditions, the observer has the ability to measure the intensity of feeling from engaged activities because their specifications are guaranteed by Theorem 1.

I illustrate the advantage for the observer who knows specifications with a hypothetical example of the machine by Professor Jay Coggins. If the observer chooses utility to study choice, she will either look for a specification that meets her goals but is not guaranteed that it represent the unobserved preference relation or test different specifications (Loomes and Sugden, 1998; Barberis et al., 2006) to assess which specification could represent the observed choice satisfactorily. Imagine now if there is a machine that gives the observer specifications of intensities of feeling from engaged activities. The machine is a metaphor for Theorem 1. Once the observer feeds the machine with its input from my Assumptions I-III, it gives her the ready-made output with the specification of the intensity of feeling from every engaged activity in which the subject is considering spending time. I would choose the machine to measure intensities of feeling from engaged activities to study choice.

Intensities of feeling from engaged activities can be used to analyze individual behavior over any period. If this period is sufficiently short, it is reasonable to assume that the interior state of the organism does not change, and hence $\alpha(t) = a$ is a constant during this period. Then intensities of feeling from engaged activities are as follows.

Corollary 1.1. *If $\alpha(t) = a$ is constant, intensities of feeling from engaged activities are: $\mathbf{f}(t) = Ke^{a\Lambda t}\mathbf{c}$. In particular, if $\alpha(t) = 1$ (the subject feels normal), the specifications are:*

$$\begin{aligned}
f_1(t) &= c_1 e^{(2-n)t} + \sum_{i=2}^{n} c_i e^{2t} \\
f_2(t) &= c_1 e^{(2-n)t} - c_2 e^{2t} \\
f_3(t) &= c_1 e^{(2-n)t} - c_3 e^{2t} \\
&\vdots \\
f_n(t) &= c_1 e^{(2-n)t} - c_n e^{2t}
\end{aligned}$$

Using the earlier example, from Corollary 1.1 with $\alpha(t) = 1$, intensities of feeling from walking the dog ($f_W(t)$), drinking coffee ($f_C(t)$), and reading ($f_R(t)$) are:

$$\begin{bmatrix} f_W(t) \\ f_C(t) \\ f_R(t) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} e^{2t} + c_3 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} e^{2t} + = \begin{bmatrix} c_1 e^{-t} + c_2 e^{2t} + c_3 e^{2t} \\ c_1 e^{-t} - c_2 e^{2t} \\ c_1 e^{-t} - c_3 e^{2t} \end{bmatrix}$$

If $f_W(0) = 2, f_C(0) = 1, f_R(0) = 0$, intensities of feeling from engaged activities (calculations in Appendix A) are:

$$f_W(t) = e^{-t} + e^{2t}, f_C(t) = e^{-t}, f_R(t) = e^{-t} - e^{2t}$$

As noted, there have been efforts to assess which specification of utility could predict an observed choice (Loomes and Sugden, 1998; Barberis et al., 2006). However, because utility is an unspecified function which does not belong to any family of functions, and different specifications are assumed and/or assessed in different contexts, it is difficult to know which preference relation this function is supposed to represent even if a particular specification predicts the observed (optimal) choice, unless the only focus is on that choice and no more.

The implication of the lack of uniqueness of utility functions in representing the preference relation as a primitive is that research continues to find that often utility functions are not capable of representing the underlying preference relation. One case where different utility specifications have been tested is by Loomes and Sugden (1998). When assessing three different specifications of risky choice—the Harless-Camerer (after Harless and Camerer, 1994), the Hey-Orme (after Hey and Orme, 1994) and a random preference model by Loomes and Sugden (1995) that generalizes the model in Becker et al. (1962) on irrational behavior—they found that these utility functions are not able to predict even the observed choice in their experiment, let alone represent the underlying individual preference relation. Loomes and Sugden (1998) report that the observed rate of dominance is lower than what is predicted by either the Harless-Camerer or the Hey-Orme specification but high enough to contradict their random preference model.

Another case is by Barberis et al. (2006) whose results show that a wide range of utility specifications, including expected utility and non-expected recursive utility functions, are not able to explain risk-averse choices for a small independent gamble, even when the gamble is actuarially favorable. They suggest that narrow-framing (Kahneman and Lovallo, 1993), an assumption which assumes that individuals isolate the risk of a single option from their overall risk (usually risk for overall wealth), could be an important factor to explain individual behavior.

Although narrow-framing, as part of the broader notion of bounded rationality

(Tversky and Kahneman, 1981; and Kahneman, 2003), is a less restrictive assumption than rationality (Mas-Colell, 1995) might be interesting to explore, as long as the role of narrow-framing or bounded rationality is evaluated through a utility function, its evaluation will have to assume rationality or a refined version thereof, otherwise there exists no function that can be used for the evaluation. Furthermore, bounded rationality is not helpful when choices are measured with continuous variables such as amounts time allocated to different activities because the agent will continue to be required to order infinitely many alternatives even with the assumption of bounded rationality.

These assessments of utility are for choice under uncertainty. When using utility to study choice involving uncertainty, there are many more requirements imposed on the agent than the requirements from already strong basic assumptions $I^u - III^u$ because the existence of a utility to represent the preference relation for choices under certainty requires additional assumptions. Furthermore, the above evaluations of different specifications of utility have been achieved with even more assumptions related to the specifications themselves. Meanwhile, Theorem 1 can be extended without extra assumptions to obtain unique intensities of feeling functions from engaged activities including both discounting and uncertainty.

Although intensities of feeling from engaged activities are functions of time, the subject's experience from each engaged activity at every instant, that is the value of $f_i(t)$ for each $i = 1, 2, \ldots, n$ at every moment in time, is neither discounted nor uncertain because it is happening at that very instant. The uncertainty is whether the subject will be engaged in an activity or not.

Suppose the subject discounts his future experience at every instant of time by a discounting factor $0 < \delta(t) \leq 1$.[13] Then for each activity i, the present value of the

[13] Exponential discounting $\delta(t) = \delta^t$ is a special case of general discounting in my theory.

intensity of feeling is $\delta(t)f_i(t)$ so that df_i/dt in the engaged system depends on $\delta(t)f_i(t)$ as opposed to $f_i(t)$ only. Therefore, the engaged system with discounting is:

The engaged system with discounting.

$$\dot{f}_i = \alpha(t)\left(\delta(t)f_i(t) - \sum \delta(t)f_k(t)\right) = \alpha(t)\delta(t)\left(f_i(t) - \sum f_k(t)\right)$$

Next suppose that the subject is uncertain about whether he will engage in an activity of not. If the subject engages in activity i, he will have the integrated experience E_i from that activity with a probability p_i. The amount of experience depends on when the subject will stop spending time on i and start spending time on k, that is the switch-time t_k^*, which is determined by the non-engaged system, as explained in the next chapter. If he does not engage in activity i, he will not have any integrated experience from i with probability $1 - p_i$. Then the rate of change df_i/dt in the intensity of feeling from each activity i depends on the probability distribution of integrated experience $P_r(E_i)$ for that activity.

$$P_r(E_i) = \begin{cases} p_i & \text{If the engaged activity is } i\text{, and } E_i = \int_{t_i^*}^t f_i(s)ds \\ 1 - p_i & \text{If the engaged activity is not } i\text{, and } E_i = 0 \end{cases}$$

Hence, if the engaged activity is i, then E_i occurs with probability p_i $(t > t_i^*)$. If the engaged activity is not i, then $E_i = 0$ with probability $1 - p_i$. So the expected value of integrated experience EF_i from activity i is: $EE_i = p_i \int_{t_i^*}^t f_i(s)ds$. Then the intensity of feeling from engaged activity i with uncertainty is:

$$\frac{d}{dt}EE_i = \frac{d}{dt}\left(p_i \int_{t_i^*}^t f_i(s)ds\right) = p_i f_i(t)$$

The rate of change itself is instantaneous, hence it is certain. So df_i/dt now depends on

$p_i f_i(t)$ as opposed to $f_i(t)$. Then the engaged system with discounting and uncertainty is:

The engaged system with discounting and uncertainty.

$$\dot{f}_i = \alpha(t)\delta(t)\left(p_i f_i(t) - \sum p_k f_k(t)\right) =$$
$$\beta(t)\left(p_i f_i(t) - \sum p_k f_k(t)\right),\ \beta(t) = \alpha(t)\delta(t)$$

In matrix notation, the engaged system with discounting and uncertainty can be written as the product of $\beta(t)$ and a matrix $B = AP$ with $P = \text{diag}(p_1, p_2, \ldots, p_n)$ the diagonal matrix with the probabilities corresponding to each activity, and $\mathcal{B}(t) = \beta(t)B$.

The engaged system with discounting and uncertainty in matrix notation.

$$\begin{bmatrix} \dot{f}_1 \\ \dot{f}_2 \\ \vdots \\ \dot{f}_n \end{bmatrix} = \beta(t) \begin{bmatrix} 1 & -1 & \cdots & -1 \\ -1 & 1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_n \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix}$$
$$= \begin{bmatrix} p_1\beta(t) & -p_2\beta(t) & \cdots & -p_n\beta(t) \\ -p_1\beta(t) & p_2\beta(t) & \cdots & -p_n\beta(t) \\ \vdots & \vdots & \ddots & \vdots \\ -p_1\beta(t) & -p_2\beta(t) & \cdots & p_n\beta(t) \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix}$$

$$\dot{\mathbf{f}} = \beta(t)AP\mathbf{f}(t) = \beta(t)B\mathbf{f}(t) = \mathcal{B}(t)\mathbf{f}(t)$$

Since $B = AP$, it is diagonalizable, and hence it has a complete set of orthogonal eigenvectors.

Remark 1. *Matrix B is diagonalizable.*

Proof. Left-multiplying both sides in $B = AP$ by the square matrix $P^{1/2}$ and right-multiplying by its inverse $P^{-1/2}$ (both sides), we get: $P^{1/2}BP^{-1/2} = P^{1/2}APP^{-1/2} = P^{1/2}AP^{1/2}$. Since A is symmetric, $P^{1/2}AP^{1/2}$ is also symmetric. Therefore, B is similar to a symmetric matrix, and hence it is diagonalizable. This completes the proof. □

Let $K^* = \begin{bmatrix} \mathbf{k}^{*1} & \mathbf{k}^{*2} & \cdots & \mathbf{k}^{*n} \end{bmatrix}$ denote the matrix with eigenvectors of B, $\Lambda^* = \text{diag}(\lambda_1^*, \lambda_2^*, \ldots, \lambda_n^*)$ the diagonal matrix with its corresponding eigenvalues, $\tilde{\beta}(t) = \int \beta(t)dt$, and $\tilde{\Lambda}^* = \tilde{\beta}\Lambda^*$. Let $\mathbf{c}^* = (c_1^*, c_2^*, \ldots c_n^*)$ denote a vector of constants and, as before, e the exponential function. Then my Assumptions I-III again guarantee the existence of unique intensities of feeling from engaged activities with discounting and uncertainty that are: expressed explicitly; real-valued; and linearly independent. With the same steps for the proof of Theorem 1, the result with discounting and uncertainty follows.

Lemma 1. *The family of intensities of feeling from engaged activities with discounting and uncertainty is:* $\mathbf{f}(t) = K^* e^{\tilde{\Lambda}^*} \mathbf{c}^*$. *Given initial conditions* $\mathbf{f}(0)$*, the specifications are unique.*

Note that from Lemma 1, the result from Corollary 1.1 with $\alpha = \frac{1}{T}$ could also be interpreted as the engaged system with uncertainty expressed by the uniform distribution $p_i = \frac{1}{T}$ for all $i = 1, 2, \ldots, n$.

While Lemma 1 provides the observer with the specifications to measure intensities of feeling from engaged activities with both discounting and uncertainty, utility is limited in its ability to represent discounting. A utility function is discounted at periods $t, t+1, \cdots$. As noted, utility represents the preference relation, and not vice versa, because the preference relation is the unknown (unobservable) primitive whereas utility is the analytical tool the observer uses to predict the agent's choice of alternatives/goods at different periods. This is why utility belongs to the researcher whereas the preference relation belongs to the agent.

If the agent does not discount the alternatives, the researcher should not discount the utility either. If he (the agent) does, so should she (the observer). When the observer includes discounting, the discounted values of the utility function chosen for a particular study represent the agent's discounted values of the preference relation ordering over different periods. Note that the earlier recursive model by Stokey et al. (1989) has a solution only if there is discounting, which is done by the observer for the subject.

In the Stokey et al. (1989) model, consumption c at $t = 0$ is represented by utility $U(c_0)$ and at $t = 1$ by discounted utility $\eta U(c_1)$. The agent is indifferent between consumption at $t = 0$ and consumption at $t = 1$ because their utility is discounted but not because good c is discounted. $U(c_0)$ and $\eta U(c_1)$ are two equal function values which are assumed to represent the same preference ordering for an individual indifferent between c_0 and c_1.

The implication from discounting utility but not the goods (alternatives) is that the observer's chosen utility function is only able to represent the agent's preference relation if it is a very special kind of function. This finding is presented as follows where I also show which kind of utility function is able to represent the preference relation with discounting.

Remark 2. *The observer has a discounting factor η for utility U and the agent a discounting factor δ for goods $\mathbf{x} = x^1, x^2, \cdots$ over different periods. Then the observer's U represents the subject's preference relation only if it is a homogeneous function of degree $k = \frac{\ln \eta}{\ln \delta} = \log_\delta \eta$.*

Proof. It is not known a priori if $\delta = \eta$. Denoting indifference by $\sim$ and equivalence by $\Leftrightarrow$, the following holds for all $t = 0, 1, \cdots$.[14]

$$\mathbf{x}_t \sim \delta \mathbf{x}_{t+1} \Leftrightarrow \frac{1}{\delta}\mathbf{x}_t \sim \mathbf{x}_{t+1} \Leftrightarrow U(\mathbf{x}_t) = \eta U(\mathbf{x}_{t+1}) \Leftrightarrow \frac{1}{\eta}U(\mathbf{x}_t) = U(\mathbf{x}_{t+1})$$

This gives:

$$\frac{1}{\eta}U(\mathbf{x}_t) = U(\frac{1}{\delta}\mathbf{x}_t) \Rightarrow \left(\frac{1}{\delta}\right)^k = \frac{1}{\eta} \Leftrightarrow \delta^k = \eta \Leftrightarrow k = \frac{\ln \eta}{\ln \delta} = \log_\delta \eta$$

This is an *only if* result, indicated by the (only) $\Rightarrow$ right implication, because not every homogeneous function of degree $\log_\delta \eta$ represents the subject's preference relation. In particular, if U is a homogeneous function of degree 1, then $\delta = \eta$. This completes the proof. $\square$

For example, these specifications might represent the preference relation with discounting:

$$U = x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n} \left(\sum_{i=1}^{n} \alpha_i = \log_\delta \eta\right), \text{ or } U = (x_1^{1/\ln \delta} + x_2^{1/\ln \delta} + \cdots + x_n^{1/\ln \delta})^{\ln \eta}$$

As Remark 2 also shows, only the specification of a homogeneous utility function of degree 1 might be able to represent the preference relation with discounting if the observer's discounting factor is the same as the subject's discounting factor. In my theory, specifications from Theorem 1 are guaranteed to measure intensities of feeling from engaged activities with discounting because I discount the subject's primitive rather than the observer's function. Although the observer might not know the subject's $\delta(t)$ in my theory, since it becomes part of the coefficient of proportionality $\beta(t) = \alpha(t)\delta(t)$, a self-assessment by the subject is sufficient and, for all purposes, practical.

[14] Given that utility U is assumed to exist, the preference relation is continuous.

By uncertainty, I mean that there exists some randomness for the integrated experience from each engaged activity i measured by p_i by the subject. Uncertainty in my theory coincides with what Knight (1921) termed as measurable uncertainty: "... a *measurable* uncertainty, or "risk" proper, ... is so far different from an *unmeasurable* one that it is not in effect an uncertainty at all." I use the term uncertainty because both discounting and uncertainty are part of coefficients of proportionality $\beta(t)p_i$ for each engaged activity i, and hence no additional assumptions are required to derive specifications that measure intensities of feeling from these activities with both discounting and uncertainty.

Again although the observer might not know the subject's probabilities p_i in my theory, these probabilities do not have to be precise but only practical. In contrast, many additional assumptions are required to derive the existence of an unspecified utility function to predict choice. How the specifications of intensities of feeling from engaged activities are applied to explain and predict time-allocations to engaged activities is explained in the next chapter.

Chapter 3

The Application

In the neoclassical economic setting, choice is a bundle at a point in time, including the bundle of time-allocations to alternatives. In my research, choice is a sequence of time-allocations to activities, a schedule of engaged activities over a period of time. The intensity of feeling function for each engaged activity obtained in the previous chapter is a useful analytical tool that the observer can use to analyze any type of choice once each time-allocation to the engaged activity is determined as during the time spent on the engaged activity the subject can consume, produce, swim, socialize, etc. But time itself is a defining ingredient of my primitive subjective feeling. Therefore, analyzing the subject's choice of the time-allocation to each activity is the immediate application of my theory.

In order to apply my subjective feeling theory, the observer also needs intensities of feeling functions from non-engaged activities. Intensities of feeling from non-engaged activities interact simultaneously with the intensity of feeling from the engaged activity to produce a time-allocation to an activity. Functions of intensities of feeling from engaged activities from Theorem 1 are the solution to the engaged system. Functions of intensities of feeling from non-engaged activities are the solution to the non-engaged system.

3.1 Non-engaged activities

From my Assumption II, the subject spends time on a single activity. From my Assumption I, the subject is considering spending time on n activities. Therefore, at every moment in time, there are $n-1$ non-engaged activities on which he is not spending time while spending time on the engaged activity. Intensities of feeling from non-engaged activities pull the subject to switch from spending time on the engaged activity to another activity.

From my Assumption III, the dynamics of the non-engaged system are similar to the engaged system except for the clock. The non-engaged system clock measures the stopwatch time because intensities of feeling from non-engaged activities $j|i$, $j \neq i$, given the engaged activity i, start to exist when the subject starts spending time on i, and they cease to exist when the subject switches from spending time on i to spending time on another activity.

Similar to the engaged system, the pull force of the intensity of feeling from each non-engaged activity has to withstand the combined force of intensities of feeling from other non-engaged activities. However, the intensity of feeling from each non-engaged activity also competes for time with the intensity of feeling from the engaged activity on which the subject is actually spending time. As for the engaged system, the non-engaged system can be written in matrix notation, where the subscript $|i$ denotes given the engaged activity i.

The non-engaged system in matrix notation.

$$\begin{bmatrix} \dot{f}_{1|i} \\ \vdots \\ \dot{f}_{i-1|i} \\ \dot{f}_{i+1|i} \\ \vdots \\ \dot{f}_{n|i} \end{bmatrix} = \alpha(t) \begin{bmatrix} 1 & \cdots & -1 & -1 & \cdots & -1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ -1 & \cdots & 1 & -1 & \cdots & -1 \\ -1 & \cdots & -1 & 1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & \cdots & -1 & -1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} f_{1|i}(t) \\ \vdots \\ f_{i-1|i}(t) \\ f_{i+1|i}(t) \\ \vdots \\ f_{n|i}(t) \end{bmatrix} + \alpha(t) \begin{bmatrix} -f_i(t) \\ \vdots \\ -f_i(t) \\ -f_i(t) \\ \vdots \\ -f_i(t) \end{bmatrix}$$

$$= \begin{bmatrix} \alpha(t) & \cdots & -\alpha(t) & -\alpha(t) & \cdots & -\alpha(t) \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ -\alpha(t) & \cdots & \alpha(t) & -\alpha(t) & \cdots & -\alpha(t) \\ -\alpha(t) & \cdots & -\alpha(t) & \alpha(t) & \cdots & -\alpha(t) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\alpha(t) & \cdots & -\alpha(t) & -\alpha(t) & \cdots & \alpha(t) \end{bmatrix} \begin{bmatrix} f_{1|i}(t) \\ \vdots \\ f_{i-1|i}(t) \\ f_{i+1|i}(t) \\ \vdots \\ f_{n|i}(t) \end{bmatrix} + \begin{bmatrix} -\alpha(t)f_i(t) \\ \vdots \\ -\alpha(t)f_i(t) \\ -\alpha(t)f_i(t) \\ \vdots \\ -\alpha(t)f_i(t) \end{bmatrix}$$

$$\dot{\mathbf{f}}_{|i} = \alpha(t)A_{|i}\mathbf{f}_{|i}(t) + \mathbf{a}_{|i} = \mathcal{A}_{|i}(t)\mathbf{f}_{|i}(t) + \mathbf{a}_{|i}(t)$$

where

$$\dot{\mathbf{f}}_{|\mathbf{i}}(t) = \left(\dot{f}_{1|i}, \cdots, \dot{f}_{i-1|i}, \dot{f}_{i+1|i}, \cdots, \dot{f}_{n|i}\right)$$

$$\mathbf{f}_{|i}(t) = \left(f_{1|i}(t), \cdots, f_{i-1|i}(t), f_{i+1|i}(t), \cdots, f_{n|i}(t)\right)$$

$$\mathbf{a}_{|i}(t) = (-\alpha(t)f_i(t), \cdots, -\alpha(t)f_i(t), \alpha(t)(t)f_i(t), \cdots, -\alpha(t)f_i(t))$$

$$A_{|i} = A_{n-1\times n-1},\ \mathcal{A}_{|i}(t) = \alpha(t)A_{|i}$$

The homogeneous part $\dot{\mathbf{f}}_{|i} = \alpha(t)A_{|i}\mathbf{f}_{|i}(t) = \mathcal{A}_{|i}(t)\mathbf{f}_{|i}(t)$ of the non-engaged system is similar to the engaged system. Therefore, its eigenvalues and eigenvectors are readily available.

As in Theorem 1, let $K_{|i} = \begin{bmatrix} \mathbf{k}_{|i}^1, \mathbf{k}_{|i}^2 & \cdots & \mathbf{k}_{|i}^{n-1} \end{bmatrix}$ denote the matrix with eigenvectors of $A_{|i}$, $\Lambda_{|i} = \operatorname{diag}(2-(n-1), 2, \cdots, 2)$ the diagonal matrix with its corresponding eigenvalues, and $\tilde{\Lambda}_{|i} = \tilde{\alpha}(t)\Lambda_{|i}$. Let $\mathbf{h}_{|i}(t) = (h_1(t), h_2(t), \ldots h_{n-1}(t))$ be the same solution (with $n-1$ functions) as in Theorem 1 to the homogeneous part and $\mathbf{f}_{|i}^p(t)$ a particular solution to the non-engaged system.

Let $\mathbf{f}_{-i}^p(t) = \left(f_1^p(t), \ldots, f_{i-1}^p(t), f_{i+1}^p(t), \ldots, f_n^p(t)\right)$ denote the specifications for intensities of feeling from engaged activities from a given set of initial conditions $\mathbf{f}(0)$ from Theorem 1 that do not include the intensity of feeling from (the engaged) activity i. Hence $f_k^p(t) = f_k(t)$, $k \neq i$, indicated by the subscript $-i$. However, the constants from Theorem 1 are the same as those obtained from the initial conditions $\mathbf{f}(0)$ because the non-engaged system clock measures stopwatch time. To emphasize this, I denote the constants from the initial conditions $\mathbf{f}(0)$ by $\mathbf{c}^0 = (c_1^0, c_2^0, c_3^0, \ldots, c_n^0) = (c_1, c_2, c_3, \ldots, c_n) = \mathbf{c}$.

Also, let $\mathbf{c}_{|i} = (c_{1|i}, c_{2|i}, c_{3|i}, \ldots, c_{n-1|i})$ be a vector of constants, and $K_{-i} = \begin{bmatrix} \mathbf{k}_{-i}^1 & \cdots & \mathbf{k}_{-i}^i & \cdots & \mathbf{k}_{-i}^n \end{bmatrix}$ the values of K by excluding its ith row but not its ith column. Then my Assumptions I-III guarantee the existence of a unique family of intensities of feeling functions from non-engaged activities that are expressed explicitly.

Theorem 2 (The Family of Intensities of Feeling from Non-engaged Activities)**.** *Given n activities, the engaged activity, and the common coefficient of proportionality $\alpha(t) > 0$ that is piecewise constant with a finite number of jump discontinuities, if the rate of change in intensity of feeling from an activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities, there exists a unique family of intensities of feeling functions from non-engaged activities expressed explicitly:*

$$\mathbf{f}_{|i}(t) = \mathbf{h}_{|i}(t) + \mathbf{f}_{-i}^p(t) = K_{|i} e^{\tilde{\Lambda}_{|i}} \mathbf{c}_{|i} + K_{-i} e^{\tilde{\Lambda}_{|i}} \mathbf{c}^0$$

Given $c_1^0, c_2^0, c_3^0, \ldots, c_n^0$ from the initial condition $\mathbf{f}(0)$, as well as $c_{1|i}, c_{2|i}, , c_{3|i}, \ldots c_{n-1|i}$ from any condition $\mathbf{f}_{|i}(t^*)$, intensities of feeling from non-engaged activities have the unique specifications:

$$
\begin{aligned}
f_{1|i}(t) &= c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} + \quad \textstyle\sum_{i\neq j=2}^{n} c_{j|i}e^{2\tilde{\alpha}t} + \quad c_1^0 e^{(2-n)\tilde{\alpha}t} + \quad \sum_{i=2}^{n} c_i^0 e^{2\tilde{\alpha}t} \\
&\vdots \\
f_{i-1|i}(t) &= c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} - \quad c_{i-1|i}e^{2\tilde{\alpha}t} + \quad c_1^0 e^{(2-n)\tilde{\alpha}t} - \quad c_{i-1}^0 e^{2\tilde{\alpha}t} \\
f_{i+1|i}(t) &= c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} - \quad c_{i+1|i}e^{2\tilde{\alpha}t} + \quad c_1^0 e^{(2-n)\tilde{\alpha}t} - \quad c_{i+1}^0 e^{2\tilde{\alpha}t} \\
&\vdots \\
f_{n|i}(t) &= c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} - \quad c_{n|i}e^{2\tilde{\alpha}t} + \quad c_1^0 e^{(2-n)\tilde{\alpha}t} - \quad c_n^0 e^{2\tilde{\alpha}t}
\end{aligned}
$$

Proof. See Appendix B. □

As in Theorem 1, if the period is sufficiently short, it is reasonable to assume that $\alpha(t) = a$ is constant. If $K_{-i} = \begin{bmatrix} \mathbf{k}_{-i}^1 & \cdots & \mathbf{k}_{-i}^i & \cdots & \mathbf{k}_{-i}^n \end{bmatrix}$ contains values of K by excluding its ith row but not its ith column, intensities of feeling from non-engaged activities are as follows.

Corollary 2.1. *If $\alpha(t) = a$ is constant, intensities of feeling from non-engaged activities are: $\mathbf{f}_{|i}(t) = K_{|i}e^{a\Lambda_{|i}t}\mathbf{c}_{|i} + K_{-i}e^{a\Lambda t}\mathbf{c}^0$. In particular, if $\alpha(t) = 1$, the specifications are:*

$$
\begin{aligned}
f_{1|i}(t) &= c_{1|i}e^{[2-(n-1)]t} + \quad \textstyle\sum_{j=2}^{n-1} c_{j|i}^0 e^{2t} + \quad c_1^0 e^{(2-n)t} + \quad \sum_{i=2}^{n} c_i^0 e^{2t} \\
&\vdots \\
f_{i-1|i}(t) &= c_{1|i}e^{[2-(n-1)]t} - \quad c_{i-1|i}e^{2t} + \quad c_1^0 e^{(2-n)t} - \quad c_{i-1}^0 e^{2t} \\
f_{i+1|i}(t) &= c_{1|i}e^{[2-(n-1)]t} - \quad c_{i+1|i}e^{2t} + \quad c_1^0 e^{(2-n)t} - \quad c_{i+1}^0 e^{2t} \\
&\vdots \\
f_{n|i}(t) &= c_{1|i}e^{[2-(n-1)]t} - \quad c_{n|i}e^{2t} + \quad c_1^0 e^{(2-n)t} - \quad c_n^0 e^{2t}
\end{aligned}
$$

In the earlier example, given $f_W(0) = 2, f_C(0) = 1, f_R(0) = 0$, intensities of feeling from engaged activities were: $f_W(t) = e^{-t} + e^{2t}, f_C(t) = e^{-t}, f_R(t) = e^{-t} - e^{2t}$. Using this example,with $\alpha(t) = 1$, from Corollary 2.1, intensities of feeling from non-engaged activities: $f_{C|W}(t), f_{R|W}(t)$ while walking the dog (W), $f_{W|C}(t), f_{R|C}(t)$ while drinking coffee (C), and $f_{W|R}(t), f_{C|R}(t)$ while reading the news (R), are:

$$\begin{bmatrix} f_{C|W}(t) \\ f_{R|W}(t) \end{bmatrix} = c_{1|W} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{0t} + c_{2|W} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t} + \begin{bmatrix} f_C(t) \\ f_R(t) \end{bmatrix} = \begin{bmatrix} c_{1|W} + c_{2|W}e^{2t} + e^{-t} \\ c_{1|W} - c_{2|W}e^{2t} + e^{-t} - e^{2t} \end{bmatrix}$$

$$\begin{bmatrix} f_{W|C}(t) \\ f_{R|C}(t) \end{bmatrix} = c_{1|C} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{0t} + c_{2|C} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t} + \begin{bmatrix} f_W(t) \\ f_R(t) \end{bmatrix} = \begin{bmatrix} c_{1|C} + c_{2|C}e^{2t} + e^{-t} + e^{2t} \\ c_{1|C} - c_{2|C}e^{2t} + e^{-t} - e^{2t} \end{bmatrix}$$

$$\begin{bmatrix} f_{W|R}(t) \\ f_{C|R}(t) \end{bmatrix} = c_{1|R} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{0t} + c_{2|R} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t} + \begin{bmatrix} f_W(t) \\ f_C(t) \end{bmatrix} = \begin{bmatrix} c_{1|R} + c_{2|C}e^{2t} + e^{-t} + e^{2t} \\ c_{1|R} - c_{2|C}e^{2t} + e^{-t} \end{bmatrix}$$

Given any initial conditions for the homogeneous part, specifications of intensities of feeling are unique because the constants will be unique. However, since intensities of feeling from non-engaged activities are conditioned on which activity is the engaged activity, they exist for as long as that engaged activity lasts. When the subject switches to spending time on another activity, the same intensities of feeling from non-engaged activities do not exist anymore because now other intensities of feeling from non-engaged activities start to exist.

For example $f_{W|C}(t), f_{R|C}(t)$ exist for as long as the subject is spending time on drinking coffee (C). When he switches to spending time on walking the dog (W), there is no more $f_{W|C}(t), f_{R|C}(t)$ because now $f_{C|W}(t), f_{R|W}(t)$ start to exist.

While there might be discounting, unlike the engaged system, the non-engaged system does not include uncertainty because the experience from each non-engaged activity $j|i$ given the engaged activity i is certain to happen for as long as the subject is spending time on i. The non-engaged system with discounting and certainty is:

The non-engaged system with discounting and certainty.

$$\dot{f}_{j|i} = \beta(t)\left[f_{j|i}(t) - \left(\sum f_{l|i}(t) + f_i(t)\right)\right],\ \beta(t) = \alpha(t)\delta(t)$$

Mirroring notation for the engaged system, the non-engaged system with discounting and certainty can be written as the product of $\beta(t)$ and $A_{|i} = B_{|i} = A_{|i}P_{|i} = A_{|i}I_{|i}$ with the identity matrix $I_{|i} = I_{n-1\times n-1} = P_{|i}$ for the probabilities equal to 1, and $\mathcal{B}_{|i}(t) = \beta(t)A_{|i}$.

The non-engaged system with discounting and certainty in matrix notation.

$$\dot{\mathbf{f}}_{|i} = \beta(t)B_{|i}\mathbf{f}_{|i}(t) + \mathbf{a}_{|i} = \beta(t)A_{|i}\mathbf{f}_{|i}(t) + \mathbf{a}_{|i} = \mathcal{B}_{|i}(t)\mathbf{f}_{|i}(t) + \mathbf{a}_{|i}(t)$$

Since $B_{|i} = A_{|i}$, its eigenvectors and corresponding eigenvalues are $K^*_{|i} = K_{|i}$ and $\Lambda^*_{|i} = \Lambda_{|i}$. Then $\tilde{\Lambda}^*_{|i} = \tilde{\beta}\Lambda^*_{|i} = \tilde{\beta}\Lambda_{|i}$, where again $\tilde{\beta}(t) = \int_{t_0}^{t} \beta(t)dt$. Let $\mathbf{c}^*_{|i} = (c^*_{1|i}, c^*_{2|i}, \dots c^*_{n-1|i})$ denote a vector of constants and again e the exponential function. Then my Assumptions I-III guarantee the existence of unique intensities of feeling from non-engaged activities with discounting and certainty that are expressed explicitly.

Although intensities of feeling from engaged activities may be uncertain from Lemma 1, as the particular solution to the non-engaged system, intensities of feeling from engaged activities must be certain. Hence, the particular solution to the non-engaged system includes the specifications in Theorem 1 but not in Lemma 1 (denoted by $f^p_k(t) = f_k(t)$). On the other hand, $f_i(t)$ might have come from Lemma 1 (with the uncertainty) because it runs on the engaged system clock that measures the calendar time. Therefore, specifications for $\mathbf{f}^p_{-i}(t)$ are those that start at $t = 0$ to match the non-engaged system clock that measures the stopwatch time whereas the specification of $f_i(t)$ continues from $t - 0$ to match the engaged system clock that measures the calendar time. With the same steps for the proof of Theorem 2, the result with discounting and certainty for the non-engaged system follows.

Lemma 2. *The family of intensities of feeling from non-engaged activities with discounting and certainty is:* $\mathbf{f}_{|i}(t) = K_{|i}e^{\tilde{\Lambda}^*_{|i}}\mathbf{c}^*_{|i} + \mathbf{f}^p_{-i}(t)$. *Given any specifications* $\mathbf{f}_{|i}(t^*)$, *the specifications are unique.*

Proof. Same steps as in Theorem 2. □

In the neoclassical economic setting, the observer maximizes a utility function to predict the optimal choice for the rational agent. In my research, I analyze choice in two parts: i) in a descriptive framework, where I find the switch-time as the zero-state (when all intensities of feeling from non-engaged activities are zero) to explain the optimal or non-optimal choice of time-allocations in a schedule for any (rational or non-rational) subject; and ii) in a normative framework, where I maximize overall experience to predict the optimal schedule for the conditionally rational subject.

Note that a rational agent is also conditionally rational, but not vice versa, because more mental effort is required to order all alternatives as opposed to only a few alternatives, that is possible schedules, given switch-times. Hence, I am able to predict more choices of time-allocations in my research than the observer in a neoclassical economic setting.

3.2 Time-Allocations

3.2.1 Descriptive Framework

In the descriptive framework, I explain the subject's choice of time-allocations. The engaged activity conditions which intensities of feeling from non-engaged activities exist, and the zero-state determines how long the engaged activity lasts. If the switch-time is an instant, there is a one-to-one correspondence between the zero-state and the constants in Theorem 2, or, equivalently, there is a one-to-one correspondence between the zero-state and specifications of intensities of feeling from non-engaged activities.

A time-allocation is the amount of time spent on the engaged activity as the difference between two consecutive switch-times (or from the start of the period to the first switch-time, or from the last switch-time to the end of the period). The switch-time is the zero-state t^* when all $f_{j|i}(t^*) = 0$. If at least one $f_{j|i}(t) \neq 0$, that is the valence of feeling from activity $j|i$ is not neutral, intensities of feeling from non-engaged activities have a role to play.

Since the existence of intensities of feeling from non-engaged activities $j|i$ is conditioned on the subject spending time on i, neutral valences for feelings from all non-engaged activities mean that they do not exist. If they existed, at least one of intensities of feeling from non-engaged activities would be non-zero. As long as the subject is spending time on i, at least one $j|i$ must have non-neutral valence (at least one $f_{j|i}(t) \neq 0$).

Hence the situation when the valence of feeling from every non-engaged activity is neutral, that is $f_{j|i}(t^*) = 0$, $\forall j$, is equivalent to the non-existence of these non-engaged activities, because other non-engaged activities have started to exist at t^* when the activity i is also not the engaged activity anymore. Therefore, t^* is the switch-time when the subject has switched from spending time on i to spending time on another activity k.

The switch-time from the engaged activity i to activity k is an instant, that is there exists a unique moment in time when the switch from spending time on activity i to spending time on activity j happens. If the switch-time were an interval of time, the subject would start spending time on k (hence the switch), and at the same time would continue spending time on i (hence the interval), a contradiction of my Assumption II because he would be spending time on both i and k during this interval. So the switch-time cannot be other than an instant.

The instant when the switch-time occurs is determined as a biological process, which instructs the subject when is the moment for him to stop spending time on the current engaged activity and to start spending time on another engaged activity. During this process, the subject creates a unique specification for intensities of feeling from all non-engaged activities. That is, intensities of feeling from all non-engaged activities are equal to this representative specification for all of them.

In order for the subject to create this representation effortlessly, he matches the intensity of feeling of his mind and body from the engaged activity with exactly one representative intensity of feeling of purely his mind from all non-engaged activities. As I noted, based on findings by Damasio (2021), feelings execute their behavioral role effortlessly. It would not be easy for feelings to play their role effortlessly if the subject were to decide when to switch from one engaged activity to another one if there were $n-1$ different intensities of feeling from non-engaged activities while he is spending time on the engaged activity.

Instead, through feelings, the organism creates the unique specification for intensities of feeling from all non-engaged activities to match with the intensity of feeling from the engaged activity for as long as this engaged activity exists, that is until the subject switches to another engaged activity. While the intensity of feeling from the engaged activity involves both the body and the mind, the representative specification for non-engaged activities is purely a mental process. I call the process that determines the switch-time *matching.*

Definition. ***Matching*** *is the biological process through which, given the intensity of feeling from every engaged activity, the organism determines the switch-time by creating the unique specification for intensities of feeling from all non-engaged activities.*

In practice, matching encapsulates the coordination of the mind and body, and it confirms that feelings play their behavioral role effortlessly (Damasio, 2021). How matching determines the switch-time is derived below formally.

Theorem 3 (Switch-time Determination)**.** *There is a one-to-one correspondence between the zero-state* t^* *and specifications of intensities of feeling from non-engaged activities, and the switch-time* t^* *is an instant. Matching determines the instant when the switch-time* t^* *occurs by the initial condition of the unique specification for intensities of feeling from all non-engaged activities given the engaged activity.*

Proof. See Appendix B. □

A feeling plays a role if it has non-neutral valence. If at some time $t = \tau$ both $f_i(\tau) = 0$ and $f_{j|i}(\tau) = 0$ for all $j|i$, then feelings would have no role to play at that time. Using part of the proof of Theorem 3, I derive below the result that valences of feelings from both the engaged activity and non-engaged activities cannot all be neutral, that is it is not possible for both $f_i(t) = 0$ and $f_{j|i}(t) = 0$ for all $j|i$ at any point in time.

Therefore, feelings always have a role to play. As noted earlier, the role of feelings was found by Damasio (2021) in their dual (informational and behavioral) role. It is reassuring to derive this neuroscientific finding mathematically.

Corollary 3.1. *Feelings always have a role to play. That is, $f_i(t) \neq 0$ and/or $f_{j|i}(t) \neq 0$.*

Proof. Suppose that at some $t = \tau$ both $f_i(\tau) = 0$ and $f_{j|i}(\tau) = 0$ for all $j|i$, in which case feelings play no role, and the intensity of feeling $f_i^*(\tau) = 0$. Then the non-engaged system equals the zero-vector: $\dot{\mathbf{f}}_{|i}(\tau) = \mathbf{0}$, because $\alpha(t) \neq 0$ (or $\beta(t) \neq 0$). Since $\mathbf{f}_{-i}(t)$ is a particular solution for the non-engaged system, $\mathbf{f}^p(\tau) = \mathbf{f}_{-i}(\tau) = \mathbf{0}$. So at $t = \tau$ intensities of feeling from all engaged activities (in the engaged system) would be zero: $\mathbf{f}(t) = \mathbf{0}$ (including $f_i(t) = 0$). Then from the proof of Theorem 1 (Appendix A), the constants must be all zero: $\mathbf{c} = \mathbf{0}$, a contradiction because $\mathbf{c}$ is a non-zero vector. Therefore, it is impossible for both $f_i(t) = 0$ and $f_{j|i}(t) = 0$ for all $j|i$, and feelings always have a role to play. This completes the proof. □

Matching tells the subject the precise moment when to switch from spending time on the engaged activity i to spending time on another (engaged) activity k. From Theorem 3, there exists a unique specification $f_{j|i}(t)$ for intensities of feeling from non-engaged activities such that $f_{j|i}(t^*) = 0$. Since the subject knows the switch-time t^* through matching, the point $(t^*, \mathbf{f}_{|i}(t^*)) = (t^*, \mathbf{0})$ has all the information needed for feelings to play their behavioral role.

Remark 3. *The moment $t = T$ at the end of the period is not a switch-time, but it acts as a forced switch–time because it is the stop-time for the last engaged activity during $[0, T]$. I use $t^* = T$ as a forced switch-time to obtain specifications of intensities of feeling from non-engaged activities during the last time-allocation.*

Although I do not expect the subject to know mathematically the functional forms for his intensities of feeling (as I do not know mine), I use expressions like "he knows how he feels" and "he knows his intensities of feeling" interchangeably because in my analysis specifications of intensities of feeling from engaged as well as non-engaged activities are unique. Therefore, as the observer, I measure intensities of feeling.

It is important for the observer to know that the switch-time tells the subject how he feels about the engaged activity and non-engaged activities, that is he knows when to switch. However, as the observer, I do not know if the subject is optimizing anything with his behavior because I have yet to assume that he optimizes something.

Based on the results so far, it is easy for the subject to know how he feels because the initial condition for the unique specification for all intensities of feeling from non-engaged activities will do the job for him. From my Assumption II, the subject is always doing something (he is always spending time on an engaged activity). Therefore, intensities of feeling from non-engaged activities are always in his mind, and matching makes it easy for him to know when to switch from spending time on one activity to spending time on another one. Suppose that you are working, and are considering spending time on other activities as well. Between focusing on work and being pulled to all of the other activities, matching tells your mind and body when you need to stop working and start doing something else.

As the observer, I have obtained useful results from the subject's switch-time because I have made as weak assumptions about his behavior as possible. At the moment, I have only imposed my Assumptions I-III on the subject. In the next section, I will also assume that the subject maximizes his overall experience, but for now these three assumptions suffice to explain his time-allocations.

Since the zero-state is the switch-time, the difference between two consecutive switch-times over $(0, T)$ gives the time-allocation to an activity. The first switch-time t_1^* also gives the time-allocation $(t_1^* - 0)$ to the first activity. Since the first engaged activity starts at 0, there are $M - 1$ switch-times when the subject engages in M activities during $[0, T]$.

As long as the number of switch-times is finite, there are no restrictions on M. It can be equal, less, or greater than n. If $M = n$ and no repeated engaged activities, the subject has engaged once in each of his n activities during $[0, T]$. If $M < n$, he has engaged in fewer activities than those on which he was considering spending time, and the time-allocation for any activity on which he did not spend time is 0. If $M > n$, at least one activity has been repeated as the engaged activity more than once.

A sequence of time-allocations gives a schedule S and its associated overall experience E_s during $[0, T]$. Given a maximum (finite) number of switch-times $M_s^* - 1$ for any schedule, the number of possible schedules (or overall experiences) N is: $N \leq M_s^*!$.

In the earlier example, suppose that the subject engages once in each activity and has the same time-allocations in any schedule.[1] Let T_W, T_C, T_R denote how many hours he spends walking the dog (W), drinking coffee (C), reading the news (R), respectively. Suppose that, given the initial conditions in Theorem 3, $T_W = 1, T_C = 0.5, T_R = 0.5$. Then $M_s^* = 3$, and there are $N = 3! = 6$ possible schedules:

$$\begin{aligned}
WCR:&\quad T_W = 1, \quad T_C = 0.5, \quad T_R = 0.5\\
WRC:&\quad T_W = 1, \quad T_R = 0.5, \quad T_C = 0.5\\
CRW:&\quad T_C = 0.5, \quad T_R = 0.5, \quad T_W = 1\\
CWR:&\quad T_C = 0.5, \quad T_W = 1, \quad T_R = 0.5\\
RWC:&\quad T_R = 0.5, \quad T_W = 1, \quad T_C = 0.5\\
RCW:&\quad T_R = 0.5, \quad T_C = 0.5, \quad T_W = 1
\end{aligned}$$

In the descriptive framework, the subject is not required to optimize anything with his behavior, and he can choose any schedule during which the switch-times tell him when to stop spending time on an activity and start spending time on another activity. He knows the switch-times from Theorem 3. In the example, the subject knows his

[1] A time-allocation to an activity does not have to be the same in every schedule.

$6 \times 2 = 12$ switch-times.

If he starts with W, after one-hour $f_{C|W}(1) = f_{R|W}(1) = 0$, and he will switch to either C or R. Suppose he switches to R. Then after half-hour $f_{C|R}(0.5) = f_{W|R}(0.5) = 0$, and he will switch to C. So he has chosen S_{WRC}: walk the dog for one-hour; then read the news for half-hour; and finally drink coffee for half-hour, during the two-hour period.

Using the earlier specifications of intensities of feeling from engaged activities

$$f_W(t) = e^{-t} + e^{2t}, \quad f_C(t) = e^{-t}, \quad \text{and} \quad f_R = e^{-t} - e^{2t}$$

his intensities of feeling from non-engaged activities (calculations in Appendix B) are:

$$\begin{gathered}
f_{C|W}(t) = c_{1|W} + c_{2|W}e^{2t} + e^{-t} = \tfrac{1}{2}e^2 - e^{-1} - \tfrac{1}{2}e^{2t} + e^{-t} \\
f_{R|W}(t) = c_{1|W} - c_{2|W}e^{2t} - e^{2t} + e^{-t} = \tfrac{1}{2}e^2 - e^{-1} - \tfrac{1}{2}e^{2t} + e^{-t} \\
f_{R|C}(t) = c_{1|C} - c_{2|C}e^{2t} - e^{2t} + e^{-t} = -e^{-\frac{1}{2}} + e^{-t} \\
f_{W|C}(t) = c_{1|C} + c_{2|C}e^{2t} + e^{2t} + e^{-t} = -e^{-\frac{1}{2}} + e^{-t} \\
f_{W|R}(t) = c_{1|R} + c_{2|R}e^{2t} + e^{2t} + e^{-t} = -\tfrac{1}{2}e - e^{-\frac{1}{2}} + \tfrac{1}{2}e^{2t} + e^{-t} \\
f_{C|R}(t) = c_{1|R} - c_{2|R}e^{2t} + e^{-t} = -\tfrac{1}{2}e - e^{-\frac{1}{2}} + \tfrac{1}{2}e^{2t} + e^{-t}
\end{gathered}$$

Intensities of feeling from non-engaged activities have the unique specification, as in Theorem 3. The intensity of feeling from the representative intensity of feeling from non-engaged activities given each engaged activity is shown below in two sets of graphs (for easier visualization with no overlaps from schedules).

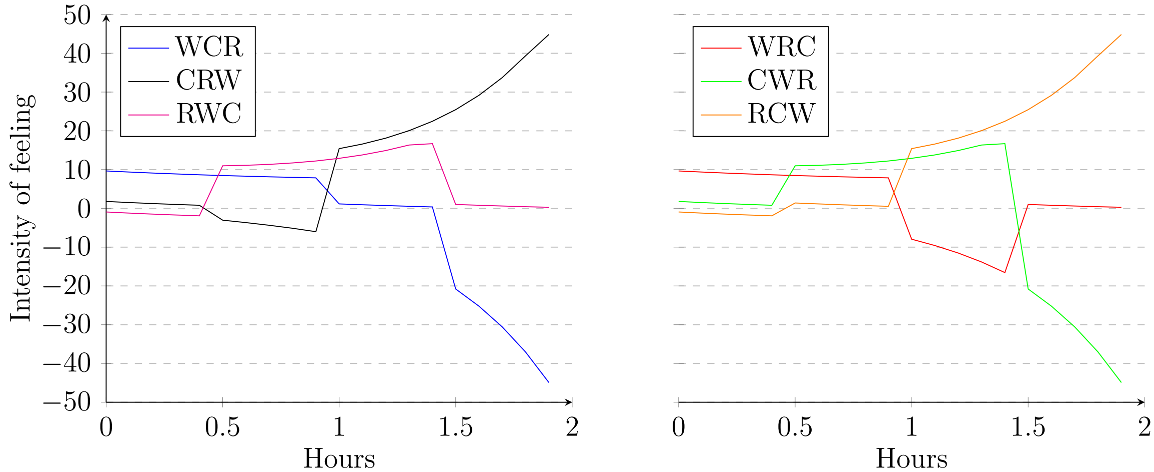

Figure 3.1: The intensity of feeling in each schedule

But what if the subject optimizes his overall experience with his behavior? Then the schedules are not equally appealing because there is no guarantee that all schedules give him the same overall experience. In this case, he needs to know which schedule gives him the maximum overall experience. The conditionally rational subject knows his optimal schedule.

3.2.2 Normative Framework

In the normative framework, I predict the subject's choice of time-allocations with my extra Assumption IV. Given a maximum number of switch-times: $M_S^* - 1$, the conditionally rational subject knows his n initial conditions for intensities of feeling from engaged activities $f_i(0)$, $\forall i = 1, 2, 3, \ldots, n$, as well as n initial conditions for representative specifications for intensities of feeling from non-engaged activities $f_{j|i}(0)$ matching each engaged activity. This knowledge is sufficient to choose the optimal schedule that gives him the maximum overall experience.

The optimal schedule is determined as an evaluative process, which informs the subject which sequence of time-allocations give him the maximum overall experience.

In order for the subject be able to choose the optimal schedule, he needs to spend some mental effort. The conditionally rational subject is able to spend the required mental effort to choose his optimal schedule. The mental effort required to order a finite number of alternatives by the subject in my research is still much smaller than the mental effort required to order an infinite number of bundles by the agent in a neoclassical economic setting.

Choosing the optimal schedule cannot be achieved simply from matching. While matching is the process tells the subject effortlessly when to switch given that he is engaged in an activity, choosing which schedule has the optimal sequence of time-allocations that gives him the maximum overall experience requires evaluating matchings as *if* he were to engage in each activity. I call this evaluative process that determines the optimal schedule *sorting*.

Definition. ***Sorting*** *is the evaluative process of all matchings during* $[0, T]$.

In reality, sorting is the mind's ability to imagine as if the subject were to engage in possible sequences of time-allocations during $[0, T]$, that is if he did every possible schedule. How sorting determines the optimal schedule is derived below formally.

Lemma 3. *Given a finite number of switch-times, the conditionally rational subject chooses the optimal schedule. If there are no switch-times during* $[0, T]$*, he maximizes his overall experience from a single engaged activity.*

Proof. Suppose the conditionally rational subject is engaged in activity i. Since he knows his n initial conditions for intensities of feeling from engaged activities, he knows $f_i(t)$ from Theorem 1. Suppose that at t_k^* he switches to activity k. Then he knows the representative specification $f_{|i}^*$ for $f_{1|i}(t), \cdots, f_{i-1|i}(t), f_{i+1|i}(t), \cdots, f_{n|i}(t)$ from Theorem 3. So he knows his intensity of feeling given i: $f_i^*(t) = f_i(t) + (n-1)f_{|i}^*$, as well as his integrated experience E_i. Since this is true for any i, he knows his overall experience from any schedule E_s.

If he switches at least once, $M_s^* - 1 \geq 1$, there are a maximum of $N \times (M_s^* - 1)$ switch-times, which the rational subject is assumed to know. But the switch-times, derived from Theorem 3, are measured with real numbers in the interval $[0, T]$, and hence he is able to order $N \times (M_S^* - 1)$ real numbers. But each overall experience is also measured with a real number. With N schedules, there are $N \leq N \times (M_S^* - 1)$ real numbers that measure overall experiences E_s from all possible schedules N. Then he is also able to order N schedules ($N > n$ in this case) by their respective overall experiences to choose the optimal schedule that gives him the maximum overall experience.

If there are no switch-times, $M_s^* - 1 = 0$, the subject would spend the whole amount of time T on one activity (Assumption II). Since there are n activities, the number of schedules is $N = n$. But the n initial conditions are measured with real numbers, and he is assumed to know them. Since he is able to order n real numbers that measure initial conditions for engaged activities, he is also able to order the same number $N = n$ of real numbers that measure overall experiences E_s from all possible schedules (n in this case) by their respective overall experiences to choose again the optimal schedule. This completes the proof. □

The care in the proof of Lemma 3 to document how much the subject knows serves two purposes. First, the observer needs to be careful with the amount of her assumptions because although they help her to study choice, assumptions also put an onus on the subject whose behavior is being studied. Second, the proof in Lemma 3 shows that the subject's optimizing mechanism is not impossible to achieve.

Although the optimizing mechanism might not be very complicated, it requires sorting as a mental process that decides about physical and mental components of feelings if the subject engaged in an activity, as well as about purely mental components of feelings when matching each engaged activity with the representative function of intensities of feeling from non-engaged activities. Therefore, my view until recently has been that the descriptive framework might be more realistic than the normative framework in daily activities.

My view has started to change recently after learning about the practical possibilities of my work from an article in Medium[2] by Pau Blasco i Roca (2023) self-reporting the effort to find which is the best daily routine by recording and analyzing the time he spent on several daily activities such as sleep, writing, reading, socializing, etc. The importance of organizing time through routines to find meaning as a desired feeling (Janoff-Bulman, 1992) was noted early on by Meyer (1922) and confirmed recently by Heintzelman and King (2019).

In practice, the subject can improve his understanding of his own behavior with my descriptive framework because matching as a biological process is done effortlessly. As the above article by Blasco (2023) shows, it is also possible to learn sorting as an evaluative process by recording and analyzing time spent on each activity. One convenient tool is the app, or the mind-app, which I mentioned earlier as a convenient tool to help the subject choose the optimal schedule. This app would have the ability to store and analyze the time spent on daily activities to help the subject learn the process of sorting through which he can use my normative framework to choose the optimal schedule for his daily routine.

In the earlier example, my normative framework helps the subject choose the opti-

[2]My Life Stats: I Tracked My Habits for a Year, and This Is What I Learned, Medium, towardsdatascience.com/my-life-stats-i-tracked-my-habits-for-a-year-and-this-is-what-i-learned-4f9c3d374889, Accessed 11/28/2023.

mal schedule (calculations in Appendix B) as follows.

$$WCR : E_{WCR} = -8.99$$
$$E_W(t) = \frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}, E_C(t) = -e^{-t} + e^{-1}, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e^3 + e^{-1.5}$$
$$WRC : E_{WRC} = -1.21$$
$$E_W(t) = \frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e^2 + e^{-1}, E_C(t) = -e^{-t} + e^{-1.5}$$
$$CRW : E_{CRW} = 17.74$$
$$E_C(t) = -e^{-t} + 1, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e + e^{-0.5}, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e^2 + e^{-1}$$
$$CWR : E_{CRW} = -4.76$$
$$E_C(t) = -e^{-t} + 1, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e + e^{-0.5}, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e^3 + e^{-1.5}$$
$$RWC : E_{RWC} = 7.01$$
$$E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{3}{2}, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e + e^{-0.5}, E_C(t) = -e^{-t} + e^{-1.5}$$
$$RCW : E_{RCW} = 18.79$$
$$E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{3}{2}, E_C(t) = -e^{-t} + e^{-0.5}, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e^2 + e^{-1}$$

As seen, the schedule with the maximum overall experience is RCW: read for half-hour; then drink coffee for half-hour; then walk the dog for one-hour. The overall experience from each schedule is presented in graphs grouped in the same sets of schedules as earlier.

Comparing starting values from the initial conditions, at least in this example, the conditionally rational subject does his least favorite thing first. After presenting this part of my research with no parallels in neoclassical economic theory, where non-engaged activities have no counterparts, I point out applicable differences focusing on theories that involve individual experience.

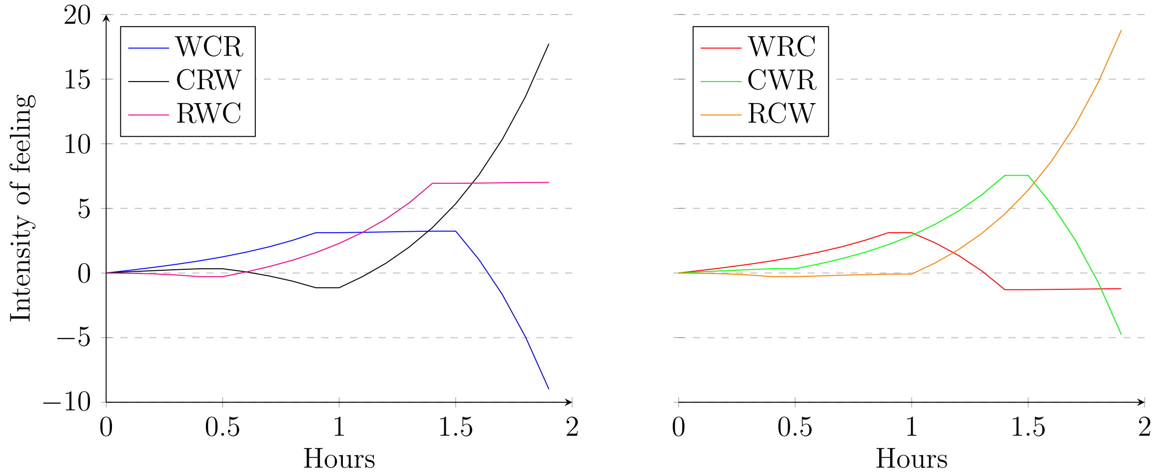

Figure 3.2: The overall experience from each schedule

3.3 Applicable differences

Choice of time-allocations in my research is derived from matching as a biological process of the subject's body and mind that determines the switch-time in a descriptive framework, and from sorting as an evaluative process of the subject's mind that selects the optimizing schedule in a normative framework. In contrast, choice of time-allocations, as well as any other choice, in a neoclassical economic setting is derived from the observer's utility-maximizing bundle believed to represent the evaluative process of outcomes of different alternatives presumed to have been done by the agent.

In my normative framework, my theory is applied with an extra assumption that the conditionally rational subject optimizes his overall experience. My theory is both descriptive and normative because it explains the subject's choice of *any* time-allocations based on matching and it predicts his choice of *the* time-allocations based on sorting.

I am able to create both descriptive and normative frameworks in the application of my theory because I have founded it with subjective feeling as a primitive that is more basic than the preference relation. Therefore, I am able to apply a theory that does not depend on the strong assumption of rationality. In contrast, rational choice theory is a theory that cannot be both "normatively adequate and descriptively accurate." Tversky and Kahneman (1986, p. S251) wrote: "Invariance and dominance [in the preference relation] are obeyed when their application is transparent and often violated in other situations. Because these rules are normatively essential but descriptively invalid, no theory of choice can be both normatively adequate and descriptively accurate."

Also, they attributed violations of the principle of invariance to: "... the rules that govern the framing of decision and to the psychophysical principles of evaluation embodied in prospect theory" (Tversky and Kahneman, 1986, p. S251). However, the point of reference, upon which the agent is assumed to make his evaluation of outcomes, in their notion of framing, is indeterminate: "Outcomes are commonly perceived as positive or negative in relation to a reference outcome that is judged neutral. Variations of the reference point can therefore determine whether a given outcome is evaluated as a gain or as a loss" (Tversky and Kahneman, 1981, p. 456).

The indeterminate nature of the reference point has also been the object of criticism of prospect theory (Kahneman and Tversky, 1979), as pointed out in a recent review article in SimplyPsychology[3] by Charlotte Nickerson (2023). Specifically, Kőszegi and Rabin (2007, p. 1047) note that: "... different specifications of the reference point ... explain many risk attitudes ... but they also generate mutually inconsistent predictions that to our knowledge have not been formally reconciled." Neither framing nor prospect theory describes the process by which the agent makes his choice.

In contrast, in the application of my subjective feeling theory, the descriptive framework explains choice with matching as the biological process by which the subject makes his choice of a time-allocation to the engaged activity, and the normative framework predicts choice with sorting as the evaluative process by which he chooses his optimal schedule. Nevertheless, framing points to the importance of ordering of alternatives. In

[3]Prospect Theory In Psychology: Loss Aversion Bias, SimplyPsychology, simplypsychology.org/prospect-theory.html, Accessed 11/26/2023.

the application of my theory, ordering is manifested in the subject's choices in both of my frameworks by the sequence of time-allocations to engaged activities in the optimal or non-optimal schedule because the engaged system clock measures the calendar time.

While prospect theory (Kahneman and Tversky, 1979) used past experience to inform the agent on his reference point, the notion of experienced utility (Kahneman et al., 1997) used instantaneous experience to represent the hedonic quality of cardinal utility introduced by Bentham (1781). Although they include experience, as noted earlier, their theory is a normative theory that is applied to extend decision/utility theory with the preference relation as the primitive. In contrast, my theory is founded and applied with subjective feeling as the primitive.

Other differences between the notion of experienced utility by Kahneman et al. (1997) and the application of my theory are: i) their theory uses only stopwatch time that restarts at 0 whereas my theory uses calendar time for the engaged system and stopwatch time for the non-engaged system; ii) they prove the existence of unspecified functional forms for experienced utility whereas I prove unique functional forms for intensities of feeling; iii) their representation of hedonic experience does not change over time or with the agent's age whereas my measurements of integrated experience can change both over time (with changing $\alpha(t)$ or $\beta(t)$) and with the subject's age; and iv) their theory does not allow discounting or uncertainty whereas my theory includes both discounting and uncertainty.

After these comparisons between select theories that involve experience and applications of my theory to time-allocations, I present next some concluding remarks.

Chapter 4

The Conclusion

In this thesis, I have shown how to measure the intensity of feeling with functions of intensities of feeling from engaged activities as well as non-engaged activities. I have also shown how to use these functions to explain and predict the choice of time-allocations to engaged activities, as well as their optimal sequence. The term activity in my research is similar to the term good in neoclassical economic theory.

Studies have reported violations of the strong assumption of rationality required for the existence of utility. However, the notion of utility as an ordinal representation of the preference relation remains the predominant analytical tool in modern economics.

I offer my measure of intensity of feeling as a more useful analytical tool than utility because, once the subject's time-allocations are known, the observer has the potential to explain and predict any behavior by analyzing economic, social, or cultural choices. Maintaining a distinction between the subject and the observer, I have founded subjective feeling theory with weaker assumptions than those in neoclassical economic theory because the primitive subjective feeling is a more basic human characteristic than the preference relation.

The fist advantage of applying my theory to analyze human behavior is that the observer can explain any choice of time-allocations with a descriptive framework that does not impose on the subject the requirement of rationality or assumes optimization. The subject's choice can be optimal or non-optimal in the descriptive framework.

The second advantage is that the observer can predict the optimal choice of time-allocations, and their sequence with a normative framework that imposes on the subject the requirement of conditional rationality and assumes optimization. The subject's choice must be the optimal schedule of maximum overall experience in the normative framework.

These advantages are illustrated in Figure 4.1 below.

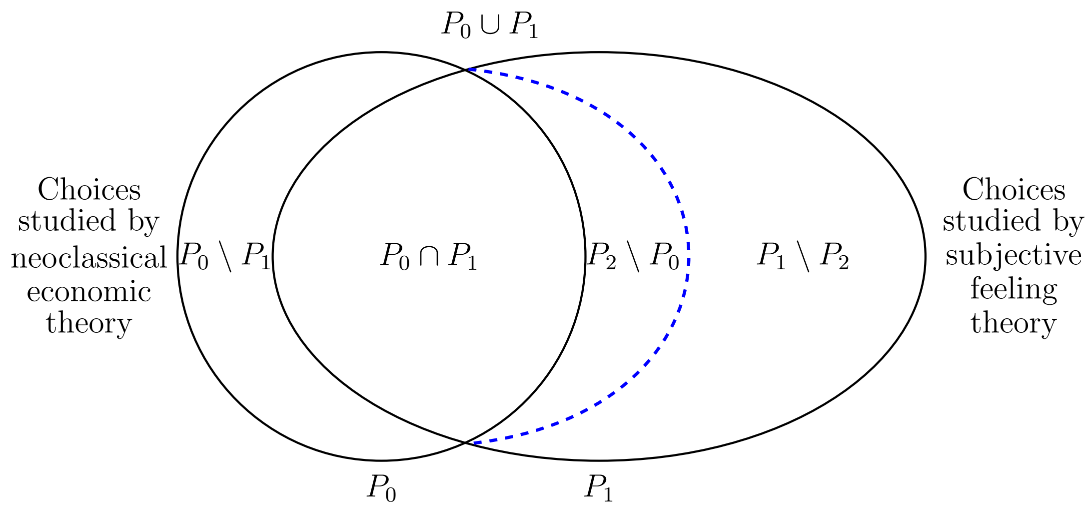

Figure 4.1: Advantages of applying subjective feeling theory to study choice

Conditional rationality imposed on the subject is a weaker requirement than rationality imposed on the agent. The rational agent in neoclassical economic theory is also the conditionally rational subject in subjective feeling theory. When someone is able to order infinitely many bundles of time allocations, he is also able to order finitely many of them.

If P_2 denotes choices made by the conditionally rational subject, then $P_2 \setminus P_0$ are additional choices predicted by the normative framework of subjective feeling theory but not by neoclassical economic theory. If the subject is not conditionally rational, then $P_1 \setminus P_2$ are further choices explained by the descriptive framework of subjective feeling theory but not by neoclassical economic theory or the normative framework of subjective feeling theory.

The input needed to apply subjective feeling theory is: 1) number of activities n; 2) time period $[0, T]$; 3) how the subject feels $\alpha(t)$; and 4) an initial condition for each intensity of feeling from the engaged activity and the respective representative specification for intensities of feeling from non-engaged activities $f_i(0), f_{j|i}(0), \forall i = 1, 2, 3, \ldots, n$. Subjective feeling theory is directly falsifiable because the observer's derived time-allocations from this input must be the subject's time-allocations observed with experimental or empirical data.

References

1. Acemoglu, D., P. Aghion, L. Bursztyn, and D. Hemous (2012): “The Environment and Directed Technical Change,” *American Economic Review*, 102, 131–66.

2. Addis, D. R., A. T. Wong, and D. L. Schacter (2007): “Remembering the Past and Imagining the Future: Common and Distinct Neural Substrates During Event Construction and Elaboration,” *Neuropsychologia*, 45, pp. 1363–77.

3. Akerlof, G. A., and R. E. Kranton (2000): “Economics and Identity,” *Quarterly Journal of Economics*, 115, 715–53.

4. Allcot, H., M. Gentzkow, and L. Song (2022): “Digital Addiction,” *American Economic Review*, 112, 2424–63.

5. Amalric, M., and S. Dehaene (2018): “Cortical Circuits for Mathematical Knowledge: Evidence for a Major Subdivision within the Brain’s Semantic Networks,” *Philosophical Transactions of the Royal Society B: Biological Sciences*, 373, Online
royalsocietypublishing.org/doi/pdf/10.1098/rstb.2016.0515,
Accessed 10/8/2023.

6. Andersen, S., G. W. Harrison, M. I. Lau, and E. E. Rutström (2008): “Eliciting Risk and Time Preferences,” *Econometrica*, 76, 583–618.

7. Andersson, J. J. (2019): “Carbon Taxes and CO2 Emissions: Sweden as a Case Study,” *American Economic Journal: Economic Policy*, 11, 1-30.

8. Andreoni, J., and J. Miller (2002): “Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism,” *Econometrica*, 70, 737–53.

9. Arrow, K. J. (1950): “A Difficulty in the Concept of Social Welfare,” *Journal of Political Economy*, 58, 328–46.

10. Arrow, K. J. (1959): “Rational Choice Functions and Orderings,” *Economica*, 102, 121–7.

11. Barberis, N., M. Huang, and R. H. Thaler (2006): "Individual Preferences, Monetary Gambles, and Stock Market Participation: A Case for Narrow Framing," *American Economic Review*, 96, 1069–90.

12. Barsky, R. B., F. T. Juster, M. S. Kimball, and M. D. Shapiro (1997): "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *Quarterly Journal of Economics*, 112, 537–79.

13. Bechara, A., A. R. Damasio, H. Damasio, and S. W. Anderson (1994): "Insensitivity to Future Consequences Following Damage to Human Prefrontal Cortex," *Cognition*, 50, 7–15.

14. Bechara, A., D. Tranel, and H. Damasio (2000): "Characterization of the Decision-making Deficit of Patients with Ventromedial Prefrontal Cortex Lesions," *Brain*, 11, 2189–202.

15. Becker, G. (1962): "Irrational Behavior and Economic Theory," *Journal of Political Economy*, 70, 1–13.

16. Becker, G. (1965): "A Theory of the Allocation of Time," *Economic Journal*, 75, 493–517.

17. Becker, G. (1973): "A Theory of Marriage: Part I," *Journal of Political Economy*, 81, 813–46.

18. Becker, G. (1974): "A Theory of Marriage: Part II, *Journal of Political Economy*, 82, 11-26.

19. Becker, G. S., and K. M. Murphy (1988): "A Theory of Rational Addiction," *Journal of Political Economy*, 96, 675–700.

20. Becker, G. S., and C. B. Mulligan (1997): "The Endogenous Determination of Time Preference," *Quarterly Journal of Economics*, 112, 729–58.

21. Bentham, J. (1781): An Introduction to the Principles of Morals and Legislation. (Reprinted by Kitchener: Batoche Books, Online historyofeconomicthought.mcmaster.ca/bentham/morals.pdf, Accessed 10/7/2023.)

22. Berhnheim, B. D., and A. Rangel (2004): "Addiction and Cue-Triggered Decision Processes," *American Economic Review*, 94, 1558–90.

23. Berridge, K. C., and J. P. O'Doherty (2014): "From Experienced Utility to Decision Utility," in *Neuroeconomics: Decision Making and the Brain*, 2nd ed., ed. by Glimcher, P. W., and E. Fehr. Academic Press, 335–51.

24. Boucher, P. O., T. Wang, L. Carceroni, G. Kane, K. V. Shenoy, and C. Chandrasekaran (2023): "Initial Conditions Combine with Sensory Evidence to Induce Decision-related Dynamics in Premotor Cortex," *Nature Communications*, 14, 6510, Online nature.com/articles/s41467-023-41752-2, Accessed 10/28/2023.

25. Bradley, M. M., and P. J. Lang (1994): "Measuring Emotion: The Self-Assessment Manikin and the Semantic Differential," *Journal of Behavior Therapy and Experimental Psychiatry*, 25, 49–59.

26. Burger, E. B., and M. Starbird (2013): The Heart of Mathematics. Fourth Edition. US: Wiley.

27. Burten, N. (2020): Heaven and Hell: The Psychology of the Emotions. Ataraxia Series Book 3. Oxford: Acheron Press.

28. Campbell, J. Y, and J. Cochrane (1999): "Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–51.

29. Camerer, C., G. Loewenstein, and D. Prelec (2005): "Neuroeconomics: How Neuroscience Can Inform Economics," *Journal of Economic Literature*, 43, 9–64.

30. Chalmers, D. J. (1995): "Facing Up to the Problem of Consciousness," *Journal of Consciousness Studies*, 2, 200–19.

31. Chambers, C. P., and F. Echenique (2016): Revealed Preference Theory. Cambridge: Cambridge University Press.

32. Chambers, C. P., and F. Echenique (2018): "On Multiple Discount Rates," *Econometrica*, 86, 1325–46.

33. Charness, G., and M. Rabin (2002): "Understanding Social Preferences with Simple Tests," *Quarterly Journal of Economics*, 117, 817–69.

34. Copeland, B. R., and M. S. Taylor (2004): "Trade, Growth, and the Environment," *Journal of Economic Literature*, 42, 7–71.

35. Cox, J. C., J. E. Ingersoll, Jr, and S. A. Ross (1985): "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53, 385–407.

36. D'Argembeau, A., and M. van der Linden (2004): "Phenomenal Characteristics Associated with Projecting Oneself Back into the Past and Forward into the Future: Influence of Valence and Temporal Distance," *Consciousness and Cognition*, 13, pp. 844-58.

37. Damasio, A. (1994): Decartes' Error: Emotion, Reason, and the Human Brain. New York: Penguin Group.

38. Damasio, A. (2018): The Strange Order of Things: Life, Feeling, and the Making of Cultures. New York: Pantheon Books.

39. Damasio, A. (2021): Feeling & Knowing: Making Minds Conscious. New York: Pantheon Books.

40. Debreu, G. (1954): "Representation of a Preference Ordering by a Numerical Function," in *Decision Processes*, ed. by Thrall, R., C. H. Coombs, and R. C. Davis. New York: John Wiley and Sons, 159–65.

41. Debreu, G. (1959): Theory of Value: An Axiomatic Analysis of Economic Equilibrium. New Haven: Yale University Press.

42. Dolan, P., and D. Kahneman (2008): "Interpretations of Utility and their Implications for the Valuation of Health," *Economic Journal*, 118, 215–34.

43. Edgeworth, F. Y. (1881): Mathematical Physics: An Essay on the Application of Mathematics to the Moral Sciences. London: C. Kegan Paul & Co. (Online historyofeconomicthought.mcmaster.ca/edgeworth/mathpsychics.pdf, Accessed 10/22/2023.)

44. Elster, J. (1979): Ulysses and the Sirens. Cambridge: Cambridge University Press.

45. Epstein, L. G, and S. E. Zin (1991): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, 99, 263–86.

46. Evren, Ö., and E. A. Ok (2011): "On the Multi-utility Representation of Preference Relations," *Journal of Mathematical Economics*, 47, 554–63.

47. Fischbacher, U., and S. Gachter (2010): "Social Preferences, Beliefs, and the Dynamics of Free Riding in Public Goods Experiments," *American Economic Review*, 100, 541–56.

48. Franzova, E., Q. Shen, K. Doyle, J. M. Chen, J. Egbebike, A. Vrosgou, J. C. Carmona, L. Grobois, G. A. Heinonen, A. Velazquez, I. J. Gonzales, S. Egawa, S. Agarwal, D. Roh, S. Park, E. S. Connolly, and J. Claassen (2023): "Injury Patterns Associated with Cognitive Motor Dissociation," *Brain*, Advanced article: published 14 August 2023.

49. Frederick, S., G. Loewenstein, and T. O'Donoghue (2002): "Time Discounting and Time Preference: A Critical Review," *Journal of Economic Literature*, 40, 351–401.

50. Frisch, R. (1926): "Sur un Probleme d'Économie Pure." Norsk Matematisk Forenings Skrifter, 1/16, Online sv.uio.no/econ/om/tall-og-fakta/nobelprisvinnere/ragnar-frisch/published-scientific-work/rf-published-scientific-works/rf1926h-transcribed.pdf, Accessed 11/3/2023 (Republished in Metroeconomica, 9(2), pp. 79–111, 1957).

51. Fudenberg, D., and D. K. Levine (2006): "A Dual-Self Model of Impulse Control," *American Economic Review*, 96, 1449–76.

52. Fuhrer, J. C. (2000): "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," *American Economic Review*, 90, 367–90.

53. Gilbert, D. T. and T. D. Wilson (2007): "Prospection: Experiencing the Future," *Science*, 317, 1351–4.

54. Gilboa, I., and D. Schmeidler (1989): "Maxmin Expected Utility with Non-unique Prior," *Journal of Mathematical Economics*, 18, 141–53.

55. Gilboa, I., A. Postlewaite, L. Samuelson, and D. Schmeidler (2019): "What Are Axiomatizations Good For?" *Theory and Decision*, 86, 339–59.

56. Grossman, G. M., and A. B. Krueger (1995): "Economic Growth and the Environment," *Quarterly Journal of Economics*, 110, 353–77.

57. Gruber, J., and B. Köszegi (2001): "Is Addiction "Rational"? Theory and Evidence," *Quarterly Journal of Economics*, 116, 1261–1303.

58. Hamermesh, D. S. (2020): Economics is Everywhere. 5th ed. New York: Macmillan Learning.

59. Harless, D., and C. F. Camerer (1994): "The Predictive Utility of Generalized Expected Utility Theories," *Econometrica*, 62, 1251–89.

60. Harrison, G. W., J. Martínez-Correa, and J. T. Swarthout, J. Todd (2015): "Reduction of Compound Lotteries with Objective Probabilities: Theory and Evidence," *Journal of Economic Behavior & Organization*, 119, 32–55.

61. Hassabis, D., D. Kumaran, and E. A. Maguire (2007): "Using Imagination to Understand the Neural Basis of Episodic Memory," *Journal of Neuroscience*, 27, 14365–74.

62. Heintzelman, S. J., and L. A. King (2019): "Routines and Meaning in Life," *Personality and Social Psychology Bulletin*, 45, 688–99.

63. Hey, J., and C. Orme (1994): "Investigating Generalizations of Expected Utility Theory Using Experimental Data," *Econometrica*, 62, 1291–326.

64. Holtz-Eakin, D., and T. M. Selden (1995): "Stoking the Fires? CO2 Emissions and Economic Growth," *Journal of Public Economics*, 57, 85-101.

65. Houthakker, H. S. (1950): "Revealed Preference and the Utility Function," *Economica* 17, 159-74.

66. Jehle, G. A, and P. J. Reny (2011): Advanced Microeconomic Theory. Third Edition. Financial Times Prentice Hall: Pearson.

67. Janoff-Bulman, R. (1992): Shattered Assumptions: Towards a New Psychology of Trauma. New York, NY: The Free Press.

68. Jevons, W. S. (1888). The Theory of Political Economy, 3d ed. Reprint of 1879, 2nd, edition, by Harriet A. Jevons. (Reprinted by Econlib (2018) Online econlib.org/library/YPDBooks/Jevons/jvnPE.html, Accessed 10/19/2023.)

69. Kahneman, D. (2003): "Maps of Bounded Rationality: Psychology for Behavioral Economics," *American Economic Review*, 93, 1449–75.

70. Kahneman, D., and D. Lovallo (1993): "Timid Choices and Bold Forecasts: A Cognitive Perspective on Risk Taking," *Management Science*, 39, 17–31.

71. Kahneman, D., and Tversky, A. (1979): "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47, 263–91.

72. Kahneman, D., and A. Tversky (1984): "Choices, values and frames," *American Psychologist*, 39, 341–50.

73. Kahneman, D., P. P. Wakker, and R. Sarin (1997): "Back to Bentham? Explorations of Experienced Utility," *Quarterly Journal of Economics*, 112, 375–406.

74. Knight, F. H. (1921): Risk, Uncertainty, and Profit, Boston: Houghton Mifflin, Online www.econlib.org/library/Knight/knRUP.html, Accessed 11/12/2023.

75. Kreps, D. M. (2013): Microeconomic Foundations I: Choice and Competitive Markets. Princeton: Princeton University Press.

76. Kroger, J. K., L. E. Nystrom, J. D. Cohen, and P. N. Johnson-Laird (2008): "Distinct Neural Substrates for Deductive and Mathematical Processing," *Brain Research*, 1243, 86-103.

77. Kőszegi, B., and M. Rabin (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 121, 1133–65.

78. Kőszegi, B., and M. Rabin (2007): "Reference-Dependent Risk Attitudes," *American Economic Review*, 97, 1047–73.

79. Lafortune, J., and C. Low (2023): "Collateralized Marriage," *American Economic Journal: Applied Economics*, 15, 252–91.

80. Laibson, D. (1997): "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 112, 443–78.

81. Lee, S., T. Parthasarathi, and J. W. Kable (2021): "The Ventral and Dorsal Default Mode Networks Are Dissociably Modulated by the Vividness and Valence of Imagined Events," *Journal of Neuroscience*, 41, pp. 5243–5250.

82. Lenharo, M. (2023): "Decades-long Bet on Consciousness Ends — and it's Philosopher 1, Neuroscientist 0," *Nature*, 619, 14–5.

83. Levitt, S. D., and J. A. List (2007): "What Do Laboratory Experiments Measuring Social Preferences Reveal About the Real World?" *Journal of Economic Perspectives*, 21, 153–74.

84. Lind, J., V. Vinken, M. Jonsson, S. Ghirlanda, and M. Enquist (2023): "A Test of Memory for Stimulus Sequences in Great Apes," *PLOS ONE*,
Online
journals.plos.org/plosone/article?id=10.1371/journal.pone.0290546,
Accessed 10/22/2023.)

85. Loewenstein, G., and D. Prelec (1992): "Anomalies in Intertemporal Choice: Evidence and an Interpretation," *Quarterly Journal of Economics*, 107, pages 573–97.

86. Loomes, G., and R. Sugden (1995): "Incorporating a Stochastic Element into Decision Theories," *European Economic Review*, 39, 641–8.

87. Loomes, G, and R. Sugden (1998): "Testing Different Stochastic Specifications of Risky Choice," *Economica*, New Series, 65, 581–98.

88. Lotka, A. J. (1910): "Contributions to the Theory of Periodic Reactions," *Journal of Physical Chemistry*, 14, 271–274.

89. Lundberg, S., and R. Pollak (1993): "Separate Spheres Bargaining and the Marriage Market," *Journal of Political Economy*, 101, 988–1010.

90. Machina, M. J., and D. Schmeidler (1992): "A More Robust Definition of Subjective Probability," *Econometrica* 60, 745–80.

91. Mas-Colell, A., M. D. Whinston, and J. R. Green (1995): Microeconomic Theory. New York: Oxford University Press.

92. Matzkin, R. L. (1991): "Axioms of Revealed Preference for Nonlinear Choice Sets," *Econometrica*, 59, 1779–86.

93. May, K. O. (1954): "Intransitivity, Utility and the Aggregation of Preference Patterns," *Econometrica*, 22, 1–13.

94. McElvain, L. E., M. Faulstich, J. M. Jeanne, J. D. Moore, and S. du Lac (2015): "Implementation of Linear Sensory Signaling via Multiple Coordinated Mechanisms at Central Vestibular Nerve Synapses," *Neuron*, 85, 1132–44.

95. McFadden, D. L. (2005): "Revealed Stochastic Preference: A Synthesis," *Economic Theory*, 26, 245-64.

96. Melloni, L., L. Mudrik, M., Pitts, and C. Koch (2021): "Making the Hard Problem of Consciousness Easier," *Science*, 372, 911–2.

97. Meyer, A. (1922): "The philosophy of Occupational Worker", *Archive of Occupational Therapy*, 1, 1–11.

98. Miettinen, T., M. Kosfeld, E. Fehr, and J. Weibull (2020): "Revealed Preferences in a Sequential Prisoners' Dilemma: A Horse-race between Six Utility Functions," *Journal of Economic Behavior & Organization*, 173, 1–25.

99. Moretti, L, D. Dragone, and G. di Pellegrino (2009): "Reward and Social Valuation Deficits following Ventromedial Prefrontal Damage," *Journal of Cognitive Neuroscience*, 21, 128–40.

100. Muller, N. Z., R. Mendelsohn, and W. Nordhaus (2011): "Environmental Accounting for Pollution in the United States Economy," *American Economic Review*, 101, 1649–75.

101. Nilsen, A. S., B. E. Juel, B. Thürer, A. Aamodt, and J. F. Storm (2022): "Are We Really Unconscious in "Unconscious" States? Common Assumptions Revisited," *Frontiers in Human Neuroscience*, 16, Online frontiersin.org/articles/10.3389/fnhum.2022.987051/full, Accessed 10/8/2023

102. Ok, E. A. (2002): "Utility Representation of an Incomplete Preference Relation," *Journal of Economic Theory*, 104, 429–49.

103. Polisson, M., J. K.-H. Quah, and L. Renou (2020): "Revealed Preferences over Risk and Uncertainty," *American Economic Review*, 110, 1782–820.

104. Rader, T. (1963): "The Existence of a Utility Function to Represent Preferences," *Review of Economic Studies*, 30, 229–32.

105. Reddy, L., B. Zoefel, J. K. Possel, J. Peters, D. E. Dijksterhuis, M. Poncet, E. C. W. van Straaten, J. C. Baayen, S. Idema, and M. W. Self (2022): "Human Hippocampal Neurons Track Moments in a Sequence of Events," *Journal of Neuroscience*, 41, pp. 6714–25.

106. Redelmeier, D. A., and D. Kahneman (1996b): "Improving the Memory of a Colonoscopy," *Working Paper*, Unpublished.

107. Samuelson, P. A. (1938): "A Note on the Pure Theory of Consumer's Behaviour," *Economica*, 5, 61–71.

108. Samuelson, P. A. (1948): "Consumption Theory in Terms of Revealed Preference," *Economica*, 15, 243–53.

109. Savage, L. J. (1954): The Foundations of Statistics. New York: John Wiley and Sons.

110. Schacter, D., D. Addis, and R. Buckner (2007): " Remembering the Past to Imagine the Future: The Prospective Brain," *National Reviews Neuroscience*, 8, 657–61.

111. Schelling, T. C. (1978): "Ergonomics, or the Art of Self-Management," *American Economic Review*, 68, 290–94.

112. Schelling, T. (2006): Strategies of Commitment and Other Essays. Cambridge, MA: Harvard University Press.

113. Schmeidler, D. (1971): "A Condition for the Completeness of Partial Preference Relations," *Econometrica*, 39, 403–04.

114. Schmeidler, D. (1989): "Subjective Probability and Expected Utility without Additivity," *Econometrica*, 57, 571–87.

115. Sharot, T., A. M. Riccardi, C. M. Raio, and E. A. Phelps (2007): "Neural Mechanisms Mediating Optimism Bias," *Nature*, 450 102–5.

116. Stern, D., M. S. Common, and E. Barbier (1996): "Economic Growth and Environmental Degradation: The Environmental Kuznets Curve and Sustainable Development," *World Development*, 1996, 24, 1151–60.

117. Stokey, N. L., R. E. Lucas, and E. C. Prescott (1989): Recursive Methods in Economic Dynamics. Cambridge: Harvard University Press.

118. Stoye, J. (2011): "Axioms for Minimax Regret Choice Correspondences," *Journal of Economic Theory*, 146, 2226–51.

119. Thaler, R. H., and H. M. Shefrin (1981): "An Economic Theory of Self-Control," *Journal of Political Economy*, 1981, 392–406.

120. Tversky, A. (1969): "Intransitivity of Preferences," *Psychological Review*, 76, 31–48.

121. Tversky, A., and D. Kahneman (1981): "The Framing of Decisions and the Psychology of Choice," *Science*, 211, 453–8.

122. Tversky, A. and D. Kahneman (1986): "Rational Choice and the Framing of Decisions," *Journal of Business*, 59, Part 2: The Behavioral Foundations of Economic Theory: S251–S278.

123. Tversky, A, P. Slovic, and D. Kahneman (1990): "The Causes of Preference Reversal," *American Economic Review*, 80, 204–17.

124. Umbach, G., P. Kantak, J. Jacobs, M. Kahana, B. E. Pfeiffer, M. Sperling, and B. Lega (2021): "Time Cells in the Human Hippocampus and Entorhinal Cortex Support Episodic Memory," *Proceedings of the National Academy of Sciences*, 117, pp. 28463–28474.

125. Volterra, V. (1926): "Fluctuations in the Abundance of a Species considered Mathematically," *Nature*, 118, 558–60.

126. von Neumann, J., and O. Morgenstern (1944): Theory of Games and Economic Behavior. Princeton: Princeton University Press.

127. Wangersky, P. J. (1978): "Lotka-Volterra Population Models," *Annual Review of Ecology and Systematics*, 9, 189–218.

128. Yaron, I., L. Melloni, M. Pitts, and L. Mudrik (2022): "The ConTraSt Database for Analysing and Comparing Empirical Studies of Consciousness Theories," *Nature Human Behaviour*, 6, 593–604.

Appendix A

The Theory: Proofs and Calculations

A.1 Proof of Theorem 1

Theorem 4 (The Family of Intensities of Feeling from Engaged Activities)**.** *Given n activities, one engaged activity at a time, and the common coefficient of proportionality $\alpha(t) > 0$ that is piecewise constant with a finite number of jump discontinuities, if the rate of change in intensity of feeling from an activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities, there exists a unique family of intensities of feeling functions from engaged activities expressed explicitly:*

$$\mathbf{f}(t) = Ke^{\tilde{\Lambda}}\mathbf{c}$$

Given $c_1, c_2, c_3, \ldots, c_n$ from the initial conditions $\mathbf{f}(0)$, intensities of feeling from engaged activities have the unique specifications:

$$\mathbf{f}(t) = Ke^{\tilde{\Lambda}}\mathbf{c}$$

Given $c_1, c_2, c_3, \ldots, c_n$ from the initial conditions $\mathbf{f}(0)$, intensities of feeling from engaged activities have the unique specifications:

$$\begin{aligned}
f_1(t) &= c_1 e^{(2-n)\tilde{\alpha}t} + \sum_{i=2}^{n} c_i e^{2\tilde{\alpha}t} \\
f_2(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_2 e^{2\tilde{\alpha}t} \\
f_3(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_3 e^{2\tilde{\alpha}t} \\
&\vdots \\
f_n(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_n e^{2\tilde{\alpha}t}
\end{aligned}$$

Proof. The engaged system in matrix notation is

$$\begin{bmatrix} \dot{f}_1 \\ \dot{f}_2 \\ \dot{f}_3 \\ \vdots \\ \dot{f}_n \end{bmatrix} = \alpha(t) \begin{bmatrix} 1 & -1 & -1 & \cdots & -1 \\ -1 & 1 & -1 & \cdots & -1 \\ -1 & -1 & 1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \\ \vdots \\ f_n(t) \end{bmatrix}$$

$$\dot{\mathbf{f}} = \alpha(t) A \mathbf{f}(t)$$

Since $\alpha(t)$ is continuous or has a finite number of jump discontinuities, $\tilde{\alpha}(t) = \int_{t_0}^{t} \alpha(t)\mathrm{d}t$ exists for any t_0 within the (sub)interval where $\alpha(t)$ is continuous.

To find solutions, I look for a fundamental set of solutions of the form: $\mathbf{v}(t) = \mathbf{k}e^{\lambda\tilde{\alpha}t}$, where $\mathbf{k} = (k_1, k_2, k_3, \ldots, k_n)$ is a vector of scalars and λ is a scalar. Substituting $\mathbf{v}(t)$ into the system:

$$\alpha(t)\lambda \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_n \end{bmatrix} e^{\lambda_i \tilde{\alpha} t} = -\alpha(t) \begin{bmatrix} 1 & -1 & -1 & \cdots & -1 \\ -1 & 1 & -1 & \cdots & -1 \\ -1 & -1 & 1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_n \end{bmatrix} e^{\lambda \tilde{\alpha} t}$$

$$(A - \lambda_i \mathbf{I})\,\mathbf{k} = \mathbf{0}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{0}$ is the zero-vector. Hence, a non-trivial solution to the engaged system exists if and only if λ is an eigenvalue of matrix A.

Since A is a real symmetric matrix, it has a complete set of real (orthogonal) eigenvectors and corresponding real eigenvalues (some of which may be repeated). If $K = \begin{bmatrix} \mathbf{k}^1 & \mathbf{k}^2 & \mathbf{k}^3 \ldots & \mathbf{k}^n \end{bmatrix}$ denotes the matrix with eigenvectors of A, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n)$ the diagonal matrix with its corresponding eigenvalues, and $\tilde{\Lambda} = \tilde{\alpha}\Lambda$, then

$$V = Ke^{\tilde{\Lambda}} = \begin{bmatrix} \mathbf{v}^1(t) & \mathbf{v}^2(t) & \mathbf{v}^3(t) & \ldots & \mathbf{v}^n(t) \end{bmatrix}$$

consists of n real-valued linearly independent functions that form a fundamental set of solutions. If $\mathbf{c} = (c_1, c_2, c_3, \ldots, c_n)$ are non-zero real constants, then the general solution is:

$$\mathbf{f}(t) = V(t)\mathbf{c} = c_1\mathbf{v}^1(t) + c_2\mathbf{v}^2(t) + c_3\mathbf{v}^3(t) + \ldots + c_n\mathbf{v}^n(t)$$

Since this is the general solution, it is the only solution.

Therefore, the solution to the engaged system consists of a unique family of functions that are expressed explicitly:

$$\mathbf{f}(t) = Ke^{\tilde{\Lambda}}\mathbf{c} = c_1\mathbf{k}^1 e^{\lambda_1\tilde{\alpha}t} + c_2\mathbf{k}^2 e^{\lambda_2\tilde{\alpha}t} + c_3\mathbf{k}^3 e^{\lambda_3\tilde{\alpha}t} + \ldots + c_n\mathbf{k} e^{\lambda_n\tilde{\alpha}t}$$

Since c_i, $i = 1, 2, 3, \ldots, n$, are also real, the solution functions are real-valued.

Since $\mathbf{v}^i(t)$ are linearly independent, for given initial conditions $\mathbf{f}(0)$, $c_1, c_2, c_3, \ldots, c_n$ are unique. Therefore, specifications of intensities of feeling from engaged activities are unique.

I find the eigenvalues and eigenvectors by inspection. For any eigenvalue λ:

$$\begin{bmatrix} 1-\lambda & -1 & -1 & \cdots & -1 \\ -1 & 1-\lambda & -1 & \cdots & -1 \\ -1 & -1 & 1-\lambda & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & 1-\lambda \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Since every row element of the matrix is equal to -1 except the diagonal elements equal to $1 - \lambda$, then $\lambda = 2$ produces

$$\begin{bmatrix} -1 & -1 & -1 & \cdots & -1 \\ -1 & -1 & -1 & \cdots & -1 \\ -1 & -1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \vdots \\ k_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$k_1 + k_2 + k_3 + \cdots + k_n = 0$$

from which $n - 1$ eigenvectors can be derived. Hence, $\lambda = 2$ is an eigenvalue of multiplicity $n-1$. It remains to find only one more eigenvalue, which becomes apparent once realizing that none of eigenvectors in the eigenspace of the eigenvalue $\lambda = 2$ can have only 1s. Hence, if the remaining eigenvector has only 1s, $\mathbf{k} = (1, 1, 1, \ldots, 1)$, then

$$1 - \lambda - (n - 1) = 0$$

and the remaining corresponding eigenvalue is $\lambda = 2 - n$ for any given n.[1] Hence, the

[1]Another way to find the eigenvalue $\lambda = 2 - n$ is to derive it from the trace of A, tr$A$. Since tr$A = n$ and $\lambda = 2$ is of multiplicity $n - 1$, the remaining eigenvalue is $\lambda = n - 2(n - 1) = 2 - n$.

eigenvalues and eigenvectors are readily available for any n, and I organize as follows.

$$K = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -1 & 0 & \cdots & 0 \\ 1 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & -1 \end{bmatrix} \text{ and } \Lambda = \begin{bmatrix} 2-n & 0 & 0 & \cdots & 0 \\ 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 2 \end{bmatrix}$$

Given $c_1, c_2, c_3, \ldots, c_n$ from the initial conditions $\mathbf{f}(0)$, intensities of feeling from engaged activities are uniquely specified:

$$\begin{aligned} f_1(t) &= c_1 e^{(2-n)\tilde{\alpha}t} + \sum_{i=2}^{n} c_i e^{2\tilde{\alpha}t} \\ f_2(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_2 e^{2\tilde{\alpha}t} \\ f_3(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_3 e^{2\tilde{\alpha}t} \\ &\vdots \\ f_n(t) &= c_1 e^{(2-n)\tilde{\alpha}t} - c_n e^{2\tilde{\alpha}t} \end{aligned}$$

This completes the proof. □

A.2 Example: Calculation of $f_W(t), f_C(t), f_R(t)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -2 & -1 & -1 \\ 0 & -1 & -2 & -2 \end{array}\right] \rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & -2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Then $c_1 = 1, c_2 = 0, c_3 = 1$, and $f_W(t) = e^{-t} + e^{2t}$, $f_C(t) = e^{-t}$, $f_R(t) = e^{-t} - e^{2t}$.

Appendix B

The Application: Proofs and Calculations

B.1 Proof of Theorem 2

Theorem 5 (The Family of Intensities of Feeling from Non-engaged Activities). *Given n activities, the engaged activity, and the common coefficient of proportionality $\alpha(t) > 0$ that is piecewise constant with a finite number of jump discontinuities, if the rate of change in intensity of feeling from an activity is proportional to the difference between the intensity of feeling from that activity and the sum of intensities of feeling from other activities, there exists a unique family of intensities of feeling functions from non-engaged activities expressed explicitly:*

$$\mathbf{f}_{|i}(t) = \mathbf{h}_{|i}(t) + \mathbf{f}^p_{-i}(t) = K_{|i}e^{\tilde{\Lambda}_{|i}}\mathbf{c}_{|i} + \mathbf{f}^p_{-i}(t)$$

Given $c^0_1, c^0_2, c^0_3, \ldots, c^0_n$ from the initial condition $\mathbf{f}(0)$, as well as $c_{1|i}, c_{2|i}, , c_{3|i}, \ldots c_{n-1|i}$ from any condition $\mathbf{f}_{|i}(t^*)$, intensities of feeling from non-engaged activities have the unique specifications:

$$\begin{array}{llll}
f_{1|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} + & \sum_{j=2}^{n-1} c_{j|i}e^{2\tilde{\alpha}t} + & c^0_1 e^{(2-n)\tilde{\alpha}t} + & \sum_{i=2}^{n} c^0_i e^{2\tilde{\alpha}t} \\
\vdots & & & \\
f_{i-1|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} - & c_{i-1|i}e^{2\tilde{\alpha}t} + & c^0_1 e^{(2-n)\tilde{\alpha}t} - & c^0_{i-1} e^{2\tilde{\alpha}t} \\
f_{i+1|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} - & c_{i+1|i}e^{2\tilde{\alpha}t} + & c^0_1 e^{(2-n)\tilde{\alpha}t} - & c^0_{i+1} e^{2\tilde{\alpha}t} \\
\vdots & & & \\
f_{n|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t} - & c_{n|i}e^{2\tilde{\alpha}t} + & c^0_1 e^{(2-n)\tilde{\alpha}t} - & c^0_n e^{2\tilde{\alpha}t}
\end{array}$$

Proof. The non-engaged system in matrix notation is

$$\begin{bmatrix} \dot{f}_{1|i} \\ \vdots \\ \dot{f}_{i-1|i} \\ \dot{f}_{i+1|i} \\ \vdots \\ \dot{f}_{n|i} \end{bmatrix} = \alpha(t) \begin{bmatrix} 1 & \cdots & -1 & -1 & \cdots & -1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ -1 & \cdots & 1 & -1 & \cdots & -1 \\ -1 & \cdots & -1 & 1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & \cdots & -1 & -1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} f_{1|i}(t) \\ \vdots \\ f_{i-1|i}(t) \\ f_{i+1|i}(t) \\ \vdots \\ f_{n|i}(t) \end{bmatrix} + \alpha(t) \begin{bmatrix} -f_i(t) \\ \vdots \\ -f_i(t) \\ -f_i(t) \\ \vdots \\ -f_i(t) \end{bmatrix}$$

$$\dot{\mathbf{f}}_{|i} = \alpha(t) A_{|i} \mathbf{f}_{|i}(t) + \mathbf{a}_{|i}$$

Let

$$V_{|i}(t) = K_{|i} e^{\tilde{\Lambda}_{|i}} = \begin{bmatrix} \mathbf{v}^1(t) & \mathbf{v}^2(t) & \mathbf{v}^3(t) & \ldots & \mathbf{v}^{n-1}(t) \end{bmatrix}$$

consists of $n-1$ real-valued linearly independent functions that form a fundamental set of solutions for the homogeneous part, as in Theorem 1. If $\mathbf{c}_{|i} = \left(c_{1|i}, c_{2|i}, c_{3|i}, \ldots, c_{n-1|i}\right)$ are real constants, then the general solution to the homogeneous part is:

$$\mathbf{h}_{|i}(t) = V_{|i}(t)\mathbf{c}_{|i} = c_{1|i}\mathbf{v}^1(t) + c_{2|i}\mathbf{v}^2(t) + c_{3|i}\mathbf{v}^3(t) + \ldots + c_{n-1|i}\mathbf{v}^{n-1}(t)$$

Since this is the general solution, it is the only solution to the homogeneous part.

Therefore, the solution to the homogeneous part consists of a unique family of functions that are expressed explicitly:

$$\mathbf{h}_{|i}(t) = K_{|i} e^{\tilde{\Lambda}_{|i}} \mathbf{c} = c_{1|i}\mathbf{k}^1 e^{\lambda_{1|i}\tilde{\alpha}t} + c_{2|i}\mathbf{k}^2 e^{\lambda_{2|i}\tilde{\alpha}t} + c_{3|i}\mathbf{k}^3 e^{\lambda_{3|i}\tilde{\alpha}t} + \ldots + c_{n-1|i}\mathbf{k}^{n-1} e^{\lambda_{n-1|i}\tilde{\alpha}t}$$

Since $c_j|i$, $j = 1, 2, 3, \ldots, n-1$, are also real, the solution functions to the homogeneous part are real-valued.

With the same reasoning as in Theorem 1, the eigenvalues and eigenvectors for the homogeneous part are readily for $n-1$, and I organize them similarly.

It remains to find a particular solution, which becomes apparent once realizing that all the specifications $f_1(t), \ldots, f_{i-1}(t), f_{i+1}(t), \ldots, f_n(t)$ for intensities of feeling from engaged activities from a given set of initial conditions $\mathbf{f}(0)$ that do not include the intensity of feeling from (the engaged) activity i, $f_i(t)$, from Theorem 1 form a particular solution to the non-engaged system. Let $\mathbf{f}^p_{-i}(t) = \left(f^p_1(t), \ldots, f^p_{i-1}(t), f^p_{i+1}(t), \ldots f^p_n(t)\right) = (f_1(t), \ldots, f_{i-1}(t), f_{i+1}(t), \ldots, f_n(t))$ denote these specifications. So $\mathbf{f}^p_{-i}(t)$ is a particular solution to the non-engaged system.

Given $c_{1|i}, c_{2|i}, , c_{3|i}, \ldots c_{n-1|i}$ from any condition $\mathbf{f}_{|i}(t^*)$, intensities of feeling from non-engaged activities are uniquely specified:

$$
\begin{array}{llll}
f_{1|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t}+ & \sum_{i\neq j=2}^{n} c_{j|i}e^{2\tilde{\alpha}t}+ & c_1^0 e^{(2-n)\tilde{\alpha}t}+ & \sum_{i=2}^{n} c_i^0 e^{2\tilde{\alpha}t} \\
\vdots & & & \\
f_{i-1|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t}- & c_{i-1|i}e^{2\tilde{\alpha}t}+ & c_1^0 e^{(2-n)\tilde{\alpha}t}- & c_{i-1}^0 e^{2\tilde{\alpha}t} \\
f_{i+1|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t}- & c_{i+1|i}e^{2\tilde{\alpha}t}+ & c_1^0 e^{(2-n)\tilde{\alpha}t}- & c_{i+1}^0 e^{2\tilde{\alpha}t} \\
\vdots & & & \\
f_{n|i}(t) = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t}- & c_{n|i}e^{2\tilde{\alpha}t}+ & c_1^0 e^{(2-n)\tilde{\alpha}t}- & c_n^0 e^{2\tilde{\alpha}t}
\end{array}
$$

This completes the proof. □

B.2 Proof of Theorem 3

Theorem 6 (Switch-time Determination)**.** *There is a one-to-one correspondence between the zero-state t^* and specifications of intensities of feeling from non-engaged activities, and the switch-time t^* is an instant. Matching determines the instant when the switch-time t^* occurs by the initial condition of the unique specification for intensities of feeling from all non-engaged activities given the engaged activity.*

Proof. In order for t^* to be a zero-state, $f_{j|i}(t^*) = 0$ for all $j|i$. Then $\mathbf{f}_{|i}(t^*) = \mathbf{0}$, where $\mathbf{0}$ is the zero-vector. From the proof of Theorem 2, $\mathbf{f}_{|i}^{p}(t)$ is a particular solution to the non-engaged system, and at $t = t^*$:

$$\mathbf{0} = c_{1|i}\mathbf{v}^1(t^*) + c_{2|i}\mathbf{v}^2(t^*) + c_{3|i}\mathbf{v}^3(t^*) + \cdots + c_{n-1|i}\mathbf{v}^{n-1}(t^*) + \mathbf{f}_{-i}^{p}(t^*)$$

$$-\mathbf{f}_{-i}^{p}(t^*) = c_{1|i}\mathbf{v}^1(t^*) + c_{2|i}\mathbf{v}^2(t^*) + c_{3|i}\mathbf{v}^3(t^*) + \cdots + c_{n-1|i}\mathbf{v}^{n-1}(t^*)$$

Since $\mathbf{v}^1(t^*), \mathbf{v}^2(t^*), \mathbf{v}^3(t^*), \ldots, \mathbf{v}^{n-1}(t^*)$ are linearly independent, there exist unique constants corresponding to the switch-time t^* denoted by $\mathbf{c}_{|i}^s = (c_{1|i}^s, c_{2|i}^s, c_{3|i}^s, \ldots, c_{n-1|i}^s)$ that solve the above system, and vice versa. Hence, $t^* \Leftrightarrow \mathbf{c}_{|i}^s$.

Let $\mathbf{c}_{|i}^s$ be the constants for specifications in Theorem 2 found from the given condition $\mathbf{f}_{|i}(t^*) = (f_{1|i}(t^*), \cdots, f_{i-1|i}(t^*), f_{i+1|i}(t^*), \cdots, f_{n|i}(t^*))$. Since $\mathbf{f}_{|i}(t^*) \Leftrightarrow \mathbf{c}_{|i}^s$, there is a one-to-one correspondence between the zero-state/switch-time t^* and specifications of intensities of feeling from non-engaged activities.

Also, $t^* \Leftrightarrow \mathbf{c}_{|i}^s$ implies that t^* is instant. If t_k^* is the switch-time from activity i to activity k, there is only one moment in time when this switch happens. If the subject engages in activity i more than once during $[0, T]$, and again switches to k, then this new switch again can only happen at another instant corresponding to a different set of $\mathbf{c}_{|i}^s = \left(c_{1|i}^s, c_{2|i}^s, c_{3|i}^s, \ldots, c_{n-1|i}^s\right)$. In particular, it is not possible for this switch to happen during an interval of instants of time, so there is always only one engaged activity at a time, as in my Assumption II.

It remains to show that matching happens by the unique specification of all intensities of feeling functions from non-engaged activities, and it is this unique specification that determines the switch-time t^*.

If $n = 1$, there is no switch-time, so the following discussion is relevant when $n \geq 2$. If $f_{j|i}$ are the same for all j, then the coefficients for $e^{2\tilde{\alpha}t}$ must be the same:

$$\sum_{i \neq j=2}^{n} c_{j|i} + \sum_{i=2}^{n} c_i^0 = \cdots = -c_{i-1|i} - c_{i-1}^0 = -c_{i+1|i} - c_{i+1}^0 = -c_{n|i} - c_n^0$$

Then subtracting sequentially from the first left-hand side of these equalities, I obtain the following system.

$$\begin{gathered} 2c_{2|i} + \textstyle\sum_{i\neq j\neq 2}^{n-1} c_{j|i} + \sum_{i=2}^{n} c_i^0 - c_2^0 = 0 \\ \vdots \\ 2c_{i-1|i} + \textstyle\sum_{i\neq j\neq (i-1)}^{n-1} c_{j|i} + \sum_{i=2}^{n} c_i^0 - c_{i-1}^0 = 0 \\ 2c_{i+1|i} + \textstyle\sum_{i\neq j\neq (i+1)}^{n-1} c_{j|i} + \sum_{i=2}^{n} c_i^0 - c_{i+1}^0 = 0 \\ \vdots \\ 2c_{n|i} + \textstyle\sum_{i\neq j\neq n}^{n-1} c_{j|i} + \sum_{i=2}^{n} c_i^0 - c_n^0 = 0 \end{gathered}$$

Hence, there are $n-2$ equations in $n-2$ unknowns $c_{2|i}, \ldots, c_{i-1|i}, c_{i+1|i}, \ldots, c_{n|i}$. In matrix notation:

$$\begin{bmatrix} 2 & \cdots & 1 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 1 & \cdots & 2 & 1 & \cdots & 1 \\ 1 & \cdots & 1 & 2 & \cdots & 1 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & \cdots & 2 \end{bmatrix} \begin{bmatrix} c_{2|i} \\ \vdots \\ c_{i-1|i} \\ c_{i+1|i} \\ \vdots \\ c_{n|i} \end{bmatrix} = \begin{bmatrix} c_2^0 - \sum_{i=2}^{n} c_i^0 \\ \vdots \\ c_{i-1}^0 - \sum_{i=2}^{n} c_i^0 \\ c_{i+1}^0 - \sum_{i=2}^{n} c_i^0 \\ \vdots \\ c_n^0 - \sum_{i=2}^{n} c_i^0 \end{bmatrix}$$

This coefficient matrix has full rank because:

$$\begin{bmatrix} 2 & \cdots & 1 & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 1 & \cdots & 2 & 1 & \cdots & 1 \\ 1 & \cdots & 1 & 2 & \cdots & 1 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & \cdots & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & -1 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & -1 \\ 0 & \cdots & 0 & 1 & \cdots & -1 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cdots & 1 & 1 & \cdots & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \cdots & 0 & 0 & \cdots & -1 \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & -1 \\ 0 & \cdots & 0 & 1 & \cdots & -1 \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & n-1 \end{bmatrix}$$

Therefore, $c_{2|i}, \ldots, c_{i-1|i}, c_{i+1|i}, \ldots, c_{n|i}$ are uniquely determined (including the possibility when they are all 0) when they form an equal coefficient for $e^{2\tilde{\alpha}t}$. I denote this equal coefficient by C in $f_{j|i}(t)$, $\forall j$. Also, since $c_{1|i}$ and c_1^0 are the same for all $f_{j|i}(t)$, then intensities of feeling from non-engaged activities are uniquely represented:

$$f_{j|i}(t) = c_{1|i} e^{[2-(n-1)]\tilde{\alpha}t} + Ce^{2\tilde{\alpha}t} + c_1^0 e^{(2-n)\tilde{\alpha}t}$$

Note that at the moment this function is a unique representation, but not a unique specification, because it represents the functional form for all intensities of feeling from non-engaged activities for any $c_{1|i}$, which is yet to be determined. The zero-state t^* implies that:

$$f_{j|i}(t^*) = 0 = c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t^*} + Ce^{2\tilde{\alpha}t^*} + c_1^0 e^{(2-n)\tilde{\alpha}t^*}$$

at some $t = t^*$. Also, from the non-engaged system, $\dot{f}_{j|i}\Big|_{t=t^*}$ implies:

$$[2-(n-1)]c_{1|i}e^{[2-(n-1)]\tilde{\alpha}t^*} + 2Ce^{2\tilde{\alpha}t^*} + (2-n)c_1^0 e^{(2-n)\tilde{\alpha}t^*} = -f_i(t^*)$$

If C_2^0 denotes the coefficient for $e^{2\tilde{\alpha}t}$ in $f_i(t)$, by eliminating $e^{[2-(n-1)]\tilde{\alpha}t^*}$ from these two equations, I obtain the following:

$$(n-1)Ce^{2\tilde{\alpha}t^*} - c_1^0 e^{(2-n)\tilde{\alpha}t^*} = -c_1^0 e^{(2-n)\tilde{\alpha}t^*} - C_2^0 e^{2\tilde{\alpha}t^*}$$

which implies that

$$C = -\frac{C_2^0}{n-1}$$

From the uniqueness property of specifications for intensities of feeling from non-engaged activities from Theorem 2, $c_{1|i}$ is uniquely determined by the initial condition $f_{j|i}(0)$. Hence, t^* is also uniquely determined by the same initial condition for the unique specification for all intensities of feeling from non-engaged activities. This completes the proof. □

B.3 Example: Calculation of intensities of feeling from non-engaged activities

Possible schedules:

$$\begin{array}{llll} WCR: & T_W = 1, & T_C = 0.5, & T_R = 0.5 \\ WRC: & T_W = 1, & T_R = 0.5, & T_C = 0.5 \\ CRW: & T_C = 0.5, & T_R = 0.5, & T_W = 1 \\ CWR: & T_C = 0.5, & T_W = 1, & T_R = 0.5 \\ RWC: & T_R = 0.5, & T_W = 1, & T_C = 0.5 \\ RCW: & T_R = 0.5, & T_C = 0.5, & T_W = 1 \end{array}$$

Specifications for intensities of feeling from engaged activities are:

$$f_W = e^{-t} + e^{2t}, f_C = e^{-t}, f_R = e^{-t} - e^{2t}$$

The subject is walking the dog: $f_W = e^{-t} + e^{2t}$, $T_W = 1$.

$$f_{C|W}(1) = 0 = c_{1|W} + c_{2|W}e^2 + e^{-1} \qquad \text{so} \qquad 2c_{2|W}e^2 = -e^2,\ c_{2|W} = \tfrac{1}{2}$$
$$f_{R|W}(1) = 0 = c_{1|W} - c_{2|W}e^2 - e^2 + e^{-1} \qquad 2c_{1|W} = e^2 - 2e^{-1},\ c_{1|W} = \tfrac{1}{2}e^2 - e^{-1}$$

The subject is drinking coffee: $f_C = e^{-t}$, $T_C = 0.5$.

$$f_{R|C}(0.5) = 0 = c_{1|C} - c_{2|C}e^1 - e^1 + e^{-\frac{1}{2}} \qquad \text{so} \qquad 2c_{2|C}e^1 = -2e^1,\ c_{2|C} = -1$$
$$f_{W|C}(0.5) = 0 = c_{1|C} + c_{2|C}e^1 + e^1 + e^{-\frac{1}{2}} \qquad 2c_{1|C} = -2e^{-\frac{1}{2}},\ c_{1|C} = -e^{-\frac{1}{2}}$$

The subject is reading the news: $f_R = e^{-t} - e^{2t}$, $T_R = 0.5$.

$$f_{W|R}(0.5) = 0 = c_{1|R} + c_{2|R}e^1 + e^1 + e^{-\frac{1}{2}} \qquad \text{so} \qquad 2c_{2|R}e^1 = -e^1, c_{2|R} = -\tfrac{1}{2}$$
$$f_{C|R}(0.5) = 0 = c_{1|R} - c_{2|R}e^1 + e^{-\frac{1}{2}} \qquad 2c_{1|R} = -e^1 - 2e^{-\frac{1}{2}}, c_{1|R} = -\tfrac{1}{2}e - e^{-\frac{1}{2}}$$

Therefore

$$f_{C|W}(t) = c_{1|W} + c_{2|W}e^{2t} + e^{-t} = \tfrac{1}{2}e^2 - e^{-1} - \tfrac{1}{2}e^{2t} + e^{-t}$$
$$f_{R|W}(t) = c_{1|W} - c_{2|W}e^{2t} - e^{2t} + e^{-t} = \tfrac{1}{2}e^2 - e^{-1} - \tfrac{1}{2}e^{2t} + e^{-t}$$
$$f_{R|C}(t) = c_{1|C} - c_{2|C}e^{2t} - e^{2t} + e^{-t} = -e^{-\frac{1}{2}} + e^{-t}$$
$$f_{W|C}(t) = c_{1|C} + c_{2|C}e^{2t} + e^{2t} + e^{-t} = -e^{-\frac{1}{2}} + e^{-t}$$
$$f_{W|R}(t) = c_{1|R} + c_{2|R}e^{2t} + e^{2t} + e^{-t} = -\tfrac{1}{2}e - e^{-\frac{1}{2}} + \tfrac{1}{2}e^{2t} + e^{-t}$$
$$f_{C|R}(t) = c_{1|R} - c_{2|R}e^{2t} + e^{-t} = -\tfrac{1}{2}e - e^{-\frac{1}{2}} + \tfrac{1}{2}e^{2t} + e^{-t}$$

The result $C = -\frac{C_2^0}{n-1}$, and the unique specification of intensities of feeling from non-engaged activities from Theorem 3 are confirmed.

B.4 Example: Calculation of the intensities of feeling and overall experiences

The intensities of feeling are:

$$f_W^*(t) = e^{-t} + e^{2t} + 2\left(\tfrac{1}{2}e^2 - e^{-1} - \tfrac{1}{2}e^{2t} + e^{-t}\right) = e^2 - 2e^{-1} + 3e^{-}t$$
$$f_C^*(t) = e^{-t} + 2\left(-e^{-\frac{1}{2}} + e^{-t}\right) = -2e^{-\frac{1}{2}} + 3e^{-}t$$
$$f_R^*(t) = e^{-t} - e^{2t} + 2\left(-\tfrac{1}{2}e - e^{-\frac{1}{2}} + \tfrac{1}{2}e^{2t} + e^{-t}\right) = -e - 2e^{-\frac{1}{2}} + 3e^{-}t$$

The integrated experiences as a function of time t are:

$$E_W(t) = \int_0^t f_W(t)\mathrm{d}t = \int_0^t e^{-t} + e^{2t}\mathrm{d}t = \tfrac{1}{2}e^{2t} - e^{-t} + \tfrac{1}{2}$$
$$E_C(t) = \int_0^t f_C(t)\mathrm{d}t = \int_0^t e^{-t}(t)\mathrm{d}t = -e^{-t} + 1$$
$$E_R(t) = \int_0^t f_R(t)\mathrm{d}t = \int_0^t e^{-t} - e^{2t}\mathrm{d}t = -\tfrac{1}{2}e^{2t} - e^{-t} + \tfrac{3}{2}$$

The overall experiences sequences in each schedule follow:

$$WCR : E_{WCR} = -8.99$$
$$E_W(t) = \frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}, E_C(t) = -e^{-t} + e^{-1}, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e^3 + e^{-1.5}$$
$$WRC : E_{WRC} = -1.21$$
$$E_W(t) = \frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e^2 + e^{-1}, E_C(t) = -e^{-t} + e^{-1.5}$$
$$CRW : E_{CRW} = 17.74$$
$$E_C(t) = -e^{-t} + 1, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e + e^{-0.5}, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e^2 + e^{-1}$$
$$CWR : E_{CRW} = -4.76$$
$$E_C(t) = -e^{-t} + 1, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e + e^{-0.5}, E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{1}{2}e^3 + e^{-1.5}$$
$$RWC : E_{RWC} = 7.01$$
$$E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{3}{2}, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e + e^{-0.5}, E_C(t) = -e^{-t} + e^{-1.5}$$
$$RCW : E_{RCW} = 18.79$$
$$E_R(t) = -\frac{1}{2}e^{2t} - e^{-t} + \frac{3}{2}, E_C(t) = -e^{-t} + e^{-0.5}, E_W(t) = \frac{1}{2}e^{2t} - e^{-t} - \frac{1}{2}e^2 + e^{-1}$$