Essays on Consumer Information Acquisition in Marketing Channels

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DEDICATION

To my parents

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Background

1. Consumer Information Acquisition

In many markets, consumers lack important information, such as product price and product quality. This kind of hidden information can hinder market operations. For example, Akerlof (1970) shows that quality uncertainty can lead to market failure despite obvious gains from trade. Therefore, it is important to understand the procedures of consumers' information acquisition and consequences for market outcomes.

In general, consumers acquire information through two modes. The first is to *actively* collect information on the market. One notable example is the consumer's costly search (e.g., Stigler 1961). This stream has become increasingly popular due to the boom in internet shopping. The lower per-search cost online has motivated consumers to search more frequently. Therefore, the active search plays a crucial role in understanding internet markets.

However, the active search cannot completely resolve information asymmetry even if its cost falls to zero. Some product information, for example, qualities of experience goods (Nelson 1970), can be difficult to investigate simply through search. In such cases, consumers must rely on the second mode of information acquisition: *passively* interpreting messages sent by firms. A prominent theoretical framework for interpreting such messages is the signaling model (e.g., Spence 1973).

1.1 Consumer Search

Stigler (1961) initiates the work on consumer search, seeking to explain price dispersions on the market. However, the early theoretical models, which examine the implications of consumer search on market outcomes, has offered counter-factual predictions. For instance, Diamond (1971) shows that consumers in equilibrium would not search even when they encounter a monopoly price provided (1) products are homogenous and (2) consumers have a common positive search cost. Two streams of theoretical frameworks have been refined to overcome these difficulties. One stream relaxes the homogenous search cost assumption; for example, Stahl (1989) presents that search cost heterogeneity can lead to price dispersions for homogenous products. In addition, Wolinsky (1986) initiates another stream of research that accommodates heterogeneous consumer taste for the product. His finding suggests that the consumer's costly search for information on product valuation can result in monopolistic competition.

Gap. Both these streams on the active search usually abstract away the marketing channel and some important features of the retail market structure. For example, internet retail search traffic is heavily concentrated (e.g., De Los Santos et al. 2012; NPR 2018). Essay 1 fills this gap by examining the implication of search traffic concentration on an intra-brand market, where identical products are sold through competing retailers.

1.2 Signaling

Spence (1973) proposes that sellers can signal product quality. A signal can be any costly action taken by a seller that does not affect its product's intrinsic property, but rather changes the consumer's belief about it. Consumers interpret signals to differentiate between sellers. A large body of literature is centered on understanding the properties of credible signals, which include but are not limited to price (e.g., Wolinsky 1983), advertising expenditures (e.g., Milgrom and Roberts 1986), branding (Wernerfelt 1988), warranties (e.g., Moorthy and Srinivansan 1995), and advertising content (Mayzlin and Shin 2010).

Gap. Most products are sold through a vertical structure, from the producer via a reseller/retailer to consumers, but the signaling literature has abstracted away from considering marketing channels, so we know little about the implications of the consumer's quality inference when channel members have their own (and possibly conflicting) goals. Essay 2 fills this gap by examining price signals in a setting that features an intermediary retailer and a producer.

2. Contributions

Overall, this dissertation employs game theoretical models to investigate consumer information acquisition. Its principal novelty is that I incorporate a richer supply structure with active retailers. This allows me to explore issues that are unique to the marketing channel. Table 1 summarizes two essays based on their information acquisition modes and issues in the marketing channel. Consider each essay in turn.

Essay 1: Prominent Retailer and Intra-brand Competition studies consumers' *active* information acquisition patterns. Specifically, I investigate the interaction between active consumer search and retailers' pricing decisions when the search share is disproportionally distributed. I am motivated by the phenomenon of "prominence". That is, internet retail search traffic tends to be concentrated on a "prominent" retailer, such as Amazon in the United States or Alibaba in China. I develop a sequential search model to provide insights into the impact of prominence in an intra-brand setting. My research sheds light on (1) how a prominent retailer's relative prices hinge on a threshold level of prominence, (2) the consequences of search traffic volumes on a retailer's profit, and (3) the impact of search traffic concentration on competition and consumer welfare. Surprisingly, I find that more search traffic can reduce a retailer's profit.

Essay 2: Retailer Reputation in a Distribution Channel investigates the consumer's *passive* acquisition of information. Specifically, I am interested in the interplay between the consumer's price-quality inference and vertical coordination between the manufacturer and the retailer. Ample empirical evidence supports that consumers infer unobserved product qualities from observed prices (e.g., Tellis and Wernerfelt 1987) and seller reputation (e.g., Purohit and Srivastava 2001). However, the prices themselves depend on the vertical supply structure. I develop a model of signals issued by firms in a vertically separated (decentralized) channel. Specifically, a manufacturer sells through a retailer with a limited reputation where consumers use the observed retail price to infer product quality. I find that the signaling role of the retail price can facilitate channel coordination. Furthermore, this effect is moderated by the retailer's reputation. Surprisingly, the manufacturer, the retailer, and consumers can all become better off when the retailer is less reputable.

Table 1. Topics of Two Essays

	Format of Information Acquisition	Issues in Marketing Channels
Essay 1	Active Search	Intra-brand Competition
Essay 2	Passive Interpretation of Signals	Vertical Control

Essay 1: Prominent Retailer and Intra-brand Competition

Online retail search traffic tends to be concentrated on a "prominent" retailer, such as Amazon in the U.S. or Alibaba in China. This research asks the following questions in an intra-brand setting: how does a prominent retailer's pricing leverage its search traffic advantage? More broadly, how does search traffic concentration affect price competition, consumer welfare, and profits? I examine these questions through a sequential search model. Customers with heterogeneous search costs conduct price searches across intra-brand competitors within their limited awareness sets. In my model, the prominent retailer enjoys two search traffic advantages (1) it has the highest first-search share and (2) it appears in all consumers' awareness sets. I find that the prominent retailer's price is stochastically lower than its competitors up to a critical first-search share level. Above this level, it charges a stochastically higher price. Furthermore, higher traffic concentration can intensify price competition and lower average prices for all retailers. Thus, consumers can be better off with more concentrated traffic. Lastly, I remark on the "curse of prominence" wherein an incremental traffic increase can reduce a prominent retailer's profit.

Keywords: Search Traffic Concentration, Intra-brand Competition, Ordered Search, Competitive Strategy

1. Introduction

Internet search traffic is often quite concentrated among selected companies. Specifically, in online retail markets, search traffic is heavily concentrated on a "prominent" retailer like Amazon.[1] There are two aspects to this traffic pattern. First, a disproportionally high percentage of online consumers start their searches with this prominent retailer. For instance, overall, 44% of U.S. online consumers start their searches at Amazon (NPR 2018). In a specific market such as books, around 65% of consumers start at Amazon (De Los Santos et al. 2012). Second, virtually all consumers are aware of the prominent retailer, notwithstanding the low general level of awareness of online outlets. For instance, NPR (2018) reports that 92% of U.S. consumers are aware of Amazon. As such, consumers are very likely to visit a prominent retailer even if they start their search at some other retailer. For instance, Bloomreach (2016) documents that 9 in 10 consumers will check Amazon even if they find a product that they want on another retailer's site earlier.

This concentration of internet search traffic raises several concerns, including the notion that this is the chief antitrust concern of the digital age (Kahn 2018). Regulatory agencies have begun reviews of the practices of large tech companies that dominates internet search traffic to ascertain whether competition is compromised to the detriment of consumers (Kendall 2019).

This research explores the implication of search traffic concentration. I study an intra-brand setting, where consumers search for prices of a specific product across

[1] The term "prominence" was first used by Armstrong et al. (2009) in ordered search literature. My "prominent retailer" notion differs in two ways. First, a prominent retailer has more traffic not only from first searches (as in Armstrong et al. 2009) but also from subsequent searches. Second, my notion of prominence is motivated by the market structure of search traffic concentration, whereas, prominence in Armstrong et al. (2009) is motivated by the order of product presentation, which is not relevant to my intra-brand competition setting.

competing retailers. Specifically, I explore the following interrelated questions about pricing, competition and profits.

First, how will a prominent retailer leverage its traffic advantage in its pricing decisions? Specifically, might it choose to charge higher or lower prices compared to its intra-brand competitors? Industry anecdotes and commentary about Amazon point to both possibilities. On the one hand, some observers consider Amazon's prices to be low. For instance, "Amazon has been crowned the cheapest of all leading online retailers" (Hanbury 2018). But other reports suggest Amazon offers higher prices Walmart (e.g., average price across 50 popular products per Peterson 2018). Can I reconcile these conflicting perceptions by uncovering some contingency?

Second, how does search traffic concentration influence competition and consumer welfare? Intuitively, it seems reasonable to conjecture that search traffic concentration should exhibit similar effects as other forms of market concentration: they all appear to endow market power to exploit consumers and soften competition. This line of reasoning is implicit in the statements of prominent government officials. For instance, Treasury Secretary Mnuchin famously noted that "…. there's no question they (Amazon) have limited competition" (Fitzgerald 2019). That said, I lack any scholarly work into this question. Is it possible that traffic concentration might actually encourage intra-brand competition and therefore benefit consumers under some circumstances?

Lastly, what is the impact of search traffic volume on a retailer's profit? Intuitively, more traffic appears to increase demand and thus profit. Casual empiricism supports this intuition as I see firms spending large sums to increase search traffic via search engine.

However, as above, I lack scholarly work on this profit issue. Indeed, it is possible that more traffic might lead to lower retailer's profit?

This research provides a micro-foundation to unpack the impact of search traffic concentration on intra-brand competitors. In this model, retailers sell the same product and consumers gather price information through a sequential search process. Consumers with heterogeneous search costs conduct price search across retailers within their limited awareness sets. My prominent retailer is characterized by (1) being searched first by the highest percentage of consumers than any other individual retailer, and (2) appearing in all consumers' awareness sets. My analytical model yields the following insights.

First, I establish that no pure strategy equilibrium exists; a mixed strategy holds featuring prices varying randomly from a distribution. This strategy comports with the well-documented temporal variation in prices at visible online retailers like Amazon.

There is a specific pattern to the prices charged. Beyond a critical crossover point in its first-search share level, the prominent retailer charges higher prices (stochastically) relative to its competitors. Below this crossover point, it charges lower prices (stochastically). The intuition behind this result is as follows. Search dominance impacts the customer mix at the prominent retailer in two ways. On one hand, as the highest first-search competitor, it captures more high-search-cost consumers. On the other hand, as the competitor with the highest awareness set inclusion, it captures more low-search-cost consumers. When the former force prevails (at sufficiently high first-search levels), the prominent retailer's mix contains a higher proportion of high-search-cost consumers relative to low-search-cost ones compared to its competitors. This less elastic demand commands higher prices. When the latter force prevails, its customer mix reverses, and the more elastic demand induces lower relative prices. The relative influence of these two forces switches at the critical cross-over point.

Second, I find that search traffic concentration can intensify price competition. When more consumers start their searches at a prominent retailer, it can capture more high-search-cost consumers. This leaves fewer such consumers for other retailers, forcing them to compete for low-search-cost consumers. It heightens competition and lowers prices at these other retailers. This falling price makes additional searches more attractive to consumers who start at the prominent retailer, who reacts by choosing a lower price itself. Consumers benefit from the intensified intra-brand competition driven by search traffic concentration.

Lastly, more traffic can lead to a lower profit for the prominent retailer; i.e., it suffers from a "curse of prominence". More first searches yield a demand enhancement effect by allowing it to capture more high-search-cost consumers. However, it also intensifies price competition, which could weaken its ability to profit from its consumers. This negative effect dominates the former one when it already has a high demand. In this case, more traffic lowers a retailer's profit. This takeaway is counter-intuitive: counter to what retailers might believe given their costly efforts to improve their traffic volumes, they should not blindly increase its exposure without considering its impact on the competitive landscape.

The remainder of the paper is organized as follows. Immediately following, I review the academic literature. Section 3 presents my model assumptions and outlines the game structure. Section 4 presents the main model's solutions and insights. Section 5 provides model extensions. Section 6 concludes with managerial implications and directions for future search.

2. Literature Review

2.1 Dominant Retailer

The marketing literature usually considers that a retailer's "dominance" comes from cost advantage and/or channel power. For example, Raju and Zhang (2005) as well as Kolay and Shaffer (2013) model the dominant retailer as a price leader, while Dukes et al. (2006) consider it as a retailer with cost advantage. Furthermore, a dominant retailer can dictate wholesale prices (Geylani et al. 2007) or have the power to determine assortments first (Dukes et al. 2009). Search traffic dominance is qualitatively different from all these previous notions, and is uniquely important in the online space. My work is among the first to consider a "search-dominant" retailer, a.k.a. a prominent retailer in an intra-brand setting.

2.2 Price Dispersion

Unlike offline markets that are dispersed geographically, thus limiting the number of effective competitors, there are many more retailers within reach of a customer in the online space. Curiously, these online intra-brand markets are also characterized by a significant degree of intra-brand price dispersion; i.e., prices for the same product sold by different retailers. Baye et al. 2006 identify two streams of theoretical literature that speak to this phenomenon.

One stream centers on the "search-theoretical" model, which emphasizes consumer heterogeneity in the cost of acquiring information (e.g., Stigler 1961; Stahl 1989; Jassen et al. 2005). The second stream centers on the "information clearinghouse" model, which explicitly assumes that a subset of consumers is able to obtain a list of prices charged by all firms (e.g., Varian 1980; Narasimhan 1988; Baye and Morgan 2001; Chen et al. 2002). My study falls into the first category. By endogenizing consumers' search decisions, I am able to capture the influence of retail price competition on consumer's incentive to search. This is the central driving force to my result on the "curse of prominence".

2.3 Consumer Search

There is an extensive literature on modeling consumer search (e.g., McCall 1979; Weitzman 1979), and its implications for pricing (e.g., Diamond 1971; Wolinsky 1986; Stahl 1989; Anderson and Renault 1999; Lal and Sarvary 199; Kuksov 2004; Janssen and Shelegia 2015; Jiang, Kumar, and Ratchford 2017; Ke, Jiang, and Sun 2017; Zou and Jiang 2018; Ke and Lin 2019). Stahl's (1989) work on search-influenced price dispersion in an intra-brand setting provides the closest point of departure for my work. I differ in my focus. Unlike his symmetric retailers who price identically, my asymmetric competitors yield asymmetric outcomes on prices and competition.

My work also connects to the ordered search literature (e.g., Arbatskaya 2007; Armstrong et al. 2009; Wilson 2010; Xu et al. 2011; Astorne-Figaria and Yankelevich 2014; Choi et al. 2018; Petrikaite 2018; Zou and Jiang 2018; Mamadehussene 2019; Mamadehussene 2020; Janssen and Ke 2020).

The Armstrong et al. (2009) work provides the closest point of departure for my work. In their work, prominence sorts consumers based on their product valuations. They find that (1) the prominent firm always charges a higher price, (2) the prominence always reduces price competition, and (3) more traffic monotonically increases the prominent firm's profit. In contrast to their homogenous cost-of-search customers who seek the best-fitting product, my work focuses on heterogeneous search-cost customers who search for prices across retailers for the same product. In my setup, prominence sorts consumers based on their search costs instead of product valuations. It fits my intra-brand focus, where product valuation is the same across sellers. My results also contrast with Armstrong et al. (2009): (1) my prominent retailer might charge higher prices relative to other retailers, (2) prominence might increase price competition, and (3) more traffic might reduce the prominent retailer's profit.

Furthermore, Xu et al. (2011) shows that the firm with cost advantage might avoid the first position in sponsored link advertising to reduce price competition. This study is different in the following ways. First, Xu et al. (2011) focuses on search advertising, where the search order can change constantly. Whereas, this research is motivated by the phenomenon of search traffic concentration, a long-term market structure under intra-brand market. Second, Xu et al. (2011) restricts the definition of "prominence" on search order. However, "prominent retailer" in this study only receives more first searches, but also more subsequent searches. Lastly, my study does not assume any cost differences. A firm might have a lower profit from becoming prominent even if there exists no cost differentiation.

3. Model

There are three retailers selling the identical product. Each retailer has an identical wholesale price, normalized to zero. The mass of consumers is normalized to be one and each consumer has a unitary demand.[2]

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[2] The consumer purchases the product when (1) she ceases searching and (2) her valuation to the product v is higher than the price. I assume v to be sufficiently high relative to the search cost. This allows us to focus on the impact of consumer search on the market outcome.

3.1 Consumer Limited Awareness

Each consumer is not aware of all retailers, because her limited cognitive resource prevents her from retrieving every retailer from memory (e.g., Hauser and Wernerfelt 1990; Nedungadi 1990; Amaldoss and He 2019). I assume she is aware of two out of three retailers. Within her limited awareness set, she endogenously chooses when to stop searching under sequential search with perfect recall.

3.2 Consumer Search Cost

As is commonly assumed (e.g., Stahl 1989; Kuksov 2004), the consumer has a free first search to ensure a full participation market. My approach extends prior work with heterogeneous free first search destination. In particular, each consumer starts her search with *only* one retailer at no initial search cost.[3] This retailer can be the "default' or familiar retailer to her. Consumers have heterogeneous "default" retailers.[4]

Beyond the first search, I assume that there are two consumer segments with heterogeneous search costs. The "shopper" segment of size $\mu \in (0,1)$ have a zero search cost and the "non-shopper" segment of size $(1-\mu)$ have a positive search cost $c > 0$.

3.3 Prominent Retailer

I assume one prominent retailer and two symmetric fringe retailers denoted as d and f respectively. They differ along two dimensions as follows.

First, with respect to first searches, denote α_i as the fraction of consumers who start their search at retailer i ($i = d, f_1, f_2$), $\sum_i \alpha_i = 1$. The prominent retailer has the highest

[3] The zero cost per-search can explain why she starts with this retailer.
[4] Such heterogeneity in "default" retailers is similar to the heterogeneous loyal segments Narasimhan (1988) and Chen et al. (2001), except that the consumer's reservation price is endogenized through their search decisions in this model.

first-search share, so $\alpha_d \in (\frac{1}{n}, 1]$. Given symmetric fringe retailers, $\alpha_f = \alpha_{f_1} = \alpha_{f_2} = \frac{1-\alpha_d}{2} < \frac{1}{3} < \alpha_d$.

Second, with respect to awareness sets, denote β_i as the fraction of consumers aware of retailer i. I characterize limited awareness as sets of size two; $i.e., \sum_i \beta_i = 2$. The prominent retailer appears in all awareness sets, so $\beta_d = 1 > \beta_f = \frac{1}{2}$.

3.4 Game Sequence and Equilibrium Concept

At the first stage of the two-stage game, retailers simultaneously set prices. I assume that retailers cannot identify a consumer's search cost, so price discrimination is ruled out. At the second stage, the consumer searches for prices and purchases the product.

The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Under PBE, retailers maximize their profits π and set prices simultaneously conditional on whether they or their rivals are prominent, their prominence levels, and their expectations on consumer behavior. These expectations are consistent with equilibrium search strategy. The consumer maximizes her utility and makes the search decision conditional on her belief on retailers' pricing strategies, which is consistent with equilibrium strategy.

4. Analysis

In this section, I first derive the property of the consumer's search and purchase strategy as well as the retailer's pricing strategy. Then I present equilibrium outcome and its implication.

4.1 Consumer's Search Strategy

Let's start with the consumer's strategy. First, the shopper segment customers learn prices at zero cost and purchase from the lowest price retailer within their personal awareness sets. Since retailer i appears in β_i awareness sets, it is searched by a proportion of β_i shoppers.

As for non-shopper segment customers, they incur a positive cost for each search beyond the first search. They stop searching when the search benefit is no higher than the cost. Suppose that a non-shopper observes a price z at retailer i, her search benefit is finding a price $p < z$. Given price distribution $F_j(p)$ and its lower bound $\underline{p}_j$ from the best alternative retailer j, the non-shopper's expected search benefit after finding a price z at retailer i is

$$EB(z) \equiv \int_{\underline{p}_j}^{z} F_j(p)dp. \tag{1}$$

$EB(z)$ monotonically increases with z. Put differently, a non-shopper customer's search benefit is higher if she encounters higher prices at retailer i.

These non-shoppers stop searching at one retailer if its price is sufficiently low. The highest acceptable price for the non-shoppers to cease searching at retailer i is r_i, which equalizes the search benefit to cost:

$$EB(r_i) = c. \tag{2}$$

I label r_i as endogenized reservation price at retailer i. If $p \leq r_i$, she stops searching and purchase from retailer i. Otherwise, she continues to search. Notice that non-shoppers who are unsatisfied with the price at the prominent retailer have the option to visit a fringe retailer, and vice versa. As such, the endogenized reservation prices r_d and r_f satisfy

$$c = \int_{\underline{p}_f}^{r_d} F_f(p)dp = \int_{\underline{p}_d}^{r_f} F_d(p)dp. \tag{3}$$

4.2 Firm's Price Strategy

Next, consider the retailer's pricing strategy.

Lemma 1. *There is no pure strategy equilibrium.*

I present proofs of all lemmas and propositions in the Appendix. Each retailer can be searched by both shoppers and non-shoppers. Thus, it faces the tradeoff between the two segments. On one hand, it has incentive to charge a high price to extract surplus from non-shoppers who visit it first. On the other hand, it is tempted to charge a low price to compete for shoppers who visit more than one retailer. Due to such conflicting incentives, retailers would deviate from any single prices. Consequently, if the equilibrium exists, then it must be mixed strategy. That is, retailer i charges prices between $p \in [\underline{p_i}, \overline{p}_i]$ following the cumulative distribution function $F_i(p)$, where $\underline{p_i}$ and $\overline{p}_i$ are the lower and upper boundary of each retailer's price support. Under any mixed strategy equilibrium, the retailer's expected profit must be constant at any $p \in [\underline{p_i}, \overline{p}_i]$.

The properties of any mixed strategy equilibrium expressed in the following lemmas serve as stepstones to the equilibrium outcome.

Lemma 2. *Under any mixed strategy equilibrium, the price support of retailer i is convex for $p \leq r_{-i}$.*

The intuition is as follows. If there is a "hole" in the price support of retailer i for $p \leq r_{-i}$, either retailer i or $-i$ would have inconstant profits within its price support. For example, the competing retailer $-i$ could have a higher profit at the upper than the lower boundary of the "hole". This would violate the definition of mixed strategy equilibrium.

Lemma 3. *Under any mixed strategy equilibrium, $\overline{p}_i \leq r_i$.*

Under a mixed strategy equilibrium, all retailers price no higher than their endogenized reservation prices to lock in the non-shoppers who visit them first. Otherwise, it would either violate lemma 2 or lead to a decrease in its profit. Lemma 3 implies the following claim:

Claim 1. *Under any mixed strategy equilibrium, non-shoppers purchase at the first retailer they visit.*

From claim 1, I know that each retailer captures the non-shoppers who inspect it first under any mixed strategy equilibrium. Given the properties of consumer's search and retailer's pricing strategy, I can characterize retailer i's profit for $p \in [\underline{p}_i, \overline{p}_i]$ given the other's pricing strategy as:

$$E\pi_d = \{(1-\mu)\alpha_d + \beta_d \mu[1 - F_f(p)]\}p, \tag{4.1}$$

$$E\pi_f = \{(1-\mu)\alpha_f + \beta_f [1 - F_d(p)]\}p. \tag{4.2}$$

$(1-\mu)\alpha_d$ and $(1-\mu)\alpha_f = \frac{(1-\mu)(1-\alpha_d)}{2}$ represent the non-shoppers who start their searches with the prominent and fringe retailer. Moreover, $\beta_d \mu = \mu$ and $\beta_f \mu = \frac{\mu}{2}$ stand for the shoppers who search each retailer. Note that the prominent retailer is searched more by both non-shoppers and shoppers due to its first-search prominence ($\alpha_d > \alpha_f$) and awareness prominence ($\beta_d > \beta_f$), respectively. The probability to acquire shoppers is $[1 - F_f(p)]$ for the prominent retailer and $[1 - F_d(p)]$ for the fringe one.

4.3 Equilibrium Price

Given the retailer's expected profit function and consumer's search strategy, I can solve the equilibrium price distribution and reservation price. Proposition 1 summarizes the equilibrium outcome and Figure 1 depicts it.

Proposition 1. (Equilibrium Price)

(1) When $\alpha_d \geq \alpha^* = \frac{1}{2}$, the prominent retailer charges higher prices stochastically relative to the fringe one.

Particularly, for $p \in [\underline{p}, r_d]$, $F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}] \leq F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ where $r_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]}$ and $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_p$.

(2) When $\alpha_d < \alpha^*$, the prominent retailer charges lower relative prices stochastically.

$F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}\left(\frac{r_f}{p} - 1\right) > F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}]$ for $p \in [\underline{p}, r_f]$, where $r_f = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}$ and $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} r_f$.

Proposition 1 shows that at sufficiently high first-search levels ($\alpha_d \geq \alpha^*$), the prominent retailer's equilibrium price first-order stochastically dominates the fringe retailer's, $F_d(p) \leq F_f(p)$. It charges higher prices stochastically. The opposite is true for first-search levels of the prominent retailer below this point; $\alpha_d < \alpha^*$.

To appreciate the intuition of this result, let's first focus on each retailer's potential customer, which includes the non-shoppers and shoppers who search it. As shown previously, shoppers search both retailers within the awareness set and non-shoppers only searches the first retailer. As such, retailer *i*'s non-shopper relative to shopper (I call it non-shopper ratio) is $\frac{\alpha_i(1-\mu)}{\beta_i \mu}$. Note that non-shoppers are less price-sensitive compared with the shoppers, because the positive search cost prevents them from finding the lowest possible price. Consequently, a higher non-shopper ratio implies that a retailer has a less price-sensitive customer.

Search dominance has two impacts on the customer mix. On one hand, the high first-search level enables the prominent retailer to capture more high-search cost consumers. On the other hand, the high awareness allows it to be searched by more low-search-cost consumers. The former force prevails at a sufficiently high first-search level($\alpha_d \geq \alpha^*$). Therefore, the prominent retailer has a higher non-shopper ratio than the fringe one, $\frac{(1-\mu)\alpha_d}{\mu} \geq \frac{(1-\mu)(1-\alpha_d)}{\mu}$, so it faces a more inelastic demand. This leads to higher prices from the prominent retailer. However, the latter force dominates when $\alpha_d < \alpha^*$. In this case, the higher awareness rate allows the prominent retailer to capture more shoppers relative to non-shoppers compared with its competitors. In order to keep these price-sensitive shoppers, the prominent retailer charges lower prices compared with its competitors. The result suggests that the more search traffic can influences the customer mix. Therefore, a retailer has to understand such impact before pricing accordingly.

Figure 1. Equilibrium Price Distribution

<table>
<tr><th>$\alpha_d < \alpha^*$</th><th>$\alpha_d \geq \alpha^*$</th></tr>
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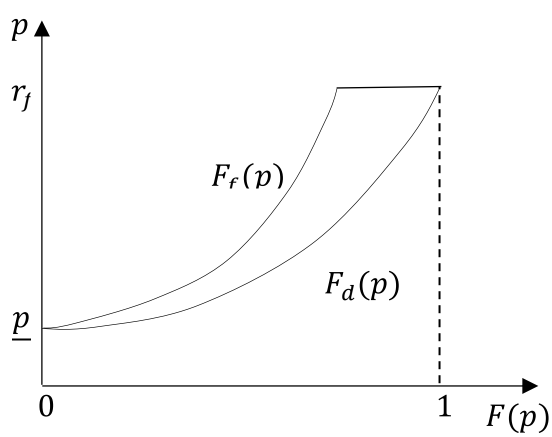

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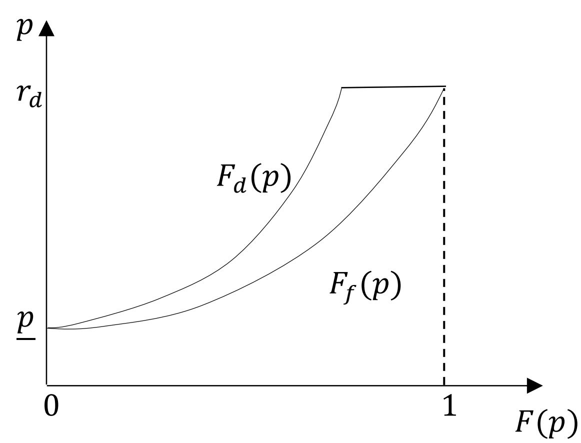

</td></tr>
</table>

Moreover, I can verify that the prominent retailer's profit is higher than the fringe one's at any α_d. The search traffic dominance gives the prominent retailer strategic advantage. Such advantage leads to a higher profit whether the prominent retailer leverage the advantage through high or low prices.

4.3 Search Traffic Concentration and Competition

Next, I examine the impact of an increase in α_d, a higher concentration on search traffic, on average prices of both retailers, Ep_d and Ep_f.

Proposition 2 (Search Traffic Concentration and Competition)

An increase in α_d lowers the average prices of both retailers when $\alpha_d < \alpha^*$; otherwise, it raises average prices.

This proposition states that more concentrated traffic from first search has a non-monotonic impact on price competition. The intuition is as follows. When the shoppers search two retailers, one retailer only needs to undercut the price of the other to acquire shoppers. In this case, the price competition level is determined by the difference in customer mix between the prominent and fringe retailer. The price competition would become higher when two retailers share more similar customers, which are measured by the non-shopper ratios. When $\alpha_d < \alpha^*$, the prominent retailer has a lower non-shopper ratio than the fringe ones. An increase in α_d increases the prominent retailer's non-shopper ratio, while decreases the fringe one's. Thus, it assimilates the customer mix between retailers. This leads to a more intensified price competition. When $\alpha_d = \alpha^*$, all retailers share the same non-shopper ratio. As a result, an increase in α_d beyond α^* further differentiates the retailers and therefore decreases the price competition.

4.3 Search Traffic and Profit

The impact of more traffic on the prominent retailer's profit is counter-intuitive. I would expect that a higher retailer's profit from more search traffic, but this is not true as shown below.

Proposition 3 (The Curse of Prominence)

The prominent retailer's profit decreases with α_d *when* $\alpha_d < \alpha^*$ *and* $\mu > \mu_1(\alpha_d) = \sqrt{\frac{1}{2} + \frac{\alpha_d{}^2}{16(1-\alpha_d)^2}} - \frac{\alpha_d}{4(1-\alpha_d)}$).

This proposition suggests that more traffic from first search can hurt the prominent retailer. This result arises from the unintended price competition effect from search traffic concentration. It captures the tradeoff between the *demand enhancement effect* and *price competition effect* from more traffic. On one hand, more traffic from first search increases the number of non-shoppers that the retailer can acquire. On the other hand, it affects the retailer's margin by its influence on the competitive landscape.

The curse of prominence occurs at mild first-search levels and when there are more shoppers. At mild first-search levels, the negative *price competition effect* activates. A decline in margin hurts the prominent retailer more if it has a larger demand. The prominent retailer has a larger demand when the market has a higher proportion of shoppers, because it has more shoppers relative to non-shoppers. As such, the negative *price competition effect* dominates the *demand enhancement effect* when the prominent retailer has a mild first-search levels and the market contains a high proportion of shoppers.

This result suggests that the prominent retailer might not necessarily benefit from more traffic. When the prominent retailer receives more traffic from first search, it leaves fewer non-shoppers to the fringe retailers. It encourages them to price more aggressively, which intensifies the price competition. This weakens the prominent retailer's ability to milk its customers.

In addition, I find that more first searches at the prominent retailer might not necessarily hurt the fringe one as shown below.

Corollary 1

An increase in α_d *leads to a higher profit for the fringe retailer when* $\alpha_d \geq \alpha^*$ *and* $\mu > \mu_2(\alpha_d) = \sqrt{\frac{1}{2} + \frac{(1-\alpha_d)^2}{16\alpha_d{}^2}} - \frac{1-\alpha_d}{4\alpha_d}$.

The result also arises from the price competition effect from traffic concentration. At high first-search levels, more traffic to the prominent retailer reduces the price competition and raises the margin for the fringe one, though it decreases the number of non-shoppers for it. The benefit is especially large when there are more shoppers. In this case, the rising margin from a massive shopper market can compensate for its loss in demand from non-shoppers.

This result provides guidance for fringe retailers competing with a prominent rival. Instead of spending efforts to attract more traffic, the fringe retailer might become better off from less aggressive efforts to attract traffic. This reduces price competition and may increase its profit.

5. Extensions

In this section, I extend the model to cases that (1) consumers are aware of all available retailers, and (2) there are more than three retailers on the market.

5.1 Full Awareness

I am interested in the full awareness case for the following reasons. First, this extension finds the boundary condition of the main results. Full awareness allows me to isolate the impact of the first-search prominence from awareness prominence. I find that first search prominence is responsible for the intensified competition due to concentrated search (Proposition 2) and “curse of prominence” (Proposition 3). Whereas, two dimensions of the prominence drives the high/low relative prominent retail price (Proposition 1). Also, this case captures the scenario that some consumers might get access to all possible retailers, with the assistance of shopping comparison websites or search engine.

Under the full awareness, if the non-shoppers are unsatisfied with the price at the prominent retailer, their alternative option is to visit a fringe retailer. Thus, their reservation price at the prominent retailer r_d satisfies $c = \int_{\underline{p}_f}^{r_d} F_f(p)dp$. On the contrary, if the non-shoppers are unsatisfied with the price of a fringe retailer, the alternative option is to inspect either the prominent or the other fringe retailer, whichever has a higher search benefit. The reservation price r_f satisfies $c = max\left\{\int_{\underline{p}_d}^{r_f} F_d(p)dp\,, \int_{\underline{p}_f}^{r_f} F_f(p)dp\right\}$.

Similar to the main model, under the equilibrium, there is no pure strategy equilibrium and no retailer would price higher than its reservation. As such, in equilibrium, the shoppers search all three retailers and buy from the lowest priced one, while the non-shoppers only search and purchase from the first one that they visit. Therefore, I can characterize the equilibrium profit of retailer i for $p \in [\underline{p}_i, \overline{p}_i]$:

$$E\pi_d = \left\{(1-\mu)\alpha_d + \mu\left[1 - F_f(p)\right]^2\right\}p, \tag{5.1}$$

$$E\pi_f = \{(1-\mu)\alpha_f + \mu[1 - F_d(p)][1 - F_f(p)]\}p. \tag{5.2}$$

$(1-\mu)\alpha_d$ and $(1-\mu)\alpha_f = \frac{(1-\mu)(1-\alpha_d)}{2}$ represent the non-shoppers who start their searches with the prominent and fringe retailer. μ stands for the shoppers who search each retailer. The probability to acquire the shoppers is $\left[1 - F_f(p)\right]^2$ for the prominent retailer and is $[1 - F_d(p)][1 - F_f(p)]$ for the fringe one.

I solve the equilibrium and summarize it in Proposition 4.

Proposition 4

Under the full awareness set, the prominent retailer charges a higher price than fringe retailers.

Specifically, the prominent retailer charges $p_d = r$ *with probability one; while the fringe retailer randomizes prices between* $p \in [\underline{p}, r)$ *following CDF* $F_f(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{2\mu}(\frac{r}{p} - 1)$, *where* $r = \frac{c}{1 - \frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}$ *and* $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+2\mu} r$.

Under the full awareness, I find that the prominent retailer always charges a higher price compared with its competitors. The intuition is as follows. When all shoppers are aware of and search all three retailers, the prominent and fringe retailer are searched by the same number of shoppers. However, due to its higher first-search level, the prominent retailer is searched by more non-shoppers than the fringe one. As such, the prominent retailer always has a higher non-shopper to shopper ratio compared with competitors. That is, the first-search prominence always prevails after I mute the awareness prominence. Consequently, it faces a more inelastic demand and thus charges a higher price.

The fringe retailers play a mixed pricing strategy due to its tradeoff between the shoppers and non-shoppers. However, such a tradeoff is muted for the prominent retailer. The price competition between fringe retailers makes it unprofitable for the prominent one to poach the shoppers. Therefore, it only acquires the non-shoppers who inspect it first. This equilibrium outcome is similar to the result of other asymmetric oligopoly models. For example, Kocas and Kiyak (2006) find that only two retailers compete for switchers in an oligopoly market. Anderson et al. (2015) report that only the top two firms with the highest quality levels advertise in an asymmetric oligopoly market.

Next, I summarize the impact of an increase in α_p on price and profit in proposition 5.

Proposition 5

Under the full awareness set, an increase in α_p *(1) always increases price competition and (2) leads to a lower profit for the prominent retailer if the size of shopper segment* (μ) *is small and* $\alpha_d \geq \frac{1}{2}$.

I find that a higher concentration on the first search always encourages price competition under the full awareness set. The intuition is as follows. In this case, shoppers search all three retailers. As a result, it is similar to the oligopolistic competition. To acquire shoppers, one fringe retailer needs to undercut not only the price of the prominent retailer, but also the price of the other fringe. When traffic from the first search is more concentrated at the prominent retailer, it leaves fewer non-shoppers to both fringe retailers. This forces them to compete for price-sensitive shoppers. It intensifies the competition between the fringes and therefore lowers their prices. It makes an additional search more appealing to the non-shoppers who starts with the prominent retailer. Thus, they are willing to cease searching at a lower reservation price. To keep these customers, the prominent retailer has to lower its price as well.

Furthermore, the curse of prominence also realizes under the full awareness set. Similar to the limited awareness, the intuition also features the prominent retailer's tradeoff between the *demand enhancement effect* and the *price competition effect* from more traffic. However, different from the limited awareness set, the curse realizes when the size of shoppers is low. When the shoppers search all three retailers, the prominent retailer always has more non-shoppers relative to shoppers than the fringes. Therefore, it has a larger demand and suffers more from *price competition effect* when there are fewer shoppers.

5.2 Comparison between Limited and Full Awareness

Now I compare the prices and profits between the limited and full awareness set. This provides additional insights on the impact of awareness set expansion on pricing, competition, and profits. I find that awareness set expansion can intensify competition and even decrease the profit for the retailer that is known by more consumers. It suggests that retailers might become better off from hiding from consumers. Proposition 6 summarizes the impact of awareness set size on the prices and profits of retailers.

Proposition 6.

Compared to the limited awareness set,

(1) a fringe retailer always has a lower average price, while the prominent retailer has a high average price when the size of shopper segment (μ) *is large under the full awareness set;*

(2) both prominent and fringe retailers have low profits under the full awareness set.

Compared with the case under limited awareness, the fringe retailer is searched by more shoppers under the full awareness set. This gives them stronger incentives to compete for shoppers, which results in a lower average price of the fringe retailer.

The awareness set expansion has a non-monotonical influence on the price of the prominent retailer. To keep up with the price of its competitors, the prominent retailer might lower its price under the full than limited awareness set. However, when there are many shoppers on the market, the price competition between the fringe retailers is so fierce that it becomes unprofitable for the prominent one to battle for shoppers. Consequently, it forgoes shoppers and focuses only on non-shoppers. This could lead to a high price at the prominent retailer under the full awareness set.

It is not surprising that the prominent retailer has a lower profit under the full than the limited awareness set, because it gains no additional consumers under the full awareness set. However, the awareness set expansion also leads to a lower profit for the fringe one. The intuition is as follows. On one hand, the awareness set expansion increases a fringe retailer's demand from shoppers. On the other hand, this demand expansion also encourages the price competition and thus lowers the margin of a fringe retailer. The incremental demand from shoppers cannot compensate for the losses in margins from both shoppers and non-shoppers. As such, it lowers the fringe retailer's profit.

5.3 More than Three Retailers

In this section, I report the robustness of my result with more than three retailers $n > 3$ on the market. Let's start with the case that consumers are aware of two out of n retailers. Proposition 7 reports the finding.

Proposition 7

When there are $n > 3$ retailers and consumers possess awareness sets of size 2,

1. *there exists a crossover point $\alpha^* = \frac{1}{2}$. Beyond α^*, the prominent retailer charges higher prices stochastically than the fringe retailers. Below α^*, it charges lower prices stochastically.*
2. *An increase in α_d lowers the average prices of both retailer when $\alpha_d < \alpha^*$; otherwise, it raises the average prices;*
3. *The prominent retailer's profit decreases with α_d when $\alpha_d < \alpha^*$ and $\mu > \mu_1(\alpha_d) = \sqrt{\frac{1}{2} + \frac{{\alpha_d}^2}{16(1-\alpha_d)^2}} - \frac{\alpha_d}{4(1-\alpha_d)}$).*

This proposition shows that my results in Propositions 1, 2, and 3 are robust with more than three retailers. Particularly, in my model, the equilibrium prices of both retailers do not change with the number of retailers n as long as consumer's awareness set remains the same. The intuition is as follows. When consumers are aware of two retailers, each fringe only competes with the prominent retailer for the shoppers, whereas the fringes do not compete with each other. As such, regardless of the number of fringe retailers being included, they can all be considered as the case that a representative fringe retailer plays the duopoly competition with the prominent retailer. Therefore, the number of fringes does not affect equilibrium prices. Furthermore, the prominent retailer's profit does not change, because its price, first-search-level, and awareness remain the same in my model. However, each fringe retailer's profit declines with the respect to the number of retailers, because they need to share the profits with more symmetric fringe competitors. These results imply that the number of retailers has little impact on market outcomes if the consumer's awareness set remains the same. It suggests the key element for a firm's successful competitive strategy is the battle within the consumer's limited awareness set.

Now let's consider the case that consumers are aware of $m > 2$ out of n retailers. Proposition 8 reports the finding.

Proposition 8

When there are $n > 3$ retailers and consumers possess awareness sets of size $m > 2$, there exists a crossover point $\alpha^* = \frac{1}{m}$. Beyond α^*, the prominent retailer charges higher prices stochastically than the fringe retailers. Below α^*, it charges lower prices stochastically.

This result suggests that the crossover point α^* exists and is determined by the size of the consumer's awareness set when there are more than three retailers available. This further strengthens the robustness of my results.

This concludes the extension section.

6. General Discussion

6.1 Summary

This study investigates the implication of search traffic concentration in an intra-brand setting with an analytical model. In my model, consumers search for prices of the same product across retailers within their own limited awareness sets. There exists a prominent retailer that receives a disproportionally high first-search share and appear in all consumers' awareness sets. I find that the prominent retailer charges higher prices than its competitors when its first-search share is sufficiently high. Otherwise, its price is low relative to other retailers. In addition, the growing search traffic concentration can intensify price competition by forcing fringe retailers to compete for low-search-cost consumers more aggressively. This can lower prices and benefit consumers. Lastly, a "curse of prominent", such that more traffic reduces a retailer's profit, might exist. It reflects the tradeoff between the *demand enhancement effect* and *price competition effect* as a result of high search traffic volumes.

6.2 Contribution, Limitation, and Future Research

This study advances the bases of a retailer's dominance (e.g., Raju and Zhang 2005; Dukes et al. 2006; Geylani et al. 2007; Dukes et al. 2009). I unpack the effects of "search-dominance" or prominence in internet search traffic. Such a retailer can leverage its search traffic advantage through either high or low relative prices, depending on its first-search level. My research also contributes to consumer search, particular the prominent search literature (e.g., Arbatskaya 2007; Armstrong et al. 2009; Wilson 2010; Xu et al. 2011; Choi et al. 2018; Petrikaite 2018). In direct contrast to Armstrong et al. (2009), I find that a prominent retailer might charge higher prices than its competitors. Also, the prominence can intensify price competition and lower the prominent retailer's profit.

This research is relevant to retailers, consumers, policy-makers. The takeaways for retailers are from two perspectives. First, prominence can change a retailer's customer mix. Given its prominence, a retailer should understand its customer mix to price accordingly. Second, a retailer might not always desire to increase its prominence. Specifically, it can avoid the curse of prominence by lowering its exposure to consumers. At lower impressions, the prominent retailer not only saves advertising cost, but also can leave more high-search-cost consumers to its competitors. This strategy can buffer price competition and lead to higher average market prices, which increases the prominent retailer's profit.

Also, I find that consumers might become better off from more concentrated search traffic. This is at odds with the conventional policy position that concentration gives large firms market power to exploit consumers. Search traffic concentration is not entirely the same as market concentration due to firm's pricing to covert the traffic to sales. Each firm's search traffic share directly influences its competitive incentives and thus its pricing decision. When traffic is more concentrated on a prominent retailer, it provides stronger incentives for others to compete. This could lead to a more intensified competition and thus benefit consumers.

To illustrate, Amazon's search traffic dominance might well force other retailers, such as Walmart and Target, to price aggressively. This puts downward pressure on pricing and might explain the low inflation in the U.S. in recent years (e.g., Berman 2017; Cohen and Tankersley 2018). Consumers can benefit from such lower prices. For instance, "*Amazon's toe-to-toe price battle with the retail giant Walmart is in the best interests of consumers*" (Fernandez 2018).

Lastly, this research suggests a possible upside side of search traffic concentration in the intra-brand setting. Policymakers should be aware of this pro-competitive outcome when fashioning regulatory policies. At the very least, it calls for more comprehensive empirical evidence to understand the link between search traffic concentration and price competition.

There are limitations of this study that would benefit from future research. First, I exogenously impose the consumer's first search destination and which retailers are included in her awareness set. Such exogenous pattern might be due to the consumer's familiarity to this prominent retailer. However, it would be worthwhile to endogenize the source of prominence by modelling the consumer's decision on which retailer to search first and to include into her awareness set. Possible primitives include informativeness or persuasiveness of the advertising, product quality or variety, etc. Also, I assume no differentiation across retailers in order to focus on the role of the search cost heterogeneity on market outcome. It would be interesting to incorporate vertical and/or horizontal differentiation into consideration in future study. Finally, I mute the strategic interaction between the manufacturer and the retailer by normalizing a constant wholesale price. It might be fruitful to explore the implication of a retailer's search traffic dominance in a vertical channel. Specifically, how does the existence of a prominent retailer influence the manufacturer's price and product choice? How should a manufacturer govern the channel when the search traffic is heavily concentrated on one retailer? How should retailers make price and assortment decision with the arise of a prominent retailer? I trust future research will advance insights into these questions.

Essay 2: Retailer Reputation in a Distribution Channel

Conventional wisdom suggests sellers' reputation reduces information asymmetry and benefits both sides of the market. As such, a reputable seller is usually rewarded with price premiums and profitability. However, I show these insights might not always hold in a decentralized channel when consumers do not directly observe yet infer product quality from the retail price. I find a non-monotonic relationship between the retailer's reputation and its price. Interestingly, a "reputation trap" exists whereby an increase in retailer reputation can not only hurt the manufacturer and consumers, but also reduce the retailer's profit. Furthermore, I find that a high-quality manufacturer might prefer to distribute through a less reputable retailer when its product quality is not too high.

Keywords: Reputation, Distribution Channel, Price Signaling, Channel Efficiency

1. Introduction

Reputation is a valuable asset for sellers. Ample empirical evidence has confirmed the importance of seller reputation in marketplaces (e.g., Jin and Kato 2006; Resnick et al. 2006; Cabral and Hortacsu 2010; Cai et al. 2014; Hui et al. 2016). In general, theoretical literature argues reputation mitigates information asymmetry, reduces market inefficiency, and therefore benefits both sides of the market. As such, a reputable seller is rewarded with price premiums and profitability (see Bar-Isaac and Tadelis, 2008, for a survey).

However, prior theoretical frameworks usually are interested in the direct-selling scenario. Many transactions occur through a decentralized distribution channel. Under a vertical channel, the (re)seller/retailer cannot directly determine the product quality, because it is not the producer/manufacturer. Also, consumers usually only observe the retailer's reputation and price yet not necessarily the manufacturer's product quality. Possible conflicting interests between the manufacturer and retailer further complicate the impact of reputation in a decentralized channel. The above differences between the vertical channel and the direct selling motivate the following questions regarding retailer reputation in a distribution channel: Would a more reputable retailer always sell the same product at a higher price? Does reputation always benefit consumers and the retailer itself? Would a manufacturer prefer to distribute through a more or less reputable retailer?

To address these questions, I develop a game-theoretical model in a decentralized channel. The manufacturer sells its product through a retailer with a limited reputation. I model the retailer's reputation as its loss in future sales if it cheats consumers. Cheating occurs if the retailer sells a low-quality product at a high-quality price. The cheating is is possible because product quality is opaque to consumers but is known to both the manufacturer and the retailer. Foreseeing the retailer's incentive, consumers infer product quality based on retail price and retailer reputation. The manufacturer and the retailer play the Stackelberg game by choosing their prices sequentially.

My model yields several results. First, I find a non-monotonic relationship between retail price and retailer reputation. Reputation has two impacts on the retail price. First, consistent with the prior literature, a *communication effect* reduces information asymmetry, which leads to the price being closer to the perfect-information price. Unique in a distribution channel, reputation also has a *coordination effect*. A less reputable retailer is easier for the manufacturer to control, thus alleviating the double-marginalization problem and potentially resulting in a low price (relative to the perfect-information price). Below a reputation threshold, the positive coordination effect is activated and results in a low price. In such cases, the communication effect of reputation increases the equilibrium retail price, which approaches the perfect-information price from below. However, beyond the threshold, the coordination effect is muted. As such, the equilibrium retail price is higher than the perfect-information price. In such cases, the communication effect of reputation results in a decline in the retail price, which approaches the perfect-information price from above. Furthermore, the non-monotonicity between the price and reputation implies consumers might not always benefit from a high retailer reputation.

Second, a "reputation trap" exists in which the equilibrium retailer profit falls with its reputation at a moderate reputation level. It reflects the tradeoff between the communication effect and the coordination effect. The communication effect increases retailer profit by reducing information asymmetry. However, the coordination effect of reputation, which is unique in a decentralized channel, can reduce a retailer's profit. An increase in retailer reputation could exacerbate channel inefficiency, which in turn can hurt the retailer. This result suggests being less reputable can be a good thing for a retailer. Though less reputation reduces a retailer's pricing flexibility, it can benefit the retailer by serving as a credible commitment to facilitate channel coordination.

Lastly, I discover that the manufacturer prefers to distribute through a less reputable retailer when its product quality is not too high. By choosing a retailer with a minimal reputation, the manufacturer is able to gain more control over the channel. More control has two positive impacts on the manufacturer's profit. The first one is the coordination effect, as I discussed previously, which improves the channel efficiency and therefore increases the entire channel profit. The second one is a *profit-sharing effect*, which allows the manufacturer to possess a larger share of total channel profit. Distributing through a less reputable retailer is profitable when the manufacturer's product quality is not too high. In such cases, the positive communication effect of reputation is marginal and is dominated by the manufacturer's gain from more vertical controls.

The remainder of the paper is organized as follows. In section 2, I review the academic literature. Section 3 presents my model assumptions and outlines the game structure. Section 4 investigates the main model and provides insights from it. Section 5 concludes with managerial implications and directions for future search.

2. Related Literature

2.1 Seller Reputation and Product Quality

In this section, I summarize the literature on reputation and product quality. Ample empirical evidence shows the correlation between seller reputation and consumers' quality perception (e.g., Dodd et al. 1991; Grewal et al. 1998; Purohit and Srivastava 2001). In the empirical research, a reputable seller is usually defined as a seller with a high consumer rating.

The micro-foundation of reputation in the theoretical literature relies on consumers' repeated purchases (e.g., Klein and Leffler 1981; Shapiro 1983; Allen 1984; Biglaiser 1993; Biglaiser and Friedman 1994; Qiu and Rao 2018; see Bar-Isaac and Tadelis, 2008, for a survey). The intuition is as follows. The consumer is assumed to be able to discover true quality after purchase. If the revealed quality is lower than her expectation, the consumer would be disappointed and boycott the seller. Consequently, the seller would lose future sales by cheating consumers. This possibility serves as a credible threat to discipline sellers to maintain their reputation.

The prior theoretical framework usually focuses on the direct-selling scenario, where the seller is also the producer and can decide the product quality. As such, they are interested in moral hazard problems. However, in a decentralized channel structure, the seller/retailer cannot directly choose the product quality. Thus, the information problem becomes an adverse selection problem. A notable exception is Biglaiser and Friedman (1994), who show a middleman can serve as a guarantor for the quality of the producer. However, they abstract away the strategic interaction between channel members. This research fills this gap by modeling the conflicting interests between the manufacturer and the retailer.

2.2 Signaling

This research also connects to extant signaling literature (e.g., Spence 1973; Wolinsky 1983; Milgrom and Roberts 1986; Wernerfelt 1988; Bagwell and Riordan 1991; Shin 2005; Miklos-Thal and Zhang 2013). Bagwell and Riordan (1991) provide a classical framework on price signaling, whereas they abstract away the vertical channel structure. This research closes this gap by examining a signaling model where both the retailer and manufacturer determine the price signal.

My paper also complements the literature on signaling within the marketing channel. Prior literature focuses on resolving information asymmetry between the manufacturer and the retailer, including the choice of the wholesale price (e.g., Chu 1992), slotting allowances (e.g., Chu 1992), franchising policy (e.g., Desai and Srinivasan 1995), and sales (e.g., Jiang, Jerath, and Srinivasan 2011). However, few studies have examined how a manufacturer signals its quality to the consumer via a retailer. A notable exception is Chu and Chu (1994), in which the manufacturer signals its quality to the consumer through the retailer *type*. By contrast, consumers in my model also use retail *price* as a quality signal. Furthermore, this research is interested in strategic interaction between channel members, which is abstracted away by Chu and Chu (1994).

3. Model

3.1 Product Quality and Information Environment

The players in my model are a manufacturer, a retailer, and consumers. The manufacturer sells a product through the retailer to consumers. Product quality is exogenously determined and can be either high or low, represented by $q_\theta \in \{q_H, q_L\}$. Without loss of generality, I normalize $q_L = 1$ and $q_H = q > 1$. Thus, q represents the quality difference between the two types. Also, I normalize the production cost of both types to be zero. Both the manufacturer and the retailer have perfect information on product quality a priori. However, consumers cannot observe quality prior to purchase, but discover it fully immediately after purchase.

3.2 Consumers

I normalize the mass of consumers to one. She has a demand of at most one unit of product. The consumption utility is assumed to be

$$u(q_\theta) = kq_\theta - p,$$

where k represents the consumer's quality preference and is a random draw from the uniform distribution $k \sim U[0,1]$. The consumer purchases the product if her expected utility from purchase is non-negative $kE(q_\theta) - p \geq 0$, which yields $k \geq \frac{p}{E(q_\theta)}$. Given $k \sim U[0,1]$, I have the downward-sloping demand for product sold at price p as $D(p) = 1 - \frac{p}{E(q_\theta)}$.

The consumer forms quality expectation based on her belief. She has a $\phi \in (0,1)$ prior-probability belief regarding product quality being high. After observing retail price p, she updates her quality belief to $\hat{\phi}(\hat{q}|p)$. Based on $\hat{\phi}(\hat{q}|p)$, the demand given retail price becomes $D(p) = 1 - \frac{p}{\hat{\phi}(\hat{q}|p)q + (1 - \hat{\phi}(\hat{q}|p))}$.

3.3 Retailer Reputation

I use $m > 0$ to represent retailer reputation.[5] Following the reputation literature (e.g., Klein and Leffler 1981), the retailer suffers losses in future sales m if the consumer believes it has "cheated". The "cheating" is defined as the case in which the realized quality after purchase is lower than the consumer's quality expectation. Specifically, when the product is of low quality, instead of pricing based on the true quality, the retailer can sell a low-quality product (henceforth, the low-quality retailer)[6] at a price that mimics that of a high-quality product.

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[5] Note the micro-foundation of reputation is beyond the scope of interest in this research. The reduced-form reputation can capture the repeated purchase in the classical reputation literature.

[6] I use the retailer that sells the low-quality product and the low-quality retailer interchangeably.

3.4 Firm's Decision

The manufacturer and the retailer play the Stackelberg game. The manufacturer chooses the wholesale price w_θ to maximize its profit $\pi_\theta = w_\theta D(p)$.[7] Given the wholesale price, the retailer chooses the retail price to maximize its profit. Note reputation enters into the retailer's profit-maximizing problem when it sells the low-quality product. Specifically, the retailer loses its reputation m if it mimics a low-quality product as a high-quality one by selling it at high-quality prices. That is,

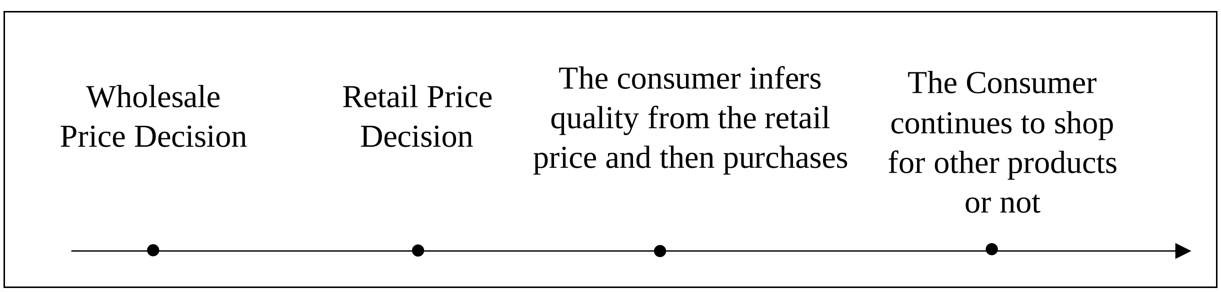
$$\pi_r(w_L, p_L) = (p_L - w_L)D(p) - \begin{cases} m, & if\ r\ mimics\ H, \\ 0, & otherwise. \end{cases}$$

When selling the high-quality product, the retailer never loses its reputation. It chooses p_H to maximize its profit from selling the focal product:

$$\pi_r(w_H, p_H) = (p_H - w_H)D(p).$$

Figure 2 summarizes the sequence of moves in the game.

Figure 2. The Sequence of Moves in the Game



3.5 Equilibrium Concept

The equilibrium concept I use is a Perfect Bayesian Equilibrium that consists of three elements: the manufacturer's wholesale pricing strategy, the retailer's pricing strategy, and

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[7] In this research, I am interested in the role of a retailer's reputation, because the retailer usually carries products from multiple manufacturers and therefore has more to lose from future sales by cheating consumers than one individual manufacturer. For simplification, I abstract away the manufacturer's reputation and its cheating incentive. In the Appendix B, I take the low-quality manufacturer's reputation and cheating incentive into consideration. I show my main results hold if the low-quality manufacturer has a sufficiently high reputation.

the consumer's belief. Given the consumer's belief, pricing decisions of the manufacturer and the retailer maximize their profits.

The consumer rationally expects the wholesale price after considering the pricing strategy of the manufacturer and the retailer. After observing the retail price, the consumer rationally updates her posterior belief $\hat{\phi}(\hat{q}|p)$ whenever possible. The off-equilibrium belief satisfies the intuitive criterion (Cho and Kreps 1987). If the retailer's *highest* possible profit from mimicry at a price p is higher than its profit from truthfully representing a low-quality product, the consumer believes the low-quality retailer might mimic the high-quality retailer, at p. Table 2 summarizes the notation employed in the model.

Table 2. Model Notation

Notation	Description
θ	Product type $\theta \in \{H, L\}$
q_θ	Product quality $q_\theta \in \{q_H, q_L\}$.
m	The retailer's reputation
ϕ	The consumer's prior belief
p_θ	Retail price
$\hat{\phi}(q|p)$	The consumer's posterior belief after observing p
w_θ	Wholesale price
$D(p)$	Demand given retail price
$\pi_r(w_\theta, \hat{\phi}(\hat{q}|p), p_\theta)$	Retailer's profit
$\Pi_\theta(w_\theta, \hat{\phi}(\hat{q}|p), p_\theta)$	Manufacturer's profit

4. Analysis

I start with a simple model of the preferment information as a benchmark. Next, I use backward induction to solve the separating equilibrium prices.[8] Then, I characterize a

[8] This research is interested in the separating equilibrium. I discuss the existence of pooling equilibrium in technical appendix b.

sufficient condition for the uniqueness of the separating equilibrium. Finally, I report the impact of retailer reputation on prices, profits, and consumer welfare and implications from results.

4.0 Benchmark: Perfect Information

When the consumer observes quality prior to purchase, the demand becomes $D_\theta(p) = 1 - \frac{p}{q_\theta}$. Also, the retailer cannot mimic the low-quality product as the high-quality one. Therefore, the retailer never loses its reputation.

Using backward induction, I can solve the equilibrium wholesale and retail prices as $w_\theta = \frac{q_\theta}{2}$ and $p_\theta = \frac{3}{4} q_\theta$. Note the problem of double marginalization exists because the manufacturer and the retailer maximize their profits separately. As such, the equilibrium retail price is higher than the price in a fully integrated channel, $\frac{q_\theta}{2}$.

4.1 Separating Equilibrium Prices

I first establish properties regarding the separating equilibrium. Use p_H^s and p_L^s to denote the separating equilibrium retail prices for high- and low-quality products. The first lemma is related to the low-quality product:

Lemma 1. *In any separating equilibrium , $p_L^s = \frac{3}{4}$ and $w_L^s = \frac{1}{2}$.*

Under any separating equilibrium, the consumer believes the low-quality product is indeed low quality. As such, its demand is the same as under perfect information. Consequently, the low-quality product's retail and wholesale prices under any separating equilibrium would be the same as under perfect informaton.

Next, we turn to the high-quality product. I first establish the property regarding the retail price of the high-quality product under the separating equilibrium.

Lemma 2. *If* $m \leq m_3(q) = \frac{q}{4} + \frac{1}{16q} - \frac{5}{16}$,[9] *then in any spearating equilibrium* , $p_H^s \notin (\underline{p}(m,q), \overline{p}(m,q))$, *where* $\underline{p}(m,q) = \frac{q}{2} + \frac{1}{4} - \frac{1}{2}\Delta$, $\bar{p}(m,q) = \frac{q}{2} + \frac{1}{4} + \frac{1}{2}\Delta$, *and* $\Delta \equiv \sqrt{-4mq + q^2 - \frac{5}{4}q + \frac{1}{4}}$. *Note the perfect-information retail price* $\frac{3}{4}q \in (\underline{p}, \overline{p})$ *if* $m < m_2(q) = \frac{3}{16}(q-1)$.

Please see detailed proofs of this lemma and subsequent analysis in the Appendix B. Lemma 2 documents that when retailer reputation is below a threshold m_3, a range of retail prices exist within which mimicry is indeed profitable for the retailer. I name it the *quality-suspicion range*.[10] The consumer cannot tell with certainty that the product sold at a price within the quality-suspicion range is of high quality. Consequently, the separating equilibrium price of a high-quality product cannot be set within this range.

In a separating equilibrium, the low-quality retailer must find mimicking the price of the high-quality retailer to be unprofitable. By engaging in mimicry, the retailer both gains and loses.[11] It gains by tricking the consumer into buying at a higher price, which increases its single-period profit. However, it loses reputation. The loss and gain from mimicry are represented by m and q. m represent the loss in reputation from cheating. q captures the discrepancy between a high- and low-quality product. The greater the discrepancy in product quality, the higher the profit from mimicry. Within the quality-

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[9] I define $m_1(q) \leq m_2(q) \leq m_3(q)$ later.

[10] Note Bagwell and Riordan (1991) also present such a range within which no separating equilibrium price can exist.

[11] When it is indifferent between mimicking and not mimicking, I assume the retailer does not mimic.

suspicion range, the gain outweighs the loss and therefore the retailer finds mimicry profitable.

The quality-suspicion range exists only when retailer reputation is sufficiently low, $m \leq m_3$. Beyond the critical point m_3, the loss from mimicry would be so high that the retailer will not find mimicking to be profitable at any price. In such a case, the retailer will always set a price that is a true reflection of product quality, a finding consistent with the role of an intermediary's reputation in solving the lemon's problem (e.g., Biglaiser and Friedman 1994).

Below the threshold m_3, the quality-suspicion range is over a moderate range of retail prices, due to the downward-sloping demand curve. When the price is low, the margin is low, so the gain from mimicry cannot compensate for the loss due to mimicry. When the price is high, the quantity sold is low, so, once again, the gain from mimicry cannot compensate for the loss due to mimicry. Also, the quality-suspicion range expands with a decline in the retailer's reputation. A lower retailer reputation reduces the cost of mimicry. As such, it makes the mimicry profitable over a wider range of prices.

Given the property on the separating equilibrium retail price, I solve the Stackelberg game by employing backward induction. Specifically, I first show the separating retail price can be discontinuous in response to the wholesale price. Next, I substitute this discontinuous retail-price response function into the manufacturer's profit function. By solving the manufacturer's profit-maximization problem, I derive the separating equilibrium wholesale price and the corresponding retail price of the high-quality product, w_H^s and p_H^s. I leave the detailed proof in the Appendix B and summarize the findings in proposition 1. Also, I present findings virtually in the space of retailer reputation and quality difference between the two type, (m, q), in Figure 3.

Proposition 1. (Separating Equilibrium Prices for High-Quality Product)

The separating equilibrium retail and wholesale prices for the high-quality product are

(1) $p_H^s = \underline{p}$ *and* $w_H^s = \frac{1}{2}$ *at low reputation levels* $m \leq m_1(q)$*;*

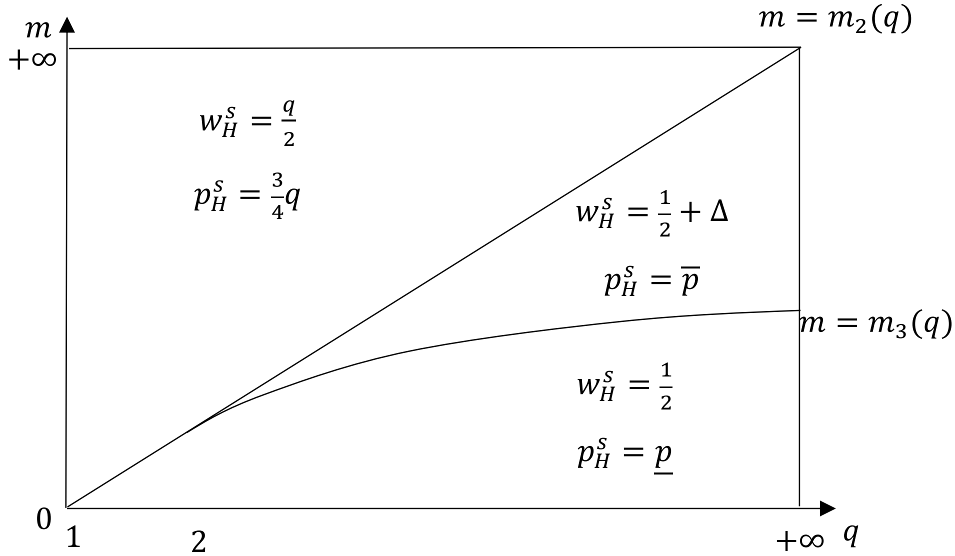
(2) $p_H^s = \overline{p}$ *and* $w_H^s = \frac{1}{2} + \Delta$ *at moderate reputation levels* $m \in (m_1(q), m_2(q)]$*;*

(3) $p_H^s = \frac{3}{4}q$ *and* $w_H^s = \frac{q}{2}$ *at high reputation levels* $m > m_2(q)$*.*

Proposition 1 states that retailer reputation plays a significant role in equilibrium prices. The impact can be ascribed to two perspectives. On one hand, reputation resolves information asymmetry on quality, which is the communication effect on consumers. On the other hand, it has a coordination effect among channel members, which is unique in a distribution channel. A more reputable retailer is harder for the manufacturer to control. A lack of control dampens the coordination between the manufacturer and the retailer. Let's consider equilibrium prices by regions.

At low reputation levels $m \leq m_1$, the retail price is set a low-level $\underline{p}$ (relative to the perfect-information price). This is result is driven by the coordination effect. To signal product quality, the retailer has to avoid pricing within the quality-suspicion range. Such avoidances handicap the retailer's pricing flexibility in response to the wholesale price, and reduces the retailer margin, thus alleviating the double-marginalization problem. Specifically, when retailer reputation is sufficiently low, the separating retail price is discontinuous in response to the wholesale price. The less smooth retail-price response enables the manufacturer to more easily "control" the retail price. Consequently, the manufacturer is willing to charge a low wholesale price. This lower wholesale price leads the equilibrium retail price to be at $\underline{p}$, which is lower than the perfect-information price. Note the retailer's price is under greater "control" when its reputation is lower. As such, the coordination effect is positive only below the critical point m_1.

Figure 3. Equilibrium Prices by (Reputation and Quality Difference)



Beyond m_1, the positive coordination effect is muted. Instead, the communication effect plays the main role in the equilibrium price. Specifically, at high reputation levels $m \geq m_2$, the communication effect is so strong that information asymmetry on quality is minor or resolved. As such, the equilibrium price would be the same as the perfect-information price $\frac{3}{4}q$.

When the reputation is moderate $m \in (m_1, m_2)$, the retailer sells the product at a high price $\overline{p}$. The intuition is as follows. Because both channel members observe product quality, the wholesale price of the low-quality product is lower than the wholesale price of the high-quality product. A lower wholesale price implies a lower selling cost per unit for the low-quality retailer. Consequently, it hurts more from a shrinking demand due to a high retail price. Hence, a high price (relative to perfect-information price) $\overline{p}$ is the most efficient way to separate the two types, and therefore becomes the equilibrium price. Note the result in this region corresponds to Bagwell and Riordan (1991), who find a high price signals high quality under a direct-selling scenario.

To further appreciate the impact of the retailer reputation on equilibrium prices, I report the relationship between the retailer's price and its reputation in Corollary 1.

Corollary 1. (Non-monotonicity between the Retailer's Reputation and Price)

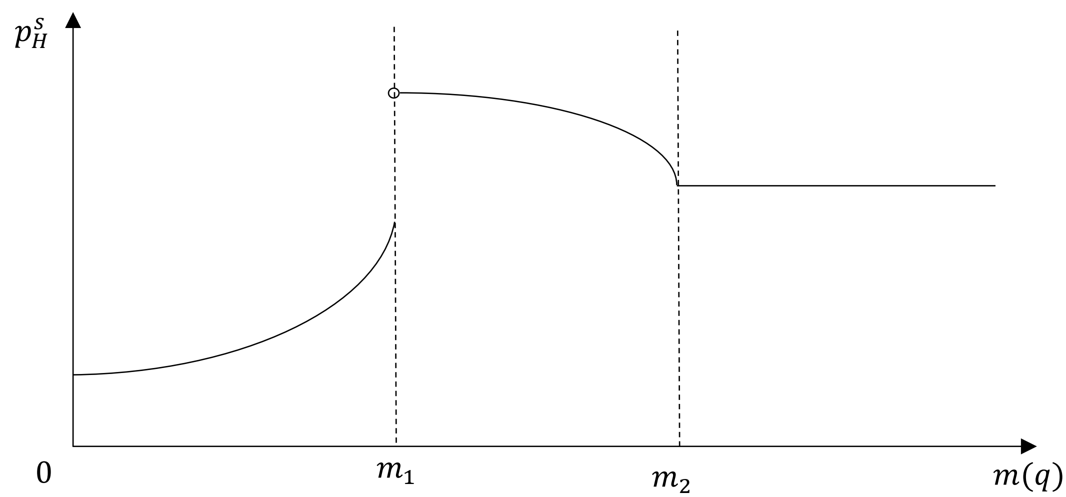
The retailer's price increases with its reputation when $m \leq m_1(q)$, yet the price falls with reputation when $m \in (m_1(q), m_2(q))$, and does not change with respect to reputation when $m \geq m_2(q)$.

Figure 4 presents the relationship between the retailer's price and its reputation (relative to quality difference), $m(q)$. This corollary reports a non-monotonic relationship between the retailer's price and reputation. Again, it reflects the coordination and communication effect of reputation. Following the intuition in Proposition 1, a "jump" occurs in the equilibrium price at the critical point m_1 because the positive coordination effect is activated below m_1 yet muted beyond m_1.

The communication effect can explain the monotonicity in $(0, m_1]$ and $(m_1, m_2]$. A higher reputation increases the retailer's cost of mimicry. A higher mimicry cost forces the retailer to be more likely to truthfully represent the product. This reduces the information asymmetry on quality. As such, the equilibrium price approaches the perfect-information price. That is, it increases from a low price (relative to the perfect-information price) for $m \in (0, m_1]$, and declines from a high price (relative to the perfect-information price) for $m \in (m_1, m_2]$.

At high reputation levels $m \geq m_2$, the consumer forms almost no quality suspicion. The information asymmetry no longer exists or is marginal. Hence, the equilibrium price is the same as the perfect-information price. As such, the price does not change with respect to a further increase in reputation.

Figure 4. The Retailer's Price and Reputation



Now consider the impact of retailer reputation on consumer welfare. I report the finding in claim 1.

Claim 1. (Non-monotonicity between Consumer Welfare and Retailer Reputation)

An increase in the retailer's reputation can hurt consumers when $m \leq m_1(q)$. Otherwise, consumer welfare increases with it when $m \in (m_1(q), m_2(q)$ and does not change with it when $m \geq m_2(q)$.

Claim 1 directly follows from Corollary 2. The consumer is better off from a lower retail price, under which she can buy more yet pay less. As such, she becomes worse off with the increase in retailer reputation when $m \leq m_1$. This result suggests consumers are not always better off from a higher seller reputation. This finding is at odds with the conventional wisdom that the reputation should benefit consumers by resolving information asymmetry. The decline in consumer welfare is due to the coordination effect of reputation, which is unique in a vertical channel. A more reputable retailer is harder for the manufacturer to control and thus can dampen the channel coordination. The channel inefficiency can spill over to and hurt consumers.

4.3 Equilibrium Refinement and Uniqueness

Thus far, I have focused on the separating equilibrium. However, a pooling equilibrium may also exist. Which equilibrium should I select? When the separating equilibrium is the unique equilibrium? I use the intuitive refinement (Cho and Kreps 1987) for equilibrium selection. Proposition 2 reports the unique condition for the separating equilibrium.

Proposition 2 (Uniqueness).

The separating equilibrium is the only equilibrium that satisfies the intuitive criterion if the consumer's prior-quality belief is sufficiently low, $\phi \leq \phi_1 = \frac{1}{q-1}[2m - \frac{3}{8} + \sqrt{\frac{9}{64} + 4m^2 + \frac{5}{2}m}]$.

The intuition from this proposition is as follows. If the consumer's prior-quality belief ϕ is sufficiently low, the pooling-quality expectation becomes lower. As such, the demand is substantially lower under a pooling than under a separating equilibrium. Consequently, a low prior-quality belief enhances the benefit from separating. Hence, the separating equilibrium is more profitable for the retailer and is the only one that survives the intuitive criterion refinement when the prior-quality belief is low.

4.4 The Retailer's Profit and Reputation

In this section, I discuss the impact of the retailer's reputation on its profit. Surprisingly, the retailer can hurt from its own reputation.

Let's start with the retailer's profit. Proposition 3 reports a "reputation trap" such that the equilibrium retailer's profit decreases with its reputation.

Proposition 3. (Reputation Trap)

The retailer's profit decreases with its reputation at $m = m_1(q)$*. Otherwise, the retailer's profit is non-decreasing with its reputation. Also, the retailer's profit is highest at a moderate level* $m_1(q)$ *if* $q \leq q_1 = 4 + 2\sqrt{2}$*, whereas its profit is the highest at high levels* $m \geq m_2(q)$ *if* $q > q_1$*.*

In general, the retailer's profit increases with its reputation, because the communication effect resolves the information asymmetry and increases the consumer's quality expectation. This finding echoes the classical literature on reputation (see Bar-Isaac and Tadelis, 2008, for a survey). However, a "reputation trap" exists at a moderate reputation level m_1. At this level, a further increase in reputation can reduce the retailer's profit. The intuition is as follows. Note the positive coordination effect is activated below m_1 but would be turned off beyond m_1. In such a case, the loss in the coordination effect is so large that it cannot be compensated by the communication effect. Therefore, a "dive" of the retailer profit with respect to incremental reputation exists at m_1. Figure 5 depicts the relationship between retailer profit and reputation, $m(q)$ (relative to quality difference).

Note that for any quality difference, the monotonicity does not change. However, the reputation level that yields the highest retailer profit is different for different quality-difference levels. Specifically, when the quality difference is low $q \leq q_1$, the retailer's profit is the highest under a moderate reputation level $m = m_1$, whereas the retailer's profit is the highest at high reputation levels $m \geq m_2(q)$ when the quality difference is high.

Figure 5. The Retailer's Profit and Reputation

Figure 5(a). When $q \leq q_1 = 4 + 2\sqrt{2}$

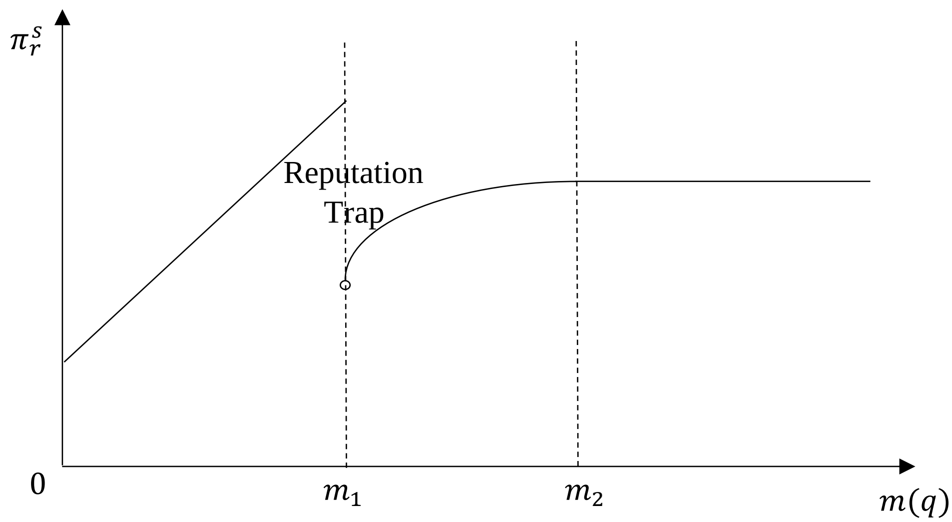
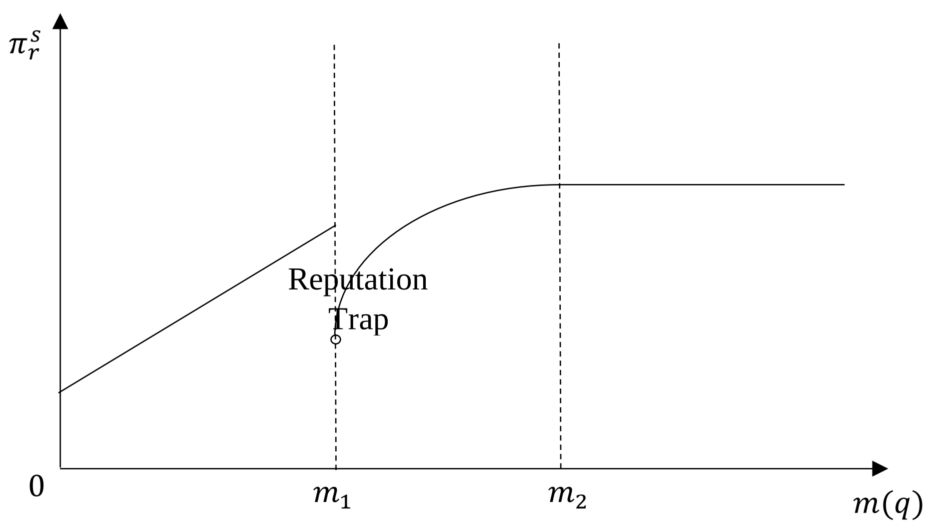


Figure 5(b). When $q > q_1$



This result also suggests the retailer might not always benefit from its reputation. Retailer reputation can backfire and hurt the retailer itself. Again, this result reflects the tradeoff between the coordination and communication effect. The coordination effect benefits a retailer with a low reputation, because it facilitates channel coordination. Whereas the communication effect benefits a reputable retailer that resolves information asymmetry. A lower quality difference decreases the benefit of separating two types, the communication effect. Consequently, the coordination effect dominates the communication effect when the quality difference is low. At a moderate reputation level, the retailer self-handicaps its pricing flexibility. The pricing inflexibility serves as a credible commitment to coordinate the channel. A better coordinated channel, in turn, can increase the retailer's profit.

4.5 The Manufacturer's Profit and Distribution Decision

In this section, I discuss the relationship between manufacturer profit and retailer reputation in proposition 4.

Proposition 4. (Manufacturer Profit and Retailer Reputation)

The manufacturer's profit decreases with m when $m \leq m_1(q)$, increases with m when $m \in (m_1(q), m_2(q)]$, and does not change with respect to m when $m \geq m_2(q)$.

The increase in retailer reputation hurts the manufacturer below a crossover point, yet it benefits the manufacturer above the crossover point. The intuition is as follows. Besides coordination and communication effects, a profit-sharing effect exists that influences the manufacturer's profit. A more reputable retailer demands a larger share of the total channel profit, thus reducing the manufacturer's profit. The profit-sharing effect is especially strong when retailer reputation increases from a low level. Consequently, below the critical point m_1, the manufacturer profit and retailer reputation have a negative relationship. However, when retailer reputation is sufficiently high, the benefit of the communication effect is so strong that it dominates the other two impacts. In such cases, retailer reputation increases the manufacturer's profit. Figure 6 presents the relationship between manufacturer profit and retailer reputation relative to the quality difference, $m(q)$. Note the retailer reputation that yields the highest manufacturer profit is $m \to 0$ in Figure 6(a) yet $m \geq m_2$ in Figure 6(b).

Figure 6. Manufacturer Profit and Retailer Reputation

Figure 6(a). When $q \leq q_2 = \frac{1}{2}(3 + \sqrt{13})$

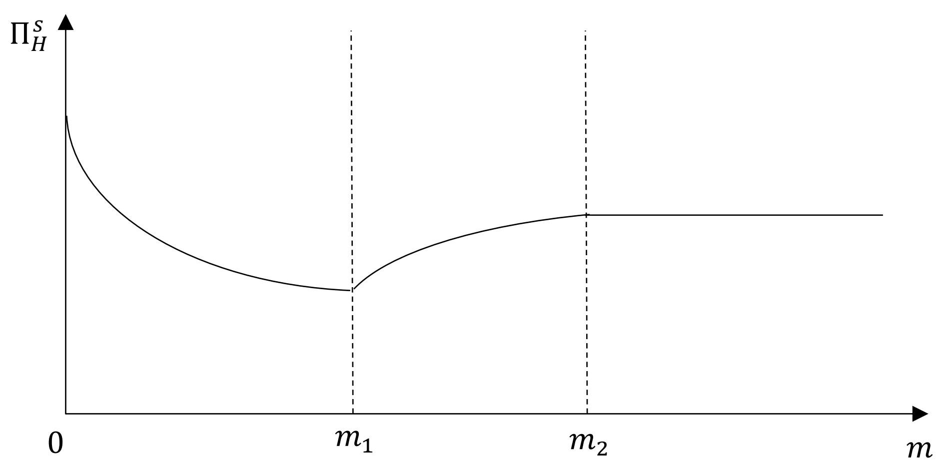
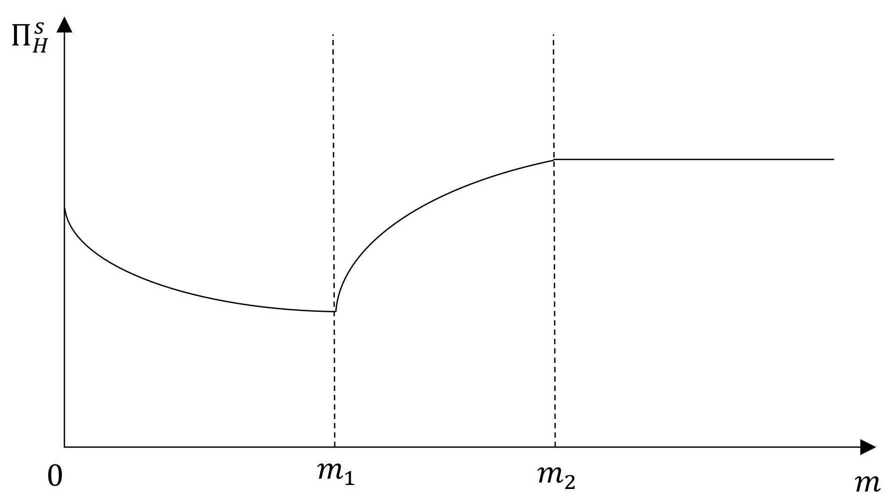


Figure 6(b). When $q > q_2$



Proposition 4 has implications for the manufacturer's distribution strategy. Corollary 2 reports that the manufacturer might prefer to distribute its product through a less reputable retailer.

Corollary 2. (The Manufacturer's Distribution Strategy)

The high-quality manufacturer prefers to distribute through a retailer with a minimal-level reputation $m \to 0$ *if* $q < q_2 = \frac{1}{2}(3 + \sqrt{13})$*; however, it prefers to distribute through a reputable retailer* $m \geq m_2$ *if* $q \geq q_2$.

Considerations on the coordination, communication, and profit-sharing effect influence the manufacturer's distribution strategy. On one hand, the manufacturer prefers to sell its product through a less reputable retailer. A retailer with a minimal reputation is easier to control and asks for a smaller share of channel profit. On the other hand, a more reputable retailer is attractive to the manufacturer because it resolves the information asymmetry and thus increases the consumer's quality expectation. When the quality difference is high, the benefit of communication is high. In such cases, the communication effect dominates: the manufacturer profits more from distributing through a retailer with a high reputation. However, when the quality difference is low, the coordination and profit-sharing effects prevail, thus motivating the manufacturer to distribute through a less reputable retailer.

Finally, my result has an implication on the information strategy for channel members. I find the manufacturer, the retailer, and consumers can all become better off under imperfect than under perfect information. This finding suggests hiding quality information from consumers might be beneficial for channel members. Corollary 4 summarizes the finding.

Corollary 3. (Win-Win-Win from Information Asymmetry)

When $m \in (\underline{m}(q), m_0(q))$, *the manufacturer, the retailer, and the consumer all benefit from information asymmetry compared with the perfect information setting where consumers directly observe product quality, where* $\underline{m}(q) = \frac{1}{16}(q-1)$ *and* $m_0 = \min\{\frac{1}{16q}[(4q-1)(q-1)-(q-1)^4], m_1(q)\}$.

Under imperfect information, the consumer does not directly observe product quality. She has to use the retailer's price and its reputation to infer quality. This price-quality inference can restrict the retailer's pricing flexibility. It can lower the retail margin and mitigate the double-marginalization problem, thus improving the channel efficiency, which benefits everyone.

This concludes the main analysis.

5. General Discussion

5.1 Summary

In this research, I explore the consequence of retailer reputation on a vertical channel that features information asymmetry between channel members and consumers. I find reputation has two main impacts. First, consistent with the classical literature (e.g., Biglaiser and Friedman 1994), a communication effect resolves the information asymmetry. However, unique in a distribution channel, a coordination effect also exists between the manufacturer and the retailer. These two effects lead to a non-monotonic relationship between the retail price and retailer reputation. A more reputable retailer might not always sell the product at a higher price than its less reputable counterparts. In addition, I report a "reputation trap" such that equilibrium retailer profit might fall with the retailer's reputation. Lastly, my research shows a high-quality manufacturer might avoid selling through a retailer with an intermediate reputation. Instead, selling through a retailer with either a minimal or a high reputation is optimal for the high-quality manufacturer.

5.2 Contributions, Limitations, and Future Research

First, this study contributes to the literature on reputation. Prior reputation literature usually abstracts away strategic interaction between channel members (e.g., Shapiro 1983; Allen 1984; Biglaiser 1994; Baiglaiser and Friedman 1994). They argue that reputation mitigates information asymmetry and improves market efficiency. Therefore, a reputable seller is usually rewarded with price premiums and profitability. However, this research uncovers that reputation can exacerbate the coordination problem in a vertical channel. A more reputable retailer might not always sell at a higher price. Furthermore, seller reputation can backfire and hurt all members in a distribution channel.

In addition, this study provides guidance on the manufacturer's distribution strategy. The manufacturer faces the tradeoff between the communication effect and issues on the vertical control when choosing the retailer to sell its product. On one hand, the manufacturer prefers a retailer with a minimal reputation, because such a retailer is easy to control and asks for a smaller share of channel profit. On the other hand, it prefers a reputable retailer because the reputation increases the consumer's quality expectation. As such, the manufacturer should avoid distributing through a retailer with an intermediary reputation. Instead, it chooses between a reputable retailer and a retailer with a minimal reputation based on its product quality.

This study has some limitations that would benefit from future research. First, it focuses on the channel inefficiency of double marginalization, which applies to many retail settings, where linear (unit) pricing is commonly observed in practice (Iyer and Villas-

Boas 2003; Gal-Or, Geylani, and Dukes 2008). Nevertheless, extending our model to a non-linear pricing regime would be interesting. More importantly, the double-marginalization problem is a representative source of channel inefficiency. However, I believe the central idea, namely, that retailer reputation exacerbates vertical control, will hold in other contexts of channel inefficiency as well. Second, I model reputation in a reduced-form way for analytical tractability. Providing a micro-foundation for reputation and seeing whether my results are robust would be worthwhile. Finally, this model examines a monopoly setting, comprising one manufacturer and one retailer. Considering the implications of competition among manufacturers, retailers, or both might be valuable.

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Appendices

Appendix A: Proofs in Essay 1

Proof of Lemma 1

In this section, I show that there is a unilateral deviation from any pure strategy equilibrium. Suppose that there exists a pure strategy equilibrium, and without loss of generality, let's assume the equilibrium price to be p_d, p_f and $p_d < p_f$. Note that the same logic goes through for $p_d \geq p_f$.

First, I want to show that $p_f - p_d < c$ in equilibrium. If $p_f - p_d \geq c$, then non-shoppers who visit the fringe retailer first would make an additional search at the prominent one. Since $p_d < p_f$, all consumers purchase from the prominent retailer. Consequently, the fringe has a zero demand and thus a zero profit. This is lower than the positive profit for the fringe under $p_f < p_d + c$. This proves $p_f - p_d < c$.

Next, I show that no equilibrium exists for $p_f - p_d < c$. In this case, non-shoppers buy from the first retailer they visit, while all shoppers purchase from the prominent one because its price is lower. If this is the equilibrium, there must exist $\epsilon \to 0$ such that $p_f - p_d < \epsilon$. Otherwise, the difference between p_f and p_p would be large enough to attract the prominent retailer to deviate to $p_d{}' \in (p_d, p_f)$, under which it has a higher margin without losing any demand. However, if there exists $\epsilon \to 0$ such that $p_f - p_d < \epsilon$, one fringe would deviate to a price slightly lower than p_d, under which it has a discontinuous demand increase by capturing all shoppers. Thus, no equilibrium exists for $p_f - p_d < c$.

The above shows that no pure strategy price equilibrium exists for $p_d < p_f$.

Proof of Lemma 2

I prove lemma 2 by contradiction. I will show that if there exists a "hole" at the price support of retailer i when $p \leq r_{-i}$, one retailer would have inconstant profits within the support. This contradicts the definition of mixed strategy equilibrium.

Suppose that retailer i and $-i$ compete for the shoppers. If there exists a "hole" (a, b) at the price support of retailer i for $p \leq r_{-i}$, it implies that retailer i puts zero probability for $p \in (a,b)$. Suppose $(\underline{p}_{-i}, \overline{p}_{-i}) \cap (a,b) = (a',b')$, where $a \leq a' \leq b' \leq b$. The hole (a,b) implies that $F_i(a) = F_i(a') = F_i(b') = F_i(b)$. Since the price support of the retailer i and $-i$ can be the same or different, there can be different cases regarding (a',b'). Consider all possible cases as follows.

First, let's consider the case that $(a',b') \neq \emptyset$. Because $b' < b < r_{-i}$, non-shoppers who start with retailer $-i$ stop searching and purchase from $-i$ at both $p = a'$ or $p = b'$. However, this leads to a profit for retailer $-i$ under a' than b':

$$E\pi_{-i}(a') = \{(1-\mu)\alpha_{-i} + \mu\beta_{-i}[1 - F_i(a')]\}a'$$
$$< E\pi_{-i}(b') = \{(1-\mu)\alpha_{-i} + \mu\beta_{-i}[1 - F_i(b')]\}b',$$

because $a' < b'$ and $F_i(a') = F_i(b')$. The inconstant profit within the price support contradicts to the definition of mixed strategy equilibrium.

Second, if $(a',b') = \emptyset$, it implies either $a < b < \underline{p}_{-i}$ or $b > a > \overline{p}_{-i}$. In both cases, I have $F_{-i}(a) = F_{-i}(b)$. In this case, there can be three possible scenarios (1) $a < r_i$, (2) $a = r_i$, and (3) $a > r_i$. Let's consider all three, respectively.

Case 1. $\boldsymbol{a < r_i}$

I can find that retailer i has a lower expected profit at a than $\min\{b, r_i\}$ as follows:

$$E\pi_i(a) = \{(1-\mu)\alpha_i + \mu\beta_i[1 - F_{-i}(a)]\}a$$
$$< E\pi_i(\min\{b, r_i\}) = \{(1-\mu)\alpha_i + \mu\beta_i[1 - F_{-i}(\min\{b, r_i\})]\}\min\{b, r_i\},$$

Because $a < \min\{b, r_i\}$ and $F_{-i}(a) = F_{-i}(\min\{b, r_i\}) = F_{-i}(b)$. This contradicts the definition of mixed strategy equilibrium.

Case 2. $\boldsymbol{a = r_i}$

If $a = r_i$, it implies that the price support of retailer i contains a. However, I can find that $E\pi_{-i}(a) < E\pi_{-i}(b)$, because $a < b < r_{-i}$ and $F_i(a) = F_i(b)$. This contradicts the definition of the mixed strategy equilibrium.

Case 3. $\boldsymbol{a > r_i}$

In this case, non-shoppers at retailer i search both retailer i and $-i$ and purchase from a lower price. I can find that retailer i has a strictly lower profit at a than b:

$$E\pi_i(a) = \{(1-\mu)\alpha_i + \mu\beta_i\}[1 - F_{-i}(a)]a$$
$$< E\pi_i(b) = \{(1-\mu)\alpha_i + \mu\beta_i\}[1 - F_{-i}(b)]b,$$

because $r_i < a < b$ and $F_{-i}(a) = F_{-i}(b)$. This contradicts the definition of mixed strategy equilibrium.

The above proves that there must exist no "hole" for the price support of retailer i if $p \leq r_{-i}$.

Proof of Lemma 3

I am interested in an equilibrium that two fringe retailers play an identical strategy. I want to show $\overline{p}_f \leq r_f$ and $\overline{p}_d \leq r_d$. Without loss of generalizability, I will show that $\overline{p}_f \leq r_f$ and $\overline{p}_d \leq r_d$ when $r_f \leq r_d$. The same logic goes through the case that $r_f > r_d$ as well.

First, I show $\overline{p}_d \leq r_d$ as follows. If the prominent retailer charges $p > r_d$, it has a zero demand, because both non-shoppers and shoppers can find a lower price from the fringe, whose price is $p \leq r_f < r_d$. Thus, the prominent retailer has a zero profit when charging $p > r_d$. However, this is strictly lower than its positive profit under $p = r_d$. This proves $\overline{p}_d \leq r_d$.

Second, I show $\overline{p}_f \leq r_f$ by contradiction. Suppose a fringe retailer charges $p = r_f + \epsilon$, where $\epsilon > 0$. In this case, non-shoppers who start with the fringe would keep searching the prominent one. There can only be two possible cases: (1) $\overline{p}_d \leq r_f$ or (2) $\overline{p}_d > r_f$. Note that the proofs vary in two cases. If $\overline{p}_d \leq r_f$, the proof is similar to Stahl (1989), the fringe retailer has a strictly lower profit under $p = r_f + \epsilon$ than r_f. However, if $\overline{p}_d > r_f$ I will show that $p = r_f + \epsilon$ would violate lemma 2.

Case 1. $\boldsymbol{\overline{p}_d \leq r_f}$

If $\overline{p}_d \leq r_f$, all consumers could find a lower price from the prominent retailer with certainty. Thus, this fringe has a zero profit at $r_f + \epsilon$. However, it is strictly lower than the positive profit at r_f. Thus, $\overline{p}_f \leq r_f$.

Case 2. $\boldsymbol{\overline{p}_d > r_f}$

If $\overline{p}_d > r_f$, non-shoppers find a lower price at the prominent retailer with probability. Therefore, they purchase from the retailer with a lower price. This gives us fringe's profit at $r_f + \epsilon$ as

$$E[\pi_f(r_f+\epsilon)] = (r_f+\epsilon)[(1-\mu)\alpha_f + \mu\beta_f][1-F_d(r_f+\epsilon)].$$

Since $F_d(p)$ is non-decreasing, I have

$$E[\pi_f(r_f+\epsilon)] \leq (r_f+\epsilon)[(1-\mu)\alpha_f + \mu\beta_f][1-F_d(r_f)].$$

Combining it with $E[\pi_f(r_f)] = r_f\{(1-\mu)\alpha_f + \mu\beta_f[1-F_d(r_f)]\}$, I can have $E[\pi_f(r_f+\epsilon)] < E[\pi_f(r_f)]$ when $\epsilon < \epsilon_1 = \frac{(1-\mu)\alpha_f F_d(r_f)}{[(1-\mu)\alpha_f + \mu\beta_f][1-F_d(r_f)]} r_f$. This suggests that $E[\pi_f(p)] < E[\pi_f(r_f)]$ if $p \in (r_f, r_f+\epsilon_1)$. This results in a "hole" $(r_f, r_f+\epsilon_1)$ for the fringe retailer when $p < r_d$. It contradicts lemma 2. Thus, $\overline{p}_f \leq r_f$ under a mixed strategy equilibrium.

This proves Lemma 3.

Proof of Proposition 1

First, let's establish some properties regarding the mass point of equilibrium price distribution:

(1) There must exist a mass point. (e.g., Narasimhan 1988; Kocas and Kiyak 2006).

(2) All retailers cannot have a mass point at the same price. Otherwise, one retailer is better off from moving probability mass to a slightly lower price than the mass point. This gives it a discontinuous increase in demand without sharing shoppers at the mass point.

(3) The mass point can only exist at the reservation prices. I prove this by contradiction. Let's consider the case $r_f < r_d$, note that the same proof goes through the opposite case.

   a. First, no retailer can have a mass point at $p < r_f < r_d$. If one retailer has a mass point at $p < r_f < r_d$, the other one has a higher profit at $p+\epsilon$, where $\epsilon \to 0$ and $\epsilon > 0$, than at p, because it has a higher margin and the same demand. This contradicts the definition of mixed strategy equilibrium.

   b. Second, no mass point can exist for $p \in (r_f, r_d)$. Since $\overline{p}_f \leq r_f$, all non-shoppers purchase from the first retailer that they visit. Consequently, the prominent retailer has no incremental demand from pricing at $p \in (r_f, r_d)$ than $p = r_d$. As a result, it has a strictly lower profit at $p \in (r_f, r_d)$ than $p = r_d$. This implies that the prominent retailer puts zero probability at $p \in (r_f, r_d)$. Hence force, there can be no mass point at $p \in (r_f, r_d)$.

Next, let's consider the lower boundary of price support. In this case, each shopper searches one prominent and one fringe retailer. Thus, the prominent retailer only needs to undercut one fringe retailer's price to acquire shoppers, and vice versa. Consequently, no retailer is willing to charge a price that is lower than the competitor's lower boundary. Therefore, two retailers share the common lower boundary, $\underline{p}_d = \underline{p}_f = \underline{p}$. (Note that this is similar to the duopoly competition in Narasimhan (1988).) By pricing at the common boundary $\underline{p}$, a retailer captures shoppers with probability one.

Regarding the upper boundary, there can be two possible cases: (1) $r_d \leq r_f$ and (2) $r_d > r_f$. I consider both and find that equilibrium holds for the first one when $\alpha_d = \alpha^* \geq \frac{1}{2}$ and for the latter case when $\alpha_d < \alpha^*$.

Case 1: $\boldsymbol{r_d \leq r_f}$

If $r_d \leq r_f$, $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_d} F_f(p)dp$, because $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_f} F_d(p)dp = \int_{\underline{p}}^{r_d} F_f(p)dp = c$. The rest of the proof in this case proceeds as the follows.

First, I find there can be two possible subcases, which depends on whether a mass point exists at r_f of $F_f(p)$ or not.

Second, I derive the equilibrium outcome in both subcases and find that equilibria exist only when $\alpha_d \geq \frac{1}{2}$. The derivation follows four steps. Step 1, identify the mass point. Step 2, derive the equilibrium price distribution given reservation price. Step 3, make sure that $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_d} F_f(p)dp$. Step 4, solve the reservation price from optimal stopping rule.

Finally, I select the equilibrium that yields higher profits for both retailers. I find the selected equilibrium has no mass point at r_f for $F_f(p)$. The intuition is as follows. No mass point at r_f implies a lower competition level. Thus, both retailers have higher profits in this scenario.

<u>*Subcase 1.1*</u>*: $F_f(p)$ has no mass point at r_f*

Step 1. If no mass point exists at r_f of $F_f(p)$, then either $F_d(p)$ or $F_f(p)$ has a mass point at r_d. I show that $F_f(p)$ does NOT have a mass point at r_d by contradiction. If $F_f(p)$ has a mass point at r_d, then $F_d(p)$ does not have a mass point at r_d. This implies that the

fringe retailer captures shoppers with zero probability at both r_d and r_f since $r_d \leq r_f$. This gives $\pi_f(r_d) = \frac{(1-\mu)(1-\alpha_d)}{2} r_p \leq \pi_f(r_f) = \frac{(1-\mu)(1-\alpha_d)}{2} r_f$.[12] However, this contradicts the definition of mixed strategy equilibrium. As a result, $F_f(p)$ does not have a mass point at r_d.

Step 2. Since $F_f(p)$ has no mass point, the prominent retailer gets shoppers with zero probability at r_d. This gives $E\pi_d = \pi_d(r_d) = (1-\mu)\alpha_d r_d$. Substitute it into (4.1), I have $F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ for $p \in [\underline{p}, r_d]$. Given $E\pi_d = \pi_d(\underline{p}) = [(1-\mu)\alpha_d + \mu]\underline{p}$, I have $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_d$. This gives $E\pi_f = \pi_f\left(\underline{p}\right) = \left[\frac{(1-\mu)(1-\alpha_d)}{2} + \frac{\mu}{2}\right]\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_d$. Substitute it into (4.2), I have $F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}]$ for $p \in [\underline{p}, r_d]$.

Step3. I show that this equilibrium only holds when $\alpha_d \geq \frac{1}{2}$ as follows. When $\alpha_d < \frac{1}{2}$, I find that $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_d} F_f(p)dp$, because $F_d(p) - F_f(p) = \frac{(1-2\alpha_d)(1-\mu)}{\mu}\left[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}\right] > 0$ for $p \in [\underline{p}, r_d]$, which contradicts to $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_d} F_f(p)dp$.

Step 4. I have $r_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\,[1+\frac{\mu}{(1-\mu)\alpha_d}]}$ and $r_f = r_d + \frac{(1-\mu)(2\alpha_d-1)}{(1-\mu)\alpha_d+\mu}c$ after substituting $F_f(p)$, $F_d(p)$, and $\underline{p}$ into $c = \int_{\underline{p}}^{r_d} F_f(p)dp = \int_{\underline{p}}^{r_{\mathrm{f}}} F_d(p)dp$. Given r_d and r_f, I have $E\pi_d = \frac{c}{-\frac{1}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]+\frac{1}{(1-\mu)\alpha_d}}$ and $E\pi_f = \frac{(1-\mu)(1-\alpha_d)+\mu}{-\frac{(1-\mu)\alpha_d+\mu}{\mu}\ln\,[1+\frac{\mu}{(1-\mu)\alpha_d}]+\frac{(1-\mu)\alpha_d+\mu}{(1-\mu)\alpha_d}}\frac{c}{2}$.

<u>Subcase 1.2</u>: *$F_f(p)$ has a mass point at r_f*

Step 1. If $F_f(p)$ has a mass point at r_f, $F_d(p)$ must have a mass point at r_d. If no mass point exists at r_d for $F_d(p)$, the fringe retailer captures shoppers with zero probability at both r_d and r_f. Since $r_d \leq r_f$, $\pi_d(r_d) \leq \pi_f(r_f)$. This violates the definition of the mixed strategy equilibrium. Therefore, a mass point must exist at r_d. In this case, given that the prominent retailer's price is no higher than r_d, the fringe retailer puts no weight on $p \in (r_d, r_f)$, under which it has a strictly lower profit than $p = r_f$ due to the lower margin yet the same demand.

[12] I define the break-even condition to violate the mixed strategy equilibrium definition. It would not change the result qualitatively.

Step 2. Since the fringe retailer gets shopper with zero probability at r_f, I have $E\pi_f = \pi_f(r_f) = \frac{(1-\mu)(1-\alpha_d)}{2} r_f$. Substitute it into (4.2), I have $F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}(\frac{r_f}{p} - 1)$ for $p \in [\underline{p}, r_d]$. Given $\pi_f(\underline{p}) = E\pi_f$, this yields $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} r_f$. Substitute $\underline{p}$ into $\pi_d(\underline{p})$, I have $E\pi_d = \pi_d(\underline{p}) = [(1-\mu)\alpha_d + \mu]\frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} r_f$. This gives $F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}]$ for $p \in [\underline{p}, r_d]$.

Step 3. I can show that this equilibrium holds only when $\alpha_d \geq \frac{1}{2}$ as follows. When $\alpha_p < \frac{1}{2}$, $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_d} F_f(p)dp$, because $F_d(p) - F_f(p) = \frac{(1-2\alpha_d)(1-\mu)}{\mu}\left[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}\right] > 0$ for $p \in [\underline{p}, r_d]$. This contradicts to $\int_{\underline{p}}^{r_d} F_d(p)dp \leq \int_{\underline{p}}^{r_d} F_f(p)dp$.

Step 4. I cannot have closed form solution for the reservation price and thus the profit in this case. Instead, let's derive the upper bound of retailers' profits in this case. Substitute $F_i(p)$ and $\underline{p}$ into $c = \int_{\underline{p}}^{r_f} F_d(p)\,dp$ and $c = \int_{\underline{p}}^{r_d} F_f(p)\,dp$, I have

$$(1-\mu)(1-\alpha_d)r_f \ln\left[\left(1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right)\frac{r_f}{r_d}\right] = \mu c - (1-\mu)(1-\alpha_d)r_d - [\mu - (1-\mu)(1-\alpha_d)]r_f.$$

$$(1-\mu)(1-\alpha_d)r_f \ln\left[\left(1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right)\frac{r_f}{r_d}\right] = \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}\mu c - [(1-\mu)(1-\alpha_d)+\mu]r_d + (1-\mu)(1-\alpha_d)r_f.$$

Combining the two equations, I have

$$r_f - r_d = \frac{(1-\mu)(2\alpha_d - 1)}{(1-\mu)\alpha_d+\mu} c.$$

Since $F_f(p)$ has a mass point at r_f, I have $F_f(r_f) < 1$. This gives $F_f(r_d) < 1$ given $r_d \leq r_f$. Consequently, $E\pi_d = \pi_d(r_f) = \{(1-\mu)\alpha_d + \mu[1 - F_f(r_d)]\}p > (1-\mu)\alpha_d r_d$. It yields $\pi_d(\underline{p}) = [(1-\mu)\alpha_d + \mu]\frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} r_f = E\pi_d > (1-\mu)\alpha_d r_d$. Substitute $r_d = r_f - \frac{(1-\mu)(2\alpha_d-1)}{(1-\mu)\alpha_d+\mu} c$ into it, I have $r_f < \frac{(1-\mu)\alpha_d[(1-\mu)(1-\alpha_d)+\mu]}{\mu[(1-\mu)\alpha_d+\mu]} c$. Hence, I have $E\pi_d < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d}{\mu} c$ and $E\pi_f < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d[(1-\mu)(1-\alpha_d)+\mu]}{2\mu[(1-\mu)\alpha_d+\mu]} c$.

Profit Comparison and Equilibrium Selection

I show that both retailers have higher profits under subcase 1.1 than 1.2 as follows. Suppose retailer $i's$ profit in 1.1 as $E\pi_i{}'$. I have $E\pi'_d > \frac{(1-\mu)\alpha_d[(1-\mu)\alpha_d+\mu]}{\mu} c$, because

$\ln(1+x) > \frac{x}{1+x}$ where $x = \frac{\mu}{(1-\mu)\alpha_p}$. Note that $E\pi_d < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d}{\mu}c$, this gives $\frac{E\pi_d'}{E\pi_d} > \frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)} > 1$ for $\alpha_d \geq \frac{1}{2}$. Following the same proof, I can show that $\frac{E\pi_f'}{E\pi_f} > 1$. Therefore, I choose the equilibrium in 1.1, under which both retailers have higher profits.

As a summary, when $\alpha_d \geq \frac{1}{2}$, $F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}]$ and $F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ $p \in [\underline{p}, r_d]$, where $r_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]}$ and $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}r_d$.

Case 2: $\boldsymbol{r_d > r_f}$

If $r_d > r_f$, I have $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_d} F_f(p)dp$, because $\int_{\underline{p}}^{r_d} F_d(p)dp > \int_{\underline{p}}^{r_f} F_d(p)dp = \int_{\underline{p}}^{r_d} F_f(p)dp$. Similar to Case 1, first, I find there can be two subcases, which depends on whether a mass point exists at r_d of $F_d(p)$ or not. Second, I derive the equilibrium outcome in both subcases and find that equilibria exist only when $\alpha_d < \frac{1}{2}$. Similar to the previous case, the derivation follows four steps. Finally, I select the equilibrium that yields higher profits for both retailers.

<u>Subcase 2.1</u>*: $F_d(p)$ has no mass point at r_d*

Step 1. If $F_d(p)$ has no mass point at r_d, then either $F_f(p)$ or $F_d(p)$ has a mass point at r_f. I show that $F_d(p)$ does NOT have a mass point at r_f by contradiction. If $F_d(p)$ has a mass point at r_f, then $F_f(p)$ does not have a mass point at r_f. The prominent retailer captures shoppers with zero probability at both r_f and r_d since $r_d > r_f$. This gives $\pi_d(r_f) = (1-\mu)\alpha_d r_f < \pi_f(r_d) = (1-\mu)\alpha_d r_d$. This contradicts the definition of mixed strategy equilibrium. As a result, no mass point exists for $F_d(p)$.

Step 2. If no mass point exists for $F_d(p)$, then the fringe retailer captures shoppers with zero probability at r_f. This gives $E\pi_f = \pi_f(r_f) = \frac{(1-\mu)(1-\alpha_d)}{2}r_f$. Substitute it into (4.2), I have $F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}(\frac{r_f}{p} - 1)$ for $p \in [\underline{p}, r_f]$. Given $\pi_f(\underline{p}) = E\pi_f$, I have

$\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} r_f$. Substitute $\underline{p}$ into $\pi_d\left(\underline{p}\right)$, $E\pi_d = \pi_d\left(\underline{p}\right) = \frac{(1-\mu)(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)+\mu} r_f$. Substitute $E\pi_d$into (4.1), I have $F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}]$ for $p \in [\underline{p}, r_f]$.

Step 3. I can show that this equilibrium holds only when $\alpha_d < \frac{1}{2}$ as follows. When $\alpha_d \geq \frac{1}{2}$, I find that $\int_{\underline{p}}^{r_f} F_d(p)dp \leq \int_{\underline{p}}^{r_f} F_f(p)dp$, because $F_d(p) - F_f(p) \leq 0$ for $p \in [\underline{p}, r_f]$. This contradicts to $\int_{\underline{p}}^{r_f} F_d(p)dp > \int_{\underline{p}}^{r_f} F_f(p)dp$.

Step 4. I have $r_f = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}$ and $r_d = r_f + \frac{(1-\mu)(1-2\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} c$ after substituting $F_d(p)$, $F_f(p)$, and $\underline{p}$ into $c = \int_{\underline{p}}^{r_f} F_d(p)\, dp = \int_{\underline{p}}^{r_d} F_f(p)\, dp$.This gives $E\pi_d = \frac{(1-\mu)\alpha_d+\mu}{1+\frac{\mu}{(1-\mu)(1-\alpha_d)}-[\frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}]\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]} c$ and $E\pi_f = \frac{c}{\frac{2}{(1-\mu)(1-\alpha_d)}-\frac{2}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}$.

<u>Subcase 2.2</u>*: $F_d(p)$ has a mass point at r_d*

Step 1. If $F_d(p)$ has a mass point at r_d, $F_f(p)$ must have a mass point at r_f. Otherwise, the prominent retailer captures shoppers with zero probability at both r_f and r_d. Since $r_f < r_d$, this gives $\pi_d\left(r_f\right) < \pi_d(r_d)$. It violates the definition of mixed strategy equilibrium. Thus, a mass point must exist at r_f of $F_f(p)$. In this case, given that the fringe retailer's price is no higher than r_f, the prominent retailer puts no weight on $p \in (r_f, r_d)$, under which it has a strictly lower profit than $p = r_d$ due to a lower margin yet the same demand.

Step 2. I have $F_f(r_d) = 1$, because the fringe retailer would not price higher than r_f and $r_f < r_d$. This gives $E\pi_d = \pi_d(r_d) = (1-\mu)\alpha_d r_d$. Substitute it into (4.1), I have $F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ for $p \in [\underline{p}, r_f]$. Given $F_f\left(\underline{p}\right) = 0$, I have $E\pi_d = \pi_d\left(\underline{p}\right) = [(1-\mu)\alpha_d + \mu]\underline{p}$, which yields $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_d$. Given $F_d\left(\underline{p}\right) = 0$, I have $E\pi_f = \pi_f\left(\underline{p}\right) = \left[\frac{(1-\mu)(1-\alpha_d)}{2} + \frac{\mu}{2}\right]\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_p+\mu} r_d$. Substitute it into (4.2), I have $F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}]$ for $p \in [\underline{p}, r_f]$.

Step 3. I can show that this equilibrium holds only when $\alpha_d < \frac{1}{2}$ as follows. When $\alpha_d \geq \frac{1}{2}$, $\int_{\underline{p}}^{r_f} F_d(p)dp \leq \int_{\underline{p}}^{r_f} F_f(p)dp$, because $F_d(p) - F_f(p) \leq 0$ for $p \in [\underline{p}, r_f]$, which contradicts to $\int_{\underline{p}}^{r_f} F_d(p)dp > \int_{\underline{p}}^{r_f} F_f(p)dp$.

Step 4. I cannot derive the closed form solution for reservation price and profit. Instead, I derive the upper boundary of the retailer's profit in this case. Substitute $F_i(p)$ and $\underline{p}$ into $c = \int_{\underline{p}}^{r_f} F_d(p)\, dp$ and $c = \int_{\underline{p}}^{r_d} F_f(p)\, dp$, I have

$$(1-\mu)\alpha_d r_d \ln\left[\left(1 + \frac{\mu}{(1-\mu)\alpha_d}\right)\frac{r_d}{r_f}\right] = (1-\mu)\alpha_d r_d - [(1-\mu)\alpha_d + \mu]r_f + \frac{\mu[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)+\mu}c.$$

$$(1-\mu)\alpha_d r_d \ln\left[\left(1 + \frac{\mu}{(1-\mu)\alpha_d}\right)\frac{r_d}{r_f}\right] = -[\mu - (1-\mu)\alpha_d]r_d - (1-\mu)\alpha_d r_f + \mu c.$$

Combining the two, I have

$$r_d - r_f = \frac{(1-\mu)(1-2\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}c.$$

Furthermore, I have $F_d(r_d) < 1$, because $F_d(p)$ has a mass point at r_d. This gives $F_d(r_f) < 1$ given $r_d > r_f$. Hence, $E\pi_f = \pi_f(r_f) = \left[\frac{(1-\mu)(1-\alpha_d)}{2} + \frac{\mu}{2}\right]\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_d > \frac{(1-\mu)(1-\alpha_d)}{2} r_f$. In addition, I have $E\pi_f = \pi_f\left(\underline{p}\right) = \left[\frac{(1-\mu)(1-\alpha_d)}{2} + \frac{\mu}{2}\right]\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_p$. It yields $\left[\frac{(1-\mu)(1-\alpha_d)}{2} + \frac{\mu}{2}\right]\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r_d > \frac{(1-\mu)(1-\alpha_d)}{2} r_f$. Substitute $r_f = r_d - \frac{(1-\mu)(1-2\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}c$ into it, I have $r_d < \frac{(1-\mu)(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{\mu[(1-\mu)(1-\alpha_d)+\mu]}c$. This yields $E\pi_d < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d[(1-\mu)\alpha_d+\mu]}{\mu[(1-\mu)(1-\alpha_d)+\mu]}c$ and $E\pi_f < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d}{2\mu}c$.

Profit Comparison and Equilibrium Selection

I show that both retailers have higher profits under subcase 2.1 than 2.2 as follows. Suppose retailer $i's$ profit in 2.1 as $E\pi_i'$. I have $E\pi_d' > \frac{(1-\mu)(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{\mu}c$, because $\ln(1+x) > \frac{x}{1+x}$ where $x = \frac{\mu}{(1-\mu)(1-\alpha_d)}$. When $\alpha_d < \frac{1}{2}$, I find that $\frac{E\pi_d'}{E\pi_d} > \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d} > 1$, because $E\pi_d < \frac{(1-\mu)^2(1-\alpha_d)\alpha_d[(1-\mu)\alpha_d+\mu]}{\mu[(1-\mu)(1-\alpha_d)+\mu]}c$. Following similar proof, I can verify that $\frac{E\pi_f'}{E\pi_f} > 1$. Thus, I select the equilibrium in 2.1, under which both retailers have higher profits.

As a summary, when $\alpha_d < \frac{1}{2}$, $F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}(\frac{r_f}{p} - 1)$ and $F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}]$ for $p \in [\underline{p}, r_f]$, where $r_f = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}$ and $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu} r_f$.

Proof of Proposition 2

In this section, I show that $\frac{\partial E\pi_i}{\partial \alpha_d} > 0$ when $\alpha_d \geq \alpha^* = \frac{1}{2}$ and $\frac{\partial E\pi_i}{\partial \alpha_d} < 0$ when $\alpha_d < \alpha^*$. First, I have $Ep_i = \int_{\underline{p}}^{r_i} p \frac{dF_i(p)}{dp} dp = pF_i(p)|_{\underline{p}}^{r_i} - \int_{\underline{p}}^{r_i} F_i(p)dp = r_i - \int_{\underline{p}}^{r_i} F_i(p)dp$.

Case 1. $\alpha_d \geq \alpha^*$

In this case, $r_d = \frac{c}{g(\alpha_d)}$, where $g(\alpha_d) = 1 - \frac{(1-\mu)\alpha_d}{\mu}\ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right]$. $g'(\alpha_d) = -\frac{\mu}{1-\mu}\left\{\ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right] - \frac{\mu}{(1-\mu)\alpha_d+\mu}\right\} < 0$, because $\ln(1+x) > \frac{x}{1+x}$ where $x = \frac{\mu}{(1-\mu)\alpha_d}$. This gives $\frac{\partial r_d}{\partial \alpha_d} = -\frac{g'(\alpha_d)}{g(\alpha_d)^2}c > 0$.

I can calculate the average price as follows. $Ep_d = r_d - \int_{\underline{p}}^{r_d} F_d(p)dp = r_d - \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu} r_d \left\{1 - \frac{(1-\mu)\alpha_d}{\mu}\ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right]\right\}$. This gives $Ep_d = r_d - \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}c$ given $c = r_d\left\{1 - \frac{(1-\mu)\alpha_d}{\mu}\ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right]\right\}$. Hence, $\frac{\partial Ep_d}{\partial \alpha_d} = \frac{\partial r_d}{\partial \alpha_d} + \frac{(1-\mu)(1+\mu)}{[(1-\mu)\alpha_d+\mu]^2}c > 0$.

Moreover, $Ep_f = r_f - \int_{\underline{p}}^{r_f} F_f(p)dp = r_f - \{\int_b^{r_d} F_f(p)dp + \int_{r_d}^{r_f} 1dp\} = r_d - c$. Consequently, $\frac{Ep_f}{\partial \alpha_d} = \frac{\partial r_d}{\partial \alpha_d} > 0$.

Case 2. $\alpha_d < \alpha^*$

In this case, $r_f = \frac{c}{h(\alpha_d)}$, where $h(\alpha_d) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right]$. I have $h'(\alpha_d) = \frac{1-\mu}{\mu}\left\{\ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right] - \frac{\mu}{(1-\mu)(1-\alpha_d)+\mu}\right\} > 0$, since $\ln(1+x) > \frac{x}{1+x}$ where $x = \frac{\mu}{(1-\mu)(1-\alpha_d)}$. Therefore, $\frac{\partial r_f}{\partial \alpha_d} = -\frac{h'(\alpha_d)}{h^2(\alpha_d)}c < 0$.

Furthermore, $Ep_d = r_d - \int_{\underline{p}}^{r_d} F_d(p)dp = r_d - \{\int_{\underline{p}}^{r_f} F_d(p)dp + \int_{r_f}^{r_d} 1dp\} = r_f - c$. It yields $\frac{\partial Ep_d}{\partial \alpha_d} = \frac{\partial r_f}{\partial \alpha_d} < 0$.

Lastly, $Ep_f = r_f - \int_{\underline{p}}^{r_f} F_f(p)dp = r_f - \frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)+\mu} r_f \left\{1 - \frac{(1-\mu)(1-\alpha_d)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right]\right\}$. Given $c = r_f \left\{1 - \frac{(1-\mu)(1-\alpha_d)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right]\right\}$, I have $Ep_f = r_f - \frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)+\mu} c$. This gives $\frac{\partial Ep_f}{\partial \alpha_d} = \frac{\partial r_f}{\partial \alpha_d} - \frac{(1-\mu)(1+\mu)}{[(1-\mu)(1-\alpha_d)+\mu]^2} c < 0$.

Proof of Proposition 3

In this section, I show that $\frac{\partial E\pi_d}{\partial \alpha_d} < 0$ when $\alpha_d < \frac{1}{2}$ and $\mu > \sqrt{\frac{1}{2} + \frac{\alpha_d^2}{16(1-\alpha_d)^2}} - \frac{\alpha_d}{4(1-\alpha_d)}$.

When $\alpha_d < \frac{1}{2}$, $\frac{\partial E\pi_d}{\partial \alpha_d} = \frac{c}{\left\{-\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]\frac{(1-\mu)(1-\alpha_d)+\mu}{\mu} + 1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right\}^2} l(\mu, \alpha_d)$, where $l(\mu, \alpha_d) = \frac{1+\mu}{1-\alpha_d} - \frac{\mu[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)^2} - \frac{(1-\mu)(1+\mu)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right] < \frac{1+\mu}{1-\alpha_d} - \frac{\mu[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)^2} = \frac{2(\alpha_d-1)\mu^2 - \alpha_d\mu + (1-\alpha_d)}{(1-\mu)(1-\alpha_d)^2}$, given $\ln\left[1 + \frac{\mu}{(1-\mu)(1-\alpha_d)}\right] > 0$. This gives a sufficient condition for $l(\mu, \alpha_d) < 0$: $2(\alpha_d - 1)\mu^2 - \alpha_d\mu + (1 - \alpha_d) < 0$. From it, I have $\mu > \mu_1(\alpha_d) = \sqrt{\frac{1}{2} + \frac{\alpha_d^2}{16(1-\alpha_d)^2}} - \frac{\alpha_d}{4(1-\alpha_d)}$. Consequently, if $\mu > \mu_1$, $l(\mu, \alpha_d) < 0$ and thus $\frac{\partial E\pi_d}{\partial \alpha_d} < 0$.

Proof of Corollary 1

In this section, I show $\frac{E\pi_f}{\partial \alpha_d} > 0$ when $\alpha_d \geq \frac{1}{2}$ and $\mu > \sqrt{\frac{1}{2} + \frac{(1-\alpha_d)^2}{16\alpha_d^2}} - \frac{1-\alpha_d}{4\alpha_d}$.

When $\alpha_p \geq \frac{1}{2}$, $\frac{\partial E\pi_f}{\partial \alpha_d} = \frac{1}{\left\{\frac{(1-\mu)\alpha_d+\mu}{(1-\mu)\alpha_d} - \frac{(1-\mu)\alpha_d+\mu}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]\right\}^2} \frac{c}{2} m(\mu, \alpha_d)$, where $m(\mu, \alpha_d) = \frac{(1-\mu)(1+\mu)}{\mu} \ln\left[1 + \frac{\mu}{(1-\mu)\alpha_d}\right] - \frac{1+\mu}{\alpha_d} + \frac{\mu[(1-\mu)(1-\alpha_d)+\mu]}{(1-\mu)\alpha_d^2} > -\frac{1+\mu}{\alpha_d} + \frac{\mu[(1-\mu)(1-\alpha_d)+\mu]}{(1-\mu)\alpha_d^2} = \frac{2\alpha_d\mu^2 + (1-\alpha_d)\mu - \alpha_d}{(1-\mu)\alpha_d^2}$. A sufficient condition for $m(\mu, \alpha_d) > 0$ is $2\alpha_d\mu^2 +$

$(1-\alpha_d)\mu - \alpha_d > 0$. It yields $\mu > \mu_2(\alpha_d) = \sqrt{\frac{1}{2} + \frac{(1-\alpha_d)^2}{16\alpha_d^2}} - \frac{1-\alpha_d}{4\alpha_d}$. Therefore, if $\mu > \mu_2$, $m(\mu, \alpha_d) > 0$ and hence $\frac{\partial E\pi_f}{\partial \alpha_d} > 0$.

Proof of Proposition 4

First, I can verify that lemma 3 holds for full awareness. Without loss of generalizability, I will show that $\overline{p}_f \leq r_f$ and $\overline{p}_d \leq r_d$ when $r_f \leq r_d$. Suppose a fringe charges $p > r_f$, then non-shoppers who visit it first would keep searching. In this case, consumers can at least find a lower price from another fringe. Consequently, this fringe has a zero demand and thus zero profit by pricing at $p = r_f + \epsilon$. However, it is strictly lower than the positive profit under r_f. Therefore, a fringe would never charge $p = r_f + \epsilon$. Similarly, I show $\overline{p}_d \leq r_d$ as follows. The prominent retailer has a zero demand and thus profit under $p > r_d$, because all consumers can find a lower price from the fringe, whose price is $p \leq r_f < r_d$. However, this I strictly lower than the positive profit at r_d. Thus, this proves $\overline{p}_d \leq r_d$.

Next, let's consider the lower boundary of price support. Under the full awareness set case, I must $\underline{p}_d \geq \underline{p}_f$. The intuition is as follows. In this case, two symmetric fringes and one prominent retailer compete for shoppers. When one fringe retailer prices at $\underline{p}_d$, it does not capture shoppers with probability one, because the other fringe's price might be lower than $\underline{p}_d$. As a result, the fringe has an incentive to price below $\underline{p}_d$. However, by pricing at $\underline{p}_f$, the prominent retailer undercuts the prices of both fringes. Therefore, the prominent retailer never prices below $\underline{p}_f$. The above logic implies that $\underline{p}_d \geq \underline{p}_f$.

Now let's consider the upper boundary. Given that the search cost equals search benefit: $c = \int_{\underline{p}_f}^{r_d} F_f(p)dp = max\left\{\int_{\underline{p}_d}^{r_f} F_d(p)dp, \int_{\underline{p}_f}^{r_f} F_f(p)dp\right\}$, there can be two possible cases: (1) if $\int_{\underline{p}_d}^{r_f} F_d(p)dp \leq \int_{\underline{p}_f}^{r_f} F_f(p)dp = \int_{\underline{p}_f}^{r_d} F_f(p)dp$, then $r_d = r_f = r$; (2) if $\int_{\underline{p}_f}^{r_d} F_f(p)dp = \int_{\underline{p}_p}^{r_f} F_d(p)dp > \int_{\underline{p}_f}^{r_f} F_f(p)dp$, then $r_d > r_f$. I consider both and find that equilibrium exists only for the first case.

Case 1: $\boldsymbol{r_d = r_f = r}$

The proof proceeds as follows. First, I identify the mass point and use it to find the equilibrium profit of one retailer. Second, I locate the lower boundary of the price support and find the equilibrium price distribution as a function of the reservation price. Lastly, I solve the reservation price.

Step 1. In this case, a mass point exists at r of either $F_d(p)$ or $F_f(p)$ but not at both of them. This implies that at least one of the fringe's competitors (either the prominent or the other fringe retailer) does not have a mass point at r. Thus, the fringe retailer captures shoppers with zero probability at r. This gives us the fringe retailer's equilibrium profit as $E\pi_f = \pi_f(r) = \frac{(1-\mu)(1-\alpha_d)}{2}r$. In addition, the prominent retailer's competitors might have a mass point at r. Consequently, the prominent retailer captures shoppers either with positive or zero probability at r, this gives $E\pi_d = \pi_d(r) \geq (1-\mu)\alpha_d r$.

Step 2. Because $E\pi_f = \pi_f\left(\underline{p}_f\right)$, I can have $\underline{p}_f = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+2\mu}r$. Given that it is assured of getting the entire shopper segment, the lowest price that a prominent retailer is willing to charge is no higher than $\frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}r$, because $E\pi_p \geq (1-\mu)\alpha_d r$. This gives $\underline{p}_d > \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}r$. In addition, I have $\underline{p}_d > \underline{p}_f$ since $\underline{p}_f < \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}r$. It implies that $F_d(p) = 0$ for $p \in [\underline{p}_f, \underline{p}_d]$. This gives the fringe retailer's profit for $p \in [\underline{p}_f, \underline{p}_d]$ as

$$E\pi_f = \left\{\frac{(1-\mu)(1-\alpha_d)}{2} + \mu\left[1 - F_f(p)\right]\right\}p.$$

Substitute $E\pi_f = \frac{(1-\mu)(1-\alpha_d)}{2}r$ into it, I have $1 - F_f(p) = \frac{(1-\mu)(1-\alpha_d)}{2\mu}\left(\frac{r}{p} - 1\right)$ for $p \in [\underline{p}_f, \underline{p}_d]$.

Next, let's solve $\underline{p}_d$. Since $\pi_d\left(\underline{p}_d\right) = E\pi_d \geq (1-\mu)\alpha_d r$, I have:

$$\left\{(1-\mu)\alpha_d + \mu\left[1 - F_f\left(\underline{p}_d\right)\right]^2\right\}\underline{p}_d \geq (1-\mu)\alpha_d r.$$

Substitute $1 - F_f\left(\underline{p}_d\right) = \frac{(1-\mu)(1-\alpha_d)}{2\mu}\left(\frac{r}{\underline{p}_d} - 1\right)$ into it, I have

$$\frac{1}{\mu}\left[\frac{(1-\mu)(1-\alpha_d)}{2}\left(\frac{r}{\underline{p}_d} - 1\right)\right]^2 \geq (1-\mu)\alpha_d\left(\frac{r}{\underline{p}_d} - 1\right).$$

From it, I have either $\underline{p}_d = r$ or $\underline{p}_d \leq \frac{(1-\mu)(1-\alpha_d)^2}{4\alpha_d\mu+(1-\mu)(1-\alpha_d)^2} r$. However, $\frac{(1-\mu)\alpha_d}{\mu+(1-\mu)\alpha_d} r > \frac{(1-\mu)(1-\alpha_d)^2}{4\alpha_d\mu+(1-\mu)(1-\alpha_d)^2} r \geq \underline{p}_d$ for $\alpha_d > \frac{1}{3}$. This contradicts to $\underline{p}_d \geq \frac{(1-\mu)\alpha_d}{\mu+(1-\mu)\alpha_d} r$. Therefore, $\underline{p}_d = r$.

Step 3. I have $r = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}$ after substituting $F_f(p)$ and $\underline{p}_f$ into $c = \int_{\underline{p}_f}^{r} F_f(p)dp$.

Case 2: $\boldsymbol{r_d > r_f}$

In this case, $\int_{\underline{p}_d}^{r_f} F_d(p)dp > \int_{\underline{p}_f}^{r_f} F_f(p)dp$. Given the mass point property of asymmetric equilibrium, I have three possible subcases: (1) only $F_d(p)$ has a mass point at r_f; (2) $F_f(p)$ and $F_d(p)$ have a mass point at r_f and r_d, respectively; (3) only $F_f(p)$ has a mass point at r_f. I will show that no equilibrium holds for any of the above cases.

Subcase 2.1. I rule out this case by showing that it contradicts the definition of mixed strategy equilibrium. If a mass point exists at r_f of $F_d(p)$, then $F_f(p)$ cannot have a mass point at r_f. Consequently, the prominent retailer captures shoppers with zero probability at both r_f and r_p. Since $r_d < r_p$ I have $E\pi_d(r_f) = (1-\mu)\alpha_d r_f < E\pi_d(r_d) = (1-\mu)\alpha_d r_d$. This contradicts the definition of mixed strategy equilibrium. Hence, no equilibrium holds in this case.

Subcase 2.2. I show that there is a unilateral deviation in this case. If there exists a mass point at r_d of $F_d(p)$, then $F_d(r_f) < 1$ because $r_f < r_d$. If both fringes have mass points at r_f, then one of them is better off from shifting mass to a price slightly below r_f. This yields a discontinuous demand increase by not sharing shoppers with the competitors at r_f. Therefore, no equilibrium holds in this case.

Subcase 2.3. Following the same process in case 1, I can try to solve the equilibrium price distribution and reservation price. However, similar to case 1, I find that the prominent retailer plays a pure strategy at r_f in this case. However, this would lead to a unilateral deviation. Given both fringes have mass point at r_f and the prominent retailer plays a pure strategy at r_f, one fringe becomes strictly better off from shifting mass to a price slightly below r_f. This yields a discontinuous demand increase by not sharing shoppers with the competitors at r_f. Hence, not equilibrium holds in this case. I leave the detailed derivation of equilibrium in web appendix. The proof is the same as case 1 except that I replace r with r_f.

Proof of Proposition 5

In this section, I show that all retailer's average prices, Ep_d and Ep_f, decreases with α_d for any $\alpha_d > \frac{1}{3}$. Then I show that $E\pi_d$ decreases with α_d when μ is sufficiently low.

Step 1. Average Price.

First, I have $Ep_d = r = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]} = \frac{c}{1-A(\alpha_d)}$.

$\frac{\partial A(\alpha_d)}{\partial \alpha_d} = \frac{(1-\mu)}{(1-\mu)(1-\alpha_d)+2\mu} - \frac{(1-\mu)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right] < 0$, because $\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right] > \frac{2\mu}{(1-\mu)(1-\alpha_d)+2\mu}$. This proves that $\frac{\partial A(\alpha_d)}{\partial \alpha_d}$ and thus $\frac{\partial Ep_d}{\partial \alpha_d} < 0$.

Second, I have $Ep_f = \int_{\underline{p}}^{r} p\frac{dF_f(p)}{dp}dp = pF_f(p)\Big|_{\underline{p}}^{r} - \int_{\underline{p}}^{r} F_f(p)dp = r - c$. It yields $\frac{\partial Ep_f}{\partial \alpha_d} = \frac{\partial r}{\partial \alpha_d} < 0$.

Step 2. Equilibrium Profit $E\pi_d$.

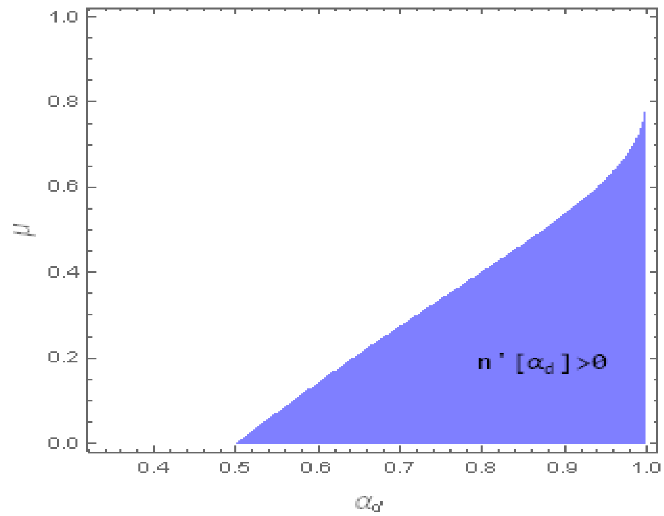
$E\pi_d = \frac{c}{\frac{1}{(1-\mu)\alpha_d}-\frac{(1-\alpha_d)}{2\alpha_d\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}$. Let $E\pi_d = \frac{c}{n(\alpha_d)}$, where $n(\alpha_d) = \frac{1}{(1-\mu)\alpha_d} - \frac{(1-\alpha_d)}{2\alpha_d\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]$.

Now let's discuss the monotonicity of $n(\alpha)$. It is easy to verify that $n(\alpha_d)' = -\frac{1+\mu}{\alpha_d^2(1-\mu)[(1-\mu)(1-\alpha_d)+2\mu]} + \frac{1}{2\mu\alpha_d^2}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]$. Unfortunately, there is no closed-form solution for $n(\alpha_d)' = -\frac{1+\mu}{\alpha_d^2(1-\mu)[(1-\mu)(1-\alpha_d)+2\mu]} > 0$.

I have to rely on the numerical results. I present the numerical result for $n(\alpha_d)'$ given any (α_d, μ) in figure 7. The shaded areas represent that $n(\alpha_d)' > 0$.

This proves that $\frac{\partial E\pi_d}{\partial \alpha_d} = -\frac{c}{n^2(\alpha_d)}n(\alpha_d)' < 0$ in the shaded area, where μ is sufficiently low compared with α_d.

Figure 7. Numerical Results for the Derivatives of Retailer Profit



Proof of Proposition 6

In this section, I compare retailer i's average price and equilibrium profit between limited and full consideration. I use superscript e to represent the full awareness case.

A6.1 Fringe Retailer's Price under Full and Limited Awareness $Ep_f < Ep_f^e$

Note that $Ep_f^e = c\{\frac{1}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]} - 1\}$.

<u>*Case 1.*</u> $\alpha_d < \frac{1}{2}$

In this case, $Ep_f = c\{\frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]} - 1\}$.

I show $Ep_f^e < Ep_f$ as follows. Let $w(x) = x\ln\left[1+\frac{1}{x}\right]$. I have $w'(x) = \ln\left[1+\frac{1}{x}\right] - \frac{1}{1+x} > 0$. This gives $w[\frac{(1-\mu)(1-\alpha_d)}{2\mu}] < \mathrm{w}[\frac{(1-\mu)(1-\alpha_d)}{\mu}]$. This proves $Ep_f^e < Ep_f$.

<u>*Case 2.*</u> $\alpha_d \geq \frac{1}{2}$

In this case, $Ep_f = c\{\frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]} - 1\}$.

I have $w\left(\frac{(1-\mu)(1-\alpha_d)}{2\mu}\right) < w(\frac{(1-\mu)\alpha_d}{\mu})$, given $\frac{(1-\mu)(1-\alpha_d)}{2\mu} < \frac{(1-\mu)\alpha_d}{\mu}$. This proves $Ep_f^e < Ep_f$.

A6.2 Prominent Retailer's Price under Full and Limited Awareness: Ep_d v.s. Ep_d^e

Note that $Ep_d^e = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}$.

Case 1. $\alpha_d < \frac{1}{2}$

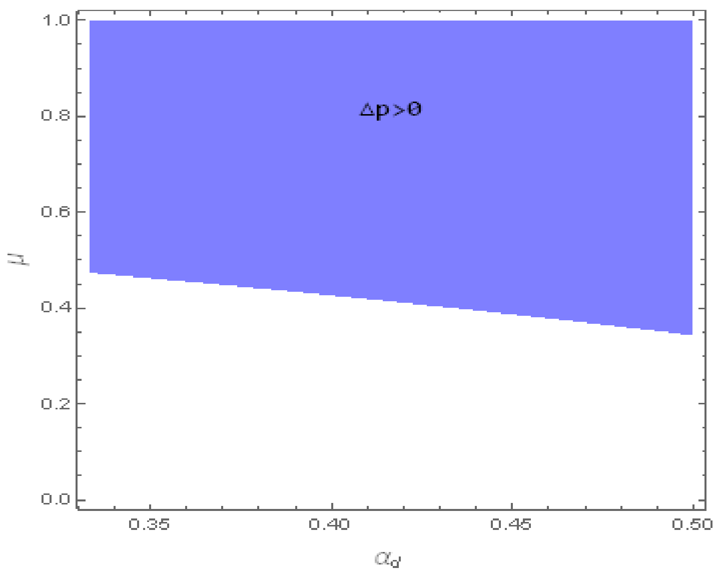
In this case, $p_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]} - \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}c$.

Let's denote the difference between two average prices as

$$\Delta p = Ep_d^e - Ep_d = c\left\{\frac{1}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]} - \frac{1}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]} + \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}\right\}.$$

From the numerical result shown in figure 8, I can verify that $\Delta p > 0$ in the shaded area, when μ is sufficiently large.

Figure 8. Numerical Results for the Prominent Retailer's Price Difference between Two Scenarios at Moderate Prominence Levels



This proves that $p_d^e > Ep_d$ when μ is sufficiently large.

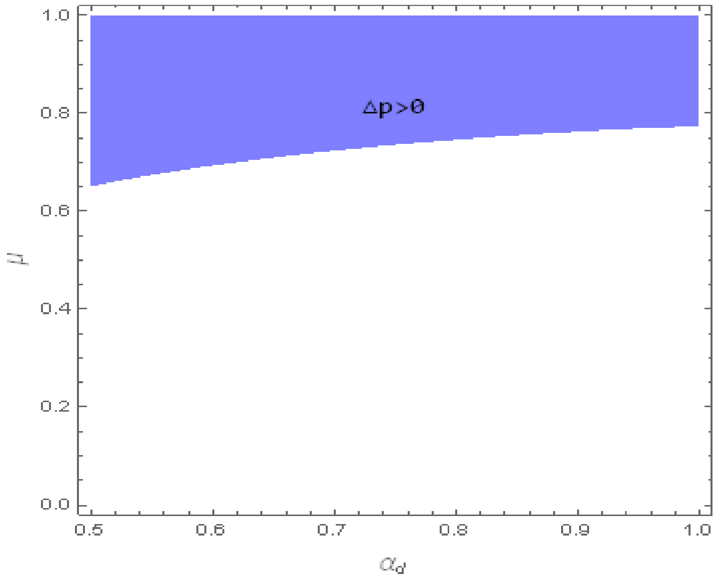
Case 2. $\alpha_d \geq \frac{1}{2}$

In this case, $Ep_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]} - \frac{(1-\mu)(1-\alpha_d)+\mu}{(1-\mu)\alpha_d+\mu}c$. This gives

$$\Delta p = Ep_d^e - Ep_d = c\left\{\frac{1}{1-\frac{(1-\mu)(1-\alpha_d)}{2\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]} - \frac{1}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]} + \frac{(1-\mu)\alpha_d+\mu}{(1-\mu)(1-\alpha_d)+\mu}\right\}.$$

From the numerical results in figure 9, I can verify that $\Delta p > 0$ in the shaded area, when μ is sufficiently large.

Figure 9. Numerical Results for the Prominent Retailer's Price Difference between Two Scenarios at High Prominence Levels



This proves that $Ep_d^e > Ep_d$ when μ is sufficiently large.

A6.3 *Fringe Retailer's Profit under Limited and Full awareness:* $E\pi_f > E\pi_f^e$

<u>*Case 1.*</u> $\alpha_d < \frac{1}{2}$

$\frac{E\pi_f}{E\pi_f^e} = \frac{\frac{2}{(1-\mu)(1-\alpha_d)} - \frac{1}{\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}{\frac{2}{(1-\mu)(1-\alpha_d)} - \frac{2}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]} = \frac{A_1}{B_1}$. I can verify that $A_1 > B_1 > 0$. This proves that $\frac{E\pi_f}{E\pi_f^e} = \frac{A_1}{B_1} > 1$. Thus, $E\pi_f > E\pi_f^e$ when $\alpha_d < \frac{1}{2}$.

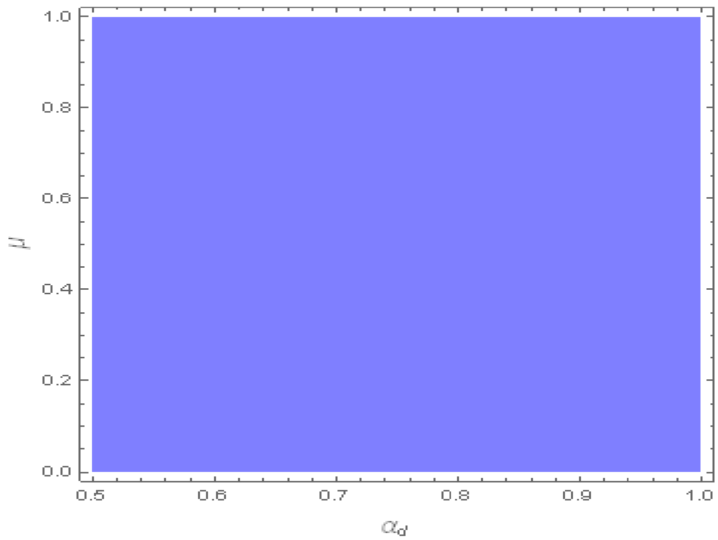
<u>*Case 2.*</u> $\alpha_d \geq \frac{1}{2}$

In this case, $\frac{E\pi_f}{E\pi_f^e} = \frac{1+\frac{\mu}{(1-\mu)(1-\alpha_d)} - \left[\frac{(1-\mu)(1-\alpha_d)+\mu}{2\mu}\right]\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}{1+\frac{\mu}{(1-\mu)\alpha_d} - \left[\frac{(1-\mu)\alpha_d+\mu}{\mu}\right]\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]} = \frac{A_2}{B_2}$.

Now let's discuss the relationship between A_2 and B_2. Figure 10 presents the numerical result for $\Delta = A_2 - B_2$ given (α_d, μ). The numerical results verify that $\Delta > 0$ for any (α_d, μ). This gives $A_2 > B_2$.

Furthermore, since $A_2 > 0$ and $B_2 > 0$, therefore $\frac{E\pi_f}{E\pi_f^e} = \frac{A_2}{B_2} > 1$. This shows $E\pi_f > E\pi_f^e$ when $\alpha_d \geq \frac{1}{2}$.

Figure 10. Numerical Results for the Fringe Retailer's Profit Difference between Two Scenarios at High Prominence Levels

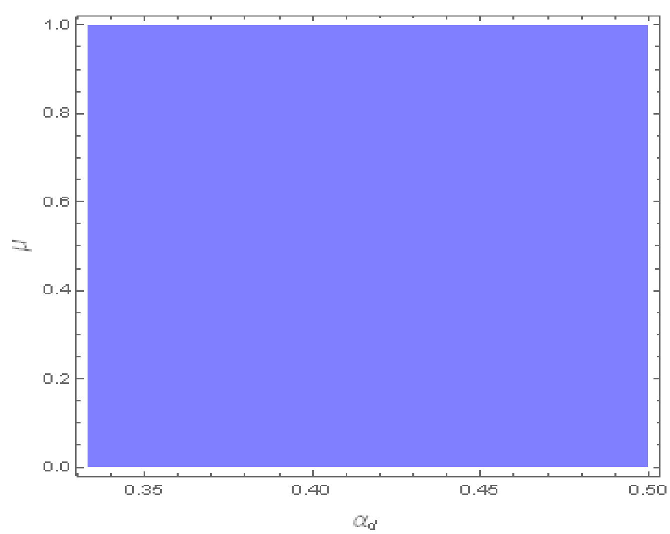


A6.4 Prominent Retailer's Profit under Full and Limited Awareness $E\pi_d > E\pi_d^e$

<u>*Case 1.*</u> $\alpha_d < \frac{1}{2}$

In this case $\frac{E\pi_d}{E\pi_d^e} = \frac{1+\frac{\mu}{(1-\mu)\alpha_d}-\frac{[(1-\mu)\alpha_d+\mu](1-\alpha_d)}{2\alpha_d\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}{1+\frac{\mu}{(1-\mu)(1-\alpha_d)}-[\frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}]\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]} = \frac{A_4}{B_4}$. Figure 11 presents the numerical results for $\Delta = A_4 - B_4$. It shows that $\Delta > 0$ for any (α_d, μ).

Figure 11. Numerical Results for the Prominent Retailer's Profit Difference between Two Scenarios at Moderate Prominence Levels



This shows that $A_4 > B_4$. This proves that $\frac{E\pi_d}{E\pi_d^e} = \frac{A_4}{B_4} > 1$ $A_4 > 0$ and $B_4 > 0$. Consequently, $E\pi_d > E\pi_d^e$.

Case 2. $\alpha_d \geq \frac{1}{2}$

In this case, $\frac{E\pi_d}{E\pi_d^e} = \frac{\frac{1}{(1-\mu)\alpha_d} - \frac{(1-\alpha_d)}{2\alpha_d\mu}\ln\left[1+\frac{2\mu}{(1-\mu)(1-\alpha_d)}\right]}{\frac{1}{(1-\mu)\alpha_d} - \frac{1}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]} = \frac{A_3}{B_3}$. It is easy to how that $A_3 > B_3 > 0$. This proves that $\frac{E\pi_d}{E\pi_d^e} = \frac{A_3}{B_3} > 1$ when $\alpha_d \geq \frac{1}{2}$.

Proof of Proposition 7

In this section, I extend the model to $n > 3$ retailers when consumers are aware of two of them. The same as assumption in the main model, I have $\beta_d = 1$ and $\alpha_d > \alpha_f$. In particular, $\alpha_d > \frac{1}{n}$.

Slightly different from the main model, I have $\alpha_f = \frac{1-\alpha_d}{n-1}$ and $\beta_f = \frac{1}{n-1}$. After substituting α_d, β_d, β_d, and β_f into (A4.1) and (A4.2), the rest of proof would be the same as the main model. I do not present the details proofs due to a limited space. Instead, I summarize the equilibrium outcome for $n > 3$ case as follows.

When $\boldsymbol{\alpha_d \geq \frac{1}{2}}$**,**

The equilibrium prices are as follows.

$F_d(p) = \frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}[1 - \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}\frac{r_d}{p}]$ and $F_f(p) = 1 - \frac{(1-\mu)\alpha_d}{\mu}(\frac{r_d}{p} - 1)$ $\quad p \in [\underline{p}, r_d]$,

where $r_d = \frac{c}{1-\frac{(1-\mu)\alpha_d}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]}$ and $\underline{p} = \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}r_d$.

The equilibrium profits are as follows.

$E\pi_d = \frac{c}{-\frac{1}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]+\frac{1}{(1-\mu)\alpha_d}}$ and $E\pi_f = \frac{(1-\mu)(1-\alpha_d)+\mu}{-\frac{(1-\mu)\alpha_d+\mu}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)\alpha_d}\right]+\frac{(1-\mu)\alpha_d+\mu}{(1-\mu)\alpha_d}}\frac{c}{n-1}$.

When $\boldsymbol{\alpha_d < \frac{1}{2}}$

The equilibrium prices are as follows.

$F_d(p) = 1 - \frac{(1-\mu)(1-\alpha_d)}{\mu}(\frac{r_f}{p} - 1)$, $F_f(p) = \frac{(1-\mu)\alpha_d+\mu}{\mu}[1 - \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}\frac{r_f}{p}]$, where $r_f = \frac{c}{1-\frac{(1-\mu)(1-\alpha_d)}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}$ and $\underline{p} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+\mu}r_f$. In addition, $p \in [\underline{p}, r_f]$.

The equilibrium profits are as follows.

$$E\pi_d = \frac{(1-\mu)\alpha_d+\mu}{1+\frac{\mu}{(1-\mu)(1-\alpha_d)}-[\frac{(1-\mu)(1-\alpha_d)+\mu}{\mu}]\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}c,\ E\pi_f = \frac{c}{\frac{n-1}{(1-\mu)(1-\alpha_d)}-\frac{n-1}{\mu}\ln\left[1+\frac{\mu}{(1-\mu)(1-\alpha_d)}\right]}.$$

Proof of Proposition 8

This section is interested in the case that there are $n > 3$ retailers and consumers are aware of $m > 2$ of them. Similar to the main model, I assume that the prominent retailer has the highest first search share and appears in all awareness sets, $\alpha_d > \frac{1}{n}$ and $\beta_d = 1$. While the symmetric fringe retailer has $\alpha_f = \frac{1-\alpha_d}{n-1} < \alpha_d$ and $\beta_f = \frac{m-1}{n-1}$.

Similar to the main model, each retailer plays a mixed strategy. Retailer i randomizes its price between $[\underline{p}_i, \overline{p}_i]$ following cumulative price distribution $F_i(p)$. Each retailer satisfies

$$E\pi_d = \left\{(1-\mu)\alpha_d + \mu\left[1-F_f(p)\right]^{m-1}\right\}p, \qquad \text{(A8.1)}$$

$$E\pi_f = \{(1-\mu)\frac{1-\alpha_d}{n-1} + \frac{m-1}{n-1}\mu[1-F_d(p)]\left[1-F_f(p)\right]^{m-2}\}p. \qquad \text{(A8.2)}$$

Each shopper searches $m \geq 3$ retailers. In this case, retailers play an oligopolistic competition. As such, the lower boundaries satisfy $\underline{p}_d \geq \underline{p}_f$.

The discussion on the upper boundary is similar to full awareness case with three retailers. No retailer prices higher than the reservation price. And the reservation price r_i satisfies

$$c = \int_{\underline{p}_f}^{r_d} F_f(p)dp = \max\left\{\int_{\underline{p}_d}^{r_f} F_d(p)dp, \int_{\underline{p}_2}^{r_f} F_f(p)dp\right\}.$$

If $\int_{\underline{p}_d}^{r_f} F_d(p)dp \leq \int_{\underline{p}_f}^{r_f} F_f(p)dp$, I have $r_d = r_f = r$. Otherwise, $r_d > r_f$. I consider both cases respectively. I find the equilibrium holds when $\alpha_d \geq \frac{1}{m}$ and $Ep_d \geq Ep_f$, while the equilibrium holds for $\alpha_d < \frac{1}{m}$ and $Ep_d < Ep_f$.

Case 1. $\boldsymbol{r_d = r_f = r}$

In this case, $\int_{\underline{p}_d}^{r_f} F_d(p)dp \leq \int_{\underline{p}_f}^{r_f} F_f(p)dp$. I solve the equilibrium in the following three steps. First, I identify the mass point. Second, I find the lower boundary of price support. Finally, I solve the equilibrium outcome and verify it exists when $\alpha_d \geq \frac{1}{m}$.

Step 1. In this case, a mass point exists at r of either $F_d(p)$ or $F_f(p)$. Therefore, at least one of the fringe retailer's competitors does not have a mass point at r. Thus, the fringe retailer captures shoppers with zero probability at r. This gives $E\pi_f = \pi_f(r) = \frac{(1-\mu)(1-\alpha_d)}{n-1}r$. In addition, the prominent retailer might capture shoppers with a positive probability at r, because its competitors are possible to have mass points at r. This gives $E\pi_d = \pi_d(r) \geq (1-\mu)\alpha_d r$.

Step 2. Similar to the full awareness under the main model, there can be two possible cases regarding the lower boundary: $\underline{p}_d = \underline{p}_f$ or $\underline{p}_d > \underline{p}_f$. I consider both.

Before exploring both cases, let's first figure out the lowest possible price p_i' that a retailer is willing to charge to be assured of getting the entire shopper segment. Given each retailer's expected profit, I have $\underline{p}_f{}' = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+(m-1)\mu}r$ and $\underline{p}_d' \geq \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}$.

<u>*Subcase 1.1.*</u> $\underline{p}_d = \underline{p}_f$

I rule out this case by contradiction.

If $\underline{p}_d = \underline{p}_f$, I must have $\underline{p}_f{}' > \underline{p}_d' \geq \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}$. This implies $\alpha_d \in (\frac{1}{n}, \frac{1}{m})$.

Also, given $\underline{p}_d = \underline{p}_f = \underline{p}_f' = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+(m-1)\mu}r$, this gives $E\pi_d = \pi_d\left(\underline{p}_d\right) = \frac{(1-\mu)(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)+(m-1)\mu}r$. Substitute $E\pi_d$ and $E\pi_f$ into (A1.1) and (A1.2), This gives

$$[1-F_d(p)][1-F_f(p)]^{m-2} = \frac{(1-\mu)(1-\alpha_d)}{(m-1)\mu}(\frac{r}{p}-1),$$

$$[1-F_f(p)]^{m-1} = \frac{(1-\mu)}{\mu}[\frac{(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)+(m-1)\mu}\frac{r}{p} - \alpha_d].$$

Combining the two, I have

$$\frac{1-F_d(p)}{1-F_f(p)} = \frac{\frac{(1-\alpha_d)}{(m-1)}(\frac{r}{p}-1)}{\frac{(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)+(m-1)\mu}\frac{r}{p}-\alpha_d} = \frac{A}{B}.$$

However, I find that $A - B = (m\alpha_d - 1)\{1 - \frac{(1-\mu)(1-\alpha_d)}{[(1-\mu)(1-\alpha_d)+\mu(m-1)]}\frac{r}{p}\} < 0$ when $\alpha_d < \frac{1}{m}$. This suggest that $F_d(p) > F_f(p)$, which contradicts to $\int_{\underline{p}_d}^{r_f} F_d(p)dp \leq \int_{\underline{p}_2}^{r_f} F_f(p)dp$. Consequently, there is no equilibrium when $\underline{p}_d = \underline{p}_f$.

<u>*Subcase 1.2.*</u> $\underline{p}_d > \underline{p}_f$

I show that equilibrium exists when $\alpha_d \geq \frac{1}{m}$.

From the previous case, I know that if $\alpha_d < \frac{1}{m}$, then $\underline{p_f}' > \underline{p}_d' \geq \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}$. This contradicts to $\underline{p_d} > \underline{p}_d' > \underline{p_f} = \underline{p}_f'$. Thus, I must have $\alpha_d \geq \frac{1}{m}$ in this case.

Also, $\underline{p_d} > \underline{p_f}$ implies that $F_d(p) = 0$ for $p \in [\underline{p_f}, \underline{p_d}]$. This gives the fringe retailer's profit for $p \in [\underline{p_f}, \underline{p_d}]$ as

$$E\pi_f = \left\{\frac{(1-\mu)(1-\alpha_d)}{n-1} + \frac{m-1}{n-1}\mu\left[1 - F_f(p)\right]^{m-2}\right\} p.$$

Substitute $E\pi_f = \frac{(1-\mu)(1-\alpha_d)}{n-1} r$ into it, I have $1 - F_f(p) = \left[\frac{(1-\mu)(1-\alpha_d)}{(m-1)\mu}\left(\frac{r}{p} - 1\right)\right]^{\frac{1}{m-2}}$ for $p \in [\underline{p_f}, \underline{p_d}]$.

Next, let's solve $\underline{p_d}$. Since $\pi_d\left(\underline{p_d}\right) = E\pi_d \geq (1-\mu)\alpha_d r$, I have:

$$\{(1-\mu)\alpha_d + \mu[1 - F_2(p)]^{m-1}\}\underline{p_d} \geq (1-\mu)\alpha_d r.$$

Substitute $1 - F_f(p) = \left[\frac{(1-\mu)(1-\alpha_d)}{(m-1)\mu}\left(\frac{r}{p} - 1\right)\right]^{\frac{1}{m-2}}$ into it, I have

$$\mu\left[\frac{(1-\mu)(1-\alpha_d)}{(m-1)\mu}\left(\frac{r}{\underline{p_d}} - 1\right)\right]^{\frac{m-1}{m-2}} \geq (1-\mu)\alpha_d\left(\frac{r}{\underline{p_d}} - 1\right).$$

From it, I have either $\underline{p_d} = r$ or $\underline{p_d} \leq \frac{(1-\mu)(1-\alpha_d)^{m-1}}{(m-1)^{m-1}\alpha_d^{m-2}\mu + (1-\mu)(1-\alpha_d)^{m-1}} r$. However, I can verify that $\underline{p}_d' \geq \frac{(1-\mu)\alpha_d}{\mu+(1-\mu)\alpha_d} r \geq \frac{(1-\mu)(1-\alpha_d)^{m-1}}{(m-1)^{m-1}\alpha_d^{m-2}\mu+(1-\mu)(1-\alpha_d)^{m-1}} r \geq \underline{p_d}$ for $\alpha_d \geq \frac{1}{m}$. This contradicts to $\underline{p_d} \geq \underline{p}_d'$. Therefore, I must have $\underline{p_d} = r$.This suggests that the prominent retailer plays a pure strategy at r in equilibrium.

Step 3. However, I cannot have closed form solution for r after substitute $F_f(p)$ and $\underline{p_f}$ into $c = \int_{\underline{p_f}}^{r} F_f(p)dp = \int_{\frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+(m-1)\mu}r}^{r} \{1 - \left[\frac{(1-\mu)(1-\alpha_d)}{(m-1)\mu}\left(\frac{r}{p} - 1\right)\right]^{\frac{1}{m-2}}\}dp$.

I can verify that $Ep_d \geq Ep_f$, because $p_d = r \geq Ep_f$, where $p_f \in [\underline{p_f}, r]$.

Case 2. $\boldsymbol{r_d > r_f}$

In this case, $\int_{\underline{p_d}}^{r_f} F_d(p)dp > \int_{\underline{p_f}}^{r_f} F_f(p)dp$. There are three possible cases regarding the mass points: (1) only $F_d(p)$ has a mass point at r_f; (2) $F_f(p)$ and $F_d(p)$ each have a mass point at r_f and r_d, respectively; (3) only $F_f(p)$ has a mass point at r_f.

Subcase 2.1. I show that this case contradicts the definition of mixed strategy equilibrium. If a mass point exists at r_f of $F_d(p)$, then $F_f(p)$ cannot have a mass point at r_f. Consequently, the prominent retailer captures shoppers with zero probability at both r_f and r_d. Since $r_f < r_d$, I have $E\pi_d(r_f) = (1-\mu)\alpha_d r_f < E\pi_d(r_d) = (1-\mu)\alpha_d r_d$. This contradicts the definition of the mixed strategy equilibrium. Hence, no equilibrium holds in this case.

Subcase 2.2. I show that there is a unilateral deviation in this case. If there exists a mass point at r_d of $F_d(p)$, then $F_d(r_f) < 1$ because $r_f < r_d$. If both fringes have mass points at r_f, then one of them is better off from shifting mass to a price slightly below r_f. This yields a discontinuous demand increase by not sharing shoppers with the competitors at r_f. Therefore, no equilibrium holds in this case.

Subcase 2.3. If only $F_f(p)$ has a mass point at r_f, it implies that $F_d(p)$ has no mass point. As such, under $p = r_f$, the fringe retailer acquires shoppers with a zero probability, while the prominent one acquires shoppers with positive probability. This gives $E\pi_f = \pi_f(r_f) = \frac{(1-\mu)(1-\alpha_d)}{(n-1)} r_f$ and $E\pi_d = \pi_d(r_d) > (1-\mu)\alpha_d r_f$. In addition, given that $F_d(p)$ has no mass point and it has zero probability for $p \in (r_f, r_d)$, the upper boundary of its price support is r_f.

Now let's consider the lower bound $\underline{p_i}$. There can be two possible cases: $\underline{p_d} = \underline{p_f}$ or $\underline{p_d} > \underline{p_f}$. I consider both.

Before exploring each case, let's first establish the lower price $p_i{}'$ that a retailer is willing to charge to be assured to get shoppers with probability one. Given the expected profit of each retailer, I have $\underline{p_d'} > \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu} r$ and $\underline{p_f'} = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+(m-1)\mu} r$.

Sub-subcase 2.3.1. $\underline{p_d} > \underline{p_f}$

I will rule out this case by contradiction.

Similar to case 1.2, $\underline{p_d} > \underline{p_f}$ implies that $\alpha_d \geq \frac{1}{m}$ and $F_d(p) = 0$ for $p \in [\underline{p_f}, \underline{p_d}]$. Following the same procedure in case 1.2, I can first solve $F_f(p)$ for $p \in [\underline{p_f}, \underline{p_d}]$, then I can find that $\underline{p_d} = r$, which implies that the prominent retailer charges r with probability one. This result suggests that the prominent retailer also has a mass point. This contradicts to this case's condition such that $F_d(p)$ has no mass point.

Sub-subcase 2.3.2. $\underline{p}_d = \underline{p}_f$

I show that equilibrium exists when $\alpha_d < \frac{1}{m}$ and $Ep_d < Ep_f$.

If $\underline{p}_d = \underline{p}_f$, I must have $\underline{p}_f{}' > \underline{p}_d' \geq \frac{(1-\mu)\alpha_d}{(1-\mu)\alpha_d+\mu}$. This implies that $\alpha_d < \frac{1}{m}$.

Also, given $\underline{p}_d = \underline{p}_f = \underline{p}_f' = \frac{(1-\mu)(1-\alpha_d)}{(1-\mu)(1-\alpha_d)+(m-1)\mu} r = \underline{p}$, I have $E\pi_d = \pi_d\left(\underline{p}_d\right) = \frac{(1-\mu)(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)+(m-1)\mu} r$. Substitute $E\pi_d$ and $E\pi_f$ into (A1.1) and (A1.2), this gives

$$[1 - F_d(p)]\left[1 - F_f(p)\right]^{m-2} = \frac{(1-\mu)(1-\alpha_d)}{(m-1)\mu}\left(\frac{r}{p} - 1\right),$$

$$\left[1 - F_f(p)\right]^{m-1} = \frac{(1-\mu)}{\mu}\left[\frac{(1-\alpha_d)[(1-\mu)\alpha_d+\mu]}{(1-\mu)(1-\alpha_d)+(m-1)\mu}\frac{r}{p} - \alpha_1\right].$$

Combining the two, I have

$$\frac{1-F_d(p)}{1-F_f(p)} = \frac{\frac{(1-\alpha_d)}{(m-1)}\left(\frac{r}{p}-1\right)}{\frac{(1-\alpha_d)[(1-\mu)\alpha_d+\mu]\,r}{(1-\mu)(1-\alpha_d)+(m-1)\mu p} - \alpha_d} = \frac{A}{B}.$$

If $\alpha_d < \frac{1}{m}$, I can verify that $A - B = (m\alpha_d - 1)\{1 - \frac{(1-\mu)(1-\alpha_d)}{[(1-\mu)(1-\alpha_d)+\mu(m-1)]}\frac{r}{p}\} < 0$. It implies that $F_d(p) > F_f(p)$ for $p \in [\underline{p}, r_f]$. This proves that $Ep_d < Ep_f$.

Appendix B: Proofs in Essay 2

The appendix starts with the proof of Lemma 2.

Proof of Lemma 2.

In this section, I prove that $p_H^s \notin (\underline{p}, \bar{p})$ when $m \leq m_3(q)$ by showing the low-quality retailer is profitable to mimic the high-quality one at $p \in (\underline{p}(m,q), \bar{p}(m,q))$.

When a low-quality retailer engages in mimicry, it earns an extra one-period profit at the cost of its reputation. I use intuitive criterion (Cho and Kreps 1987) to define the profitable mimicry: it occurs when the highest possible mimicry profit is higher than the separating profit, $\frac{1}{16}$. The highest mimicry profit at p for the retailer is when consumer believes that the product is of high quality, $\hat{\phi}(\hat{q}, p) = 1$. Given $w_L = \frac{1}{2}$, this gives the highest mimicry profit as $\left(p - \frac{1}{2}\right)\left(1 - \frac{p}{q}\right) - m$. Thus, mimicry is profitable at price p if

$$\left(p - \frac{1}{2}\right)\left(1 - \frac{p}{q_H}\right) - m > \frac{1}{16}.$$

It holds for $p \in (\underline{p}(m,q), \bar{p}(m,q))$, where $\underline{p}(m,q) = \frac{q}{2} + \frac{1}{4} - \frac{1}{2}\Delta$, $\bar{p}(m,q) = \frac{q}{2} + \frac{1}{4} + \frac{1}{2}\Delta$, and $\Delta \equiv \sqrt{-4mq + q^2 - \frac{5}{4}q + \frac{1}{4}}$. Note that Δ exists, $-4mq + q^2 - \frac{5}{4}q + \frac{1}{4} \geq 0$, when $m \leq m_3(q) = \frac{q}{4} + \frac{1}{16q} - \frac{5}{16}$.

This proves that $p_H^s \notin (\underline{p}, \bar{p})$ when $m \leq m_3(q)$.

Proof of Proposition 1.

In this section, I use backward induction to solve the separating equilibrium price. First, I solve the optimal separating retail price in response to the wholesale price, $p_H(w_H)$. Next, I substitute it into the manufacturer's profit function to derive the equilibrium prices.

Optimal Separating Retail Price in Response to Wholesale Price

There are two cases, consider each in turn.

<u>Case 1.</u> $m > m_3$

In this case, $(\underline{p}, \bar{p})$ does not exist. The optimal separating retail price given wholesale price maximizes $\pi_r = (p - w_H)D(p)$. This gives $p_H(w_H) = \frac{w_H+q}{2}$, which is the same as that under the perfect information.

<u>*Case 2.*</u> If $m \le m_3$

In this case, $\left(\underline{p}, \bar{p}\right)$ exists.

For $\frac{q_H+w_H}{2} \notin \left(\underline{p}, \bar{p}\right)$, $p_H(w_H) = \frac{w_H+q}{2}$.

However, for $\frac{w_H+q}{2} \in \left(\underline{p}, \bar{p}\right)$, that is, $w_H \in (\frac{1}{2} - \Delta, \frac{1}{2} + \Delta)$, $p_H(w_H) \neq \frac{w_H+q}{2}$, because $p_H^s \notin (\underline{p}, \bar{p})$. Instead, for $w_H \in (\frac{1}{2} - \Delta, \frac{1}{2} + \Delta)$, the retailer would charge a price p such that (1) $p \notin \left(\underline{p}, \bar{p}\right)$ and (2) $p = \arg\{\min |p - \frac{w_H+q}{2}|\}$. This gives $p_H(w_H) = \overline{p}$ for $(\frac{1}{2}, \frac{1}{2} + \Delta)$ and $p_H(w_H) = \underline{p}$ for $(\underline{w}, \frac{1}{2}]$[13], where $\underline{w} = max\{0, \frac{1}{2} - \Delta\}$ to make sure that the wholesale price is non-negative.

To sum up, when $m > m_3$, $p_H(w_H) = \frac{w_H+q}{2}$. When $m \le m_3$, I have

$$p_H(w_H) = \begin{cases} \underline{p}, \ w_H \in (\underline{w}, \frac{1}{2}] \\ \overline{p}, \ w_H \in (\frac{1}{2}, \frac{1}{2} + \Delta) \\ \frac{w_H+q_H}{2}, \ othewise. \end{cases}$$

Solving Equilibrium Prices

Substituting $p_H(w_H)$ into the manufacturer's profit maximization problem:

$$\max_{w_H}: \Pi_H = w_H D\big(p_H(w_H)\big). \tag{B1.1}$$

From (B1.1), I solve the equilibrium wholesale price w_H^s. Then I can derive the corresponding retail price p_H^s. Note that there are two cases, consider each case in turn.

<u>*Case 1.*</u> $m > m_3$

Given $p_H(w_H) = \frac{w_H+q}{2}$, (B1.1) becomes $\frac{w_H(q-w_H)}{2q}$. Solving through F.O.C., I have $w_H^s = \frac{q}{2}$ and $p_H^s = \frac{3}{4}q$. Note that these are the same as prices under the perfect information.

<u>*Case 2.*</u> $m \le m_3$

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[13] I assume that the retailer charges $p_H\left(w_H = \frac{1}{2}\right) = \underline{p}$ under the break-even condition. It would not change the result qualitatively.

In this case, $p_H(w_H)$ is not continuous. There are three separate regions, $w_H \in (\underline{w}, \frac{1}{2}]$, $w_H \in (\frac{1}{2}, \frac{1}{2}+\Delta)$, and $w_H \notin (\underline{w}, \frac{1}{2}+\Delta)$. I solve w_H^S as follows. First, I find "locally optimal prices" by solving the wholesale price that yields the highest manufacturer's profit within each region. Next, I compare the "locally optimal prices" across regions and select the one that leads to the highest manufacturer profit.

Step 1. *Finding "Locally Optimal Prices"*

First, for $w_H \in (\underline{w}, \frac{1}{2}]$, $p_H(w_H) = \underline{p}$. Therefore, (B1.1) becomes $w_H D(\underline{p})$, which monotonically increases with w_H. Consequently, $\frac{1}{2}$ is the "locally optimal" price for $w_H \in (\underline{w}, \frac{1}{2}]$.

Second, for $w_H \in (\frac{1}{2}, \frac{1}{2}+\Delta)$, $p_H(w_H) = \overline{p}$. As such, (B1.1) becomes $w_H D(\overline{p})$, which monotonically increases with w_H. Thus, $\frac{1}{2}+\Delta-\epsilon$, where $\epsilon \to 0$, is the "locally optimal" price for $w_H \in (\frac{1}{2}, \frac{1}{2}+\Delta)$.

Lastly, for $w_H \notin (\underline{w}, \frac{1}{2}+\Delta)$, (B1.1) becomes $\frac{w_H(q-w_H)}{2q}$. The manufacturer's profit-maximizing problem becomes

$$\max_{w_H}: \Pi_H = \frac{w_H(q-w_H)}{2q}$$

$$s.t.: w_H \geq \frac{1}{2}+\Delta \text{ or } w_H \leq \underline{w}$$

From it, I have the "locally optimal price" as $\frac{q_H}{2}$ and its corresponding retail price as $\frac{3}{4}q_H$ if $m > m_2(q) = \frac{3}{16}(q-1)$; whereas, the "local optimal price" is $\frac{1}{2}+\Delta$ and its corresponding retail price is $\overline{p}$ if $m \leq m_2$.

Step 2. *Comparing "Locally Optimal Prices"*

When $m > m_2$, let's compare $w_H = \frac{q}{2}, \frac{1}{2}$, and $\frac{1}{2}+\Delta-\epsilon$. I find that $w_H = \frac{q}{2}$ yields the highest manufacturer profit as follows.

$$\Pi_H\left(\frac{q}{2}, \frac{3}{4}q\right) - \Pi_H\left(\frac{1}{2}, \underline{p}\right) = \frac{1}{8q}\left[(q-1)^2 - \sqrt{(2q-1)^2 - q_H - 16qm}\right] > 0,$$

$$\Pi_H\left(\frac{q}{2}, \frac{3}{4}q\right) - \Pi_H\left(\frac{1}{2}+\Delta-\epsilon, \overline{p}\right)$$

$$> \Pi_H\left(\frac{q}{2}, \frac{3}{4}q\right) - \Pi_H\left(\frac{1}{2}+\Delta, \overline{p}\right) = \frac{1}{8q}\left[(q-1)^2 + 4\Delta(\Delta+q+1)\right] > 0.$$

As such, $w_H^s = \frac{q}{2}$ and $p_H^s = \frac{3}{4}q$ for $m \in (m_1, m_2]$

When $m \le m_2$, let's compare $w_H = \frac{1}{2} + \Delta, \frac{1}{2}$, and $\frac{1}{2} + \Delta - \epsilon$. First, $\Pi_H\left(\frac{1}{2} + \Delta, \bar{p}\right)$-$\Pi_H\left(\frac{1}{2} + \Delta - \epsilon, \bar{p}\right) > 0$, this leaves only $\frac{1}{2} + \Delta$ and $\frac{1}{2}$ as two possible candidates. I have

$$\Delta\Pi_H = \Pi_H\left(\frac{1}{2}, \underline{p}\right) - \Pi_H\left(\frac{1}{2} + \Delta, \bar{p}\right) = \frac{\Delta}{q}\left(\frac{\Delta}{2} + \frac{3}{4} - \frac{q}{2}\right).$$

$\Delta\pi_H \ge 0$ if and only if $\sqrt{-4mq + q^2 - \frac{5}{4}q + \frac{1}{4}} \ge q - \frac{3}{2}$. Solving this inequality, I have $\Delta\pi_H \ge 0$ if $\{m \le \frac{7}{16} - \frac{1}{2q} \& q \ge \frac{8}{7}\}$ or $q \le \frac{3}{2}$. Note that $m \le m_2$ as well. This gives $w_H^s = \frac{1}{2}$ and $p_H^s = \underline{p}$ when $m \le m_1(q)$, where $m_1(q) = \frac{7}{16} - \frac{1}{2q}$ when $q \ge 2$ and $m_1(q) = \frac{3}{16}(q-1)$ if $q < 2$. When $m > m_1(q)$, $\Delta\Pi_H < 0$, $w_H^s = \frac{1}{2} + \Delta$ and $p_H^s = \bar{p}$.

To summary, I have equilibrium wholesale and retail prices as follows

$$(w_H^s, p_H^s) = \begin{cases} (\frac{1}{2}, \underline{p}),\ m \le m_1(q) \\ (\frac{1}{2} + \Delta, \overline{p}),\ m \in (m_1(q), m_2(q)] \\ (\frac{q}{2}, \frac{3}{4}q),\ m > m_2(q). \end{cases}$$

Proof of Corollary 1.

This section shows the relationship between retailer's price and its reputation under separating equilibrium by presenting $\frac{\partial p_H^s}{\partial m}$. First, I have $\frac{\partial \Delta}{\partial m} < 0$, given $\Delta = \sqrt{-4mq + q^2 - \frac{5}{4}q + \frac{1}{4}}$.

When $m \le m_1(q)$, $\frac{\partial p_H^s}{\partial m} = \frac{\partial(\frac{q}{2} + \frac{1}{4} - \frac{1}{2}\Delta)}{\partial m} = -\frac{1}{2}\frac{\partial \Delta}{\partial m}$. This gives $\frac{\partial p_H^s}{\partial m} > 0$.

When $m \in (m_1(q), m_2(q))$, I have $\frac{\partial p_H^s}{\partial m} = \frac{\partial(\frac{q}{2} + \frac{1}{4} + \frac{1}{2}\Delta)}{\partial m} = \frac{1}{2}\frac{\partial \Delta}{\partial m} < 0$.

When $m > m_2(q)$, $\frac{\partial p_H^s}{\partial m} = \frac{\partial(\frac{3}{4}q)}{\partial m} = 0$.

Proof of Proposition 2

In this section, I derive a sufficient condition for the separating equilibrium to be the only equilibrium that survives intuitive criterion. The sufficient condition is that for *any* wholesale price, the retailer's optimal price in response is a separating price, $p_H(w_H) \notin \left(\underline{p}, \bar{p}\right)$. I show that it occurs when the consumer's prior quality belief ϕ is sufficiently low.

I have shown that $p_H(w_H) = \frac{q+w_H}{2} \notin \left(\underline{p}, \bar{p}\right)$ for $w_H \notin (\underline{w}, \frac{1}{2} + \Delta)$ in the previous section. Now I need to show that $p_H(w_H) \notin \left(\underline{p}, \bar{p}\right)$ for $w_H \in (\underline{w}, \frac{1}{2} + \Delta)$ when $\phi < \phi_1 = \frac{1}{q-1}[2m - \frac{3}{8} + \sqrt{\frac{9}{64} + 4m^2 + \frac{5}{2}m}]$.

First, I show that $p_H(w_H) = \underline{p}$ for any $w_H \in (\underline{w}, \frac{1}{2}]$ when $\phi < \phi_1$ as follows. It is equivalent to show that for any $w_H \in (\underline{w}, \frac{1}{2}]$,

$$\Delta\pi_r = \pi_r\left(w_H, \underline{p}\right) - \max_{p \in \left(\underline{p}, \bar{p}\right)} \pi_r(w_H, p) \geq 0. \quad \text{(B2)}$$

Since $p \in \left(\underline{p}, \bar{p}\right)$, $\hat{\phi}(\hat{q}|p) \leq \phi$ and thus $\pi_r(w_H, p) \leq (p - w_H)[1 - \frac{p}{\phi q + (1-\phi)}]$. Thus, a sufficient condition for (B2) is

$$\Delta\pi_r = \left(\underline{p} - w_H\right)\left(1 - \frac{\underline{p}}{q}\right) - \max_{p \in \left(\underline{p}, \bar{p}\right)} \left\{(p - w_H)\left[1 - \frac{p}{\phi q + (1-\phi)}\right]\right\} \geq 0. \quad \text{(B2.1)}$$

There are two possible cases, consider both in turn.

Case 1. $\frac{w_H}{2} + \frac{\phi q + (1-\phi)}{2} \in \left(\underline{p}, \bar{p}\right)$

In this case, $p_H(w_H) = \frac{w_H}{2} + \frac{\phi q + (1-\phi)}{2}$. Therefore, (B2.2) becomes

$$\Delta\pi_r = \left(\underline{p} - w_H\right)\left(1 - \frac{\underline{p}}{q}\right) - \frac{[\phi q + (1-\phi) - w_H]^2}{4[\phi q + (1-\phi)]} \geq 0. \quad \text{(B2.2)}$$

We can show that $\frac{\partial \Delta\pi_r}{\partial w_H} < 0$ as follows. First, in this case, the wholesale price must satisfy $w_H > (1-\phi)(q-1) + \frac{1}{2} - \Delta$. I can show that $\frac{\partial \Delta\pi_r}{\partial w_H} < 0$ as follows.

$$\frac{\partial \Delta\pi_r}{\partial w_H} = \frac{\frac{1}{2} - \Delta}{2q} - \frac{w_H}{2[\phi q + (1-\phi)]} < \frac{\frac{1}{2} - \Delta}{2q} - \frac{(1-\phi)(q-1) + \frac{1}{2} - \Delta}{2[\phi q + (1-\phi)]} = -\frac{(1-\phi)(q-1)}{2q[\phi q + (1-\phi)]}\ (q - \frac{1}{2} + \Delta) < 0.$$

Since $\frac{\partial \Delta\pi_r}{\partial w_H} < 0$, a sufficient condition for (B2.2) to hold for any $\{w_H \in \left(\underline{w}, \frac{1}{2}\right]$ & $w_H > (1-\phi)(q-1) + \frac{1}{2} - \Delta\}$ is that it holds for $w_H = \frac{1}{2}$. This occurs when $\phi < \phi_1 = \frac{1}{q-1}[-\frac{3}{8} + 2m + \sqrt{\frac{9}{64} + 4m^2 + \frac{5}{2}m}]$.

Case 2. $\frac{w_H}{2} + \frac{\phi q + (1-\phi)}{2} \notin \left(\underline{p}, \bar{p}\right)$

In this case, $p_H(w_H) = \underline{p} + \epsilon$, because $\underline{p} + \epsilon = \arg\max |p - \frac{w_H}{2} - \frac{\phi q + (1-\phi)}{2}|$ for $p \in \left(\underline{p}, \bar{p}\right)$.

As such, I show that $\Delta\pi_r > 0$ as follows. $\Delta\pi_r = \lim_{\epsilon\to 0}[\pi_r\left(w_H, \underline{p}\right) - \pi_r\left(w_H, \underline{p} + \epsilon\right)] = \frac{(1-\phi)(q-1)\left(\underline{p}-w_H\right)}{q[\phi q+(1-\phi)]} > 0$. Thus, (B2) always holds in this case.

The above proves that $p_H(w_H) = \underline{p}$ for any $w_H \in (\underline{w}, \frac{1}{2}]$ if $\phi < \phi_1$. Due to the symmetry, it is to verify that $p_H(w_H) = \overline{p}$ for $w_H \in (\frac{1}{2}, \frac{1}{2} + \Delta)$.

As a summary, $p_H(w_H) = \frac{q+w_H}{2} \notin \left(\underline{p}, \bar{p}\right)$ for any w_H if $\phi < \phi_1$. This proves that the separating equilibrium would be the only equilibrium to survive intuitive criterion.

Proof of Proposition 3.

In this section, I first present $\frac{\partial \pi_r^s}{\partial m}$ and then find the reputation that yields the highest retailer profit.

<u>*Relationship between Retailer Profit and Reputation*</u> $\frac{\partial \pi_r^s}{\partial m}$.

First, I calculate the retailer's profit as a function of reputation under different scenarios. When $m \le m_1$, $\pi_r^s = \pi_r\left(\frac{1}{2}, \underline{p}\right) = \frac{1}{16} + m$. When $m \in (m_1, m_2]$, $\pi_r^s = \pi_r\left(\frac{1}{2} + \Delta, \overline{p}\right) = \frac{\left(q-\frac{1}{2}-\Delta\right)^2}{4q}$. When $m > m_2$, $\pi_r^s = \pi_r\left(\frac{q}{2}, \frac{3}{4}q\right) = \frac{q}{16}$.

Next, I can verify that $\frac{\partial \pi_r^s}{\partial m} > 0$ for $m < m_1$ and $m \in (m_1, m_2]$ and $\frac{\partial \pi_r^s}{\partial m} = 0$ for $m > m_2$. Now, let's focus on the discontinuity point $m = m_1$. I prove $\frac{\partial \pi_r^s}{\partial m} < 0$ at m_1 by showing that $\lim_{\epsilon\to 0}\{\pi_r^s(m_1 + \epsilon) - \pi_r^s(m_1)\} < 0$ where $\epsilon \to 0$ as follows. Note that there are two possible scenarios for $m_1(q)$. Let's consider each in turn

When $q < 2$, $m_1(q) = \frac{3}{16}(q-1)$. Therefore, I have $\pi_r^s(m_1) = \frac{1}{16} + m_1 = \frac{3}{16}q - \frac{1}{8}$ and $\pi_r^s(m_1 + \epsilon) = \frac{1}{4q}\left(q - \frac{1}{2} - \Delta\right)^2 = \frac{1}{4q}\left(q - \frac{1}{2} - \sqrt{\frac{1}{4}(q-1)^2 - 4q\epsilon}\right)^2$. This gives $\lim_{\epsilon\to 0}\{\pi_r^s(m_1 + \epsilon) - \pi_r^s(m_1)\} = \frac{(1-q)}{8} < 0$.

When $q \geq 2$, $m_1(q) = \frac{7}{16} - \frac{1}{2q}$. Thus, $\pi_r^s(m_1) = \frac{1}{16} + m_1 = \frac{1}{2} - \frac{1}{2q}$ and $\pi_r^s(m_1 + \epsilon) = \frac{1}{4q}\left(q - \frac{1}{2} - \Delta\right)^2 = \frac{1}{4q}\left(q - \frac{1}{2} - \sqrt{\left(q - \frac{3}{2}\right)^2 - 4q\epsilon}\right)^2$. This gives $\lim_{\epsilon \to 0}\{\pi_r^s(m_1 + \epsilon) - \pi_r^s(m_1)\} = \frac{3}{4q} - \frac{1}{2} < 0$ when $q \geq 2$.

The above proves that the retailer's profit decreases with its reputation at $m = m_1$.

<u>Highest Retailer Profit.</u>

Now I find the reputation that yields the highest retailer's profit given q.

In the previous section, I have shown that $\frac{\partial \pi_r^s}{\partial m} > 0$ for $m < m_1$ and $m \in (m_1, m_2]$, $\frac{\partial \pi_r^s}{\partial m} = 0$ for $m > m_2$, and $\frac{\partial \pi_r^s}{\partial m} < 0$ at $m = m_1$, it implies that π_r^s is the highest under m_1 for $m \leq m_1$ and under $m \geq m_2$ for $m > m_1$. Thus, π_r^s is the highest profit for either m_1 or m_2. Let's compare them.

First, I have $\pi_r^s(m_2) = \frac{q}{16}$.

Note that there are two possible cases for $m_1(q)$.

First, when $q < 2$, $m_1 = \frac{3}{16}(q-1)$, $\pi_r^s(m_1) = \frac{1}{16} + m_1 = \frac{3}{16}q - \frac{1}{8}$. The profit of retailer is higher at m_1: $\pi_r^s(m_1) - \pi_r^s(m_2) = \frac{q-1}{8} > 0$.

Second, when $q \geq 2$, $m_1(q) = \frac{7}{16} - \frac{1}{2q}$. $\pi_r^s(m_1) = \frac{1}{16} + m_1 = \frac{1}{2} - \frac{1}{2q}$. $\Delta\pi = \pi_r^s(m_1) - \pi_r^s(m_2) = -\frac{q^2 - 8q + 8}{16q}$. $\Delta\pi \geq 0$ when $q \leq 4 + 2\sqrt{2}$; $\Delta\pi < 0$ when $q > 4 + 2\sqrt{2}$.

To summary, the retailer's profit is the highest under m_1 when $q \leq 4 + 2\sqrt{2}$ and its profit is the highest at $m \geq m_2$.

Proof of Proposition 4.

In this section, I derive the relationship between the high-quality manufacturer's profit Π_H^s and the retailer's reputation m by presenting $\frac{\partial \Pi_H^s}{\partial m}$.

First, I calculate the manufacturer's profit.

When $m \leq m_1$, $\Pi_H^s\left(\frac{1}{2}, \underline{p}\right) = \frac{1}{4q}\left(q - \frac{1}{2} + \Delta\right)$. When $m \in [m_1, m_2)$, $\Pi_H^s\left(\frac{1}{2} + \Delta, \overline{p}\right) = \frac{4mq - q^2 + \frac{7}{4}q - \frac{1}{2} + (q-1)\Delta}{2q}$. When $m \geq m_2$, $\Pi_H^s\left(\frac{q}{2}, \frac{3}{4}q\right) = \frac{q}{16}$.

Now I present the derivatives.

Note that $\frac{\partial \Delta}{\partial m} = -\frac{2q}{\sqrt{-4mq + q^2 - \frac{5}{4}q + \frac{1}{4}}} < 0$. As such, I have $\frac{\partial \Pi_H^s}{\partial m} = \frac{\partial \Delta}{\partial m} < 0$ when $m \leq m_1$. When $m \in [m_1, m_2)$, $\frac{\partial \Pi_H^s}{\partial m} = 2 - \frac{q-1}{\sqrt{-4mq + q^2 - \frac{5}{4}q + \frac{1}{4}}} > 0$ for $m < m_2(q) = \frac{3}{16}(q-1)$. When $m \geq m_2$, $\frac{\partial \Pi_H^s}{\partial m} = 0$.

This proves Proposition 4.

Proof of Corollary 2.

In this section, I find the reputation m that yields the highest Π_H^s given q.

In the previous section, I have shown that $\frac{\partial \Pi_H^s}{\partial m} < 0$ for $m \leq m_1$, $\frac{\partial \Pi_H^s}{\partial m} > 0$ for $m \in (m_1, m_2]$, and $\frac{\partial \Pi_H^s}{\partial m} = 0$ for $m > m_2$, it implies that Π_H^s is the highest under $m \to 0$ for $m \leq m_1$ and under $m \geq m_2$ for $m > m_1$. Thus, π_r^s is the highest profit under either $m \to 0$ or m_2. Let's compare them.

Note that $\lim_{m \to 0} \Pi_H^s(m) = \frac{1}{4q}[q - \frac{1}{2} + \sqrt{q^2 - \frac{5}{4}q + \frac{1}{4}}]$ and $\Pi_H^s(m_2) = \frac{q}{8}$.

$\Delta\Pi = \lim_{m \to 0} \Pi_H^s(m) - \Pi_H^s(m_2) = \frac{\sqrt{4q^2 - 5q + 1} - (q-1)^2}{8q}$. $\Delta\Pi \geq 0$ if $q < \frac{1}{2}(3 + \sqrt{13})$. $\Delta\Pi < 0$ if $q \geq \frac{1}{2}(3 + \sqrt{13})$.

To summary, the manufacturer's profit is the highest under $m \to 0$ if $q < \frac{1}{2}(3 - \sqrt{13})$ and is highest under $m \geq m_2$ when $q \geq \frac{1}{2}(3 + \sqrt{13})$. This proves Corollary 2.

Proof of Corollary 3.

In this section, I show that when $m' < m < m_0 = \min\{f(q), m_1\}$, the manufacturer's profit, the retailer's profit, and consumer welfare are all higher under the imperfect than perfect information.

I am interested in the case that $m \leq m_1$. In such cases, $p_H^s = \underline{p} = \frac{q}{2} + \frac{1}{4} - \frac{1}{2}\Delta$, $\pi_r^s = \frac{1}{16} + m$, and $\Pi_H^s = \frac{q - \frac{1}{2} + \Delta}{4q}$. Also, note that under the perfect information, $p_H = \frac{3}{4}q$, $\pi_r = \frac{q}{16}$, and $\Pi_H = \frac{q}{8}$.

Let's start with a comparison of consumer welfare. I show that consumer welfare is higher under the imperfect information by proving the retail price is lower as follows. $\Delta p = p_H^s - \frac{3}{4}q = \frac{1}{4}(1-q) - \frac{1}{2}\Delta < 0$.

Second, the retailer's profit under the imperfect and perfect information when $\Delta\pi_r = \pi_r^s - \frac{q}{16} = \frac{1}{16} + m - \frac{q}{16} > 0$. This gives $m > m' = \frac{1}{16}(q-1)$.

Finally, the manufacturer's profit is higher under the imperfect than perfect information if $\Delta\Pi_H = \Pi_H^s - \frac{q}{8} = \frac{\sqrt{-16mq + 4q^2 - 5q + 1} - (q-1)^2}{8q} > 0$. It occurs if $m < f(q) = \frac{(4q-1)(q-1) - (q-1)^4}{16q}$. Note that $f(q) > 0$ when $q \in (1, \frac{3+\sqrt{13}}{2})$. In addition, I have $f(q) > m'$ when $q < 3$.

To summary, the manufacturer', retailer's profit, and consumer welfare are all higher under imperfect than perfect information if $m' < m < m_0 = \min\{f(q), m_1\}$

Technical Appendix 1. (Manufacturer's Cheating Incentive)

In this section, I show that after considering the manufacturer's incentive to mimic, my main results are stable if the manufacturer's reputation is sufficiently high $R \geq R_1(q,m) = \frac{1}{8} + \frac{1}{4q}[\Delta - \frac{1}{2}]$. I show it by proving that $\underline{p}$ is stable. Specifically, the manufacturer's profit is lower to induce the retailer to mimic at $\underline{p}$ when $R \geq R_1(q,m)$.

First, I need to characterize consumer's belief at $\underline{p}$. If the consumer purchases and finds that product is low quality at $\underline{p}$, she would rationally infer that the manufacturer must induce the retailer to mimic at this price. Otherwise, the retailer is not profitable to do so.[14] As a result, both retailer and manufacturer loss their reputations m and R, respectively.

[14] In the model, the consumer can rationally infer whether the mimic is conducted by the retailer alone or by both the manufacturer and the retailer. Such belief is similar to Janssen and Shelegia (2015), in which the consumer forms a reservation price for the search threshold, p^*. The consumer rationally interprets that a price above p^* is a deviation by the retailer alone.

Next, I show that the manufacturer's profit is lower when inducing mimicry at $\underline{p}$ than separating equilibrium profit when R is sufficiently high. First, the retailer's incentive to mimicry at $\underline{p}$ given w_L must satisfy $\left(\underline{p} - w_L\right)\left(1 - \frac{\underline{p}}{q}\right) - m > \frac{1}{4}(\frac{3}{4} - w_L)$. This gives $w_L < \frac{1}{2}$. This gives the manufacturer's mimicry profit to satisfy

$$\Pi_L^{mic} = w_L\left(1 - \frac{\underline{p}}{q}\right) < \frac{1}{2}\left(1 - \frac{\underline{p}}{q}\right) - R.$$

As such, the manufacturer's profit is lower from mimicry when if $\Pi_L^{mic} \leq \Pi_L^s = \frac{1}{8}$. This gives $R \geq R_1 = \frac{1}{8} + \frac{1}{4q}[\Delta - \frac{1}{2}]$.

This proves that $\underline{p}$ is unaffected when $R \geq R_1$

Technical Appendix 2. (Pooling Equilibrium)

In this section, I derive pooling equilibrium and report the condition for it to survive the intuitive criterion.[15]

Under the pooling equilibrium, the high and low-quality products are sold at the same price p^p. As such, the consumer remains her prior quality belief ϕ when observing p^p. This gives the demand at pooling price as $1 - \frac{p}{A(\phi)}$, where $A(\phi) = \phi q + 1 - \phi$. Given the pooling demand, we can derive pooling prices.

Low-quality Product

Under a pooling equilibrium, the low-quality retailer must have a higher profit from pooling than separating. Hence, p^p must satisfy

$$\left(p^p - \frac{1}{2}\right)\left(1 - \frac{p}{A(\phi)}\right) - m > \frac{1}{16}. \qquad \text{(TB1)}$$

This gives $p^p \in (\underline{p}_p, \overline{p}_p)$ $\underline{p}_p = \frac{A}{2} + \frac{1}{4} - \frac{1}{2}\Delta(A)$ and $\overline{p}_p = \frac{A}{2} + \frac{1}{4} + \frac{1}{2}\Delta(A)$, where $\Delta(A) = \sqrt{-4mA + A^2 - \frac{5}{4}A + \frac{1}{4}}$. $\Delta(A)$ exists only if $-4mA + A^2 - \frac{5}{4}A + \frac{1}{4} \geq 0$. This gives a sufficient condition for the existence of pooling equilibrium $\phi \geq \phi_1 = \frac{1}{q-1}[2m - \frac{3}{8} +$

[15] Note that similar to Jiang et al. (2014), single crossing property does not hold in my setting. Consequently, the intuitive criterion cannot eliminate the pooling equilibrium under these conditions.

$\sqrt{\frac{9}{64}+4m^2+\frac{5}{2}m]}$. Furthermore, note that $\phi_1 > 1$ when $m \leq m_3(q) = \frac{q}{4}+\frac{1}{16q}-\frac{5}{16}$, therefore, pooling equilibrium never exists if $m \geq m_3$.

High-quality Product

Now let's turn to the high-quality product. First, I derive pooling equilibrium prices through backward induction. The profit maximization problems of the high-quality manufacturer and retailer are as follows:

$$\max_{p^p}(p^p - w^p)\left(1-\frac{p}{A(\phi)}\right), \tag{TB2}$$

$$\max_{w^p} w^p\left(1-\frac{p}{A(\phi)}\right). \tag{TB3}$$

Note that pool price also must satisfy

$$p^p \in (\underline{p}_p, \overline{p}_p). \tag{TB4}$$

Conditions (TB2) to (TB4) yield the pooling equilibrium wholesale and retail prices as $(\frac{A}{2}, \frac{3}{4}A)$ if $\phi > \frac{16m}{3(q-1)}$ and $q > 2$; or $(\frac{1}{2}+\Delta(A), \frac{A}{2}+\frac{1}{4}+\frac{1}{2}\Delta(A))$ if otherwise.

Under a pooling equilibrium, profits from both the manufacturer and the retailer must be higher than those under separating equilibrium. Thus, I can compare pooling and separating profits. There are two cases, consider each in turn.

<u>Case 1</u>. $\phi \leq \frac{16m}{3(q-1)}$

In this case, I will rule out pooing equilibrium by proving that the manufacturer's profit is lower under the pooling than separating equilibrium.

Let's define $g(x) = \frac{1}{x}\left(\frac{A}{2}-\frac{\Delta(x)}{2}-\frac{1}{4}\right)\left[\frac{1}{2}+\Delta(x)\right]$. I show that $\Pi_H^p = g(A) \leq \Pi_H^s = g(q)$, as follows.

First, I show that $\frac{\partial g}{\partial x} = \frac{r(\Delta(x))}{2x\Delta(x)} > 0$ for $x = A$. I prove it by showing that $r(\Delta(x)) > 0$, where $r(\Delta(x)) = (x+1)\Delta(x)^2 - (2x^2-1)\Delta(x) + (x-1)(x^2-\frac{1}{4})$. When $\phi \leq \frac{16m}{3(q-1)}$, $A = \phi(q-1)+1 \leq \frac{16m}{3}+1$. This gives $m \geq \frac{3}{16}(x-1)$. Substitute it into $\Delta(x)$, this gives $\Delta(x) \leq \frac{x-1}{2}$. Consequently, $\frac{\partial r(\Delta(x))}{\partial \Delta(x)} = 2(x+1)\Delta(x) - (2x^2-1) \leq \frac{x-1}{2}2(x+1) - (2x^2-1) = -x^2 < 0$. This gives $r(\Delta(x)) \geq r\left(\frac{x-1}{2}\right) > 0$. As such, this proves $\frac{\partial g}{\partial x} > 0$.

Next, since $q \geq A$, the monotonicity condition yields $\Pi_H^p = g(A) \leq \Pi_H^s = g(q)$.

Case 2. $\phi > \frac{16m}{3(q-1)}$

In this case, pooling equilibrium prices are $(\frac{A}{2}, \frac{3}{4}A)$ and pooling profit is $\Pi^p = \frac{A}{8}$ for any m.

The Manufacturer's Profit.

Let's start to check the condition such that the manufacturer's profit is higher under the pooling and separating equilibrium. Note that separating prices (w_H^s, p_H^s) and profit Π_H^s vary across m. Let's consider each case in turn.

First, when $m > m_2$, $(w_H^s, p_H^s) = (\frac{q}{2}, \frac{3}{4}q)$ and $\Pi_H^s = \frac{q}{8}$. I can verify that $\Pi_H^s \geq \Pi^p$ for any ϕ, because $q \geq A$. Hence, I rule out pooling equilibrium any $m \geq m_2$.

Second, when $m \in (m_1, m_2]$, $(w_H^s, p_H^s) = (\frac{1}{2} + \Delta, \overline{p})$ and $\Pi_H^s = \frac{1}{q}(\frac{1}{2} + \Delta)\left(\frac{q}{2} - \frac{\Delta}{2} - \frac{1}{4}\right)$. I have $\Pi^p > \Pi_H^s$ if $\phi > \phi_a = \frac{[1+2\Delta](2q-2\Delta-1)-q}{q(q-1)}$. Also, I can verify that $\phi_a < 1$ by showing that $1 - \phi_a = \frac{(q-2\Delta-1)^2}{q(q-1)} > 0$. This shows that pooling equilibrium survives intuitive criterion if $\phi > \phi_a$ for $m \in (m_1, m_2]$.

Lastly, when $m \leq m_1$, $(w_H^s, p_H^s) = (\frac{1}{2}, \underline{p})$ and $\Pi_H^s = \frac{1}{2q}\left(\frac{q}{2} - \frac{\Delta}{2} + \frac{1}{4}\right)$. I have $\Pi^p > \Pi_H^s$ when $\phi > \phi_b = \frac{q-1+2\Delta}{q(q-1)}$. I check that $\phi_b < 1$ if $m < f(q) = \frac{(2q-1)^2 - q - (q-1)^4}{16q}$. This proves that pooling equilibrium survives intuitive criterion if $\phi > \phi_b$ and $m < f(q)$ for $m \leq m_1$.

The Retailer's Profit.

In this section, I check that the condition that the retailer is unprofitable to deviate from a pooling retail price $p^p = \frac{3}{4}A$ to a separating one $p \notin (\underline{p}, \overline{p})$ given the pooling wholesale price $w^p = \frac{A}{2}$.

First, I show that $\overline{p} = \arg \max_{p \notin (\underline{p}, \overline{p})} \pi_r\left(\frac{A}{2}, p\right)$ as follows. First, I have $p_H\left(\frac{A}{2}\right) = \frac{A}{4} + \frac{q}{2}$ for $D(p) = 1 - \frac{p}{q}$. Next, I can verify that $\overline{p} = \arg \min_{p \notin (\underline{p}, \overline{p})} |p - (\frac{A}{4} + \frac{q}{2})|$ by showing that $\frac{A}{4} + \frac{q}{2} \in (\frac{\underline{p} + \overline{p}}{2}, \overline{p})$. This proves $\overline{p} = \arg \max_{p \notin (\underline{p}, \overline{p})} \pi_r\left(\frac{A}{2}, p\right)$.

Next, I compare the retailer's profit between $\Pi^s(\frac{A}{2},\overline{p})$ and $\Pi^p = \frac{A}{8}$. The retailer would not deviate to $\overline{p}$ if

$$\Pi_s(\frac{A}{2},\overline{p}) = \left(\overline{p} - \frac{A}{2}\right)\left(1 - \frac{\overline{p}}{q}\right) < \Pi^p = \frac{A}{8}.$$

This gives $\phi > \phi_c = \frac{16mq}{(5q-4\Delta-2)(q-1)}$. I show $\phi_c < 1$ as the follows. To show $\phi_c < 1$ is equivalent to show that $4\Delta < (5q-2) - \frac{16mq}{(q-1)}$. I can verify that $16\Delta^2 < \left[(5q-2) - \frac{16mq}{(q-1)}\right]^2$. In addition, I can show that $(5q-2) - \frac{16mq}{(q-1)} > 0$ when $m \leq m_2(q) = \frac{3}{16}(q-1)$. This proves that $\phi_c < 1$. This shows that pooling equilibrium survives intuitive criterion when $\phi > \phi_c$.

Summarize from all previous sections, I find that pooling equilibrium survives the intuitive criterion if (a) $\phi > \phi_2$ when $m \in (m_1, m_2]$ or (b) $\phi > \phi_3$ when $m \leq m_0 = \min\{f(q), m_1\}$, where $\phi_3 = \max\{\phi_1, \phi_b, \phi_c\}$ and $\phi_2 = \max\{\phi_3, \phi_a\}$.

Note that $\phi_1 = \frac{1}{q-1}[2m - \frac{3}{8} + \sqrt{\frac{9}{64} + 4m^2 + \frac{5}{2}m}]$, $\phi_a = \frac{[1+2\Delta](2q-2\Delta-1)-q}{q(q-1)}$, $\phi_b = \frac{q-1+2\Delta}{q(q-1)}$, and $\phi_c = \frac{16mq}{(5q-4\Delta-2)(q-1)}$.

References for Technical Appendix

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