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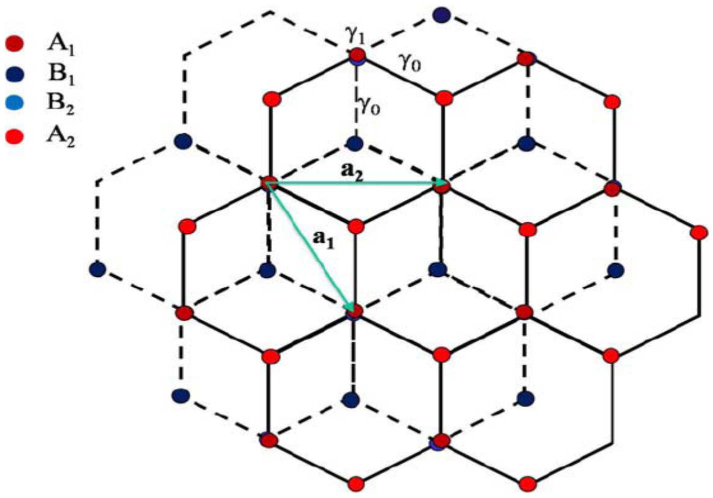
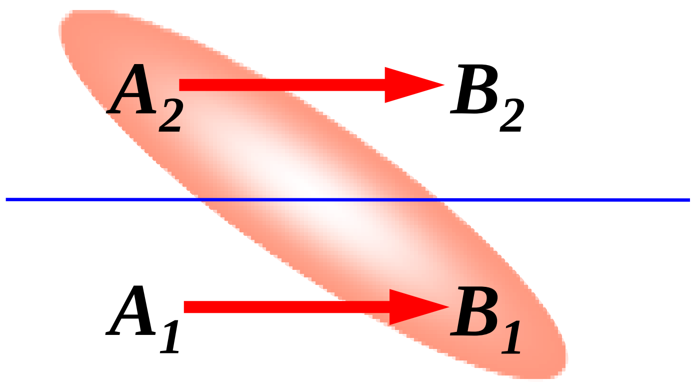
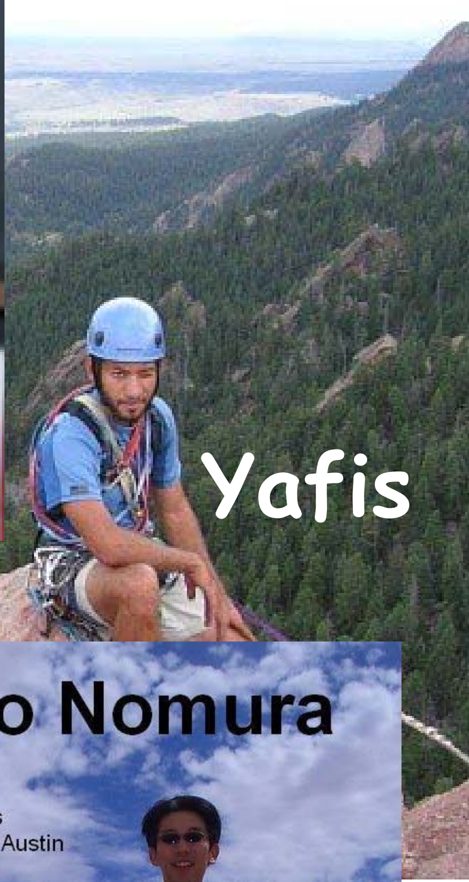
Graphene Bilayer Quantum Hall Ferromagnets

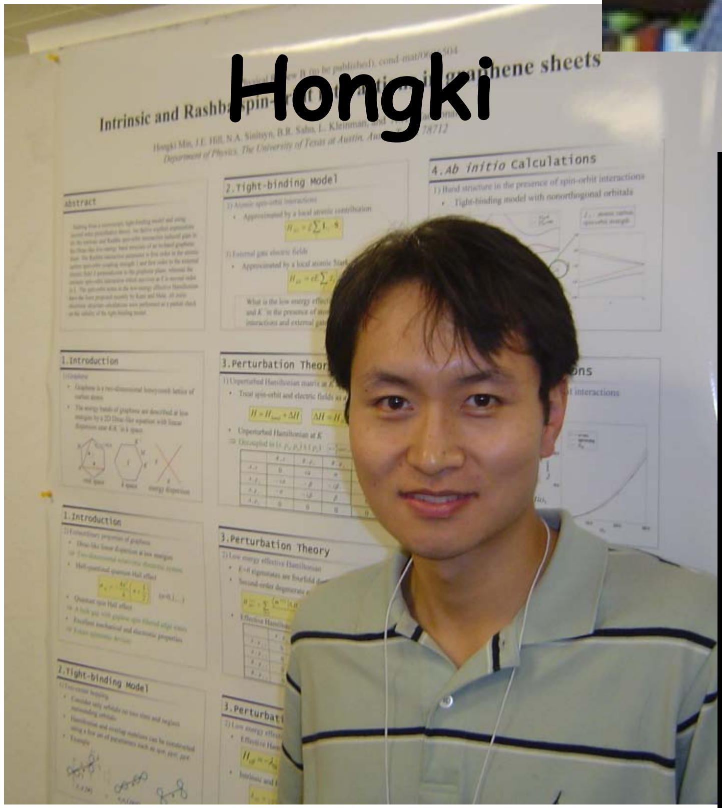




PRL 2008, 2010, arXiv:1003.5679


Rene


Yafis


Hongki
4. Ab initio Calculations
2. Tight-binding Model
Abstract
1. Introduction
3. Perturbation Theory
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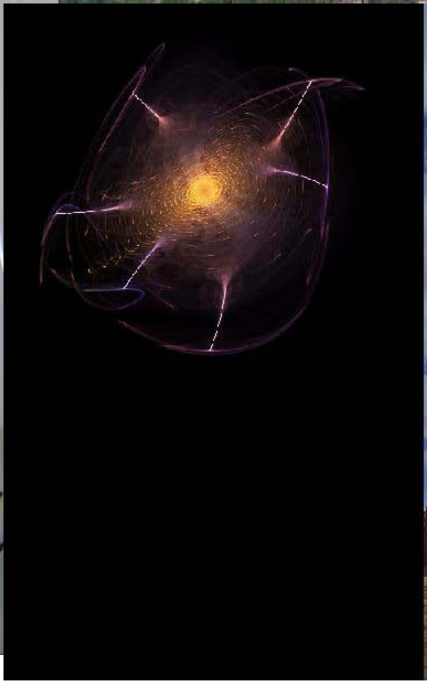
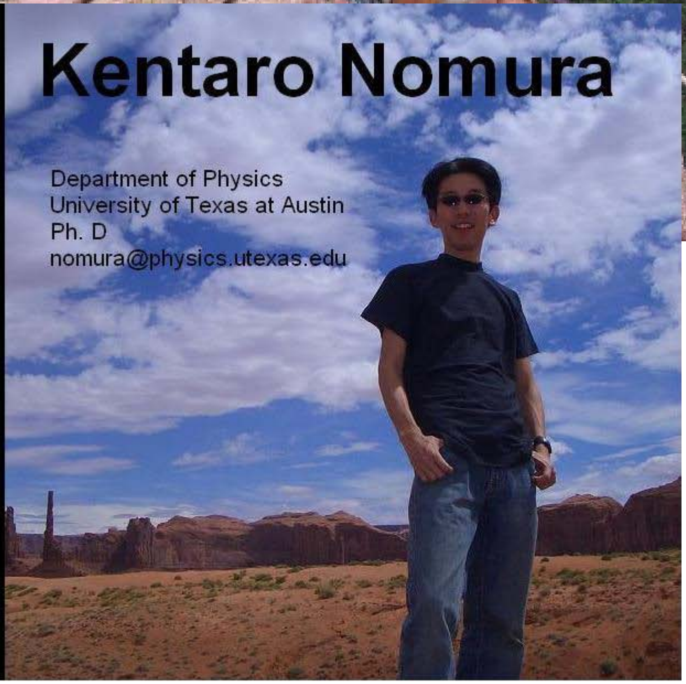
3. Perturbation Theory
2. Tight-binding Model




Kentaro Nomura
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University of Texas at Austin
Ph. D
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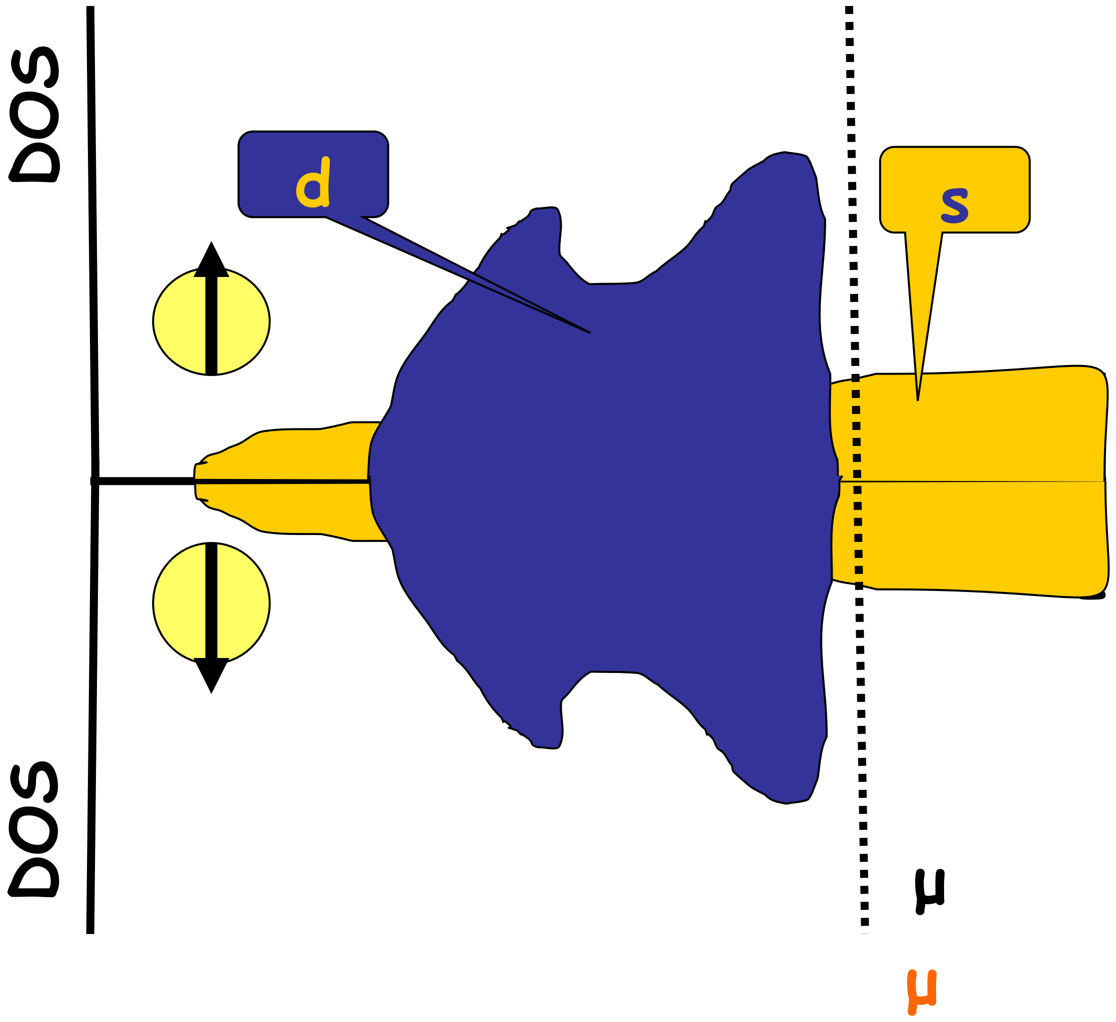
Quantum Hall Ferromagnets

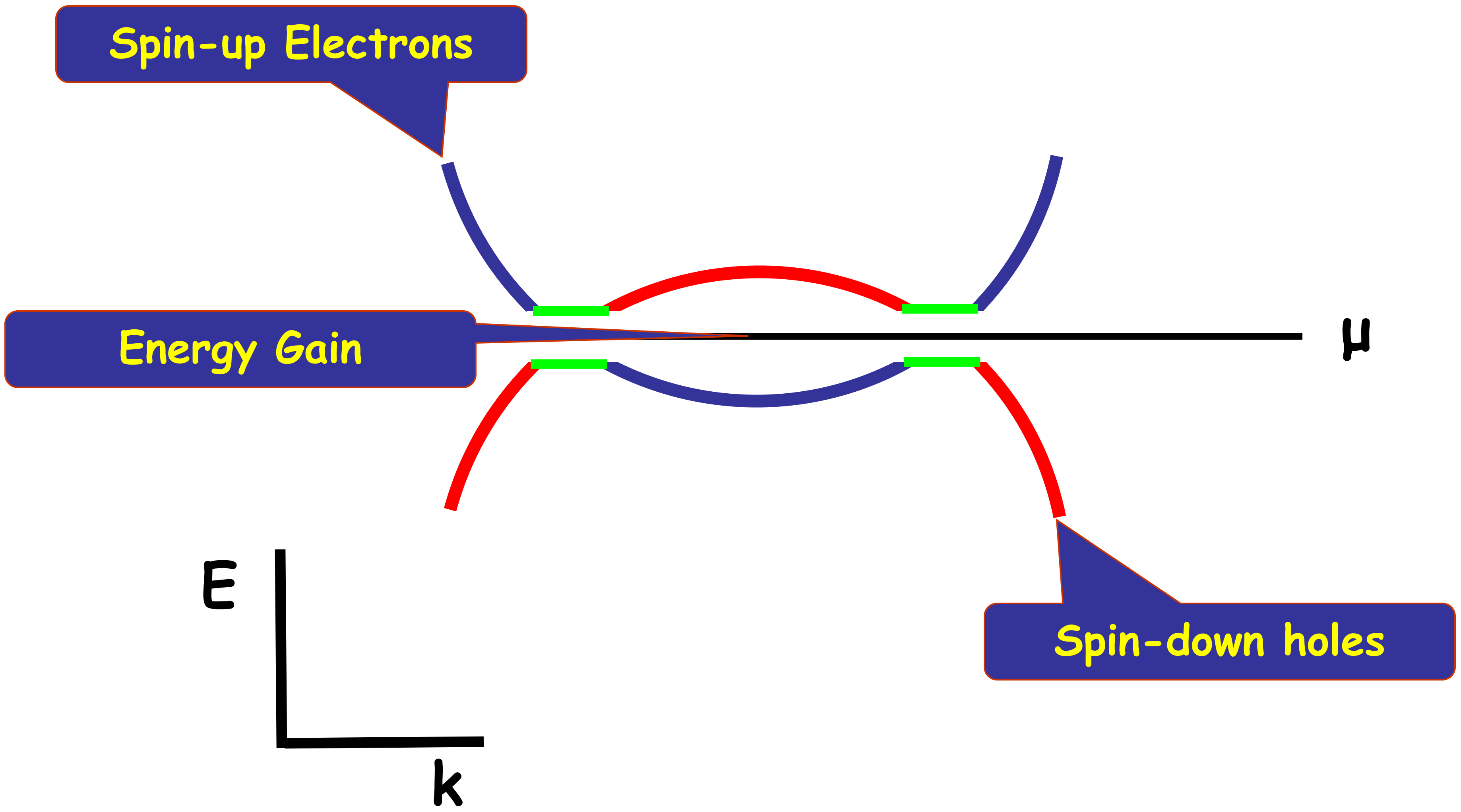
Multilayer Graphene Electronic Structure

Bilayer Graphene Quantum Hall Ferromagnetism

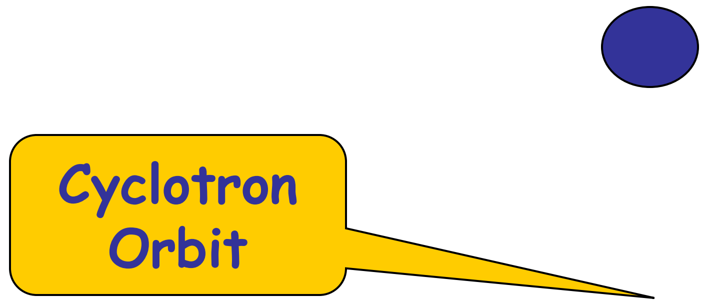
Stoner Physics

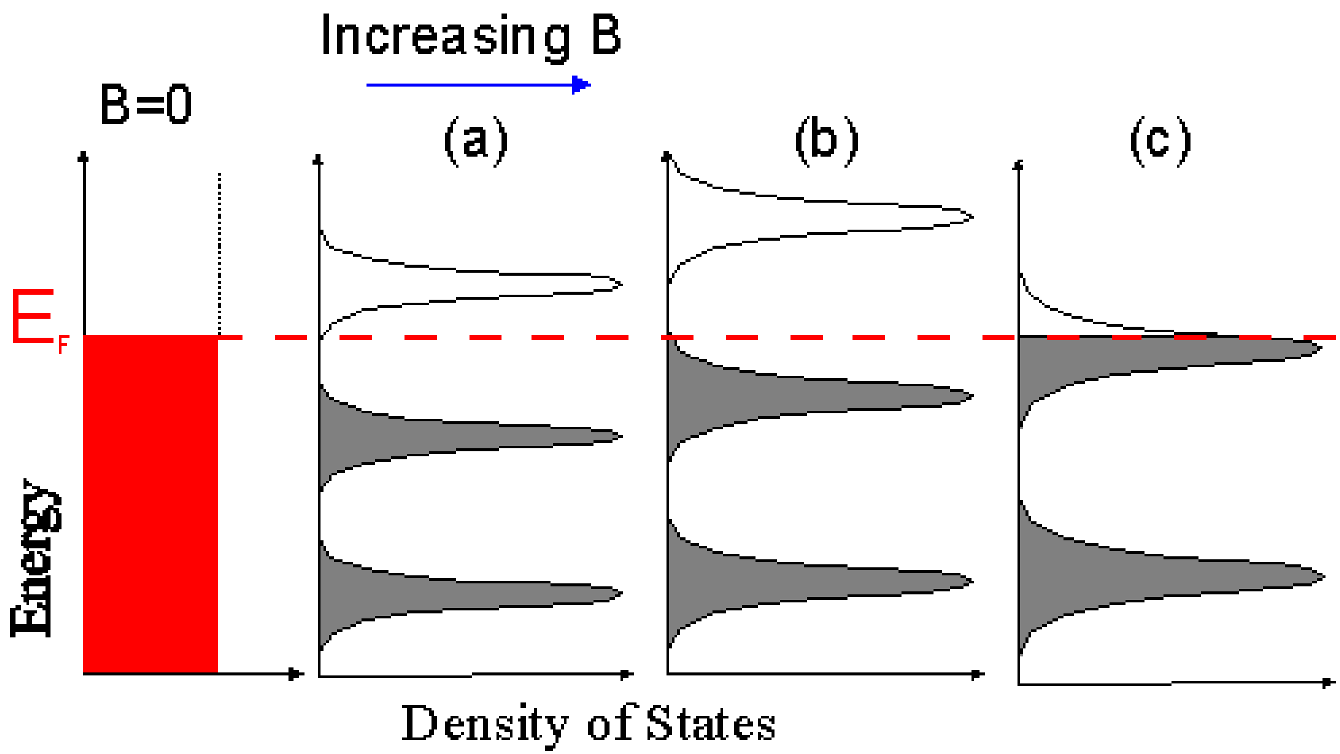


BCS Physics

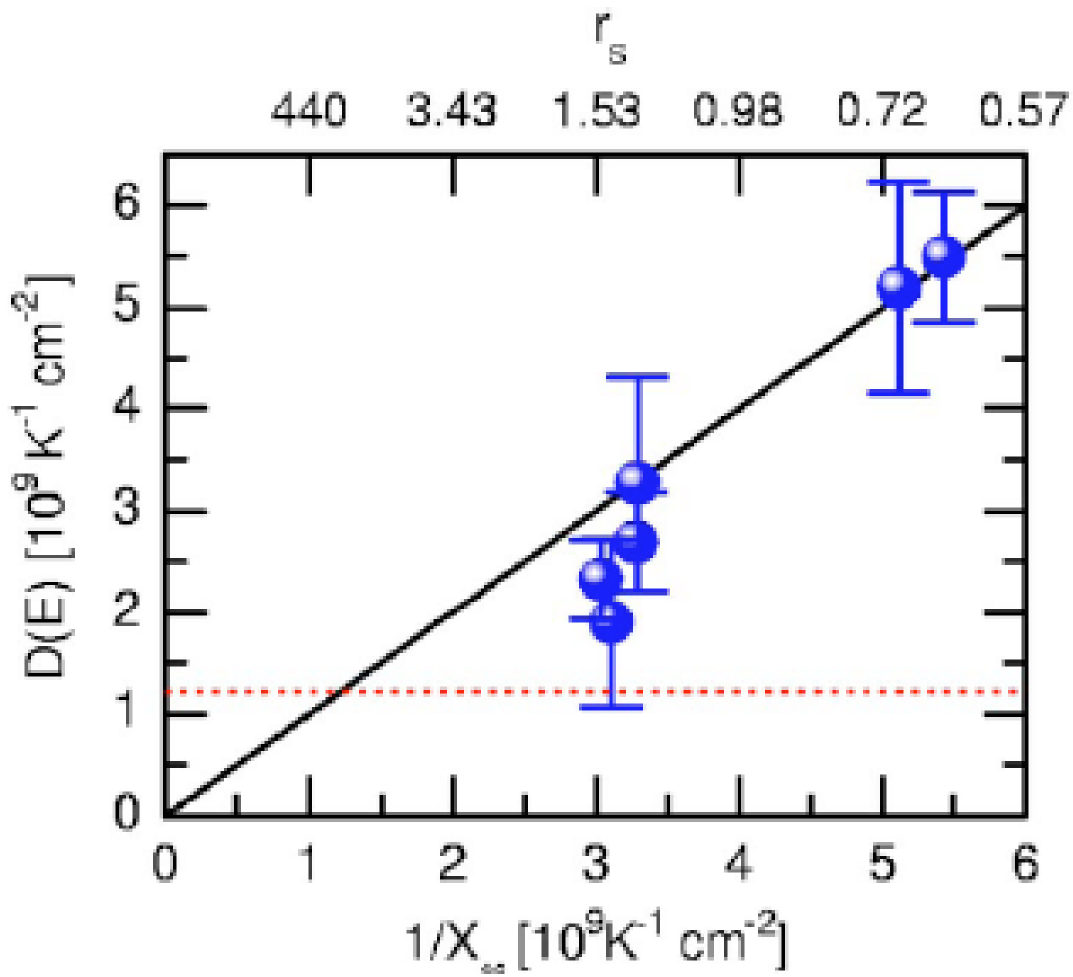


Landau Levels





QHF- Stoner-Criterion



Maude et al. PRB (2005)
Fogler and Shklovskii PRB (1995)

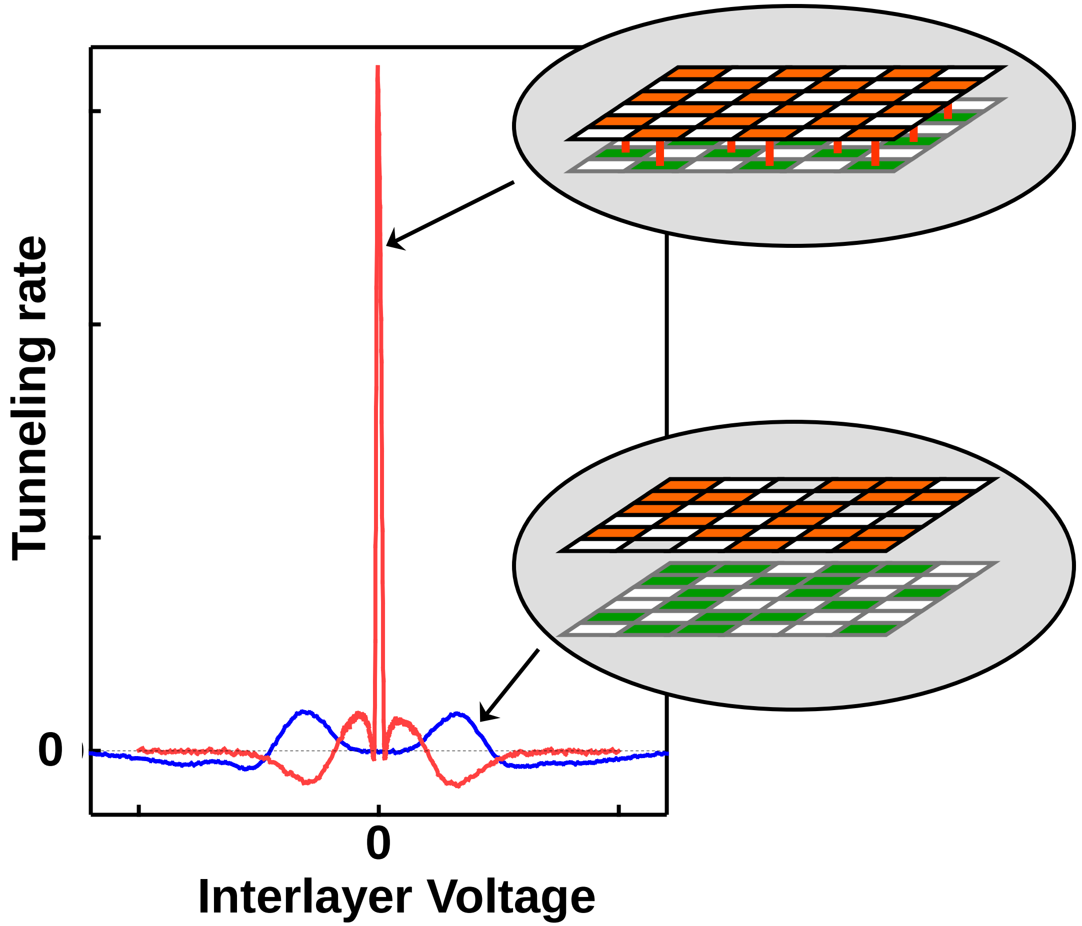
Spontaneous Interband Coherence

$$\langle \psi_t^\dagger(\vec{r})\, \psi_b(\vec{r}) \rangle = |\Psi(\vec{r})|\, e^{i\phi(\vec{r})} \neq 0$$

is

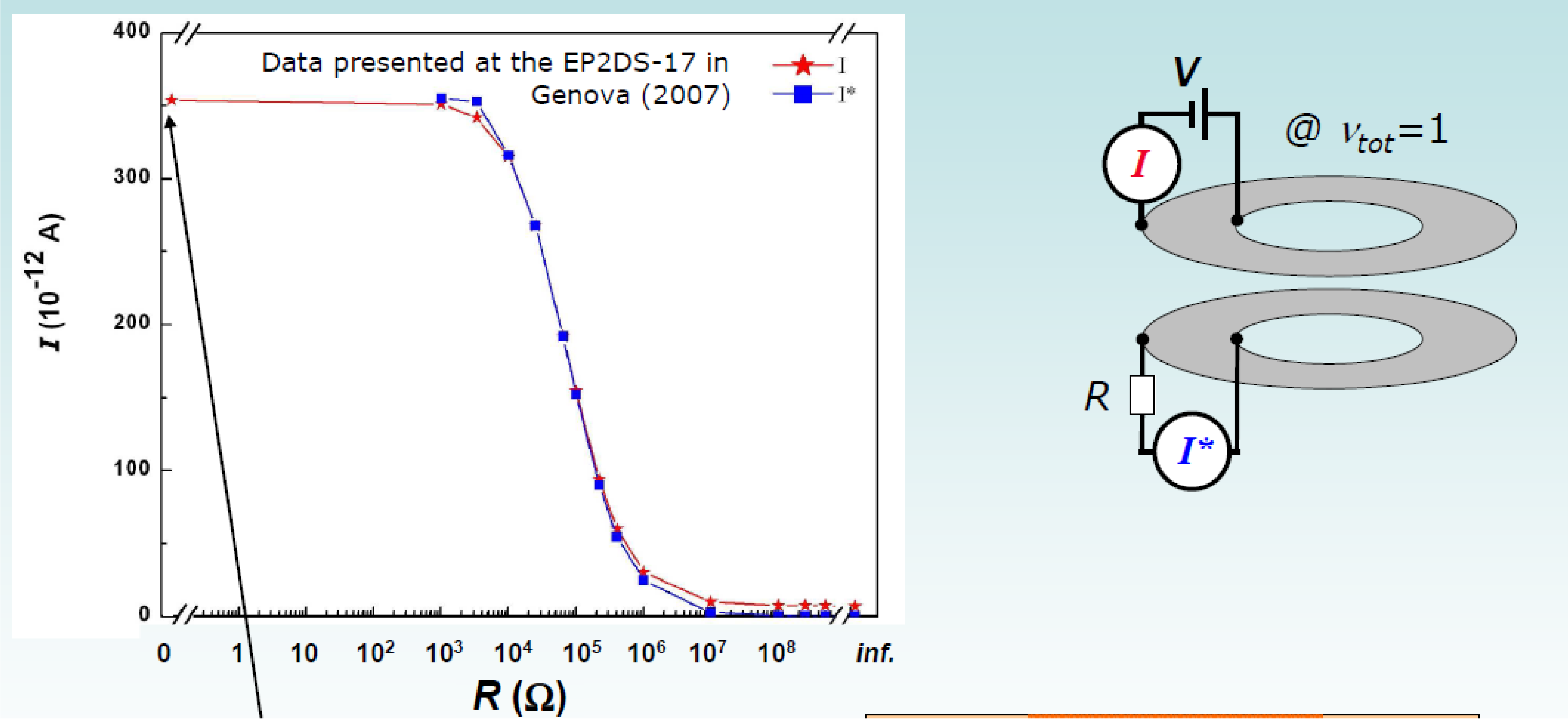
Exciton Condensation

Tunnel Experiment



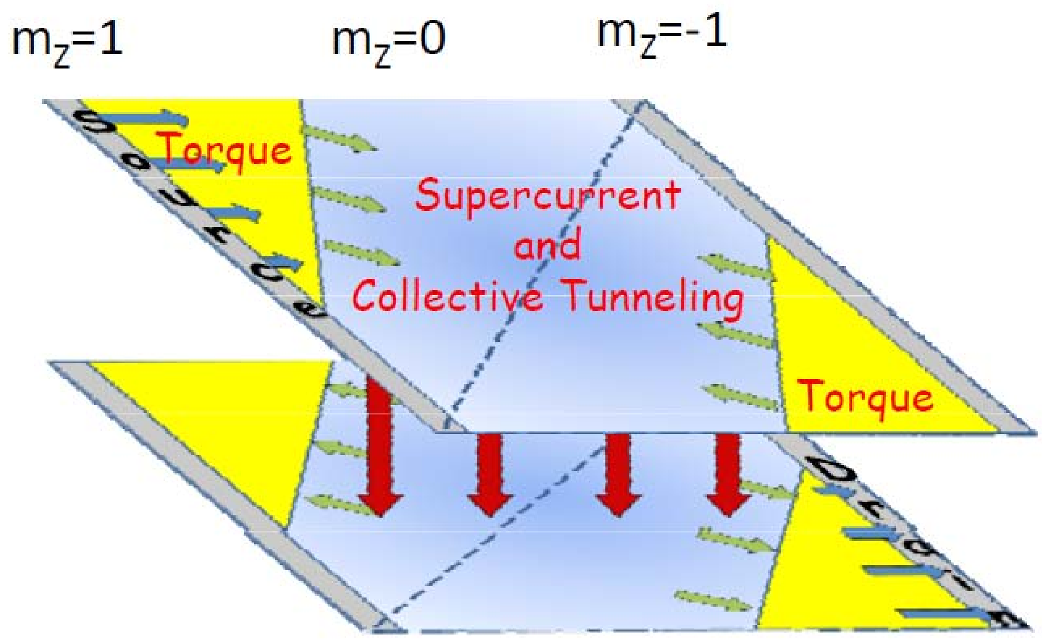
Spielman et al. PRL (2000)

Drag Counterflow Experiment



Tiemann et al. NJP (2008)

Landau-Liftshitz-Slonczweski



$$0 = -\frac{\rho_s}{\hbar}\vec{\nabla}^2\phi + \frac{1}{2}\frac{\Delta_t n}{\hbar}\sin\phi - \frac{1}{2}\vec{j}\cdot\vec{\nabla}\, m_z\,.$$

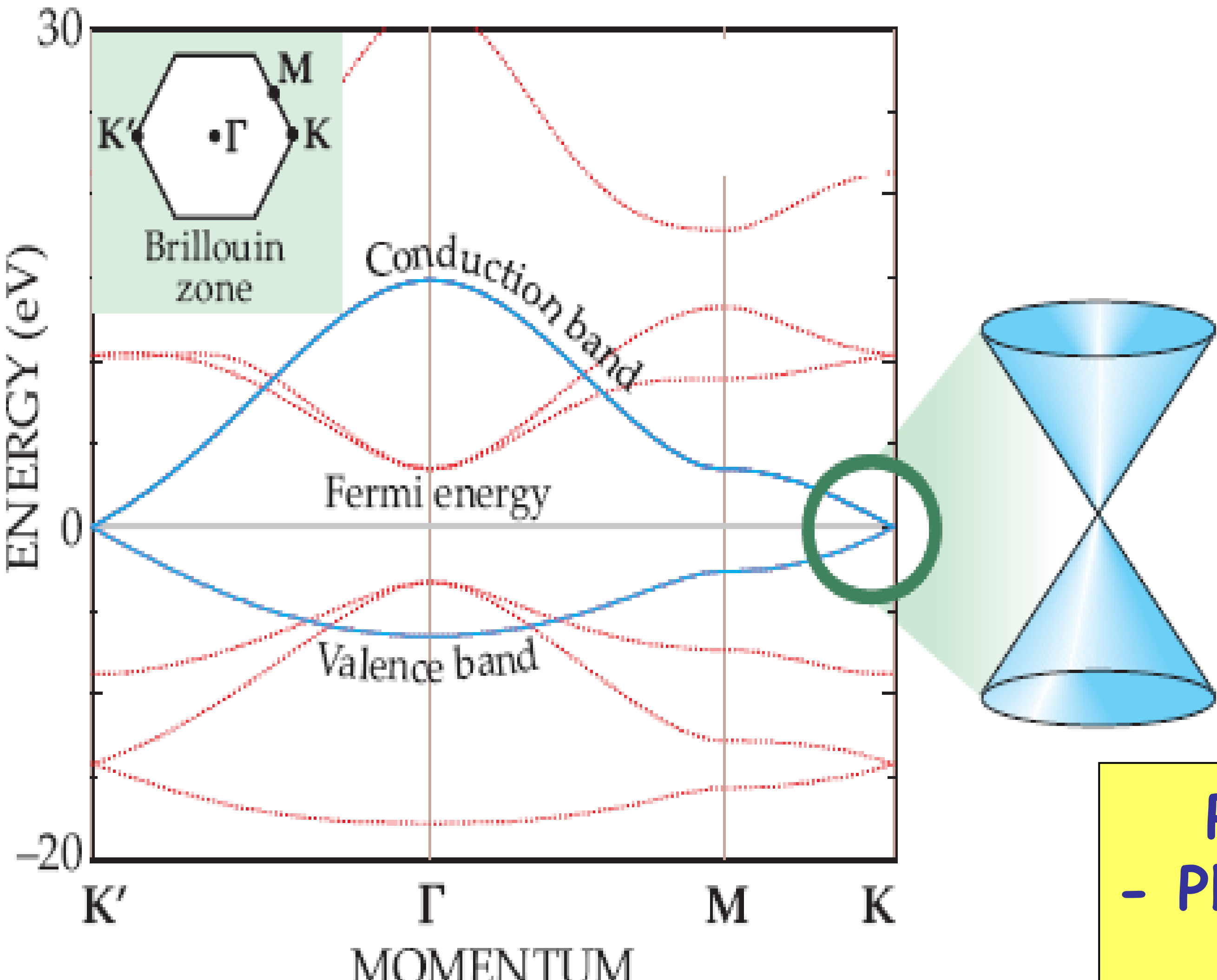
Jung-Jung Su and AHM arxiv:1001.2923

Quantum Hall Ferromagnets

Multilayer Graphene Electronic Structure

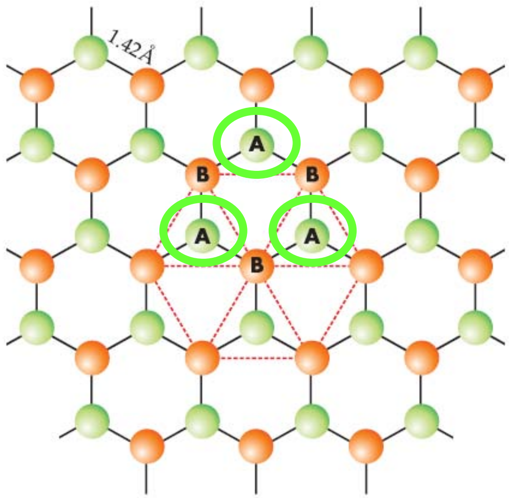
Bilayer Graphene Quantum Hall Ferromagnetism

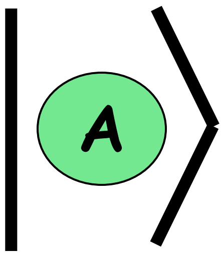
flatland band structure

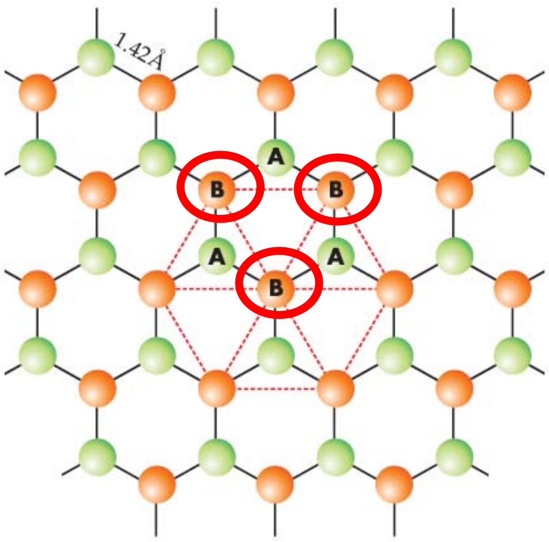


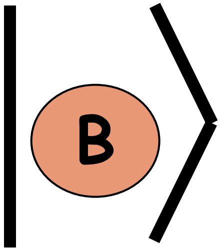
Phil Wallace
– Physical Review –
1947

Pseudospins - Graphene



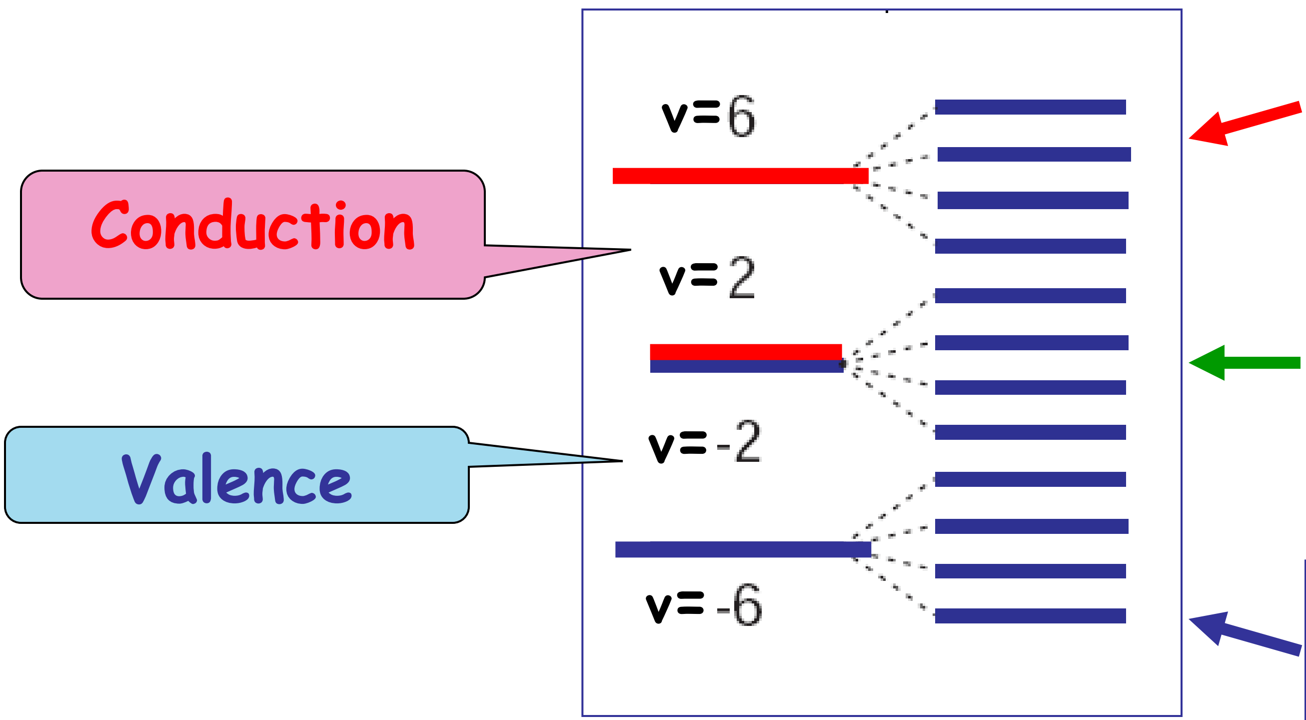






Graphene Landau Levels - Quantum

$$\mathcal{H}_0 = \frac{\sqrt{2}\hbar v}{\ell_B} \begin{pmatrix} 0 & a^\dagger \\ a & 0 \end{pmatrix} \rightarrow \sqrt{2\hbar\omega_c \times mcv} \begin{pmatrix} 0 & \sqrt{n} \\ \sqrt{n} & 0 \end{pmatrix}$$



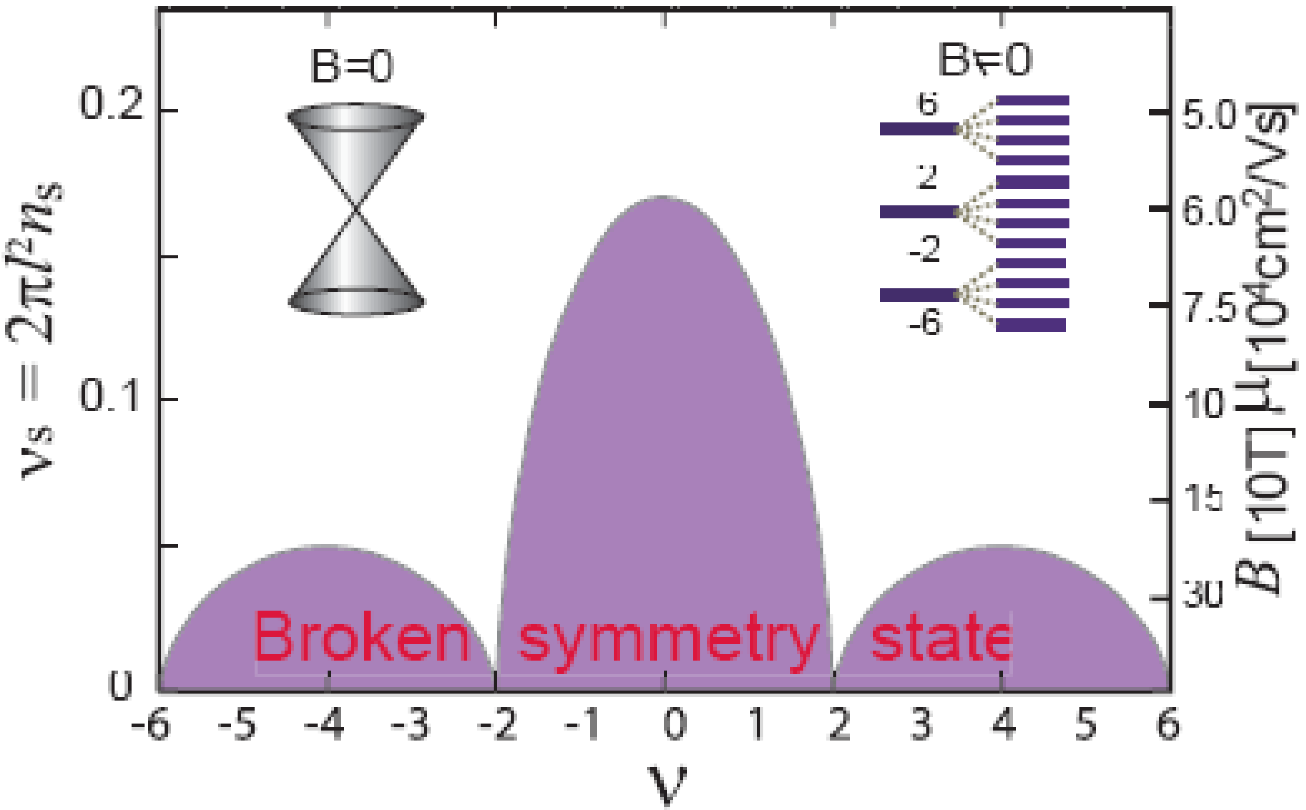
$$|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_0 \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}$$

$$|-1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ -\phi_0 \end{pmatrix}$$

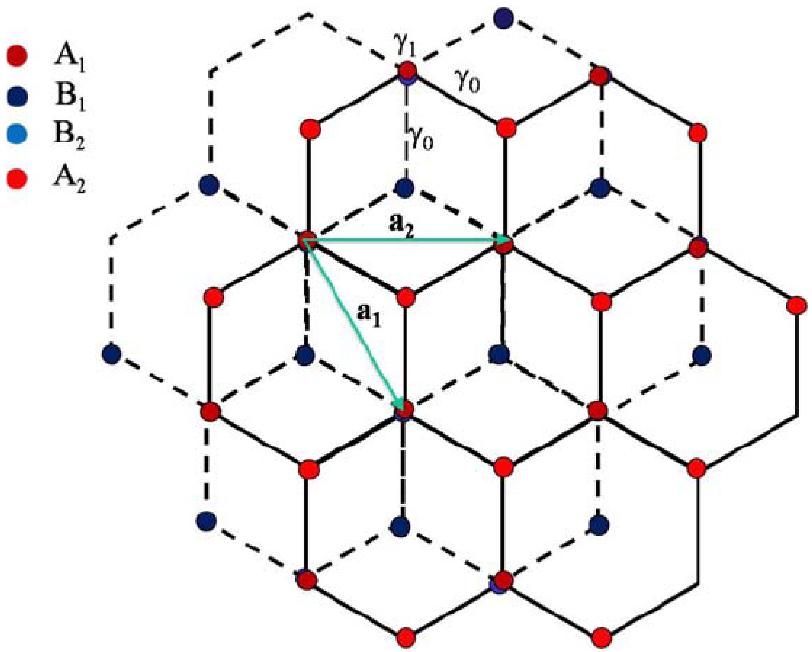
Phase Diagram

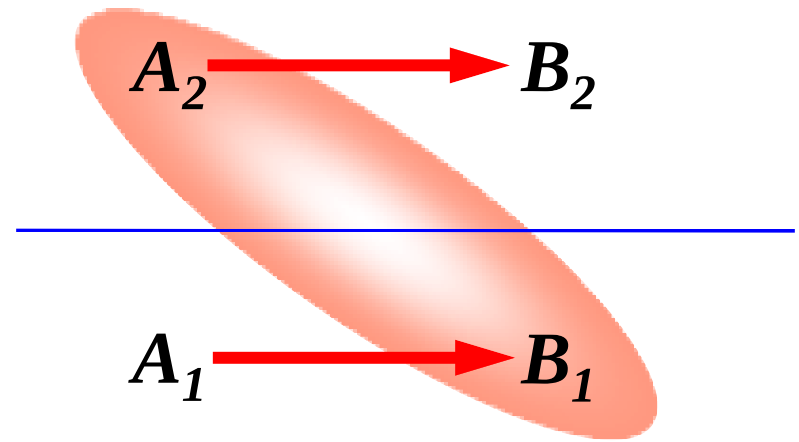
Nomura & AHM, PRL (2006)
Moessner, Alicea, Yang, Abanav



Manchester μ ~ 1.5
Columbia μ ~ 5.0

Bilayer AB stacking

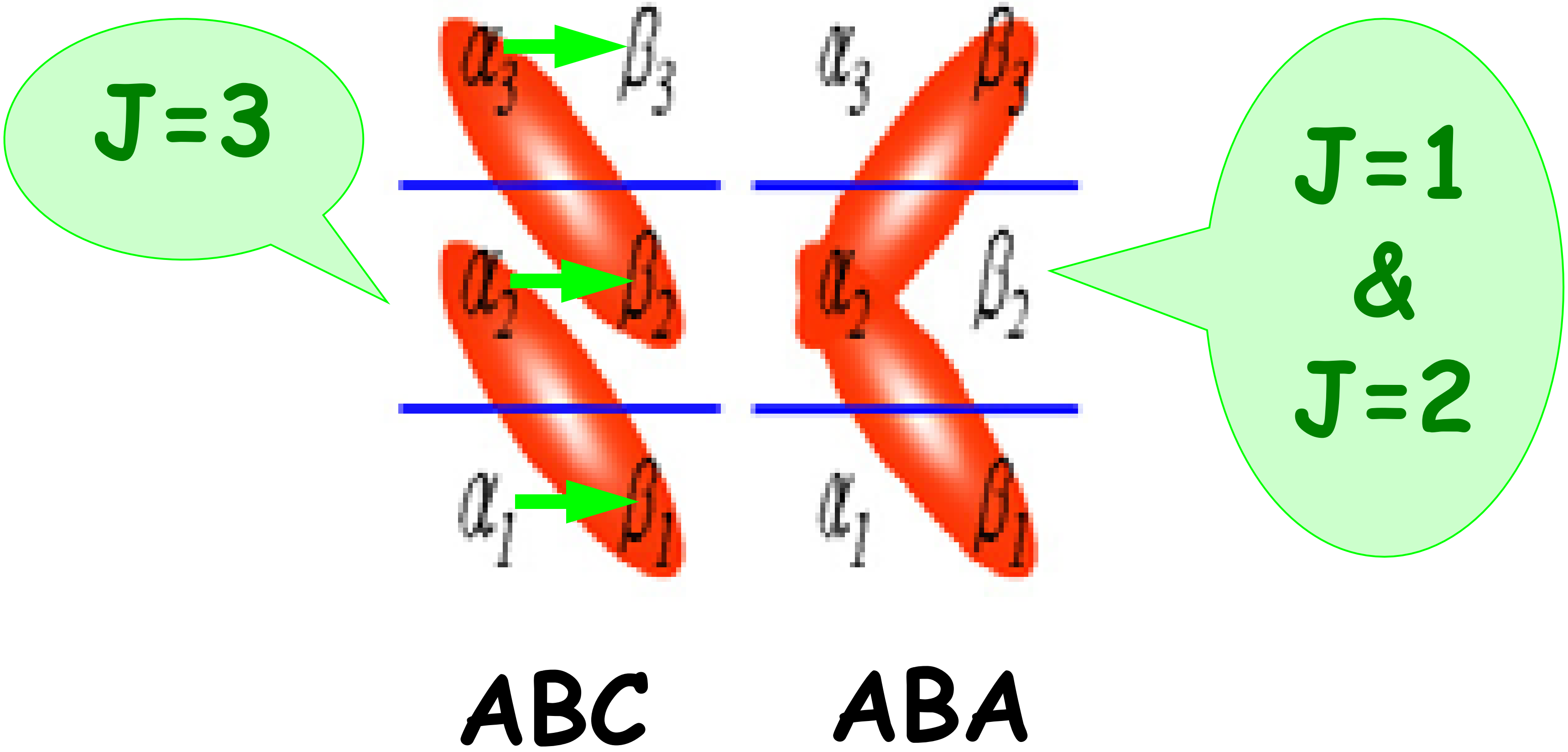




$\pi = \pi_x + i\pi_y$
=perturbation

$$\mathcal{H}_{MD}(\boldsymbol{k}) = -\begin{pmatrix} 0 & v\pi^\dagger \\ v\pi & 0 \end{pmatrix} \begin{matrix} |A\rangle \\ |B\rangle \end{matrix}$$

3 layer stacks

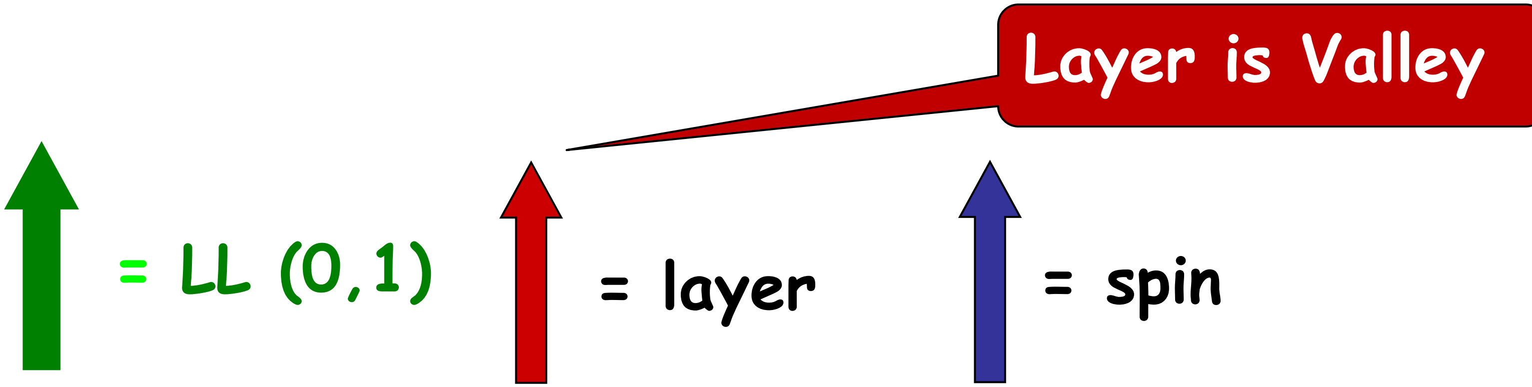


Quantum Hall Ferromagnets

Multilayer Graphene Electronic Structure

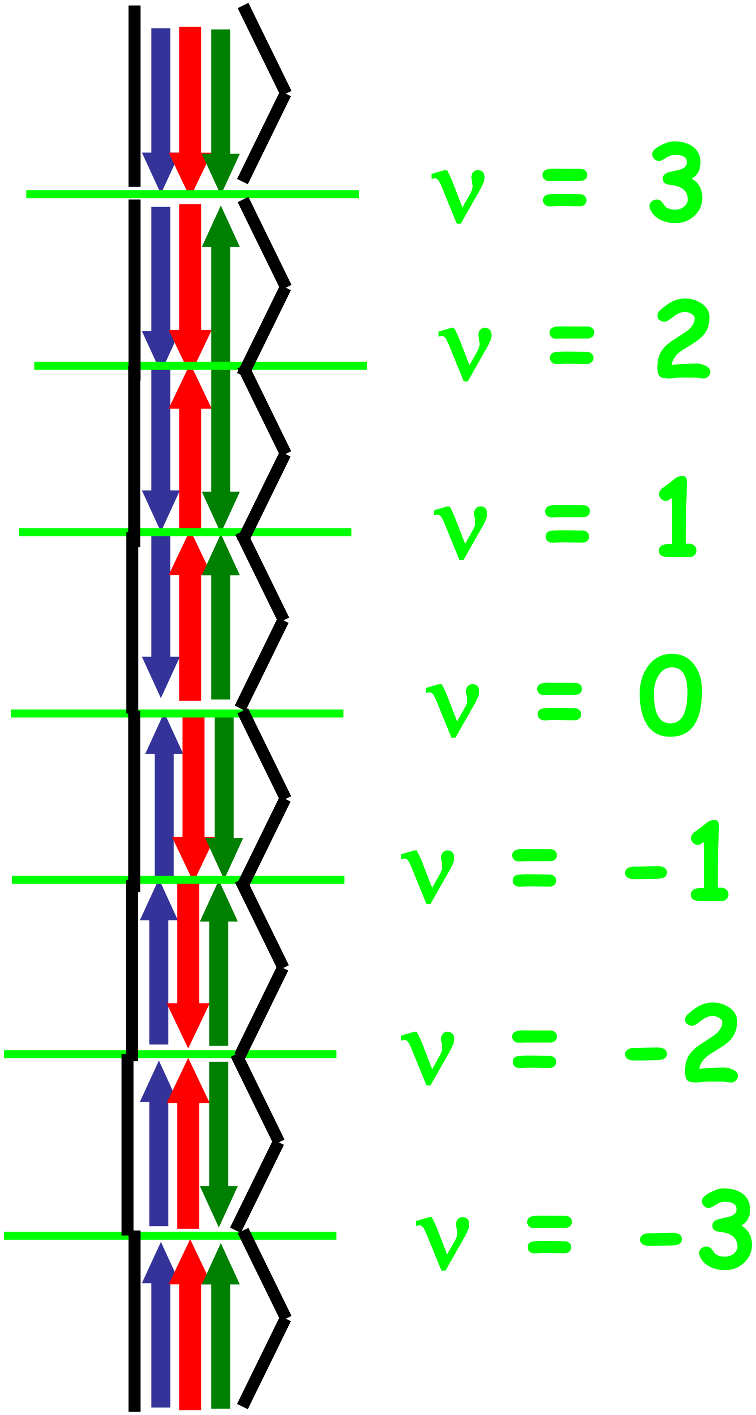
Bilayer Graphene Quantum Hall Ferromagnetism

N=8 Quantum Hall Ferromagnets



$$\mathcal{H} = -\hbar\omega_c \begin{pmatrix} 0 & a^2 \\ (a^\dagger)^2 & 0 \end{pmatrix} \begin{matrix} |A\rangle \\ |B\rangle \end{matrix}$$

The Bilayer Octet - Barlas et al. - PRL (2008)



Hunds Rules:

Spin

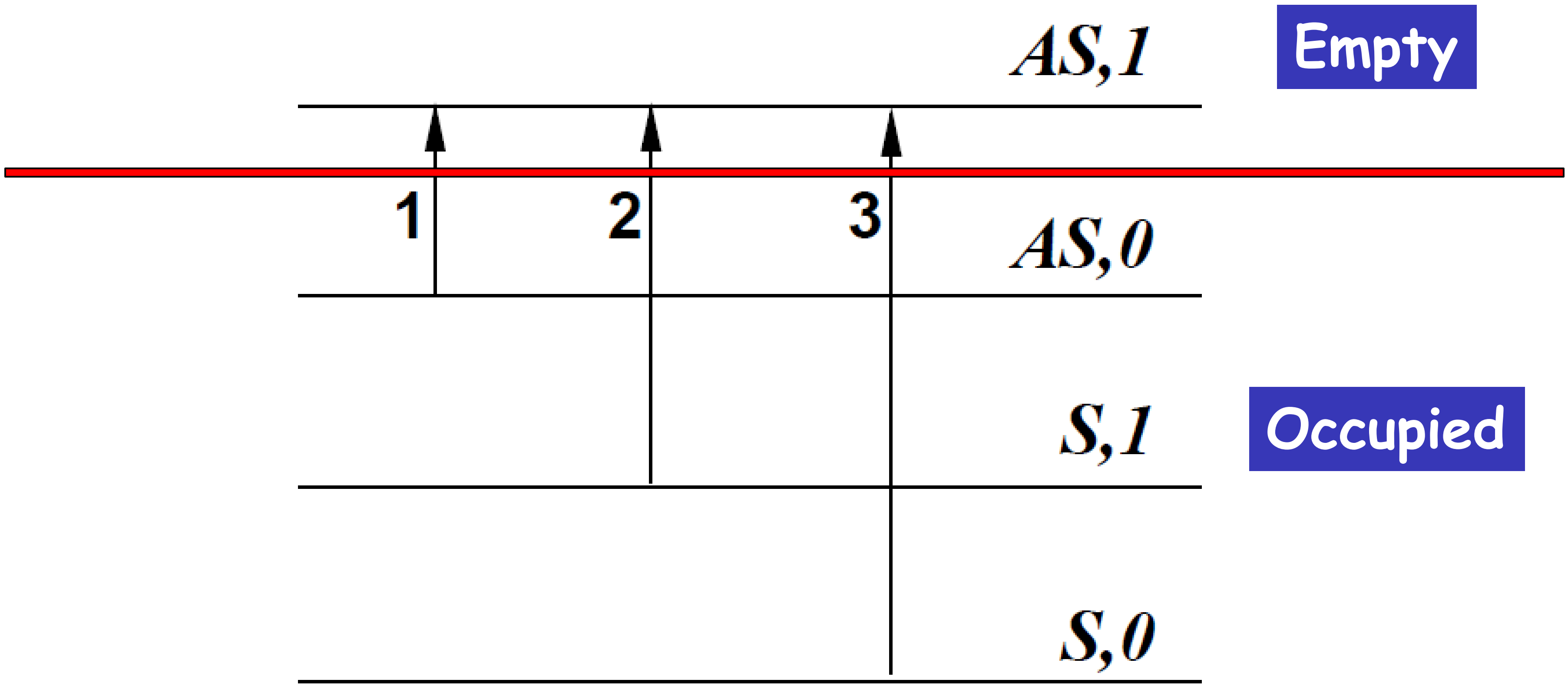
then

layer

then

LL

Time-Dependent Mean-Field Theory



v=-1,3 State

Quantum Fluctuations

$$S[\bar{z}, z] = S_B + \mathcal{E}$$

$$|\psi[z]\rangle = \prod_X (z_{0X\sigma} c^\dagger_{0X\sigma} + z_{1X\sigma} c^\dagger_{1X\sigma})|0\rangle;$$

Energy Functional

$$\sum_{XX'} (\frac{1}{2} \sum_{n_i} [H(X-X') - F(X-X')] \bar{z}_{n_1 X,\sigma} z_{n_3 X\sigma} \bar{z}_{n_2 X',\sigma} z_{n_4 X'\sigma} + \xi \Delta_{LL} \bar{z}_{1X\sigma} z_{1X'\sigma})$$

LL Pseudospin Dependence

$$F_{n_1,n_2,n_3,n_4}(X) = \frac{1}{L^2} \sum_{\mathbf{q}} v_{\mathbf{q}} \delta_{q_y,X} F_{n_1 n_3}(\mathbf{q}) F_{n_2 n_4}(-\mathbf{q})$$

$$h_{12} = H_{1,0,1,1}(\mathbf{q}) = -h_8^*, \tag{A13}$$
$$h_{13} = H_{1,1,0,0}(q) = h_4, \tag{A14}$$
$$h_{14} = H_{1,1,0,1}(\mathbf{q}) = -h_8, \tag{A15}$$
$$h_{15} = H_{1,1,1,0}(\mathbf{q}) = h_8^*, \tag{A16}$$

where $\theta_{\mathbf{q}}$ is the angle between the wavevector $\mathbf{q}$ and the x axis and $\Lambda(q) = \exp\left(\frac{-q^2\ell^2}{2}\right)$. The interactions $\widetilde{h}_n(\mathbf{q})$ are obtained by multiplying h_n by e^{-qd} where d is the inter-layer separation. The interactions $\widehat{h}_n(\mathbf{q})$ and $\widehat{\widetilde{h}}_n$ are obtained by removing the term i and the phase factor $e^{\pm i\theta_{\mathbf{q}}}$ or $e^{\pm 2i\theta_{\mathbf{q}}}$ in h_n and $\widetilde{h}_n$. For example $\widehat{h}_2 = -\Lambda(q)/\sqrt{2}$ while $h_2(\mathbf{q}) = -\frac{i}{\sqrt{2}}e^{i\theta_{\mathbf{q}}}\Lambda(q)$.

The Fock interactions are defined by

$$x_1(q) = X_{0,0,0,0}(\mathbf{q}) = \int_0^\infty dy e^{-y^2/2} J_0(q\ell y), \tag{A17}$$

$$x_2(\mathbf{q}) = X_{0,0,0,1}(\mathbf{q}) = \frac{i}{\sqrt{2}}e^{i\theta_{\mathbf{q}}}\int_0^\infty dy y e^{-y^2/2} J_1(q\ell y), \tag{A18}$$

$$x_4(q) = X_{0,0,1,1}(\mathbf{q}) = \int_0^\infty dy \left(1 - \frac{y^2}{2}\right) e^{-y^2/2} J_0(q\ell y), \tag{A19}$$

$$x_6(\mathbf{q}) = X_{0,1,0,1}(\mathbf{q}) = \frac{1}{2}e^{2i\theta_{\mathbf{q}}}\int_0^\infty dy y^2 e^{-y^2/2} J_2(q\ell y), \tag{A20}$$

$$x_7(q) = X_{0,1,1,0}(\mathbf{q}) = \frac{1}{2}\int_0^\infty dy y^2 e^{-y^2/2} J_0(q\ell y), \tag{A21}$$

$$x_8(\mathbf{q}) = X_{0,1,1,1}(\mathbf{q}) = -\frac{i}{\sqrt{2}}e^{i\theta_{\mathbf{q}}}\int_0^\infty dy y \times \left(1 - \frac{y^2}{2}\right) e^{-y^2/2} J_1(q\ell y), \tag{A22}$$

$$x_{16}(q) = X_{1,1,1,1}(\mathbf{q}) = \int_0^\infty dy \left(1 - \frac{y^2}{2}\right)^2 e^{-y^2/2} J_0(q\ell y), \tag{A23}$$

and

$$x_3 = X_{0,0,1,0}(\mathbf{q}) = x_2^*, \tag{A24}$$
$$x_5 = X_{0,1,0,0}(\mathbf{q}) = -x_2, \tag{A25}$$
$$x_9 = X_{1,0,0,0}(\mathbf{q}) = -x_2^*, \tag{A26}$$
$$x_{10} = X_{1,0,0,1}(q) = x_7, \tag{A27}$$
$$x_{11} = X_{1,0,1,0}(\mathbf{q}) = x_6^*, \tag{A28}$$

$$x_{12} = X_{1,0,1,1}(\mathbf{q}) = x_8^*, \tag{A29}$$
$$x_{13} = X_{1,1,0,0}(q) = x_4, \tag{A30}$$
$$x_{14} = X_{1,1,0,1}(\mathbf{q}) = -x_8, \tag{A31}$$
$$x_{15} = X_{1,1,1,0}(\mathbf{q}) = -x_8^*. \tag{A32}$$

The interactions $\widetilde{x}_n$ are obtained by multiplying the integrand by $e^{-yd/\ell}$. The interactions $\widehat{x}_n$ and $\widehat{\widetilde{x}}_n$ are obtained by removing the imaginary term i and the phase factor.

The combinations:

$$H_i = h_i - \widetilde{h}_i, \tag{A33}$$
$$T_i = h_i + \widetilde{h}_i, \tag{A34}$$
$$X_i = x_i + \widetilde{x}_i, \tag{A35}$$
$$U_i = x_i - \widetilde{x}_i. \tag{A36}$$

To define $\widehat{H}_n, \widehat{T}_n, \widehat{X}_n, \widehat{U}_n$, we follow the same procedure as for $\widehat{h}_n, \widehat{\widetilde{h}}_n, \widehat{x}_n, \widehat{\widetilde{x}}_n$.

Some useful constants:

$$x_1(0) = \sqrt{\frac{\pi}{2}}, \tag{A37}$$

$$x_4(0) = \frac{1}{2}\sqrt{\frac{\pi}{2}}, \tag{A38}$$

$$x_7(0) = \frac{1}{2}\sqrt{\frac{\pi}{2}}, \tag{A39}$$

$$x_{16}(0) = \frac{3}{4}\sqrt{\frac{\pi}{2}}. \tag{A40}$$

Appendix B: MATRIX F_1 FOR THE INTER-LAYER-COHERENT MODES

The collective modes at $\nu = -1$ and $\Delta_B < \Delta_B^{(1)}$ involve the matrices $F_1(q)$ in Eq. (54). The elements of this matrix are defined by

$$A(q) = \widetilde{x}_4(0) - \widetilde{x}_1(0) - \frac{3}{4}x_1(0) - \widetilde{x}_{16}(0) + X_4(q) - H_1(q) + H_4(q), \tag{B1}$$

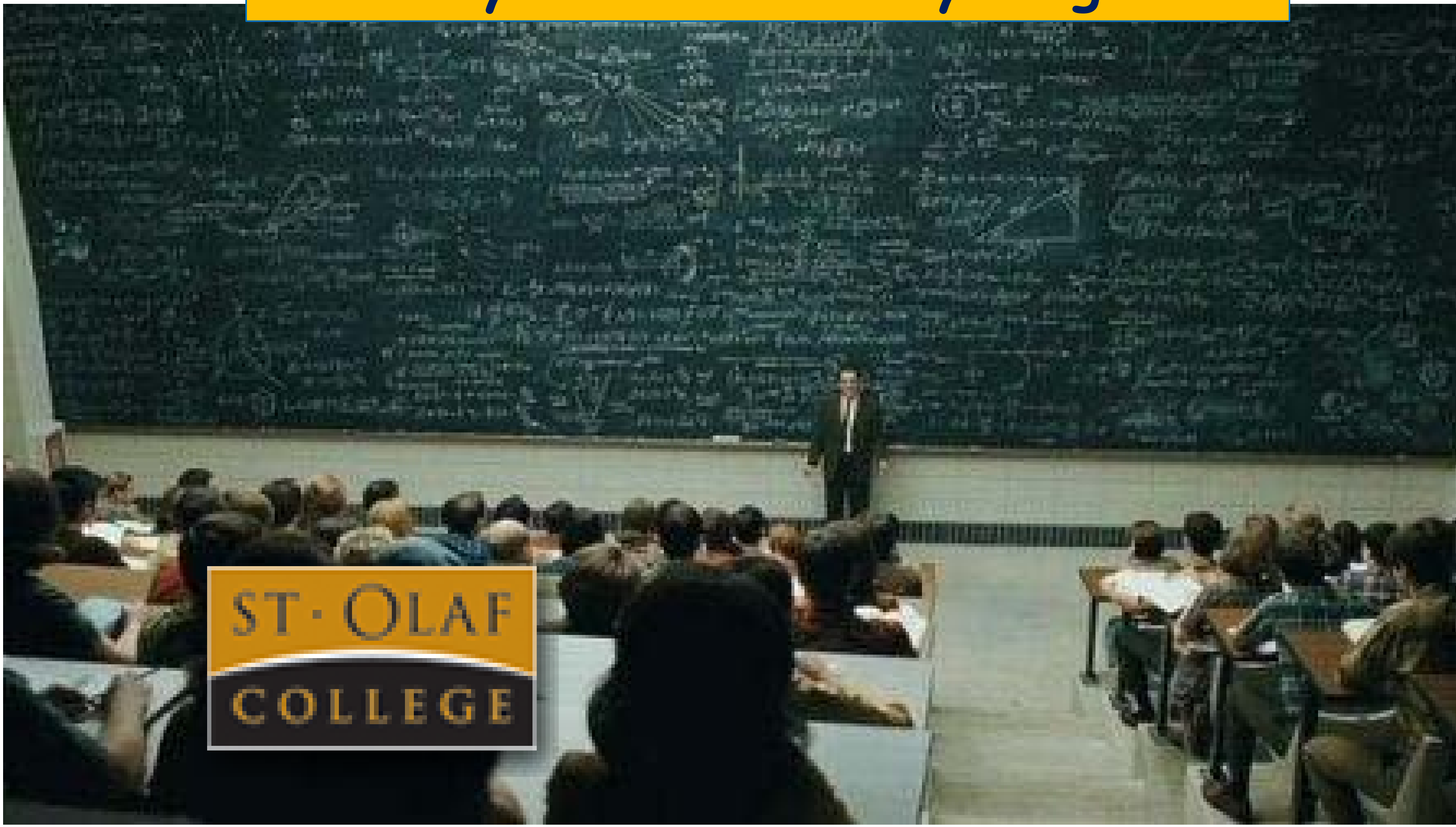
$$B(q) = 2\Delta_B^c(\beta - 1) + H_1(q) - H_4(q) - U_4(q) + \widetilde{x}_1(0) - \frac{5}{4}x_1(0) - \widetilde{x}_4(0) + \widetilde{x}_{16}(0) + 2\frac{d}{\ell}, \tag{B2}$$

$$C(q) = -H_{16}(q) + X_{16}(q) - 2\widetilde{x}_{16}(0), \tag{B3}$$

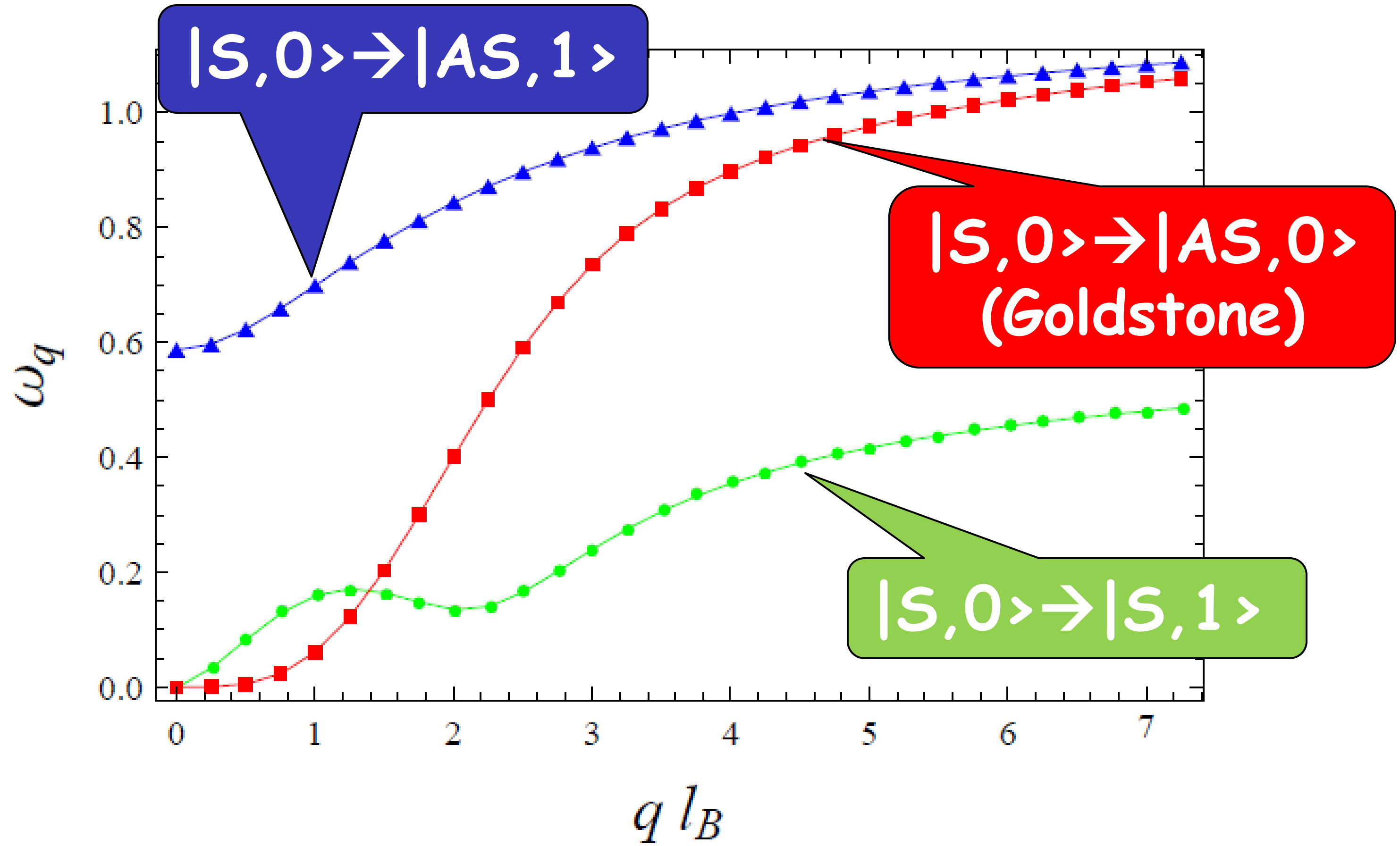
$$D(q) = 2\Delta_B^c(2\beta - 1) + H_{16}(q) - U_{16}(q) + 2\widetilde{x}_{16}(0) - \frac{3}{2}x_1(0) + 2\frac{d}{\ell}, \tag{B4}$$

$$E(q) = T_4(q) + X_4(q) - T_1(q) + \widetilde{x}_1(0) - \widetilde{x}_4(0) - \frac{3}{4}x_1(0) - \widetilde{x}_{16}(0), \tag{B5}$$

` ... and this proves we don't really know about anything ...'

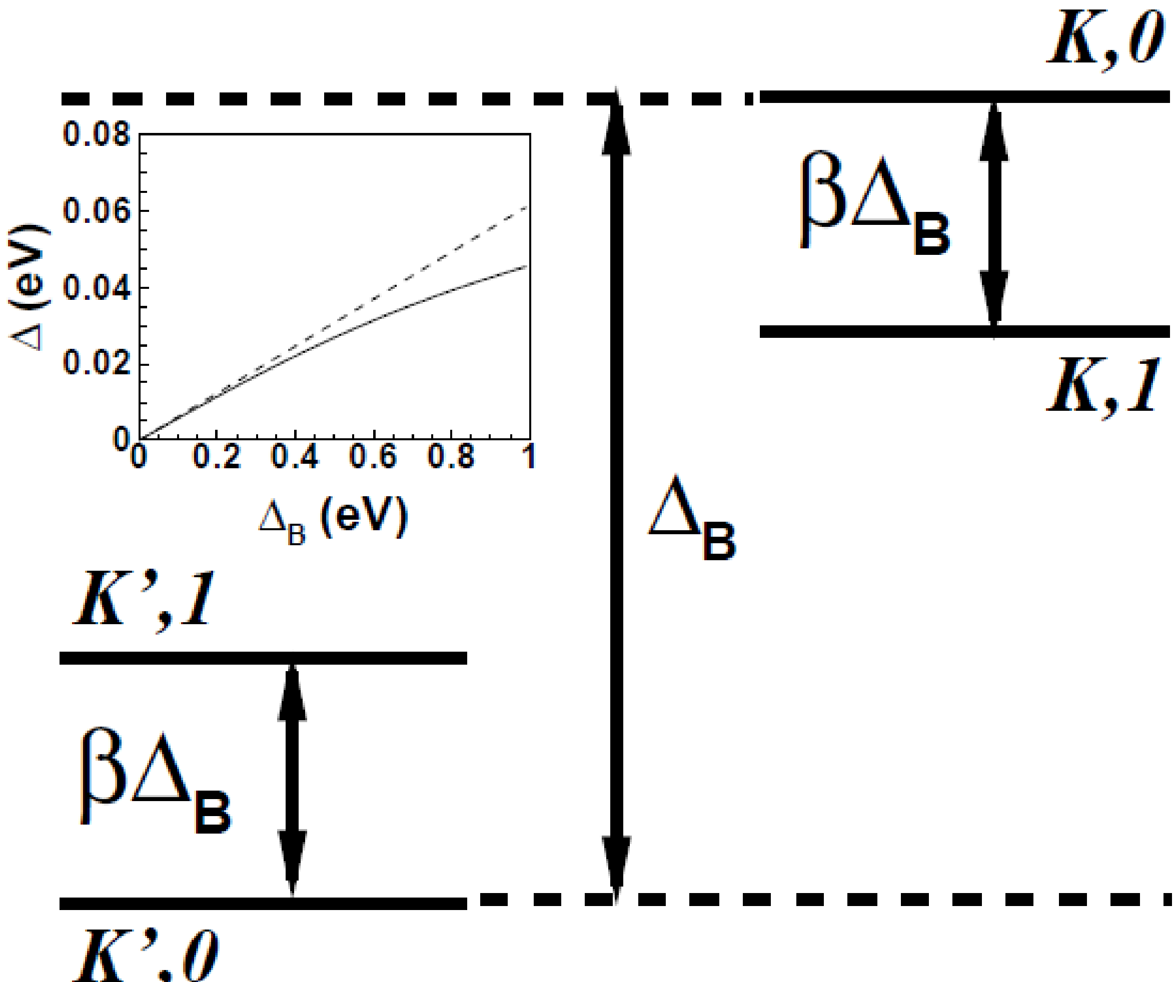


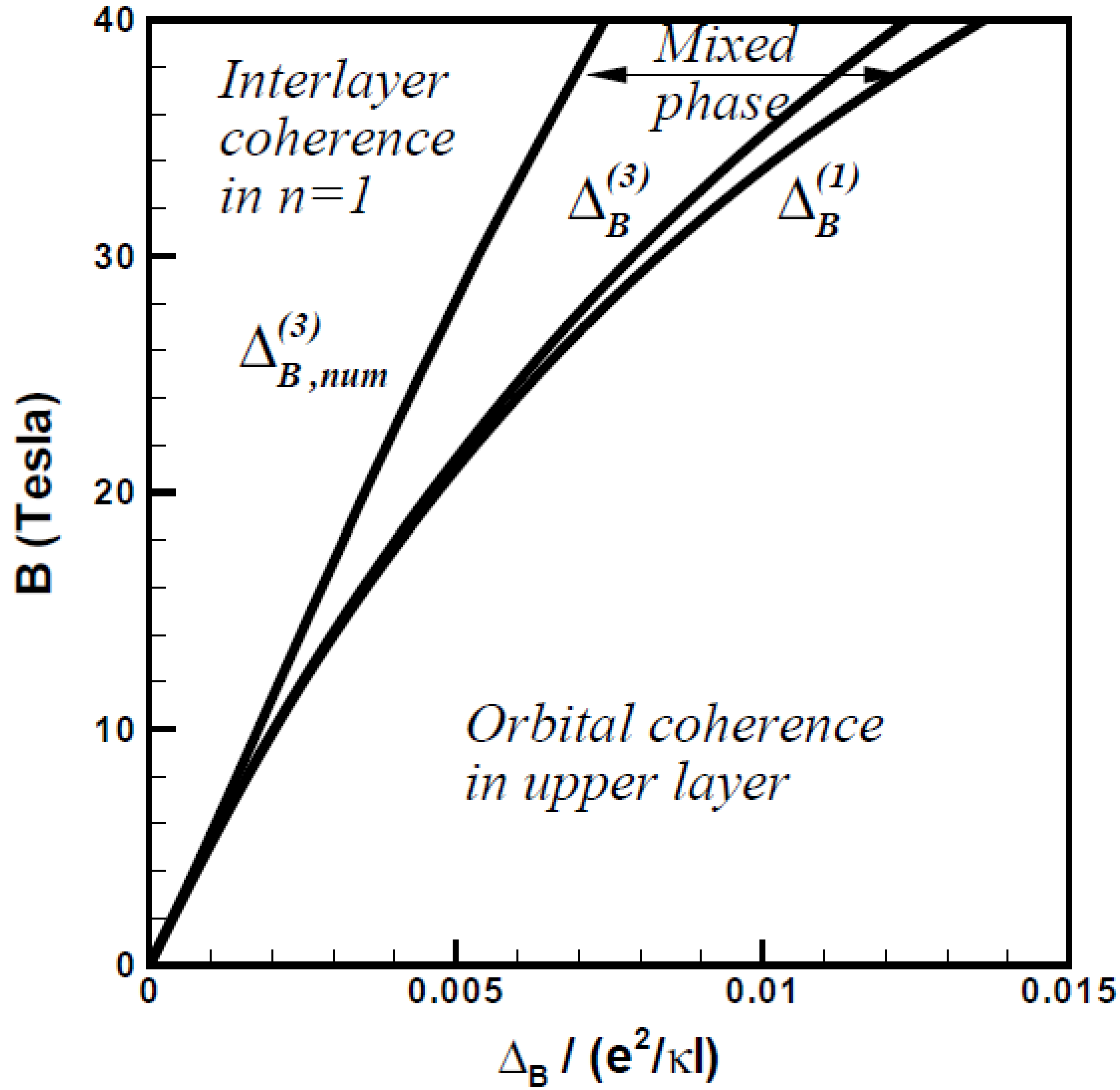
Balanced Bilayer ν=-3,1



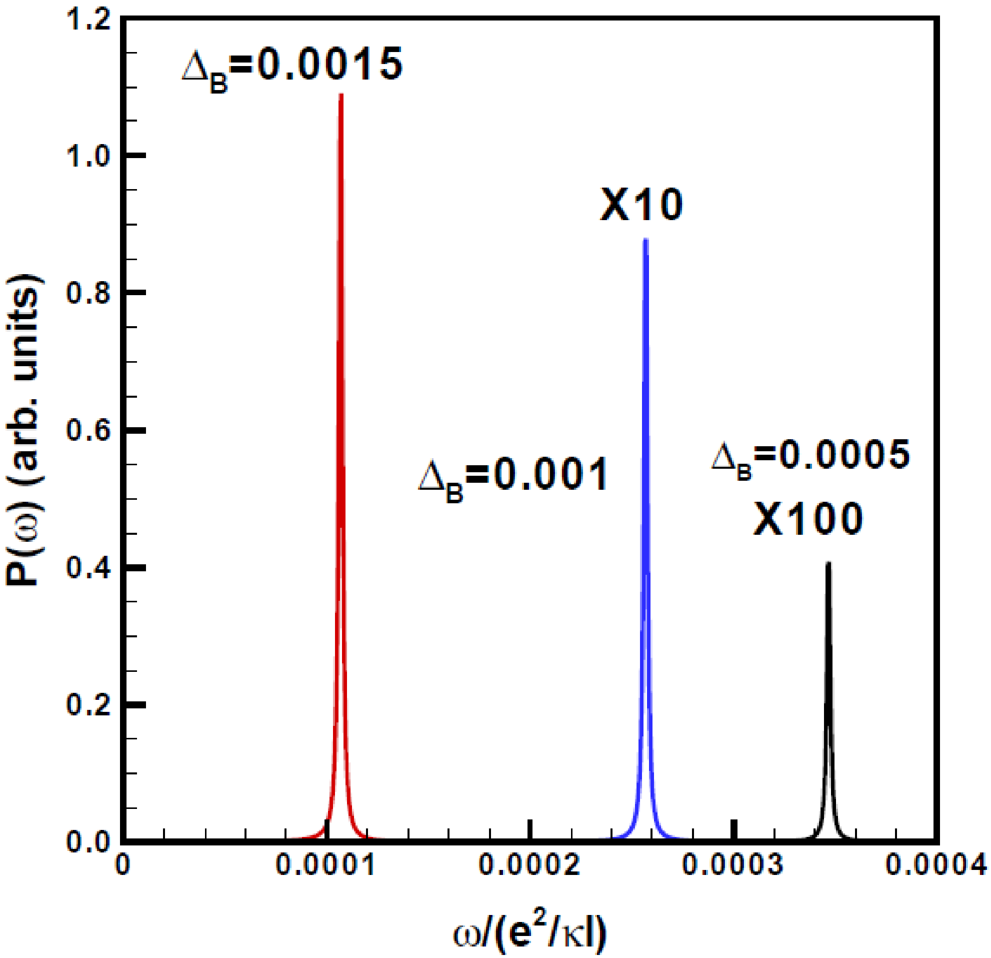
Mean Field Spontaneous Coherence State

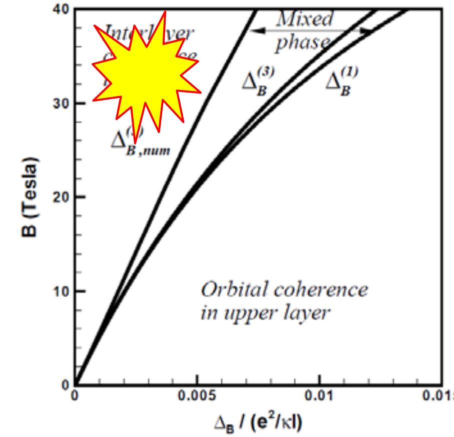
Layer and `Orbital' Fields





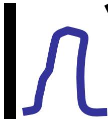
ν=-1,3: B/Δ Phase Diagram

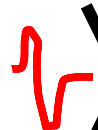


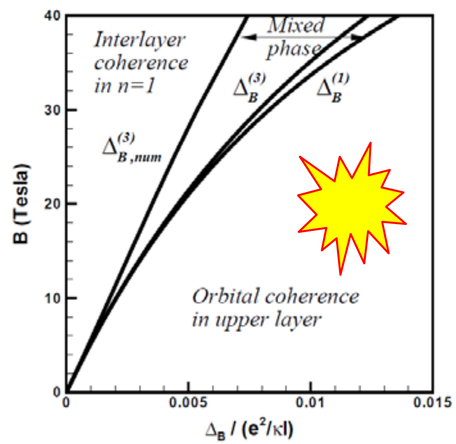


Intra-Landau-Level Cyclotron Resonance

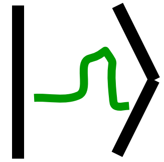
LL Pseudospins

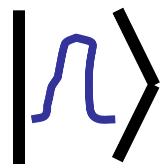
$|n=0\rangle$ = |  ⟩

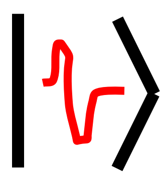
$|n=1\rangle$ = |  ⟩



$m_{\perp}$ = current !!!

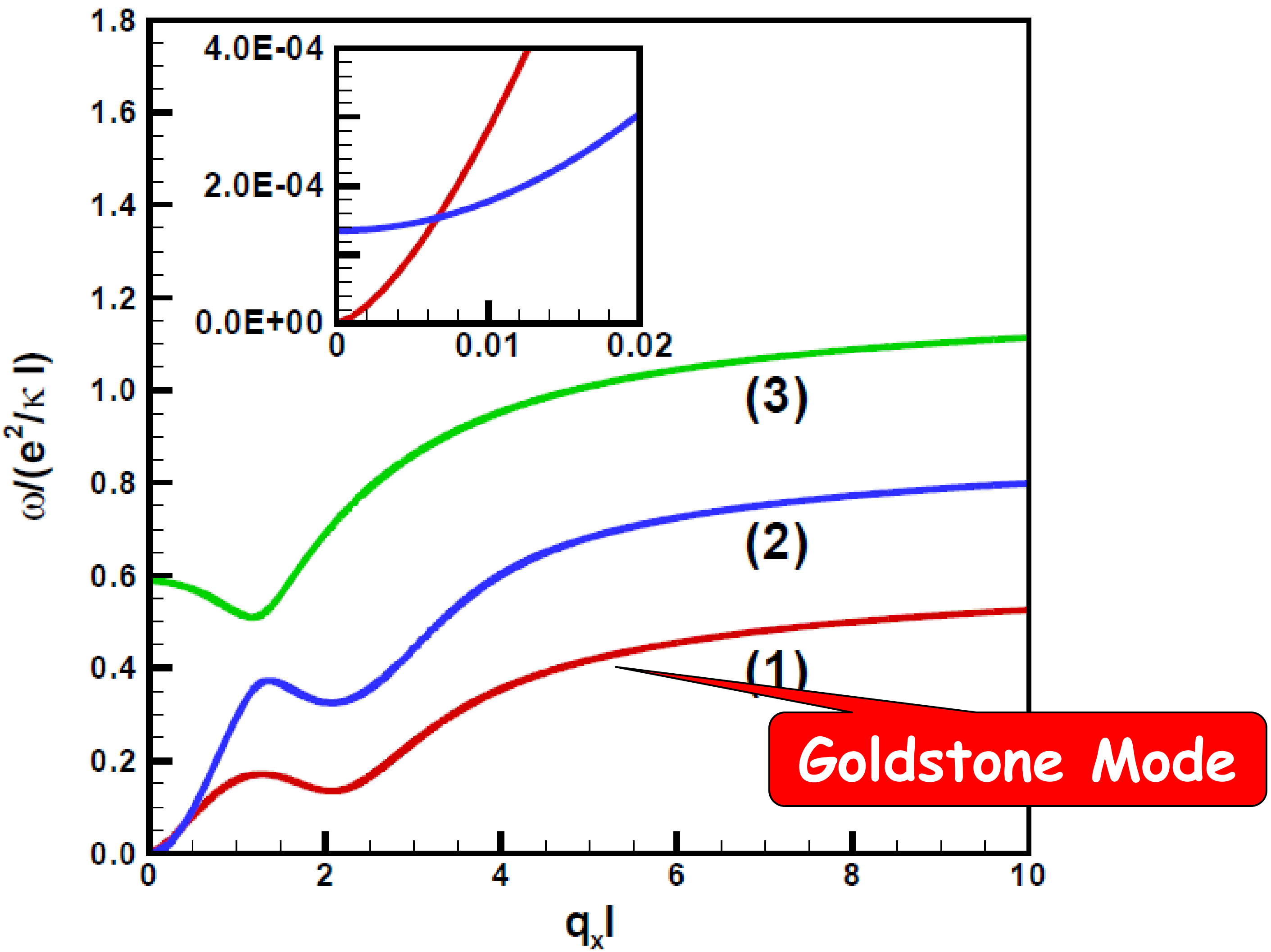
$$|n=1\rangle + |n=0\rangle =$$ 

$$|n=0\rangle =$$ 

$$|n=1\rangle =$$ 

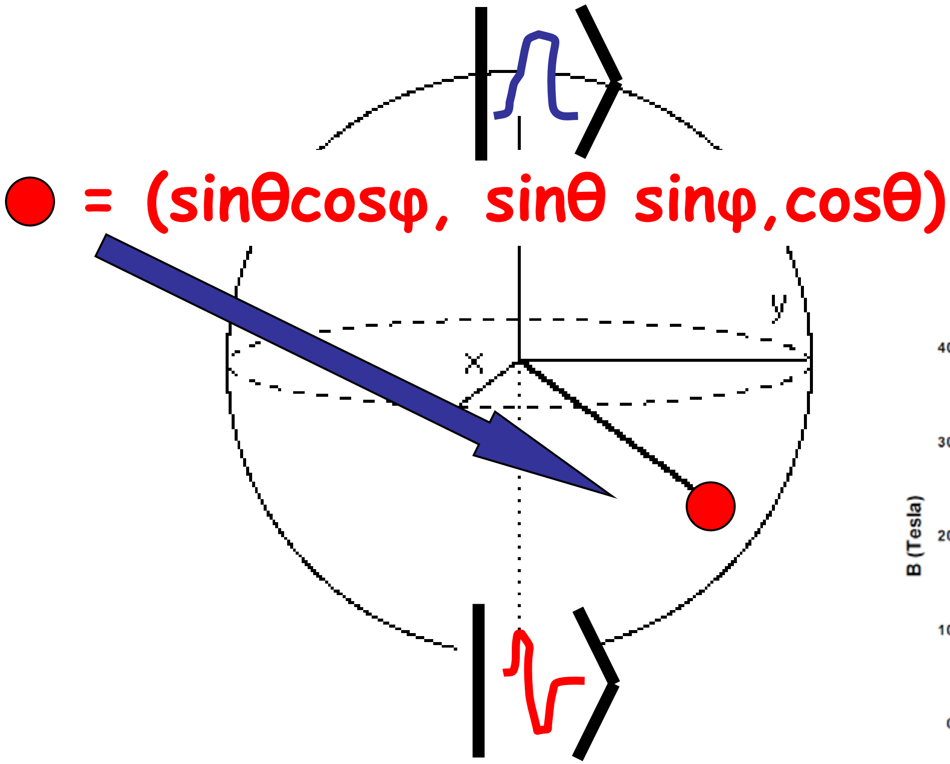
- Exciton condensation occurs in semiconductor *and* graphene bilayers in a strong field
- Dipole-allowed intra-LL transitions
- Spontaneous Current ground states

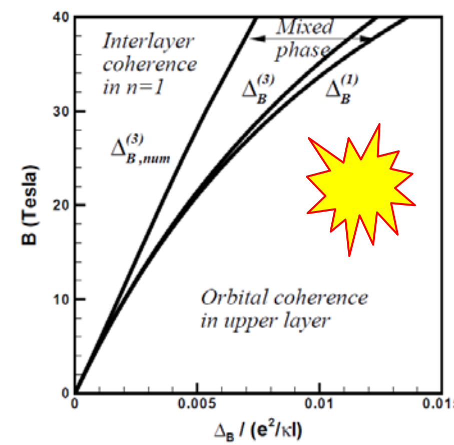
Unbalanced ν=-1,3



Orbital Ferromagnet Collective Modes

LL (Pseudo)Spin Coherent States





$\cos(\theta/2)\,|\ \rangle + \sin((\theta/2)\, e^{i\varphi}\,|\ \rangle$