EXPONENTIAL STABILITY OF A THERMOELASTIC SYSTEM WITHOUT MECHANICAL DISSIPATION II: THE CASE OF SIMPLY SUPPORTED BOUNDARY CONDITIONS

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Exponential Stability of a Thermoelastic System Without Mechanical Dissipation II: The Case of Simply Supported Boundary Conditions

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Abstract

We continue here the work initiated in [3] to show the uniform stability of a thermoelastic plate model with no added dissipative mechanism on the boundary; in this paper, the original simply supported boundary conditions are considered (uniform stability of a thermoelastic plate with added boundary dissipation was shown in [8], as was that of the analytic case–where rotational forces are neglected–in [17]). Many of the computations performed in [3] are still pertinent for this special choice of boundary conditions, but in addition, an abstract "trace" analysis must be invoked, with respect to a particular component of the solution to the coupled system, so as to generate the desired estimates.

1 Introduction

1.1 Statement of the Problem

Let Ω be a bounded open subset of $\mathbb{R}^2$ with sufficiently smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$, Γ_0 and Γ_1 both nonempty and $\overline{\Gamma_0} \cap \overline{\Gamma_1} = \varnothing$. We consider here the following thermoelastic system taken from J. Lagnese's monograph [8]:

$$\begin{cases} \left\{ \begin{array}{l} \omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega + \alpha\Delta\theta = 0 \\ \beta\theta_t - \eta\Delta\theta + \sigma\theta - \alpha\Delta\omega_t = 0 \end{array} \right. \quad \text{on } (0,\infty)\times\Omega; \\ \omega = \dfrac{\partial\omega}{\partial\nu} = 0 \quad \text{on } (0,\infty)\times\Gamma_0; \\ \left\{ \begin{array}{l} \Delta\omega + (1-\mu)B_1\omega + \alpha\theta = 0 \\ \dfrac{\partial\Delta\omega}{\partial\nu} + (1-\mu)\dfrac{\partial B_2\omega}{\partial\tau} - \gamma\dfrac{\partial\omega_{tt}}{\partial\nu} + \alpha\dfrac{\partial\theta}{\partial\nu} = 0 \end{array} \right. \quad \text{on } (0,\infty)\times\Gamma_1; \\ \dfrac{\partial\theta}{\partial\nu} + \lambda\theta = 0 \qquad \text{on } (0,\infty)\times\Gamma,\ \lambda \geq 0; \\ \omega(t=0) = \omega^0,\ \omega_t(t=0) = \omega^1,\ \theta(t=0) = \theta^0 \text{ on } \Omega; \end{cases} \tag{1}$$

Here, α, β and η are strictly positive constants; positive constant γ is proportional to the thickness of the plate and assumed to be small with $0 < \gamma \leq M$; the constant $\sigma \geq 0$ and the boundary operators B_i are given by

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$$B_1\omega \equiv 2\nu_1\nu_2\frac{\partial^2\omega}{\partial x\partial y} - \nu_1^2\frac{\partial^2\omega}{\partial y^2} - \nu_2^2\frac{\partial^2\omega}{\partial x^2};$$

$$B_2\omega \equiv (\nu_1^2 - \nu_2^2)\frac{\partial^2\omega}{\partial x\partial y} + \nu_1\nu_2\left(\frac{\partial^2\omega}{\partial y^2} - \frac{\partial^2\omega}{\partial x^2}\right); \tag{2}$$

the constant μ is the familiar Poisson's ratio $\in (0, \frac{1}{2})$, and $[\nu_1, \nu_2]$ denotes the outward unit normal to the boundary. The given model mathematically describes a Kirchoff plate, the displacement of which is represented by the function ω, subjected to a thermal damping as quantified by θ. We are concerned here with the uniform stability of solutions $[\omega, \theta]$ to (1).

1.2 Preliminaries and Abstract Formulation

As a departure point for obtaining the proofs of well–posedness and of exponential stability, we will consider the system (1) as an abstract evolution equation in a certain Hilbert space, for which we introduce the following definitions and notations.

- With $H^k_{\Gamma_0}(\Omega) \equiv \left\{\omega \in H^k(\Omega) : \left.\frac{\partial^j\omega}{\partial\nu^j}\right|_{\Gamma_0} = 0 \text{ for } j = 0, ..., k-1\right\}$, we define $\mathring{\mathbf{A}}$: $L^2(\Omega) \supset D\left(\mathring{\mathbf{A}}\right) \to L^2(\Omega)$ to be $\mathring{\mathbf{A}} = \Delta^2$, with domain

$$D(\mathring{\mathbf{A}}) = \Big\{\omega \in H^4(\Omega) \cap H^2_{\Gamma_0}(\Omega) : \Delta\omega + (1-\mu)B_1\omega = 0 \text{ on } \Gamma_1 \text{ and } \frac{\partial\Delta\omega}{\partial\upsilon} + (1-\mu)\frac{\partial B_2\omega}{\partial\tau} = 0 \text{ on } \Gamma_1\Big\}. \tag{3}$$

- $\mathring{\mathbf{A}}$ is then positive definite, self–adjoint, and consequently from [5] we have the characterizations

$$D(\mathring{\mathbf{A}}^{\frac{1}{4}}) = H^1_{\Gamma_0}(\Omega).$$
$$D(\mathring{\mathbf{A}}^{\frac{1}{2}}) = H^2_{\Gamma_0}(\Omega); \tag{4}$$
$$D(\mathring{\mathbf{A}}^{\frac{3}{4}}) = \left\{\omega \in H^3(\Omega) \cap H^2_{\Gamma_0}(\Omega) : \Delta\omega + (1-\mu)B_1\omega = 0\right\}$$

Moreover, using the Green's formula in [8], we have that for ω, $\widehat{\omega}$ "smooth enough",

$$\int_\Omega (\Delta^2\omega)\widehat{\omega}d\Omega = a\left(\omega, \widehat{\omega}\right) + \int_\Gamma \left[\frac{\partial\Delta\omega}{\partial\nu} + (1-\mu)\frac{\partial B_2\omega}{\partial\tau}\right]\widehat{\omega}d\Gamma - \int_\Gamma \left[\Delta\omega + (1-\mu)B_1\omega\right]\frac{\partial\widehat{\omega}}{\partial\nu}d\Gamma, \tag{5}$$

where $a(\cdot,\cdot)$ is defined by

$$a\left(\omega, \widehat{\omega}\right) \equiv \int_\Omega \left[\omega_{xx}\widehat{\omega}_{xx} + \omega_{yy}\widehat{\omega}_{yy} + \mu\left(\omega_{xx}\widehat{\omega}_{yy} + \omega_{yy}\widehat{\omega}_{xx}\right) + 2(1-\mu)\omega_{xy}\widehat{\omega}_{xy}\right]d\Omega. \tag{6}$$

In particular, this formula and the second characterization in (4) give that for all $\omega, \widehat{\omega} \in D(\mathring{\mathbf{A}}^{\frac{1}{2}})$,

$$\left\langle \mathring{\mathbf{A}}\omega, \widehat{\omega}\right\rangle_{[D(\mathring{\mathbf{A}}^{\frac{1}{2}})]' \times D(\mathring{\mathbf{A}}^{\frac{1}{2}})} = \left(\mathring{\mathbf{A}}^{\frac{1}{2}}\omega, \mathring{\mathbf{A}}^{\frac{1}{2}}\widehat{\omega}\right)_{L^2(\Omega)} = a\left(\omega, \widehat{\omega}\right)_{L^2(\Omega)}, \tag{7}$$

and in addition,

$$||\omega||^2_{D(\mathring{\mathbf{A}}^{\frac{1}{2}})} = \left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} = a\left(\omega, \omega\right). \tag{8}$$

- We define $A_D : L^2(\Omega) \supset D(A_D) \to L^2(\Omega)$ to be $A_D = -\Delta$, with Dirichlet boundary conditions, viz.

$$D(A_D) = H^2(\Omega) \cap H_0^1(\Omega). \tag{9}$$

  A_D is also positive definite, self–adjoint, and by [5]

$$D(A_D^{\frac{1}{2}}) = H_0^1(\Omega). \tag{10}$$

- The space $L^2_{\sigma+\lambda}(\Omega)$ will be defined as

$$L^2_{\sigma+\lambda}(\Omega) \equiv \begin{cases} L^2(\Omega), & \text{if } \sigma+\lambda > 0 \\ L_0^2(\Omega), & \text{if } \sigma+\lambda = 0, \end{cases} \tag{11}$$

  where $L_0^2(\Omega) = \left\{\theta \in L^2(\Omega) \ni \int_\Omega \theta = 0\right\}$.

- We designate as $A_R : L^2(\Omega) \supset D(A_R) \to L^2(\Omega)$ the following second order elliptic operator:

$$\begin{aligned} A_R &= -\Delta + \frac{\sigma}{\eta}\mathbf{I}, \\ D(A_R) &= \left\{\theta \in H^2(\Omega) : \frac{\partial\theta}{\partial\nu} + \lambda\theta = 0\right\}; \end{aligned} \tag{12}$$

  A_R is self–adjoint, positive semidefinite on $L^2(\Omega)$, and once more by [5]

$$D(A_R^{\frac{1}{2}}) = H^1(\Omega); \tag{13}$$

  When $\lambda = \sigma = 0$, we shall denote the corresponding operator as A_N.

  Furthermore, as the bilinear form $\left(\nabla\theta, \nabla\tilde{\theta}\right)_{L^2(\Omega)}$ is $H^1(\Omega)$–elliptic on $H^1(\Omega) \cap L_0^2(\Omega)$, we can define the norm-inducing inner product on $H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$ as

$$\left(\theta, \tilde{\theta}\right)_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)} \equiv \left(\nabla\theta, \nabla\tilde{\theta}\right)_{L^2(\Omega)} + \lambda\left(\theta, \tilde{\theta}\right)_{L^2(\Gamma)} + \frac{\sigma}{\eta}\left(\theta, \tilde{\theta}\right)_{L^2(\Omega)}. \tag{14}$$

- (γ_0, γ_1) will denote the Sobolev trace maps, which yield for $f \in C^\infty(\overline{\Omega})$

$$\gamma_0 f = f|_\Gamma; \ \gamma_1 f = \left.\frac{\partial f}{\partial\nu}\right|_\Gamma. \tag{15}$$

- We define the elliptic operators G_1, G_2 and D as thus:

$$G_1 h = v \Longleftrightarrow \begin{cases} \Delta^2 v = 0 \ \text{ in } (0,\infty)\times\Omega \\ v = \dfrac{\partial v}{\partial \nu} = 0 \text{ on } (0,\infty)\times\Gamma_0; \\ \begin{cases} \Delta v + (1-\mu) B_1 v = h \\ \dfrac{\partial \Delta v}{\partial \nu} + (1-\mu)\dfrac{\partial B_2 v}{\partial \tau} = 0 \end{cases} \quad \text{on } (0,\infty)\times\Gamma_1; \end{cases} \tag{16}$$

$$G_2 h = v \Longleftrightarrow \begin{cases} \Delta^2 v = 0 \ \text{ in } (0,\infty)\times\Omega \\ v = \dfrac{\partial v}{\partial \nu} = 0 \text{ on } (0,\infty)\times\Gamma_0; \\ \begin{cases} \Delta v + (1-\mu) B_1 v = 0 \\ \dfrac{\partial \Delta v}{\partial \nu} + (1-\mu)\dfrac{\partial B_2 v}{\partial \tau} = h \end{cases} \quad \text{on } (0,\infty)\times\Gamma_1; \end{cases} \tag{17}$$

$$Dh = v \Longleftrightarrow \begin{cases} \Delta v = 0 \ \text{ on } (0,\infty)\times\Omega \\ v|_\Gamma = h \ \text{ on } (0,\infty)\times\Gamma. \end{cases} \tag{18}$$

The classic regularity results of [15] then provide that for $s \in \mathbb{R}$,

$$\begin{cases} D \in \mathcal{L}\left(H^s(\Gamma), H^{s+\frac{1}{2}}(\Omega)\right); \\ G_1 \in \mathcal{L}\left(H^s(\Gamma), H^{s+\frac{5}{2}}(\Omega)\right); \\ G_2 \in \mathcal{L}\left(H^s(\Gamma), H^{s+\frac{7}{2}}(\Omega)\right). \end{cases} \tag{19}$$

With the operators $\mathring{\mathbf{A}}$ and G_i as defined above, one can readily show with the use of the Green's formula (5) that $\forall\, \omega \in D(\mathring{\mathbf{A}}^{\frac{1}{2}})$ the adjoints $G_i^* \mathring{\mathbf{A}}_1 \in \mathcal{L}\left(D(\mathring{\mathbf{A}}^{\frac{1}{2}}), L^2(\Gamma)\right)$ satisfy respectively

$$G_1^* \mathring{\mathbf{A}} \omega = \begin{cases} \left.\dfrac{\partial \omega}{\partial \nu}\right|_{\Gamma_1} & \text{on } (0,\infty)\times\Gamma_1 \\ 0 & \text{on } (0,\infty)\times\Gamma_0; \end{cases}$$

$$G_2^* \mathring{\mathbf{A}} \omega = \begin{cases} -\omega|_{\Gamma_1} & \text{on } (0,\infty)\times\Gamma_1 \\ 0 & \text{on } (0,\infty)\times\Gamma_0. \end{cases} \tag{20}$$

- We define the operator P_γ by

$$P_\gamma \equiv \mathbf{I} + \gamma A_N, \tag{21}$$

and:

(i) For $\gamma > 0$, we define a space $H^1_{\Gamma_0,\gamma}(\Omega)$ equivalent to $H^1_{\Gamma_0}(\Omega)$ with *its inner product as*

$$(\omega_1, \omega_2)_{H^1_{\Gamma_0,\gamma}(\Omega)} \equiv (\omega_1, \omega_2)_{L^2(\Omega)} + \gamma\, (\nabla \omega_1, \nabla \omega_2)_{L^2(\Omega)} \quad \forall\, \omega_1, \omega_2 \in H^1_{\Gamma_0}(\Omega), \tag{22}$$

and with its dual denoted as $H^{-1}_{\Gamma_0,\gamma}(\Omega)$. Recalling that $H^1(\Omega) = D(A_N)$, two extensions by continuity will then yield that

$$P_\gamma \in \mathcal{L}\left(H^1_{\Gamma_0,\gamma}(\Omega), H^{-1}_{\Gamma_0,\gamma}(\Omega)\right), \text{ with} \tag{23}$$

$$\langle P_\gamma \omega_1, \omega_2 \rangle_{H^{-1}_{\Gamma_0,\gamma}(\Omega)\times H^1_{\Gamma_0,\gamma}(\Omega)} = (\omega_1, \omega_2)_{H^1_{\Gamma_0,\gamma}(\Omega)}. \tag{24}$$

Furthermore, the obvious $H^1_{\Gamma_0,\gamma}(\Omega)$–ellipticity of P_γ and Lax–Milgram give us that P_γ is boundedly invertible, i.e.

$$P_\gamma^{-1} \in \mathcal{L}\left(H^{-1}_{\Gamma_0,\gamma}(\Omega), H^1_{\Gamma_0,\gamma}(\Omega)\right). \tag{25}$$

Moreover, P_γ being positive definite, self–adjoint as an operator $P_\gamma : L^2(\Omega) \supset D(P_\gamma) \to L^2(\Omega)$, the square root $P_\gamma^{\frac{1}{2}}$ is well–defined with $D(P_\gamma^{\frac{1}{2}}) = H^1_{\Gamma_0,\gamma}(\Omega)$ (using the interpolation theorem in [15], p. 10); it then follows from (28) that for ω and $\widehat{\omega} \in H^1_{0,\gamma}(\Omega)$,

$$\left\| P_\gamma^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} = \|\omega\|^2_{L^2(\Omega)} + \gamma \|\nabla\omega\|^2_{L^2(\Omega)} = \|\omega\|^2_{H^1_{\Gamma_0,\gamma}(\Omega)}; \tag{26}$$

$$\left(P_\gamma^{\frac{1}{2}}\omega, P_\gamma^{\frac{1}{2}}\widehat{\omega}\right)_{L^2(\Omega)} = (\omega, \widehat{\omega})_{H^1_{\Gamma_0,\gamma}(\Omega)}. \tag{27}$$

(ii) Finally, inasmuch as Green's formula yields for $\omega, \widehat{\omega} \in D(\mathring{\mathbf{A}}^{\frac{1}{2}})$,

$$\begin{aligned}\gamma \left\langle \left(\Delta + \mathring{\mathbf{A}} G_2\gamma_1\right)\omega, \widehat{\omega}\right\rangle_{[D(\mathring{\mathbf{A}}^{\frac{1}{2}})]'\times D(\mathring{\mathbf{A}}^{\frac{1}{2}})} &= -\gamma\left(\nabla\omega, \nabla\widehat{\omega}\right)_{L^2(\Omega)} + \gamma\left(\frac{\partial\omega}{\partial\nu}, \widehat{\omega}\right)_{L^2(\Gamma_1)} + \gamma\left(\gamma_1\omega, G_2^*\mathring{\mathbf{A}}\widehat{\omega}\right)_{L^2(\Gamma_1)} \\ &= -\gamma\left(\nabla\omega, \nabla\widehat{\omega}\right)_{L^2(\Omega)},\end{aligned} \tag{28}$$

after using (20), we thus obtain after two extensions by continuity to $H^1_{\Gamma_0,\gamma}(\Omega)$,

$$\langle P_\gamma\omega, \widehat{\omega}\rangle_{H^{-1}_{\Gamma_0,\gamma}(\Omega)\times H^1_{\Gamma_0,\gamma}(\Omega)} = \left\langle \mathbf{I} - \gamma\left(\Delta + \mathring{\mathbf{A}}G_2\gamma_1\right)\omega, \widehat{\omega}\right\rangle_{H^{-1}_{\Gamma_0,\gamma}(\Omega)\times H^1_{\Gamma_0,\gamma}(\Omega)}. \tag{29}$$

- We denote the Hilbert space $\mathbf{H}_\gamma$ to be

$$\mathbf{H}_\gamma \equiv D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega) \times L^2_{\sigma+\lambda}(\Omega), \tag{30}$$

with the inner product

$$\begin{aligned}&\left(\begin{bmatrix}\omega_1\\ \omega_2\\ \theta\end{bmatrix}, \begin{bmatrix}\widehat{\omega}_1\\ \widehat{\omega}_2\\ \widehat{\theta}\end{bmatrix}\right)_{\mathbf{H}_\gamma} \\ &= \left(\mathring{\mathbf{A}}^{\frac{1}{2}}\omega_1, \mathring{\mathbf{A}}^{\frac{1}{2}}\widehat{\omega}_1\right)_{L^2(\Omega)} + \left(P_\gamma^{\frac{1}{2}}\omega_2, P_\gamma^{\frac{1}{2}}\widehat{\omega}_2\right)_{L^2(\Omega)} + \beta\left(\theta, \widehat{\theta}\right)_{L^2(\Omega)}.\end{aligned} \tag{31}$$

- With the above definitions, we then set $\mathcal{A}_\gamma : \mathbf{H}_\gamma \supset D(\mathcal{A}_\gamma) \to \mathbf{H}_\gamma$ to be

$$\mathcal{A}_\gamma \equiv \begin{pmatrix} 0 & \mathbf{I} & 0 \\ -P_\gamma^{-1}\mathring{\mathbf{A}} & 0 & (\clubsuit) \\ 0 & -\frac{\alpha}{\beta}A_D(\mathbf{I}-D\gamma_0) & -\frac{\eta}{\beta}A_R \end{pmatrix}$$

$$\begin{aligned} &\text{where } (\clubsuit) \equiv \alpha P_\gamma^{-1}\left(A_R - \frac{\sigma}{\eta}\mathbf{I} - \mathring{\mathbf{A}}G_1\gamma_0 + \lambda\mathring{\mathbf{A}}G_2\gamma_0\right), \\ &\text{with } D(\mathcal{A}_\gamma) = \Big\{[\omega_1,\omega_2,\theta] \in D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times D(A_R) \cap L^2_{\sigma+\lambda}(\Omega) \\ &\qquad \text{such that } \mathring{\mathbf{A}}\omega_1 + \alpha\mathring{\mathbf{A}}G_1\gamma_0\theta - \alpha\lambda\mathring{\mathbf{A}}G_2\gamma_0\theta \in H^{-1}_{\Gamma_0,\gamma}(\Omega) \\ &\qquad \text{and } \alpha\,\Delta\omega_2 + \eta\Delta\theta \in L^2_{\sigma+\lambda}(\Omega)\Big\}. \end{aligned} \tag{32}$$

If we take the initial data $[\omega^0, \omega^1, \theta^0]$ to be in $\mathbf{H}_\gamma$, then the coupled system (1) becomes the operator theoretic model

$$\begin{aligned} &\frac{d}{dt}\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} = \mathcal{A}_\gamma \begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} \\ &\begin{bmatrix} \omega(0) \\ \omega_t(0) \\ \theta(0) \end{bmatrix} = \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^0 \end{bmatrix}. \end{aligned} \tag{33}$$

Remark 1 *For initial data $[\omega^0, \omega^1, \theta^0]$ in $D(\mathcal{A}_\gamma)$, the two equations of (1) may be written pointwise as*

$$P_\gamma\omega_{tt} = -\mathring{A}\omega - \alpha\mathring{A}G_1\gamma_0\theta + \alpha\lambda\mathring{A}G_2\gamma_0\theta - \alpha\Delta\theta \ \ in\ H^{-1}_{\Gamma_0,\gamma}(\Omega); \tag{34}$$

$$\beta\theta_t = \eta\Delta\theta - \sigma\theta + \alpha\Delta\omega_t \ \ in\ L^2_{\sigma+\lambda}(\Omega). \tag{35}$$

1.3 Previous Literature

In recent years, questions related to the controllability and stabilization of thermoelastic plates have drawn considerable attention in the recent past (see [7], [8], [6], [17], [18] and [19]); we shall concentrate here on detailing results of strong and uniform stability related to the present model, that of the two dimensional Kirchoff plate coupled with the heat equation. This particular model, associated with simply supported boundary conditions, was introduced by J. Lagnese in [8]. In that work, he established the well–posedness and exponential stability of (1) with γ strictly positive, and with the appropriately chosen feedback mechanisms $[\mathcal{F}_1(\omega_t), \mathcal{F}_2(\omega_t)]$ inserted into the natural boundary conditions of the Kirchoff plate component of the system, viz.

$$\begin{cases} \omega = \dfrac{\partial\omega}{\partial\nu} = 0 \text{ on } (0,\infty)\times\Gamma_0 \\ \Delta\omega + (1-\mu)B_1\omega + \alpha\theta = \mathcal{F}_1(\omega_t) \text{ on } (0,\infty)\times\Gamma_1 \\ \dfrac{\partial\Delta\omega}{\partial\nu} + (1-\mu)\dfrac{\partial B_2\omega}{\partial\tau} - \gamma\dfrac{\partial\omega_{tt}}{\partial\nu} + \alpha\dfrac{\partial\theta}{\partial\nu} = \mathcal{F}_2(\omega_t) \text{ on } (0,\infty)\times\Gamma_1; \end{cases} \tag{36}$$

the proof of Lagnese is based on the use of differential multipliers, and it exploits the fact that $\gamma > 0$. Since, from a physical point of view, the thermal effects present should induce some measure of energy dissipation (in fact, one can show the system's strong stability by routine methods; see [8] Chap. 7, including the remark at the end of Sect. 2.3 on p. 161), a natural question arising in this context is whether the system is actually (uniformly) stable without the boundary feedbacks $\mathcal{F}_1(\omega_t)$, $\mathcal{F}_2(\omega_t)$ in place, i.e. when there are no added mechanical forces. Indeed, in the case $\gamma = 0$ and with different boundary conditions imposed upon the system, the answer to the question is in the affirmative and has been provided by several authors. With $\gamma = 0$, J. Kim in [7] showed the uniform stability of (1) with the clamped boundary conditions $\omega = \frac{\partial \omega}{\partial \nu} = \theta = 0$ on Γ, as did J. Rivera and R. Racke in [20] with instead the hinged boundary conditions $\omega = \Delta\omega = \theta = 0$. Also with $\gamma = 0$, Z. Liu and S. Zheng in [17] proved the exponential stability of (1) with the boundary conditions

$$\begin{cases} \omega = \dfrac{\partial \omega}{\partial \nu} = 0 \text{ on } (0,\infty) \times \Gamma_0 \\ \\ \omega = \Delta\omega + (1-\mu)B_1\omega + \alpha\theta = 0 \text{ on } (0,\infty) \times \Gamma_1, \end{cases} \tag{37}$$

leaving the case of simply supported boundary conditions as an open question, even in the case $\gamma = 0$. The proof of Liu and Zheng is indirect in the sense that it is based on a contradiction argument applied to the exponential decay stability criterion (due to L.A. Monauni, R. Nagel and F.L. Huang), a criterion essentially dictating the uniform estimate for that part of the resolvent which lies on the imaginary axis. On the other hand, it is now known that the case $\gamma = 0$ is rather special as the corresponding system (at least for certain boundary conditions) generates an *analytic* semigroup (see [16]), a consequence of which will be the exponential stability of the system (recall that the system is strongly stable). Given these results, the question of interest now is whether the given thermoelastic system (without any additional boundary dissipation) is *uniformly* stable in the *nonanalytic* case, viz. $\gamma > 0$, with consequently the elastic part of the system being of hyperbolic character.

The main goal of this paper is to provide an affirmative answer to the question posed above, with the simply supported boundary conditions in place, and pertaining to the case $\gamma > 0$. The fact that the presence of the simply supported boundary conditions greatly complicates the analysis was duly noted in [17], and the arguments employed in that work do not carry over for simply supported plates, even when $\gamma = 0$; our proof is "direct", based on pseudodifferential (or operator theoretic) multipliers, in contrast to the contradiction argument supplied in [17]. Another advantage of the direct proof provided herein is that it leads to explicit estimates of the decay rates. The most peculiar aspect of our stability proof is probably the decomposition of the solution ω into three separate components, and the subsequent application of a recently derived trace regularity result to one of these, exploiting the fact that this particular component (modulo a change of variable) solves a certain wave equation; this scrutiny of boundary traces for the hyperbolic component of the dynamics is a *sine qua non* for obtaining the necessary estimates. We reiterate that this particular difficulty of the problem is due to the specific boundary conditions being considered, and does not appear for other combinations of B.C.'s.

1.4 Statement of the Results

We shall begin by giving preliminary results regarding the well–posedness of the system (1) and the regularity of its solutions.

Theorem 1 *(well–posedness) Again with the parameter $\gamma > 0$, $\mathcal{A}_\gamma$, given by (32), generates a C_0–semigroup of contractions $\{e^{\mathcal{A}_\gamma t}\}_{t\geq 0}$ on the energy space $\boldsymbol{H}_\gamma$; therefore for initial data $[\omega^0, \omega^1, \theta^0]$*

$\in \boldsymbol{H}_\gamma$, *the solution* $[\omega, \omega_t, \theta]$ *to (33), and consequently to (1) is given by*

$$\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} = e^{\mathcal{A}_\gamma(\cdot)} \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^0 \end{bmatrix} \in C([0,T], \boldsymbol{H}_\gamma). \tag{38}$$

The following regularity result is needed to justify the computations performed below.

Theorem 2 ***(i)*** *For intial data* $[\omega^0, \omega^1, \theta^0] \in D\left(\mathcal{A}_\gamma^2\right)$, *we have that the solution* $[\omega, \omega_t, \theta]$ *to (1) is an element of* $C([0,T]; H^4(\Omega) \times H^3(\Omega) \times H^2(\Omega))$.

(ii) $\omega - \gamma G_2 \gamma_1 \omega_{tt} + \alpha G_1 \gamma_0 \theta - \alpha \lambda G_2 \gamma_0 \theta \in C([0,T]; D(\mathring{\boldsymbol{A}}))$.

Our main result is:

Theorem 3 *(uniform stability) With* $\gamma > 0$, *the solution* $[\omega, \omega_t, \theta]$ *of (1) decays exponentially; that is to say, there exist constants* $\delta > 0$ *and* $M_\delta \geq 1$ *such that for all* $t > 0$

$$\left\| \begin{bmatrix} \omega(t) \\ \omega_t(t) \\ \theta(t) \end{bmatrix} \right\|_{\boldsymbol{H}_\gamma} \leq M_\delta e^{-\delta t} \left\| \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^1 \end{bmatrix} \right\|_{\boldsymbol{H}_\gamma}. \tag{39}$$

Remark 2 *The case* $\gamma = 0$ *can be treated similiarly; it is even easier. However, the estimates obtained in* ***Theorem 3*** *are not "robust" with respect to* γ; *this situation is unlike that presented by the imposition of any other combination of boundary conditions, be they clamped or hinged.*

2 Proofs

The proofs of well–posedness and of regularity (**Theorems 1** and **2**) are by now fairly routine (see Chap. 7 in [8] for related well–posedness/regularity results). However, since these preliminaries are critical for our ultimate end of uniform stability, we provide their concise proofs for the sake of completeness.

2.1 Proof of Theorem 1

In establishing the semigroup generation of $\mathcal{A}_\gamma$, we will show that the conditions of the Lumer–Phillips Theorem are satisfied; namely, we demonstrate here that $\mathcal{A}_\gamma$ is maximal dissipative.

To show the dissipativity of $\mathcal{A}_\gamma$: For $[\omega_1, \omega_2, \theta] \in D(\mathcal{A}_\gamma)$ we have

$$\begin{aligned}
&\left(\mathcal{A}_\gamma \begin{bmatrix} \omega_1 \\ \omega_2 \\ \theta \end{bmatrix}, \begin{bmatrix} \omega_1 \\ \omega_2 \\ \theta \end{bmatrix} \right)_{\mathbf{H}_\gamma} = \\
&\left(\mathring{\mathbf{A}}^{\frac{1}{2}} \omega_2, \mathring{\mathbf{A}}^{\frac{1}{2}} \omega_1 \right)_{L^2(\Omega)} \\
&+ \left(P_\gamma^{\frac{1}{2}} P_\gamma^{-1} (-\mathring{\mathbf{A}} \omega_1 + \alpha A_R \theta - \frac{\sigma}{\eta} \theta - \alpha \mathring{\mathbf{A}} G_1 \gamma_0 \theta + \alpha \lambda \mathring{\mathbf{A}} G_2 \gamma_0 \theta), P_\gamma^{\frac{1}{2}} \omega_2 \right)_{L^2(\Omega)} \\
&- \alpha \left(A_D (\mathbf{I} - D\gamma_0) \omega_2, \theta \right)_{L^2(\Omega)} - (\eta A_R \theta, \theta)_{L^2(\Omega)};
\end{aligned} \tag{40}$$

Using the characterizations (4) and the standard result that for every $\omega^* \in H^{-1}_{\Gamma_0,\gamma}(\Omega)$ and $\omega \in D(\mathring{\mathbf{A}}^{\frac{1}{2}})$

$$\langle \omega^*, \omega \rangle_{H^{-1}_{\Gamma_0,\gamma}(\Omega) \times H^1_{\Gamma_0,\gamma}(\Omega)} = \langle \omega^*, \omega \rangle_{\left[D(\mathring{\mathbf{A}}^{\frac{1}{2}})\right]' \times D(\mathring{\mathbf{A}}^{\frac{1}{2}})}, \tag{41}$$

we have upon taking adjoints and using the characterizations (20) in the second term on the right hand side of (40),

$$\begin{aligned}
(40) \quad = \quad & \left(\mathring{\mathbf{A}}^{\frac{1}{2}}\omega_2, \mathring{\mathbf{A}}^{\frac{1}{2}}\omega_1\right)_{L^2(\Omega)} - \left\langle \mathring{\mathbf{A}}\omega_1, \omega_2 \right\rangle_{[D(\mathring{\mathbf{A}}^{\frac{1}{2}})]' \times D(\mathring{\mathbf{A}}^{\frac{1}{2}})} \\
& + \alpha\left(A_R\theta - \frac{\sigma}{\eta}\theta, \omega_2\right)_{L^2(\Omega)} - \alpha\left(\theta, \frac{\partial\omega_2}{\partial\nu}\right)_{L^2(\Gamma_1)} - \alpha\lambda\left(\theta, \omega_2\right)_{L^2(\Gamma_1)} \\
& - \alpha\left(A_D(\mathbf{I} - D\gamma_0)\omega_2, \theta\right)_{L^2(\Omega)} - \left(\eta A_R\theta, \theta\right)_{L^2(\Omega)} \\
= \quad & \left(\mathring{\mathbf{A}}^{\frac{1}{2}}\omega_2, \mathring{\mathbf{A}}^{\frac{1}{2}}\omega_1\right)_{L^2(\Omega)} - \left(\mathring{\mathbf{A}}^{\frac{1}{2}}\omega_1, \mathring{\mathbf{A}}^{\frac{1}{2}}\omega_2\right)_{L^2(\Omega)} - \alpha\left(\Delta\theta, \omega_2\right)_{L^2(\Omega)} \\
& - \alpha\left(\theta, \frac{\partial\omega_2}{\partial\nu}\right)_{L^2(\Gamma_1)} - \alpha\lambda\left(\theta, \omega_2\right)_{L^2(\Gamma_1)} + \alpha\left(\Delta\omega_2, \theta\right)_{L^2(\Omega)} + \left(\eta\Delta\theta - \sigma\theta, \theta\right)_{L^2(\Omega)} \\
= \quad & \left(\mathring{\mathbf{A}}^{\frac{1}{2}}\omega_2, \mathring{\mathbf{A}}^{\frac{1}{2}}\omega_1\right)_{L^2(\Omega)} - \left(\mathring{\mathbf{A}}^{\frac{1}{2}}\omega_1, \mathring{\mathbf{A}}^{\frac{1}{2}}\omega_2\right)_{L^2(\Omega)} + \alpha\left(\nabla\theta, \nabla\omega_2\right)_{L^2(\Omega)} \\
& - \alpha\left(\nabla\omega_2, \nabla\theta\right)_{L^2(\Omega)} - \eta\left\|\nabla\theta\right\|^2_{L^2(\Omega)} - \lambda\eta\left\|\theta\right\|^2_{L^2(\Gamma)} - \sigma\left\|\theta\right\|^2_{L^2(\Omega)} \\
\leq \quad & 0 \qquad (42)
\end{aligned}$$

(here, we are using the fact that $\frac{\partial\theta}{\partial\nu} = -\lambda\theta$); i.e. $\mathcal{A}_\gamma$ is dissipative.

To show the maximality of $\mathcal{A}_\gamma$: if for some $\xi > 0$ and arbitrary $[f_1, f_2, f_3] \in \mathbf{H}_\gamma$, $[\omega_1, \omega_2, \theta] \in D(\mathcal{A}_\gamma)$ solves the equation

$$(\xi\mathbf{I} - \mathcal{A}_\gamma)\begin{bmatrix} \omega_1 \\ \omega_2 \\ \theta \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \tag{43}$$

then this relation holds if and only if

$$\begin{cases} \xi\omega_1 - \omega_2 = f_1 \text{ in } D(\mathring{\mathbf{A}}^{\frac{1}{2}}), \\ \xi\omega_2 + P_\gamma^{-1}\left(\mathring{\mathbf{A}}\omega_1 + \alpha\mathring{\mathbf{A}}G_1\gamma_0\theta - \alpha\lambda\mathring{\mathbf{A}}G_2\gamma_0\theta - \alpha A_R\theta + \frac{\alpha\sigma}{\eta}\theta\right) = f_2 \text{ in } H^1_{\Gamma_0,\gamma}(\Omega), \\ \xi\theta + \frac{\alpha}{\beta}A_D(\mathbf{I} - D\gamma_0)\omega_2 + \frac{\eta}{\beta}A_R\theta = f_3 \text{ in } L^2_{\sigma+\lambda}(\Omega) \end{cases} \tag{44}$$

$$\Longleftrightarrow$$

$$\begin{cases} \xi^3 P_\gamma\omega_1 + \xi\mathring{\mathbf{A}}\omega_1 + \alpha\xi\mathring{\mathbf{A}}G_1\gamma_0\theta - \alpha\lambda\xi\mathring{\mathbf{A}}G_2\gamma_0\theta - \alpha\xi A_R\theta + \frac{\alpha\xi\sigma}{\eta}\theta = \xi P_\gamma f_2 + \xi^2 P_\gamma f_1 \text{ in } H^{-1}_{\Gamma_0,\gamma}(\Omega), \\ \alpha\xi A_D(\mathbf{I} - D\gamma_0)\omega_1 + \beta\xi\theta + \eta A_R\theta = \beta f_3 + \alpha A_D(\mathbf{I} - D\gamma_0)f_1 \text{ in } L^2(\Omega). \end{cases} \tag{45}$$

At this point we bring forth the following:

Proposition 1 *The operator $\boldsymbol{F}$ defined by*

$$\boldsymbol{F} \equiv \begin{bmatrix} \xi^3 P_\gamma + \xi\mathring{\boldsymbol{A}} & \alpha\xi\mathring{\boldsymbol{A}}G_1\gamma_0 - \alpha\lambda\xi\mathring{\boldsymbol{A}}G_2\gamma_0 - \alpha\xi A_R + \frac{\alpha\xi\sigma}{\eta}\boldsymbol{I} \\ \alpha\xi A_D(\boldsymbol{I} - D\gamma_0) & \beta\xi\boldsymbol{I} + \eta A_R \end{bmatrix}, \tag{46}$$

is an element of $\mathcal{L}\left(D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega), \left[D(\mathring{\mathbf{A}}^{\frac{1}{2}})\right]' \times \left[H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\right]'\right)$ *and is boundedly invertible.*

Proof of Proposition 1. We first note by Green's Theorem that for arbitrary $\theta \in D(A_R)$ and $\omega \in D(\mathring{\mathbf{A}}^{\frac{1}{2}})$,

$$\left\langle A_R\theta - \lambda\mathring{\mathbf{A}}G_2\gamma_0\theta, \omega\right\rangle_{\left[D(\mathring{\mathbf{A}}^{\frac{1}{2}})\right]' \times D(\mathring{\mathbf{A}}^{\frac{1}{2}})} = -\left(\nabla\theta, \nabla\omega\right)_{L^2(\Omega)} - \frac{\sigma}{\eta}\left(\theta, \omega\right)_{L^2(\Omega)}; \tag{47}$$

the characterization (12) and an extension by continuity will then have that (47) holds $\forall\, \theta \in H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$. (47) in turn, when coupled with (14), (28) and (20), will yield the asserted boundedness of $\mathbf{F}$, and moreover (41), (28), (14), (20), (47) and Green's formula will provide the following coercivity inequality for all $[\omega, \theta] \in D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$:

$$\begin{aligned}
\left\langle \mathbf{F}\begin{bmatrix}\omega\\ \theta\end{bmatrix}, \begin{bmatrix}\omega\\ \theta\end{bmatrix}\right\rangle &= \xi^3\left\|\omega\right\|^2_{L^2(\Omega)} + \xi^3\gamma\left\|\nabla\omega\right\|^2_{L^2(\Omega)} + \xi\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} \\
&\quad -\alpha\xi\left(\nabla\theta, \nabla\omega\right)_{L^2(\Omega)} + \alpha\xi\left(\nabla\theta, \nabla\omega\right)_{L^2(\Omega)} \\
&\quad +\eta\left\|\nabla\theta\right\|^2_{L^2(\Omega)} + \lambda\eta\left\|\theta\right\|^2_{L^2(\Gamma)} + (\sigma + \beta\xi)\left\|\theta\right\|^2_{L^2(\Omega)} \\
&\quad \text{(after noting the cancellation of boundary terms)} \\
&\geq C\left[\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} + \left\|\theta\right\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\right]
\end{aligned} \tag{48}$$

(where $\langle\cdot,\cdot\rangle$ in (48) denotes the pairing between $D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$ and its dual, and where the constant $C > 0$). Thus, by Lax–Milgram, $\mathbf{F}^{-1}$ exists as an element of $\mathcal{L}\left(\left[D(\mathring{\mathbf{A}}^{\frac{1}{2}})\right]' \times \left[H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\right]', D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\right)$, and the Proposition is proved.

To complete the proof of the maximality of $\mathcal{A}_\gamma$, we apply the inverse assured by **Proposition 1** to both sides of (45) to obtain

$$\left\{\begin{array}{l}
\begin{bmatrix}\omega_1\\ \theta\end{bmatrix} \equiv \mathbf{F}^{-1}\begin{bmatrix}\xi P_\gamma f_2 + \xi^2 P_\gamma f_1\\ \beta f_3 + \alpha A_D(\mathbf{I} - D\gamma_0)f_1\end{bmatrix} \\
\omega_2 \equiv \xi\omega_1 - f_1,
\end{array}\right. \tag{49}$$

and *a fortiori*, one has, by using the second equation in (45), that

$$A_R\theta = -\frac{\beta\xi}{\eta}\theta - \frac{\alpha\xi}{\eta}A_D(\mathbf{I} - D\gamma_0)\omega_1 + \frac{\beta}{\eta}f_3 + \frac{\alpha}{\eta}A_D(\mathbf{I} - D\gamma_0)f_1 \ \in L^2(\Omega),$$

viz. $\theta \in D(A_R) \cap L^2_{\sigma+\lambda}(\Omega)$. This additional regularity of θ, in conjunction with that implied in the first equation of (45) (namely, $\mathring{\mathbf{A}}\omega_1 + \alpha\mathring{\mathbf{A}}G_1\gamma_0\theta - \alpha\lambda\mathring{\mathbf{A}}G_2\gamma_0\theta \in H^{-1}_{\Gamma_0,\gamma}(\Omega)$), and along with the third equation of (44), gives that our constructively acquired solution $[\omega_1, \omega_2, \theta]$ to (43) is in $D(\mathcal{A}_\gamma)$ as defined in (32). Hence, $\mathcal{A}_\gamma$ is maximal dissipative and the proof of **Theorem 1** is complete.

2.2 Proof of Theorem 2

By definition, if $[\omega^0, \omega^1, \theta^0] \in D(\mathcal{A}_\gamma)$, then $\omega^1 \in D(\mathring{\mathbf{A}}^{\frac{1}{2}})$ and $\theta^0 \in D(A_R)$, and

$$\mathring{\mathbf{A}}\omega^0 + \alpha\mathring{\mathbf{A}}G_1\gamma_0\theta^0 - \alpha\lambda\mathring{\mathbf{A}}G_2\gamma_0\theta^0 = g \in H^{-1}_{\Gamma_0,\gamma}(\Omega) = \left[D(\mathring{\mathbf{A}}^{\frac{1}{4}})\right]'; \tag{50}$$

as $\mathring{\mathbf{A}}^{-1} : \left[D(\mathring{\mathbf{A}}^{\frac{1}{4}})\right]' \to D(\mathring{\mathbf{A}}^{\frac{3}{4}}) \subset H^3(\Omega)$ (this containment deduced by the last characterization in (4)), we have after applying $\mathring{\mathbf{A}}^{-1}$ to (50), the use of trace theory and the regularity posted in (19) that

$$\omega^0 = \mathring{\mathbf{A}}^{-1}g - \alpha G_1\gamma_0\theta^0 + \alpha\lambda G_2\gamma_0\theta^0 \in H^3(\Omega). \tag{51}$$

Thus, for $[\omega^0, \omega^1, \theta^0] \in D(\mathcal{A}_\gamma^2)$,

$$\mathcal{A}_\gamma \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^0 \end{bmatrix} = \begin{bmatrix} \omega^1 \\ -P_\gamma^{-1}\mathring{\mathbf{A}}\omega^0 - \alpha P_\gamma^{-1}\mathring{\mathbf{A}}G_1\gamma_0\theta^0 + \alpha\lambda P_\gamma^{-1}\mathring{\mathbf{A}}G_2\gamma_0\theta^0 + \alpha P_\gamma^{-1}\left(A_R\theta^0 - \frac{\sigma}{\eta}\theta^0\right) \\ -\frac{\eta}{\beta}A_R\theta^0 - \frac{\alpha}{\beta}A_D(\mathbf{I} - D\gamma_0)\omega^1 \end{bmatrix} \in D(\mathcal{A}_\gamma), \tag{52}$$

and (52) coupled with (51) implies that

$$\omega^1 \in H^3(\Omega). \tag{53}$$

In addition, where $h \in H^2(\Omega)$; applying $A_R{}^{-1}$ to both sides of (60) thus yields

$$\theta^0 \in H^3(\Omega). \tag{54}$$

Moreover, (52) also has that

$$P_\gamma^{-1}\mathring{\mathbf{A}}\omega^0 + \alpha P_\gamma^{-1}\mathring{\mathbf{A}}G_1\gamma_0\theta^0 - \alpha\lambda P_\gamma^{-1}\mathring{\mathbf{A}}G_2\gamma_0\theta^0 = g + \alpha P_\gamma^{-1}\left(A_R\theta^0 - \frac{\sigma}{\eta}\theta^0\right) \tag{55}$$

where $g \in D(\mathring{\mathbf{A}}^{\frac{1}{2}})$, or equivalently

$$\mathring{\mathbf{A}}\omega^0 + \gamma\mathring{\mathbf{A}}G_2\gamma_1 g + \alpha\mathring{\mathbf{A}}G_1\gamma_0\theta^0 - \alpha\lambda\mathring{\mathbf{A}}G_2\gamma_0\theta^0 = g - \gamma\Delta g - \alpha\Delta\theta^0 \in L^2(\Omega). \tag{56}$$

A fortiori then, $\omega^0 + \gamma G_2\gamma_1 g + \alpha G_1\gamma_0\theta^0 + \alpha G_2\gamma_0\theta^0 \in D(\mathring{\mathbf{A}}) \subset H^4(\Omega)$. But trace theory and the smoothing specified in (19) give that $G_2\gamma_1 g$, $G_1\gamma_0\theta^0$ and $G_2\gamma_0\theta^0 \in H^4(\Omega)$, and thus $D(\mathcal{A}_\gamma^2) \subset H^4(\Omega) \times H^3(\Omega) \times H^3(\Omega)$ with the inclusion being continuous. The solution $[\omega, \omega_t, \theta]$ will consequently have the asserted regularity upon consideration of the fundamental property that for $\xi \geq 0$, $[\omega^0, \omega^1, \theta^0] \in D(\mathcal{A}_\gamma^\xi) \Rightarrow$

$$\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} = e^{\mathcal{A}_\gamma(\cdot)} \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^0 \end{bmatrix} \in C\left([0,T]; D(\mathcal{A}_\gamma^\xi)\right). \tag{57}$$

To prove **(ii)**, we note that with $[\omega^0, \omega^1, \theta^0] \in D(\mathcal{A}_\gamma^2)$, $\omega_{tt} \in C\left([0,T]; D(\mathring{\mathbf{A}}^{\frac{1}{2}})\right)$, so the solution $[\omega, \omega_t, \theta]$ to (1) satisfies

$$-\mathring{\mathbf{A}}\omega + \gamma\mathring{\mathbf{A}}G_2\gamma_1\omega_{tt} - \alpha\mathring{\mathbf{A}}G_1\gamma_0\theta + \alpha\lambda\mathring{\mathbf{A}}G_2\gamma_0\theta = \omega_{tt} - \gamma\Delta\omega_{tt} + \alpha\Delta\theta \tag{58}$$

in $C([0,T]; L^2(\Omega))$, which establishes the result. □

Remark 3 *Because of the regularity result posted in **Theorem 2 (ii)**, we have for sufficiently smooth initial data the valid pointwise representation*

$$\omega_{tt} + \Delta^2\omega - \gamma\Delta\omega_{tt} + \alpha\Delta\theta = 0. \tag{59}$$

Remark 4 *If $\sigma > 0$, then for initial data $\left[\omega^0, \omega^1, \theta^0\right] \in D\left(\mathcal{A}_\gamma^2\right)$, we will also have that the solution component θ of 1 is in $C([0,T]; H^3(\Omega))$; indeed, the last component on the right hand side of (52), the definition of $D\left(\mathcal{A}_\gamma\right)$, and (53) give that*

$$A_R\theta^0 = h + \frac{\alpha}{\eta}\Delta\omega^1 \in H^1(\Omega), \tag{60}$$

where $h \in H^2(\Omega)$. Applying $A_R{}^{-1}$ (which exists for $\sigma > 0$) to both sides of (60) thus yields

$$\theta^0 \in H^3(\Omega), \tag{61}$$

and the result will follow from the semigroup property posted in (57).

In proving **Theorem 3**, we begin with a preliminary energy identity.

Lemma 1 *Again, with initial data $\left[\omega^0, \omega^1, \theta^0\right] \in \boldsymbol{H}_\gamma$, we have that the component θ of the solution of (1) is an element of $L^2\left(0, \infty; H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\right)$; indeed, we have the following relation $\forall\ T > 0$:*

$$-2\eta\int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt = E_\gamma(T) - E_\gamma(0), \tag{62}$$

where the "energy" $E_\gamma(t)$ is defined by

$$E_\gamma(t) \equiv \left\|\mathring{A}^{\frac{1}{2}}\omega(t)\right\|^2_{L^2(\Omega)} + \left\|P_\gamma^{\frac{1}{2}}\omega_t(t)\right\|^2_{L^2(\Omega)} + \|\theta\|^2_{L^2_{\sigma+\lambda}(\Omega)}, \tag{63}$$

and where the norm of $H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$ is as defined in (14).

Proof: Starting with initial data in $D(\mathcal{A}_\gamma)$ which will provide $\forall\ T > 0$ that the solution $[\omega, \omega_t, \theta] \in C([0,T]; D(\mathcal{A}_\gamma))$ and $[\omega_t, \omega_{tt}, \theta_t] \in C([0,T]; \mathbf{H}_\gamma)$, we have pointwise on $(0,T)$

$$\frac{d}{dt}\left\|\begin{bmatrix}\omega(t)\\ \omega_t(t)\\ \theta(t)\end{bmatrix}\right\|^2_{\mathbf{H}_\gamma} = 2\left(\mathcal{A}_\gamma\begin{bmatrix}\omega(t)\\ \omega_t(t)\\ \theta(t)\end{bmatrix}, \begin{bmatrix}\omega(t)\\ \omega_t(t)\\ \theta(t)\end{bmatrix}\right)_{\mathbf{H}_\gamma},$$

and for this special choice of initial data we will have the desired equality (62) upon integration and using the fact from (12) that

$$\begin{aligned}(A_R\theta, \theta)_{L^2(\Omega)} &= \left(-\Delta\theta + \frac{\sigma}{\eta}\theta, \theta\right)_{L^2(\Omega)}\\ &= \|\nabla\theta\|^2_{L^2(\Omega)} + \frac{\sigma}{\eta}\|\theta\|^2_{L^2(\Omega)} + \lambda\|\theta\|^2_{L^2(\Gamma)} \quad \text{for } \theta \in D(A_R).\end{aligned} \tag{64}$$

The asserted L^2–regularity follows immediately from (62), using the norm definition (14) for $H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$, and the fact that $\left\{e^{\mathcal{A}_\gamma t}\right\}_{t\geq 0}$ is a contraction semigroup. A density argument concludes the proof. □

Remark 5 *J. Lagnese in [8] first showed the dissipativity property (62) through a formal integration and a subsequent justification through variational arguments, and the alternate proof is included here as a simple consequence of contractive semigroups.*

We next derive a trace regularity result for the clamped model which does not follow from the standard Sobolev trace theory, and which is critical in our estimates of uniform decay. We note that related trace regularity results for Euler–Bernoulli plates were proved in [14], and for Kirchoff plates in [10].

Lemma 2 *One has the component ω of the solution $[\omega,\omega_t,\theta]$ of (1) satisfies $\Delta\omega|_{\Gamma_0}\in L^2(0,T;L^2(\Gamma_0))$ with the estimate*

$$\int_0^T \|\Delta\omega\|^2_{L^2(\Gamma_0)}\,dt \le C\left(\int_0^T \left[\left\|\mathring{A}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} + \left\|P_\gamma^{\frac{1}{2}}\omega_t\right\|^2_{L^2(\Omega)} + \|\theta\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\right]dt \right.$$

$$\left. +E_\gamma(T)+E_\gamma(0)\right), \tag{65}$$

where C does not depend on the parameter γ.

Proof: If we take initial data $[\omega^0,\omega^1,\theta^0]$ in $D(\mathcal{A}_\gamma^2)$, then **Theorem 2** provides that $[\omega,\omega_t,\theta]$ is a classical pointwise solution of (1). We will work to extract the desired estimate (65) in this special case–and consequently for all initial data after an extension by continuity–by multiplying the first equation of (1) by the quanity $h\cdot\nabla\omega$, where $h(x,y)\equiv[h_1(x,y),h_2(x,y)]$ is a $\left[C^2(\overline{\Omega})\right]^2$ vector field[1] which satisfies

$$h|_\Gamma = \begin{cases} [\nu_1,\nu_2] & \text{on } \Gamma_0 \\ 0 & \text{on } \Gamma_1, \end{cases} \tag{66}$$

followed by an integration from 0 to T; i.e. we will work with the equation

$$\int_0^T \left(\omega_{tt}-\gamma\Delta\omega_{tt}+\Delta^2\omega+\alpha\Delta\theta, h\cdot\nabla\omega\right)_{L^2(\Omega)}dt = 0. \tag{67}$$

(i) First,

$$\int_0^T (\omega_{tt},h\cdot\nabla\omega)_{L^2(\Omega)}\,dt = (\omega_t,h\cdot\nabla\omega)_{L^2(\Omega)}\Big|_0^T - \int_0^T (\omega_t,h\cdot\nabla\omega_t)_{L^2(\Omega)}\,dt$$

$$= (\omega_t,h\cdot\nabla\omega)_{L^2(\Omega)}\Big|_0^T - \frac{1}{2}\int_0^T\int_\Omega div\left(\omega_t^2 h\right)dt d\Omega$$

$$+\frac{1}{2}\int_0^T\int_\Omega \omega_t^2\left[h_{1x}+h_{2y}\right]dt d\Omega$$

$$= (\omega_t,h\cdot\nabla\omega)_{L^2(\Omega)}\Big|_0^T + \frac{1}{2}\int_0^T\int_\Omega \omega_t^2\left[h_{1x}+h_{2y}\right]dt d\Omega, \tag{68}$$

after making use of the divergence theorem and the fact that $\omega_t=0$ on Γ_0.

[1] Here is where we use the fact that Γ_0 and Γ_1 are separated.

(ii) Next

$$
\begin{aligned}
&\int_0^T \left(-\Delta\omega_{tt}, h\cdot\nabla\omega\right)_{L^2(\Omega)} dt \\
= \quad & \left(\nabla\omega_t, \nabla\left(h\cdot\nabla\omega\right)\right)_{L^2(\Omega)}\Big|_0^T - \int_0^T \left(\nabla\omega_t, \nabla\left(h\cdot\nabla\omega_t\right)\right)_{L^2(\Omega)} dt \\
= \quad & \left(\nabla\omega_t, \nabla\left(h\cdot\nabla\omega\right)\right)_{L^2(\Omega)}\Big|_0^T - \frac{1}{2}\int_0^T \int_\Omega div\left(\left|\nabla\omega_t\right|^2 h\right) dt d\Omega \\
& -\int_0^T\int_\Omega \left[\frac{\omega_{tx}^2 h_{1x}}{2} + \frac{\omega_{ty}^2 h_{2y}}{2}\right] dt d\Omega - \int_0^T\int_\Omega \left[\omega_{tx}\omega_{ty}h_{2x} + \omega_{tx}\omega_{ty}h_{1y}\right] dt d\Omega \\
& +\int_0^T\int_\Omega \left[\frac{\omega_{tx}^2 h_{2y}}{2} + \frac{\omega_{ty}^2 h_{1x}}{2}\right] dt d\Omega \\
= \quad & \left(\nabla\omega_t, h\cdot\nabla\omega\right)_{L^2(\Omega)}\Big|_0^T \\
& +\int_0^T\int_\Omega \left[\frac{\omega_{tx}^2 h_{2y}}{2} + \frac{\omega_{ty}^2 h_{1x}}{2} - \frac{\omega_{tx}^2 h_{1x}}{2} - \frac{\omega_{ty}^2 h_{2y}}{2}\right] dt d\Omega \\
& -\int_0^T\int_\Omega \left[\omega_{tx}\omega_{ty}h_{2x} + \omega_{tx}\omega_{ty}h_{1y}\right] dt d\Omega, \qquad (69)
\end{aligned}
$$

after again using the divergence theorem and the fact that $\int_\Omega div\left(\left|\nabla\omega_t\right|^2 h\right) d\Omega = \int_{\Gamma_0}\left|\nabla\omega_t\right|^2 d\Gamma_0 = 0$ (as $\omega_t(t) \in H^2_{\Gamma_0}(\Omega)$).

(iii) To handle the fourth order term, we use the Green's Theorem (5), the given boundary conditions of (1), (66), and the fact that $\omega \in H^2_{\Gamma_0}(\Omega)$ to obtain

$$
\begin{aligned}
&\int_0^T \left(\Delta^2\omega, h\cdot\nabla\omega\right)_{L^2(\Omega)} dt = \int_0^T a\left(\omega, h\cdot\nabla\omega\right) dt \\
&+\alpha\int_0^T\int_{\Gamma_1} \theta\cdot\frac{\partial h\cdot\nabla\omega}{\partial\nu} d\Gamma_1 dt \; - \int_0^T\int_{\Gamma_0}\left(\Delta\omega + (1-\mu)B_1\omega\right)\frac{\partial^2\omega}{\partial\nu^2} d\Gamma_0 dt. \qquad (70)
\end{aligned}
$$

We note at this point that we can rewrite the first term on the right hand side of (70) as

$$
\begin{aligned}
\int_0^T a\left(\omega, h\cdot\nabla\omega\right) dt = \frac{1}{2}\int_0^T\int_\Omega h\cdot\nabla\left[\omega_{xx}^2 + \omega_{yy}^2 + 2\mu\omega_{xx}\omega_{yy} + 2(1-\mu)\omega_{xy}^2\right] dt d\Omega \\
+\mathcal{O}\left(\int_0^T \left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} dt\right), \qquad (71)
\end{aligned}
$$

where $\mathcal{O}\left(\int_0^T \left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} dt\right)$ denotes a series of terms which can be majorized by the $L^2(0, T; D(\mathring{\mathbf{A}}^{\frac{1}{2}}))$–

norm of ω; we consequently have by the divergence theorem that

$$\int_0^T a(\omega, h\cdot\nabla\omega)\,dt =$$

$$\frac{1}{2}\int_0^T\int_\Omega h\cdot\nabla\left[\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2\right]dtd\Omega$$

$$= \frac{1}{2}\int_0^T\int_\Omega div\left\{h\left[\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2\right]\right\}$$

$$+\mathcal{O}\left(\int_0^T\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2dt\right)$$

$$= \frac{1}{2}\int_0^T\int_{\Gamma_0}\left[\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2\right]dtd\Gamma_0$$

$$+\mathcal{O}\left(\int_0^T\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2dt\right)$$

$$= \frac{1}{2}\int_0^T\int_{\Gamma_0}(\Delta\omega)^2\,dt+\mathcal{O}\left(\int_0^T\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2dt\right), \tag{72}$$

where in the last step above, we have used the fact (as reasoned in [8], Ch. 4) that $\omega|_{\Gamma_0} = \left.\frac{\partial\omega}{\partial\nu}\right|_{\Gamma_0} = 0$ implies that $\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2 = (\Delta\omega)^2$ on Γ_0.

To handle the last term on the right hand side of (70), we note that $B_1\omega = 0$ on Γ_0, which implies that

$$\Delta\omega+(1-\mu)B_1\omega = \Delta\omega = \frac{\partial^2\omega}{\partial\nu^2}\ \text{ on } \Gamma_0\ ; \tag{73}$$

we consequently have upon the insertion of (72) into (70), as well as the consideration of (73) that

$$\int_0^T\left(\Delta^2\omega, h\cdot\nabla\omega\right)_{L^2(\Omega)}dt = -\frac{1}{2}\int_0^T\|\Delta\omega\|_{L^2(\Gamma_0)}^2\,dt$$

$$+\alpha\int_0^T\int_{\Gamma_1}\theta\cdot\frac{\partial h\cdot\nabla\omega}{\partial\nu}\,d\Gamma_1dt\ +\ \mathcal{O}\left(\int_0^T\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2dt\right). \tag{74}$$

(iv) To handle the last term on the left hand side of equation (67), we again use Green's theorem and the boundary conditions in (1) to obtain

$$\int_0^T(\Delta\theta, h\cdot\nabla\omega)_{L^2(\Omega)}\,dt = -\int_0^T(\nabla\theta,\nabla(h\cdot\nabla\omega))_{L^2(\Omega)}\,dt. \tag{75}$$

To finish the proof, we rewrite (67) by collecting the relations given above in (68), (69), (74) and (75) to thereby attain the inequality (65), upon the taking of norms and a subsequent majorization. □

In showing the exponential decay of the semigroup $\left\{e^{\mathcal{A}_\gamma t}\right\}_{t\geq 0}$ (**Theorem 3**) it will suffice as usual, to prove that there exists a time $0 < T < \infty$ and a corresponding constant C_T which satisfies for all initial data in $\mathbf{H}_\gamma$,

$$E_\gamma(T) \leq \xi E_\gamma(0) \text{ with } \xi < 1. \tag{76}$$

By a density argument, it will then be enough by Lemma 1 to show the existence of a time T, $0 < T < \infty$, and a positive constant C_T (independent of γ) for initial data in $\left[\omega^0, \omega^1, \theta^0\right] \in D\left(\mathcal{A}_\gamma^2\right)$ such that

$$E_\gamma(T) \leq C_T \int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt, \tag{77}$$

to which end we will proceed to work.

2.3 Proof of the Inequality (77)

Because of **Theorem 2**, we have for initial data $\left[\omega^0, \omega^1, \theta^0\right] \in D\left(\mathcal{A}_\gamma^2\right)$ a classical pointwise solution $[\omega, \omega_t, \theta]$ of (1); we can thus multiply the first equation in (1) by $A_D^{-1}\theta$, integrate from 0 to T to obtain

$$\int_0^T \left[\left(\omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega + \alpha\Delta\theta, A_D^{-1}\theta\right)_{L^2(\Omega)}\right] dt = 0; \tag{78}$$

and the bulk of the work from here on out will be the scrutiny of the left hand side of (78).
(A.1) *Dealing with* $\int_0^T \left(\omega_{tt} - \gamma\Delta\omega_{tt}, A_D^{-1}\theta\right)_{L^2(\Omega)} dt$: Using an integration by parts, the second differential equation of (1) and the fact that $A_R\theta = -\Delta\theta + \frac{\sigma}{\eta}\theta = -\Delta\theta + \Delta D\gamma_0\theta + \frac{\sigma}{\eta}\theta = A_D(\mathbf{I} - D\gamma_0)\theta + \frac{\sigma}{\eta}\theta$ produces

$$\begin{aligned}
&\int_0^T \left(\omega_{tt} - \gamma\Delta\omega_{tt}, A_D^{-1}\theta\right)_{L^2(\Omega)} dt \\
= \;& \left(\omega_t, A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T + \gamma\left(\nabla\omega_t, \nabla A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T \\
& - \int_0^T \left[\left(\omega_t, A_D^{-1}\theta_t\right)_{L^2(\Omega)} + \gamma\left(\nabla\omega_t, \nabla A_D^{-1}\theta_t\right)_{L^2(\Omega)}\right] dt \\
= \;& \alpha\beta^{-1} \int_0^T \left[\|\omega_t\|^2_{L^2(\Omega)} + \gamma\|\nabla\omega_t\|^2_{L^2(\Omega)}\right] dt \\
& -\alpha\beta^{-1} \int_0^T \left[(\omega_t, D\gamma_0\omega_t)_{L^2(\Omega)} + \gamma(\nabla\omega_t, \nabla D\gamma_0\omega_t)_{L^2(\Omega)}\right] dt \\
& +\eta\beta^{-1} \int_0^T \left[(\omega_t, (\mathbf{I} - D\gamma_0)\theta)_{L^2(\Omega)} + \gamma(\nabla\omega_t, \nabla(\mathbf{I} - D\gamma_0)\theta)_{L^2(\Omega)}\right] dt \\
& +\sigma\beta^{-1} \int_0^T \left[\left(\omega_t, A_D^{-1}\theta\right)_{L^2(\Omega)} + \gamma\left(\nabla\omega_t, \nabla A_D^{-1}\theta\right)_{L^2(\Omega)}\right] dt \\
& + \left(\omega_t, A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T + \gamma\left(\nabla\omega_t, \nabla A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T;
\end{aligned} \tag{79}$$

A further integration by parts and application of Green's Theorem (5) to the term $\int_0^T (\nabla\omega_t, \nabla D\gamma_0\omega_t)_{L^2(\Omega)}\,dt$ yield (after a consideration of the boundary conditions posted in (1))

$$\begin{aligned}
&-\gamma\int_0^T (\nabla\omega_t, \nabla D\gamma_0\omega_t)_{L^2(\Omega)}\,dt \\
= \;& -\gamma\,(\nabla\omega_t, \nabla D\gamma_0\omega)_{L^2(\Omega)}\Big|_0^T + \gamma\int_0^T (\nabla\omega_{tt}, \nabla D\gamma_0\omega)_{L^2(\Omega)}\,dt \\
= \;& -\gamma\,(\nabla\omega_t, \nabla D\gamma_0\omega)_{L^2(\Omega)}\Big|_0^T - \gamma\int_0^T (\Delta\omega_{tt}, D\gamma_0\omega)_{L^2(\Omega)}\,dt \\
& +\gamma\int_0^T \left(\frac{\partial\omega_{tt}}{\partial\nu}, \gamma_0\omega\right)_{L^2(\Gamma_1)} dt \\
= \;& -\gamma\,(\nabla\omega_t, \nabla D\gamma_0\omega)_{L^2(\Omega)}\Big|_0^T - \int_0^T (\omega_{tt} + \Delta^2\omega + \alpha\theta, D\gamma_0\omega)_{L^2(\Omega)}\,dt \\
& +\gamma\int_0^T \left(\frac{\partial\omega_{tt}}{\partial\nu}, \gamma_0\omega\right)_{L^2(\Gamma_1)} dt \\
= \;& -\gamma\,(\nabla\omega_t, \nabla D\gamma_0\omega)_{L^2(\Omega)}\Big|_0^T - (\omega_t, D\gamma_0\omega)_{L^2(\Omega)}\Big|_0^T + \int_0^T (\omega_t, D\gamma_0\omega_t)_{L^2(\Omega)}\,dt \\
& -\int_0^T a\,(D\gamma_0\omega, \omega)\,dt - \int_0^T \left(\alpha\theta, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_1)} dt - \int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt \\
& +\alpha\int_0^T (\nabla\theta, \nabla D\gamma_0\omega)_{L^2(\Omega)}\,dt.
\end{aligned} \tag{80}$$

Given that $D\gamma_0 \in \mathcal{L}(H^s(\Omega))$ for all real s and the fact that A_D^{-1} is "smoothing", viz. $\left\|A_D^{-1}\theta\right\|_{H^2(\Omega)} \leq C\,\|\theta\|_{L^2(\Omega)}$, we have the following estimates for the solution $[\omega, \omega_t, \theta]$ of (1) corresponding to arbitrary initial data in $\mathbf{H}_\gamma$:

$$\|(I - D\gamma_0)\theta\|_{L^2(\Omega)} + \left\|A_D^{-1}\theta\right\|_{L^2(\Omega)} \leq C\,\|\theta\|_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\,; \tag{81}$$

$$\|\nabla(I - D\gamma_0)\theta\|_{L^2(\Omega)} + \left\|\nabla A_D^{-1}\theta\right\|_{L^2(\Omega)} \leq C\,\|\theta\|_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\,; \tag{82}$$

$$\|\nabla D\gamma_0\omega\|_{L^2(\Omega)} \leq C\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}; \tag{83}$$

$$\left\|\frac{\partial D\gamma_0\omega}{\partial\nu}\right\|_{H^{\frac{1}{2}}(\Gamma)} \leq C\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}; \tag{84}$$

thus a substitution of (80) into (79) and its subsequent majorization, keeping in mind the inequalities (81)–(84), will give the estimate

$$\left|\int_0^T \left(\omega_{tt} - \gamma\Delta\omega_{tt}, A_D^{-1}\theta\right)_{L^2(\Omega)} dt - \alpha\beta^{-1}\int_0^T \left[\|\omega_t\|^2_{L^2(\Omega)} + \gamma\|\nabla\omega_t\|^2_{L^2(\Omega)}\right] dt\right|$$

$$\leq C\int_0^T \left[\|\omega_t\|_{L^2(\Omega)}\|\theta\|_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)} + \gamma\|\nabla\omega_t\|_{L^2(\Omega)}\|\theta\|_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\right] dt$$

$$+C\left[E_\gamma(0) + E_\gamma(T)\right] + \left|\int_0^T a\left(D\gamma_0\omega, \omega\right) dt\right|$$

$$+\left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right|$$

$$\leq \epsilon\int_0^T \left[\|\omega_t\|_{L^2(\Omega)} + \gamma\|\nabla\omega_t\|_{L^2(\Omega)}\right] dt + C_\epsilon\int_0^T \|\theta\|_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)} dt$$

$$+C\left[E_\gamma(0) + E_\gamma(T)\right] + \left|\int_0^T a\left(D\gamma_0\omega, \omega\right) dt\right|$$

$$+\left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right|, \tag{85}$$

where the constants C and C_ϵ do not depend on γ, $0 < \gamma \leq M$.

(A.2) *Dealing with* $\int_0^T \left(\Delta^2\omega, A_D^{-1}\theta\right) dt$: Yet another application of Green's theorem in (5) and the use of the enforced boundary conditions give

$$\int_0^T \left(\Delta^2\omega, A_D^{-1}\theta\right) dt = \int_0^T a\left(\omega, A_D^{-1}\theta\right) dt - \int_0^T \left(\Delta\omega, \frac{\partial A_D^{-1}\theta}{\partial\nu}\right)_{L^2(\Gamma_0)} dt$$

$$+\alpha\int_0^T \left(\theta, \frac{\partial A_D^{-1}\theta}{\partial\nu}\right)_{L^2(\Gamma_1)} dt. \tag{86}$$

Estimating the right hand side of (86) yields, after the use of trace theory, elliptic regularity and the mean inequality,

$$
\begin{aligned}
&\left|\int_0^T \left(\Delta^2\omega, A_D^{-1}\theta\right) dt\right| \\
\leq\; & C_0 \int_0^T \left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)} ||\theta||_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt \\
& + \frac{\epsilon}{2C}\int_0^T ||\Delta\omega||^2_{L^2(\Gamma_0)}\, dt + C_\epsilon \int_0^T ||\theta||^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt \\
& \text{(where the inverted } C \text{ is the same constant present in (65))} \\
\leq\; & C_0 \int_0^T \left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)} ||\theta||_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt \\
& + \frac{\epsilon}{2}\left[\int_0^T \left(\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} + \left\|P_\gamma^{\frac{1}{2}}\omega_t\right\|^2_{L^2(\Omega)}\right) dt + \right. \\
& \left. + E_\gamma(0) + E_\gamma(T)\right] + C_\epsilon \int_0^T ||\theta||^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt \\
& \text{(by } \textbf{Lemma 2}\text{)} \\
\leq\; & \epsilon \int_0^T \left[\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} + \left\|P_\gamma^{\frac{1}{2}}\omega_t\right\|^2_{L^2(\Omega)}\right] dt \\
& + C\left[E_\gamma(0) + E_\gamma(T)\right] + C_\epsilon \int_0^T ||\theta||^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt,
\end{aligned}
\tag{87}
$$

after the mean inequality.

(A.3) *Dealing with* $\int_0^T \left(\alpha\Delta\theta, A_D^{-1}\theta\right)_{L^2(\Omega)} dt$: Finally, for the last term of (78), again using the fact that $A_R\theta = A_D(\mathbf{I}-D\gamma_0)\theta + \frac{\sigma}{\eta}\theta$, we have easily

$$
\alpha\int_0^T \left(A_D(\mathbf{I}-D\gamma_0)\theta + \frac{\sigma}{\eta}\theta, A_D^{-1}\theta\right)_{L^2(\Omega)} dt = \alpha\int_0^T \left[||\theta||^2_{L^2(\Omega)} - (D\gamma_0\theta,\theta)_{L^2(\Omega)} + \left(\frac{\alpha\sigma}{\eta}A_D^{-1}\theta,\theta\right)_{L^2(\Omega)}\right] dt
$$

$$
\leq C\int_0^T ||\theta||^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\,. \tag{88}
$$

(A.4) *Combining (78), (85), (87) and (88) thus results in the following*: For $\epsilon > 0$ small enough there exists a constant $C > 0$ (independent of γ) such that the solution $[\omega, \omega_t, \theta]$ of (1) satisfies

$$
\begin{aligned}
&\left(\frac{\alpha}{\beta} - 2\epsilon\right)\int_0^T \left[||\omega_t||^2_{L^2(\Omega)} + \gamma\,||\nabla\omega_t||^2_{L^2(\Omega)}\right] dt \\
\leq\; & C\left[\int_0^T ||\theta||^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\, dt + E_\gamma(T) + E_\gamma(0)\right] \\
& + \epsilon\int_0^T \left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} dt + \left|\int_0^T a\left(D\gamma_0\omega,\omega\right) dt\right| \\
& + \left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right|,
\end{aligned}
\tag{89}
$$

where the noncrucial dependence of C upon ϵ has not been noted.

(A.5) *Estimating the residual terms* $\left|\int_0^T a\left(D\gamma_0\omega,\omega\right)dt\right|$ and $\left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right|$.[2] At this point we will find it advantageous to consider a decomposition of the solution component $[\omega,\omega_t]$ into $\omega = \omega^{(1)}+\omega^{(2)}+\omega^{(3)}$ (again with the corresponding initial data $[\omega_0,\omega_1]\in D(\mathcal{A}_\gamma^2)$), where the $\omega^{(i)}$ solve respectively:

$$\begin{cases} -\gamma\Delta\omega_{tt}^{(1)} + \Delta^2\omega^{(1)} = -\alpha\Delta\theta \ \text{ on } (0,\infty)\times\Omega \\ \omega^{(1)} = \dfrac{\partial\omega^{(1)}}{\partial\nu} = 0 \ \text{ on } (0,\infty)\times\Gamma_0 \\ \begin{cases} \Delta\omega^{(1)} + (1-\mu)B_1\omega^{(1)} + \alpha\theta = 0 \\ \dfrac{\partial\Delta\omega^{(1)}}{\partial\nu} + (1-\mu)\dfrac{\partial B_2\omega^{(1)}}{\partial\tau} - \gamma\dfrac{\partial\omega_{tt}^{(1)}}{\partial\nu} = 0 \end{cases} \text{ on } (0,\infty)\times\Gamma_1 \\ \omega^{(1)}(t=0) = \omega_t^{(1)}(t=0) = 0; \end{cases} \tag{90}$$

$$\begin{cases} -\gamma\Delta\omega_{tt}^{(2)} + \Delta^2\omega^{(2)} = -\omega_{tt} \ \text{ on } (0,\infty)\times\Omega \\ \omega^{(2)} = \dfrac{\partial\omega^{(2)}}{\partial\nu} = 0 \ \text{ on } (0,\infty)\times\Gamma_0 \\ \begin{cases} \Delta\omega^{(2)} + (1-\mu)B_1\omega^{(2)} = 0 \\ \dfrac{\partial\Delta\omega^{(2)}}{\partial\nu} + (1-\mu)\dfrac{\partial B_2\omega^{(2)}}{\partial\tau} - \gamma\dfrac{\partial\omega_{tt}^{(2)}}{\partial\nu} + \alpha\dfrac{\partial\theta}{\partial\nu} = 0 \end{cases} \text{ on } (0,\infty)\times\Gamma_1 \\ \omega^{(2)}(t=0) = \omega_t^{(2)}(t=0) = 0. \end{cases} \tag{91}$$

$$\begin{cases} -\gamma\Delta\omega_{tt}^{(3)} + \Delta^2\omega^{(3)} = 0 \ \text{ on } (0,\infty)\times\Omega \\ \omega^{(3)} = \dfrac{\partial\omega^{(3)}}{\partial\nu} = 0 \ \text{ on } (0,\infty)\times\Gamma_0 \\ \begin{cases} \Delta\omega^{(3)} + (1-\mu)B_1\omega^{(3)} = 0 \\ \dfrac{\partial\Delta\omega^{(3)}}{\partial\nu} + (1-\mu)\dfrac{\partial B_2\omega^{(3)}}{\partial\tau} - \gamma\dfrac{\partial\omega_{tt}^{(3)}}{\partial\nu} = 0 \end{cases} \text{ on } (0,\infty)\times\Gamma_1 \\ \omega^{(3)}(0) = \omega^0; \ \ \omega_t^{(3)}(0) = \omega^1. \end{cases} \tag{92}$$

---

[2] Notice that at this point, one might be tempted to apply a straightforward estimate which gives $\left|\int_0^T a\left(D\gamma_0\omega,\omega\right)\right| \leq C\int_0^T \left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega(t)\right\|_{L^2(\Omega)}^2 dt$. However, this will not suffice as we do not have control over the constant C (C may not be small $<< 1$). Therefore, we need a different, more complex argument which will lead to the estimate (113) below; likewise for the term $\left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right|$.

Through a semigroup formulation, the well–posedness of (91) and (92) can be handled just as easily as the entire system (1); to wit, defining on the state space $D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega)$ the operator $\widetilde{\mathcal{A}}_\gamma$ as

$$\widetilde{\mathcal{A}}_\gamma \equiv \begin{pmatrix} 0 & \mathbf{I} \\ -\widetilde{P}_\gamma^{-1}\mathring{\mathbf{A}} & 0 \end{pmatrix} \tag{93}$$

$$(\text{where } \widetilde{P}_\gamma \equiv \gamma A_N \in \mathcal{L}(H^1_{\Gamma_0,\gamma}(\Omega), H^{-1}_{\Gamma_0,\gamma}(\Omega)) \tag{94}$$

$$\text{with domain } D(\widetilde{\mathcal{A}}_\gamma) = \left\{[\omega_1, \omega_2] \in D(\mathring{\mathbf{A}}^{\frac{3}{4}}) \times D(\mathring{\mathbf{A}}^{\frac{1}{2}})\right\} \tag{95}$$

(note that A_N^{-1} exists as A_N is elliptic on $H^1_{\Gamma_0,\gamma}(\Omega)$); then with the same sort of effort as in the proof of **Theorem 1**, we can show that $\widetilde{\mathcal{A}}_\gamma$ generates a unitary C_0–group $\left\{e^{\widetilde{\mathcal{A}}_\gamma t}\right\}_{t\geq 0}$ on $D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega)$. Consequently we have that $\omega^{(2)} \in C([0,T]\,; D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega))$ with

$$\begin{bmatrix} \omega^{(2)}(t) \\ \omega_t^{(2)}(t) \end{bmatrix} = \int_0^t e^{\widetilde{\mathcal{A}}_\gamma(t-s)} \begin{bmatrix} 0 \\ \widetilde{P}_\gamma^{-1}\left(-\omega_{tt}(s) + \alpha\lambda\mathring{\mathbf{A}}G_2\gamma_0\theta(s)\right) \end{bmatrix} ds, \tag{96}$$

where again ω_{tt} is the second time derivative of the solution component ω; recall that the intial data $[\omega^0, \omega^1, \theta^0] \in D(\mathcal{A}_\gamma^2)$, so $\omega_{tt} \in H^1_{\Gamma_0,\gamma}(\Omega)$ pointwise, and $\lambda G_2\gamma_0\theta(t) \in D(\mathring{\mathbf{A}}^{\frac{3}{4}})$ pointwise (from (17), (19) and (4)) providing that $\mathring{\mathbf{A}}G_2\gamma_0\theta(t) \in H^{-1}_{\Gamma_0,\gamma}(\Omega)$, and hence the formula (96)) is well–defined. Likewise, $\omega^{(3)} \in C([0,T]\,; D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega))$ with

$$\omega^{(3)}(t) = e^{\widetilde{\mathcal{A}}_\gamma t}\begin{bmatrix} \omega^0 \\ \omega^1 \end{bmatrix}. \tag{97}$$

Regarding the well–posedness of the system (90), we have the following result from [10]:

Regularity Theorem: *For arbitrary initial data* $[\omega_0, \omega_1] \in D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega)$, *parameter* $\xi \geq 0$, $f \in L^2(0,T; H^{-1}_{\Gamma_0,\gamma}(\Omega))$ *and* $g \in L^2(0,T; H^{\frac{1}{2}}(\Gamma_1))$, *the following system is well-posed:*

$$\begin{cases} \xi\omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega = f \quad \text{on } (0,\infty)\times\Omega \\ \omega = \dfrac{\partial\omega}{\partial\nu} = 0 \quad \text{on } (0,\infty)\times\Gamma_0 \\ \left\{ \begin{array}{l} \Delta\omega + (1-\mu)B_1\omega = g \\ \dfrac{\partial\Delta\omega}{\partial\nu} + (1-\mu)\dfrac{\partial B_2\omega}{\partial\tau} - \gamma\dfrac{\partial\omega_{tt}}{\partial\nu} = 0 \end{array} \right. \quad \text{on } (0,\infty)\times\Gamma_1 \\ \omega(0) = \omega_0, \;\; \omega_t(0) = \omega_1, \end{cases} \tag{98}$$

with the solution $[\omega, \omega_t] \in C([0,T]\,; D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega))$.

To make use of the above theorem for the resolution of (90) with arbitrary θ in $H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)$, we note that $-\Delta = A_R - \dfrac{\sigma}{\eta} \in \mathcal{L}\left(H^1(\Omega), [H^1(\Omega)]'\right)$ and consequently $\Delta\theta \in L^2(0,T; H^{-1}_{\Gamma_0,\gamma}(\Omega))$; moreover $\theta|_\Gamma \in L^2(0,T; H^{\frac{1}{2}}(\Gamma))$ by the trace theorem, and so the **Regularity Theorem** will give us that

$$\omega^{(1)} \in C([0,T]\,; D(\mathring{\mathbf{A}}^{\frac{1}{2}}) \times H^1_{\Gamma_0,\gamma}(\Omega)), \tag{99}$$

with the pointwise estimate

$$\left\| \begin{array}{c} \omega^{(1)}(t) \\ \omega_t^{(1)}(t) \end{array} \right\|^2_{D(\mathring{A}^{\frac{1}{2}})\times H^1_{\Gamma_0,\gamma}(\Omega)} \leq C\left[\int_0^T \|\Delta\theta(t)\|^2_{H^{-1}_{\Gamma_0,\gamma}(\Omega)}\,dt + \alpha\int_0^T \|\theta(t)\|^2_{H^{\frac{1}{2}}(\Gamma_1)}\,dt\right] \leq C\int_0^T \|\theta(t)\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}\,dt. \tag{100}$$

A simple uniqueness argument that uses the **Regularity Theorem** verifies that indeed the solution component $\omega \equiv \omega^{(1)} + \omega^{(2)} + \omega^{(3)}$. Moreover, concerning the explicit representation (96), an integration by parts has that

$$\begin{aligned} \int_0^t e^{\tilde{\mathcal{A}}_\gamma(t-s)}\left[\begin{array}{c} 0 \\ \widetilde{P}_\gamma^{-1}\omega_{tt}(s) \end{array}\right] ds &= e^{\tilde{\mathcal{A}}_\gamma(t-s)}\left[\begin{array}{c} 0 \\ \widetilde{P}_\gamma^{-1}\omega_t(s) \end{array}\right]\Bigg|_0^t \\ &- \int_0^t e^{\tilde{\mathcal{A}}_\gamma(t-s)}\widetilde{\mathcal{A}}_\gamma\left[\begin{array}{c} 0 \\ \widetilde{P}_\gamma^{-1}\omega_t(s) \end{array}\right] ds \\ = \; e^{\tilde{\mathcal{A}}_\gamma(t-s)}\left[\begin{array}{c} 0 \\ \widetilde{P}_\gamma^{-1}\omega_t(s) \end{array}\right]\Bigg|_0^t &- \int_0^t e^{\tilde{\mathcal{A}}_\gamma(t-s)}\left[\begin{array}{c} \widetilde{P}_\gamma^{-1}\omega_t(s) \\ 0 \end{array}\right] ds, \end{aligned} \tag{101}$$

where the equality above makes sense in $\left[D\left(\widetilde{\mathcal{A}_\gamma}^*\right)\right]' = \left[D(\mathring{A}^{\frac{3}{4}})\right]' \times \left[D(\mathring{A}^{\frac{1}{2}})\right]'$; hence upon majorizing (96) with the expression (101) in mind (and using the contraction of the semigroup $\left\{e^{\tilde{\mathcal{A}}_\gamma(t)}\right\}_{t\geq 0}$) we have,

$$\left\| \begin{array}{c} \omega^{(2)}(t) \\ \omega_t^{(2)}(t) \end{array} \right\|^2_{D(\mathring{A}^{\frac{1}{2}})\times H^1_{\Gamma_0,\gamma}(\Omega)} \leq C_T\left[\|\theta\|^2_{L^2(0,T;H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega))} + \|\omega_t\|^2_{C([0,T];L^2(\Omega))}\right]. \tag{102}$$

Thus, using (100), (102) and the explicit representation (97), we have:

$$\left\| \left[\begin{array}{c} \omega^{(1)}(t) + \omega^{(2)}(t) \\ \omega^{(1)}(t) + \omega_t^{(2)}(t) \end{array}\right] \right\|^2_{D(\mathring{A}^{\frac{1}{2}})\times H^1_{\Gamma_0,\gamma}(\Omega)} \leq C_T\left[\|\theta\|^2_{L^2(0,T;H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega))} + \|\omega_t\|^2_{C([0,T];L^2(\Omega))}\right] \tag{103}$$

$$\left\| \left[\begin{array}{c} \omega^{(3)}(t) \\ \omega_t^{(3)}(t) \end{array}\right] \right\|^2_{D(\mathring{A}^{\frac{1}{2}})\times H^1_{\Gamma_0,\gamma}(\Omega)} \leq E_\gamma(0). \tag{104}$$

Further analyzing $\omega^{(3)}$, if we make the substitution $z \equiv \Delta\omega^{(3)}$, we then note that z solves the wave equation

$$\gamma z_{tt} = \Delta z, \tag{105}$$

with $[z, z_t] \in C([0,T]; L^2(\Omega) \times H^{-1}(\Omega))$. Consequently, the recent regularity result of [21] (specifically, apply Theorem 3 together with Remark 2.3 and the remark after Theorem 9 in [21]) reveals that z has a "trace" on Γ with a positive constant $C(T,\gamma)$ and a $\rho > 0$ such that the following estimate holds[3]:

$$\|z|_\Gamma\|_{L^2(0,T;H^{-\frac{1}{2}+\rho}(\Gamma))} \leq C(T,\gamma)\,\|[z, z_t]\|_{C([0,T];L^2(\Omega)\times H^{-1}(\Omega))}; \tag{106}$$

and as pointwise we have

$$\|z(t)\|^2_{L^2(\Omega)} + \|z_t(t)\|^2_{H^{-1}(\Omega)} \leq CE_\gamma(0) \tag{107}$$

[3] We note that the value of ρ depends on the geometry; however, we always have $\rho > 0$.

(from the estimate (104)), we end up with

$$\left\| \Delta\omega^{(3)}\Big|_{\Gamma} \right\|^2_{L^2(0,T;H^{-\frac{1}{2}+\rho}(\Gamma))} \leq C(T,\gamma)E_\gamma(0). \tag{108}$$

Recall that $\omega^{(3)}$, as the solution of (92), satisfies

$$\Delta\omega^{(3)} - (1-\mu)\frac{\partial^2\omega^{(3)}}{\partial\tau^2} = (1-\mu)\kappa\frac{\partial\omega^{(3)}}{\partial\nu} \quad \text{on } (0,T)\times\Gamma_1, \tag{109}$$

where κ denotes the curvature, and so (109), coupled with the estimates (??) and

$$\left\| \frac{\partial\omega^{(3)}}{\partial\nu} \right\|_{L^\infty(0,T;H^{\frac{1}{2}}(\Gamma_1))} \leq C\left\| \omega^{(3)} \right\|_{L^\infty(0,T;H^2(\Omega))} \leq C(T)E_\gamma(0),$$

gives that $\dfrac{\partial^2\omega^{(3)}}{\partial\tau^2} \in L^2(0,T;H^{-\frac{1}{2}+\rho}(\Gamma_1))$ with

$$\left\| \frac{\partial^2\omega^{(3)}}{\partial\tau^2} \right\|_{L^2(0,T;H^{-\frac{1}{2}+\rho}(\Gamma_1))} \leq C(T,\gamma)E_\gamma(0), \tag{110}$$

and (110) is in turn equivalent to

$$\left\| \gamma_0\omega^{(3)} \right\|_{L^2(0,T;H^{\frac{3}{2}+\rho}(\Gamma_1))} \leq C(T,\gamma)E_\gamma(0). \tag{111}$$

Remark 6 *The estimate in (111) can also be derived independently by decomposing problem (92) microlocally into elliptic and hyperbolic parts. In the elliptic sector, we can use standard elliptic regularity and the boundary conditions on Γ_1 to deduce the regularity of the trace $\gamma_0\omega^{(3)} \in H^2(0,T\times\Gamma_1)$. In the hyperbolic sector, we apply the transformation $z \equiv \Delta\omega^{(3)}$, and we are thus led to the study of the wave equation with its forcing term in $L^2(0,T;H^{-1}\Omega)$ (due to microlocalization). The arguments presented in [12] and (see also [13]) apply to the hyperbolic sector only and provide the estimate (111) valid in that sector. Combining elliptic and hyperbolic estimates yields (111) with the value of ρ being at least $\frac{1}{10}$. Instead, the estimate obtained by using Tataru's result [21] leads to the optimal improvement of $\rho = \frac{1}{6}$.*

Given this extra regularity for the trace of $\omega^{(3)}\big|_{\Gamma_1}$, we can hence invoke a classical pde interpolation inequality to finally obtain

$$\left\| \gamma_0\omega^{(3)} \right\|^2_{L^2(0,T;H^{\frac{3}{2}}(\Gamma_1))} \leq C(T,\gamma)^{-1}\left\| \gamma_0\omega^{(3)} \right\|^2_{L^2(0,T;H^{\frac{3}{2}+\rho}(\Gamma_1))} + C_{T,\gamma}\left\| \gamma_0\omega^{(3)} \right\|^2_{L^2(0,T;H^{\frac{1}{2}}(\Gamma_1))}$$

(where $C(T,\gamma)$ is as in (111), and $C_{T,\gamma}$ denotes another positive constant depending on T and γ)

$$\leq E_\gamma(0) + C_{T,\gamma}\left\| \omega^{(3)} \right\|^2_{L^2(0,T;H^1(\Omega))}$$

(after using the estimate (111) and trace theory)

$$\leq E_\gamma(0) + C_{T,\gamma}\|\omega\|^2_{L^2(0,T;H^1(\Omega))} + C_{T,\gamma}\left\| \omega^{(1)} + \omega^{(2)} \right\|^2_{L^2(0,T;H^1(\Omega))}$$

(after using the decomposition $\omega = \omega^{(1)} + \omega^{(2)} + \omega^{(3)}$)

$$\leq E_\gamma(0) + C_{T,\gamma}\left[\|\theta\|^2_{L^2(0,T;H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega))} + \|\omega\|^2_{L^2(0,T;H^1(\Omega))} + \|\omega_t\|^2_{C([0,T];L^2(\Omega))} \right], \tag{112}$$

after using the inequality (103).

With the decomposition of ω in hand, along with the accompanying norm estimates, particularly that of the trace $\gamma_0\omega^{(3)}$ in (112), we can now tackle the recalcitrant terms $\left|\int_0^T a\left(D\gamma_0\omega,\omega\right)dt\right|$ and $\left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right|$:

(A5.i) *Dealing with* $\left|\int_0^T a\left(D\gamma_0\omega,\omega\right)dt\right|$:

$$
\begin{aligned}
&\left|\int_0^T a\left(D\gamma_0\omega,\omega\right)dt\right| \\
= &\left|\int_0^T a\left(D\gamma_0\left(\omega^{(1)}+\omega^{(2)}+\omega^{(3)}\right),\omega\right)dt\right| \\
\leq &\int_0^T C\left\|D\gamma_0\left(\omega^{(1)}+\omega^{(2)}+\omega^{(3)}\right)\right\|_{H^2(\Omega)}\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}dt \\
&\text{(after using the fact that } D\gamma_0\in\mathcal{L}(H^2(\Omega))\text{)} \\
\leq &\ \epsilon\int_0^T \left\|\mathring{\mathbf{A}}\omega\right\|^2_{L^2(\Omega)}dt + C_{T,\gamma}\left[\int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2(\Omega)}dt + \|\omega\|^2_{L^2(0,T;H^1(\Omega))} + \|\omega_t\|^2_{C([0,T];L^2(\Omega))}\right] \\
&+C\left[E_\gamma(T)+E_\gamma(0)\right],
\end{aligned}
\tag{113}
$$

using the boundedness of the Dirichlet map D followed by the standard mean inequality and crucial estimates (112) and (103) (here we have not noted the noncrucial dependence of ϵ in the constant $C_{T,\gamma}$).

(A.5ii) *Dealing with* $\left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right|$: By **Lemma 2**, $\Delta\omega| \in L^2(0,T;L^2(\Gamma_0))$, and hence

$$
\begin{aligned}
&\left|\int_0^T \left(\Delta\omega, \frac{\partial D\gamma_0\omega}{\partial\nu}\right)_{L^2(\Gamma_0)} dt\right| \\
\leq &\ C\int_0^T \|\Delta\omega\|_{L^2(\Gamma_0)}\|D\gamma_0\omega\|_{H^2(\Omega)}dt \\
&\text{(by the trace theorem)} \\
= &\ C\int_0^T \|\Delta\omega\|_{L^2(\Gamma_0)}\left\|D\gamma_0\left(\omega^{(1)}+\omega^{(2)}+\omega^{(3)}\right)\right\|_{H^2(\Omega)}dt \\
\leq &\ \frac{\epsilon}{C}1\int_0^T \|\Delta\omega\|^2_{L^2(\Gamma_0)}dt + C_{T,\gamma}\left[\int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2(\Omega)}dt + \|\omega\|^2_{L^2(0,T;H^1(\Omega))} + \|\omega_t\|^2_{C([0,T];L^2(\Omega))}\right] \\
&+C\left[E_\gamma(T)+E_\gamma(0)\right] \\
&\text{(again using the mean inequality followed by (112) and (103),} \\
&\text{and where the inverted positive constant } C \text{ is as in (65))} \\
\leq &\ \epsilon\int_0^T\left[\left\|\mathring{\mathbf{A}}^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} + \int_0^T\left\|P_\gamma^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)}\right]dt + C_{T,\gamma}\left[\int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2(\Omega)}dt\right. \\
&\left.+\ \|\omega\|^2_{L^2(0,T;H^1(\Omega))} + \|\omega_t\|^2_{C([0,T];L^2(\Omega))}\right] + C\left[E_\gamma(0)+E_\gamma(T)\right].
\end{aligned}
\tag{114}
$$

Combining (89), (113) and (114), we finally have

$$\left(\frac{\alpha}{\beta} - 3\epsilon\right) \int_0^T \left[\|\omega_t\|^2_{L^2(\Omega)} + \gamma \|\nabla \omega_t\|^2_{L^2(\Omega)} \right] dt$$

$$\leq 3\epsilon \int_0^T \left\| \mathring{\mathbf{A}}^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} dt + C_{T,\gamma} \left[\int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)} dt \right.$$

$$\left. + \|\omega\|^2_{L^2(0,T;H^1(\Omega))} + \|\omega_t\|^2_{C([0,T];L^2(\Omega))} \right]$$

$$+ C\left[E_\gamma(0) + E_\gamma(T)\right]. \tag{115}$$

(B) *Conclusion of the Proof*: To majorize the norm of the component ω, we multiply (34) by ω, integrate from 0 to T and employ Green's Theorem to obtain (after accounting for the boundary conditions and using (20))

$$\left(P_\gamma^{\frac{1}{2}} \omega_t, P_\gamma^{\frac{1}{2}} \omega \right)_{L^2(\Omega)} \Big|_0^T - \int_0^T \left\| P_\gamma^{\frac{1}{2}} \omega_t \right\|^2_{L^2(\Omega)} dt$$

$$= - \int_0^T \left\| \mathring{\mathbf{A}}^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} dt - \alpha \int_0^T \left(\theta, \frac{\partial \omega}{\partial \nu} \right)_{L^2(\Gamma_1)} dt$$

$$+ \alpha \int_0^T (\nabla \theta, \nabla \omega)_{L^2(\Omega)} dt; \tag{116}$$

since by the trace theorem we have pointwise

$$\left| \left(\theta, \frac{\partial \omega}{\partial \nu} \right)_{L^2(\Gamma)} \right| + \left| (\nabla \theta, \nabla \omega)_{L^2(\Omega)} \right|$$

$$\leq C \left[\|\theta\|_{H^{\frac{1}{2}}(\Gamma)} \left\| \frac{\partial \omega}{\partial \nu} \right\|_{H^{\frac{1}{2}}(\Gamma)} + \|\theta\|_{H^1(\Omega)} \|\omega\|_{H^1(\Omega)} \right]$$

$$\leq C \|\theta\|_{H^1(\Omega)} \|\omega\|_{H^2(\Omega)} \leq \epsilon \left\| \mathring{\mathbf{A}}^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} + C_\epsilon \|\theta\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)}, \tag{117}$$

we thus arrive at:

There exists a constant $C > 0$ such that for $\epsilon > 0$ small enough, the the solution $[\omega, \omega_t, \theta]$ of (1) satisfies

$$(1-\epsilon) \int_0^T \left\| \mathring{\mathbf{A}}^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} dt \leq C \int_0^T \left[\|\omega_t\|^2_{L^2(\Omega)} + \gamma \|\nabla \omega_t\|^2_{L^2(\Omega)} \right] dt$$

$$+ C \left(\int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)} dt + E_\gamma(T) + E_\gamma(0) \right), \tag{118}$$

where the noncrucial dependence of C upon ϵ has not been noted.

Thus, if ϵ is small enough, we then have, upon combining (115) and (118), the existence of a constants C and $C_{T,\gamma}$ such that

$$\int_0^T \left[\left\| \mathring{\mathbf{A}}^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} + ||\omega_t||^2_{L^2(\Omega)} + \gamma ||\nabla \omega_t||^2_{L^2(\Omega)} + ||\theta||^2_{L^2(\Omega)} \right] dt$$

$$\leq C_{T,\gamma} \int_0^T ||\theta||^2_{H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)} dt + C \left[E_\gamma(T) + E_\gamma(0) \right]$$

$$+ C_{T,\gamma} \left[||\omega||^2_{L^2(0,T;H^1(\Omega))} + ||\omega_t||^2_{C([0,T];L^2(\Omega))} \right]. \qquad (119)$$

From here, we apply the relation (62) and its inherent dissipativity property (that is, $E_\gamma(T) \leq E_\gamma(t) \; \forall \; 0 \leq t \leq T$) to (119) to finally attain the preliminary inequality; namely, for $T > 2C$ (with C as in (119) independent of T),

$$E_\gamma(T) \leq \frac{C_{T,\gamma} + 2C\eta}{T - 2C} \int_0^T ||\theta||^2_{H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)} dt$$

$$+ C_{T,\gamma} \left[||\omega||^2_{L^2(0,T;H^1(\Omega))} + ||\omega_t||^2_{C([0,T];L^2(\Omega))} \right]. \qquad (120)$$

A straightforward compactness–uniqueness argument similar to that employed [11] and [1] will subsequently eliminate the lower order terms in (120), viz.

Proposition 2 *The presence of the inequality (120) implies that there exists a constant C_T which satisfies*

$$||\omega||^2_{L^2(0,T;H^1(\Omega))} + ||\omega_t||^2_{C([0,T];L^2(\Omega))} dt \leq C_T \int_0^T ||\theta||^2_{H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)} dt. \qquad (121)$$

Hence, the inequalities (120) and (121) give the desired estimate (77) (and consequently (76)), and the proof of **Theorem 3** is now complete.

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