Authentication and Obfuscation of Digital Signal Processing Integrated Circuits

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA

BY

Yingjie Lao

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

Advisor: Keshab K. Parhi

July, 2015
Acknowledgements

First and foremost, I want to thank my advisor Prof. Keshab K. Parhi, for his continuing and fatherly encouragement, tremendous guidance, and financial support throughout my entire Ph.D. study at the University of Minnesota. Throughout the five years that he has been my advisor, he has served as an incredible inspiration and talented mentor in my educational and career development. He also gives me a lot of guidance and advice on my career development. As a student of him, I not only have developed expertise in several areas, but also have learned some important skills in research.

I also would like to thank Prof. Chris Kim, Prof. Marc Riedel, and Prof. Yousef Saad at the University of Minnesota, for their support as members of my Ph.D. committee and kind help throughout my graduate study.

My sincere thank also goes to my research group. I am grateful to Weikang Qian, Manohar Ayinala, Qianying Tang, Saroj Satapathy at the University of Minnesota, for their collaborations and valuable feedback to my research. I also would like to thank Renfei Liu, Te-Lung Kung, Zisheng Zhang, Yin Liu, Sohini Roychowdhury, Sayed Ahmad Salehi, Bo Yuan, Tingting Xu, Jieming Yin, Cong Ma, and Yang Su, for their support in my Ph.D. life.

Last but not the least, I am forever grateful to my mother Yuanfeng Xu, my father Youhong Lao, my mother in law Yajun Di, and my father in law Guolin Yang. Without their support, I would not have earned my Ph.D. degree. I would like to express my
dearest thanks to my wife Yuzhe Yang who has always been extremely understanding and supportive during my studies, and to my little daughter Clover for bringing so much happiness and joy into our life. My career and life are more meaningful because of the love and care that I have been privileged to receive from my whole family.
Dedication

To my wife, Yuzhe, and my daughter, Clover, for their endless love.
Abstract

As electronic devices become increasingly interconnected and pervasive in people’s lives, security, trustworthy computing, and intellectual property (IP) protection have notably emerged as important challenges for the next decade. The assumption that hardware is trustworthy and that security effort should only be focused on networks and software is no longer valid given globalization of integrated circuits and systems design and fabrication. In 2011, the Semiconductor Industry Association pegged the cost of electronics counterfeiting at US $7.5 billion per year in lost revenue and tied it to the loss of 11,000 U.S. jobs [1]. From a national defense perspective, unsecured devices can be compromised by the enemy, putting military personnel and equipment in danger. Therefore, securing integrated circuit (IC) chips, in other words, hardware security, is extremely important.

This dissertation considers the design of highly secure digital signal processing circuits by employing both authentication-based and obfuscation-based approaches. In the first part of the dissertation, we focus on one emerging authentication-based solution: Physical Unclonable Function (PUF). We present novel reconfigurable PUF designs which could simultaneously achieve better reliability and security. We also present a systematic statistical analysis to quantitatively evaluate the performances of various multiplexer (MUX)-based PUFs. The statistical analysis results can be used to predict the relative advantages of various MUX-based PUF designs. These results can be used by the designer to choose a proper type of PUF or appropriate design parameters for a certain PUF based on the requirements of a specific application. Furthermore, a lightweight PUF-based local authentication scheme is also proposed, which eliminates the use of error correcting codes.
In the next part of the dissertation, we consider another hardware protection method: obfuscation. Hardware obfuscation is a technique by which the description or the structure of electronic hardware is modified to intentionally conceal its functionality, which makes it significantly more difficult to reverse engineer. Unlike these prior works, we start to look at Digital Signal Processing (DSP) circuits. In the literature, security aspects of DSP circuits have only attracted little attention. However, high-level transformations of DSP circuits are intrinsically suitable for hardware obfuscation, as these techniques only alter the structure of a circuit, while maintaining the original functionality. Based on this finding, we present a novel design methodology for obfuscated DSP circuits by hiding functionality via high-level transformations. The key idea is to generate meaningful and non-meaningful design variations by using high-level transformations.

In the final part of the dissertation, we consider the design and analysis of True Random Number Generator (TRNG), which is also an important topic in hardware security. We examine the modeling and statistical aspects of the proposed TRNG circuit. According to our model, we show that the performance of the beat-frequency detector based TRNG can be improved by appropriately adjusting design parameters. Motivated by our analysis, we propose several alternate BFD-TRNG designs which could achieve improved performance. Various post-processing methods which are specific to the proposed designs are also studied.
Contents

Acknowledgements i

Dedication iii

Abstract iv

List of Tables xii

List of Figures xiv

1 Introduction 1
   1.1 Introduction .................................. 1
   1.2 Summary of Contributions ......................... 3
      1.2.1 Authentication ............................ 3
      1.2.2 Obfuscation ............................... 5
      1.2.3 True Random Number Generator ............... 6
   1.3 Outline of the Dissertation ........................ 7

2 Novel Reconfigurable Silicon Physical Unclonable Functions 9
   2.1 Introduction .................................. 9
   2.2 Background .................................... 11
      2.2.1 Silicon Physical Unclonable Function ........ 11

vi
3.5 Performance Analysis of Feed-Forward MUX PUFs and MUX/DeMUX PUF

3.5.1 Statistical Properties of Feed-Forward MUX PUF ........................................ 52
3.5.2 Statistical Properties of Modified Feed-Forward MUX PUF ......................... 53
3.5.3 Statistical Properties of Different Types of Modified Feed-forward MUX PUFs .......................................................... 59
3.5.4 Statistical Properties of MUX/DeMUX PUF .................................................. 62

3.6 Performance Comparison of Various MUX-based PUFs .................................. 64

3.7 Experiments

3.7.1 Experimental Setup ........................................... 65
3.7.2 Results ..................................................... 65
3.7.3 Discussion .................................................. 67

3.8 Conclusion ..................................................... 67

4 Lightweight, Secure, and Reliable PUF-based Local Authentication with Self-Correction

4.1 Introduction .................................................. 69
4.2 Background .................................................. 70
4.2.1 PUF-based Authentication ........................................... 70
4.2.2 Error Correction ........................................... 72
4.3 Two-Level FSM Architecture ........................................... 74
4.4 Error Correction based on the Two-Level FSM ........................................... 78
4.4.1 Self-Correcting Functionality ........................................... 78
4.4.2 Advantages .................................................. 81
4.5 Other Applications ........................................... 83
4.5.1 Two-Factor Authentication ........................................... 84
4.5.2 Signature Generation ........................................... 84
4.5.3 Obfuscation .................................................. 86
6 Beat Frequency Detector based True Random Number Generators:

**Statistical Modeling and Analysis**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>127</td>
</tr>
<tr>
<td>6.2</td>
<td>Beat Frequency Detector based TRNG</td>
<td>129</td>
</tr>
<tr>
<td>6.3</td>
<td>Physical Component Modeling of RO TRNGs</td>
<td>131</td>
</tr>
<tr>
<td>6.4</td>
<td>Statistical Analysis of BFD-TRNG</td>
<td>133</td>
</tr>
<tr>
<td>6.4.1</td>
<td>BFD Model</td>
<td>133</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Effect of Counter Value</td>
<td>134</td>
</tr>
<tr>
<td>6.4.3</td>
<td>Bounds on Bias of Each Bit</td>
<td>139</td>
</tr>
<tr>
<td>6.4.4</td>
<td>Post-Processing</td>
<td>145</td>
</tr>
<tr>
<td>6.4.5</td>
<td>Online Test and Feedback Control</td>
<td>147</td>
</tr>
<tr>
<td>6.5</td>
<td>Alternate BFD-TRNG Architectures</td>
<td>148</td>
</tr>
<tr>
<td>6.5.1</td>
<td>Parallel Structure</td>
<td>148</td>
</tr>
<tr>
<td>6.5.2</td>
<td>Cascade Structure</td>
<td>149</td>
</tr>
<tr>
<td>6.5.3</td>
<td>Parallel-Cascade Structure</td>
<td>151</td>
</tr>
<tr>
<td>6.6</td>
<td>Comparison with Other Existing ROSC based TRNGs</td>
<td>153</td>
</tr>
<tr>
<td>6.6.1</td>
<td>Two-Oscillator TRNG</td>
<td>153</td>
</tr>
<tr>
<td>6.6.2</td>
<td>ROSC TRNG with XOR Tree</td>
<td>154</td>
</tr>
<tr>
<td>6.6.3</td>
<td>Comparison</td>
<td>156</td>
</tr>
<tr>
<td>6.7</td>
<td>Conclusion and Future Work</td>
<td>160</td>
</tr>
</tbody>
</table>

7 Conclusion and Future Directions

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Conclusion</td>
<td>161</td>
</tr>
<tr>
<td>7.2</td>
<td>Future Directions</td>
<td>162</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Evaluate and Attack PUFs</td>
<td>162</td>
</tr>
<tr>
<td>7.2.2</td>
<td>Obfuscation CAD Tool</td>
<td>163</td>
</tr>
<tr>
<td>7.2.3</td>
<td>DSP Circuit Reverse Engineering</td>
<td>164</td>
</tr>
<tr>
<td>7.2.4</td>
<td>Security in Emerging Technologies based Computing Systems</td>
<td>165</td>
</tr>
</tbody>
</table>
# List of Tables

<table>
<thead>
<tr>
<th>Table No.</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Simulation Results: Variations</td>
<td>32</td>
</tr>
<tr>
<td>2.2</td>
<td>Simulation Results: Reconfigurability</td>
<td>33</td>
</tr>
<tr>
<td>3.1</td>
<td>Notation Used in the Chapter</td>
<td>39</td>
</tr>
<tr>
<td>3.2</td>
<td>Predicted Performance of the Original MUX PUFs for Different Number of Stages (N) Obtained by Statistical Analysis Using the Model Derived from Experimental Results with N=100.</td>
<td>51</td>
</tr>
<tr>
<td>3.3</td>
<td>Performances of Different Feed-Forward MUX PUFs</td>
<td>54</td>
</tr>
<tr>
<td>3.4</td>
<td>Theoretical Performance Indicators Comparison</td>
<td>64</td>
</tr>
<tr>
<td>3.5</td>
<td>Results of Inter-Chip and Intra-Chip Variations for 100-Stage PUFs</td>
<td>66</td>
</tr>
<tr>
<td>3.6</td>
<td>Results of Performance Indicators for 100-Stage PUFs</td>
<td>67</td>
</tr>
<tr>
<td>4.1</td>
<td>Key Values Can Successfully Authenticate the Corresponding PUF Response</td>
<td>76</td>
</tr>
<tr>
<td>4.2</td>
<td>Key Values Can Successfully Authenticate the Corresponding PUF Response with Self-Correction</td>
<td>80</td>
</tr>
<tr>
<td>4.3</td>
<td>Key Values Can Successfully Authenticate the Corresponding PUF Response with Self-Correction</td>
<td>81</td>
</tr>
<tr>
<td>4.4</td>
<td>Area (Gate Count) of the Proposed Self-Correcting FSM that Can Correct m Bits of an N-Bit PUF Response</td>
<td>88</td>
</tr>
<tr>
<td>4.5</td>
<td>Power (μW) of the Proposed Self-Correcting FSM that Can Correct m Bits of an N-Bit PUF Response</td>
<td>88</td>
</tr>
<tr>
<td>4.6</td>
<td>Area (gate count) of the BCH Codes</td>
<td>90</td>
</tr>
</tbody>
</table>
4.7 Power ($\mu W$) of the BCH Codes ........................................... 91
5.1 Switch Configurations .............................................................. 104
5.2 Switch Configurations Example .................................................. 117
5.3 Overhead (%) of the (3l)th-order IIR Filter Benchmark ................. 118
5.4 Overhead (%) of the (12l)-Tap FIR Filter Benchmark .................... 118
6.1 The Probability of "1" Occurrence $p_1$ for Each Bit ..................... 140
6.2 Bias $\varepsilon$ for Each Bit under Different $\sigma_G$ in the Worst Case .... 142
6.3 Comparison of Block-Wise XOR and Von Neumann Corrector .......... 145
6.4 Number of Bits per Sample for Different $\sigma_G$ in the Worst Case .... 146
6.5 Correlation Coefficients of Each Bit among the Outputs for a 4-Parallel Structure .............................................. 149
6.6 Correlation Coefficients of Each Bit among the Outputs for a 4-Parallel-Cascade Structure .................................................... 152
6.7 Comparison of $\sigma_{acc}^2$ for Different ROSC based TRNG Designs .......... 158
6.8 Summary of Different ROSC based TRNG Designs ......................... 159
6.9 Area and Power Consumptions for ROSC based TRNG Components .... 159
6.10 Cost for Different ROSC based TRNG Designs ............................ 159
List of Figures

2.1 Silicon MUX physical unclonable function. ............................. 13
2.2 Feed-forward silicon MUX PUF structure. ................................. 13
2.3 Reconfigurable challenge and response PUF structure. .................. 20
2.4 Challenge XOR PUF structure. ........................................... 21
2.5 PUF structure of using LFSR to configure the challenge. .................. 22
2.6 Two parallel MUX PUF structure. .......................................... 24
2.7 MUX PUF structure of output recombination. ............................... 25
2.8 Feed-forward MUX PUF overlap structure. ................................. 27
2.9 Feed-forward MUX PUF cascade structure. ................................. 27
2.10 Feed-forward MUX PUF separate structure. .............................. 27
2.11 Logic-reconfigurable feed-forward MUX PUF. ............................ 28
2.12 MUX and DeMUX PUF structure. .......................................... 29
3.1 Modified feed-forward MUX PUF structure. ............................... 37
3.2 Modified feed-forward MUX PUF overlap structure. ....................... 38
3.3 Modified feed-forward MUX PUF cascade structure. ...................... 38
3.4 Modified feed-forward MUX PUF separate structure. ..................... 38
3.5 Scatter plot of outputs. ................................................... 44
3.6 Gaussian fit curve of delay difference distribution. ....................... 44
3.7 Gaussian fit curve of inter-chip variation distribution. ................... 49
3.8 Stage variation probability of the ending stage of the second feed-forward arbiter. 61
5.7 An obfuscated structure, which can be configurable as a 3rd-order IIR filter shown in either Figure 5.1 or Figure 5.4. .......................... 101
5.8 Switch implementation. ............................................. 101
5.9 The complete reconfigurable switch design. .......................... 103
5.10 Configuration key containing an initialization key and a configure data. ........ 103
5.11 (a) A DSP data-flow graph containing $M$ cascaded stages. Block Alg.$^i$ represents $i$-th stage of the cascade. (b) A folded architecture where $M$ stages are folded to same hardware. $D_F$ represents the number of folded delays. ............... 106
5.12 Relationship between the obfuscated design and the original design. ........ 110
5.13 Architecture of the proposed obfuscated DSP circuit. .......................... 111
5.14 Key encoding. .......................................................... 114
5.15 The obfuscated design of the original 18th-order IIR filter. ...................... 117
5.16 Normalized power consumption for different meaningful modes (%) of the (3$l$)th-order IIR filter benchmark (normalized to the power consumption when considering $l$ is a variable). ........................................ 120
5.17 Normalized area and power cost for different numbers of meaningful modes. . 121
5.18 Normalized area and power cost for different numbers of non-meaningful modes. 122
5.19 State registers sharing. ..................................................... 124
6.1 Two-oscillator TRNG. ...................................................... 130
6.2 BFD-TRNG: (a) basic principle, (b) die microphotograph in 65nm. .............. 131
6.3 Counter value distribution ($\Delta \mu = 0.28\%, \sigma = 0.0006$). ......................... 134
6.4 Counter value distribution ($\Delta \mu = 0.4\%, \sigma = 0.0006$). ......................... 135
6.5 Counter value distribution ($\Delta \mu = 0.28\%, \sigma = 0.0012$). ......................... 137
6.6 The relationship between the dynamic range of counter values and $\Delta \mu$ ($\sigma = 0.0006\%$). .............................................. 138
6.7 The relationship between the dynamic range of counter values and $\sigma$ ($\Delta \mu = 0.28\%$). 138
6.8 Counter value distribution ($\Delta \mu = 0.391\%$ and $\sigma = 0.0009$). ................. 140
6.9 The biasedness of $b_3$ for different counter values. ............................... 141

xvi
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.10</td>
<td>The relation between the number of unbiased bits and the value of $\sigma_G$.</td>
<td>143</td>
</tr>
<tr>
<td>6.11</td>
<td>Values of Equations (6.13) and (6.14) for different $\Delta \mu$’s ($\sigma = 0.0006$).</td>
<td>144</td>
</tr>
<tr>
<td>6.12</td>
<td>An M-parallel TRNG structure.</td>
<td>149</td>
</tr>
<tr>
<td>6.13</td>
<td>A cascade TRNG structure.</td>
<td>150</td>
</tr>
<tr>
<td>6.14</td>
<td>Counter values of the original BFD-TRNG (top figure) and the cascade structure (bottom figure).</td>
<td>150</td>
</tr>
<tr>
<td>6.15</td>
<td>A 4-parallel-cascade structure.</td>
<td>152</td>
</tr>
<tr>
<td>6.16</td>
<td>ROSC TRNG with XOR tree.</td>
<td>155</td>
</tr>
<tr>
<td>6.17</td>
<td>Enhanced ROSC TRNG with XOR tree.</td>
<td>156</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Introduction

Hardware security - whether for attack or defense - differs from software, network, and data security in that attackers may find ways to physically tamper with devices without leaving a trace, misleading the user to believe that the hardware device is authentic and trustworthy. The fact that computing devices are becoming distributed, unsupervised, and physically exposed, obviously aggravates the problem of hardware security. General speaking, hardware security has emerged as an important field of study aimed at addressing critical hardware-based security challenges, which include:

a) protect intellectual property from piracy,

b) prevent unauthorized access,

c) protect secrets from being stolen,

d) prevent fraud.

However, hardware security is very challenging. Due to the fact that adversary can physically tamper with the device, there are massive, various types of post-silicon attacking methods available, which include side-channel attacks, software attacks, fault
generation, microprobing, reverse engineering, and so forth. Especially, invasive and semi-invasive attacking methods provide more tools for the adversary to compromise the devices. For example, the side-channel attacks, information is gained from the hardware implementation rather than from the weakness of certain algorithm. In order to design secure ICs, we need to take all of these attacking methods into considerations. Furthermore, the advent of new attack modes, illegal recycling, and hard-to-detect Trojans make hardware protection an increasingly challenging task.

Moreover, industry business model has shifted from vertical to horizontal, as IC design and fabrication process becomes globalized. With the increasing complexity and cost of modern design and fabrication, many designs involves a number of Intellectual Property (IP) resellers, system on a chip (SOC) design houses, and fabrication houses that span multiple countries. Many critical hardware components are manufactured in foundries that are located in countries where the cost to build and run a foundry is competitive. ICs that are outsourced for fabrication are especially vulnerable to piracy from overproduction. For example, it is conceivable that a dishonest manufacturing plant could create more chips than ordered and sell the additional chips at a lower cost, subverting the profits of the legitimate owner.

There is no static definition of "security" in hardware. The security is dependent on the potential profit that an adversary can earn and the corresponding cost to compromise a device. Nothing is 100% secure. Given enough time, motivation, resources, an attacker can break any system. Therefore, what we can do to improve the security is essentially to increase the cost for an adversary. So the question is: what we can do to increase the attacking cost for the adversary?

The design of secure hardware ICs requires novel approaches for authentication that are ideally based on multiple factors that is difficult to compromise. Equally important is the need for protecting intellectual property and design of integrated circuits that are harder to reverse engineer. In general, hardware protection methods can be broadly classified as authentication-based approaches and obfuscation-based approaches. Another
important field of hardware security is *True Random Number Generator (TRNG)*, which is a critical building block in systems that require high levels of security as it provides a unique and unpredictable sequence of bits for encrypting messages or providing secrets for other cryptographic applications.

## 1.2 Summary of Contributions

### 1.2.1 Authentication

Traditionally, secret keys, which are used as unique identifiers for devices, are embedded into ICs in a ROM immediately after manufacturing process. Unfortunately, digital keys stored in a non-volatile memory are vulnerable to physical attacks. Several invasive and semi-invasive physical tampering methods have been developed; these include techniques such as reverse engineering, micro-probing (access to the silicon to manipulate the internals of system), and power analysis (predict the secret keys from power consumption analysis). These approaches have made it possible to learn the ROM-based keys through attacks and compromise systems by using the keys from the counterfeit copies. The fact that the devices should be inexpensive, mobile, and cross-linked obviously aggravates the problem.

Physical Unclonable Function (PUF) is one emerging security primitive, which has successfully addressed the problems faced by traditional ROM-based authentication techniques. Contrary to standard digital systems, PUFs extract secrets from complex properties of a physical material rather than storing them in a non-volatile memory. PUFs are compact circuits that exploit inherent manufacturing process variations to generate an output response that is unique to each chip. Analogous to human *fingerprint*, PUFs are powerful tools for authentication and cryptographic applications.

Given the advantages of PUFs, we are interested in how we can enhance the security and reliability. A good PUF should be nearly impossible to predict, clone, or duplicate. When a PUF is provided with an input (or challenge), the output (or response) should
satisfy the following three properties: (i) unique output due to chip-to-chip variation, (ii) random output that is impossible or difficult to model, and (iii) reliable output that is consistent across temperature, voltage and aging conditions. In this dissertation, I explore the multiplexer (MUX)-based PUFs in depth - both the practical implications and the theoretical underpinnings.

On the practical front, we propose several novel reconfigurable PUF architectures, where the challenge-response pairs are updatable. It is shown that these novel reconfigurable PUFs can embed more non-linearity into the challenge-response mapping functions and lead to more secure PUFs without degrading reliability and robustness. We also develop a novel modified feed-forward PUF structure to improve the reliability of PUFs, while maintaining the same level of security. Unlike the standard feed-forward PUF, the output of a feed-forward arbiter from an intermediate stage is input as the challenge bit to two consecutive MUX stages in the modified feed-forward PUF structure. The non-linearity provided by multiple feed-forward paths in our design makes it difficult for attackers to predict the PUF behavior. PUF chips are fabricated in 32nm SOI IBM process that can be reconfigured as simple MUX PUF, feed-forward MUX PUF, and modified feed-forward MUX PUF. Our experimental results show that using the proposed modified feed-forward path can achieve significantly better reliability than using standard feed-forward path. Moreover, another contribution is the design of a novel two-arbiter PUF structure. Redundancy is introduced to the response to improve the reliability. These proposed PUF designs are extremely suitable for applications that mandate lightweight and cost-effective hardware with very secure and reliable authentication.

On the theoretical front, we present a systematic statistical analysis to quantitatively evaluate the performances of various MUX-based PUFs for the first time in the literature. The proposed statistical analysis approach is conceptually very general. The statistical analysis results can be used to predict the relative advantages of various MUX-based PUF designs. These results can be used by the designer to choose a proper
type of PUF or appropriate design parameters for a certain PUF based on the requirements of a specific application. This eliminates the need for fabrication and testing of many PUFs for selecting an appropriate PUF.

We also propose a novel lightweight PUF-based local authentication scheme. One major issue for PUF-based local authentication is the robustness of the system, since environmental variations (e.g., temperature, voltage and aging variations) will affect the PUF response. In a server-based remote authentication system, this issue can be easily resolved by tolerating certain number of bit errors. For example, the server could authenticate a PUF response whose Hamming distance (HD) to the desired response is less than a certain threshold. However, this cannot be used in a local authentication scheme due to the high overhead. Additionally, it is not feasible to store a large amount of CRPs on chip. Error correcting codes such as BCH codes and fuzzy extractors have been incorporated into PUF-based authentication protocols to improve their robustness, which could also be used in local authentication. However, the use of error correcting techniques significantly increases the cost and design complexity. The proposed novel lightweight finite-state machine (FSM) based local authentication scheme could overcome these problems. The advantage is the inherent redundancy built into the self-correcting FSM by contiguously entering the authentication key twice that eliminates the need for an error correcting code.

1.2.2 Obfuscation

Digital Signal Processing (DSP) plays a critical role in numerous applications such as video compression, portable systems/computers, multimedia, wired and wireless communications, speech processing, and biomedical signal processing. However, the security aspect for DSP applications has only attracted little attention in the literature. While PUFs can be used as authentication-based methods to improve the security of DSP circuits, obfuscation-based approaches are also obliged to protect the intellectual property. Design obfuscation is a technique that transforms an application or a design into one
that is functionally equivalent to the original but is significantly more difficult to reverse engineer.

In this dissertation, we propose a novel design methodology for obfuscated DSP circuits by hiding functionality via high-level transformations. The DSP circuits are obfuscated by introducing a finite-state machine (FSM) whose state is controlled by a key. The FSM enables a reconfigurator that configures the functionality mode of the DSP circuit. High-level transformations lead to many equivalent circuits and all these create ambiguity in the structural level. High-level transformations also allow design of circuits using same datapath but different control-flows. Different variation modes can be inserted into the DSP circuits for obfuscation. While some modes generate outputs that are functionally incorrect, these may represent correct outputs under different situations, since the output is meaningful from a signal processing point of view. Other modes would lead to non-meaningful outputs. The initialization key and the configure data must be known for the circuit to work properly. Consequently, the proposed design methodology generates a both structurally and functionally obfuscated DSP circuit. While high-level transformations have been exploited for area-speed-power tradeoffs, our work is the first work to exploit the security perspective of high-level transformations.

1.2.3 True Random Number Generator

Another important field of hardware security is True Random Number Generator (TRNG), which is a critical building block in systems that require high levels of security as it provides a unique and unpredictable sequence of bits for encrypting messages or providing secrets (e.g., the challenges for a PUF) for cryptographic applications. Similar to PUFs, TRNGs also extract randomness from physical phenomena. However, manufacturing process variations are key to generating unique signatures in PUFs, while environmental variations are key to creating truly random outputs in TRNGs.

Evaluating TRNGs is a difficult task. Clearly, it should not be limited to testing the TRNG output bitstream. One important requirement in TRNG security evaluation is
the existence of a mathematical model of the physical noise source and the statistical properties of the digitized noise derived from it. However, creating a model of a TRNG is difficult as the model parameters are unknown. Thus, it is impossible to predict performance of new TRNG designs as their models cannot be created. Furthermore, it can be argued that TRNG performance can only be measured from fabricated chips. Therefore, how good a new TRNG design can only be determined by measurements from a fabricated design. We are interested in exploiting the synergy between a model and the measurements of the real device. We present a rigorous analysis of the so-called beat-frequency detector TRNG (BFD-TRNG). The key contribution of the proposed approach lies in fitting the model to measured data, and the ability to use the model to predict performance of BFD-TRNGs that have not been fabricated. Motivated by the statistical analysis results, we propose several novel BFD-TRNG architectures, which could achieve further improved performances.

1.3 Outline of the Dissertation

The dissertation is outlined as follows. Physical Unclonable Function (PUF) is introduced in Chapter 2. Afterwards, we introduce several novel reconfigurable PUF structures which could achieve better security. The key idea in our method is that the challenge-response pairs can be updatable by pre-processing the challenge and response, or altering the PUF circuit, without leaking security information to the adversary. Then the methodology to simulate PUFs is described.

Chapter 3 introduces the design of a novel modified feed-forward PUF structure to improve the reliability of PUFs, while maintaining the same level of security. Then a systematic statistical analysis of various MUX-based PUFs is presented.

Chapter 4 introduces the idea of PUF-based local authentication. Afterwards, a two-level finite-state machine (FSM) is proposed to correct erroneous bits generated by environmental variations.
In Chapter 5, we introduce the concept of hardware obfuscation. Afterwards, a novel design methodology for obfuscated DSP circuits by hiding functionality via high-level transformations is presented.

Chapter 6 introduces the a novel BFD-TRNG design. The statistical modeling of the BFD-TRNG is discussed.

Finally, Chapter 7 concludes with a summary of total contributions of this dissertation and future research directions.
Chapter 2

Novel Reconfigurable Silicon Physical Unclonable Functions

2.1 Introduction

In today’s world, as electronic devices become increasingly interconnected and pervasive in people’s lives, security, trustworthy computing, and privacy protection have emerged over the past decade as hardware design objectives of great significance. It is demanded that semiconductor devices be resistant not only to computational attacks, but also to physical attacks. Traditionally, secret keys, which are used as unique identifiers, are embedded into integrated circuits (ICs) in a ROM immediately after manufacturing. Unfortunately, digital keys stored in a non-volatile memory are vulnerable to physical attacks. Several invasive and semi-invasive physical tampering methods have been developed; these include techniques such as micro-probing (access to the silicon to manipulate the internals of system), power analysis (predict the secret keys from power consumption analysis) and so forth. These approaches have made it possible to
learn the ROM-based keys through attacks and compromise systems by using counterfeit copies of the secret information. The fact that the devices should be inexpensive, mobile, and cross-linked obviously aggravates the problem.

The described problem has become more intense recently, and this motivated the idea of using intrinsic random features of physical objects for identification and authentication. The concept of physical unclonable function (PUF) proposed in [2, 3, 4] has successfully addressed the problems faced by traditional techniques. A PUF is a function that exploits the unique intrinsic uncontrollable physical features by process variations during manufacturing. Signatures generated by PUFs are determined by device manufacturing variations and the so-called external challenges. Contrary to standard digital systems, physical unclonable functions enable significantly higher secure authentication by extracting secrets from complex properties of a physical material rather than storing them in non-volatile memory. Due to the uncontrollable random components, PUFs are easy to measure but almost impossible to clone, predict, or reproduce. Furthermore, it is infeasible for an adversary to mount an attack to counterfeit the secret information without changing the physical randomness. Taking coating PUF as an example, which is a function built in the top layer of an IC by filling the space between and above the comb structure with an opaque material and randomly doping with dielectric particles, any physical attack on a coating PUF would damage the protective coating and destroy the cryptographic key. Based on these advantages, PUFs can efficiently and reliably generate volatile secret keys for cryptographic operations and enable low-cost authentication of ICs.

The first PUF in the literature is the optical PUF [2], which utilizes the randomness in the placement of the light scattering particles and the complexity of the interaction between the laser and the particles. After that, several PUF hardware structures have been proposed [3, 4, 5, 6, 7]. Most PUFs are built on conventional silicon techniques so that they do not require any special fabrication and can be easily integrated into IC chips, except a few types such as coating PUF and magnetic PUF. Among these
PUFs, silicon PUFs are of the most interest, as these exploit manufacturing variability of nanoscale silicon structure to generate a unique challenge-response mapping for each IC. These unique properties of each IC are easy to measure through the circuits but hard to copy without changing the challenge-response pairs (CRPs).

The delay-based silicon PUFs in previous work have always considered a static challenge-response behavior. In those protocols, PUFs should always generate the same or error tolerated response to a randomly selected challenge. Unfortunately, recent analysis has demonstrated that those PUF structures are vulnerable to several security attacks including emulation, replay (man-in-the-middle attack), reverse engineering and modeling attack [8]. Moreover, updatable cryptographic keys are very attractive in some applications [9]. Therefore, a dynamic PUF that can alter the CRPs every time the data is modified to prevent the hidden information leaked out is very desirable.

In this chapter, we mainly focus on the design of reconfigurable silicon PUFs and their analysis. We propose several novel reconfigurable PUFs and discuss their performance. We also examine the reliability and the security of different PUF structures. The key idea in our approach is that we try to make CRPs updatable. By doing this, the challenge-response behavior of a PUF can be altered to generate a more secure hardware system. Furthermore, we discuss the techniques to improve the reliability of silicon PUFs.

2.2 Background

2.2.1 Silicon Physical Unclonable Function

There are several subtypes of PUFs, each with its own applications and security features. A major type is the so-called silicon PUFs, which exploit the delay variations of CMOS logic components to generate a unique signature for each IC. Silicon PUFs can be integrated into chips very conveniently since there are implemented with standard digital logic and without any special fabrication. There are two main types of delay-based
silicon PUFs: Ring Oscillator (RO) PUF [3] and Multiplexor (MUX) PUF [10]. However, MUX PUF has better performance than RO PUF from the security perspective, as the frequencies of the ring oscillators can be relatively easily evaluated by attackers; moreover, a MUX PUF is more suitable for resource-constrained applications. Instead of duplicating the hardware N times, we can use N different challenges to obtain a N-bit long response in a MUX PUF, as illustrated in Figure 2.1. Each challenge creates two paths through the circuit that are excited simultaneously. The output is generated by the delay difference between the two paths. The silicon MUX PUF consists of N stages MUXs and one arbiter which connects the final stage of the two paths. MUXs in each stage act as a switch to either cross or straight propagate the rising edge signals, based on the corresponding challenge bit. Each MUX should be designed equivalently, while the variations will be introduced only during the manufacturing process. For transistors, manufacturing randomness exists due to variations in transistor length, width, gate oxide thickness, doping concentration density, metal width, metal thickness, and ILD (interlevel dielectric) thickness, etc [11]. These manufacturing variations show a significant amount of variability, which are sufficient to generate unique challenge-response pairs for each IC by comparing the delays of two paths. Finally, the arbiter (always simply a D flip-flop) translates the analog timing difference into a digital value. For instance, if the rising edge signal arrives at the above input of the arbiter earlier than the signal arriving at the bottom input, the output will be one, otherwise if the bottom path is fast, the output will be zero otherwise. The output response depends on the applied challenge bits and will be permanent for each IC or only vary in a small range under different environmental conditions.

2.2.2 Feed-Forward Structure

Later on, in order to improving the security of silicon PUF, a feed-forward structure has been proposed in [12] to prevent attacks by linear modeling. Figure 2.2 shows one
basic structure of feed-forward MUX PUF, which uses the racing result of an intermediate stage as the select signal for a block of MUXs in a later stage. This structure provides nonlinearity to the arbiter-PUF, which increases the complexity for numerical modeling attacks. However, the reliability of the PUF has been degraded in this feed-forward structure since an error in the output of an internal feed-forward arbiter caused by environmental variation can increase the noise probability in the final response.

**Figure 2.2: Feed-forward silicon MUX PUF structure.**

### 2.2.3 Reconfigurable PUF

Recently, in order to satisfy the needs of updatable keys for PUF-based authentication systems, reconfigurable PUFs have emerged as a new class of PUFs. A good example is the protection of sensitive data in untrusted non-volatile storage. The confidentiality and integrity of such data can be protected with an encryption and authentication algorithm respectively, but a refresh mechanism is needed to protect the system from the replay of old versions of the data. One possible solution is to alter the key
material of the protection scheme every time that data is modified, which leads to the
use of reconfigurable PUFs. A reconfigurable PUF has the properties that it could
preserve the properties of original PUF but have unpredictable challenge-response be-
haviors after every reconfiguration. The first reconfigurable PUF [10] was presented in
2004, which is an arbiter PUF built with floating gate transistors. It was also shown
that the reconfigurability for a PUF is a very desirable characteristic.

Recently, two types of reconfigurable PUFs have been proposed in [13]: reconfig-
urable optical PUF and phase change memory based reconfigurable PUF. These two
types of PUF can be updated and can inherit the properties of the original PUF. How-
ever, these PUFs have certain limitations and constraints in practical use; for instance,
the reconfigurable optical PUF needs a special material, which is not as widely used as
silicon PUFs in industry.

Several implementations of reconfigurable silicon PUFs have also been published [9,
14, 15, 16]. However, all of these efforts are based on the reconfigurability of FPGA,
and most of them are built on the Ring Oscillator PUF. In these existing FPGA-based
solutions, many assumptions are required as significant amount of information regarding
the underlying VLSI layout of the reconfigurable fabric is lost. Moreover, many PUF
designs require a symmetrical routing that is difficult to implement on a FPGA platform
as a designer can only manipulate the higher level design blocks such as the LUTs, the
memory blocks, and the connection matrices. Additionally, it is possible to undo the
reconfigurations of contemporary FPGAs, which makes the reconfigurable PUF more
vulnerable to attacks.

Therefore, non-FPGA based architectures for reconfigurable silicon PUF are very
desirable. Our major contribution is to present several novel structures of silicon PUFs
which can be implemented on non-reconfigurable hardware, that is which can be de-
signed at transistor level and fabricated and integrated into chips. Moreover, we also
address the security and the reliability perspectives of the reconfigurable PUFs.
2.2.4 PUF-based Key Generation and Authentication

Due to its complex and disordered structure, a PUF can avoid some of the shortcomings associated with digital keys. For example, it is usually harder to read out, predict, or derive its responses than to obtain the values of digital keys stored in non-volatile memory. This fact has been exploited for various PUF-based security protocols. Prominent examples include schemes for identification and authentication [2, 3], key exchange or digital rights management purposes [4]. PUFs are often assumed to be evaluated on a challenge $c$ which is sent to the device. Upon receiving $c$, the response is computed as $r = PUF(c)$ and is assumed to be unpredictable to anyone without access to the device. Schemes exist for use in different contexts (e.g., for protection of intellectual property and authentication), where the inability to clone the function improves the properties of a solution.

Although PUF-based authentication system is an efficient and powerful solution for counterfeit IC prevention, this technique also suffers from several security drawbacks. In a PUF-based authentication system, the challenge-response pairs of an authentic IC are stored in the database of a trusted party. To check the authenticity of an IC later, the chip needs to communicate with the trusted party, thus revealing a security loophole to the adversary. Then the system can be compromised by some modeling attack method when multiple CRPs are obtained by attackers. One possible solution that still hasn’t been investigated is a novel PUF-based self-authenticated system, which can avoid communications between the chip and server. As only one challenge-response pairs will be needed in this kind of authentication protocol, therefore, the system would be resistant to software model building attacks.

From another point of view, if the adversary is unable to create the model of the underlying PUF or successfully predict the response for a challenge even with a large number of CRPs, the authentication protocol could also remain strong and secure. Reconfigurable PUFs could be a suitable solution for this problem. In these reconfigurable
PUF-based systems, the response will be computed as \( r = PUF_d(c) \), where \( d \) is the configure data. Therefore, the PUF model will be hard to model, as the PUF mapping function is not deterministic but varies according to the configure data. Furthermore, the reconfigurability also enables systems to use multiple CRPs for authentication, which can increase the reliability and security.

### 2.3 Methodology

#### 2.3.1 PUF Model

As shown in Figure 2.1, a MUX PUF consists of a sequence of N-stage MUXs and an arbiter. The rising edge signal will excite the two parallel paths simultaneously. The actual propagated paths will be determined by the external applied challenge bits. After the last stage, the arbiter will generate the output bit by comparing the arrival time of the two different paths. It has become standard to model the MUX PUF via an additive linear delay model. According to the efforts in the field of Statistical Static Timing Analysis (SSTA) [11], the manufacturing process parameter variations for transistors can be modeled by a Gaussian distribution. Therefore, the variations of delay will be also approximately Gaussian.

Process variations can be classified as follows: inter-die variations are the variations form die to die, while intra-die variations correspond to variability within a single chip. Inter-die variations affect all the devices on the same chip similarly, while intra-die variations affect different devices differently on the same chip. A very widely used model for delay spatial correlation is the "Grid model" [11], which assume perfect correlations among the devices in the same grid, high correlations among those in nearby grids and low or zero correlations in faraway grids, since devices close to each other are more likely to have more similar characteristics than those placed far away.

Additionally, experimental results have already shown that the inter-chip variation across the wafers is similar to that within a single wafer. Thus, inter-chip variation is not
strongly dependent on the location of dies on wafers. Moreover, the output of the arbiter in silicon PUF is only based on the difference of two selected paths. Therefore, these die-to-die, wafer-to-wafer, and lot-to-lot manufacturing variations will have minimum effect on the output response of the silicon PUF.

For simplicity, as every MUX is design equivalently, we can model the delay of each single MUX an i.i.d. random variable $D_i$, which follows $N(\mu, \sigma^2)$; therefore, the total delay of the N stages will be $N(N\mu, N\sigma^2)$. Since the output of arbiter only depends on the delay difference between the two paths, thus the time difference will also follow a Gaussian distribution $\Delta \sim N(0, 2N\sigma^2)$.

We denote the delay in the top path of the i-th stage as $Dt_i$, the delay in the bottom path of the i-th stage as $Db_i$, and the challenge bit for each stage as $C_i$. Thus the delay difference of the i-th stage will be:

$$Dt_i - Db_i \sim N(0, 2\sigma^2)$$

Then if the challenge is 0, then the delay difference added into the whole path will be $Dt_i - Db_i$, otherwise, if the challenge bit for the i-th stage is 1, the additive delay difference will be $Db_i - Dt_i$. It can be expressed as:

$$\Delta_i = (-1)^{C_i}(Dt_i - Db_i) \sim N(0, 2\sigma^2)$$

As a result, the final arrival time difference into the arbiter is:

$$\Delta_z = \sum_{i=1}^{N} (-1)^{C_i}(Dt_i - Db_i) \sim N(0, 2N\sigma^2)$$

Thus, the final output bit is:

$$r = sign(\Delta_z)$$

where we make technical convention of saying that $r = sign(a) = 0$ when $a < 0$, and $r = sign(a) = 1$ when $a \geq 0$. 
2.3.2 Simulation Model

In our experiment, we use simulation method to test and analyze the efficiency of PUFs instead of real fabrication in our experiment. There are several advantages of using simulation method: First, fabrication is expensive. Second, a good simulation method can be used as a pre-fabrication test, which can predict the performance of a new PUF design. Moreover, we can analyze all the possible performance and characteristics of the PUFs by setting different environmental conditions. Additionally, it is also convenient to follow the shrinking of technology scale.

In our simulation, we apply the Gaussian model which has already been described above for manufacturing process variations. We set up the process parameters and their max percentages of deviations based on the predictions from [17, 18]. For spatial correlation, we assume perfect spatial correlation in one MUX. The process variations will have the same effect on the PMOS and NMOS devices in each MUX, while the parameters among the different stages of MUXs have no correlation.

In our simulation result, the total delay deviation of 100 stages is \( \leq \pm 0.4\% \). Since

\[
\frac{\sigma_z}{\mu_z} = \sqrt{\frac{1}{N}} \left( \frac{\sigma}{\mu} \right)
\]

and \( \mu_z \) increases linearly with \( N \), we can conclude that our result conforms with other published result of 65nm technology, according to the experimental results in [9] that \( 3\sigma/\mu \approx 5\% \) for a single stage of MUXs. Furthermore, our simulation result of inter-chip variation a Hamming distance from 22 to 59 bits for a total of 100 stages, while the intra-chip variation is 5.8 bits on average, with a maximum value 13 bits. These results are also in agreement with published results for fabricated chips. Thus, we believe that our simulation delay model is consistent with the industrial manufacturing process variations.
2.4 Novel Reconfigurable PUFs

In order to add reconfigurable property into general MUX based silicon PUFs, we must make the challenge-response pairs (CRPs) reconfigurable, which can be used to update the identification database. The methods can be classified into two categories:

(a) Make the challenge-response pairs reconfigurable directly, by adding some extra circuits into the structure, but without configuring the main PUF circuit. This can be achieved by utilizing some techniques to pre-process the challenge before applying to PUF or pre-process the response before using it for authentication.

(b) Make the PUF circuit reconfigurable, therefore the challenge-response pairs will be reconfigurable as well.

We propose several novel non-FPGA reconfigurable PUFs implementations for the above two categories, which would be more suitable for practical use comparing with FPGA-based techniques. Furthermore, we address the reliability and the security of the PUF performance, as some information of the hidden secrets that an adversary can take advantage may leak out during reconfigurations.

2.4.1 Reconfigurable Challenge and/or Response Structure

The reconfigurable structures of PUFs are built on the prior work in Physical Unclonable Function, which can also be applied to various types of silicon PUFs as well as other challenge-response based PUFs. Our goal is to develop a reconfigurable PUF which is a PUF with a mechanism to transform it into a new PUF with a new unpredictable and uncontrollable challenge-response behavior, even if the challenge-response behavior of the original PUF is already known. Additionally, the new PUF inherits all the security properties of the original one.

An early reconfigurable design PUF [10] in the literature treated some challenge bits as configure data. As an example, the last 10 bits of a 100-bit challenge can be
fixed as configure data, leaving only 90 bits for actual challenge. When a user wants to
update the CRPs, just apply another 10-bit stream to the last 10 stages of the PUF.
However, it is very clear that the reconfigured PUF will be have high correlation and will
be vulnerable to attacks, as this method is similar to adding a certain time difference
between the two paths or introducing an interval between the two rising edge signals.
Even worse, the performance of the PUF will be greatly degraded, if the cumulative
variations in the last 10 stages are relatively large. Due to these disadvantages, this
architecture of reconfigurable PUFs cannot generate unpredictable challenge-response
behaviors.

Intuitively, adding reconfigurable elements before the challenges are applied to the
PUF can definitely make the PUF reconfigurable. At the same time, the performance
of original PUF will be preserved. The main structure of this type of reconfigurable
PUF is shown in Figure 2.3.

![Figure 2.3: Reconfigurable challenge and response PUF structure.](image)

**Challenge XOR and LFSR**

We start from some very simple implementation of reconfigurable circuits to examine
the properties of MUX-based silicon PUF. Pre-process the challenge bits before applying
to the MUXs by exclusive-or operations with one certain bit stream is a very simple
method to reconfigure the challenge, as in Figure 2.4. To reconfigure this circuit, we
can just XOR the challenge bits with a different bit stream. This is similar to applying a different challenge to the MUXs; thus, we can expect that the characteristics and performance will remain the same as the original MUX PUF.

![Figure 2.4: Challenge XOR PUF structure.](image)

However, this structure will not be reliable, since the correlation would be extremely high between the reconfigured challenge-response pairs. We can adopt other more complex and secure circuits as the reconfigurable element, such as reconfigurable linear feedback shift register (LFSR). Such a structure is shown in Figure 2.5. LFSR is an important part of sequence cipher and be used to generate pseudo-random key stream. In past years, several designs of reconfigurable linear feedback shift register have been published in [19, 20]. A designer can apply various random patterns generated using different seeds to the IC, or alter the generating polynomial. Furthermore, the length of signatures can also be changed by applying different patterns. Such capability makes it extremely difficult for adversaries to obtain PUF signature. It is important to point out that we can improve the security of the PUF system, by benefiting from the property of the LFSR in cryptography.
Hash Function

The purpose of hash function is to produce a "fingerprint" of a file message, or other block of data. A cryptographic hash function is a deterministic procedure that takes an arbitrary block of data and returns a fixed-size bit string, the (cryptographic) hash value, such that an accidental or intentional change to the data will change the hash value. The data to be encoded is often called the "message", and the hash value is sometimes called the message digest or simply digest. Hash function is a kind of "one-way" function, which means it is easy to compute the hash value for any given message. However, it is infeasible to find a message that has a given hash, and it is infeasible to find two different messages with the same hash.

Due to the security property of hash function, we can employ a hash function as the reconfigurable element in reconfigurable PUF. This structure can be reconfigured very easily, such as by adding several different lengths of 0's at the end of every challenge. Additionally, the security of PUF can be increased, due to the "one-way" property of hash function. Many hash algorithms have been investigated and developed in the last years. Currently, the SHA-1 algorithm is the National institute of Standards and
technology (NIST) secure hash standard. Several hash function unit architectures have been published in past years [21].

This structure has already been named as Control Physical Unclonable Function in [22], which was described as using a strong PUF as a building block, but adding control logic to prevent challenges from being applied freely to the PUF and hinder direct read out of the response. Instead of performing a simple hash before the challenges are applied to the PUF, we can consider adding another control logic, which would make the CRPs updatable. We propose several reconfiguration methods:

(a) Adding different bit streams into the challenges, e.g., adding different numbers of 0’s at the end of the challenges.

(b) Reordering the challenge stream by certain rules.

(c) Reconfiguring the hash function, by using the reconfigurability of these reconfigurable Hash Function implementations.

Due to the property of hash function, it is extremely hard for adversary to model the PUF, even after we configure it several times, as each output of the hash function is unpredictable.

**Output Recombination**

Another idea is to add extra reconfigurable element to pre-process the output of the arbiter before using it as an authentication key. One simple example is to use two parallel MUX PUFs to update the CRPs, as shown in Figure 2.6. In this case, 4 paths are selected by challenges through which the signal (rising edge) will propagate. Then we can select two of the four paths using the configuration data and forward to the arbiter to generate the response. We will have total 12 possible combinations if we use a 2 level parallel structure. Therefore, we can reconfigurable this architecture 12 times. However, there will be very high correlation between some of these 12 different
possible combinations. For example, if we know that path 1 is faster than path 2, and path 2 is faster than path 3, we can know definitely that path 1 will be faster than path 3. Therefore, there should be some constraint for the reconfiguration, and the total possible reconfigurations will decrease. Actually, there are $N!$ possible cases for ordering $N$ paths based on their arrival time. Therefore, $\log_2(N!)$ independent bits can be produced by $N$ paths.

![Figure 2.6: Two parallel MUX PUF structure.](image)

However, there will be a problem by employing this structure, since the pre-processing component after the last stage also has variations, which will affect the performance of the PUF. To solve this problem, we can pre-process the data after the arbiters, as in structure of Figure 2.7.

If we use $N$ MUX-based PUF circuits, we will need $2N-1$ arbiters, while we only compare the neighbor paths. This is a concept borrowed from ring oscillator PUF which could ensure there will be no correlation between the output bits of the arbiters, as the comparison pairs are non-cyclic. If we want to achieve the bits’ entropy limit, we need to choose the output comparison pairs adaptively, which would increase the design complexity and fabrication area significantly.
Figure 2.7: MUX PUF structure of output recombination.
2.4.2 Reconfigurable Circuit Structure

Instead of only making the CRPs reconfigurable by processing the challenge and response directly, we can reconfigure the main circuit to update the challenge-response behavior. This kind of reconfigurable PUFs will have better performance from security perspective, since after reconfiguration, it leads to a different PUF circuit, while the previous method only changes the CRPs.

The most important thing in these structures is that, we must ensure the extra circuit will not affect the PUF performance, or more generally, the extra circuit will have identical effect on the delay of different paths statistically.

Reconfigure Feed-Forward PUF

It has been shown that the security of the MUX PUF in Figure 2.2 can be improved by adding feed-forward arbiters to it [12]. However, in previous literature, how to choose the feed-forward startpoints and endpoints, or how many paths (sometimes referred as feed-forward loop in the literature [8]) are chosen for feed-forward purpose have not been clearly presented. One constraint is trivial: the signal produced by the feed-forward arbiter should arrive earlier than the two signals propagating through the MUX paths. Therefore, we should ensure that the feed-forward path has at least 3-5 stages between the input and output of the feed-forward arbiter. We denote the stages from the input of a feed-forward arbiter to the output of the feed-forward arbiter as a feed-forward loop. We consider the following three feed-forward structures:

(a) Feed-forward Overlap (FFO): This structure has at least one stage overlap between two feed-forward paths.

(b) Feed-forward Cascade (FFC): In this structure, the endpoint of a feed-forward path will be the starting stage of another feed-forward path.
(c) Feed-forward Separate (FFS): In this case, the different feed-forward paths will be separated. Thus, there is no stage overlap between any two of the feed-forward paths.

Figure 2.8: Feed-forward MUX PUF overlap structure.

Figure 2.9: Feed-forward MUX PUF cascade structure.

Figure 2.10: Feed-forward MUX PUF separate structure.

In our experimental results, the intra-chip variations are increased by adding non-linearity to the circuits. Among the 3 different structures, the feed-forward cascade structure has the largest intra-chip variation, with 10.7 bit Hamming distance on average with response length of 100 bits, compared with 5.8 bits for non-feed-forward structure (see Section 2.5).
By utilizing this property, we propose the Reconfigurable Feed-forward MUX PUF structure. A basic example that combines overlap and separate approaches is shown in Figure 2.11. The structure can be configured among the 3 different structures ((a) overlap, (b) cascade, (c) separate), which will make the PUF model more complex. By configuring the PUF, the mathematical model for the PUF will be altered. This makes it infeasible for attackers to break the PUF by only using one single uniform linear model. The delay of MUXs connected after the feed-forward structure (normally just an arbiter) may also affect the delay difference of the two paths. However, this time difference could add into the total path delay difference both positively and negatively, depending on the select signal. Therefore, the effect of these MUXs would be statistically equivalent to the two paths of original MUX PUF, even if the delay of the added two MUXs vary quite significantly. From above, we conclude that the MUX based PUF will be more secure when feed-forward arbiter is reconfigurable.

![Figure 2.11: Logic-reconfigurable feed-forward MUX PUF.](image)

**DeMUX and MUX PUF**

The function of MUX is multiplexing; it selects one of many input signals and forwards the selected signal into a single line. The DeMUX is a device that takes a single input signal and distributes it over multiple output signals. Using DeMUX enables us to select the direction of the propagating signal, and makes the PUF reconfigurable.
The select signal applied to the DeMUX can also be regarded as the challenge, which would be similar to the structure of reconfigurable feed-forward PUF.

A basic reconfigurable structure is shown in Figure 2.12. Instead of propagating the rising edge signal successively, we can choose to skip some stages by adding DeMUX components, which could make the challenge-response behavior reconfigurable and hard to predict. This structure will be harder for attackers to model than the silicon PUF only based on MUX.

![MUX and DeMUX PUF structure.](image)

Figure 2.12: MUX and DeMUX PUF structure.

### 2.5 Experimental Results

All of our experiments are carried out using SPICE simulations on a 65-nm technology process. We use Monte Carlo method to simulate the effect of process variations and environmental variations. In our simulation, we set up the transistor parameters and process variations based on a major industrial standard model. Each proposed structure has been simulated over at least 20 Monte Carlo runs in SPICE. The simulated MUX based Physically Unclonable Functions all have 100 stages. Accordingly, we need to apply a 100-bit challenge to the PUF to produce a 1-bit response, and we apply 100 different challenges to generate the final 100-bit digital signature for each IC.

**Measuring inter-chip variations:** The inter-chip variations were determined by comparing the digital signature of each IC to each other and calculate the Hamming Distance between the two signatures. Since we had 20 chip instances, we had 20*19/2,
i.e., 190 possible digital signature comparisons. We use the maximum and the minimum of these numbers as measures of the inter-chip variation.

**Measuring intra-chip variations:** The intra-chip variations were determined by comparing the digital signal processing of the same IC under different environmental conditions. In our case, we use temperature as the primary factor. We also simulated the intra-chip variations under different voltages, from 1V to 1.2V. However, it is shown that the intra-chip variations introduced by different temperatures from 0°C to 100°C were more significant compared to the intra-chip variations caused by voltage variations. The digital signatures of the PUF at 0°C, 20°C, 40°C, 80°C, 100°C were obtained; however, we only present the comparisons of Hamming Distance between 0°C and 100°C, as those exhibit the largest variations. We simulated 10 different CRPs for each IC, and simulated 20 different IC instances. Therefore we have 200 comparisons in total. We provide the maximum and the average of the Hamming distances for the intra-chip variations.

**Simulation circuit structures:** As presented in the previous section, we simulated the digital signature for each proposed Physical Unclonable Function structure. We add 10 feed-forward arbiters for each feed-forward PUF. For instance, the feed-forward arbiters are from stage 1 to stage 11, from stage 11 to stage 21 ... from stage 91 to stage 100 in a feed-forward cascade structure. The feed-forward arbiters are from stage 1 to stage 7, from stage 11 to stage 17 ... from stage 91 to stage 97 in feed-forward overlap structure. In a feed-forward cascade structure, the feed-forward arbiters are from stage 1 to stage 51, from stage 6 to stage 56 ... from stage 46 to stage 96. For the reconfigurable feed-forward structure, we also add 10 such arbiters and MUX structures into the original PUF circuit, which can switch among the 3 different feed-forward structures. Moreover, we also simulated 10 DeMUX components in the MUX and DeMUX PUF, the inputs and the outputs of the DeMUXs are from stage 3 to stage 8, from stage 13 to stage 18 ... from stage 93 to stage 98. Finally, we simulated 20 parallel MUX PUFs for the output recombination structure. Therefore, we have 40
paths in total, and we derived the digital signature by comparing adjacent paths. In other words, the delay of the first path was compared with that of the second path, which is the same as a single PUF. The delay of the second path was compared with that of the third path, and so on. Finally, the delay of the 39th path was compared with that of the 40th path. Therefore, except for the first and the last paths, each path was compared to two other paths.

**Measuring reconfigurability:** We randomly choose the configure data of the different structures, and then fix the configure data when examining the inter-chip variations and the intra-chip variations. However, when we add the reconfigurable components into the circuits, the challenge-bit lengths are decreased; therefore, we need to adjust the challenge bits when simulating these reconfigurable structures. To test the reconfigurability, we simply fix the challenge bits, but change the configure data to compare the digital signatures. All the simulations were carried out under the environmental condition of 25°C and 1.1V.

Table 2.1 presents the inter-chip variations and intra-chip variations for different MUX Physical Unclonable Function structures. First, it can be observed that the minimum inter-chip variation is larger than the maximum intra-chip variation for all of the simulated structures. Thus, we can conclude that the variations caused by the randomness in manufacturing process are more significant than the variations under different environmental conditions. Therefore, these PUFs can be used as reliable secret keys with some error correcting techniques. Second, it can also be observed that by adding feed-forward arbiters into the MUX PUF circuit, the inter-chip variations and intra-chip variations are both increased, since the noise can have influence on the select signals of some intermediate stages. By comparing the inter-chip variations and the intra-chip variations, we conclude the feed-forward separate structure is the most reliable structure while the feed-forward cascade is the least reliable one among the 3 feed-forward structures. The reconfigurable feed-forward structure has very close performance to the 3 types of feed-forward structures, since its functionality is switching among the 3.
Moreover, the reconfigurable DeMUX and MUX PUF has similar inter-chip variation as the non-feed-forward structure, but the intra-chip variation is increased, as the number of stages is reduced by some configurations. Therefore, the reliability of this structure is decreased.

Table 2.1: Simulation Results: Variations

<table>
<thead>
<tr>
<th>Structures</th>
<th>Inter-chip Variation</th>
<th>Intra-chip Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>Non-feed-forward</td>
<td>59</td>
<td>22</td>
</tr>
<tr>
<td>Feed-forward Overlap</td>
<td>66</td>
<td>27</td>
</tr>
<tr>
<td>Feed-forward Cascade</td>
<td>64</td>
<td>25</td>
</tr>
<tr>
<td>Feed-forward Separate</td>
<td>65</td>
<td>26</td>
</tr>
<tr>
<td>Reconfigurable Feed-forward</td>
<td>65</td>
<td>25</td>
</tr>
<tr>
<td>MUX and DeMUX</td>
<td>57</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 2.2 shows the reconfigurability of each reconfigurable structure. It can be seen that the output recombination structure has the best reconfigurability, that is, by fixing the challenge bits and only changing the configure data, the digital signature would vary most significantly. In our simulation result, the average variation is 38.7 bits. The MUX and DeMUX PUF exhibits the least reconfigurability; that is because the function of the DeMUX is only to determine whether to skip some stages or not. When the process variations of other stages are relatively large, the difference of digital signatures with two different configure data may only vary a little bit. For output recombination structure, it is similar to comparing different paths with different configurations, so its performance is close to the inter-chip variation of the non-feed-forward MUX PUF. It also can be observed that although the challenge hash structure and the challenge LFSR structure both pre-process the challenge before it is applied to the circuit, their reconfigurability still have some difference. As the challenge LFSR appears to have better reconfigurability, we can conclude that the number generated by the LFSR in our case may have better randomness than that of the Hash Function. Finally, the proposed reconfigurable feed-forward MUX PUF has the average Hamming distance 32.4 bits by
different configurations, which will be sufficient to be used as a secure and reliable secret key storage method, considering its complex and nonlinear functionality.

<table>
<thead>
<tr>
<th>Structures</th>
<th>Variation</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenge LFSR</td>
<td>44</td>
<td>34.6</td>
<td>28</td>
</tr>
<tr>
<td>Challenge Hash</td>
<td>42</td>
<td>28.3</td>
<td>19</td>
</tr>
<tr>
<td>Output recombination</td>
<td>57</td>
<td>38.9</td>
<td>25</td>
</tr>
<tr>
<td>Reconfigurable Feed-forward</td>
<td>47</td>
<td>32.4</td>
<td>22</td>
</tr>
<tr>
<td>MUX and DeMUX</td>
<td>33</td>
<td>24.7</td>
<td>13</td>
</tr>
</tbody>
</table>

Overall, all the proposed reconfigurable structures have considerable reconfigurability, and can be used for reliable authentication and identification within certain error tolerance, as the minimum of the inter-chip variations would be larger than the maximum intra-chip variation. The output recombination structure has the best reconfigurable ability; however, we consider the reconfigurable feed-forward MUX PUF to have the best performance due to its security, as it is extremely hard to be modeled by certain linear modeling methods.

### 2.6 Conclusion

We have presented several reconfigurable silicon MUX Physical Unclonable Functions based on two major approaches and demonstrated their effectiveness by experimental results via inter-chip variation and intra-chip variation. We also have discussed the reliability perspective of PUFs and proposed several methods to increase the security.
Chapter 3

Statistical Analysis of
MUX-based Physical Unclonable Functions

3.1 Introduction

Physical Unclonable Functions (PUFs) [2, 22, 4] are novel security primitives which store secret keys in physical objects by exploiting the uncontrollable randomness due to manufacturing process variations. PUFs generate signatures based on the unique intrinsic uncontrollable physical features, which can then be used for hardware authentication or the generation of secret keys. Contrary to standard digital systems, PUFs extract secrets from complex properties of a physical material rather than storing them in a non-volatile memory. It is nearly impossible to predict, clone or duplicate PUFs. Furthermore, an adversary cannot easily mount an attack to counterfeit the secret information without changing the physical randomness. Based on these advantages, PUFs can efficiently and reliably generate volatile secret keys for cryptographic operations and enable lightweight and cost-effective authentication of ICs.
The performance of a PUF depends on both process variations and environmental conditions. Designing a PUF that is close to truly random in nature and that can operate reliably over a wide range of operating conditions is still a challenge. Some metrics have been introduced to evaluate the performances of PUFs by analyzing the outputs of PUF instances. These considered metrics include reliability, uniqueness, and randomness. PUF reliability captures how efficient a PUF is in reproducing the response bits of an IC chip. When the same challenge is applied repetitively to a MUX-based PUF, the responses are expected to be identical. Uniqueness represents the ability of a PUF to uniquely distinguish a particular chip among a group of chips of the same type. When the same challenge sets are applied to different PUFs, the output responses are expected to be different. Ideally, the Hamming distances between the responses of different PUFs should be 50%. Randomness indicates the unbiasedness of the PUF response. However, these metrics need to be characterized over a large population of chips to validate the effectiveness of PUFs. This can involve a long and costly chip manufacturing process followed by many measurements after the circuits are fabricated. Furthermore, since the manufacturing process variation and the environmental variation are uncontrollable, it is hard to get a very accurate estimation of the performance during the design stage. Note that security is another performance metric of PUFs, which is not addressed in this chapter. A PUF is more secure, if an adversary finds it harder to break in.

Knowledge about the circuit-level behavior such as process variation pattern, variation of circuit parameters (e.g., delay, threshold voltage) over changing operating conditions could help designers to predict the performance comparisons among different PUF designs. Conducting the performance comparison among detailed PUF designs before fabrication would guarantee robust on-chip PUF performance. Monte-Carlo simulations of netlists that take process and environment variations into account can be used for this purpose. These simulations can provide approximate results, which can be used as
indicators of the true performances of different PUFs. An alternate approach to evaluating the performance of the PUFs is by modeling the physical components of PUFs in a statistical manner. A number of such efforts have been developed in the literature. Statistical analysis on Coating PUF has been presented in [23]. In [24], entropy analysis of Optical PUF has been discussed. The statistical models of Ring Oscillator PUF [25, 26] and MUX PUFs [27, 28, 29] have also been studied in the literature. Additionally, the work in [30] relates the statistical analysis of PUFs to circuit-level optimization and architecture-level optimization, which leads to interesting results that could improve the design and implementation of reliable and efficient PUFs.

The objective of this chapter is to theoretically compare the performances of different MUX-based PUFs to predict the relative advantages of various MUX-based PUF designs. In previous works, such as the efforts in [27, 28, 29], only the statistical modeling with respect to the input-output mappings of PUF structures was presented. However, theoretical performances of PUFs based on these models were not studied. Additionally, there seems little consensus about which PUF is more suitable for a specific application or a particular device in the existing literature. The work presented in this chapter differs from existing efforts in several respects. First, to the best of our knowledge, this work, for the first time, presents a systematic statistical analysis of the performances of various MUX-based PUFs. These include the original MUX PUF, feed-forward MUX PUFs, and multiplexer-demultiplexer (MUX/DeMUX) PUF. Moreover, the focus of our work is on the comparison of performance of various MUX PUFs, which could help the designer to select an appropriate PUF during the design stage. Finally, instead of only modeling the structures of various MUX-based PUFs, statistical analysis is performed to provide a deeper insight into the nature of these PUFs. Equations about the PUF performances are derived; these equations allow the designer to estimate the PUF performance metrics theoretically. In addition, we also introduce a class of modified feed-forward MUX PUFs obtained by modifying the standard feed-forward path. These
structures are also analyzed statistically. It is shown that the modified feed-forward MUX PUFs have less intra-chip variations than standard feed-forward MUX PUFs.

3.2 Modified Feed-Forward MUX PUFs

3.2.1 Modified Feed-Forward Path

In this section, we propose a novel modified feed-forward MUX PUF structure shown in Figure 3.1, which is motivated by our statistical analysis results. In this structure, the output of a feed-forward arbiter from an intermediate stage is input as the challenge bit to two consecutive late MUX stages. By employing this modified feed-forward path, the reliability of the feed-forward PUF structure can be improved, while the same level of security will be retained. This structure is analyzed statistically in this chapter.

The complexity of the modified feed-forward MUX PUFs can be further improved by using several modified feed-forward paths in a PUF circuit. Note that if we want to maintain the length of challenge bits as $N$, we need to increase the number of MUX stages to $N + 2M$ for the modified feed-forward structure, compared to $N + M$ of the standard feed-forward PUF, where $M$ represents the number of feed-forward paths. Additionally, the design overhead will also include $M$ arbiters for both the standard feed-forward MUX PUF and the modified feed-forward MUX PUF.
3.2.2 Different Types of Modified Feed-Forward MUX PUFs

Similar to the three types of the standard feed-forward MUX PUFs as discussed in Chapter 2, the modified feed-forward MUX PUFs can also be classified as Modified Feed-Forward Overlap (MFFO), Modified Feed-Forward Cascade (MFFC), and Modified Feed-Forward Separate (MFFS) as shown in Figure 3.2, Figure 3.3, and Figure 3.4, respectively.

![Figure 3.2: Modified feed-forward MUX PUF overlap structure.](image)

These three different structures also have different inter-chip and intra-chip behaviors, which are analyzed in Section 3.5. Additionally, the modified feed-forward paths can also be used in the Logic-Reconfigurable Feed-forward MUX PUF as presented in Section 2.4.2 to improve the reliability while retaining the high security.
3.3 Definition of PUF Performance

As discussed in Section 3.1, Monte-Carlo simulation can be used to provide performance indicators for different PUF designs. For example, by simulating the three feed-forward MUX PUF structures with the same parameter variations and environmental conditions, we were able to conclude in [31, 32] that the FFO structure is the most reliable among the three feed-forward structures. We focus on analyzing the quantitative performance of various MUX-based PUFs through statistical modeling of the delay variations and environmental variations. Performance indicators ranging from 0 to 1 with 1 representing the best performance are generated through a theoretical analysis. The notations used in this chapter are listed in Table 3.1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of MUX stages in a PUF instance</td>
</tr>
<tr>
<td>M</td>
<td>Number of feed-forward paths</td>
</tr>
<tr>
<td>L</td>
<td>Length of a response</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Challenge bit of the i-th MUX stage</td>
</tr>
<tr>
<td>$D^t_i$</td>
<td>Delay of the top element of the i-th MUX stage</td>
</tr>
<tr>
<td>$D^b_i$</td>
<td>Delay of the bottom element of the i-th MUX stage</td>
</tr>
<tr>
<td>$\Delta_i$</td>
<td>Delay difference between top and bottom elements of the i-th MUX stage</td>
</tr>
<tr>
<td>$\Delta_{Arb}$</td>
<td>Skew effect of the arbiter</td>
</tr>
<tr>
<td>$r_N$</td>
<td>Delay difference of N stages</td>
</tr>
<tr>
<td>$R$</td>
<td>Response</td>
</tr>
</tbody>
</table>

In this section, we introduce three PUF metrics to quantify the performance of MUX-based PUFs. The relative performance behaviors of the PUF structures are the main concern of this chapter rather than the absolute value of each indicator.

3.3.1 Reliability

Intra-chip variation is a measure of the reliability of PUF, which is determined by comparing the digital signatures of the PUF to the same challenge under different environmental conditions.
Let $P_{\text{intra}}$ represent the probability that a certain bit of a response will flip when applying a randomly selected challenge multiple times. All the bits of a PUF response have the same value of $P_{\text{intra}}$, since each bit is generated independently by a same PUF instance (i.e., the effects of manufacturing process variation and environmental variation for all the bits are the same). As a result, $P_{\text{intra}}$ can be used to represent the intra-chip variation for the entire $L$-bit response. In particular, the average Hamming distance (HD) between the responses is used to measure the intra-chip variations of MUX-based PUFs. The $P_{\text{intra}}$ and the averaged HD are described by:

$$E(HD_{\text{intra}}) = P_{\text{intra}} = E\left(\frac{1}{m} \sum_{i=1}^{m} \frac{HD(R_i, R'_i)}{L} \times 100\%\right),$$

(3.1)

where $m$ is the number of HD comparisons, and $R$ and $R'$ represent two measurements of the PUF response under different conditions. The expected value of $HD_{\text{intra}}$ is equal to $P_{\text{intra}}$. If the responses are sampled sufficient number of times, the averaged intra-chip variation would be close to the value of $P_{\text{intra}}$.

As smaller intra-chip variation means better reliability, the reliability indicator is defined as

$$\text{Reliability} = 1 - P_{\text{intra}}.$$  

(3.2)

### 3.3.2 Uniqueness

Inter-chip variation is a measure of the uniqueness of PUF, which is determined by comparing the digital signature of a PUF to that of another. Similarly, we can also define $P_{\text{inter}}$ as the probability that the bits generated by the same challenge for different PUF instances are different. Since uniqueness is a measure of inter-chip performance, all possible chip-combinations should be considered. Therefore, the average inter-chip
HD of $K$ PUFs can be described as:

$$E(HD_{inter}) = P_{inter}$$

$$= E\left( \frac{2}{(K-1)K} \sum_{i=1}^{K-1} \sum_{j=i+1}^{K} \frac{HD(R(i),R(j))}{L} \times 100\% \right).$$  \hspace{1cm} (3.3)$$

It can also be seen that $P_{inter}$ represents the expected value of the inter-chip variation. Since $P_{inter} = 50\%$ represents the best uniqueness for a PUF, the uniqueness indicator can be defined by:

$$Uniqueness = 1 - |2P_{inter} - 1|. \hspace{1cm} (3.4)$$

3.3.3 Randomness

A MUX-based PUF is expected ideally to produce unbiased 0’s and 1’s. Randomness represents the ability of the PUF to output 0 and 1 response with equal probability. One measurement of the randomness can be expressed as:

$$Randomness = 1 - |2P(R = 1) - 1|. \hspace{1cm} (3.5)$$

Therefore, a randomness of 1 indicates unbiased PUF responses.

3.4 Performance Analysis of the Original MUX PUF

3.4.1 Physical Component Modeling of MUX-based PUFs

As shown in Figure 2.1, a MUX PUF consists of a sequence of MUXs and an arbiter. The rising edge signal excites the two paths at the first stage simultaneously. The actual propagated paths are determined by the external challenge bits. After the last stage, the arbiter will generate the output bit by comparing the arrival time of the two paths at its input. It has become standard to model the MUX PUF via an additive linear delay model [27, 28]. According to the efforts in the field of statistical static timing analysis (SSTA) [11], the manufacturing process variations for the parameters of transistors can
be modeled as Gaussian distributions. As a result, the variations of the delays will also be approximately Gaussian.

Process variations are classified as follows: inter-die variations are the variations from die to die, while intra-die variations correspond to variability within a single chip. Inter-die variations affect all devices on the same chip similarly, while intra-die variations affect different devices differently on the same chip. A very widely used model for delay spatial correlation is the "grid model" [11], which assumes perfect correlations among the devices in the same grid, high correlations among those in nearby grids, and low or zero correlations in faraway grids, since devices close to each other are more likely to have similar characteristics than those placed far away.

Additionally, experimental results in [10] have already shown that the inter-chip variation for MUX PUF across the wafers is similar to that within a single wafer, as the output of the arbiter in silicon MUX PUF is only based on the difference of two selected paths. Therefore, these die-to-die, wafer-to-wafer, and lot-to-lot manufacturing variations will have minimum effect on the output response.

Based on the facts discussed above, for simplicity, we can model the delay of each single MUX as an independent identically distributed (i.i.d.) random variable $D_i$, modeled by a Gaussian random variable $N(\mu, \sigma^2)$, where $\mu$ represents the mean and $\sigma$ represents the standard deviation of the delay of each MUX. Therefore, the total delay of the $N$ stages is modeled by $N(N\mu, N\sigma^2)$. The delay difference between top and bottom MUXs of the $i$-th stage will also follow a Gaussian distribution, and can be expressed as:

$$\Delta_i = D_i^t - D_i^b \sim N(0, 2\sigma^2).$$ (3.6)

For the original MUX PUF, the response is dependent on the delay difference of the two selected paths. The sign of the delay difference of each stage is determined by the external challenge bits. Consequently, the delay difference after the last stage can be modeled as

$$r_N = \sum_{i=1}^{N} (-1)^{C_i} \Delta_i,$$ (3.7)
where $C'_i = \oplus_{j=i+1}^N C_j$ and $C'_N = 0$. The output bit is generated by

$$R = \text{sign}(r_N) = \begin{cases} 1, & r_N \geq 0 \\ 0, & r_N < 0 \end{cases}. \quad (3.8)$$

It can be seen that the original MUX PUF forms an additive linear model.

In a real PUF circuit, the arbiter would not be ideal. The skew effect of the arbiters also affects the performance of MUX-based PUFs by reducing the uniqueness, producing a biased response, and even degrading the security. If we assume that the threshold of the arbiter is $\Delta_{Arb}$, the response is given by

$$R = \text{sign}(r_N) = \begin{cases} 1, & r_N \geq \Delta_{Arb} \\ 0, & r_N < \Delta_{Arb} \end{cases}, \quad (3.9)$$

since the arbiter is preset to 0 and requires a setup time constraint to switch to 1.

### 3.4.2 Probability Distribution of Output Delay Difference

Figure 3.5 shows a scatter plot of output samples from the simulations of 100-stage original MUX PUFs. Note that there are overlaps between the regions of output 1’s and output 0’s, which makes it difficult to estimate $\Delta_{Arb}$ accurately. This could be because the measured delay differences (measured for each path at the 50% point in the transition) and the actual delay differences that the arbiter operates at are different. Since the delay difference can be modeled by $N(0, 2N\sigma^2)$, we fit a Gaussian distribution to the delay differences, as shown in Figure 3.6. The standard deviation $\sqrt{2N\sigma^2}$ of the generated Gaussian distribution is $5.2936 \times 10^{-11}$. It can be seen that the skew effect of the arbiter leads to biased outputs with 32.8% 1’s and 67.2% 0’s.

Moreover, the average of the total delay of one path is $1.2667 \times 10^{-8}$s in our simulation results. Therefore, the percent of delay deviation of 100 stages is about 0.4% (i.e., $\frac{5.2936 \times 10^{-11}}{1.2667 \times 10^{-8}}$), which conforms with other published results of 65nm technology (e.g., [9]).
Figure 3.5: Scatter plot of outputs.

Figure 3.6: Gaussian fit curve of delay difference distribution.
3.4.3 Effect of Number of Stages

The probability that the output is equal to 1 can be derived as:

\[
P(R = 1) = P\left(\sum_{i=1}^{N} (-1)^{C_i} \Delta_i \geq \Delta_{Arb}\right)
\]

\[
= \int_{\Delta_{Arb}}^{\infty} \frac{1}{\sqrt{2\pi N 2\sigma^2}} e^{x^2/(2N 2\sigma^2)} dx
\]

\[
= \frac{1}{2} - \frac{1}{2} erf\left(\frac{\Delta_{Arb}}{\sqrt{2N 2\sigma^2}}\right).
\]

(3.10)

It can be seen that \(P(R = 1)\) is also dependent on the number of stages in a MUX PUF. If we only consider the number of stages \(N\) as a variable, the above equation can be rewritten as

\[
P(R = 1) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{K}{\sqrt{N}}\right),
\]

(3.11)

where \(K\) is a constant and is equal to \(\frac{\Delta_{Arb}}{\sqrt{4\sigma^2}}\). Thus, although the value of \(\Delta_{Arb}\) is unclear, we can still estimate the \(K\) based on the experimental results:

\[
K = \sqrt{N} \times erfinv(1 - 2P(R = 1)).
\]

(3.12)

In our simulations, the 50-stage MUX PUF structure is only able to generate 1’s with probability 25.6%, which is very close to the value of \(P(R = 1) = 26.4\%\) that is calculated theoretically from Equation (3.11).

3.4.4 Statistical Properties of the Original MUX PUF

Reliability

In order to analyze the reliability, we need to consider the effect of environmental noise. As a usual practice, we assume that the noise \(n_i\) of the \(i\)-th stage follows a zero-mean Gaussian distribution with variance \(\sigma_n^2\). Then, \(P_{intra}\) can be described as:

\[
P_{intra} = P\left[\text{sign}\left(\sum_{i=1}^{N} (-1)^{C_i} \Delta_i + \sum_{i=1}^{N} n_i\right) \neq \text{sign}\left(\sum_{i=1}^{N} (-1)^{C_i} \Delta_i + \sum_{i=1}^{N} n'_i\right)\right].
\]

(3.13)
where \( n_i \) and \( n'_i \) represent the noise under different environmental conditions. As each individual stage follows the zero-mean i.i.d. Gaussian distribution, then the intra-chip variation probability is equivalent to a single stage intra-chip variation probability:

\[
P_{\text{intra}} = P[\text{sign}(s_i + n_i) \neq \text{sign}(s_i + n'_i)]
\]

(3.14)

where \( s_i = (-1)^{C_i}(D^i_t - D^b_i) \) has a variance that is equal to \( 2\sigma^2 \).

As manufacturing process variation and environmental noise of the delay difference both follow zero-mean Gaussian distribution, their probability density functions (PDF) are given by:

\[
f_s(s) = \frac{1}{\sqrt{2\pi\sigma^2_s}} \exp\left(-\frac{s^2}{2\sigma^2_s}\right),
\]

\[
f_n(n) = \frac{1}{\sqrt{2\pi\sigma^2_n}} \exp\left(-\frac{n^2}{2\sigma^2_n}\right).
\]

(3.15)

Note that \( \sigma^2_s = 2\sigma^2 \) in Equation (3.6). \( P_{\text{intra}} \) of an original MUX PUF can be calculated as

\[
P_{\text{intra}} = P[\text{sign}(s_i + n_i) \neq \text{sign}(s_i + n'_i)]
\]

\[
= 4 \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2_s}} \exp\left(-\frac{s^2}{2\sigma^2_s}\right) \int_{-\infty}^{-s} \frac{1}{\sqrt{2\pi\sigma^2_n}} \exp\left(-\frac{n^2}{2\sigma^2_n}\right) dn \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2_n}} \exp\left(-\frac{n'^2}{2\sigma^2_n}\right) dn'ds
\]

\[
= 4 \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2_s}} \exp\left(-\frac{s^2}{2\sigma^2_s}\right) \left(\frac{1}{4} - \frac{1}{4} \text{erf}\left(\frac{s}{\sqrt{2\sigma^2_n}}\right)\right) ds
\]

\[
= \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2_s}} \exp\left(-\frac{s^2}{2\sigma^2_s}\right) ds - \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2_s}} \exp\left(-\frac{s^2}{2\sigma^2_s}\right) \text{erf}\left(\frac{s}{\sqrt{2\sigma^2_n}}\right) ds
\]

\[
= \frac{1}{2} - \frac{1}{\pi} \arctan\left(\sqrt\frac{\sigma^4_s}{\sigma^4_s + 2\sigma^4_n}\right),
\]

(3.16)

where the first of the two integrals in the fourth line represents integrating just a Gaussian over half of the space, and the second is a known definite integral [33].

It can be seen that by increasing the ratio of manufacturing process variation to environmental variation, the intra-chip variation can be reduced to close to 0.

If we consider the \( P_{\text{intra}} \) of the PUF response with a given challenge, the conditional probability can be derived as

\[
P[\text{sign}(s_i + n_i) \neq \text{sign}(s_i + n'_i)|s_i] = \frac{1}{2} - \frac{1}{2} \text{erf}\left(\frac{|s_i|}{\sqrt{2\sigma^2_n}}\right).
\]

(3.17)
The reliability for a certain challenge-response pair is greatly dependent on the manufacturing process variation between the two generated paths. If a challenge selects two paths with $r_N \approx 0$, the variation is large, since the above equation achieves maximum at $s_i = 0$. Otherwise, if $|r_N|$ is relatively large, the manufacturing process variation would be the primary factor to determine the output and the noise would hardly flip the response.

However, if we take the skew effect of non-ideal arbiters into consideration, the intra-chip variation behaviors would also be dependent on the number of stages and the performance of the arbiter. The response then can be described as $\text{sign}(\sum_{i=1}^{N}(s_i + n_i) - \Delta_{\text{Arb}})$. Therefore, $P_{\text{intra}}$ is given by

$$P[\text{sign}(\sum_{i=1}^{N}(s_i + n_i) - \Delta_{\text{Arb}}) \neq \text{sign}(\sum_{i=1}^{N}(s_i + n_i') - \Delta_{\text{Arb}})].$$

(3.18)

If we combine $\sum_{i=1}^{N}s_i$ and $\Delta_{\text{Arb}}$ as a variable $X \sim (\Delta_{\text{Arb}}, N\sigma_s^2)$, $P_{\text{intra}}$ can be expressed as $P[\text{sign}(x + n) \neq \text{sign}(x + n')]$, where $x \sim N(-\frac{\Delta_{\text{Arb}}}{\sqrt{N}}, \sigma_s^2)$ and $n \sim N(0, \sigma_n^2)$. Therefore, according to Equation (3.18), the intra-chip variation probability decreases with the increase of $\Delta_{\text{Arb}}$. Intuitively we would expect this as when $\Delta_{\text{Arb}}$ is relatively large and the number of stages is small, $\sum_{i=1}^{N}(s_i + n_i)$ will have a high probability of unaltered $\text{sign}()$ value. However, if the number of stages is relatively large that $\frac{\Delta_{\text{Arb}}}{\sqrt{N}}$ approaches to 0, $P_{\text{intra}}$ will be reduced to Equation (3.16).
A closed-form expression for \( P_{\text{intra}} \) (i.e., Equation (3.18)) does not exist, but we can derive the expression by using a first-order approximation of the exponential function:

\[
P[\text{sign}(\sum_{i=1}^{N} (s_i + n_i) - \Delta_{\text{Arb}}) \neq \text{sign}(\sum_{i=1}^{N} (s_i + n_i') - \Delta_{\text{Arb}})]
\]

\[
= 2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{s^2}{2\sigma_s^2}\right) \left(\frac{1}{4} - \frac{1}{4} \frac{1}{\sqrt{2\pi\sigma_n^2}} \text{erf}\left(\frac{|s - \Delta_{\text{Arb}}|}{\sqrt{2\sigma_n^2}}\right)\right) ds
\]

\[
= 2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(x + \Delta_{\text{Arb}})^2}{2\sigma_s^2}\right) \left(\frac{1}{4} - \frac{1}{4} \frac{1}{\sqrt{2\pi\sigma_n^2}} \text{erf}\left(\frac{|x|}{\sqrt{2\sigma_n^2}}\right)\right) dx
\]

\[
\approx 2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{x^2}{2\sigma_s^2}\right) \left(1 - \frac{\Delta_{\text{Arb}}}{\sigma_s^2\sqrt{N}}\right) \left(\frac{1}{4} - \frac{1}{4} \frac{1}{\sqrt{2\pi\sigma_n^2}} \text{erf}\left(\frac{|x|}{\sqrt{2\sigma_n^2}}\right)\right) dx
\]

\[
= \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\sigma_s^2}{2\sigma_s^2\sigma_n^2 + \sigma_n^2}\right) - \frac{\Delta_{\text{Arb}}}{\sigma_s^2\sqrt{N}} \int_{0}^{\infty} \frac{x}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{x^2}{2\sigma_s^2}\right) \left(\frac{1}{4} - \frac{1}{4} \frac{1}{\sqrt{2\pi\sigma_n^2}} \text{erf}\left(\frac{x}{\sqrt{2\sigma_n^2}}\right)\right) dx
\]

\[
= \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\sigma_s^2}{2\sigma_s^2\sigma_n^2 + \sigma_n^2}\right) - \frac{\Delta_{\text{Arb}}}{\sigma_s^2\sqrt{N}} \left(1 - \frac{2}{\pi} \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} \arctan\left(\frac{\sigma_s^2}{\sqrt{\sigma_s^2 + \sigma_n^2}}\right)\right)
\]

(3.19)

It can be seen that \( P_{\text{intra}} \) increases with the number of stages, since

\[
\frac{2}{\pi} \sqrt{\frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}} \arctan\left(\sqrt{\frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}}\right) < 1.
\]

(3.20)

Additionally, the term of \( \sqrt{\frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}} \) is close to 1, while the ratio of \( \frac{\sigma_s}{\sigma_n} \) is relatively large. As a result, \( \left(1 - \frac{2}{\pi} \sqrt{\frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}} \arctan\left(\sqrt{\frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}}\right)\right) \) will be close to 0. Therefore, in this case, the number of the stages only has a minor influence on the intra-chip variation of the original MUX PUF.

**Conclusion 1:** The reliability indicator of an original MUX PUF is

\[
\text{Reliability} = 1 - P_{\text{intra}}
\]

\[
= \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{\sigma_s}{2\sigma_s^2\sigma_n^2 + \sigma_n^2}\right) + \frac{\Delta_{\text{Arb}}}{\sqrt{2\pi N\sigma_s^2}} \left(1 - \frac{2}{\pi} \sqrt{\frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}} \arctan\left(\frac{\sigma_s^2}{\sqrt{\sigma_s^2 + \sigma_n^2}}\right)\right)
\]

(3.21)
where $\sigma_s$ is the standard deviation of manufacturing process variation for a single stage, $\sigma_n$ is the standard deviation of environmental noise, $\Delta_{Arb}$ is the skew effect of the arbiter, and $N$ is the number of stages in an original MUX PUF.

**Uniqueness**

In order to compute inter-chip variation based on the same mathematical model, we need to compare the responses of different PUFs. The Gaussian fit curve for the inter-chip variations of the 100-stage MUX PUF is shown in Figure 3.7. The average of the inter-chip variation is 43.2%.

![Gaussian fit curve of inter-chip variation distribution.](image)

**Figure 3.7:** Gaussian fit curve of inter-chip variation distribution.

Theoretically, if the PUFs are uncorrelated, the expected inter-chip variation of the original MUX PUF is simply given by

\[
P_{\text{inter}} = 2P(R = 1)(1 - P(R = 1)) = \frac{1}{2} - \frac{1}{2} e^{-2\frac{K}{\sqrt{N}}},
\]

\[(3.22)\]
where \( K = \frac{\Delta_{Arb}}{\sqrt{4\sigma_s^2}} = \frac{\Delta_{Aarb}}{\sqrt{2\sigma_n^2}} \). Therefore, the value of uniqueness indicator can be expressed as

\[
Uniqueness = 1 - |2P_{inter} - 1| = 4P(R = 1)(1 - P(R = 1)) \\
= 1 - erf^2\left( \frac{\Delta_{Aarb}}{\sqrt{2N\sigma_n^2}} \right).
\] (3.23)

According to the value of \( P(R = 1) \), we could expect the average of the inter-chip variation to be \( 2 \times 0.328 \times (1 - 0.328) = 44\% \), which is also consistent with our experimental results.

**Randomness**

Similarly, according to Equation (3.10) in Section 3.4.3, the randomness of an original MUX PUF is

\[
Randomness = 1 - |2P(R = 1) - 1| = 1 - erf\left( \frac{\Delta_{Aarb}}{\sqrt{2N\sigma_n^2}} \right).
\] (3.24)

### 3.4.5 Design Example

The above equations are useful for designing a PUF that could meet the specific application requirement. Consider the scenario that the PUF designer has fabricated a PUF and tested its performance. However, the designer found the performance of the current PUF cannot satisfy the application requirement. Instead of fabricating a large number of different PUF designs, the designer could utilize the statistical analysis results to predict the performances of other PUF designs theoretically, since some of the parameters can be calculated from the results of the current PUF. For example, if we fabricate a 100-stage original MUX PUF which has the performance similar to our experimental results, we can calculate the value of \( K \) by Equation (3.12) based on the performance of the 100-stage MUX PUF. We can then substitute different values of \( N \) to predict the performances of the original MUX PUFs for different numbers of stages. For the intra-chip variation, we can obtain the relations among \( \sigma_s, \sigma_n, \) and \( \Delta_{Aarb} \).
by utilizing Equations (3.19) and (3.24) together. The various performance results for
different values of \( N \) are calculated theoretically and are summarized in Table 3.2 (the
bold line indicates the results obtained from the current measured or simulated PUF).

Table 3.2: Predicted Performance of the Original MUX PUFs for Different Number of Stages
\( (N) \) Obtained by Statistical Analysis Using the Model Derived from Experimental Results with
\( N=100 \).

<table>
<thead>
<tr>
<th>( N )</th>
<th>Intra-chip</th>
<th>Reliability</th>
<th>Inter-chip</th>
<th>Uniqueness</th>
<th>( P(R=1) )</th>
<th>Randomness</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>5.3%</td>
<td>94.7%</td>
<td>30.3%</td>
<td>60.7%</td>
<td>18.7%</td>
<td>37.3%</td>
</tr>
<tr>
<td>50</td>
<td>5.6%</td>
<td>94.4%</td>
<td>38.9%</td>
<td>77.8%</td>
<td>26.4%</td>
<td>52.9%</td>
</tr>
<tr>
<td>75</td>
<td>5.7%</td>
<td>94.3%</td>
<td>42.3%</td>
<td>84.6%</td>
<td>30.4%</td>
<td>60.7%</td>
</tr>
<tr>
<td>100</td>
<td>5.8%</td>
<td>94.2%</td>
<td>44.1%</td>
<td>88.2%</td>
<td>32.8%</td>
<td>65.6%</td>
</tr>
<tr>
<td>150</td>
<td>5.9%</td>
<td>94.1%</td>
<td>46.0%</td>
<td>91.9%</td>
<td>35.8%</td>
<td>71.6%</td>
</tr>
<tr>
<td>200</td>
<td>5.9%</td>
<td>94.1%</td>
<td>46.9%</td>
<td>93.9%</td>
<td>37.7%</td>
<td>75.3%</td>
</tr>
</tbody>
</table>

Therefore, the statistical analysis can be used to decide how many MUX stages
should be used to design the PUF to match the application requirements. Note that
the values may not be exact, but we can obtain the trends of the performance metrics
from the statistical analysis results. These results show that inter-chip variation is
strongly dependent on \( N \) for \( N < 100 \), while intra-chip variation is almost independent
of \( N \), for a relatively large \( N \) (i.e., \( N > 25 \)).

### 3.5 Performance Analysis of Feed-Forward MUX PUFs
and MUX/DeMUX PUF

In Section 3.4, we demonstrated that MUX-based PUFs can be statistically analyzed
by modeling of the physical components. In this section, we continue to analyze the
performance of feed-forward MUX PUFs and MUX/DeMUX PUF. We compare the
performance of these MUX-based PUFs with respect to the three indicators described
in Section 3.3.
3.5.1 Statistical Properties of Feed-Forward MUX PUF

Reliability

As shown in Figure 2.2, some of the challenge bits of a feed-forward MUX PUF will be the intermediate stage arbiter outputs instead of the external bits in a feed-forward MUX PUF. For instance, if there is only one feed-forward path in a MUX PUF, which is from the $a$-th stage to the $b$-th stage, the time difference of the $b$-th stage could be expressed as:

$$\Delta_b = (-1)^{sign(r_a)}(D_b^a - D_b^b).$$  \hspace{1cm} (3.25)

It can be expected intuitively that the intra-chip variation will be greatly dependent on the location of the ending stage of the feed-forward path. If the challenge of the last stage flips, the output bit will be $sign(-\sum_{i=1}^{N-1}(-1)^{C_i}\Delta_i + \Delta_N)$ compared to $sign(\sum_{i=1}^{N-1}(-1)^{C_i}\Delta_i + \Delta_N)$ while there is no error. We can also illustrate this characteristic of the feed-forward PUF mathematically. As an example, if the challenge of the $k$-th stage flips in an $N$-stage structure, the probability that the output bit will change, $P_e$, is given by (without considering noise):

$$P_e = P[sign(\sum_{i=1}^{k-1}(-1)^{C_i}\Delta_i + \sum_{i=k}^{N}(-1)^{C_i}\Delta_i) \neq sign(-\sum_{i=1}^{k-1}(-1)^{C_i}\Delta_i + \sum_{i=k}^{N}(-1)^{C_i}\Delta_i)]$$

$$= 2\int_{0}^{\infty} \frac{1}{\sqrt{2\pi(N-k+1)\sigma_s^2}}exp\left(-\frac{s^2}{2(N-k+1)\sigma_s^2}\right)ds \left(1 - \int_{-s}^{s} \frac{1}{\sqrt{2\pi(k-1)\sigma_s^2}}exp\left(-\frac{w^2}{2(k-1)\sigma_s^2}\right)dw\right)$$

$$= 2\int_{0}^{\infty} \frac{1}{\sqrt{2\pi(N-k+1)\sigma_s^2}}exp\left(-\frac{s^2}{2(N-k+1)\sigma_s^2}\right)(1 - erf\left(\frac{s}{\sqrt{2\pi(k-1)\sigma_s^2}}\right))ds$$

$$= 1 - \frac{2}{\pi}arc\tan\left(\frac{\sqrt{N-k+1}}{k-1}\right)$$

$$= \frac{2}{\pi}arc\tan\left(\frac{k-1}{\sqrt{N-k+1}}\right)$$ \hspace{1cm} (3.26)

where the second integral in the third line is also a known definite integral in [33].
It can be seen that the probability $P_e$ increases with $k$. Obviously, the problem for this structure is that if the ending stage of a feed-forward path is close to the last stage, the reliability of the PUF will be degraded significantly. If $k = N$, i.e., the ending stage of the feed-forward path is the last stage, $P_e$ will be $\frac{2}{\pi} \arctan(\sqrt{N - 1})$, which is close to 1.

Therefore, $P_{\text{intra}}$ of this feed-forward MUX PUF with single feed-forward path can be described as

$$\left(1 - P_1\right) P_1 + P_1 \frac{2}{\pi} \arctan\left(\frac{k - 1}{N - k + 1}\right),$$

where $P_1$ is equal to $P_{\text{intra}}$ of the original MUX PUF. Note that, for simplicity, the following analysis on reliability will focus on the case without considering the skew effect of arbiters (i.e., $P_1$ represents Equation (3.16) instead of Equation (3.19)), since the number of stages and the skew effect do not have a significant impact on intra-chip variation, as described in Section 3.4. Additionally, there could be a number of feed-forward paths in one PUF design. In these PUFs, if the ending stage of a feed-forward path is close to the last stage, the reliability of the PUF will be degraded significantly.

### 3.5.2 Statistical Properties of Modified Feed-Forward MUX PUF

Motivated by this analysis, we propose the modified feed-forward MUX PUF structure shown in Figure 3.1, which is presented in Section 3.2. The modified feed-forward path mitigates the effect of the locations of feed-forward paths. In this structure, the modified feed-forward path only affects the delay difference of one stage. For example, if the two consecutive ending stages of a feed-forward path are the $k$-th and $(k + 1)$-th stages respectively, we can derive $C'$ for the modified feed-forward MUX PUF as follows:

$$C'_{k-1} = \bigoplus_{j=k+2}^{N} C_j \oplus C_k \oplus C_{k+1},$$

$$C'_k = \bigoplus_{j=k+2}^{N} C_j \oplus C_{k+1}.$$
\[ C_{k+1}' = \oplus_{j=k}^{N} C_j. \]  

(3.30)

Since \( C_k = C_{k+1} \) in this structure, the modified feed-forward path will only affect the value of \( C_k' \).

As a result, by employing this modified feed-forward path, only one stage will be affected by each feed-forward arbiter. Thus, this structure will have lower intra-chip variation, compared to the standard feed-forward MUX PUF. However, the nonlinearity of mathematical models for the modified feed-forward MUX PUF and the standard feed-forward MUX PUF are similar, except the challenge mapping \( C_i' \). Therefore, we can conclude that one benefit of using the proposed modified feed-forward path is that the reliability of the feed-forward MUX PUF can be improved. Furthermore, the modified feed-forward path can lead to higher security, as the degree of nonlinearity can be increased without significant increase of intra-chip variation.

The designer can predict the performances of different implementations of the feed-forward MUX PUFs based on above statistical analysis results. For example, we consider the 100-stage feed-forward MUX PUFs with one feed-forward path (assuming that the feed-forward path starts from the output of the 20th stage, and the ends at the \( k \)-th stage). If error occurs in the feed-forward path, the probabilities that the output bit will change \( P_e \) and the intra-chip variation probabilities, \( P_{\text{intra}} \), are summarized in Table 3.3 (\( P_1 \) is equal to 5.8\% in our experimental results). Note that the probabilities of errors in the feed-forward paths are the same for all the feed-forward MUX PUFs with single feed-forward path, since there is no feed-forward path in previous stages.

<table>
<thead>
<tr>
<th>( k )</th>
<th>Standard Feed-Forward MUX PUF</th>
<th>Modified Feed-Forward MUX PUF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50</td>
<td>70</td>
</tr>
<tr>
<td>( P_e )</td>
<td>49.4%</td>
<td>62.4%</td>
</tr>
<tr>
<td>( P_{\text{intra}} )</td>
<td>8.33%</td>
<td>9.09%</td>
</tr>
</tbody>
</table>
It can be seen that the modified feed-forward path can reduce the intra-chip variation of the feed-forward MUX PUF. Compared to the original MUX PUF, the intra-chip variation of the modified feed-forward MUX PUF with single feed-forward path is only increased very slightly. Based on Table 3.3, it can also be concluded that if the designer want to design a standard feed-forward MUX PUF, the designer could adjust the locations of the feed-forward paths according to the particular design performance requirement.

Reliability

In this structure, the delay difference of the ending stage of the first feed-forward path (from stage $a$ to stage $b$) will be $(-1)^{C_{b+1}'}(D_b^i - D_b^h)$ with probability $1 - P_1$, and will be $(-1)^{C_{b+1}'}(D_b^i - D_b^h)$ with probability $P_1$. We define the stage variation probability as the probability that the sign of the delay difference for each stage changes from positive to negative or vice versa. Note that the stage variation probability is a good indicator of the effect of the noise at each MUX stage. The greater the stage variation probability, the less reliable the response. It is obvious that for the MUX PUF, the stage variation probability for each stage is equal to the intra-chip variation probability $P_{\text{intra}} = P_1$.

For the modified feed-forward structure, the stage variation probability for the stage whose select signal is from the first feed-forward arbiter can be calculated as

$$P[\text{sign}(s_i + n_i)\text{sign}(s_b + n_b) \neq \text{sign}(s_i + n_i')\text{sign}(s_b + n_b')]$$

$$= 2(1 - P_1)P_1 = \frac{1}{2} - \frac{2}{\pi^2} \arctan^2\left(\sqrt{\frac{\sigma_s^4}{2\sigma_s^2\sigma_n^2 + \sigma_n^4}}\right). \quad (3.31)$$
Since $\arctan(\sqrt{\frac{\sigma^2}{2\sigma^2\sigma_n^2 + \sigma_n^4}}) \leq \frac{\pi}{2}$, we can conclude that

$$\frac{1}{2} - \frac{2}{\pi^2} \arctan^2(\sqrt{\frac{\sigma_n^4}{2\sigma^2\sigma_n^2 + \sigma_n^4}}) = \frac{1}{2} - \frac{2}{\pi} \arctan(\sqrt{\frac{\sigma_n^4}{2\sigma^2\sigma_n^2 + \sigma_n^4}}) \frac{1}{\pi} \arctan(\sqrt{\frac{\sigma_n^4}{2\sigma^2\sigma_n^2 + \sigma_n^4}}) \geq \frac{1}{2} - \frac{1}{\pi} \arctan(\sqrt{\frac{\sigma_n^4}{2\sigma^2\sigma_n^2 + \sigma_n^4}}). \quad (3.32)$$

Therefore, it can be concluded that the stage variation probability is increased by introducing feed-forward arbiters. This is similar to the scenario where the environmental noise can cause large variations of the time difference to the ending stages of feed-forward paths. The intra-chip variation probability of a feed-forward MUX PUF can be expressed as

$$P_{\text{intra}} = \frac{1}{2} - \frac{1}{\pi} \arctan(\sqrt{\frac{\sigma_n^4}{2\sigma^2\sigma_n^2 + \sigma_n^4}}), \quad (3.33)$$

where $\tilde{\sigma}^2 = \sigma_n^2 + \frac{1}{N} \sum_{k=1}^{M} \sigma_k^2$, and $M$ is the number of feed-forward paths and $\sigma_k^4$ is the additional deviation introduced by the feed-forward paths. The value of $\sigma_k^4$ for each feed-forward path is different, which is dependent on the noise in previous stages. Therefore, unlike the original MUX PUF, the feed-forward MUX PUF structure has a large number of variants. The differences in both the number and the locations of the feed-forward paths result in different mathematical models, which will lead to different values of $P_{\text{intra}}$.

**Conclusion 2:** Although a general expression cannot be derived for $P_{\text{intra}}$ of the modified feed-forward MUX PUF, we can still conclude from Equation (3.33) and Table 3.3 that

$$P_{\text{intra}}(\text{feed-forward MUX PUF}) > P_{\text{intra}}(\text{modified feed-forward MUX PUF}) > P_{\text{intra}}(\text{original MUX PUF}). \quad (3.34)$$
Thus, the modified feed-forward MUX PUF has lower value of the reliability indicator than the original MUX PUF, since \( \text{Reliability} = 1 - P_{\text{intra}} \). If we take skew effect of the arbiter into consideration, the same conclusion above can also be reached, since the feed-forward PUF has larger stage variations (assuming all other parameters to be the same).

\[ \blacksquare \]

**Uniqueness**

If we still assume that the arbiters are ideal, the inter-chip variation will not be affected by the modified feed-forward paths. The delay differences of the ending stages of feed-forward paths still follow a zero-mean Gaussian distribution. Thus, the mean of total time difference of the two generated paths is 0. Since the manufacturing process variations are uncorrelated for different PUFs, \( P_{\text{inter}} \) will remain 50\% for the modified feed-forward MUX PUFs.

However, if we consider the skew effect of arbiters, the inter-chip variation behaviors would be different for the original MUX PUFs and the feed-forward MUX PUFs. We consider the Gaussian random variable \( Y \) that follows the distribution \( N(0, N\sigma_s^2 + \sum_{i=1}^{N} \sigma_{n_i}^2) \). Without loss of generality, we assume \( \Delta_{\text{Arb}} \geq 0 \). Thus, the probability that the PUF output is 1 is given by:

\[
P(R = 1) = P(Y > \Delta_{\text{Arb}}) = \frac{1}{2} - \frac{1}{2}erf\left(\frac{\Delta_{\text{Arb}}}{\sqrt{2N\sigma_s^2 + 2\sum_{i=1}^{N} \sigma_{n_i}^2}}\right).
\]

(3.35)

If we consider two variables \( Y \) and \( Y' \), where \( Y' \) has larger stage variations (i.e., larger \( \sum_{i=1}^{N} \sigma_{n_i}^2 \)), we can obtain the relation that \( P(Y > \Delta_{\text{Arb}}) < P(Y' > \Delta_{\text{Arb}}) < \frac{1}{2} \) from Equation (3.35). In this case, \( P_{\text{inter}} \) for the two different PUFs are \( 2P(Y > \Delta_{\text{Arb}})(1 - P(Y > \Delta_{\text{Arb}})) \) and \( 2P(Y' > \Delta_{\text{Arb}})(1 - P(Y' > \Delta_{\text{Arb}})) \), respectively. We can
show that

\[
2P(Y > \Delta_{Arb})(1 - P(Y > \Delta_{Arb})) - 2P(Y' > \Delta_{Arb})(1 - P(Y' > \Delta_{Arb})) \\
= 2(P(Y > \Delta_{Arb}) - P(Y' > \Delta_{Arb}))(1 - P(Y > \Delta_{Arb}) - P(Y' > \Delta_{Arb})) \\
< 0. \quad (3.36)
\]

Thus we conclude that the PUF structure with larger stage variations has a larger inter-chip variation. In particular, we can conclude that the modified feed-forward MUX PUF has a greater inter-chip variation probability $P_{inter}$ than the original MUX PUF, since the stage variation probability for a modified feed-forward MUX PUF is larger.

**Conclusion 3:** The values of $P_{intra}$ and $P_{inter}$ of a modified feed-forward MUX PUF are both greater than those of the original MUX PUF. Therefore, it can be concluded that the feed-forward MUX PUF has higher uniqueness than the original MUX PUF, as $P_{inter}$ of the modified feed-forward MUX PUF is closer to $\frac{1}{2}$.

**Randomness**

If we still consider the variable $Y$, we can get $P(R = 1) = P(Y > \Delta_{Arb})$ while taking the skew effect of the arbiters into consideration. Since $P(Y > \Delta_{Arb}) < \frac{1}{2}$, we can obtain the expression for the randomness as

\[
Randomness = 1 - |2P(R = 1) - 1| = 2P(Y > \Delta_{Arb}). \quad (3.37)
\]

Therefore, we can also conclude that the modified feed-forward MUX PUF has better randomness than the original MUX PUF, as the value of $P(Y > \Delta_{Arb})$ for the modified feed-forward MUX PUF is greater.
3.5.3 Statistical Properties of Different Types of Modified Feed-forward MUX PUFs

As discussed in Section 3.2, modified feed-forward MUX PUFs can be classified into three different structures, which have different inter-chip and intra-chip characteristics. We examine the relations of these structures statistically in this subsection.

Reliability

From Conclusion 2, the stage variation probability of the ending stage of the first modified feed-forward path is given by

\[ P_2 = (1 - P_1)P_1 + P_1(1 - P_1) > P_1. \]  

(3.38)

Similarly, the stage variation probability of the ending stage of the second modified feed-forward path can be written as

\[ P_3 = (1 - P_2)P_1 + P_2(1 - P_1). \]  

(3.39)

Generally speaking, if we have the stage variation probability for the ending stages of the first \( m \) modified feed-forward paths and \( P_1 < P_2 < \ldots < P_m \), then the stage variation probability for the ending stage of the \( (m + 1) \)-th modified feed-forward path is given by

\[ P_{m+1} = (1 - P_m)P_1 + (1 - P_1)P_m > P_m. \]  

(3.40)

This can be proved as follows:

\[
P_{m+1} - P_m = (1 - P_m)P_1 + (1 - P_1)P_m - (1 - P_m)(1 - P_1)P_1 - (1 - P_1)P_{m-1} \\
= (P_m - P_m)P_1 + (1 - P_1)(1 - P_{m-1}) \\
= (P_m - P_{m-1})(1 - 2P_1).
\]

(3.41)

As we have already shown that \( P_1 < \frac{1}{2} \) and we have \( P_m > P_{m-1} \), therefore, we can conclude that \( P_{m+1} > P_m \).
Conclusion 4: In a modified feed-forward MUX PUF structure, the stage variation probability of the ending stage of a modified feed-forward path is greater than those of previous modified feed-forward paths.

It can be expected that the stage variation probability of a modified feed-forward overlap structure is less than the separate or cascade structure, since there is no feed-forward arbiter in previous path for any of the modified feed-forward paths in the overlap structure. We can show this property by combining the feed-forward paths and other stages together. The stage variation probability of the first feed-forward arbiter is equal to $P_1$, since there is no feed-forward path involved. Then, the stage variation probability of the second feed-forward arbiter is given by (assuming there are $K_2$ stages before the second feed-forward path and the $b$-th stage is the ending stage of the first feed-forward path):

$$P_2 = P\left[\text{sign}\left(\sum_{i=1, i \neq b}^{K_2} s_i + s_b + \sum_{i=1}^{K_2} n_i\right) \neq \text{sign}\left(\sum_{i=1, i \neq b}^{K_2} s_i + x_b + \sum_{i=1}^{K_2} n'_i\right)\right]$$

where $x_b = s_b$, with probability $1 - P_1$, and $x_b = -s_b$, with probability $P_1$.

In the expression above, we have $\sum_{i=1, i \neq b}^{K_2} s_i \sim N(0, (K_2 - 1)\sigma_s^2)$, $s_b \sim N(0, \sigma_s^2)$ and $\sum_{i=1}^{K_2} n_i \sim N(0, K_2\sigma_n^2)$. This involves triple integrals over Gaussian distributions and does not have a closed-form expression. Therefore, we use Monte-Carlo simulation method to examine the performance. Figure 3.8 shows the stage variation probabilities of the second feed-forward arbiter for different $K_2$.

It can be observed from Figure 3.8 that the stage variation probability is decreased with the increase of $K_2$, since the error in feed-forward path is more likely to be averaged out by more stages. For the third feed-forward arbiter, the stage variation probability is

$$P_3 = P\left[\text{sign}\left(\sum_{i=1, i \neq b,d}^{K_3} s_i + s_b + s_d + \sum_{i=1}^{K_3} n_i\right) \neq \text{sign}\left(\sum_{i=1, i \neq b,d}^{K_3} s_i + x_b + x_d + \sum_{i=1}^{K_3} n'_i\right)\right]$$ 

(3.43)
Figure 3.8: Stage variation probability of the ending stage of the second feed-forward arbiter.

where $x_b = s_b$, with probability $1 - P_1$, and $x_b = -s_b$, with probability $P_1$; $x_d = s_d$, with probability $1 - P_2$, and $x_d = -s_d$, with probability $P_2$. We can find that $P_3$ also decreases with $K_3$.

In the modified feed-forward separate structure, $K_i$ is greater than the corresponding $K_i$ in the cascade structure. Therefore, the stage variations of the ending stages of feed-forward paths in the cascade structure are larger. Furthermore, large stage variation probability in previous path is more likely to lead to a large stage variation probability of the following feed-forward arbiters. As a result, we can conclude that the modified feed-forward separate structure is more reliable than the modified feed-forward cascade structure.

**Conclusion 5:** Based on the stage variation properties, we conclude that $P_{intra}$ of the three structures satisfy

$$P_{intra}(MFFO) < P_{intra}(MFFS) < P_{intra}(MFFC).$$

The MFFO structure has the best reliability, while the MFFC structure is the least reliable. Note that the above statistical analysis approach can also be applied to the three different types of standard feed-forward MUX PUFs. These feed-forward MUX
PUFs exhibit similar characteristics as the modified feed-forward MUX PUFs.

Uniqueness

According to the derivation in Section 3.5.2, the MFFC structure has the largest stage variation probability and, thus, has the best uniqueness, while the MFFO has the lowest value of the uniqueness indicator among the three modified feed-forward structures.

Randomness

Similarly, as discussed in Section 3.5.2, the randomness of the three structures satisfy the relation:

\[
\text{Randomness}(MFFC) > \text{Randomness}(MFFS) > \text{Randomness}(MFFO). \tag{3.45}
\]

3.5.4 Statistical Properties of MUX/DeMUX PUF

Reliability

The analysis of the MUX/DeMUX PUF is similar to the feed-forward structure. If the select signal of a DeMUX flips, the intra-chip variation of the response is given by
(assuming the DeMUX acts as skipping $K$ stages, while there are $N$ stages in total): 

\[
P[\text{sign}(\sum_{i=1}^{N} s_i + n_i) \neq \text{sign}(\sum_{i=1}^{N-K} s_i + n_i)]
\]

\[
= 2 \int_{0}^{\infty} \frac{1}{\sqrt{2\pi(N-K)(\sigma_s^2 + \sigma_n^2)}} \exp\left(-\frac{x^2}{2(N-K)(\sigma_s^2 + \sigma_n^2)}\right) dx
\]

\[
= 2 \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi K(\sigma_s^2 + \sigma_n^2)^2}} \exp\left(-\frac{y^2}{2K(\sigma_s^2 + \sigma_n^2)}\right) dy
\]

\[
= \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\sqrt{N-K}}{K}\right).
\]

(3.46)

It can be seen that the probability $P[\text{sign}(\sum_{i=1}^{N} s_i + n_i) \neq \text{sign}(\sum_{i=1}^{N-K} s_i + n_i)]$ increases with $K$.

If we also employ feed-forward path to generate the select signal of the DeMUXs, then $P_{\text{intra}}$ of the MUX/DeMUX PUF with a single feed-forward path can be expressed as

\[
P_{\text{intra}} = (1 - P_1)P_1 + P_1 \times \left(\frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\sqrt{N-K}}{K}\right)\right),
\]

(3.47)

where $P_1 = \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\sigma_s^2}{\sigma_n^2 + \sigma_n^2}\right)$, which is equal to $P_{\text{intra}}$ of the original MUX PUF. If $\frac{N-K}{K} < \frac{\sigma_s^4}{2\sigma_s^2\sigma_n^2 + \sigma_n^4}$, $P_{\text{intra}}$ of MUX/DeMUX PUF will be greater than $P_{\text{intra}}$ of the original MUX PUF. Therefore, similar to the feed-forward MUX PUF, the intra-chip variation of the MUX/DeMUX PUF will also depend on the number and the locations of the signal propagation paths.

**Uniqueness**

As shown in Section 3.4, the greater of the number of stages in the original MUX PUF, the less biased the output will be. Therefore, the MUX/DeMUX PUF will have less uniqueness, as some stages will be skipped under certain configurations. In other
words, \( P_{\text{inter}} \) of the MUX/DeMUX PUF is expected to be smaller than the \( P_{\text{inter}} \) of the original MUX PUF, if their total numbers of stages are the same.

**Randomness**

Since the output will be more biased, we can conclude that the randomness will also be degraded by introducing DeMUXs into the original MUX PUF.

### 3.6 Performance Comparison of Various MUX-based PUFs

Based on the statistical analysis results, the performance comparisons of the MUX-based PUFs are summarized in Table 3.4.

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Expression</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability</td>
<td>( 1 - P_{\text{intra}} )</td>
<td>original MUX &gt; MFFO &gt; MFFS &gt; MFFC</td>
</tr>
<tr>
<td>Uniqueness</td>
<td>( 1 - \left</td>
<td>2P_{\text{inter}} - 1 \right</td>
</tr>
<tr>
<td>Randomness</td>
<td>( 1 - \left</td>
<td>2P(R = 1) - 1 \right</td>
</tr>
</tbody>
</table>

These analysis results enable deeper understandings of the MUX-based PUFs, which could be exploited to improve the performance during PUF design. A number of claims are listed below:

(a) In a MUX PUF, if we increase the number of stages, uniqueness and randomness will improve while reliability will be degraded.

(b) Smaller skew of the arbiter will lead to higher uniqueness and randomness.

(c) The stage variation probability will increase with the number of previous feed-forward paths in a feed-forward structure, as the error in previous path would propagate to later stages.

(d) When designing the feed-forward PUF, an appropriate tradeoff point should be achieved based on the particular application and the performance requirement.
Designer should be careful in selecting the type, the number, and the locations of feed-forward paths.

(e) The number and the locations of the paths in the MUX/DeMUX PUF also provide a tradeoff between reliability and uniqueness.

3.7 Experiments

3.7.1 Experimental Setup

Experiments were carried out by SPICE simulations on a 65-nm technology process. We use the Monte-Carlo method to simulate the effect of process variations and environmental variations. In our simulation, we set up the transistor parameters and process variations based on a major industrial standard model, according to the findings in the area of statistical static timing analysis [11, 34]. All of the simulated MUX-based PUFs have 100 MUX stages. We placed 10 feed-forward paths regularly on the MUX stages for each PUF structure with feed-forward paths and 10 DeMUXs regularly for MUX/DeMUX PUFs. We generated 100-bit responses for measurement in our experiments. All the structures were tested by at least 1000 Monte-Carlo runs.

The inter-chip variations and the intra-chip variations are computed according to the Hamming distances obtained for different chips and the same chip under different readouts, respectively. Part of these results have already been presented in [31, 32]. The randomness values are calculated based on the total numbers of 0’s and 1’s for each MUX-based PUF structure.

3.7.2 Results

Table 3.5 presents the results of inter-chip variations, intra-chip variations, and the percentage of 1’s in the response., while Table 3.6 presents the results of the three performance indicators: reliability, uniqueness, and randomness. First, it can be observed
that the minimum inter-chip variation is larger than the maximum intra-chip variation for all of the simulated structures. Thus, we can conclude that the variations caused by the randomness in manufacturing process are more significant than the variations under different environmental conditions. Therefore, these PUFs can be used as reliable secret keys with some error correcting techniques. Second, it can also be observed that by adding feed-forward arbiters into the MUX PUF circuit, the inter-chip variations and intra-chip variations are both increased, since the noise influences the select signals of some of the intermediate stages. Furthermore, it can be seen that the modified feed-forward structures lead to better reliability than the standard feed-forward MUX PUFs. Compared to the original MUX PUF, the intra-chip variation of the standard feed-forward MUX PUF is increased by 68% on average. But the intra-chip variation of the modified feed-forward MUX PUF is only increased by 17% on average, which is only \( \frac{1}{4} \) of the standard feed-forward PUFs. Therefore, we can conclude that the reliability is improved by adopting the proposed modified feed-forward path. Finally, it can also be observed that the randomness is improved by introducing feed-forward paths into the original MUX PUF.

<table>
<thead>
<tr>
<th>Structures</th>
<th>Inter-Chip Variation</th>
<th>Intra-Chip Variation</th>
<th>( P(R = 1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original MUX</td>
<td>Max 59%</td>
<td>Min 22%</td>
<td>Max 13%</td>
</tr>
<tr>
<td>Feed-forward Overlap</td>
<td>Max 66%</td>
<td>Min 27%</td>
<td>Max 15%</td>
</tr>
<tr>
<td>Feed-forward Cascade</td>
<td>Max 64%</td>
<td>Min 25%</td>
<td>Max 20%</td>
</tr>
<tr>
<td>Feed-forward Separate</td>
<td>Max 65%</td>
<td>Min 26%</td>
<td>Max 17%</td>
</tr>
<tr>
<td>Modified Feed-forward Overlap</td>
<td>Max 61%</td>
<td>Min 25%</td>
<td>Max 14%</td>
</tr>
<tr>
<td>Modified Feed-forward Cascade</td>
<td>Max 64%</td>
<td>Min 25%</td>
<td>Max 15%</td>
</tr>
<tr>
<td>Modified Feed-forward Separate</td>
<td>Max 61%</td>
<td>Min 27%</td>
<td>Max 15%</td>
</tr>
<tr>
<td>MUX/DeMUX</td>
<td>Max 57%</td>
<td>Min 23%</td>
<td>Max 16%</td>
</tr>
</tbody>
</table>
Table 3.6: Results of Performance Indicators for 100-Stage PUFs

<table>
<thead>
<tr>
<th>Structures</th>
<th>Reliability</th>
<th>Uniqueness</th>
<th>Randomness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original MUX</td>
<td>94.2%</td>
<td>88.2%</td>
<td>65.6%</td>
</tr>
<tr>
<td>Feed-forward Overlap</td>
<td>91.3%</td>
<td>95.0%</td>
<td>77.6%</td>
</tr>
<tr>
<td>Feed-forward Cascade</td>
<td>89.3%</td>
<td>97.5%</td>
<td>84.2%</td>
</tr>
<tr>
<td>Feed-forward Separate</td>
<td>90.1%</td>
<td>96.2%</td>
<td>80.6%</td>
</tr>
<tr>
<td>Modified Feed-forward Overlap</td>
<td>93.4%</td>
<td>93.5%</td>
<td>74.6%</td>
</tr>
<tr>
<td>Modified Feed-forward Cascade</td>
<td>93.0%</td>
<td>95.9%</td>
<td>79.8%</td>
</tr>
<tr>
<td>Modified Feed-forward Separate</td>
<td>93.1%</td>
<td>94.6%</td>
<td>76.8%</td>
</tr>
<tr>
<td>MUX/DeMUX</td>
<td>92.9%</td>
<td>83.8%</td>
<td>59.8%</td>
</tr>
</tbody>
</table>

3.7.3 Discussion

By comparing the experimental results presented in Table 3.5 and Table 3.6, it can be concluded that the relations between the performances of different types of MUX-based PUFs are consistent with the theoretical results shown in Table 3.4. Note that the value of $P(R = 1)$ which is more close to 0.5 indicates better randomness. It can also be observed from Table 3.5 and Table 3.6 that the feed-forward separate structure is the most reliable structure while the feed-forward cascade is the least reliable one among the three feed-forward structures. Moreover, the MUX/DeMUX PUF has relatively low inter-chip variations; as a result, the uniqueness of this structure is decreased.

These experimental results validate the correctness of our statistical analysis. Overall, all the MUX-based PUF structures can be used as reliable secret keys for authentication and identification within certain error tolerance, as the PUFs exhibit sufficient gaps between the minimum of the inter-chip variations and the maximum of intra-chip variations.

3.8 Conclusion

We have presented a systematic statistical approach to quantitatively evaluate various types of MUX-based PUFs. We defined three performance indicators - reliability, uniqueness, and randomness - to compare the performances of these MUX-based PUFs.
These indicators are also validated by the corresponding simulation results. The experimental results show that the proposed statistical analysis approach effectively reflects the characteristics of various PUF designs. We have also proposed a novel modified feed-forward MUX PUF structure, which has better reliability than the standard feed-forward MUX PUF.
Chapter 4

Lightweight, Secure, and Reliable PUF-based Local Authentication with Self-Correction

4.1 Introduction

As we discussed in previous chapters, Physical Unclonable Function (PUF) is a powerful tool for chip authentication and cryptographic applications [2, 22]. Unfortunately, PUFs are noisy in nature. The response of a PUF would be affected by intra-chip variation sources such as temperature changes, voltage drifts, and aging effects. The reported PUFs in the literature, such as optical PUF [35], multiplexer PUF [10], ring oscillator PUF [22], butterfly PUF [36], SRAM PUF [16], and sensor PUF [37], are not 100% stable. However, cryptography in general relies on the existence of precisely reproducible keys. As a result, it is clear that the plain PUF responses are not suitable as cryptographic keys. One solution to this problem is to add a stage to correct the errors after collecting the PUF response based on error correcting codes (ECC) such as Bose-Chaudhuri-Hocquenghem (BCH) codes [38, 39] or fuzzy extractors [40, 41]. In
order to obtain reliable responses from PUFs, while still keeping PUFs attractive for low-cost hardware applications, the error correction technique must be implemented in hardware as well. Moreover, its implementation should be area-efficient; otherwise, it will defeat the purpose of using PUFs in lightweight hardware devices.

This chapter presents a two-level finite-state machine (FSM) architecture which can be used to authenticate a chip. Besides, a novel self-correcting approach is also proposed based on the two-level FSM which eliminates the use of high overhead error correcting techniques. In the literature, FSM-based techniques have been incorporated into a number of works on hardware device protection, which include active metering [42], remote IC activation [43], FPGA IP binding [44], and obfuscation [45, 46]. The major advantage of using FSM is that it is not extractable from the synthesized design. Thus, even for an adversary who has access to the synthesized hardware IP, extracting or changing the FSM would need an effort equivalent to redoing all the stages of design and implementation [47]. By utilizing the benefit of FSM, our proposed approach can achieve a lightweight, secure and reliable authentication scheme. Different from previous works, we incorporate self-correction into the FSM, which could eliminate the use of high-overhead error correcting methods. This work perhaps is the first work to develop an error correcting method that is tailored for PUF-based local authentication. In fact, the BCH codes are usually used in storage devices and communication systems [48], and the first fuzzy extractor construction is aimed at biometric applications [49].

4.2 Background

4.2.1 PUF-based Authentication

Remote Authentication

As the PUF response is unique and unpredictable for each IC, it is straightforward to use PUF for IC authentication. In the literature, authentication using PUF response
bits as secret keys has been explored in many previous works [38, 39, 50, 51]. Most of the existing PUF-based authentication schemes are remote authentications, which involve a device and a trusted party or so-called server. A communication link should be established between the server and the devices. During the enrollment phase, the trusted party applies randomly chosen challenges to the device to collect unpredictable responses. Then, the trusted party stores these CRPs in a database for future authentication operations. Later, if a device initiates an authentication request, the trusted party will send a challenge to the device and obtain the PUF response through the communication link. The device will be considered as authentic only if the response is the same as the previously recorded one or only vary in a predefined range. However, remote authentication schemes suffer from man-in-the-middle attacks. The adversary will be able to perform modeling attacks to create a software program after collecting a set of CRPs.

Local Authentication

The problem can be resolved by using a local authentication scheme, as CRPs will not be transmitted through a communication link. Local authentication can be used to verify that each component inside a system is authentic and has not been tampered with. Local authentication is applicable to different layers of the design hierarchy: for instance, a controller can authenticate each IC in a system, an IC can authenticate each IP block, an IP can authenticate each functional unit, and so on. Moreover, local authentication is particularly useful in the applications where the communication link between the server and the device cannot be established or the server is not available. Note that local authentication scheme can also achieve IC metering [42] and IP binding [44]. Figure 4.1 illustrates the basic process of PUF-based local authentication. Unlike the remote authentication that authenticated CRPs are kept on a trusted server, secret information has to be stored on chip in a local authentication scheme. For example, after fabrication, an authenticated response for a given challenge can be programmed
into the memory of the chip after fabrication. During the authentication phase, a re-generated response of the PUF is compared to values in the on-board memory to check the authenticity of the device. Alternatively, the designer can provide the authenticated CRPs to the customers which can be considered as a key that is required to be entered during authentication process, which could protect their ownership.

![Figure 4.1: PUF-based local authentication.](image)

### 4.2.2 Error Correction

One major issue for PUF-based local authentication is the robustness of the system, since environmental variations will also affect the PUF response. In a server-based remote authentication system, this issue can be easily resolved by tolerating certain number of bit errors. For example, the server could authenticate a PUF response whose Hamming distance to the desired response is less than a certain threshold. However, this method cannot be used in a local authentication scheme due to the high overhead. Additionally, it is not feasible to store a large amount of CRPs on chip. As mentioned in Section 4.1, ECC and fuzzy extractor can be employed into PUF-based authentication protocols to improve their robustness, which could also be used in the local authentication scenario. However, given the fact that PUF is usually a compact circuit, the use of error correcting techniques significantly increases the design complexity. Moreover, there is a security concern that the syndrome or helper bits will reveal information about the secret bits. Therefore, error correction has to be secure, robust and efficient.
In a typical error correction setting for PUF, during an initialization phase, a PUF response is generated and the error correcting syndrome is computed based on this response. The syndrome or helper data is public information which is later sent to the PUF along with the challenges to perform correction on response bits. Equivalently, the syndrome can be stored locally on chip. To re-generate the same PUF output, the PUF first produces a response from the circuit. Then, the PUF uses the syndrome from the initialization step to correct any errors in the circuit output. In this way, the PUF can consistently reproduce the output from the initialization step if the device is authentic.

However, in most of previous works on correcting PUF responses, the area complexities of the ECCs or the fuzzy extractors were not considered and there was no detailed explanation on how to choose the algorithm and the parameters [38, 39]. In fact, the implementation costs and hardware overheads of the commonly used ECCs are relatively high, compared to the PUF circuit [52]. Besides of using ECC or fuzzy extractor, other error correcting methods have also been exploited in the literature. The method of utilizing majority voting on reducing errors has been demonstrated in [53]. The use of repetition codes along with conventional syndrome generation using XOR masking has been proposed for PUFs in [41]. Soft-decision encoders and decoders have also been employed to correct PUF response errors [54]. However, the hardware implementations of these methods have been only based on FPGAs and the area complexities have not been addressed [53, 41, 54, 55]. Furthermore, authentications using pattern matching algorithms [52, 51] need to maintain a database to store pattern information, which are also not suitable for local authentication. In contrast to the above works, this chapter proposes a novel secure, reliable and efficient error correcting method which can be used for PUF-based local authentication.
4.3 Two-Level FSM Architecture

This section presents a basic two-level FSM scheme which could be used for PUF-based authentication, IP binding, and IC metering. The general concept is illustrated in the state transition graph (STG) of Figure 4.2. The PUF response and an authentication key (i.e., Key in Figure 4.2) are used to determine the state of the FSM. The PUF response is the input of the first level FSM, while key is the input of the second level FSM. The first level FSM is designed to transit to a unique intermediate state for each unique PUF response. Only if the actual PUF response is the same as the desired PUF response, the FSM will enter into the correct intermediate state. Furthermore, only one key value will transit the second level FSM from a certain intermediate state to the desired state (i.e., Auth in Figure 4.2). The desired state then can be used to authenticate the circuit or activate the correct functionality of certain blocks. Basically, this architecture generates unique mapping pairs between PUF response and key, i.e., \((R_i, K_i)\).

![Figure 4.2: Two-level FSM.](image)

It is important to note that \((R_i, K_i)\) can be arbitrarily designed. Thus, all such possible pairs are only known to the designer. For an \(N\)-bit PUF response, there will be
intermediate states. The length of the key will at least be $N$, if we ensure only one value of the key could transit the FSM into the desired state. Note that the lengths of PUF response and the key are not necessarily identical, i.e., $N$-to-$N$ mappings. A longer key can be used to increase the complexity of the structure. At the expense of increasing the probability of key collision, multiple PUF responses can also be mapped into one intermediate state or multiple key values can be designed as correct inputs to a PUF response. Moreover, the $(R_i, K_i)$ mappings can be designed differently for different chips. As a result, an adversary with access to the response and key authentication records from other devices is still unable to authenticate a new device.

For example, we consider an example of a 3-bit PUF response as shown in Figure 4.3. Without loss of generality, we can assume the values of $R_i$ as marked in Figure 4.3, respectively. The correct $(R_i, K_i)$ pairs are summarized in Table 4.1, where the values of $K_i$ can be arbitrarily chosen.

![Figure 4.3: A 3-bit example of the two-level FSM.](image)

We can add another state Unauth as shown in Figure 4.4. If the key entered is wrong, the FSM will transit into the Unauth state. This stage can be used to lock the chip or trigger an alarm to report a possible attack.
Table 4.1: Key Values Can Successfully Authenticate the Corresponding PUF Response

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>correct $K_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>$K_1$</td>
</tr>
<tr>
<td>001</td>
<td>$K_2$</td>
</tr>
<tr>
<td>010</td>
<td>$K_3$</td>
</tr>
<tr>
<td>011</td>
<td>$K_4$</td>
</tr>
<tr>
<td>100</td>
<td>$K_5$</td>
</tr>
<tr>
<td>101</td>
<td>$K_6$</td>
</tr>
<tr>
<td>110</td>
<td>$K_7$</td>
</tr>
<tr>
<td>111</td>
<td>$K_8$</td>
</tr>
</tbody>
</table>

Figure 4.4: Two-level FSM that also has a lock or alarm state.
The global flow of the local authentication scheme by utilizing the proposed technique is shown in Figure 4.5. For example, Company X designs the circuit and integrates a PUF and the self-correcting FSM into the design. The \((R_i, K_i)\) pairs are designed at this stage. After that, Company X sends the detailed manufacturable design specifications to Foundry Y who makes the mask and manufactures multiple chips implementing this design. Each chip will be uniquely locked after fabrication due to the inter-chip variations of the PUFs. For each chip, the PUF response needs to be tested, which can be conducted by either Foundry Y or Company X if Foundry Y sends the manufactured chips back to Company X. If the tests are done by Foundry Y, the PUF responses should be sent back to Company X. According to the PUF response (which will be considered as the desired PUF response for the authentication phase), only Company X has knowledge of calculating the key for entering into the correct authenticated state. The key could be programmed by another honest vendor after the chips have been fabricated or sold to the customer, i.e., Company Z. Therefore, the correct key is a strong proof of ownership. At the beginning of the authentication phase, the FSM enters into an intermediate state based on the variability-induced response of PUF, as shown in Figure 4.6. Only Company Z, who has the possession of the correct key, can authenticate the chip.

The global flow can also prevent the ICs piracy from overbuilding, as modern chip designs are usually outsourced for fabrication. For example, it is conceivable that a dishonest manufacturing plant could create more chips than ordered and sell the additional chips at a lower cost, subverting the profits of the legitimate owner. However, by employing the proposed two-level FSM, over-produced chips that without the correct keys cannot function properly, since the manufacturing plant (Foundry Y, for example) does not know the correct \((R_i, K_i)\) pairs.
4.4 Error Correction based on the Two-Level FSM

4.4.1 Self-Correcting Functionality

We propose a novel FSM structure which not only has the capability for PUF-based authentication, but also could correct certain number of PUF response bit errors, caused due to environmental variations, to improve the robustness.

The two-level FSM structure presented in Section 4.3 can be extended to incorporate the error correcting functionality by allowing a second key attempt, as shown in Figure 4.6. For instance, in the conventional two-level FSM authentication scheme, if the PUF response varies due to environmental noise, the FSM will enter into a different intermediate state and the device cannot be authenticated with the correct key. The problem can be solved by introducing a pathway from intermediate state $S'_i$ to the authenticated state, where $S'_i$ represent a state whose Hamming distance of the corresponding PUF response to the correct PUF response is less than or equal to $m$ bits. Note that $S'_i$ and $S_j$ as shown in Figure 4.6 may represent multiple candidate states. If the PUF response varies within $m$ bits in the first level of the FSM structure, the second attempt of the correct key $K_i$ will bring the FSM back into the desired intermediate state.
state $S_i$. The advantage of the proposed approach is the inherent redundancy built into the self-correcting FSM by contiguously entering the key twice that eliminates the need for an extra error correcting code. We can also design a scheme that the key will always be internally entered to the FSM twice.

Figure 4.6: Two-level FSM can fix up to $m$-bit intra-chip errors (i.e., $1 \leq HD \leq m$ bit).

A 3-bit example of such a structure is shown in Figure 4.7. For example, if the response of the PUF has been tested to be 010 during the enrollment phase, the designer can calculate the correct key as $K_3$. However, during the authentication phase, the PUF response may vary 1 bit due to environmental variations, e.g., 000 instead of 010. In the proposed structure, the FSM could transit into the desired black state $S_3$ from the grey state $S_1$ by entering the correct key $K_3$ for the first time. Then, if the authenticated user enters $K_3$ once more, the FSM will transit to the authenticated state. For other PUF responses that have Hamming distances of 2 or greater, $K_3$ would not be able to bring the FSM back into the authenticated state. Note that only the edges associated with $K_3$ are shown in the STG, while other edges are omitted in Figure 4.7. The complete next state table of the FSM design is presented in Table 4.2.

Similar to error correcting codes or fuzzy extractor, adding the error correcting functionality will degrade the level of security. The probability for the adversary to
Figure 4.7: A simplified 3-bit example for the proposed scheme.

Table 4.2: Key Values Can Successfully Authenticate the Corresponding PUF Response with Self-Correction

<table>
<thead>
<tr>
<th>Present State</th>
<th>K_1</th>
<th>K_2</th>
<th>K_3</th>
<th>K_4</th>
<th>K_5</th>
<th>K_6</th>
<th>K_7</th>
<th>K_8</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_0</td>
<td>S_1</td>
<td>S_2</td>
<td>S_3</td>
<td>S_4</td>
<td>S_5</td>
<td>S_6</td>
<td>S_7</td>
<td>S_8</td>
</tr>
<tr>
<td>S_1</td>
<td>Auth</td>
<td>S_2</td>
<td>S_3</td>
<td>Unauth</td>
<td>S_4</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
</tr>
<tr>
<td>S_2</td>
<td>S_1</td>
<td>Auth</td>
<td>Unauth</td>
<td>S_4</td>
<td>Unauth</td>
<td>S_6</td>
<td>Unauth</td>
<td>Unauth</td>
</tr>
<tr>
<td>S_3</td>
<td>Unauth</td>
<td>S_1</td>
<td>Auth</td>
<td>S_4</td>
<td>Unauth</td>
<td>Unauth</td>
<td>S_7</td>
<td>Unauth</td>
</tr>
<tr>
<td>S_4</td>
<td>Unauth</td>
<td>S_2</td>
<td>S_3</td>
<td>Auth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>S_8</td>
</tr>
<tr>
<td>S_5</td>
<td>S_1</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Auth</td>
<td>S_6</td>
<td>S_7</td>
<td>Unauth</td>
</tr>
<tr>
<td>S_6</td>
<td>Unauth</td>
<td>S_2</td>
<td>Unauth</td>
<td>Unauth</td>
<td>S_5</td>
<td>Auth</td>
<td>Unauth</td>
<td>S_8</td>
</tr>
<tr>
<td>S_7</td>
<td>Unauth</td>
<td>Unauth</td>
<td>S_3</td>
<td>Unauth</td>
<td>S_5</td>
<td>Unauth</td>
<td>Auth</td>
<td>S_8</td>
</tr>
<tr>
<td>S_8</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>S_4</td>
<td>Unauth</td>
<td>S_6</td>
<td>S_7</td>
<td>Auth</td>
</tr>
<tr>
<td>Auth</td>
<td>Auth</td>
<td>Auth</td>
<td>Auth</td>
<td>Auth</td>
<td>Auth</td>
<td>Auth</td>
<td>Auth</td>
<td>Auth</td>
</tr>
<tr>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
<td>Unauth</td>
</tr>
</tbody>
</table>
guess the key value for a given PUF response will increase from $\frac{1}{2^N}$ to $\frac{1+N}{2^N}$, if one bit error can be corrected for an $N$-bit PUF response. More generally, for $m$ bits correction in an $N$-bit PUF response, $1+\sum_{i=1}^{m} \binom{N}{m}$ key values can bring a certain intermediate state to the correct Auth state, while there are $2^N$ possible key values in total. In the example of Figure 4.7, 4 out of 8 possible key values can authenticate the corresponding PUF response. The details of successful key values that can authenticate the corresponding PUF response are presented in Table 4.3. For example, besides $K_3$, another three key values $K_1$, $K_4$, $K_7$ can also authenticate the PUF response 010. However, when $N$ is large (e.g., $N = 256$), the value of $\frac{1+N}{2^N}$ will still be very small for a 1-bit correction scheme. Furthermore, a requirement for a practical PUF is that the PUF responses should have a large inter-chip variation (50% Hamming distance ideally) so that even in the presence of noise it is possible to distinguish responses originating from different devices. Therefore, key collision of the proposed self-correcting approach would not be an issue for PUF-based authentication. A set of distinguishable keys can be obtained for different chips even if the $(R_i, K_i)$ pairs are designed equivalently. Moreover, as discussed in Section 4.3, we can increase the length of key to improve the security.

Table 4.3: Key Values Can Successfully Authenticate the Corresponding PUF Response with Self-Correction

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>correct $K_i$</th>
<th>other successful $K_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>$K_1$</td>
<td>$K_2, K_3, K_5$</td>
</tr>
<tr>
<td>001</td>
<td>$K_2$</td>
<td>$K_1, K_4, K_6$</td>
</tr>
<tr>
<td>010</td>
<td>$K_3$</td>
<td>$K_1, K_4, K_7$</td>
</tr>
<tr>
<td>011</td>
<td>$K_4$</td>
<td>$K_2, K_3, K_6$</td>
</tr>
<tr>
<td>100</td>
<td>$K_5$</td>
<td>$K_1, K_6, K_7$</td>
</tr>
<tr>
<td>101</td>
<td>$K_6$</td>
<td>$K_2, K_5, K_8$</td>
</tr>
<tr>
<td>110</td>
<td>$K_7$</td>
<td>$K_3, K_5, K_8$</td>
</tr>
<tr>
<td>111</td>
<td>$K_8$</td>
<td>$K_4, K_6, K_7$</td>
</tr>
</tbody>
</table>

4.4.2 Advantages

Generally speaking, if we want to correct up to $m$ bits of an $N$-bit PUF response, $\sum_{i=1}^{m} \binom{N}{m}$ extra transition edges need to be inserted for each intermediate state in the
state transition graph of the conventional two-level FSM. A FSM is usually defined by a 6-tuple \((I, O, S, S_0, F, G)\), where \(S\) is a finite set of internal states, \(I\) and \(O\) represent finite set of inputs and outputs of the FSM, respectively, \(F\) is the next-state function, \(G\) is the output function, and \(S_0\) is the initial state. However, the extra transition edges introduced by the error correcting functionality only affect the next state logic of the FSM, while the output logic and the size of state registers remain the same, as shown in Figure 4.8. As a result, we can expect the extra design complexity would be relatively small for the self-correcting FSM structure, which only involves an \(N\)-to-\(N\) combinational logic synthesis. Furthermore, although there are \(2^N\) intermediate states in the proposed two-level FSM, the state registers only need to store \(N + 1\) bits in total (including \(S_0\), \textit{Auth}, and \textit{Unauth}). For example, when \(N = 3\) as shown in Figure 4.7, only a 4-bit state registers are required, as there are 11 states in the design. Therefore, the proposed self-correcting FSM scheme could enable lightweight yet reliable PUF-based authentication.

![Figure 4.8: Only the output function of the FSM needs to be redesigned after adding the self-correcting functionality.](image)

Another advantage is that the proposed scheme is more secure, compared to ECC or fuzzy extractor. For example, in the applications of ECC, when testing a PUF response, an error correcting syndrome (helper data, if for a fuzzy extractor) for that response is also computed and saved for later use. The syndrome is information that allows for correcting bit-flips in regenerated PUF outputs. The generated syndrome
is public information and can be stored anywhere (on-chip, off-chip, or remotely on a server). Clearly, the syndrome reveals information about the PUF response. In general, given the $b$-bit syndrome, attackers can learn at most $b$ bits of the $N$-bit PUF response, where $b$ is less than $N$ and is dependent on the specific parameters of the employed ECC. The soft-decision syndrome coding scheme proposed in [54] could limit the amount of leaked information to improve the security of PUF-based authentication. However, the generated syndrome is still associated with the PUF response.

As opposed to these prior works, the key in our proposed self-correcting FSM is not public that only the designer or the authenticated users have the knowledge of the correct key. Furthermore, the fact that the PUF response and authentication key pairs $(R_i, K_i)$ can be arbitrarily designed, which are not based on any algorithm such as ECC or fuzzy extractor, also enhances the security. In other words, even if the adversary knows the PUF response (or key) in a $(R_i, K_i)$ pair, it is still infeasible to guess the corresponding key (or PUF response). Another advantage is that the successful key values are not close, i.e., Hamming distance is less than a certain threshold. For example, even for two pairs $(R_i, K_i)$ and $(R_j, K_j)$ that the Hamming distance of the two PUF responses $R_i$ and $R_j$ is only 1, the Hamming distance of the two keys $K_i$ and $K_j$ could be very large. Additionally, along with the fact as discussed above that extraction of the corresponding STG of the FSM is a computationally intractable task, the proposed scheme could achieve a very high level of security.

### 4.5 Other Applications

In fact, the proposed self-correcting two-level FSM can be extended for other applications. In this section, we discuss a number of other possible applications.
4.5.1 Two-Factor Authentication

The proposed self-correcting two-level FSM can also be used for the so-called two-factor authentication [56]. The challenge of a PUF is combined with the key to achieve stronger hardware protection. The authenticated device, correct PUF challenge, and correct key are required for the two-factor authentication. In other words, the \((R_i, K_i)\) pair is extended to \((C_i, R_i, K_i)\). The state of the proposed two-level FSM is determined by \(R_i\) and \(K_i\), while \(R_i\) can be calculated by the challenge \(C_i\) for a given PUF. However, PUF is a one-way function in the sense that it is hard to reconstruct the challenge from the response. Therefore, even if the adversary knows the desired \((R_i, K_i)\) pair, it is still infeasible for the adversary to compromise the device without knowing the correct challenge. Additionally, as described in Section 4.3, the \((R_i, K_i)\) pairs can be designed differently for different devices. As a result, \((C_i, R_i)\) and \((R_i, K_i)\) pairs will be unique for each chip. The security can be greatly improved by the proposed two-factor authentication. The security properties are summarized below:

(a) The device cannot be duplicated.

(b) The user is unable to authenticate without the device.

(c) The device cannot be used by someone else to successfully authenticate as the user without the correct key.

(d) An adversary with access to the response and key authentication records from other devices is still unable to authenticate a new device without the correct challenge.

(e) The device does not need to store any information.

4.5.2 Signature Generation

The proposed self-correcting FSM architecture can also be utilized for reliable signature generation. The FSM can be modified as shown in Figure 4.9 to regenerate
the same PUF output, which would be more straightforward from the error correction aspect. Instead of having two states (i.e., Auth and Unauth) at the last stage, the FSM structure can be extended to enable an $N$-bit output by adding extra $2^N$ output states (OSs) at the last stage. Note that $OS_i'$ and $OS_j$ may represent multiple states, while $R_i'$ and $R_j$ may represent multiple incorrect PUF responses. Each output state $OS_i$ will generate a unique output $R_i$, which corresponds to the desired PUF response. For example, if the actual PUF response $R_i'$ vary within a tolerated range as the correct PUF response $R_i$, the FSM will enter into an intermediate state $S_i'$. By adding the transition edges of the proposed self-correcting functionality into the STG, the FSM could transit to the desired intermediate state $S_i$ by entering the correct key $K_i$ for the first time. When entering $K_i$ the second time, the FSM will transit into $OS_i$ and the correct PUF response $R_i$ will be generated. Key values other than $K_i$ cannot bring the FSM to the last stage of the FSM from state $S_i$. As a result, the same PUF response cannot be regenerated. In this case, the key value can be made public which will be similar to the functionality of syndrome in ECC or helper data in fuzzy extractor. As mentioned in Section 4.4, it is still infeasible to predict the corresponding PUF response even if the adversary knows the key value.

![Figure 4.9: Reliable signature generation by utilizing the self-correcting FSM.](image)
4.5.3 Obfuscation

A FSM-based obfuscation approach is proposed in [46]. The obfuscating FSM can be replaced by the proposed two-level FSM. An example of the architecture is shown in Figure 4.10. The correct PUF response and key pair will activate the reconfigurator (i.e., FSM will transit into the output state \( OS_1 \)), which controls the functionality of the circuit. In this example, the output states determine the mode of a switch. The switch will work correctly (i.e., output \{1, 1, 2, 0\} in a period of 4) only with the correct configure data \( D_1 \). The error correcting functionality can also be incorporated into the structure.

![Proposed Two-Level FSM](image)

Figure 4.10: Architecture with authentication and obfuscation.

The functionality of the circuit is integrated into the self-correcting FSM. It is infeasible to isolate the correct functionality of the circuit, because the extraction of the corresponding state transition graph is a computationally intractable task. Consequently, the proposed architecture enables authentication as well as achieves obfuscation.
4.5.4 Correcting Other Errors

Although the proposed design of self-correcting two-level FSM is tailored for PUF-based local authentication, this technique is not limited to correcting PUF response errors. For example, the two-level FSM can be used for correcting communication errors, where the key in our design will be treated as the syndrome in an ECC. The proposed FSM architecture is also applicable to authentication for the Internet of Things (IoT), where security is an important concern. As discussed in Section 4.4, the proposed error correcting method could achieve higher security than the commonly used ECC or fuzzy extractor, as syndrome will leak information of the secret bits to the adversary, while \((R_i, K_i)\) pairs in the two-level FSM can be arbitrarily chosen.

4.6 Hardware Implementation

In this section, we present the performance of the proposed two-level self-correcting FSM. All the circuits are synthesized using Synopsys Design Compiler with optimization parameters set for minimum area and mapped to a 65 nm standard cell library. Note that we use a same bit-length for the PUF response and key in our implementations. We first examine the performance with respect to the PUF response bit-length \(N\), and the number of tolerated error bits \(m\). Then we compare the proposed error correcting technique to one commonly used ECC for PUF-based authentication, i.e., the BCH codes.

4.6.1 Area and Power

Table 4.4 and Table 4.5 show the area and power consumptions of the FSM as shown in Figure 4.6, respectively, for different design parameters (i.e., \(N\) and \(m\)). Note that when \(m = 0\), the implemented structure is reduced to the FSM without self-correction as shown in Figure 4.4. The results include average area and power overheads over a number of different implementations, where the PUF response and key value mappings
are randomly designed (include both simple bitwise comparison and highly random perturbation).

Table 4.4: Area (Gate Count) of the Proposed Self-Correcting FSM that Can Correct $m$ Bits of an $N$-Bit PUF Response

<table>
<thead>
<tr>
<th>$m$</th>
<th>$N$</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>70</td>
<td>114</td>
<td>197</td>
<td>367</td>
<td>709</td>
<td>1399</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>85</td>
<td>134</td>
<td>235</td>
<td>470</td>
<td>919</td>
<td>1810</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>147</td>
<td>265</td>
<td>507</td>
<td>985</td>
<td>1909</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>281</td>
<td>514</td>
<td>994</td>
<td>1963</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Power ($\mu W$) of the Proposed Self-Correcting FSM that Can Correct $m$ Bits of an $N$-Bit PUF Response

<table>
<thead>
<tr>
<th>$m$</th>
<th>$N$</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0.60</td>
<td>0.95</td>
<td>1.66</td>
<td>3.03</td>
<td>5.79</td>
<td>11.44</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>0.63</td>
<td>1.00</td>
<td>1.91</td>
<td>3.61</td>
<td>6.83</td>
<td>13.47</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1.12</td>
<td>2.08</td>
<td>3.80</td>
<td>7.47</td>
<td>14.55</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>2.11</td>
<td>3.87</td>
<td>7.61</td>
<td>15.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As expected, the area and power consumptions are not very significant. For example, the area of the proposed FSM for a 128-bit PUF response with 7 bits error correction is only equivalent to 2061 NAND2 gates, while the power consumption is about 16$\mu W$.

As expected, the area and power consumptions are not very significant. For example, the area of the proposed FSM for a 128-bit PUF response with 7 bits error correction is only equivalent to 2061 NAND2 gates, while the power consumption is about 16$\mu W$. This can be compared to 1399 gates and 11$\mu W$ with no error correction for a 128-bit PUF response.

For better illustrations, we plot the gate counts for $m = 0$ (without self-correction) and $m = 2$ (with 2 bits error correction) as shown in Figure 4.11. It can be observed
that when the bit-length $N$ is doubled, the area is also almost doubled for a fixed $m$. The power consumption exhibits a similar trend.

![Figure 4.11: Area (gate count) for different bit-length $N$ without self-correction and with 2 bits error correction.](image)

We also plot the gate counts for different $m$ for $N = 64$ as shown in Figure 4.12, which is normalized to the gate count of the FSM when $m = 0$ and $N = 64$. It can be seen that the overhead is about 30% for adding 1-bit error correcting functionality into the conventional two-level FSM. However, as $m$ increases, the additional overhead in area or power consumption becomes less and less. For example, the overhead is 47% when $m = 7$, while the overhead is already 39% when $m = 2$. Therefore, we can expect that overhead of the proposed self-correcting FSM would be reasonable even for a large $m$. Note that we can draw similar conclusions for area and power consumptions of the FSM with other values of $N$, as shown in Table 4.4 and Table 4.5.

### 4.6.2 Comparison to BCH Codes

As illustrated in Section 4.2, the hardware implementation of the error correcting techniques in prior works on PUF-based authentication were either not considered or only implemented on FPGAs. However, error correcting techniques that incur relatively large overhead, compared to PUF circuit, would not be suitable for lightweight and low-cost devices. Furthermore, to the best of our knowledge, there is no other work on
efficient hardware implementation of error correcting method that is particularly well suited for PUF-based authentication in the literature. As a result, it is important to examine the performance comparison with other state-of-the-art low-cost error correcting techniques. In order to achieve fair comparison, we also synthesized comparable BCH decoders using the same 65 nm standard cell library. The area and power consumptions for different parameters are presented in Table 4.6 and Table 4.7, respectively. Note that the values of \( N \) are chosen to be 1 less than \( 2^n \), as that is the highest value permitted in the finite field of \( 2^n \).

Table 4.6: Area (gate count) of the BCH Codes

<table>
<thead>
<tr>
<th>( m )</th>
<th>( N )</th>
<th>31</th>
<th>63</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1031</td>
<td>1370</td>
<td>1760</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1816</td>
<td>2410</td>
<td>3092</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2612</td>
<td>3460</td>
<td>4435</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3419</td>
<td>4519</td>
<td>5789</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4231</td>
<td>5580</td>
<td>7153</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5039</td>
<td>6646</td>
<td>8527</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5854</td>
<td>7720</td>
<td>9908</td>
<td></td>
</tr>
</tbody>
</table>

It can be seen from our experimental results that the proposed self-correcting FSM consumes about \( 2 \times \) to \( 10 \times \) less area and about \( 20 \times \) to \( 100 \times \) less power than the BCH codes. Therefore, it can be concluded that the cost of correcting PUF response can be
Table 4.7: Power ($\mu W$) of the BCH Codes

<table>
<thead>
<tr>
<th>$m$</th>
<th>$N$</th>
<th>$31$</th>
<th>$63$</th>
<th>$127$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>181.67</td>
<td>228.86</td>
<td>286.60</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>319.54</td>
<td>401.99</td>
<td>503.07</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>458.77</td>
<td>576.39</td>
<td>721.18</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>598.77</td>
<td>751.31</td>
<td>940.61</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>739.70</td>
<td>926.44</td>
<td>1161.15</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>881.98</td>
<td>1102.21</td>
<td>1382.33</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>1024.90</td>
<td>1238.89</td>
<td>1604.58</td>
</tr>
</tbody>
</table>

significantly reduced by the proposed approach. Particularly, the power consumption can be reduced to $1\% \sim 5\%$ of the BCH codes. Additionally, as discussed above, the extra overhead of the proposed self-correcting FSM will be small for a large number of tolerated error bits $m$. However, for the BCH codes, it can be observed from Table 4.6 and Table 4.7 that both the area and power consumptions increase linearly with the number of tolerated error bits, which is also consistent with the results in [57]. Therefore, we can expect that the area consumption of the proposed method will be significantly less than the BCH codes for a large $m$.

Furthermore, it is important to note that the proposed FSM architecture not only corrects the errors, but also has the capability for PUF-based authentication. If we only consider the design complexity for the error correcting functionality itself, the proposed approach would be much more lightweight and low-cost compared to the BCH codes. For example, we consider the overhead of introducing 4 bits error correcting functionality to the two-level FSM without error correction. The area and power overhead results for both the proposed self-correction FSM and the BCH codes are normalized to the cost of the two-level FSM without error correction, as shown in Figure 4.13 and Figure 4.14, respectively. It can be seen that the normalized overheads of the BCH codes are significantly greater than those of the proposed self-correcting FSM. For instance, when $N = 128$ and $m = 4$, the normalized area overhead for the proposed self-correcting
FSM is $9 \times$ less than the BCH codes, while the normalized power overhead for the proposed self-correcting FSM is $167 \times$ less than the BCH codes. It can also be observed that the overhead incurred by BCH codes will decrease as $N$ increases. However, the length of PUF response used for authentication is usually relatively small ($N \leq 256$). Therefore, it can be concluded that the overhead of the proposed self-correcting FSM is significantly less than the BCH codes for the PUF-based authentication.

Figure 4.13: Normalized area overheads of introducing 4 bits error correcting functionality for the proposed self-correcting FSM and the BCH codes.

Figure 4.14: Normalized power overheads of introducing 4 bits error correcting functionality for the proposed self-correcting FSM and the BCH codes.
4.7 Conclusion

This chapter has presented a novel two-level self-correcting FSM, which can be used for PUF-based authentication while tolerating a certain number of bit errors that are generated by environmental variations. The applications of the proposed method for local authentication and reliable signature generation have also been discussed. The performances of the proposed two-level self-correcting FSM with respect to the PUF response bit-length and the number of tolerated error bits have been studied. We have also shown that the proposed technique achieves significantly lower cost than the error correcting methods that are previously used for PUF-based authentication.
Chapter 5

Obfuscatng DSP Circuits via High-Level Transformations

5.1 Introduction

Digital Signal Processing (DSP) plays a critical role in numerous applications such as video compression, portable systems/computers, multimedia, wired and wireless communications, speech processing, and biomedical signal processing. However, as electronic devices become increasingly interconnected and pervasive in people’s lives, security, trustworthy computing, and privacy protection have emerged as important challenges for the next decade. Therefore, DSP system designers have to pay more attention to the security perspective of DSP circuits, since the adversary can easily learn the functionality using massive attacking methods.

The problem of hardware security is a serious concern that has led to a lot of work on hardware prevention of piracy and intellectual property (IP), which can be broadly classified into two main categories: 1) authentication-based approach, and 2) obfuscation-based approach. The authentication-based approaches include physical unclonable functions (PUFs) based authentication [38], digital watermarking [58, 59, 60, 61], key-locking
scheme [62, 63], and hardware metering [42]. The focus of this chapter is on obfuscation, which is a technique that transforms an application or a design into one that is functionally equivalent to the original but is significantly more difficult to reverse engineer. Some hardware protection methods are achieved by altering the human readability of the hardware description language (HDL) code, or by encrypting the source code based on cryptographic techniques [64]. Recently, a number of hardware obfuscation schemes have been proposed that modify the finite-state machine (FSM) representations to obfuscate the circuits [65, 66, 45].

However, to the best of our knowledge, no obfuscation-based IP protection approach has been proposed specifically for DSP circuits in the literature. Our work, for the first time, presents design of obfuscated DSP circuits via high-level transformations that are harder to reverse engineer. From this standpoint of view, a DSP circuit is more secure, if it is harder for the adversary to discover its functionality even if the adversary can physically tamper the device. In other words, a high level of security is achieved if the functionality of a DSP circuit is designed to be hidden from the adversary.

The key contribution of this chapter is a novel approach to design obfuscated DSP circuits by high-level transformations during the design stage. The DSP circuits are obfuscated by introducing a finite-state machine (FSM) whose state is controlled by a key. The FSM enables a reconfigurator that configures the functionality mode of the DSP circuit. High-level transformations lead to many equivalent circuits and all these create ambiguity in the structural level. High-level transformations also allow design of circuits using same datapath but different control circuits. Different variation modes can be inserted into the DSP circuits for obfuscation. While some modes generate outputs that are functionally incorrect, these may represent correct outputs under different situations, since the output is meaningful from a signal processing point of view. Other modes would lead to non-meaningful outputs. The initialization key and the configure data must be known for the circuit to work properly. Consequently, the proposed
design methodology leads to a DSP circuit that is both structurally and functionally obfuscated.

Furthermore, the approach presented in the chapter will prevent piracy from overproduction and mask theft, because the manufacturer would not have access to either the initialization key or the configure data. These keys could be programmed by another honest vendor after the chips have been fabricated or provided to the customers by the designer. Therefore, overproduced chips without the correct keys cannot function properly.

5.2 Hiding Functionality by High-Level Transformations

High-level transformations [67] have been known for a long time and have been used in a wide range of applications, such as pipelining [68], interleaving [68], folding [69], unfolding [70] and look-ahead transformations (e.g., quantizer loops [71], multiplexer loops [72, 73], relaxed look-ahead [74], annihilation reordering look-ahead [75]), and have been used in synthesis of DSP systems [76]. These techniques can be applied at the algorithm or the architecture level to achieve a tradeoff among different metrics of performance, such as area, speed, and power [77]. However, the use of high-level transformations from a security perspective has not been studied before. High-level transformations alter the structure of a DSP circuit, while maintaining the original functionality. These transformations may lead to architectures whose functionalities are not obvious. Take an extreme case for example, many filters can be folded into one multiply-accumulator (MAC), but their functionalities are not the same. In other words, one MAC with proper switches can implement many different digital filters. Therefore, we can conclude that high-level transformations naturally provide a means to obfuscate DSP circuits both structurally and functionally. Structural obfuscation and functional obfuscation are defined as follows:

---

96
(a) **Structural Obfuscation**: any algorithm can be implemented by a family of architectures by using high-level transformations. These architectures enable structural obfuscation where the functionalities of the algorithms can be hidden. This can be considered as a "passive" model from attacker’s perspective.

(b) **Functional Obfuscation**: this is realized by encrypting the normal functionality of a DSP circuit with one or more sets of keys. The DSP circuit cannot function correctly without the keys. This corresponds to an "active" model from attacker’s perspective.

Folding is such an example of high-level transformation which could be utilized to achieve design obfuscation. The folding transformation generates folded variants based on the folding set, which is the reverse of the unfolding transformation [70]. The choice of folding set is critical to the performance of the folded structure, since an appropriate choice of folding order can lead to an architecture with lower area and power. Folding sets can be designed intuitively to meet the performance requirements or can be obtained from a high-level synthesis system [76]. The details and other examples (e.g., interleaving) of how to hide the functionalities of DSP circuits by high-level transformations are described in [78, 79]. We can observe that: 1) Circuits with different functionalities can have a similar structure, and circuits with the same functionality may have very different structures; 2) **Structural obfuscation** can be achieved by high-level transformations; 3) If the switch instances are invisible to the adversary, then the DSP systems will be harder to reverse engineer, since the functionality of a DSP circuit is not obvious due to obfuscation achieved by high-level transformations. As a result, the adversary who only has knowledge of the structural information but lacks knowledge of the switch instances cannot easily discover the functionality of a DSP circuit.

As an example, we consider a 3rd-order IIR digital filter given by transfer function $H(z) = \frac{1 + m_2 z^{-1} + m_3 z^{-2}}{1 - m_0 z^{-1} - m_1 z^{-2}}$, as shown in Figure 5.1. The coefficients $m_i$ correspond to the multiplication $M_i$. We assume the availability of one 1-stage pipelined adder and one
3-stage pipelined multiplier. The filter is folded with folding factor $N = 4$ using the following folding sets:

\[ M = \{ M_0, M_1, M_2, M_3 \}, \]
\[ A = \{ A_0, A_1, A_2, A_3 \}. \]

Folding sets represent the order of operations executed by the same hardware. For a folded system to be realizable, the folding equations, \( DF(U \xrightarrow{e} V) = Nw(e) - PU + v - u, \)

must be greater or equal to 0 for all the edges in the diagram, where \( N \) is the folding factor, \( w(e) \) is the number of delays from \( U \) to \( V \), \( PU \) represents the pipelining level of hardware functional unit for operation type \( U \), \( u \) and \( v \) represent the folding orders of \( U \) and \( V \), respectively. Retiming and pipelining can be used to satisfy this property (or it can be determined that the folding sets are not feasible), as a preprocessing step prior to folding. The folded architecture is shown in Figure 5.2. Figure 5.3 presents the structure that the switch instances are designed to be invisible. Null operations are incorporated into the switches.

We consider the implementation of another 3rd-order IIR filter given by transfer function \( H(z) = \frac{1 + m_2 z^{-1} + m_3 z^{-2}}{1 - m_1 z^{-3}} \) as shown in Figure 5.4. In order to achieve obfuscation, an architecture can be designed to be configurable as a 3rd-order IIR filter shown in either Figure 5.1 or Figure 5.4. These two modes are considered as meaningful modes.
Figure 5.2: A folded structure of the 3rd-order IIR filter in Fig. 1. The switch instance "i" corresponds to clock cycle $4l + i$.

Figure 5.3: A folded structure of the 3rd-order IIR filter with invisible switches.
In fact, a folded architecture of Figure 5.4 using the following folding sets can be obtained by assigning different switch instances to the structure in Figure 5.3, which is shown in Figure 5.5.

$$M = \{\emptyset, M_1, M_2, M_3\},$$

$$A = \{\emptyset, A_1, A_2, A_3\}.$$ 

The folding factor is 4, while there are only 3 multipliers and 3 adders in the DSP circuit. Therefore, if we consider the functionality of Figure 5.4 as the desired mode, one computation cycle is wasted every 4 cycles. The latency will also be increased. Note that we could use clock gating techniques to reduce the power for the null operation cycles.

![Figure 5.4: Another 3rd-order IIR filter.](image)

![Figure 5.5: A folded structure of the 3rd-order IIR filter in Figure 5.4. The switch instance "i" corresponds to clock cycle 4l + i.](image)
However, we can extend the periodicity of the switches to overcome the hardware underutilization. For instance, we can fold the 3rd-order IIR filter in Figure 5.4 by folding factor 3 with the folding sets:

\[ M = \{M_3, M_1, M_2\}, \]

\[ A = \{A_2, A_1, A_3\}. \]

The folded structure is shown in Figure 5.6. We can accommodate the two meaningful modes with no increase in latency for the second mode by extending the periodicity of the switches to the least common multiple of the folding factors of the two modes (i.e., lcm(3,4)=12). Similar extensions of switch periods have been considered in design of digit-serial DSP architectures [80]. For example, switch instance \(4l + i\) can be rewritten as \(12l + i\), \(12l + 4 + i\), and \(12l + 8 + i\), for \(i\) ranging from 0 to 3, in Figure 5.2; while switch instance \(3l + i\) can be rewritten as \(12l + i\), \(12l + 3 + i\), \(12l + 6 + i\), and \(12l + 9 + i\), for \(i\) ranging from 0 to 2, in Figure 5.6. As a result, for each meaningful mode as the desired mode, the latency remains the same as the original folded structure. This is achieved by increasing the complexity of the switch and the expense of hardware overhead associated with this step. The final obfuscated architecture for these two meaningful modes is shown in Figure 5.7. The switch instances are obfuscated and the two correct configurations of the switches correspond to two meaningful modes.

Figure 5.6: Another folded structure of the 3rd-order IIR filter in Figure 5.4. The switch instance \("i\) corresponds to clock cycle \(3l + i\).
5.3 Obfuscated Design via High-Level Transformations

5.3.1 Secure Switch Design

From Section 5.2, it can be seen that the DSP circuits can be obfuscated via high-level transformations by appropriately designing the switches in a secure manner. The switches generated by high-level transformations are periodic N-to-1 switches. These switches can be implemented as multiplexers (MUXs), whose control signals are obtained from ring counters (as shown in Figure 5.8). Thus, the security of the switch relies upon design of the ring counters such that the outputs of the ring counters can be obfuscated.
A ring counter is often modeled as a FSM. A FSM is usually defined by a 6-tuple \((I, O, S, S_0, F, G)\), where \(S\) is a finite set of internal states, \(I\) and \(O\) represent the inputs and outputs of the FSM, respectively, \(F\) is the next-state function, \(G\) is the output function, and \(S_0\) is the initial state. However, unlike general FSMS, the FSM of a ring counter is input-independent, such that it always transits to the next state based on the current state. As a result, the control signal of the switches (i.e., output of the FSM) will be periodic.

### 5.3.2 Reconfigurable Switch Design

Indeed, existing works have demonstrated that functional obfuscation can be achieved by embedding a well-hidden FSM (i.e., obfuscating FSM) in the circuit to control the functionality based on a key [66, 43, 45]. In order to achieve design obfuscation by using high-level transformations, we propose a reconfigurable switch design. The detailed implementation is shown in Figure 5.9, where ”SR” represents the state registers that store the information of the current state. We employ the idea of hardware design obfuscation as an activation sequence required before configuration by inserting an obfuscating FSM. The FSM enables a reconfigurator that controls the functionality mode of the DSP circuit by configuring the output function \(G\), next-state function \(F\), and the initial state \(S_0\). In our design, the configuration key must be known for the circuit to work properly, which consists of two parts: an \(L\)-bit initialization key and a \(K\)-bit configure data, as shown in Figure 5.10. The initialization key is used as the input of the obfuscating FSM, while the configure data are applied to the reconfigurator to control the operation of the switches. As the configuration of the switch is only enabled after receiving a correct initialization key, hostile attempts of the configure data cannot be processed by the reconfigurator as the reconfigurator is not activated. Note that other secure switch designs, whose detailed switch instances are hidden to the adversary, can also be adopted in the framework.
Figure 5.9: The complete reconfigurable switch design.

Figure 5.10: Configuration key containing an initialization key and a configure data.
The number of possible variations of ring counters is limited by the length of the configure data, $K$. We can create $M$ variation modes of the original circuits that have different functionalities, while $\log_2 M$ should be less than or equal to the length of configure data, $K$. Different configure data can be mapped into the same mode. An example of the mapping between the configure data and the associated modes is illustrated in Table 5.1. Note that this only involves simple combinational logic synthesis.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Configure Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$data_1, data_2$</td>
</tr>
<tr>
<td>2</td>
<td>$data_3$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$M$</td>
<td>$data_{2^{K-1}}, data_{2^K}$</td>
</tr>
</tbody>
</table>

### 5.4 Generation of Variation Modes

#### 5.4.1 Security Perspective of Variation Modes

The cost for an adversary to find the correct key of an obfuscated DSP circuit by adopting the proposed architecture is not only dependent on the length of the configuration key ($L + K$), but also on the number of required input-vectors for learning the functionality of each variation mode (note that in this chapter, the inputs represent the original inputs of the DSP circuits, while the key represents the data to control the switches). For a certain variation mode, the adversary attempts to attack the DSP circuit by generating input-vectors until the functionality could be discovered. The most intuitive way to mask the desired functionality is to modify the switch instances arbitrarily. However, the functionality of the resulting structure may not be meaningful from a signal processing point of view, which would be easier for the adversary to distinguish whether the circuit is operating correctly. The numbers of required input-vectors for these non-meaningful variation modes would be less than the numbers of required input-vectors for meaningful variation modes. If one mode of a DSP circuit is
obfuscated to output a large portion of invalid values, the adversary can figure out this mode is non-meaningful with a relatively small number of input-vectors.

The challenge of this problem is how to generate meaningful variation modes from a signal processing point of view such that the DSP circuit can also be operated in a reconfigurable manner. Most often, we perform a high-level transformation based on the design requirements and constraints by taking an existing design and generating the resulting design during high-level synthesis stage. We may continue to add variation modes into the design, as discussed in Section 5.2. Some of the variation modes may correspond to meaningful modes (e.g., different order filters or filters of same order with different coefficients) which can be exploited for different applications, while others correspond to non-meaningful modes.

Design of obfuscated DSP circuits requires extra efforts during the design phase. In fact, variations of the algorithm (e.g., different folding sets) can also be utilized to produce several obfuscated versions. Therefore, it is possible to generate meaningful variation modes simultaneously with the high-level transformation during design stage instead of modifying the switch instances after performing high-level transformations. As a result, the variations of the structures are indeed obtained from the variations of the selected algorithm. Furthermore, the secure switches can also be designed systematically based on the variations of algorithms. Using this approach, the extra design efforts can be reduced, while reconfigurable design with a number of different meaningful modes is ensured.

Note that meaningful modes are not mandatory for the proposed obfuscated design. While meaningful modes achieve higher security, obfuscation only with non-meaningful modes could also attain considerable protection of the DSP circuit against reverse engineering.
5.4.2 A Case Study: Hierarchical Contiguous Folding Algorithm

The variation modes are generated based on the selected transformation algorithm, which are different for various high-level transformations. It is difficult to cover very large number of existing high-level transformations in this chapter. We just present an example to demonstrate how to generate variation modes. The proposed design methodology can also be extended to other high-level transformations.

Hierarchical folding approach is a novel folding technique that combines folding of $M$ cascaded stages to one hardware block, and folding $N$ operations inside each section to a hardware functional unit, as shown in Figure 5.11. Two hierarchical folding algorithms are presented in [78], which include hierarchical interleaved folding (HIF) and hierarchical contiguous folding (HCF). In this section, we only address hierarchical contiguous folding, while it is also applicable to hierarchical interleaved folding. Hierarchical contiguous folding transformation executes all operations of one section before starting execution of operations of next section. Reader is referred [78] for further details.

![Diagram of DSP data-flow graph containing M cascaded stages](image)

Figure 5.11: (a) A DSP data-flow graph containing $M$ cascaded stages. Block $Alg^i$ represents $i$-th stage of the cascade. (b) A folded architecture where $M$ stages are folded to same hardware. $DF$ represents the number of folded delays.

Hierarchical contiguous folding transformation executes all operations of one section before starting execution of operations of next section. The folding sets are described as follows:

$$\{X_0^0, X_0^1, \ldots X_{N-1}^0, X_0^1, X_1^1, \ldots X_{N-1}^1, \ldots, X_0^{M-1}, X_1^{M-1}, \ldots X_{N-1}^{M-1}\}, \quad (5.1)$$
The operation $X^j_i$, i.e., the $i$-th operation of $j$-th block, will be executed at time $N_j + i$. The operations in a later section will be executed only after all the operations from the previous section are completed. The algorithm is described below:

**Hierarchical Contiguous Folding (HCF) Algorithm**

1. Fold $Alg^j_i$ by factor $NM$, with the folding set 
   \[ \{X_0, X_1, ...X_{N-1}, \emptyset, \emptyset, ...\} \], where the number of null operations corresponds to $(NM - N)$.

2. Replace each switch $s$ by $s, s + N, s + 2N, ... s + (M - 1)N$.

3. Compute $DF(Alg^j_i \xrightarrow{\circ} Alg^{j+1})$, for $j = 0, 1, 2, ... M - 2$, and use these folded edges to replace the external inputs.

We propose an algorithm for obfuscation that generates variation modes by varying the number of sections in the cascade structure based on the HCF algorithm. For example, if the number of sections for a DSP system is $l$, then the algorithm can be described as below (the total number of operations is still $NM$, where $M \geq l$).

**Design Obfuscation Algorithm based on the HCF Algorithm**

1. Fold $Alg^j_i$ by factor $NM$, with the folding set 
   \[ \{X_0, X_1, ...X_{N-1}, \emptyset, \emptyset, ...\} \], where the number of null operations corresponds to $(NM - N)$.

2. Replace each switch $s$ by $s, s + N, s + 2N, ... s + (l - 1)N$, and set switch instances from $lN$ to $MN - 1$ to null operations.

3. Compute $DF(Alg^j_i \xrightarrow{\circ} Alg^{j+1})$, for $j = 0, 1, 2, ... l - 2$, and use these folded edges to replace the external inputs.
If $l = M$, this algorithm reduces to the hierarchical contiguous folding algorithm. From this algorithm, we can generate $M$ meaningful modes correspond to $l = 1, 2, ..., M$. Furthermore, the reconfigurator can also be designed based on the variations of the HCF algorithm, which is a simple $2^K$-to-$M$ combinational logic design problem. Note that this algorithm can be easily extended to other types of DSP systems where the sub-circuits are not directly connected.

5.5 Design Flow of the Proposed DSP Circuit Obfuscation Approach

5.5.1 Design Methodology

In this section, we propose a novel DSP hardware protection methodology through obfuscation by hiding functionality via high-level transformations. This approach helps the designer to protect the DSP design against piracy. The detailed design flow is described below:

**Step 1: DSP algorithm.** This step generates the DSP algorithm based on the DSP application.

**Step 2: High-level transformation selection.** Based on the specific application, appropriate high-level transformation should be chosen according to the performance requirement (e.g., area, speed, power or energy).

**Step 3: Obfuscation via high-level transformation.** Selected high-level transformations are applied simultaneously with obfuscation where variation modes, and different configurations of the switch instances are designed.

**Step 4: Secure switch design.** The secure switch is designed based on the variations of high-level transformations. Note that different configure data could be mapped into the same mode, which only involves simple combinational logic synthesis.
**Step 5: Two-level FSM generation.** The reconfigurator and the obfuscating FSM are incorporated into the DSP design as shown in Figure 5.9. The *configuration key* is generated at this step.

**Step 6: Design specification.** This step includes the HDL and netlist generation and synthesis of the DSP system.

After these design steps, designer sends the obfuscated design to the foundry that manufactures the DSP circuit. By using the proposed design methodology, the manufacturer will not gain access to the desired functionality or the *configuration key*. Unauthorized copies of the obfuscated DSP circuits would provide little information to the adversary. The relationship between the obfuscated design and the original design via high-level transformation is illustrated in Figure 5.12. The only difference between the obfuscated design and the original design via high-level transformation is the control of the DSP circuit. The main datapath is unaltered. As a result, the critical path would not increase for the obfuscated design. Furthermore, the proposed design methodology does not require significant changes to established verification and testing flows. In fact, the obfuscated DSP circuit with the correct key behaves just like the original circuit.

### 5.5.2 Architecture of the Obfuscated DSP Circuits

The complete system of the proposed obfuscated DSP circuit is illustrated in Figure 5.13. The reconfigurator will be enabled only by the correct *initialization key*. Only the correct *configure data* leads to the desired design. A wrong *configure data* activates an obfuscated mode (either a meaningful or non-meaningful). The obfuscating FSM and a portion of non-meaningful variation modes (i.e., we denote as *alarm modes*) can both be utilized for security check purpose. For example, some undesired modes in Table 5.1 can be designed as *alarm modes* by adding another output signal to the combinational logic. We can improve the security by mapping a larger number of *configure data* to this *alarm mode*, while keeping the portion of functional *configure data* to be
relatively small. If the circuit continuously receives wrong initialization key or configure data whose number exceeds the pre-defined threshold, the adversary is prevented from further attempts of the configuration key by a denial of use block.

5.6 Security and Resiliency against Attacks

5.6.1 Attacks and Countermeasures

The goal of the proposed methodology is to ensure the designer’s intellectual property would not be stolen against reverse engineering. Generally speaking, a hacker trying to determine the functionality of a DSP circuit can resort to either of the following ways: 1) structural analysis of the netlist to identify and isolate the original design from the obfuscated design or 2) simulation-based reverse engineering to determine functionality of the design.
Our proposed obfuscation methodology protects the hardware against the first type of attack (i.e., structural analysis) from two perspectives: 1) structural obfuscation by high-level transformation, and 2) integration with obfuscation modes. As presented in Section 5.2, high-level transformations lead to \textit{structural obfuscation} at the HDL level or gate-level netlist. Without knowing the correct configuration of the switches, it is hard for the adversary to learn the functionality of the original design. Furthermore, although the obfuscating FSMs could be isolated, the obfuscation of configuration switches cannot be separated from the original functionalities. Since the obfuscation variation modes are integrated to the reconfigurator in the synthesized DSP circuit, the adversary cannot remove the design obfuscation achieved by high-level transformations. Additionally, meaningful variation modes also create ambiguity when the adversary performs the structural analysis attacks.

For a simulation-based approach where random key vectors are sequentially applied to take the circuit to the correct mode, the probability of discovering the \textit{configuration}
key sequence is \( \frac{1}{2^{L+K}} \) for a circuit with a length-\((L + K)\) configuration key sequence. For example, the probability is only \( 5.4 \times 10^{-20} \) for a circuit with a length \( L = 32 \) initialization key sequence and a length \( K = 32 \) configure data sequence.

Moreover, the cost of the simulation based attack approach is also dependent on the size of inputs. Various input patterns need to be tested to determine the complete functionality of a DSP circuit. In practice, most DSP circuits will have considerable size of inputs and the length of configuration keys can be made larger. Therefore, brute-force attack for learning the key sequences and input/output patterns is computationally infeasible.

5.6.2 Measure of Obfuscation Degree

Structural Obfuscation Degree (SOD)

Manual attacks can be performed by visual inspection and structural analysis. In these types of manual attacks, the adversary has to analyze the RTL or gate-level structure as well as the layouts. This is a weak attack, as the adversary has very little chance of figuring out the obfuscation scheme for large DSP circuits.

The obfuscation degree of the structural obfuscation is dependent on the number of independent switches \((N_s)\), the period of switch instances after high-level transformations \((P)\), and the number of connections for each independent switch \((C_m)\). To estimate the obfuscation degree against these manual attacks, we propose a metric called Structural Obfuscation Degree (SOD):

\[
SOD = \prod_{m=1}^{N_s} (C_m + 1)^P,
\]

(5.2)

where the additional "1" corresponds to the null operation. Note that a higher SOD value implies better obfuscation, as the SOD value indicates the number of possible functionalities generated by the combinations of these switches. A circuit is more secure, if there is more ambiguity in the structure. For example, the SOD value of the structure
in Figure 5.3 can be calculated as $3^4 \times 4^4 \times 5^4 = 12,960,000$ (excluding the output switch), which is already very large.

For a small sub-circuit with a small SOD value, it may be feasible to figure out the original functionality from the obfuscated design, as the number of combinations is small and most of these combinations may result in non-meaningful functionalities. However, the structure of a DSP circuit after high-level transformations usually has a larger SOD value (as illustrated by the example above). Thus, it will be extremely hard to distinguish the original functionality from other possible functionalities of these DSP circuits.

Functional Obfuscation Degree (FOD)

The proposed methodology also achieves functional obfuscation by encrypting the correct functionality with a key sequence. The cost for an adversary to discover the correct key is dependent on the bit-length of the key ($L + K$), the number of meaningful modes ($N_m$), the number of non-meaningful modes ($N_n$), and the size of input bits ($N_I$). To estimate the obfuscation degree against simulation based attacks, we propose a metric denoted as Functional Obfuscation Degree (FOD):

$$ FOD = f(N_I)2^L[N_m + \alpha N_n + \beta(2^K - N_m - N_n)], \quad (5.3) $$

where $f(N_I)$ represents the input cost coefficient that is the number of input vectors required for learning the functionality of the DSP circuit, which is proportional to $N_I$; and $\alpha$, $\beta$ represent the cost coefficients of learning a new non-meaningful variation mode and learning a previously known mode compared to learning a new meaningful mode, respectively. These coefficients are dependent on the particular applications. As discussed in Section 5.4.1, it can be expected that they would follow the relation that $0 < \beta < \alpha < 1$. Future work will be directed towards validating the values of $\alpha$ and $\beta$ through experimental results. The higher the FOD value, more secure the obfuscated
design. Note that in practice, the computation cycles and the clock periods of the DSP circuits would also affect the cost of simulation based attacks.

5.6.3 Improving the Security by Key Encoding

In the proposed obfuscation scheme, the key consists of two parts: *initialization key* and *configure data*. However, in the scenario that the adversary has found a key that can successfully pass the initialization but is still in an incorrect configuration, the adversary will only try different *configure data* while fixing the *initialization key*. This could weaken the scheme. An encoder could be added to improve the security of the system, as shown in Figure 5.14. The encoder could be a Hash function, a linear feedback shift register (LFSR), or a Physical Unclonable Function (PUF). By incorporating the encoder, the *user key* and the *configuration key* are no longer bit-to-bit mapped.

![Figure 5.14: Key encoding.](image)

Moreover, if we use a PUF as the encoder, key collisions could also be avoided in different chips. PUFs can be used to give unique *user keys* for different DSP circuits even though they are all obfuscated with the same *configuration key*.

5.6.4 Security Properties

The main objective of our work is to protect DSP circuits against reverse engineering. The obfuscated DSP circuits will only operate in the desired mode with a negligible probability that others would be able to find. Thus, the correct functionality is hidden to the adversary even when the adversary can access the DSP circuits. Moreover, the proposed obfuscating scheme also satisfies a set of following properties to ensure security and resiliency against attacks:
(a) Unobtrusiveness: The obfuscation is invisible to the functional DSP circuits. Its presence would not interfere with regular operation of the design.

(b) Unambiguity: The probability of finding the correct key for a DSP circuit is low by employing our proposed obfuscation scheme. The chance for an adversary to enter a DSP circuit into the correct mode by random guessing is \( \frac{1}{2^{L+K}} \), which will be negligible when the bit-length of the key is long. Therefore, the correct key is a strong proof of ownership.

(c) Robustness: Since the obfuscation modes are generated along with the high-level transformation design phase, all the stages after this phase in the high-level synthesis flow would contain the obfuscation. The embedded obfuscation is extremely difficult to remove, since the variation modes are integrated to the switches. Moreover, we could combine PUFs to ensure that the keys of different DSP circuit designers would not collide.

(d) Universality: The proposed obfuscating methodology can be used for all common DSP designs. In this chapter, we have only described a few examples of high-level transformations for hardware obfuscation. However, other types of high-level transformations and other transformation algorithms can also be used to achieve hardware obfuscation.

5.7 Evaluation of the Proposed Methodology

5.7.1 Overhead Impact

Component overhead of the proposed obfuscation design includes: (a) additional control logic of switches, (b) reconfigurator, and (c) obfuscating FSM. These additional circuits only affect the switches of an obfuscated DSP circuit, while the main datapath stays the same as the original design, as shown in Figure 5.12. In this section, we present the area overhead results of the proposed obfuscating methodology for two
DSP benchmark circuits: \((3l)\)th-order IIR filter and \((12l)\)-tap FIR filter. All circuits were synthesized using Synopsys Design Compiler with optimization parameters set for minimum area and mapped to a 65 nm standard cell library. We employ the design obfuscation algorithm based on the HCF algorithm to obfuscate the circuits. In our experiments, the \((3l)\)th-order IIR filter is folded to 1 multiplier and 1 adder, while the \((12l)\)-tap FIR filter is folded to 3 multiply-accumulators.

We take the \((3l)\)th-order IIR filter benchmark as an example to illustrate the obfuscated design approach. Here, one section of the \((3l)\)th-order IIR filter is a 3rd-order IIR filter as shown in Figure 5.1. We assume the desired functionality is an 18th-order IIR filter realized as a cascade of six 3rd-order IIR filters. In our experiment, the proposed design obfuscation algorithm based on the HCF algorithm is applied to the original 18th-order IIR filter to obfuscate this DSP circuit. In order to generate 8 meaningful variation modes, the parameters \(M = 8\) and \(N = 4\) are used to the structure with 6 sections of 3rd-order IIR filter (i.e., the original 18th-order IIR filter) and 2 additional sections of null operations. The switch instances of this folded design are periodic with period 32. The 8 meaningful modes correspond to \((3l)\)th-order IIR filters where \(l = 1, 2, ..., 8\), respectively. 8 non-meaningful variation modes are also incorporated. Each secure switch is controlled by the reconfigurator independently. Figure 5.15 shows an example of the switch connected to the input of the multiplier in the obfuscated design. This switch has 5 possible input paths, as the null operations are also integrated to the switches. Based on the algorithm, the control signals of this switch within one period for the intended mode should be

\[
(3,1,4,2, 3,1,4,2, 3,1,4,2, 3,1,4,2, 3,1,4,2, 0,0,0,0, 0,0,0,0).
\]

For other generated meaningful variation modes whose functionalities are \((3l)\)th-order IIR filters, the control signals should be periodic with a length-32 sequence that consists of \(l\) \((3,4,1,2)\) in the beginning and \(8 - l\) \((0,0,0,0)\) in the end.
In our experiment, 8 non-meaningful variation modes are also incorporated. As a result, the bit-length of the configure data is at least 4, since the number of variation modes $N_m + N_n$ should be less than $2^K$. For instance, a simple design example of the reconfigurator with configure data $K = 4$ is presented in Table 5.2. Note that multiple configure data can be mapped to the same mode, if we increase $K$.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Configure Data</th>
<th>Functionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0000</td>
<td>3rd-order IIR filter</td>
</tr>
<tr>
<td>2</td>
<td>0001</td>
<td>6th-order IIR filter</td>
</tr>
<tr>
<td>3</td>
<td>0010</td>
<td>9th-order IIR filter</td>
</tr>
<tr>
<td>4</td>
<td>0011</td>
<td>12th-order IIR filter</td>
</tr>
<tr>
<td>5</td>
<td>0100</td>
<td>15th-order IIR filter</td>
</tr>
<tr>
<td>6</td>
<td>0101</td>
<td>18th-order IIR filter</td>
</tr>
<tr>
<td>7</td>
<td>0110</td>
<td>21st-order IIR filter</td>
</tr>
<tr>
<td>8</td>
<td>0111</td>
<td>24th-order IIR filter</td>
</tr>
<tr>
<td>9</td>
<td>1000</td>
<td>non-meaningful</td>
</tr>
<tr>
<td>10</td>
<td>1001</td>
<td>non-meaningful</td>
</tr>
<tr>
<td>11</td>
<td>1010</td>
<td>non-meaningful</td>
</tr>
<tr>
<td>12</td>
<td>1011</td>
<td>non-meaningful</td>
</tr>
<tr>
<td>13</td>
<td>1100</td>
<td>non-meaningful</td>
</tr>
<tr>
<td>14</td>
<td>1101</td>
<td>non-meaningful</td>
</tr>
<tr>
<td>15</td>
<td>1110</td>
<td>non-meaningful</td>
</tr>
<tr>
<td>16</td>
<td>1111</td>
<td>non-meaningful</td>
</tr>
</tbody>
</table>

An obfuscating FSM is also added into the secure switch design to provide the second-level protection of the obfuscated DSP circuit. The number of the states of the
obfuscating FSM should be less than or equal to $2^L$, where $L$ is the length of initialization key.

Area Overhead

We present the area overhead for the two DSP circuit benchmarks as shown in Table 5.3 and Table 5.4, respectively. Note that the overhead percentages presented in Tables 5.3 and 5.4 are computed based on the folded designs instead of the original circuits. The results include average area overheads over a number of different implementations. For certain lengths of initialization key and configure data, the patterns of the state transition graph in the design of obfuscating FSM and the input-output mappings in the design of reconfigurator would also affect the design overhead of the proposed obfuscated DSP circuit.

Table 5.3: Overhead (%) of the (3l)th-order IIR Filter Benchmark

<table>
<thead>
<tr>
<th>L</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.8</td>
<td>4.0</td>
<td>4.3</td>
<td>4.8</td>
<td>5.9</td>
</tr>
<tr>
<td>8</td>
<td>4.9</td>
<td>5.0</td>
<td>5.4</td>
<td>5.9</td>
<td>6.9</td>
</tr>
<tr>
<td>16</td>
<td>6.6</td>
<td>6.7</td>
<td>7.0</td>
<td>7.6</td>
<td>8.7</td>
</tr>
<tr>
<td>32</td>
<td>9.7</td>
<td>9.8</td>
<td>10.2</td>
<td>10.8</td>
<td>11.9</td>
</tr>
<tr>
<td>64</td>
<td>15.4</td>
<td>15.7</td>
<td>16.0</td>
<td>16.6</td>
<td>17.7</td>
</tr>
</tbody>
</table>

Table 5.4: Overhead (%) of the (12l)-Tap FIR Filter Benchmark

<table>
<thead>
<tr>
<th>L</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>32</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.6</td>
<td>1.7</td>
<td>1.8</td>
<td>1.9</td>
<td>2.1</td>
</tr>
<tr>
<td>8</td>
<td>2.0</td>
<td>2.1</td>
<td>2.2</td>
<td>2.4</td>
<td>2.5</td>
</tr>
<tr>
<td>16</td>
<td>2.8</td>
<td>2.9</td>
<td>3.0</td>
<td>3.2</td>
<td>3.5</td>
</tr>
<tr>
<td>32</td>
<td>4.1</td>
<td>4.2</td>
<td>4.3</td>
<td>4.4</td>
<td>4.7</td>
</tr>
<tr>
<td>64</td>
<td>6.6</td>
<td>6.7</td>
<td>6.8</td>
<td>6.9</td>
<td>7.1</td>
</tr>
</tbody>
</table>

It can be seen from Tables 5.3 and 5.4 that the overall overhead is about 17.7% for the (3l)th-order IIR filter with a 128-bit (64+64) configuration key, while the overhead is only about 7.1% for the (12l)-tap FIR filter also with a 128-bit configuration key. However, a strong obfuscation is achieved, as the chance for an adversary to enter the
DSP circuit into the desired mode is only \( \frac{1}{2^{L+K}} = \frac{1}{2^{L}} = 2.94 \times 10^{-39} \). Note that these two DSP circuit benchmarks are both small circuits. In practice, as the DSP circuits are more complex, the overhead percentage would be even smaller under the assumption that we want to create a certain degree of obfuscation (i.e., maintain an approximately same number of switches to obfuscate, even though there are more switches).

Moreover, when we compare the effects between \( L \) and \( K \), it can be seen that the overhead increases more significantly with the increase of \( L \). For the (3l)th-order IIR filter example, the overhead is 3.8% when \( L = 4 \) and \( K = 4 \). If we fix \( K = 4 \), the overhead is 15.4% when \( L = 64 \), which is increased by 305%. However, if we fix \( L = 4 \), the overhead is only increased by 55% when \( K = 64 \). Thus, in order to achieve lower overhead, we should employ a longer configure data in designing obfuscated DSP circuits when the total length of the configuration key is bounded. Indeed, this is another advantage of our proposed methodology, as obfuscating DSP circuits through secure switches incur smaller overhead, compared to the methods only based on obfuscating FSMs.

**Timing Overhead**

As the main datapath in the obfuscated design stays unaltered as the original design via high-level transformations, the critical path will not be increased. The obfuscating FSM and the reconfigurator can be pipelined such that the critical path is only dependent on the main datapath.

There can be a degradation of the performance with respect to latency if null operations are inserted into the obfuscated design. However, obfuscation does not require introduction of null operations. The parameters of an obfuscated design should be selected based on the application requirements and constraints. In addition, at the expense of the control complexity, the latency of the obfuscated design can be guaranteed to be the same as the original design by expanding the periodicity of the switches as discussed in Section 5.2.
Power and Energy Overhead

We present the power consumptions for different meaningful modes of the (3l)th-order IIR filter benchmark in Figure 5.16.

![Figure 5.16: Normalized power consumption for different meaningful modes (%) of the (3l)th-order IIR filter benchmark (normalized to the power consumption when considering l is a variable).](image)

When comparing the obfuscated design to the original folded design without these additional circuits (i.e., folded design of the 18th-order IIR filter), the power is only slightly increased by 1.1%. As a result, if we compare the obfuscated design running in the desired mode and the folded design, the power overhead is 3.9%. Additionally, if an obfuscated DSP circuit is designed with no increase of latency as presented in Section 5.2, the energy overhead will also be small.

Number of Meaningful Modes

As we pointed out above, the number of modes is bounded by $2^K$. Additionally, if we apply the design obfuscation algorithm based on the HCF algorithm, the number of meaningful modes is also limited by the selected number of sections of the DSP circuits. For instance, we could only implement at most 8 meaningful modes by choosing the parameter $M = 8$ in the (3l)th-order IIR filter benchmark example. Recall that the hierarchical folding algorithms are not only limited to cascade systems whose sections have very similar structure, but can also be extended to other types of DSP systems.
where the sub-circuits are not directly connected. In general, the maximum number of meaningful modes is dependent on the sum of the number of sub-circuits and the number of inserted null sections.

Therefore, in order to increase the number of meaningful modes, we need to add more null sections. We still use the 18th-order IIR filter as an example. If we fold the 18th-order IIR filter directly without any null section, we could only implement 6 meaningful variation modes in the obfuscated design, i.e., 3rd-order, 6th-order, 9th-order, 12th-order, 15th-order, and 18th-order IIR filter. If we want to further increase the number of meaningful modes, we have to insert null operations into the original circuit, but at the price of additional power consumption and computation cycles. Consequently, the area and power of the folded design will also increase. Note that the length of configure data $K$ should be always greater than or equal to $\log_2(\text{number of modes})$. In our experiments, we set $K = 16$ to ensure all the possible meaningful modes can be realized. The measurements of the area and power for the $(3l)$th-order IIR filter benchmark are shown in Figure 5.17.

![Figure 5.17: Normalized area and power cost for different numbers of meaningful modes.](image)

From Figure 5.17, it can be seen that area and power do not increase very significantly with the increase of the number of meaningful modes (e.g., for the obfuscated
circuit with 32 meaningful modes, the area and the power area are increased by 48% and 19%, respectively, compared to the example with 6 meaningful modes). Therefore, in the proposed methodology, we can increase the number of meaningful modes to improve the security while maintaining relatively low power and area overheads.

**Number of Non-Meaningful Modes**

As discussed above, if the number of meaningful modes increases to a value that the total number of modes is greater than $2^K$, we need to increase the length of configure data as well to realize all the possible meaningful variation modes. In this scenario, however, there is another feasible solution that is to reduce the number of non-meaningful modes, if we want to maintain the same length of configure data.

We present the experimental results of the normalized area and power for different numbers of non-meaningful modes in Figure 5.18. All of the results are based on the $(3l)$th-order IIR filter benchmark with 8 meaningful modes and $K = 16$.

![Figure 5.18: Normalized area and power cost for different numbers of non-meaningful modes.](image)

Furthermore, since the latency is not affected by the obfuscating FSM and the reconfigurator, the performance of energy has the same trends as the power with the increase
in number of variation modes and key length.

**Discussion.** Note that all the experimental results presented above are only based on the *design obfuscation algorithm based on the HCF algorithm* for the two particular benchmarks. These results are just aimed to provide an example of the performance for the proposed methodology. Null operations are inserted in this example. However, as discussed in Section 5.2, the obfuscated circuit can also be designed to maintain the same latency as the original structure obtained by performing certain high-level transformation based on the application requirements and constraints. In this case, the overheads of timing, power and energy would be minimal, which is suitable for the application where diminished runtime performance is not acceptable.

The actual performance of an obfuscated DSP circuit may vary significantly according to multiple design parameters, which include the target DSP algorithm/application, the specific variation of the algorithm, the relation between the desired mode and the obfuscated design, the number of modified switches, the key length, the numbers of meaningful and non-meaningful modes, and so forth. In the design of obfuscated DSP circuits, the designer should carefully select these parameters according to the specific application requirements and constraints.

### 5.7.2 Overhead Reduction

Generally, since the variation modes are designed during the high-level transformation phase, the reconfigurator logic can be optimized for minimum area during the following design stages. Furthermore, we can generate additional logic designs with lower overheads by choosing appropriate algorithms.

State register sharing is one possible approach to reduce the area consumption by managing the relation among different switches in a sub-circuit. For example, as shown in Figure 5.19, 4 flip-flops can be used to implement six 4-state ring counters, in contrast to 12 if no sharing of state registers is exploited. The saving could be significant for
a large circuit. In Figure 5.19, the registers marked in bold represent the active state
registers for corresponding ring counters.

Furthermore, exploiting the relation among the state registers is more effective than
managing the output function or the next-state function to control the relations among
different switches in a sub-circuit. However, note that by using these design optimization
techniques, the structural obfuscation degree (SOD) will be degraded, as the number of
independent switches ($N_s$) is decreased.

5.8 Comparison to existing obfuscation methods

As our work is the first attempt to develop a methodology to obfuscate DSP cir-
cuits by utilizing high-level transformations, it is hard to compare with other existing
obfuscation methods which are general to a wide variety of designs. Therefore, we have
introduced two metrics to analyze the security, which are discussed in Section 5.6.
Most of the hardware obfuscation techniques in the literature can also be applied to DSP circuits. However, the use of high-level transformations from a security perspective has not been incorporated into any of these prior hardware obfuscation techniques. Moreover, other circuit locking techniques only achieve protection at one-level (i.e., encrypt the normal functionality by a key), while our proposed methodology provides a two-level protection (i.e., *structural obfuscation* and *functional obfuscation*). The main advantage of the proposed methodology is the generation of meaningful variation modes from a signal processing point of view, since the meaningful modes create ambiguity to the adversary such that it is hard for the adversary to distinguish the desired functionality from other variation modes. Other existing methods, such as [62, 61], are not specific to DSP circuits, which would not be able to ensure meaningful variation modes from a signal processing point of view. Moreover, meaningful variation modes enable our proposed design methodology to be adaptable to reconfigurable applications.

Finally, when considering the metrics of the design performance, our proposed methodology is also superior. While our proposed approach only alters the logic of switches, most of the existing methods are based on explicit FSM modifications (e.g., the technique proposed in [45]), which are not scalable since the construction of the FSM is not practical for even moderate-sized circuits, not to mention that the number of added obfuscation states can be relatively large as compared to the original FSM. In our proposed methodology, area consumption is slightly increased due to the increased cost of the control logic for the obfuscated switches.

### 5.9 Conclusion and Future Work

This chapter presents a novel low-overhead solution to design DSP circuits that are obfuscated both *structurally* and *functionally* by utilizing high-level transformation techniques. It is shown that verifying the equivalence of digital signal processing circuits by employing high-level transformations will be harder if some switches can be
designed in such a way that are difficult to trace. A secure reconfigurable switch design is incorporated into the proposed design scheme to improve the security. A complete design flow is presented. In the proposed obfuscation methodology, the variation modes and the additional obfuscating circuits could also be designed systematically based on the high-level transformations. Compared to other existing obfuscation methods, another advantage of the proposed methodology is the generation of meaningful variation modes from a signal processing point of view, since the meaningful modes create ambiguity to the adversary such that it is hard for the adversary to distinguish the correct functionality from other variation modes. Experimental results have demonstrated the effectiveness of the proposed methodology.
Chapter 6

Beat Frequency Detector based True Random Number Generators: Statistical Modeling and Analysis

6.1 Introduction

The security of most cryptographic systems relies on unpredictability and irreproducibility of digital key-streams that are used for encryption and/or signing of confidential information. These key-streams are generated by random number generators (RNG), which can be further classified into two categories: true random number generators (TRNG) and pseudo random number generators (PRNG). The key difference between TRNG and PRNG lies in the entropy source component. A TRNG derives randomness from an analog physical process (electronic thermal noise, radioactive decay, etc.), while a PRNG relies on computational complexity, whose outputs are completely determined by the seed. TRNGs are used for authentication and encryption purposes
in systems requiring a high level of security. On-chip TRNGs typically harvest randomness from a circuit that converts transistor level noise such as random telegraph noise (RTN), flicker noise and thermal noise [81, 82, 83, 84, 85, 86, 87] into a voltage or delay signal.

A source of randomness commonly used in FPGA and ASIC implementations of TRNGs is the unpredictability of signal propagation time across logic gates. This unpredictability is typically accumulated in so-called ring oscillators (ROSCs), consisting of a series of inverters or delay elements connected in a ring. The phase jitter of a ring oscillator is then extracted by another ring oscillator or by an external clock signal. Ring oscillators and the underlying physical phenomena have been widely studied in the literature as building blocks for many on-chip TRNGs [88, 89, 90, 91, 92, 93, 94, 95]. One major advantage of these TRNG designs is that no analog component is required, while conventional delay based TRNGs typically involve extensive analog components for amplifying the device noise [96], which makes them less suitable for practical TRNG devices.

Evaluating TRNGs is a difficult task. Clearly, it should not be limited to testing the TRNG output bitstream. The physical characteristics of the source of randomness and the randomness extraction method determine the principal parameters of the generated bit stream: the bias of the output bit stream, correlation between subsequent bits, visible patterns, etc. While some of the non-randomness can be corrected by efficient post-processing, it is better if the generator inherently produces a good quality random bitstream. Furthermore, passing NIST [97] or DIEHARD [98] tests does not guarantee a TRNG, as these tests were originally designed to check the performance of PRNGs.

One important requirement in TRNG security evaluation is the existence of a mathematical model of the physical noise source and the statistical properties of the digitized noise derived from it [99]. If a stochastic model of the physical randomness source is available, it can be used in combination with the raw signal to estimate the entropy and the bias depending on the random input variables and the TRNG principle. Therefore,
in order to provide a proof of security for a TRNG, an analysis of the statistical property of the underlying mathematical model is needed. However, creating a model of a TRNG is difficult as the model parameters are unknown. Thus, it is impossible to predict performance of new TRNG designs as their models cannot be created. On the other hand, it can be argued that TRNG performance can only be measured from fabricated chips. Therefore, how good a new TRNG design can only be determined by measurements from a fabricated design. This chapter exploits the synergy between a model and the measurements of the real device. A new ROSC based BFD-TRNG was fabricated and tested [100]. Based on NIST tests, this TRNG was demonstrated to be an effective TRNG. Our work, for the first time, presents a model of this BFD-TRNG. The model parameters are derived by fitting the data measured from the fabricated device. Based on this created model, a rigorous analysis of the BFD-TRNG is presented. Furthermore, several new BFD-TRNG architectures are proposed and their performances are predicted based on the proposed model.

6.2 Beat Frequency Detector based TRNG

The oscillator sampling method extracts randomness from phase noise in free-running oscillators [88, 89, 92]. An example of this technique is shown in Figure 6.1, where the output of a fast oscillator is sampled on the rising edge of a slower ring oscillator using a D flip-flop (DFF). Note that the design parameters for the inverters of the two ROSCs are not necessarily the same. The timing fluctuations of the edges of the slow signal relative to the fast oscillator is the source of the randomness in the ROSC based TRNG. Oscillator jitter causes uncertainty in the exact sample values, ideally producing a random bit for each sample. Additionally, randomness can be artificially enhanced by carefully selecting the ratio of the fast and slow oscillator frequencies. Periods of these oscillations vary from cycle to cycle causing jitter in the rising and falling edges. The goal is to sample the signal at a point in time that is in close proximity of a transition
zone thereby making sampled value unpredictable. In order to accumulate sufficient jitter when the fast ring oscillator is sampled, a large ratio of the fast and slow oscillator frequencies is usually desired. Note that the slow oscillator can also be substituted by an external clock, such as in the IBM $M$-parallel structure [101].

![Figure 6.1: Two-oscillator TRNG.](image)

Built on the prior work of ROSC based TRNGs, we have proposed a novel TRNG design to harvest randomness from jitter variation based on the beat frequency detector (BFD) [100]. A beat frequency detector captures the frequency difference between the two ROSCs [102] with a very high resolution, which was originally used to measure frequency degradation of digital circuits. As shown in Figure 6.2, the ROSC A is continuously sampled by a ROSC B whose frequency is slightly different from ROSC A. The output of the DFF exhibits the beat frequency $\Delta f$, which is determined by the frequency difference of the two ROSCs. A counter measures the beat frequency with ROSC B as the clock. The counter output increments every ROSC period until it reaches the beat frequency interval after which the count is sampled and reset. The mean of the frequency difference of the two ROSCs is caused by manufacturing process variations, and can be further adjusted by trimming capacitors associated with the ring oscillators [100]. The output count will fluctuate due to the random jitter in the circuit. Under the presented setting, we can generate approximately 3.25 bits per sample by using first 3 least significant bits (LSBs) directly and processing the 4th LSB with the von Neumann corrector [103].
Figure 6.2: BFD-TRNG: (a) basic principle, (b) die microphotograph in 65nm.

Overall, there are a number of circuit level advantages of the BFD-TRNG compared to other existing ROSC based TRNGs [100]:

1. Higher tolerance to temperature or voltage drifts.
2. Fully digital operation.
3. Increased number of random bits per sample.
4. Reduced sampling time.
5. Minimal calibration, i.e., one-time calibration of the average count during start up is needed.

6.3 Physical Component Modeling of RO TRNGs

As discussed above, the statistical tests such as NIST and DIEHARD are designed to check the performance of PRNGs. The core of a TRNG is its randomness source, which usually generates a time-continuous analog signal that is digitized by certain harvest mechanism. In order to validate a TRNG, characterization of the randomness source
and the harvest mechanism are needed. In this section, we investigate the statistical properties of the BFD-TRNG.

The randomness source of the ring oscillator based TRNGs is the timing jitter in each ROSC, which is a stochastic phenomenon caused by internal random noise such as thermal, shot, and random telegraph noise in the transistors of a ring oscillator. Jitter can be considered as a short-term variation of a digital signal from their ideal position in time. The size of the jitter is determined by the properties of the hardware device and the operating environment. In these ROSC based TRNG designs, two or more oscillators are combined to produce a random bitstream. This jitter will create an accumulated phase drift in each ring so that the transition region in the sampling period is assumed to be unpredictable. In the literature, several studies of the jitter in ring oscillators have been presented [104, 105, 106, 107, 108, 109, 110]. More precisely, the jitter model should incorporate a Gaussian variable, $1/f$ noise, and a coupling sinusoidal signals [89]. However, existing works [105, 106] report that the durations between the transition times appear in many cases to be independent and identically distributed Gaussian, as it is the most dominant component. This allows us to create simple model for ROSC based TRNGs by characterizing the jitter as a Gaussian random variable with zero mean. Moreover, there are two major reasons that we do not consider Random Telegraph Noise (RTN) as the major random noise source: First, due to the averaging effect, the RTN induced jitter is much smaller than that on a single transistor. Second, the occurrence of RTN with large amplitude and high frequency is rare [81, 111].

A ROSC consists of an odd number of inverters connected together in a ring configuration. This causes the output of the oscillator to change with a period of approximately $2kD$, where $k$ is the number of inverters in a ROSC and $D$ is the delay of a single inverter. If we consider the delay of each inverter as a Gaussian random variable $D_i \sim N(\mu_i, \sigma_i^2)$, a period of the ROSC can be written as

$$T = 2 \sum_{i=1}^{k} D_i \sim N(\mu, \sigma^2), \quad (6.1)$$
which is also a Gaussian random variable. For simplicity, we directly consider a period of the ROSC as a Gaussian random variable in this chapter. Periods vary from cycle to cycle causing jitter in the rising and falling edges. Note that this model can incorporate different operating conditions (e.g., temperature, supply voltage) by modifying $\sigma$ accordingly.

6.4 Statistical Analysis of BFD-TRNG

Based on the illustrated model above, this section presents a comprehensive statistical analysis to help resolve some important BFD-TRNG design issues:

(a) How much of the frequency difference is required to produce sufficient random numbers?

(b) How many bits of the counter value can be used?

(c) How can the TRNG performance be further improved?

6.4.1 BFD Model

As shown in Figure 6.2, the BFD-TRNG consists of two ROSCs whose frequencies are slightly different. The period of the two ROSCs can be modeled as $T_A \sim N(\mu_A, \sigma_A^2)$ and $T_B \sim N(\mu_B, \sigma_B^2)$, respectively. Note that the two ring oscillators are implemented identical, therefore their free-running frequencies are very close but not identical due to the process variation. To prevent injection locking phenomenon or any other unintended coupling between the two ROSCs, we separated the frequencies of the two ROSCs using trimming capacitors prior to the testing. Experimental data showed no signs of correlation between the ROSC frequencies [100]. An output will be generated once the beat frequency is obtained, i.e., the faster ROSC completes one more cycle than the slower ROSC. The output will be the number of cycles completed by ROSC B at this moment. Without loss of generality, we always assume ROSC A is faster than
ROSC B in this chapter, i.e., $\mu_A < \mu_B$. Since the inverters in ROSC A and ROSC B are almost equivalently designed with only slight frequency difference and operated under the same environmental condition, we can assume $\sigma = \sigma_A = \sigma_B$. Therefore, the probability density function (pdf) of the counter value $N$ can be expressed as

$$pdf(N) = \{ \min N : \sum_{i=1}^{N} TA_i < \sum_{i=1}^{N-1} TB_i \} = \{ \min N : TA_N < \sum_{i=1}^{N-1} (TB_i - TA_i) \}. \quad (6.2)$$

However, the $N$ in Equation (6.2) does not have a standard probability density function. Instead, we perform Monte Carlo simulations to study the statistical properties of this model. Model parameters extracted from experimental measurements in [100] imply

$$\frac{\Delta \mu}{\mu_B} = \frac{\mu_B - \mu_A}{\mu_B} = 0.28\% \text{ and } \sigma_A = \sigma_B = 0.0006.$$

Note that in this chapter, without loss of generality, we always assume the mean of the clock signal of the DFF to be 1 (i.e., $\mu_B = 1$). Therefore, $\Delta \mu = 0.0028$ and $\mu_A = 0.9972$ in this setup. The distribution of the counter values is shown in Figure 6.3.

![Figure 6.3: Counter value distribution ($\Delta \mu = 0.28\%, \sigma = 0.0006$).](image)

### 6.4.2 Effect of Counter Value

**Mean of Counter Value**

It can be seen from Figure 6.3 that the mean of the counter values is close to

$$\frac{1}{\Delta \mu} = 357.$$  

We repeat the simulation with $\Delta \mu = 0.4\%$ as shown in Figure 6.4. The mean is also close to $\frac{1}{\Delta \mu} = 250$. Thus, we observe that the mean of counter values is inversely proportional to the value of $\Delta \mu$. 
This property can also be derived by mathematically. Since Equation (6.2) does not have a closed-form expression, we consider a simpler case: $TA$ and $TB$ remain unchanged during one measurement time. In fact, this is the original function of a beat frequency detector, i.e., to measure the frequency difference of ROSC A and ROSC B. In this case, Equation (6.2) can be simplified to

$$N = \left\{ \min N : \left| N - \frac{NTB}{TA} \right| \geq 1 \right\}.$$  \hspace{1cm} (6.3)

By solving the equation, we can get

$$N = \left\lceil \frac{TB}{|TB - TA|} \right\rceil.$$ \hspace{1cm} (6.4)

There is one trivial observation that $|TB - TA|$ cannot be very small; otherwise, the counter value will be very large (i.e., only one counter value can be obtained in very large number of cycles). If we consider $TA$ and $TB$ as the average periods of $N$ cycles, these can be characterized as Gaussian random variables $\frac{\sum_{i=1}^{N}TA_i}{N} \sim N(1 - \Delta \mu, \frac{\sigma^2}{N})$ and $\frac{\sum_{i=1}^{N}TB_i}{N} \sim N(1, \frac{\sigma^2}{N})$, respectively. Thus,

$$\frac{TB}{TB - TA} \sim \frac{N(1, \frac{\sigma^2}{N})}{N(\Delta \mu, \frac{2\sigma^2}{N})},$$  \hspace{1cm} (6.5)
which can be described as a ratio of two Gaussian random variables. We can approximate the expected value of $\frac{T_B}{T_B - T_A}$ by a second order Taylor expansion [112]:

$$E(\frac{T_B}{T_B - T_A}) \approx E(T_B) - \frac{Cov(T_B, T_B - T_A)}{E^2(T_B - T_A)} + \frac{Var(T_B - T_A)E(T_B)}{E^3(T_B - T_A)}$$

$$= \frac{1}{\Delta\mu} - \frac{\sigma^2}{\Delta\mu^2} + \frac{2\sigma^2}{N\Delta\mu^3}$$

$$= \frac{1}{\Delta\mu} + \frac{1}{N} \frac{\sigma^2}{\Delta\mu^2}(\frac{2}{\Delta\mu} - 1). \quad (6.6)$$

Based on the assumption that $T_A$ and $T_B$ are uncorrelated, we can obtain that $Cov(T_B, T_B - T_A) = Var(T_B) = \sigma^2$. For the parameters $\Delta\mu = 0.28\%$, $\sigma = 0.0006$, the above equation will equal to

$$\frac{1}{\Delta\mu} + \frac{1}{N} \frac{\sigma^2}{\Delta\mu^2}(\frac{2}{\Delta\mu} - 1) = 357.14 + \frac{32.75}{N}. \quad (6.7)$$

It can be seen from Figure 6.3 that $330 < N < 390$. Consequently, the second term of Equation (6.7) (i.e., $\frac{32.75}{N}$) is less than 0.1. Consequently, the mean of counter values is approximately $\frac{1}{\Delta\mu} \approx 357$ in this case.

Typically, the second term of Equation (6.7) will be relatively small compared to $\frac{1}{\Delta\mu}$, since $\Delta\mu$ is a very small number and $\frac{\sigma^2}{\Delta\mu^2}$ is generally less than 1.

As a result, we can conclude

$$E(N) \approx \frac{1}{\Delta\mu}. \quad (6.8)$$

Thus, in contrast to the original function of the BFD, i.e., to measure the slight frequency difference of two signals, we should set up an appropriate frequency difference for the two ROSCs. In other words, trimming capacitors should be used to set an appropriate $\Delta\mu$ for the two ROSCs, instead of equalizing their frequencies or making the frequency differences as small as possible. Therefore, we mainly harvest randomness from the jitter noise instead of the metastability in our design, as the random numbers are generated from the delay difference variations of the two ROSCs, instead of from sampling one ROSC with another ROSC. Note that our current test chips provide a
frequency trimming resolution of 0.1%, this can be further reduced with more trimming capacitor bank controls.

**Dynamic Range of Counter Value**

The randomness of the BFD-TRNG comes from the counter values. If the dynamic range is larger, we could use more bits of the counter value as random numbers. Therefore, it is important to examine the statistics of the dynamic range of the counter values. We attempt to use a Gaussian distribution to fit the counter values from experimental measurements and consider $6\sigma_G$ as the dynamic range based on the fitting result $N(\mu_G, \sigma^2_G)$. An example is shown in Figure 6.3, with parameters $\Delta\mu = 0.28\%$, $\sigma = 0.0006$. It is shown that the distribution of the counter values is close to a Gaussian distribution. In this case, the dynamic range $6\sigma_G$ is equal to 34.6.

We repeat the simulation for different parameters as shown in Figure 6.4 and Figure 6.5, whose dynamic ranges are 20.3 and 68.9, respectively. The relationship between the dynamic range of counter values and $\Delta\mu$ is shown in Figure 6.6, where the mean of counter values is equal to $\frac{1}{2\Delta\mu}$. It can be seen that the dynamic range of counter values will increase with the increase of $\sigma$, while it will decrease with the increase of $\Delta\mu$. This observation is also conformed from our measured chip data [100]. Moreover, it can be seen that only slight change of $\Delta\mu$ will affect the counter values significantly.

![Figure 6.5: Counter value distribution ($\Delta\mu = 0.28\%, \sigma = 0.0012$).](image)
Figure 6.6: The relationship between the dynamic range of counter values and $\Delta \mu$ ($\sigma = 0.0006\%$).

Figure 6.7: The relationship between the dynamic range of counter values and $\sigma$ ($\Delta \mu = 0.28\%$).
We can also examine the variance of counter values by a second order Taylor expansion [112]:

\[
\text{Var}(\frac{TB}{TB-TA}) \approx \frac{E^2(TB)}{E^2(TB-TA)} \left( \frac{\text{Var}(TB)}{E^2(TB)} - 2 \frac{\text{Cov}(TB, TB - TA)}{E(TB)E(TB-TA)} + 2 \frac{\text{Var}(TB - TA)}{E^2(TB-TA)} \right)
\]

\[
= \frac{1}{\Delta \mu^2} \left( \frac{\sigma^2}{N} - \frac{2\sigma^2}{N \Delta \mu} + \frac{2\sigma^2}{N \Delta \mu^2} \right)
\]

\[
= \frac{\sigma^2}{N \Delta \mu^2} \left( 1 - \frac{2}{\Delta \mu} + \frac{2}{\Delta \mu^2} \right).
\]

Since we have demonstrated that the expected value of \( N \) is approximately \( \frac{1}{\Delta \mu} \) and \( \Delta \mu \) is a very small value, we can further approximate the above equation as

\[
\text{Var}(\frac{TB}{TB-TA}) \approx \frac{\sigma^2}{N \Delta \mu^2} \left( 1 - \frac{2}{\Delta \mu} \right)
\]

\[
\approx \frac{\sigma^2}{\Delta \mu^2} \left( 2 \left( 1 - \frac{1}{2} \right)^2 + \frac{1}{2} \right)
\]

\[
\approx \frac{2\sigma^2}{\Delta \mu^3}.
\]

Consequently, we can obtain

\[
\sigma_G \approx \sqrt{\text{Var}(\frac{TB}{TB-TA})} \approx \frac{\sqrt{2}\sigma}{\Delta \mu}. \quad (6.11)
\]

Thus, we can conclude that the dynamic range of counter values increases linearly with \( \sigma \) and is inversely proportional to \( \Delta \mu^3 \).

### 6.4.3 Bounds on Bias of Each Bit

According to the NIST test, the probability of ”1” occurrence, \( p1 \), should satisfy 49.91% \( \leq p1 \leq 50.09% \) to pass the frequency test of NIST tests [97]. For any bit of the counter values, if the probability of ”1” occurrence is within the acceptance range [49.91%, 50.09%], then this bit might be used as random numbers directly. Otherwise, certain techniques are required to post-process the bit.  Note that we only consider the biasedness metric in our simulation, as other metrics are completely dependent on the performance of the employed pseudo random number generator for simulation. The \( p1 \)
of each bit for the distribution in Figure 6.3 is presented in the second row of Table 6.1. Since the counter values are less than 512 in our setup, we only need to consider the first 9 LSBs. It can be seen that the first 3 LSBs are within the acceptance range. In fact, our chip experimental results show that all of the first 3 LSBs can pass the NIST test individually or collectively after serializing them.

We also present the value of $p_1$ of each bit for the distribution as shown in Figure 6.8 with parameters $\Delta \mu = 0.391\%$ and $\sigma = 0.0009$ in Table 6.1. The dynamic range $6\sigma_G$ is 30.2 which is less than the dynamic range of the distribution in Figure 6.3. However, the first 4 LSBs are within the acceptance range. Furthermore, the higher bits are also less biased compared to counter values in Figure 6.3. Therefore, the counter values in Figure 6.8 have better randomness than the counter values in Figure 6.3, even though the dynamic range of the counter values in Figure 6.8 is smaller.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>$p_1(b_8)$</th>
<th>$p_1(b_7)$</th>
<th>$p_1(b_6)$</th>
<th>$p_1(b_5)$</th>
<th>$p_1(b_4)$</th>
<th>$p_1(b_3)$</th>
<th>$p_1(b_2)$</th>
<th>$p_1(b_1)$</th>
<th>$p_1(b_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 6.3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.8384</td>
<td>0.1979</td>
<td>0.4564</td>
<td>0.5003</td>
<td>0.4991</td>
<td>0.4995</td>
</tr>
<tr>
<td>Figure 6.8</td>
<td>0.5173</td>
<td>0.4827</td>
<td>0.4827</td>
<td>0.4827</td>
<td>0.4838</td>
<td>0.4991</td>
<td>0.4998</td>
<td>0.5007</td>
<td>0.5005</td>
</tr>
</tbody>
</table>

Figure 6.8: Counter value distribution ($\Delta \mu = 0.391\%$ and $\sigma = 0.0009$).

As a result, we can conclude that the number of bits that we can use is not only dependent on the dynamic range, but also dependent on the mean of the counter values. For example, we consider the two cases as shown in Figure 6.9. The counter values in the top and the bottom panels of Figure 6.9 have the same dynamic range. However,
the mean of counter values in the top panel is 15.5, which is just at the boundary of $b_3 = 0$ and $b_3 = 1$. As a result, the expected bias $\varepsilon$ will be 0, where $\varepsilon$ is defined as $\varepsilon = |0.5 - p|$. For the counter values in the bottom panel whose mean is 19.5, $p1 = 0.11$. Thus, the bias is $|0.5 - p1| = 0.39$. This is because the mean of the counter values is in the middle of the region where $b_3 = 0$.

For a certain dynamic range of counter values, in the best case, $E(N) = 2^k m - 0.5$ for $b_k$ ($m$ is an integer), which leads to $\varepsilon = 0$. In the worst case, $E(N) = 2^k m + 2^{k-1} - 0.5$, which generates the largest bias $\varepsilon$. Ideally, we can extract more randomness from the counter values by carefully adjusting the mean. However, in order to ensure the quality of each bit when taking noise and operating environmental change into consideration, we have to consider the bias in the worst case. Table 6.2 presents the corresponding bias $\varepsilon$ for each bit under different $\sigma_G$ in the worst case.

It can be seen from Table 6.2 that the first LSB is guaranteed to be unbiased if the dynamic range of counter values is greater than $6\sigma_G = 12$. Furthermore, if the dynamic range is greater than 30, the first 3 LSBs might be used as random numbers without any post-processing. As a result, we have to ensure at least a dynamic range of 30 for the BFD-TRNG [100], if we output the first 3 LSBs directly. In addition, we
cannot use $b_4$ to $b_8$ directly while the dynamic range is less than 90. Since the dynamic range increases with the increase of the mean of the counter values, we have to set an appropriate $\Delta \mu$ to attain sufficient randomness of the TRNG design.

Figure 6.10 shows the relationship between the number of bits that we can output directly and the dynamic range. The number of unbiased bits can be expressed by fitting the curve perfectly as

$$\text{number of unbiased bits} = \lfloor \log_2 \left( \frac{40}{23} + \log_2 \sigma_G \right) \rfloor = \lfloor \log_2 \left( \frac{40}{23} + \log_2 \sigma - \frac{3}{2} \log_2 \Delta \mu + \frac{1}{2} \right) \rfloor.$$  

(6.12)

Therefore, we can conclude that the number of bits that we can use is logarithmically proportional to the dynamic range of the counter values.

**Summary**

The observations are summarized below.

1. The mean of the counter values is inversely proportional to $\Delta \mu$.

2. The dynamic range of counter values is inversely proportional to $\Delta \mu^3$ and is proportional to $\sigma$.  

<table>
<thead>
<tr>
<th>$\sigma_G$</th>
<th>$\varepsilon(b_8)$</th>
<th>$\varepsilon(b_7)$</th>
<th>$\varepsilon(b_6)$</th>
<th>$\varepsilon(b_5)$</th>
<th>$\varepsilon(b_4)$</th>
<th>$\varepsilon(b_3)$</th>
<th>$\varepsilon(b_2)$</th>
<th>$\varepsilon(b_1)$</th>
<th>$\varepsilon(b_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4999</td>
<td>0.4545</td>
<td>0.1854</td>
<td>0.0046</td>
<td>0.00046</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4999</td>
<td>0.4545</td>
<td>0.1854</td>
<td>0.0046</td>
<td>0.00046</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4923</td>
<td>0.3176</td>
<td>0.0397</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4999</td>
<td>0.4545</td>
<td>0.1854</td>
<td>0.0046</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4986</td>
<td>0.3904</td>
<td>0.0926</td>
<td>0.0003</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4923</td>
<td>0.3176</td>
<td>0.0397</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4777</td>
<td>0.2475</td>
<td>0.0146</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4999</td>
<td>0.4545</td>
<td>0.1854</td>
<td>0.0046</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4996</td>
<td>0.4246</td>
<td>0.1336</td>
<td>0.0012</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4986</td>
<td>0.3904</td>
<td>0.0926</td>
<td>0.0003</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4964</td>
<td>0.3542</td>
<td>0.0618</td>
<td>0.0001</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4923</td>
<td>0.3176</td>
<td>0.0397</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4862</td>
<td>0.2816</td>
<td>0.0245</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>0.5</td>
<td>0.4999</td>
<td>0.4777</td>
<td>0.2475</td>
<td>0.0146</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>15</td>
<td>0.5</td>
<td>0.4999</td>
<td>0.4671</td>
<td>0.2153</td>
<td>0.0083</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
3. The sampling rate is inversely proportional to the mean of counter values.

4. The number of unbiased bits is logarithmically proportional to the dynamic range of counter values; thus, it is inversely logarithmically proportional to $\Delta \mu^{\frac{3}{2}}$.

5. The sampling rate is proportional to $\Delta \mu$.

As we are only able to control the value of $\Delta \mu$, we should set an appropriate $\Delta \mu$ to achieve better throughput. Note that a higher $\sigma$ can be obtained by appropriately sizing the transistor and choosing number of stage in the ROSCs, but this is out of the scope of this chapter. For post-fabrication throughput optimization, we are able to achieve better throughput with the following two methods based on $\Delta \mu$ controlling.

(a) Use a higher $\Delta \mu$ (i.e., lower counter values), which could improve the sampling rate. But we may only be able to use limited number of bits from each counter value as random numbers.

(b) Use a lower $\Delta \mu$ (i.e., higher counter values). The sampling rate is reduced. But we can use more bits from the counter values as random numbers. Moreover, we can further post-process the higher bits of the counter values to generate more bits.

Based on the summary above, we can obtain the relationship between the throughput (i.e., rate $\times$ number of unbiased bits) of the BFD based design and parameters $\Delta \mu, \sigma$.

![Figure 6.10: The relation between the number of unbiased bits and the value of $\sigma_G$.](image)
as below:

$$\text{throughput} \propto \Delta \mu \left[ \log_2 \frac{40}{23} + \log_2 \sigma G \right] \approx \Delta \mu \left[ \log_2 \frac{40}{23} + \log_2 \sigma - \frac{3}{2} \log_2 \Delta \mu + \frac{1}{2} \right]. \quad (6.13)$$

If we consider Equation (6.13) without the floor function as

$$\text{throughput} \propto \Delta \mu \left( \log_2 \frac{40}{23} + \log_2 \sigma - \frac{3}{2} \log_2 \Delta \mu + \frac{1}{2} \right). \quad (6.14)$$

Equation (6.14) achieves maximum value at

$$\Delta \mu = 2 \left( \frac{2}{3} \log_2 \frac{40}{23} \sqrt{2} \sigma - \frac{1}{\ln 2} \right). \quad (6.15)$$

For example, the values of Equations (6.13) and (6.14) are shown in Figure 6.11 when $\sigma = 0.0006$.

![Figure 6.11: Values of Equations (6.13) and (6.14) for different $\Delta \mu$'s ($\sigma = 0.0006$).](image)

It can be seen from the top panel of Figure 6.11 that the BFD-TRNG achieves the best throughput when $\Delta \mu = 0.0050$ (2 bits can be used as random numbers), while the maximum value of Equation (6.14) is achieved at $\Delta \mu = 2 \left( \frac{2}{3} \log_2 \frac{40}{23} \sqrt{2} \sigma - \frac{1}{\ln 2} \right) = 0.004768$ as shown in the bottom panel. Therefore, in this case, we can adjust the $\Delta \mu$ such that the mean of the counter values is about 200 to achieve higher throughput. Generally, the $\Delta \mu$ should be adjusted according to the environmental noise $\sigma$ based on Equation (6.15).
6.4.4 Post-Processing

As discussed above, the number of bits that can be treated as random numbers is determined by both the dynamic range and the mean of counter values. For those bits with some randomness that do not meet the NIST test requirement (i.e., $0.0009 < \varepsilon < 0.5$), post-processing techniques can be used to generate more random bits.

Post-processing techniques for TRNGs are used to ameliorate non-randomness in the raw bitstream, which are basically compression functions that are applied to the raw bitstream before the output of the TRNG. Furthermore, post-processing techniques can improve the stability of a TRNG, as it is able to correct the raw bitstream if operating conditions change. In the BFD-TRNG design, if a certain bit starts out with a high bias, the post-processing step would transform the bitstream such that the bias becomes more acceptable. The two common techniques we consider in the post-processing step include block-wise XOR and Von Neumann corrector [103]. Other techniques, such as linear compression functions based on good linear codes can also be used for de-biasing [93, 113, 114]. The comparison of block-wise XOR and Von Neumann corrector is illustrated in Table 6.3, where XOR $d$ corresponds to XOR operation with a block size of $d$.

Table 6.3: Comparison of Block-Wise XOR and Von Neumann Corrector

<table>
<thead>
<tr>
<th></th>
<th>XOR $d$</th>
<th>Von Neumann</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate</td>
<td>$\frac{1}{d}$</td>
<td>$\frac{1}{d} - \varepsilon^2$</td>
</tr>
<tr>
<td>Bias</td>
<td>$2^{d-1}\varepsilon^d$</td>
<td>0</td>
</tr>
</tbody>
</table>

Each of these post-processing techniques has its pros and cons. Using Von Neumann corrector will produce perfect correction with 0 bias but throughput is reduced to less than 25% of its original. XOR may achieve better throughput with a small $d$. However, we have to ensure that $2^{d-1}\varepsilon^d$ is within the acceptance range. For example, the compression rate is 50% when $d = 2$. However, the bias is only improved from $\varepsilon$ to $2\varepsilon^2$.

Table 6.4 presents the number of bits that we can generate for different $\sigma_G$ in the worst case. Each value in Table 6.4 represents how many bits can be used as random
numbers for a certain counter value. For example, if a bit has a bias $\varepsilon < 0.0009$, the bit can produce 1 random bit per sample. However, if the bias $\varepsilon$ exceeds the threshold, this bit of the counter value can be used to generate $\frac{1}{4}$ random bit per sample by using block-wise XOR or $\frac{1}{4} - \varepsilon^2$ bit per sample by the Von Neumann corrector. Entropy is essentially an upper bound on the number of bits that we can generate for a given dynamic range. Note that the total entropy can be obtained by the sum of the entropy for each individual bit, according to Table 6.2.

Table 6.4: Number of Bits per Sample for Different $\sigma_G$ in the Worst Case

<table>
<thead>
<tr>
<th>$\sigma_G$</th>
<th>Von Neumann</th>
<th>XOR</th>
<th>Entropy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.51</td>
<td>0.64</td>
<td>2.17</td>
</tr>
<tr>
<td>2</td>
<td>1.51</td>
<td>1.64</td>
<td>3.17</td>
</tr>
<tr>
<td>3</td>
<td>2.41</td>
<td>2.4</td>
<td>3.75</td>
</tr>
<tr>
<td>4</td>
<td>2.51</td>
<td>2.64</td>
<td>4.17</td>
</tr>
<tr>
<td>5</td>
<td>3.34</td>
<td>3.29</td>
<td>4.49</td>
</tr>
<tr>
<td>6</td>
<td>3.41</td>
<td>3.4</td>
<td>4.75</td>
</tr>
<tr>
<td>7</td>
<td>3.46</td>
<td>3.61</td>
<td>4.97</td>
</tr>
<tr>
<td>8</td>
<td>3.51</td>
<td>3.64</td>
<td>5.17</td>
</tr>
<tr>
<td>9</td>
<td>3.55</td>
<td>3.7</td>
<td>5.34</td>
</tr>
<tr>
<td>10</td>
<td>4.34</td>
<td>4.29</td>
<td>5.49</td>
</tr>
<tr>
<td>11</td>
<td>4.37</td>
<td>4.3</td>
<td>5.62</td>
</tr>
<tr>
<td>12</td>
<td>4.41</td>
<td>4.4</td>
<td>5.75</td>
</tr>
<tr>
<td>13</td>
<td>4.43</td>
<td>4.42</td>
<td>5.86</td>
</tr>
<tr>
<td>14</td>
<td>4.46</td>
<td>4.61</td>
<td>5.97</td>
</tr>
<tr>
<td>15</td>
<td>4.49</td>
<td>4.63</td>
<td>6.07</td>
</tr>
</tbody>
</table>

It can be seen that the block-wise XOR and Von Neumann corrector have comparable performances for the BFD-TRNG. However, for the bit with a small bias, XOR will be more favored than Von Neumann corrector. As we always try to utilize the bits with smaller biases, block-wise XOR could outperform Von Neumann corrector in general. From the simulation results, besides the bits of counter values that we can output directly, we may only be able to use 2 more bits by post-processing, as the rate will be too low for the higher bits. According to Table 6.2, for example, we can use the first 3 LSBs directly when $\sigma_G = 7$. If we only want to post-process the 4th LSB to generate more bits, we can generate 3.5 bits per sample by XOR or approximately 3.25
bits per sample by Von Neumann corrector. Moreover, the output of block-wise XOR is synchronous while the output of Von Neumann corrector is asynchronous. The challenge of using block-wise XOR is that we need to determine the value of $d$ for each bit based on the dynamic range of the counter values.

6.4.5 Online Test and Feedback Control

Due to tolerances of the components, aging effects, a total breakdown of the noise source or possible active attacks, the TRNG may output considerably weaker random numbers. Online tests can be very useful for a TRNG to detect non-tolerable weaknesses while the TRNG is in operation [115]. Based on our statistical analysis results, the counter values can be easily used as a performance indicator. We can monitor the counter values during operation. Note that we will monitor higher bits of the counter value to achieve feedback control, while LSBs are used as random numbers. As a result, the problem of output manipulation can be avoided. If the dynamic range or mean of the counter values changes significantly, there is a high possibility of attack or significant operating environmental change. In this case, TRNG should stop outputting random numbers.

Furthermore, feedback control can be used to neutralize the environmental change or aging effects. Trimming capacitors in ring oscillators can be adjusted adaptively to keep the same value of $\Delta \mu$ based on our statistical analysis results. For example, we can adjust the trimming capacitor according to the higher bits of the counter values, while the lower bits are considered as random numbers.

Feedback control can also improve the efficiency of the BFD-TRNG:

(a) Based on the optimal relationship of $\Delta \mu$ and environmental noise $\sigma$ as expressed in Equation (6.15), we may adjust the trimming capacitor according to the distribution of counter values during setup. For example, we can estimate the $\sigma$ after collecting a number of counter value samples based on Equation (6.11). Then
we will be able to determine whether the current $\Delta \mu$ is desired or it needs to be adjusted.

(b) According to the dynamic range, the TRNG can decide the number of unbiased bits that can be output directly in the worst case. Moreover, the type of post-processing techniques can also be determined to extract random numbers from higher bits of the counter values.

6.5 Alternate BFD-TRNG Architectures

Motivated by the statistical analysis results, in this section, we propose a number of alternate BFD-TRNG designs, which can further improve the performances.

6.5.1 Parallel Structure

Parallelizability is also a desired metric of a TRNG. In fact, the BFD-TRNG is notably easy to parallelize by adding as many extra ROSCs instead of 2 ROSCs to generate multiple outputs, as shown in Figure 6.12. Our experimental results show that the counter values of the two adjacent outputs are highly correlated. However, the advantage of the BFD-TRNG is that we can consider bits of the counter values individually. For example, the simulated correlation coefficients for a 4-parallel structure are presented in Table 6.5. Note that the $\Delta \mu$ is assumed to be same for all the ROSC pairs. According to NIST test, only the bitstreams with correlation coefficients less than 0.073 can pass the test [97]. It can be seen that the correlation coefficients of the counter values of two adjacent outputs are large, i.e., around 0.5. However, the correlations of the first 4 LSBs from two adjacent outputs are very small, while the counter values and individual bits are not correlated for non-adjacent outputs. Therefore, it can be concluded that the first 4 LSBs from the outputs in Figure 6.12 can still be used as random numbers. In general, we can generate $3.25M$ bits per sample by using $M + 1$
ROSCs and $M$ DFFs, since we can use the first 3 LSBs directly and generate $\frac{1}{4}$ bits per count by postprocessing the 4th LSB with von Neumann corrector.

![Diagram](image)

**Figure 6.12**: An M-parallel TRNG structure.

<p>| Correlation Coefficients of Each Bit among the Outputs for a 4-Parallel Structure |
|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|-------------------------------------------------|</p>
<table>
<thead>
<tr>
<th>$b_8 : b_0$</th>
<th>Output (1,2)</th>
<th>Output (2,3)</th>
<th>Output (3,4)</th>
<th>Output (1,3)</th>
<th>Output (2,4)</th>
<th>Output (1,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4967</td>
<td>0.4970</td>
<td>0.4974</td>
<td>0.0018</td>
<td>-0.0006</td>
<td>-0.0007</td>
<td></td>
</tr>
<tr>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0008</td>
<td>-0.0003</td>
<td>0.0000</td>
<td>0.0009</td>
<td></td>
</tr>
<tr>
<td>0.0003</td>
<td>0.0008</td>
<td>0.0016</td>
<td>-0.0006</td>
<td>-0.0004</td>
<td>-0.0018</td>
<td></td>
</tr>
<tr>
<td>0.0010</td>
<td>0.0013</td>
<td>0.0005</td>
<td>0.0004</td>
<td>0.0001</td>
<td>-0.0016</td>
<td></td>
</tr>
<tr>
<td>0.0027</td>
<td>0.0049</td>
<td>0.0023</td>
<td>0.0014</td>
<td>-0.0006</td>
<td>-0.0004</td>
<td></td>
</tr>
<tr>
<td>0.1193</td>
<td>0.1177</td>
<td>0.1190</td>
<td>-0.0008</td>
<td>-0.0009</td>
<td>0.0002</td>
<td></td>
</tr>
<tr>
<td>0.2974</td>
<td>0.2966</td>
<td>0.2971</td>
<td>0.0006</td>
<td>-0.0013</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>0.3023</td>
<td>0.2998</td>
<td>0.3001</td>
<td>0.0009</td>
<td>-0.0010</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>0.3023</td>
<td>0.2998</td>
<td>0.3001</td>
<td>0.0009</td>
<td>-0.0010</td>
<td>0.0001</td>
<td></td>
</tr>
</tbody>
</table>

### 6.5.2 Cascade Structure

A novel cascade structure which could achieve better randomness is shown in Figure 6.13. Note that ROSC B also connects to the clock signal of the counter (not shown in Figure 6.13). The dynamic range of this cascade structure is higher than that of the original BFD-TRNG. Figure 6.14 shows the counter value distributions of the original BFD-TRNG and the cascade structure. Note that in our simulation, we set the
\[ |f_A - f_B| - |f_B - f_C| = 0.28\% \]

to maintain the same mean of the counter values. In other words, the frequencies of the 2 DFFs in the first stage are not very close to each other.

\[ \Delta f = |f_A - f_B| \]

\[ \Delta f = |f_B - f_C| \]

\[ \Delta f = |f_A - 2f_B + f_C| \]

Figure 6.13: A cascade TRNG structure.

The dynamic range is increased from 34.6 to 64.3 by adopting the cascade structure. As a result, we can also use the 4th LSB directly according to Table 6.2, while maintaining the same mean of the counter values. Therefore, the randomness is improved by using the cascade structure. Alternatively, we can reduce the mean of the counter values to increase the sampling rate, while maintaining considerable randomness. The
cascade structure will be extremely useful when the environmental noise is small. The BFD-TRNG may even be adaptively configured between 3 ROSCs and 2 ROSCs. For example, if the noise is relatively small, we could use 3 ROSCs as shown in Figure 6.13 to increase the dynamic range of counter values; otherwise, we could use 2 ROSCs as shown in Figure 6.12 to output 2 bitstreams of random numbers. Furthermore, this cascade structure provides higher flexibility for adjusting trimming capacitors associated with the ring oscillators.

6.5.3 Parallel-Cascade Structure

We can also parallelize the cascade structure, which leads to the so-called parallel-cascade structure of the BFD-TRNG. For example, a 4-parallel-cascade structure is shown in Figure 6.15. Two adjacent outputs share two ring oscillators, while the outputs that are separated from one output share one ring oscillator. We also need to examine the correlation coefficients of the outputs to ensure the bits that are used as random numbers are not correlated. The simulated correlation coefficients for the counter values and the individual bits among different outputs are presented in Table 6.6. Note that we set all the $\Delta f$’s in Figure 6.15 as 0.0028. It can be seen that the correlation coefficients for the first 4 LSBs between any two of the outputs are still very small and satisfy the NIST criteria. The correlations of the counter values and higher bits (i.e., $b_4$ to $b_8$) between two adjacent outputs are large, while the correlation coefficients of the counter values and higher bits between the outputs 1 and 3 or the outputs 2 and 4 are smaller but still exceed the threshold (i.e., 0.0073). The outputs 1 and 4 are not correlated for both the counter values and the individual bits, since they do not share any ring oscillator. Therefore, an $M$-parallel cascade structure can generate $4M$ bits per sample by using $(M + 2)$ ROSCs and $(2M + 1)$ DFFs. Note that in order to generate 4 bits from each output, we need to ensure the frequencies are either descending or ascending from the first ROSC to the last ROSC, as $\Delta f = ||f_A - f_B|| - |f_B - f_C|| = |f_A - 2f_B + f_C|$ only if $f_A > f_B > f_C$ or $f_A < f_B < f_C$. Otherwise, $\Delta f$ will equal to $|f_A - f_C|$, which
leads to the same dynamic range as the original BFD-TRNG. As a result, only 3.25 bits can be obtained from each output in this case. However, there is a problem when $M$ is large that the frequency difference between the first ROSC and the last ROSC will be fairly large if we want to generate 4 bits from each output, which may exceed the capability of the trimming capacitors. Therefore, the frequencies need not necessarily be set as either descending or ascending from the first ROSC to the last ROSC, which leads to a parallel-cascade structure where some of the outputs can generate 4 bits each and the others can generate 3.25 bits each. The performance is still improved.

Figure 6.15: A 4-parallel-cascade structure.

Table 6.6: Correlation Coefficients of Each Bit among the Outputs for a 4-Parallel-Cascade Structure

<table>
<thead>
<tr>
<th>Correlation Coefficients</th>
<th>Output (1,2)</th>
<th>Output (2,3)</th>
<th>Output (3,4)</th>
<th>Output (1,3)</th>
<th>Output (2,4)</th>
<th>Output (1,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[b_8 : b_9]$</td>
<td>0.6640</td>
<td>0.6634</td>
<td>0.6650</td>
<td>0.1666</td>
<td>0.1664</td>
<td>-0.0027</td>
</tr>
<tr>
<td>$b_8$</td>
<td>0.0002</td>
<td>-0.0004</td>
<td>-0.0003</td>
<td>0.0001</td>
<td>0.0008</td>
<td>0.0000</td>
</tr>
<tr>
<td>$b_9$</td>
<td>-0.0009</td>
<td>0.0012</td>
<td>0.0005</td>
<td>-0.0014</td>
<td>0.0005</td>
<td>-0.0003</td>
</tr>
<tr>
<td>$b_{10}$</td>
<td>-0.0007</td>
<td>0.0011</td>
<td>-0.0007</td>
<td>-0.0006</td>
<td>0.0010</td>
<td>-0.0002</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>-0.0029</td>
<td>-0.0015</td>
<td>-0.0023</td>
<td>0.0005</td>
<td>0.0006</td>
<td>-0.0004</td>
</tr>
<tr>
<td>$b_{12}$</td>
<td>0.0377</td>
<td>0.0382</td>
<td>0.0388</td>
<td>0.0197</td>
<td>0.0181</td>
<td>-0.0001</td>
</tr>
<tr>
<td>$b_{13}$</td>
<td>0.3539</td>
<td>0.3534</td>
<td>0.3542</td>
<td>0.1032</td>
<td>0.1013</td>
<td>0.0009</td>
</tr>
<tr>
<td>$b_{14}$</td>
<td>0.1771</td>
<td>0.1673</td>
<td>0.1701</td>
<td>0.0163</td>
<td>0.0189</td>
<td>-0.0015</td>
</tr>
<tr>
<td>$b_{15}$</td>
<td>0.1817</td>
<td>0.1721</td>
<td>0.1760</td>
<td>0.0145</td>
<td>0.0177</td>
<td>-0.0011</td>
</tr>
<tr>
<td>$b_{16}$</td>
<td>0.1817</td>
<td>0.1721</td>
<td>0.1760</td>
<td>0.0145</td>
<td>0.0177</td>
<td>-0.0011</td>
</tr>
</tbody>
</table>
6.6 Comparison with Other Existing ROSC based TRNGs

Furthermore, by adopting the proposed statistical model, we could also analyze prior ring oscillator based TRNG designs. In this section, we present the performance comparisons of the BFD-TRNG with other existing ring oscillator based TRNGs.

6.6.1 Two-Oscillator TRNG

The most comprehensive model of a two-oscillator TRNG is presented in [109]. In this section, we analyze the two-oscillator TRNG as shown in Figure 6.1 based on our simple model, i.e., assume a Gaussian random variable for the period of a ring oscillator. As discussed in Section 6.2, the frequency ratio between the two ROSCs plays a very important role in the randomness of the output. Experimental results have shown that the randomness is the worst when the fast oscillator frequency is an integer multiple of half the slow oscillator frequency [104]. In practice, the ratio is often carefully selected to achieve better randomness [92].

However, even if the TRNG design was originally designed to operate at a suitable oscillator frequency/sampling frequency ratio, a change in environmental conditions or worse adversarial influences may shift the frequency ratio to a weak operating point. It is claimed that the amount of accumulated jitter $6\sigma_{acc}$ should be at least six times as large as the period of the fast oscillator to attain sufficient randomness [116]:

$$6\sigma_{acc} \geq 6\mu_A,$$  \hspace{1cm} (6.16)

where $\sigma_{acc}^2 \approx \sigma_B^2 + L\sigma_A^2$, since the randomness is generated from the timing fluctuations of the edges of the slow signal relative to the fast oscillator. Let $L$ represent the number of periods ROSC A is completed before it is sampled. If we assume the design parameters for the inverters of the two ROSCs are the same, $\sigma_B^2$ will equal $L\sigma_A^2$, since the two ROSCs accumulated approximately with the same amount of jitter. Consequently, the
value of $L$ can be calculated as:

$$L \geq \frac{\mu_A^2}{2\sigma_A^2}. \quad (6.17)$$

In order to ensure sufficient randomness, a large frequency ratio is required. For example, $L$ should be greater than 1 million when $\sigma_A = 0.0006$. However, in the application of two-oscillator TRNG, the value of $\sigma_A$ is usually much larger. Frequency dividers can also help to achieve a large frequency ratio [117, 118]. Furthermore, a smaller ratio is sufficient to pass the NIST test in practice (i.e., NIST test is not that strict, compared to the statistical analysis). For example, experimental results [119] show that the period of a 7-stage ring oscillator implemented with a 65 nm CMOS process is 220ps from circuit simulation; thus, $220 \times 6 = 1320$ps of jitter is required. On the other hand, the jitter amount of a 251-stage ring oscillator with 64-frequency dividers is measured as 100ps, which is much smaller than the necessary value. Moreover, the results in [101] demonstrate that at least a ratio of 500 is required to achieve sufficient randomness to pass the NIST test.

### 6.6.2 ROSC TRNG with XOR Tree

A ROSC TRNG with XOR tree has been proposed in [93], which does not require large frequency separation of the fast and slow ring oscillators. The outputs from the oscillator rings are XOR-ed together and sampled with a DFF. A series of ring oscillators are combined to compensate for the imbalance between the number of zeros and ones in the random signal. In this structure, the jitter is accumulated spatially instead of temporarily. The TRNG structure is shown in Figure 6.16.

A stochastic approach of this TRNG is presented in [93]. It shows that in order to increase the entropy of the generated binary raw signal and to make the generator provably secure, large number of ROSCs needs to be employed. Experimental results show that the outputs of at least 114 supposedly independent ROSCs are XOR-ed and sampled using a reference clock with a fixed frequency can pass the NIST test. Only a small frequency ratio of 5 to 20 is required (e.g., approximately 6 in [93]).
However, some weakness of this TRNG design has been pointed out in [120]. The main concern is that the XOR-tree and the sampling D flip-flop cannot handle the high number of transitions from the oscillator rings. With many oscillator rings in parallel, the number of transitions during a sampling period will be too high to meet the setup/hold-time requirements. Experimental results show that approximately 50% of the transitions get lost [121]. To cope with the problem with many transitions in the sampling period, an enhanced TRNG based on the ROSCs has been proposed in [94] by adding an extra DFF after each ring oscillator before the XOR gate Figure 6.17. This TRNG design can generate desirable raw bitstream with a significantly reduced number of ROSCs. Its outputs can pass the NIST and DIEHARD tests without postprocessing.

The mathematical models for the ROSC TRNG with XOR tree as shown in Figure 6.16 and the enhanced structure as shown in Figure 6.17 are the same [122]. Similar to the two-oscillator based TRNG, the variance of the accumulated jitter of the ROSC TRNG with XOR tree can be expressed as

\[ \sigma_{acc}^2 \approx \sigma_B^2 + mL\sigma_A^2, \]  

(6.18)

where \( M \) is the number of ROSCs in parallel and \( L \) is the frequency ratio.
The number of ROSCs can be reduced by using the enhanced ROSC TRNG with XOR tree [94]. Experimental results in [94] show that 50 ROSCs in parallel are required to achieve sufficient randomness to pass the NIST test. However, this TRNG design is still not very efficient, since most of the ROSCs in this structure do not improve the entropy of random numbers if their transition regions are not sampled.

6.6.3 Comparison

There are a number of advantages of the BFD-TRNG designs. First of all, the random numbers of the BFD-TRNG are generated from counter values, which is a better harvest mechanism that can utilize more of the entropy. The bits per sample can be increased by post-processing or appropriately adjusting the counter values, while other existing ROSC based TRNGs are only able to generate maximum 1 bit per sample. Moreover, we could also choose to post-process with the counter values instead of individual bits.

Furthermore, other existing ROSC based TRNGs are sampled continuously. If the accumulated jitter is not sufficient between consecutive samplings, these samples will
be correlated. However, for the BFD-TRNG, the counter will be reset after collecting the data. As a result, the correlation between consecutive samples is reduced.

We continue to compare their performances according to evaluation metrics as below.

**Randomness**

In fact, the BFD-TRNG can be considered as a faster ROSC B that is sampled by a slower ROSC with frequency $|f_A - f_B|$. Therefore, the variance of the accumulated jitter between two consecutive samplings is

$$\sigma_{acc}^2 \approx 2^2 \sigma_B^2 + L\sigma_A^2,$$

(6.19)

where $L$ is equal to the counter value $N$ in this case. This is similar to the sum of the jitter in ROSC A and two times of the jitter in ROSC B. If we still assume the clock signal is generated from a slower ROSC and the design parameters for the inverters in the two ROSCs are the same (i.e., $\sigma_B^2 = L\sigma_A^2$), the value of $\sigma_{acc}^2$ for the BFD-TRNG is

$$\sigma_{acc}^2 \approx 5L\sigma_A^2.$$  

(6.20)

Similarly, the cascade structure as shown in Figure 6.13 can be considered as a faster ROSC B which is sampled by a slower ROSC with frequency $|f_A - 2f_B + f_C|$. In this case, the accumulated jitter will be the sum of the jitter in ROSC A, the jitter in ROSC C, and three times of the jitter in ROSC B. As a result, the value of $\sigma_{acc}^2$ for the cascade structure will be

$$\sigma_{acc}^2 \approx (1 + 1 + 3^2)L\sigma_A^2 = 11L\sigma_A^2.$$  

(6.21)

The value of $\sigma_{acc}^2$ for each TRNG design is summarized in Table 6.7.

It can be seen that the BFD-TRNG has greater $\sigma_{acc}^2$ per ROSC than prior ROSC based TRNGs, which could lead to better randomness, as it accumulates a larger amount of jitter before it is sampled. Moreover, it can be seen that the $\sigma_{acc}^2$ of BFD-TRNG is
Table 6.7: Comparison of $\sigma_{acc}^2$ for Different ROSC based TRNG Designs

<table>
<thead>
<tr>
<th>Design</th>
<th>$\sigma_{acc}^2$</th>
<th>$\sigma_{acc}^2$ per ROSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Oscillator TRNG (Figure 6.1)</td>
<td>$2L\sigma_A^2$</td>
<td>$L\sigma_A^2$</td>
</tr>
<tr>
<td>ROSC TRNG with XOR tree (Figure 6.16, Figure 6.17)</td>
<td>$(M + 1)L\sigma_A^2$</td>
<td>$L\sigma_A^2$</td>
</tr>
<tr>
<td>BFD-TRNG (Figure 6.2)</td>
<td>$5L\sigma_A^2$</td>
<td>$2.5L\sigma_A^2$</td>
</tr>
<tr>
<td>$M$-parallel BFD-TRNG (Figure 6.12)</td>
<td>$5ML\sigma_A^2$</td>
<td>$\frac{5M}{M+1}L\sigma_A^2$</td>
</tr>
<tr>
<td>Cascade BFD-TRNG (Figure 6.13)</td>
<td>$11L\sigma_A^2$</td>
<td>$3.67L\sigma_A^2$</td>
</tr>
<tr>
<td>$M$-parallel Cascade BFD-TRNG (Figure 6.15)</td>
<td>$11ML\sigma_A^2$</td>
<td>$\frac{11M}{M+2}L\sigma_A^2$</td>
</tr>
</tbody>
</table>

150% higher than the $\sigma_{acc}^2$ of two-oscillator TRNG. The parallel, cascade, and parallel-cascade structures of the BFD-TRNG can further improve the randomness. Note that the $\sigma_{acc}^2$ is just a rough estimate of the randomness when the TRNG is sampled.

**Cost**

We summarize the performance of different ROSC based TRNG designs in Table 6.8. We measure the area and power consumptions for the 7-stage ROSC, DFF, and 10-bit counter from the test chip in 65nm, as shown in Table 6.9. Consequently, the cost comparisons (only considering the components) for different ROSC based TRNGs are presented in Table 6.10. It can be seen that the BFD-TRNGs can generate more bits per sample. Furthermore, the BFD-TRNGs have less cost per bit in general, compared to prior ROSC based TRNG designs. We can further improve the performance by setting an appropriate $\Delta \mu$ as discussed in Section 6.4. Moreover, the parallel and the parallel-cascade structures of the BFD-TRNG can further reduce the cost per bit, as only one extra ROSC is required for each extra output. When $M$ is large, the costs of the parallel and the parallel-cascade structures will be significantly less than prior existing ROSC based TRNG designs.

We now compare the area and power performance of the $M$-parallel BFD-TRNG and the 64-parallel IBM TRNG in [101]. Since the $M$-parallel BFD generates $3.25M$ bits per count, for 64 parallel bits, $M = 64/3.25 \approx 20$. With $M = 64$ for IBM TRNG and $M = 20$ for BFD-TRNG, the (power)(sample period)/bit products for the two
designs are given by 522.2125 and 182.2308, respectively. The (area)(sample period)/bit products for the two designs are given by 534.0625 and 285.0000, respectively. Thus, we conclude that the $M$-parallel BFD-TRNG has approximately 3 times power advantage and 2 times area advantage for a specified number of bits per same period, compared to the IBM TRNGs. Similar calculations show that the power and area consumptions of $M$-parallel cascade BFD-TRNG are only 30.9% and 45.4% of the IBM TRNG, respectively. However, we caution that the $M$-parallel and $M$-parallel-cascade BFD-TRNG results are not based on actual measurements, but are predicted from models.

Table 6.8: Summary of Different ROSC based TRNG Designs

<table>
<thead>
<tr>
<th></th>
<th># Bits per Sample</th>
<th>Sample Period</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Oscillator TRNG (Figure 6.1)</td>
<td>1</td>
<td>&gt; 500</td>
<td>2 ROSCs, 1 DFF</td>
</tr>
<tr>
<td>$M$-parallel Two-Oscillator TRNG ([101])</td>
<td>$M$</td>
<td>&gt; 500</td>
<td>$(M + 1)$ ROSCs, $M$ DFFs</td>
</tr>
<tr>
<td>ROSC TRNG with XOR tree (Figure 6.16)</td>
<td>1</td>
<td>5 ~ 20</td>
<td>115 ROSCs, 1 DFF‡</td>
</tr>
<tr>
<td>Enhanced ROSC TRNG with XOR tree (Figure 6.17)</td>
<td>1</td>
<td>5 ~ 20</td>
<td>51 ROSCs, 50 DFFs‡</td>
</tr>
<tr>
<td>BFD-TRNG (Figure 6.2)</td>
<td>3.25</td>
<td>500</td>
<td>2 ROSCs, 1 DFF, 1 Counter</td>
</tr>
<tr>
<td>$M$-parallel BFD-TRNG (Figure 6.12)</td>
<td>3.25$M$</td>
<td>500</td>
<td>$(M + 1)$ ROSCs, $M$ DFFs, $M$ Counters</td>
</tr>
<tr>
<td>Cascade BFD-TRNG (Figure 6.13)</td>
<td>4</td>
<td>500</td>
<td>3 ROSCs, 3 DFFs, 1 Counters</td>
</tr>
<tr>
<td>$M$-parallel Cascade BFD-TRNG (Figure 6.15)</td>
<td>4$M$</td>
<td>500</td>
<td>$(M + 2)$ ROSCs, $(2M + 1)$ DFFs, $M$ Counters</td>
</tr>
</tbody>
</table>

‡ the cost of XOR is negligible

Table 6.9: Area and Power Consumptions for ROSC based TRNG Components

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Normalized Power</th>
<th>Area</th>
<th>Normalized Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROSC</td>
<td>21.19μ</td>
<td>1</td>
<td>40 × 10$μ^2$</td>
<td>1</td>
</tr>
<tr>
<td>DFF</td>
<td>0.61μ</td>
<td>0.0288</td>
<td>3 × 7$μ^2$</td>
<td>0.0525</td>
</tr>
<tr>
<td>Counter</td>
<td>2.24μ</td>
<td>0.1057</td>
<td>30 × 10$μ^2$</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 6.10: Cost for Different ROSC based TRNG Designs

<table>
<thead>
<tr>
<th></th>
<th>Total Power</th>
<th>(Power)(Sample Period)/Bit</th>
<th>Total Area</th>
<th>(Area)(Sample Period)/Bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Oscillator TRNG (Figure 6.1)</td>
<td>2.0286</td>
<td>&gt; 1014.4</td>
<td>2.0525</td>
<td>&gt; 1026.25</td>
</tr>
<tr>
<td>$M$-parallel Two-Oscillator TRNG ([101])</td>
<td>1.8288M + 1</td>
<td>&gt; 514.4 + 500/M</td>
<td>1.0525M + 1</td>
<td>&gt; 526.25 + 500/M</td>
</tr>
<tr>
<td>ROSC TRNG with XOR tree (Figure 6.16)</td>
<td>115,0288</td>
<td>570.144 ~ 2300.576</td>
<td>115,025</td>
<td>570,2425 ~ 2301.59</td>
</tr>
<tr>
<td>Enhanced ROSC TRNG with XOR tree (Figure 6.17)</td>
<td>52.44</td>
<td>262.2 ~ 1098.6</td>
<td>52.06</td>
<td>263.725 ~ 1093.8</td>
</tr>
<tr>
<td>BFD-TRNG (Figure 6.2)</td>
<td>2.134</td>
<td>328.3846</td>
<td>2.8025</td>
<td>431.3588</td>
</tr>
<tr>
<td>$M$-parallel BFD-TRNG (Figure 6.12)</td>
<td>1.1465M + 1</td>
<td>145.5885 + 154.5861/M</td>
<td>1.8025M + 1</td>
<td>277.3077 + 154.5861/M</td>
</tr>
<tr>
<td>Cascade BFD-TRNG (Figure 6.13)</td>
<td>1.192</td>
<td>399.0125</td>
<td>2.9025</td>
<td>498.4375</td>
</tr>
<tr>
<td>$M$-parallel Cascade BFD-TRNG (Figure 6.15)</td>
<td>1.1633M + 2</td>
<td>145.4123 + 253.6/M</td>
<td>1.855M + 2</td>
<td>251.36 + 256.5625/M</td>
</tr>
</tbody>
</table>
6.7 Conclusion and Future Work

This chapter has presented a comprehensive statistical analysis for BFD-TRNG. The relationships of period difference of the two ROSCs, environmental noise and the counter values have been investigated. Furthermore, how the counter values affect the number of random bits per sample that we can use has also been examined. We have concluded that an appropriate frequency difference of the two ROSCs should be set based on the environmental noise to achieve higher throughput. Other aspects of the BFD-TRNG design, such as post-processing techniques, have also been explored. Based on statistical analysis results, we have proposed several alternate BFD-TRNG designs, which include the parallel structure, the cascade structure, and the parallel-cascade structure. These novel structures could achieve improved performances. Comparisons of the BFD-TRNG with other existing ROSC based TRNGs have also been conducted. We have shown that the BFD-TRNG designs have better performances from both the randomness and the cost perspectives.
Chapter 7

Conclusion and Future Directions

7.1 Conclusion

This dissertation has considered the authentication and obfuscation of digital signal processing integrated circuits. Furthermore, the design and analysis of True Random Number Generator have also been discussed.

Several novel reconfigurable PUF structures have been proposed, where the challenge-response pairs are updatable. We have shown that these novel reconfigurable PUFs can embed more non-linearity of the challenge-response mapping functions and lead to more secure PUFs without degrading reliability and robustness. One example of the reconfigurable PUFs is the reconfigurable feed-forward MUX PUF, whose logic is updated by adding reconfigurable feed-forward paths into a MUX PUF. We have also presented a systematic statistical analysis to quantitatively evaluate the performances of various MUX-based PUFs. Furthermore, we have also presented an approach to examine the PUF structures theoretically. Our statistical analysis and our experimental results show great consistency. Motivated by the statistical analysis results, we have proposed several novel PUF structures with improved performance.
We are the first to tackle the problem of obfuscating DSP circuits. The synthesis results for proposed methodology are convincing. The hardware cost is less than that of conventional obfuscation methods which are not specific to DSP circuits. We believe our work will lead to a number of interesting work in this new research field of exploiting the high-level transformations for hardware security applications.

TRNG plays a significant role in cryptographic applications. However, the secondary metrics of a TRNG are also very important, which include area, power, latency, and so forth. We have presented an analysis of BFD-TRNG. We have shown that the performance can be improved if we could appropriately choose the design parameters of the BFD-TRNG.

### 7.2 Future Directions

#### 7.2.1 Evaluate and Attack PUFs

Each PUF design has its own advantages and drawbacks. The performance of a PUF depends on both process variations and environmental conditions. Some metrics have been introduced to characterize the performances of PUFs by analyzing the outputs of PUF instances, such as intra-chip variation, inter-chip variation, and randomness. However, *unclonability*, the core property of Physical Unclonable Functions, is only informally described in at most an ad-hoc manner at this moment. In order to make strong claims on the security of PUFs and PUF applications, it is necessary to come up with a formalized version of PUF performances.

Therefore, one promising research direction is to formalize the unclonability of PUFs, which should include *statistical unclonability* and *physical unclonability*. By using statistical modeling, an adversary might be able to correctly predict the response for a new challenge after collecting sufficient CRPs. My research suggests that the number of unpredictable CRPs can be increased by introducing reconfigurability into the PUF. However, it is still limited to an amount at best polynomial in the size of the PUF. To
assess the usefulness of a particular PUF in some applications, we believe it is worth examining how many unpredictable response bits one can optimally obtain from a PUF of a given size. For the practical properties, i.e., physical unclonability and tamper evidence, it is more elusive to fit into a quantitative theoretical framework. It will be of great interests to address the feasibility of physical tampering for different delay-based PUFs. The statistical models of the delay and response variations under invasive and semi-invasive attacking methods can also be further exploited.

Another exciting direction of future research is to study the attacking techniques. In spite of the fact that various machine learning methods have already been applied to attack different types of PUFs, methods that are suitable to break all the PUFs remain unexploited. One promising approach is to develop an attacking technique that does not any require prior knowledge of the PUF. It is reasonable to consider a PUF as a black box, as an adversary cannot obtain the general model of the PUF even with access to the PUF. The prediction of future responses will be only determined by the collected challenge and response information. Future work can be directed towards trying data-driven data mining techniques rather than employing arbitrary models based on intuition, or even expert elicitation.

### 7.2.2 Obfuscation CAD Tool

As described by the Moore’s law, the number of transistors in an integrated circuit doubles approximately every two years. Today, some of the largest chips in the world already contain more than one billion transistors. The design of such large-scale circuits requires a considerable amount of work, which is far beyond the amount of human labor that a design team can afford. Indeed, many problems in circuit design are computationally-intensive optimization problems. The great success of the semiconductor industry is predicated on computer-aided design (CAD) programs to perform many routine tasks.
Within the new context of DSP circuit design obfuscation, future work can be directed towards developing a CAD synthesis tool which can incorporate large number of design obfuscation algorithms based on high-level transformations for DSP system design. The algorithmic aspect of different high-level transformations for design obfuscation need to be exploited. Each of those transformation techniques needs to be examined from the perspectives of (i) the suitability for design obfuscation, (ii) the generation of variation modes, and (iii) the performance tradeoff. Another exciting direction for future research is the development of a design obfuscation methodology at the behavioral level, for instance within the framework of hardware description languages such as Verilog and VHDL.

### 7.2.3 DSP Circuit Reverse Engineering

There is a old Chinese sayings: know your opponent and yourself, emerge victorious in every battle. When we are talking about security, we should always think from the adversary’s perspective. To the best of my knowledge, attacking methods that target DSP circuits are still very nascent. We can explore reverse engineering methods of DSP circuits by utilizing frequency domain information. For example, assume we know the device that we intend to attack implements an FIR filter design using windows, but we have no idea about the detailed information of the filter. As far as we know, each type of window used in the FIR filter design has distinct property in the frequency domain. We can use the frequency domain information to determine the window type according to the peak sidelobe amplitude and peak approximation error by collecting a series of outputs. In addition, we can also estimate the cut-off frequency of the FIR filter from the frequency domain information. Once we know the type of window, we could estimate the filter order based on the filter properties such as approximate width of mainlobe and transition bandwidth. Consequently, we can solve the linear equations of the FIR filter system to obtain the filter coefficients and the actual order. Future
work can be directed towards generalizing the reverse engineering approach by utilizing the frequency domain information to other types of digital signal processing circuits.

7.2.4 Security in Emerging Technologies based Computing Systems

With CMOS technology scaling down to deep nanometer realm, process variation has become more and more pronounced. The increasing process variations lead to considerable uncertainty in circuit performance and large spread in chip speed. The imprecision provided by the deeply downscaled nanometer CMOS technology no longer holds on the widely accepted practice of designing circuits that operate on deterministic zeros and ones. One interesting direction is to exploit the inherent randomness for cryptographic purposes. A broad objective is to embed security and cryptographic primitives into customized circuits with negligible overhead by making use of the inherent randomness. For example, in the sub-threshold or near-threshold applications, soft errors can be directly used as PUF response and/or random numbers. The advantage is manifest in the fact that this approach does not incur any overhead, since these soft errors are created by voltage over-scaling for the low-power purpose that will be corrected by certain compensation algorithm at a later stage.

At meanwhile, as the semiconductor industry contemplates the end of Moore’s Law, there has been a groundswell of interest in technologies that offer a path to scale beyond the limits of the current CMOS technology. Emerging technologies that might help extend the life of Moore’s Law, such as carbon nanotubes [123], nanowire arrays [124], and molecular FETs [125], present both challenges and opportunities for digital circuit design. Most of these technologies are characterized by very high defect rates, as well as large amounts of inherent randomness. For example, it is nearly impossible to guarantee perfect alignment and accurate positioning of all carbon nanotubes at VLSI scale. Mispositioned and misaligned carbon nanotubes can result in incorrect digital logic functionality. Hardware security of emerging technologies can be a very important research topic in the future. For example, future non-volatile memory systems, such
as phase change memory (PCM), spin torque transfer magnetoresistive RAM (STT-MRAM), and resistive RAM (RRAM), typically have large number of intrinsic defects. A non-volatile storage system includes solutions to mask intrinsic defects, read and erase endurance and cell coupling problems. When used appropriately, such information allows unique identification of non-volatile memory. Different technologies might to be considered within an overall framework for security applications. Future work can be directed towards developing a general method to inherently extract randomness on non-volatile memory systems for security signature or random number applications.
References


