Word Problem Solving Tasks in Third-Grade Mathematics Textbooks: How Well Do they Align with Effective Teaching Practices?

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Dedication

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Abstract

Learning mathematics is difficult for many students, especially students with mathematics learning disabilities MLD. To improve the academic achievement for students with MLD, educational legislations (IDEA 2004; ESSA 2015) mandate that schools must use evidence-base core curricula. Textbooks conveys the intended values and principles for mathematics instruction and used by teachers to guide their goals and activities. In this study, I conducted a quantitative content analysis to examine the incorporation of five effective teaching practice (clear goals, reasoning and problem solving, visual representations, discourse and conceptual understanding, and strategy instruction) in word-problem solving tasks in six commonly used mathematics textbooks (*Everyday Mathematics*, *Eureka Math*, *EnVision Math*, *Go Math*, *Math Expressions*, *and My Math*). Results from the examined 1457 WPS tasks showed variations in the incorporation of practices with discourse and conceptual understanding being the most incorporated and strategy instruction the least incorporated. A series of ANOVA tests revealed significant differences between textbooks in incorporating effective teaching practices (visual representations, discourse and conceptual understanding, strategy instruction). Thus, teachers should assess textbooks activities with respect to effective teaching practices and use their professional knowledge to supplement the activities to ensure the implementation of effective core curricula for students with MLD.

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Chapter 1

Introduction

Learning mathematics is difficult for many students, especially students with disabilities (National Assessment of Educational Progress [NAEP], 2019). About 10% of school-age children have persistent and significant difficulties with mathematics, while many more (25% to 35%) are at risk for mathematics difficulties and do not reach basic levels of mathematics achievement (Mazzocco, 2007; National Center for Education Statistics [NCES], 2019). According to the Condition of Education report, students with disabilities are spending increasing lengths of time in general education classrooms (NCES, 2019). In fall 2008, approximately two-thirds of students with specific learning disabilities (72%) spent most of their school day in general education classrooms. State and federal mandates (Individuals with Disabilities Education Improvement Act, 2004; No Child Left Behind [NCLB] Act of 2001, 2002) require that all students, including the increasing numbers of students with learning disabilities receiving mathematics instruction in general education classrooms, have access to more challenging and high-quality instructional curricula and that schools be responsible for greater student accountability.

Mathematics reform initiatives have emerged in response to the lack of curriculum coherence and criticisms of mathematics instruction in the United States (Hiebert, 1999; Schmidt et al., 2005). Efforts to increase the rigor of mathematics instruction have led to an emphasis on higher order thinking and problem solving versus memorization and procedural knowledge (e.g., National Mathematics Advisory Panel, 2008). As a result, research reports from professional organizations have highlighted the importance of adapting mathematics instruction to reflect recommendations from mathematics education research and preparing students for college and career readiness (National Research Council & Mathematics Learning Study Committee, 2001; National Council of Teachers of Mathematics Standards [NCTM], 2000). For example, the Common Core State Standards-Mathematics (CCSS-M; National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010) specifies the content to be taught at each grade level to unify the content learned across states and reflect students' learning progression in mathematics. Curriculum developers have revised mathematics textbooks to align with the CCSS-M (NGA & CCSSO, 2010) to address recent calls for rigorous mathematics performance standards. Given that students with mathematics difficulties spend most of their day in general education classrooms and that mathematics textbooks are the most important learning resource for elementary teachers' mathematics instruction (Mullis et al., 2012; Sievert et al., 2019), this study aims to evaluate mathematics textbooks in terms of the extent to which they incorporate effective mathematics teaching practices and provide instructional strategies that are likely to help students learn. This study will assess whether textbooks are satisfactory for use in classrooms where literacy in mathematics is a goal for all students.

The Focus of the Study

This study focuses on word-problem solving tasks involving whole number multiplication and division. Word-problem solving is critical to developing general problem-solving skills as well as proficiency with whole numbers and plays an integral part in doing mathematics (Verschaffel et al., 2007). Despite the importance of problem-solving, many elementary students encounter difficulties with contextual problems. For example, the NAEP (2019) reported that only 22% of the U.S. fourth graders who participated in the assessment were able to solve a contextual mathematical problem compared to 58% who were able to solve non-contextual problems. Solving arithmetic word problems requires creating a mental representation of the problem situation (i.e., a situational model) and transforming that situational model into a mathematical model (Hegarty et al., 1995; Verschaffel et al., 2020). For students with mathematics learning disabilities or difficulties (MLD), WPS is even more challenging. Students with MLD tend to have difficulties with mental representation, reading comprehension, and performing basic arithmetic operations (Andersson, 2008; Fuchs et al., 2010; van Garderen & Montague, 2003; Vukovic & Siegel, 2010). In addition, students with MLD show difficulties with domain-general skills that are critical for WPS such as working memory, attentive behavior, metacognition, and language (Andersson, 2008; Cirino et al., 2016; Fuchs et al., 2006, 2010; Geary, 2004, 2013; Geary et al., 2007; Hansen et al., 2015; Jordan et al., 2013; Seethaler et al., 2016; Zheng et al., 2011). Core mathematics textbooks must promote teachers to implement high-quality core instruction in word-problem solving to improve mathematics outcomes for students with MLD. In the following sections, I provide an overview for effective teaching practices in mathematics. Then, I briefly review the literature and discuss the rationale and research questions for the current study.

Effective Teaching Practices in Mathematics

Instructional design principles are important to consider when developing interventions for students receiving special education. However, with increasing numbers of students with MLD receiving instruction in general education classrooms and being held to the same standards as their non-disabled peers, it is important to consider the key mathematical practices for building mathematical understanding and improving problem solving. The Process Standards adopted by the National Council of Teacher Mathematics (2001) have been integrated into the CCSS-M Standards for Mathematical Practice and include the following: (1) Make sense of problems and persevere in solving them, (2) reason abstractly and quantitatively, (3) construct viable arguments and critique the reasoning of others, (4) model with mathematics, (5) use appropriate tools strategically, (6) attend to precision, (7) look for and make use of structure, and (8) look for and express regularity in repeated reasoning.

Identifying effective teaching practices for all students, including students with MLD, receiving instruction in general education classrooms, would require examining recommended teaching practices from both mathematics education and special education. The NCTM (2014) published the *Principle to Actions Ensuring Success for All* to clarify the essential features to successful learning (e.g., teaching, equity, tools, technology, and assessment). The NCTM (2014) describes eight effective teaching practices to improve all students' learning of mathematics. In addition, Park et al. (2021) and Woodward et al. (2018) describe effective practices gleaned from the mathematics education and special education literature to enhance mathematics teaching. Below I provide a brief overview of relevant mathematical effective teaching practices gleaned from the NCTM (2014), Park et al. (2021), and Woodward et al. (2018).

Clear Goals to Focus Learning

Mathematical goals that are clear and conspicuous indicate, "what mathematics students are to learn and understand as a result of instruction" (NCTM, 2014, p. 12). Explicitly stating the goal can make the lesson engaging and motivating for all students (Buzza & Dol, 2015; Wiliam, 2007; Marzano, 2003; McTighe & Wiggins, 2013). Goals should be connected to rigorous standards to support teachers in their instructional decisions (Charles & Carmel, 2005; NCTM, 2014). Research suggests that classrooms with clear goals and expectations is associated with improved learning outcomes, indicating the importance of including clear lesson goals (Hattie, 2008; Haystead & Marzano, 2009).

Reasoning and Problem Solving

Reasoning and problem solving are critical for the learning of mathematics. High-cognitive demand tasks promote reasoning and problem solving through actively engaging students in doing mathematics (e.g., reversibility tasks, multiple-step problems, real-world or non-routine problems). In addition, mathematics instruction could promote reasoning and problem solving through discussions that build flexibility or draw students' attention to general mathematical patterns or structures (Dougherty et al., 2015; NCTM, 2014; Woodward et al., 2018). Research shows that students with MLD benefit from instruction that involves high cognitive demand tasks and tasks that incorporate elements of flexibility and doing mathematics (Jitendra et al., 2017; Yang & Xin, 2021)

Visual Representations

Understanding arithmetic WPS, which is necessary for proficient problem solving, involves recognizing the underlying mathematical structure of the problem situation. Knowledge of the mathematical structures, which consist of elements and relations between those elements, is critical to constructing a mental representation that is coherent and complete (Carpenter et al., 2015). Representations involving the use of symbols, drawings, or physical objects to represent mathematical ideas play a major role in understanding mathematics (Goldin & Kaput, 1996). Representations can help students to record information in the problem text, which can offload memory storage and allow students to engage in the problem-solving processes of visualizing the abstract relations between the elements in the problem and reasoning about the problem situation to solve the problem (Zahner & Corter, 2010). There is evidence that instruction that emphasizes connecting representations with mathematical ideas, provides opportunities to select from multiple representations, and has students generate diagrams can improve student learning (NCTM, 2014; Park et al., 2021;Woodward et al., 2018). Results of several meta-analyses indicate that students with MLD benefit from instruction that focuses on the underlying mathematical structure of problem situations using representations (Gersten et al., 2009; Jitendra et al., 2015; Lein et al., 2020; Zhang & Xin, 2012).

Discourse and Conceptual Understanding

Mathematical discourse involves having students share their ideas about their mathematical thinking in verbal, written, or visual formats. Mathematical discourse is critical for students' learning as it provides opportunities for students to exchange ideas, correct misunderstandings, and develop mathematical arguments and language (NCTM, 2014; Park et al., 2021; Woodward et al., 2018). Mathematical discourse does not simply involve having students share their ideas but also includes posing questions that require reasoning and making connections between students' thinking and core mathematical ideas (NCTM, 2014). Meaningful discourse may require posing questions that allow students to reason, defend, and explain their approaches and connect their approaches to alternative and perhaps more efficient approaches. Meaningful discourse (e.g., discussion around why some procedures work to solve particular problems) can also promote students' conceptual understanding of mathematical ideas and procedures leading to better retention and generalization of the procedures (Fuson et al., 2005). Research shows that students who struggle in mathematics may have some difficulties participating in mathematical discourse (e.g., they often use one-word response) and that engaging all students in mathematical discourse requires thoughtful preparation (Baxter et al., 2001). Recent experimental studies reported promising results for engaging students with MLD in mathematics discourse or features of mathematical discourse (e.g., student's explanation) (Fuchs et al., 2016; Jayanthi et al., 2021; Xin et al., 2020).

Strategy Instruction

Cognitive strategies are search and transformation strategies used to understand and solve the problem and increase the chance of reaching an accurate solution (Verschaffel et al., 2020). Meta-cognitive strategies involve monitoring and regulating thinking when solving the tasks. Students use cognitive and metacognitive strategies to understand, plan, and solve problems and to monitor their thinking (Dinsmore, 2017). However, students' use of cognitive strategies may differ in terms of their accuracy and efficiency. Students with MLD demonstrate difficulties in identifying, applying, and monitoring strategies during problem solving (Montague & Applegate, 1993; Powell et al., 2020). Multiple studies reported positive effects for instruction that incorporated cognitive and meta-cognitive strategies during whole classroom instruction (Mevarech & Kramarski, 1997; Verschaffel et al., 1999) and for students with MLD (Jitendra et al., 2016; Jitendra et al., 2017a; Jitendra et al., 2017b).

Prior Research

Prior research on mathematics textbook analysis (e.g., Bryant et al., 2008;Doabler et al., 2012) has mainly focused on textbooks' adherence to instructional design principles (e.g., systematic instruction, discrimination practice, corrective feedback). Three studies (Jitendra, Griffin, et al., 2005; Sood & Jitendra, 2007) examined whether textbooks incorporated standards from mathematics education. Jitendra et al. (2005) and Jitendra, Griffin et al. (2007) assessed textbooks' adherence to the five Process Standards (NCTM, 2001) and generally found low incorporation of reasoning, communication, connections, or representation across the five textbooks. Recentally, Nelson et al. (2020) evaluated the extent to which one core textbook promoted the Standards for Mathematical Practice (CCSS, 2010). Although Nelson et al. (2020) concluded that the textbook adhered to Standards for Mathematical Practice, the result of the study is inconclusive given the limited sample (one textbook) and the ambiguous process used to rate the textbook (i.e., it was unclear whether the authors rated individual lessons).

Three studies assessed the incorporation of high-cognitive demand tasks in mathematics textbooks in the United States. (Jones & Tarr, 2007; Polikoff, 2015; Son, 2012). The studies differed in their coding of high cognitive demand tasks (e.g., making conjectures and generalization, doing mathematics, or strategic and extended thinking). Despite the discrepancies in how these studies defined high-cognitive demand tasks, findings from these studies were consistent in that the incorporation of high-cognitive demand tasks was generally low in the textbooks.

In addition, studies examined textbooks' use of visual representations, including the nature (given or generated), quantity, and type of the representation tasks as well as the level of involvement for students and teachers in the representation tasks. Collectively, results of these studies indicate that representations tended to be completely or partially given to students and rarely generated by students (Van Garderen et al., 2012), were more concrete and less symbolic (Sood & Jitendra, 2007), and were used more for solving application problems than during instruction (Bryant et al., 2008; van Garderen et al., 2012).

Rationale and Research Questions

Mathematics textbooks can influence teachers' instructional decisions and students' outcomes in mathematics (Blazar et al., 2020; Woodward & Brown, 2006). Given that an increasing number of students in general education classrooms have learning disabilities in mathematics or are at-risk for mathematics difficulties, it is important to judge the quality of mathematics textbooks mainly on their effectiveness in helping teachers implement high-quality instruction that supports students to achieve important mathematics learning goals. Although most of the mathematics textbooks claim to be aligned with contemporary standards in mathematics (e.g., CCSS, 2010), the background of designing and publishing mathematics textbooks differ which might have led to differences in incorporating effective teaching practices. For example, some of the mathematics textbooks were funded by federal agencies such as the National Science Foundation to develop mathematics curriculum aligned with contemporary reforms in mathematics (e.g., *Math Expressions*) and others by commercial publishers (*enVision Mathematics*). In addition, the developers of some textbooks were mainly researchers (e.g., *Everyday Mathematics*) and for other textbooks the developers were mainly mathematics teachers (e.g., *Eureka Mathematics*). To this end, this is the first study to evaluate the extent to which mathematics textbooks promote the use of contemporary effective teaching practices.

This study extends the literature in several ways. First, this study examines the extent to which instruction in textbooks reflects mathematics teaching practices informed by recent research and policy recommendations that are effective for all students, including students with MLD (Dougherty et al., 2015; NCTM, 2014; Park et al., 2021; Woodward et al., 2018). Specifically, the current study examines the following effective teaching practices gleaned from special and mathematics education literature: (1) establish mathematical goals to focus learning, (2) promote reasoning and problem solving, (3) use and connect mathematical representations, (4) facilitate discourse and pose purposeful questions, and (5) use cognitive and metacognitive strategies. Second, given that the intended curriculum is a factor that influences the experienced curriculum, it is important to assess the extent to which textbooks differ in their use of mathematical practices. This study will examine WPS tasks to provide insight into the status of teaching practices promoted in mathematics textbooks after the CCSS-M (NGO & CCSSO, 2010).

The purpose of this study is to evaluate textbooks on their use of effective mathematics teaching practices to enhance students' WPS performance. Specifically, this study will assess whether textbooks differ in their use of mathematical practices. The study will address the following research questions:

1. To what extent are mathematics teaching practices incorporated in WPS tasks in third-grade mathematics textbooks?
2. Are there significant differences between textbooks in incorporating effective teaching practices?

This study's textbook evaluation has several distinct features (e.g., focus on an important learning goal, use of research-based instructional criteria) that will contribute to the literature. First, an examination of the extent to which effective teaching practices are incorporated in mathematics textbooks will help identify the strengths and weaknesses of the textbook as a whole in its instructional design and support and hence inform curriculum developers and teachers whether certain practices need to be included to improve students' word problem-solving performance. Second, by providing ratings for each textbook on five criteria derived from research on learning and teaching mathematics, teachers can consider the quality of the textbook and whether it offers advice on how to use the criteria to guide their classroom activities and support student learning. If textbooks incorporate some practices or instructional strategies less often (or not at all) than others, do teachers have to rely on their own craft knowledge to guide their instruction? Such instruction may be straightforward when establishing mathematical goals to focus learning but may require knowledge of experienced teachers to address the more challenging practices and strategies.

Chapter 2

Literature Review

Studies show that mathematics skills and achievement may predict college success and future income (Dougherty, 2003; Leuwerke et al., 2004; Watts, 2020). For example, Watts (2020) reported that a one standard deviation increase in mathematics achievement at age 16 is associated with a 14% to 18% increase in earnings at age 33 and 50 respectively. The association between mathematics achievement and future earnings remain large and significant even when controlling for academic (reading achievement), family background (e.g., parents' occupation and income), and behavioral variables (e.g., attention and social skills), which indicates the strength of the association between mathematics achievement and future income. In addition, proficiency with mathematics can play a crucial role in promoting equity as it provides access to social mobility for disfranchised groups including students from minoritized backgrounds and students with disabilities (Schoenfeld, 2004). Given the role of mathematics on future achievement and social mobility, researchers designed programs aiming to improve students' mathematics outcomes (e.g., Watts, 2020).

Attempts to improve students' achievements in mathematics in the U.S spanned over the last century. Following the Second World War and in response to the Soviet Union's technological advances, mathematics education in the U.S received criticisms (students' lack of conceptual understanding and inability to generalize learned concepts to different disciplines) resulting in several programs aiming to prepare students for college and scientific workforce (Kilpatrick, 2014; Schoenfeld, 2004; Woodward, 2004). These programs collectively are called the "New Math Era" that emphasizes structure, abstraction, and generalization as early as elementary level. The perceived decline in college admission scores and lack of teacher preparation to implement the New Math accelerated the “Back to Basics” movement that focused largely on procedural skills, followed by other reform efforts focusing problem-solving and standards. These are other instances of reform efforts in mathematics education in the U.S. and a full review of the reform attempts is outside of the scope of this paper.

Among several factors in mathematics education (e.g., professional development), mathematics textbooks received special attention in the reform efforts due to their central role in the educational process. In the following sections, I provide an overview of contemporary reform efforts related to mathematics textbooks and policies in the U.S. I also discuss theoretical frameworks for the role of textbooks in mathematics instruction, and the role and theoretical frameworks for problem solving and its relation with students with disabilities. This paper concludes with reviewing the literature on effective teaching practices and mathematics textbooks analysis.

Policies and Historical Background in Mathematics Textbooks

Several influential publications and policy documents provided frameworks for reforming and improving mathematics education in schools. Some of these documents are directly related to mathematics instruction and curricula (e.g., NCTM, 1989, 2001; CCSS, 2010) and others are related to core instruction in general (NCLB, 2001; IDEA, 2004; Every Student Succeeds Act [ESSA, 2015]). Below I discuss these documents and their relation to core mathematics curricula.

The NCTM and CCSS Standards

The NCTM led the reform efforts in core mathematics instruction as it published a series of documents over the last two decades outlining standards for mathematics curricula in terms of content, processes, proficiency, assessment, and teaching methods. The NCTM (1989) released the influential standards for curriculum and evaluation that outlined goals for society (e.g., mathematically literate worker) and mathematics standards (e.g., problem solving, connection) for bands of grade levels (K-4, 5-8). The NCTM (2000) revised ideas addressed at some of the grade levels in the NCTM (1989) and provided more comprehensive expectations for five mathematical processes (problem solving, reasoning and proof, connections, communication, connection, and representation).

The NCTM (1989, 2000) prompted discussions and mandates that incentivized states to adopt challenging standards for performance and assessment. For example, receiving the supplemental assistance provided under Title I of the Elementary and Secondary Education Act (1994) was tied to the states' adoption of challenging standards. Ultimately, the Common Core States Standards (CCSS, 2010) was introduced as a set of rigorous expectations for all states that aim to prepare students for college and workfare. For mathematics, the CCSS (2010) outlined the content to be addressed at each grade level and the standards for mathematical practices that teachers should seek to develop in their students. The CCSS (2010) combined the NCTM (2000) standards with five strands for proficiency in mathematics outlined in *Adding it Up* (adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition) and described eight standards for mathematical practices: (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure; and (8) Look for and express regularity in repeated reasoning.

These standards provided a framework for developing a nationwide coherent curriculum, instruction, materials, and assessments (Hekimoglu & Sloan, 2005; Kilpatrick et al., 2001). The standards delineated what students should learn, what processes to develop in students, and what it means to be proficient in mathematics. Following the publications of the standards, many textbook developers claimed that their textbooks aligned with the standards, although these claims need further investigation as studies showed some misalignments between the standards and mathematics textbooks or minimal changes in the textbooks following the standards (Polikoff, 2015). In sum, the NCTM standards (1989, 2000) and CCSS in mathematics (2010) laid the foundations for developing coherent mathematics curricula, and there is a need to study the extent to which these standards are integrated into mathematics textbooks.

Policies

In addition to the standards outlined by the NCTM (1989, 2000) and CCSS (2010), several pieces of legislation were mandated in attempts to improve the core mathematics instruction and subsequently students' mathematics outcomes. These legislations are extensive and in this section, I discuss aspects related to core instruction. The NCLB Act (2001) mandated that schools should use scientifically based curriculum and tied federal funding to improved scores in high-stakes assessments. In the NCLB reauthorization, the ESSA (2015) further reiterated that schools should use evidence-based instruction in the core curriculum to help all students achieve at the highest level possible.

For students with special needs, the IDEA (2004) introduced a new framework for delivering services to students with learning disabilities (LD): Response to Intervention (RTI). In the RTI framework, a greater emphasis is directed toward implementing a *scientifically based comprehensive "core curriculum"* (Tier 1) for students with LD before they are considered for small group intervention (Tier 2) or special education evaluation and placement (Tier 3). Since the passage of IDEA (2004), most of the Tier 1 related research in special education focused on assessing the effects of Tier 1 researcher-developed programs on mathematics outcomes for students with LD. Only a few studies examined what teaching practices are being implemented in the core curriculum and whether these practices are scientifically based as mandated by IDEA and ESSA. As such, this study is important as it provides an insight into the extent to which effective teaching practices are communicated to teachers through textbooks, which might have an impact on using effective teaching practices in the core curriculum.

The Role of Mathematics Textbooks in Mathematics Education

Mathematics textbooks are critical for conveying the intended values and principles for mathematics instruction. For example, the School Mathematics Study Group published a textbooks series to convey the reform ideas during the New Math era. Similarly, the University of Chicago School Mathematics Project developed a series of textbooks that are aligned with the reform standards in the last two decades. Below I discuss theoretical frameworks explaining the role of textbooks in mathematics instruction.

Theoretical Frameworks for the Role of Mathematics Textbook

Interest in exploring the relations between various components of mathematics curricula has increased over the last two decades and several frameworks explaining the role of textbooks in mathematics instruction have emerged (Rezat et al., 2021; Stein et al., 2007). One of the earliest frameworks differentiates between four levels of curricula: the written, intended, enacted, and experienced curriculum (Stein et al., 2007). In this framework, textbooks are placed at the written level along with policy documents and other resources that might guide teacher planning of lessons (intended curriculum). Another framework proposes similar but different layers of curriculum, which includes the intended, potentially implemented, implemented, and attained curriculum (Rezat et al., 2021). In this framework, mathematics textbooks are placed in the potentially implemented curriculum level and are separated from policies that guide the education system.

Regardless of the framework, the relation between the different components of the curriculum is interactive in nature (Remillard & Heck, 2014; Stein et al., 2007). Teachers bring their own beliefs and goals as they use textbooks (potentially implemented curriculum) to plan and implement their lessons (implemented curriculum). Further, teachers' experiences during the implementation of the lessons and students' learning and outcomes (attained curriculum) shape their future intended curriculum and adaptations of the textbooks. Textbooks as a potentially implemented curriculum serve a link between national educational policies and teachers' instructional decisions before and during the lessons, underscoring the importance of examining textbooks as they are the conveyers of the principles and values in policies and the standards related to mathematics.

Mathematics Textbooks and Teacher Decisions

As mentioned earlier, the relation between textbooks and teacher implementation is interactive. Thus, researchers studied how teachers use textbooks as well as teachers' decisions associated with different textbooks (Fan et al., 2021; Rezat et al., 2021). The former line of research focused on the changes teachers make as they use textbooks and the latter examined whether teachers' decisions differed based on textbooks they used. Teachers' adaptations of textbooks include omitting, replacing, changing, and creating new instructional components (Sherin & Drake, 2009). Additionally, teachers differ in how they used textbooks, with some teachers following textbooks closely and other teachers using textbooks as a source of ideas among other sources (Haggarty & Pepin, 2002). Research shows that more experienced teachers use the textbook as a source of ideas and less experienced teachers follow the textbook closely

(Haggarty & Pepin, 2017), underscoring the importance of selecting sound mathematics textbooks especially for novice teachers.

Mathematics Textbooks and Teaching Practices

Several studies attempted to explore the link between textbooks and instructional practices used by teachers. Krammer (1985) assessed the correlation between the teaching practices incorporated in three mathematics textbooks and the observed teaching practices implemented by teachers. They found significant correlations for four practices: posing high-cognitive demand tasks, independent work, mathematics discourse, and remedial support—indicating these practices are more likely to be implemented by teachers if they are emphasized in the textbooks. However, textbooks' emphasis on other teaching practice (e.g., reviewing prerequisite skills) was not associated with greater implementation by teachers. Possible explanations for discrepancies between the practices emphasized in the textbook and teacher decisions include teachers' beliefs, orientation, training, or experience regarding these practices (Haggarty & Pepin, 2017; Remillard & Heck, 2014).

Recently, Son and Kim (2015) found similar results regarding high-cognitive demand tasks as teachers who used textbooks that incorporated high-cognitive demand tasks tend to pose high-cognitive demand tasks more often and vice versa. Fan (2000) assessed the extent to which teachers use teaching strategies aligned with their textbooks' orientation (reform vs. traditional) and found that teachers who used a reform-oriented textbook are more likely to have students work in small groups and less likely to use demonstration in the classrooms compared to teachers using traditional textbooks. The results of these studies showed a link between textbooks' emphasis of some teaching practices and teachers' use of these practices, although this relation is not experimentally tested.

Mathematics Textbooks and Student Outcomes

Researchers have also examined the relation between features of mathematics textbooks and students' mathematics achievement (attained curriculum). Blazar et al. (2020) assessed whether students' achievement was dependent on the alignment of textbooks used in their schools with the CCSS (2010). Using the data and ratings of the non-profit organization Edreports (see https://www.edreports.org/), textbooks were designated as meeting the CCSS expectations, partially meeting the expectation, or not meeting the expectations. Results showed minimal correlations between students' achievement and textbooks' alignment with CCSS (2010). It is important to note that Blazar et al. (2020) did not examine the quality of textbooks and relied on Edreport's rating of these textbooks, which covers various aspects of the CCSS (2010) including mathematical practices, differentiated instruction, assessment, and use of technology. As such, textbooks might have received high- or low-quality ratings due to assessment or technological qualities.

In a more focused study, Sievert et al. (2021) conducted a secondary data analysis of the association between the quality of four textbooks on the topic of addition and subtraction of whole numbers and student outcomes in a quantitative comparison test. They rated the lessons on aspects related to representation (e.g., use of manipulatives, iconic representation, symbolic representation, and meaningful representations) and discourse. Sievert et al. (2021) found that higher quality textbooks were associated with higher students' achievement in quantitative comparison. Similarly, Sievert et al. (2019) found that the quality of textbooks in promoting adaptive reasoning (use of multiple strategies) was associated with higher student outcomes in adaptive reasoning. Although these studies are correlational and do not allow casual inference about the relation between textbook quality and student outcome, the results of these studies are promising and warrant further investigation concerning the relation between textbooks and students' outcomes in mathematics.

Remillard et al. (2014) conducted an experimental study to assess the effects of textbooks on students' mathematics outcomes. Remillard et al. (2014) randomly assigned four textbooks to schools. The textbooks differed on their emphases in terms of instructional approach (e.g., student-centered, teacher-directed) and the type of tasks emphasized (high or low cognitive demand task). After two years of implementation, outcomes of students whose teachers used textbooks that were balanced in terms of instructional orientation and type of task outperformed students who received instruction from textbooks that were imbalanced (e.g., high focus on challenging tasks or routine skills). This study shed light on the importance of adopting a balanced emphasis on conceptual understanding, mathematical thinking and reasoning, and procedural fluency when teaching mathematics.

Summary of The Role of Mathematics Textbooks in Mathematics Education

Teachers' adherence to textbooks differs although inexperienced teachers tend to rely on textbooks more heavily. Further, the practices emphasized in textbooks tend to impact teachers' implementation of these practices although this relation might be practice dependent. Results from studies examining the relation between textbooks and students' outcomes were inconsistent with one study reporting no effect and three studies finding effects of textbook quality and features on student's outcomes. Discrepancies in findings between the studies might be due to difference in the design (correlational vs. experimental) or the scope (examining all instructional features vs. focusing on relevant features) of the studies. This study assesses whether textbooks convey messages consistent with effective teaching practices, which can impact teachers' decisions and implementation of some practices and ultimately students' mathematics outcomes.

Mathematical Problem Solving

Problem solving, which is the focus of this study, is defined as goal-oriented tasks that require a sequence of cognitive operations with an unclear pathway to the answer (Jonassen, 2000). Developing proficient problem-solving skills is emphasized in mathematics standards (NCTM, 2000; CCSS, 2010) as these skills are essential for doing and learning mathematics and for everyday life (Bostic et al., 2016). To simulate real-world problem solving, typical mathematics curricula present problem-solving tasks in text format (Hwang & Riccomini, 2016).

There are different conceptualizations for the role of problem solving in learning mathematics. A traditional conception of problem solving is that it is used for application and practice after a new concept is introduced. This notion of problem-solving distinguishes between acquiring knowledge and applying knowledge to real-life situations and uses problem solving as a vehicle for the latter. A more contemporary view of problem solving is learning mathematics in that students develop new mathematical knowledge through problem solving. Introducing new concepts through problem solving is more likely to evoke thinking and reasoning and result in a more robust understanding of the concept (Schoen & Charles, 2003).

Theoretical Frameworks for Problem Solving Instruction

Problem-solving tasks can be categorized into two types: (a) ill-structured non-routine problems that simulate real-life situations and (b) well-structured routine problems that are typically present in mathematics school curricula (Jonassen, 1997). Instruction in ill-structured problems is rooted in situated cognition, which posits that the processes involved in solving a given problem are dependent on the context in which the problem is presented (Jonassen, 1997) and the domain of the problem (Bransford, 1994). As such, processes involved in solving a problem differed based on context and domain.

Jonassen (1997) illustrates that instruction in routine problems is mainly derived from the information-processing theory of cognitive psychology, which states that problem solving involves two cognitive processes: problem understanding and search processes. For example, instruction in problem-solving strategy involves a sequence of steps a problem solver can take including understanding the problem, planning and executing a solution, and checking the accuracy of the answer. Other instruction in routine problem-solving draws from schema theories of cognitive psychology. Schema theory states that students can acquire a given problem type or schema through instruction that draws their attention to the semantic features of the problem type or schema (Sweller et al., 1990). Sufficient opportunities to solve a given problem type might lead to increased fluency and ultimately proficiency in solving that class of problem.

Research on Cognitively Guided Instruction (CGI) identified multiple mathematical structures (e.g., additive and multiplicative) and subsequent problem types (e.g., combine and equal groups). Learners at various developmental levels may evoke certain solution strategies and representations of the problem that range from less efficient to more efficient (Carpenter et al., 2015). The hierarchy of developmental levels for solution strategies include direct modeling, counting, and using number facts. Each solution strategy can be represented at different levels based on the student's level of proficiency (e.g., physical objects, drawings, memory). Carpenter et al. (2015) argue that carefully designed WPS instruction should take into account the learner's current solution strategy and representation as well as the posed problem types.

Problem Solving and Students with LD

Students with LD encounter challenges related to different aspects of learning mathematics. Core difficulties for students with LD involve fact retrieval and number reasoning, including difficulties with recalling, understanding, and abstraction of number knowledge (Gersten et al., 2007; McCloskey, 2007). Difficulties with number reasoning may hinder students' development of conceptual knowledge and their ability to do advanced mathematics(Gersten et al., 2007; McCloskey, 2007). Students with LD also demonstrate difficulties with constructing coherent mental representations for the problem situation, which is critical for solving word problems (Boonen et al., 2013; Peake et al., 2015; van Garderen, 2006). Additionally, students with LD rarely engage in classroom discussions, which is important for making connections and consolidating mathematical ideas (Baxter et al., 2002; Baxter et al., 2001). Additionally, studies show that students with LD tend to use strategies that are developmentally immature or demonstrate an emergent level of strategic competence, such as hand counting (Hunt & Empson, 2015; Ostad & Sorensen, 2007).

Summary

Problem-solving tasks play a central role in mathematics education and differ based on the complexity (routine vs. non-routine) and purpose (application vs. learning) of tasks. Students with LD encounter severe challenges with all types of problem-solving tasks, which limits their opportunity to learn and apply mathematical concepts. This study assesses the extent to which effective teaching practices are conveyed in textbooks to positively impact teachers' implementation of these practices and improve the mathematics outcomes of students with LD.

Effective Mathematics Teaching Practices

Despite the growing interest in designing effective teaching practice, several factors make drawing causal inferences between teaching practices and learning outcomes challenging for researchers (Hiebert & Grouws, 2007). First, the effectiveness of teaching practices might be dependent on the learning goals. In other words, different practices are effective for different learning goals (memorization, conceptual understanding, procedures), which makes it difficult to identify universally effective teaching practices for all learning goals. Second, any given teaching practice (e.g., posing high-cognitive demand tasks) is situated within a teaching system that involves multiple teaching methods that interact with one another (e.g., small group instruction or teacher demonstration). Third, other mediating variables inside and outside of the classroom (e.g., students' attentive behavior, classroom size) interact with teaching practices and methods and affects students learning outcomes. In addition, studies on teaching practices and learning outcomes face methodological challenges in the context of examining causal relations between teaching methods and learning outcomes including designing rigorous studies that account for relevant factors (random assignment) and employ appropriate measures of teaching and learning.

Despite these challenges in inferring causal relations between teaching methods and learning outcomes, general patterns linking teaching practices and learning outcomes can be observed in the literature. In this section, I review evidence from empirical studies that examined the effects of teaching practices on students' learning outcomes.

Clear Goals and Objectives

A growing body of literature underscores the importance of clear and shared learning goals in mathematics instruction. Clear learning goals not only draw students' attention toward the salient concept, method, and ideas to be learned, but also motivate and engage students to work toward the goals. In addition, clear learning goals inform teachers' decisions regarding activities to be included in the lessons, assessment, and adaptations to the lessons. Research suggests that it is insufficient to state only the overall goals of the lesson, instead overall goals should be unpacked to subgoals to make the relation between teaching and learning for the lesson observable (Hattie, 2008).

Empirical studies focusing on learning goals in mathematics are very limited. However, inferences about the role of setting clear and explicit learning goals can be drawn from published meta-analyses and syntheses. Hattie (2008) synthesized the results of over 800 syntheses and meta-analyses regarding factors influencing achievement of school-aged students. Hattie (2009) reviewed a wide array of factors influencing student outcomes such as student, home, school, and teaching approach. Learning goals, which was analyzed under teaching approach, was found to have a large effect on achievement (d=0.56). Although Hattie (2009) focused on the relation between learning goals and achievement in general (not in mathematics), the results of the study support the recommendation in mathematics literature regarding the importance of clear goals for learning.

Reasoning and Problem Solving

Developing a deep understanding of mathematical ideas requires engaging students in high-cognitive demands tasks that require reasoning and problem solving (Hiebert & Grouws, 2007). High-cognitive demand tasks are those that do not have a straight pathway to solutions but are within students' zones of proximal development. Cognitive theorists argue that learners form a new and deep understanding of a topic when solving a problematic situation that induces doubt and perplexity (Hatano, 1988).

Researchers have reported that students who received instruction that place high-cognitive demands performed better on solving complex problems and developed a better conceptual understanding. Carpenter et al. (1989) randomly assigned teachers to receive training in CGI that emphasizes students’ development of problem-solving skills or training in posing and solving nonroutine problems. Teachers in the CGI condition posed more problem-solving tasks (high-cognitive demand tasks) compared to teachers in the nonroutine problems condition who posed more tasks related to number facts. Results showed that students in the CGI condition performed better than students in the nonroutine problems condition in solving complex word problems and in recalling number facts. Problem-solving tasks are only one form of high-cognitive demand tasks and other forms of high cognitive demand tasks (e.g., doing mathematics) were not examined in Carpenter et al. (1989).

Stein and Lane (1996) extended this line of research by including a variety of tasks that are more representative of the construct of high-cognitive demand task. Stein and Lane (1996) examined different types of tasks presented in middle-school classrooms and their relation to enhancing students' problem-solving performance. Tasks were categorized as either high-cognitive (e.g., doing mathematics and procedures with connections) or low-cognitive (e.g., procedures without connections to meaning) demand tasks. Results showed that instruction focused on posing high-cognitive demand tasks was associated with higher gains for students on measures of problem solving, reasoning, and communication.

Similar results emerged from special education research. Fuchs et al. (2009) randomly assigned students with MLD to one of three tutoring conditions (control, automatic retrieval of number combination, or word problem instruction). Students in the automatic retrieval of number combination received instruction that focused on memorizing number combinations, whereas students in the word-problem instruction condition received tutoring that mainly involved solving word problems, which are typically more challenging to students. Results of this study showed that instruction on number combination improved students' number combination skills, but that effect did not transfer to problem-solving outcomes. In comparison, students receiving instruction that focused on problem solving performed better in problem solving and number combination skills. Powell et al. (2015) conducted a similar study and randomly assigned students to two tiers of arithmetic computation instruction or two tiers of problem-solving instruction. The results of this study were consistent with Fuchs et al. (2009) as students receiving problem-solving instruction performed better on problem-solving, calculation, and pre-algebraic knowledge measures, whereas students in the calculation group only performed better on calculation skills.

Procedural Flexibility

The importance of procedural flexibility and the ability to choose among multiple strategies when solving a problem is well established in mathematics education research. Theories from cognitive psychology explain the processes in which children develop and use multiple strategies. The development of new strategies is typically a result of a successful learning experience (Bransford et al., 2000). That is, when students have a good conceptual understating of a given concept, they recognize the usefulness (e.g., efficiency) of using and generating new strategies that are more accurate, efficient, feasible, or generalizable.

Flexibility with procedures or solution methods was studied extensively in mathematics education research. Blöte et al. (2001) compared the effects of instruction that promote procedural flexibility (considering and discussing multiple procedures) to traditional instruction in which solution methods are introduced rigidly. Blöte et al. (2001) found that students who received flexibility-focused instruction performed better on measures of use and preferences of procedures as well as solving problems using multiple strategies.

In the topic of algebraic equations, Star and Seifert (2006) directly compared the effects of multiple strategy instruction (students solve the problem in two different ways) to having students solve a series of distinct problems. The findings showed no difference between the groups on accuracy; however, students receiving multiple strategy instruction used multiple strategies and invented new strategies more frequently. Rittle-Johnson and Star (2007) extended this line of research by examining the effects of comparing different solution methods on solving equations. Rittle-Johnson and Star (2007) randomly assigned students to receive instruction that focused on comparing different solution methods or reflecting on solution methods. Results showed that although students in the two treatment conditions performed similarly on a measure of conceptual understanding, students in the treatment condition performed better on measures of procedural knowledge and flexibility. These studies showed that students benefited from instruction geared toward the development of procedural flexibility. However, it is unclear whether the studies included students with MLD and intervention was effective for these students.

Instruction designed to build procedural flexibility has not been examined with students with MLD. Jitendra et al. (2013, 2017a) incorporated instructional components that build procedural flexibility in their line of inquiry examining the effects of schema-based instruction on word-problem solving outcomes for students with LD. Although schema-based intervention has shown to be effective in improving the problem-solving outcomes for students with LD, inferences cannot be made about the effects of the procedural flexibility instruction as it was one component of a larger instructional package, and the researchers did not include measures of procedural flexibility. Zhang et al. (2013, 2014) examined the effects of instruction that is geared toward helping students with LD develop more efficient procedures when solving multiplicative word problems. Results from their studies showed that individualized and responsive instruction that is tailored to student level of procedural knowledge is a promising practice to build student knowledge and use of more efficient procedures. However, Zhang et al. focused on developing student use of more efficient strategies rather than flexible use of procedures.

Visual Representations

The process of mathematical representation is important for learning mathematics. Representations give students access to mathematical ideas that would be otherwise somewhat abstract and obscure (National Research Council, 2001). In addition, the use of representations is supported by cognitive theorists who argue that the use of external representations supports students doing mathematics as it reduces the cognitive efforts needed to understand the problem and allocate the mental resources to other aspects of problem solving (Sweller, 1994). For arithmetic word problems, using external representations can help with problem comprehension, which is critical based on the text-comprehension theory (Kintsch & Greeno, 1985).

It is difficult to empirically examine the effects of use of representations on students' learning outcomes because typically mathematics instruction includes several teaching practices. Here I provide an overview of a few studies that assessed the effects of representation instruction. Cramer et al. (2002) randomly assigned 66 classrooms to receive instruction on fraction derived from Rational Number Project (RNP) or commercial curriculum. In the NRP condition, concrete representation played a central role to develop conceptual understanding of fractions while in the commercial curricula the primary goal was to develop student's competency with symbolic representations. Results showed that students in the NRP condition significantly outperformed students in the commercial curriculum condition in various fraction assessments.

Witzel (2005) randomly assigned middle school classes to receive algebra instruction using multiple representations (concrete and semi-concrete representations) or abstract instruction (instruction only using abstract notation). Following 19 lessons of instruction, students in the multiple representations condition outperformed their peers in the abstract condition on a researcher-created measure that assessed students' ability to solve linear functions with unknowns on both sides. The results were consistent for average and low achieving students. Witzel et al. (2003) conducted a similar study with students with LD and found that students who received instruction using multiple representations outperformed students who received instruction using abstract notation on measures of solving linear equations at post-treatment and follow-up tests.

Terwel et al. (2009) examined whether student-generated or teacher-provided representation had a greater impact on students' learning. Fifth-grade students were randomly assigned to teacher-provided or student-generated representations. The two treatment conditions had other instructional features (e.g., classroom discussion, multiple strategies) and the only difference between the conditions was the generator of the representation. Results of posttest and transfer measures of solving percentage problems showed that students in the student-generated condition outperformed their counterparts in the teacher-provided representation condition.

Mathematical Discourse

Another effective teaching practice is engaging students in mathematical discourse (NCTM, 2017). Mathematical discourse involves classroom discussions that provide an opportunity for students to reveal their thinking and understandings (Chapin et al., 2009). Mathematical discourse as a practice is commonly understood in the sociocultural theory for learning, which emphasizes that learning is a sociocultural activity that is shaped by three interrelated processes – communication, thinking, and learning (Vygotsky, 1986). As such, students learn mathematical concepts when they make conjectures, talk, question, and reflect (NCTM 2000).

Mathematics discourse is a cornerstone of contemporary effective teaching practices as it develops students' ability to communicate in mathematics and develop and evaluate their mathematical understanding (Forman et al., 1997; Stein et al., 2008). A few experimental studies examined the effects of mathematical discourse on students' outcomes. Hiebert and Wearne (1993) analyzed the quantity of classrooms discussions and the type of questions asked by teachers in six elementary classrooms. Instruction in the classrooms differed in their pedagogical orientation with four classrooms providing traditional instruction and two classrooms involving reform-based instruction. With regard to mathematical discourse, Hiebert and Wearne (1993) found that reform-oriented classrooms spent more time on classroom discussion compared to traditional classrooms. The quality of discussions also differed, with reform-oriented classrooms spending more time analyzing and justifying ideas and traditional classrooms spending more time recalling facts and procedures. Regarding learning outcomes, the results showed that students in the reform-oriented classrooms scored reliably higher than students in the traditional classrooms on measures of place value, computation, story problem, and transfer. Pierson (2008) assessed the associations between teacher-student interaction and students' learning outcomes. The findings showed that a 10% increase in teacher responsive moves and classroom discourse was associated with a 11% increase in student performance on a posttest.

The research on using discourse-oriented instruction with students with LD shows mixed results. Two studies examined the effects of reform-oriented instruction on students' outcomes. Woodward and Baxter (1997) found that the mathematics outcomes of students with LD did not improve after receiving reform-oriented instruction that focused on mathematical discourse compared to their peers who received traditional instruction. Further, Baxter et al. (2001) analyzed the engagement of students with LD in mathematical discourse during reform-oriented instruction and found that these students rarely participated in classrooms discussion. Baxter et al. (2001) proposed several explanations for the disengagement of students with LD in mathematical discourse, including the limited opportunities for discourse and difficulties with metacognition that affect students with LD in terms of following the ideas of their peers and productively responding to them.

Recent studies in special education research showed promising results for carefully designed instruction that is based on teacher-student discourse. Xin et al. (2020) conducted a teaching experiment in which they provided mathematical instruction that fosters mathematical reasoning through adaptive teacher-student discourse. In Xin et al. (2020), teachers adjusted the cognitive demand of the task to the student level of mathematical reasoning and used teacher-student discourse to nurture students' reasoning throughout the task. Results from this experiment showed that adaptive teacher-student discourse had promising positive effects on students' reasoning and learning outcomes. Fuchs et al. (2016) examined the effects of instruction that involved a feature of mathematical discourse (student explanation) on magnitude comparison of fraction, word-problem solving, and quality of explanation of students with at-risk for mathematics difficulties. Fuchs et al. (2016) randomly assigned students to an intervention that provided an opportunity to explain their representations with word-problem solving instruction or word-problem solving instruction alone. The findings showed that students receiving explanation intervention scored better on measures of magnitude comparison of fraction and quality of explanation measures and students receiving word-problem instruction scored better on a word-problem solving measure. It is important to note that the explanation intervention provided minimal opportunity for students to engage in mathematical discourse (only 7 minutes per session) which might explain the student's underperformance on the word-problem solving measure.

Strategy Instruction

Cognitive strategy instruction, derived from the information-processing theory of cognitive psychology, posits that problem solving involves two processes – creating problem space (problem representation) and searching for solution methods through the problem space(Newell & Simon, 1972). Additionally, cognitive strategies involve metacognitive and self-regulation skills which involve the active monitoring of the cognitive process and associated decisions during problem solving. Knowledge about basic skills alone is insufficient to support children as they solve nonroutine problems and that students need to learn to decide when to use the basic skills and apply appropriate strategies based on the problem at hand (Mayer, 2002).

Instruction in cognitive strategies has been studied and shown promising results for improving student outcomes. Kramarski and Mevarech (2003) randomly assigned eighth-grade students to one of four conditions: cooperative learning with metacognitive strategies, cooperative learning without meta-cognitive strategies, individual learning with metacognitive strategies, or individual learning without meta-cognitive strategies. Students received instruction in linear graphing followed by small groups or individual activity—hence, the cooperative or individualized nature of the conditions. The metacognitive component of instruction in the conditions involved comprehension questions, strategic questions, and connection questions. Results showed that students who received metacognitive instruction either in a cooperative or individual condition outperformed students in the control conditions on measures of mathematical reasoning as measured by graph interpretation and construction.

Montague et al. (2011) examined the effects of cognitive strategy instruction on the problem-solving outcomes of middle school students. Students were assigned randomly to a cognitive strategy instruction or a business-as-usual condition. In the cognitive strategy instruction students learned to solve word problems using problem-solving heuristics (e.g., estimate, hypothesize) as well as self-regulation strategies (e.g., asking self-questions such as, "Did I write the estimate?"). At posttest, students in the treatment condition outperformed students in the control condition on curriculum-based measures of problem solving. Interestingly, the intervention was similarly effective for average and low achieving students as well as students with LD.

Wang et al. (2019) compared the effects of meta-cognitive strategies embedded in word-problem solving instruction to word-problem solving instruction alone on the mathematics outcomes of third-grade students with MLD. The word-problem solving instruction was constant in the two conditions except for the metacognitive instruction component (planning learning and tracking progress) in the metacognitive strategies embedded in word-problem solving instruction condition. Students in the metacognitive strategy instruction showed better outcomes compared to students in the word-problem solving instruction on measures of calculation, problem solving, and ordering of fractions. Additionally, students' outcomes in the word-problem solving instruction alone was dependent on their pretest score. That is, students with higher pretest scores benefited more from the intervention than students with lower pretest scores. In comparison, metacognitive strategy instruction was similarly effective for students with low and high pretest scores.

Mathematics Textbooks Analysis Literature

Researchers have examined various aspects of mathematics textbooks, including their alignment with instructional design principles, intervention programs, and progression in mathematics. In this section, I review the textbook analysis literature in relation to the effective teaching practices I discussed in the previous section.

Three studies examined high-cognitive demand tasks in mathematics textbooks. Polikoff (2015) assessed the alignment of four fourth-grade mathematics textbooks with CCSS (2010) in terms of content and cognitive demand. Polikoff (2015) applied the Survey of Enacted Curriculum on the selected textbooks and the CCSS (2010). Alignment indices were calculated to assess the alignment and the sources of misalignment between the textbooks and CCSS (2010). They defined the cognitive demand using Bloom's taxonomy (memorization; performing procedures; demonstrating understating; making conjectures, generalization, or proof; solving non-routine problems; and making connections). Results showed a degree of misalignment between the standards and the textbooks in terms of cognitive demand:15% of the content covered in the textbooks did not align the with cognitive demand emphasized in the CCSS (2010) for that content. Analysis of the sources of misalignment revealed that compared to the CCSS (2010), textbooks put more emphasis on low cognitive demand skills (memorization and performing procedures) and less emphasis on high cognitive demand skills (e.g., making conjectures, generalization, solving non-routine problems).

Jones and Tarr (2007) assessed the cognitive demand of probability tasks in middle school mathematics textbooks. Their analysis was conducted from a historical perspective (New Math vs. Back to Basic vs. Standards textbooks); however, I focus on the relevant results pertaining to the Standards textbooks. Jones and Tarr (2007) coded the cognitive demand of the tasks along with four categories: Memorization, Procedures without Connections, Procedures with Connections, or Doing Mathematics. The results showed textbooks published after the standards varied in terms of the cognitive demand of the tasks, as one textbook emphasized low cognitive demand tasks (83% low cognitive demand tasks) and the other textbook emphasized high cognitive demand (59% high cognitive demand tasks). Son (2012) conducted a comparative analysis between mathematics textbooks in the U.S. and South Korea in terms of cognitive demands. In this review, I focus on the results regarding the U.S. mathematics textbooks. Son (2012) examined the cognitive demand in fraction lessons in one fifth-grade reform-oriented U.S. textbook. For coding the cognitive demand of the tasks, they used a coding scheme that has three categories – low (recall/reproduce), moderate (basic application of skill/concept), and high (strategic and extended thinking). Findings showed that the majority of tasks in the textbook were devoted to low cognitive demand tasks (85%).

Van Garderen et al. (2012) examined four sixth- and seventh-grade mathematics textbooks with regard to the types of representations in lessons involving number operations, algebra, and geometry. Representations were categorized as: (1) tables or charts; (2) graphs; (3) diagrams; (4) and manipulatives. Results of a *t*-test showed that representations were used significantly more when solving application problems than during instruction. Further, representations during instruction were used mostly for delivering content rather than providing instruction about representations per se. With regard to solving application problems, results indicated that there were significantly more text-provided representations than student-generated representations.

Other studies assessed aspects of representations in mathematics textbooks although their focus was primarily on instructional design. Sood and Jitendra (2007) examined representations in number-sense lessons in traditional and reform-based mathematics first-grade textbooks. The results showed that the reform-oriented textbook focused more on concrete and semi-concrete representations, whereas traditional textbooks were more balanced across the three types of representations (concrete, semi-concrete representations, abstract). Doabler et al. (2012) and Bryant et al. (2008) assessed how textbooks incorporated representations in different grade levels. In their studies, they examined multiple textbooks (three to four different textbooks) across different grade levels (K, first, second, and fourth grade). A consistent finding from the two studies was that the incorporation of mathematical representations tended to be insufficient in older grade levels (second and fourth grade) compared to early grade levels (K and second grade).

Two studies examined problem-solving lessons in third-grade textbooks with regard to their adherence to mathematical practices articulated by NCTM (2000) (Jitendra et al., 2005a; Jitendra et al., 2005b). The examined mathematical practices included problem solving, reasoning, communication, connection, and representation. The five textbooks provided sufficient opportunities for communication and applying textbook-provided representations. However, opportunities were limited for applying problem-solving strategies, reasoning, making connections, and generating or selecting among different representations .Interestingly, textbooks published after the NCTM (2000) demonstrated minimal changes compared to textbooks published before the NCTM (2000) except for a substantial decrease in strategy instruction and an increase in textbook-provided representations (Jitendra et al., 2005a).

Bryant et al. (2008) examined the incorporation of cognitive strategies in four elementary-level textbooks. Their analysis reported very low incorporation of strategy instruction in the four textbooks. Fan and Zhu (2007) furthered the understating of strategy instruction in mathematics textbooks as they examined what strategy instruction and problem-solving heuristics are included in secondary-level textbooks in the U.S, China, and Singapore. They examined the problem-solving procedures and coded the extent to which they included Pólya's general problem-solving strategies (i.e., understand the problem, devise a plan, carry out the plan, and look back). Results pertaining to the U.S. textbook revealed that carrying-out a plan is included almost in all of the tasks followed by looking back (43% of the tasks). Understanding the problem and devising a plan were present only in 20% and 27% of the tasks respectively. Drawing a diagram, restating the problem, and using an equation are the most common form of problem-solving heuristics in the textbook.

Conclusion

An increased number of students with LD spend most of their school time in general education classrooms. The National Center for Education Statistics, U.S. Department of Education (2020) reported that 73.5% of students with LD spend 80% or more of their school days in general education classroom in 2019 marking an increase of eight percent since 2010. To improve the academic achievement for students LD while studying in general education classrooms, educational legislations (IDEA 2004; ESS 2015) mandate that schools must use evidence-base core curricula. Research from mathematics and special education have identified several effective teaching practices that should incorporated in core curricula which include posing tasks that promote reasoning and problem solving, representation, discourse, problem-solving strategies (Fuchs et al., 2009, 2016; Hiebert & Wearne, 1993; Jitendra et al., 2013, 2017; Kramarski & Mevarech, 2003; Montague et al., 2011; Powell et al., 2015; Star & Seifert, 2006; Terwel et al., 2009; Wang et al., 2019; Witzel, 2005; Woodward & Baxter, 1997; Xin et al., 2020b). Mathematics textbooks came under scrutiny given their role as a written curriculum

(Rezat et al., 2021; Stein et al., 2007). Although, previous textbook analysis studies in mathematics examined aspects of contemporary effective teaching practices (e.g., high-cognitive demand tasks or representation), this is the first study to examine the incorporation of contemporary effective teaching practices in mathematics textbooks as defined in recent publications in mathematics and special education research (Dougherty et al., 2015; NCTM, 2014; Park et al., 2021; Woodward et al., 2018). Results of this analysis might give an insight into the state of the current written mathematics curriculum in the U.S. and whether they promote effective teaching practice that support mathematics achievement for students with LD. In addition, this analysis might inform teachers and curricula developers of the practices with unsatisfactory incorporation that need to be included by curricula developers or supplemented by teachers. The study will address the following research questions:

1. To what extent are mathematics teaching practices incorporated in WPS tasks in third-grade mathematics textbooks?
2. Are there significant differences between textbooks in incorporating effective teaching practices?

Chapter 3

Method

I used quantitative content analysis to examine the incorporation of effective teaching practices in mathematics textbooks (Neuendorf, 2020). In this section, I describe textbook selection process, codebook's development, data extraction, and data analysis.

Textbook Selection

I selected a sample of mathematics textbooks that are widely used in elementary schools in the United States (Neuendorf, 2020). To determine the textbooks that are widely used in the U.S., I used two recent studies that surveyed a large sample of schools from 15 geographically and demographically diverse U.S. states (California, Delaware, Florida, Louisiana, Maryland, Massachusetts, Mississippi, Nebraska, New Jersey, Washington, New Mexico, New York, Rhode Island, Tennessee, Wisconsin) with respect to the mathematics textbooks used in schools (Blazar et al., 2020; Kaufman et al., 2020). The researchers collected data from over 17000 teachers and school administrators regarding the textbooks and instructional material used in their schools. The teacher edition of the top six textbooks with highest use as reported by the surveys were included in this study: (1) *Eureka Math* (Great Minds, 2015); (2) *My Math* (McGraw-Hill Education, 2018); (3) *enVision Math* (Pearson, 2016); (4) *Go Math* (Houghton Mifflin Harcourt, 2015); (5) *Math Expressions* (Houghton Mifflin Harcourt, 2012); and (6) *Everyday Mathematics* (McGraw-Hill Education, 2015). The teacher's edition of the textbooks was selected because in addition to its coverage of student's edition material, the teacher's edition includes prompts to teacher about using effective teaching practices that might not be observed in the student's edition. Table 1 describes the instructional approaches in the textbooks which draws Remillard et al. (2014) framework for instructional approaches.

Table 1

Instructional Approaches in the Textbooks

Textbook	Teacher Role	Expected Interaction	Pathway to learning
Everyday Mathematics	Explain and facilitate production of ideas	Teacher-student and student-student	Student reasoning followed by guided instruction of key mathematical ideas
Eureka Math	Explain and facilitate production of ideas	Teacher-student and student-student	Guided instruction of key mathematical ideas
enVision Math	Explain, demonstrate, Guide	Teacher-student	Demonstrating key mathematical ideas followed opportunity to practice
Go Math	Explain, demonstrate, guide	Teacher-student and student-student	Demonstrating key mathematical ideas followed opportunity to practice
Math Expressions	Explain and facilitate production of ideas	Teacher-student and student-student	Guided instruction of key mathematical ideas
My Math	Explain, demonstrate, guide	Teacher-student and student-student	Demonstrating key mathematical ideas followed opportunity to practice

WPS Tasks Extraction Training and Reliability

Lessons addressing the domain of operation and algebraic thinking in the textbooks were included in this study. The data source was WPS tasks, defined as problem situations presented in verbal format for which the solution required the application of mathematical operations. Within the included lessons, whole number WPS tasks addressing the four operations within the domain of operation and algebraic thinking in the CCSS (2010) in key components of the lessons (e.g., concept building, application problems) were included to keep domain constant and account for domain as factor influencing the incorporation of the examined teaching practice (see Appendix A). In addition, word-problem solving tasks used as homework, assessment, or extended activities were excluded in the analysis because it is not feasible to observe some of the examined practices (see Appendix A) in assessment and homework tasks (e.g., discourse).

First, I reviewed all of the lessons and included lessons addressing operation and algebraic thinking as stated by the textbooks. Second, two coders (doctoral students in special education) were trained on identifying WPS tasks in 12 lessons (two lessons from each textbook). The training session involved defining WPS task, providing examples and non-examples of WPS tasks, and explaining the sections of the textbooks included in the study. Third, the two coders independently extracted WPS tasks in a sample of 78 lessons divided equally across textbooks (>30% of the included sample of lessons) to assess the reliability of WPS tasks extraction. The calculated Cohen's Kappa for WPS tasks extraction was 0.89 (95% confidence interval [CI]: [0.87, 0.91]) indicating almost perfect agreement (McHugh, 2012). Sources of disagreement included the domain of the problem (e.g., measurement, data, place value), non-contextual problem (e.g., Carlos said that 6 times 0 equals 6. Do you agree?), structure of the task (multiple WPS tasks in one question), section (assessment or differentiation), technology (tasks requiring use of technology). The two coders discussed and resolved the disagreements. Since the agreement was high, I extracted the WPS tasks in the remaining lessons independently.

Effective Teaching Practices

Recent publications from special and mathematics education were used to identify effective teaching practices in mathematics (NCTM, 2014; Park et al., 2021; Woodward et al., 2018). In consultation with experts in mathematics and special education, I reviewed eight effective teaching practices in NCTM (2014), 10 practices in Park et al. (2021), and five practices in Woodward et al. (2018) for their relevance to this study. Practices from the publications were combined such as that practices from the three publications that addressed the same concepts were integrated in one code (see Table 2).

Some practices that were reported in NCTM (2014) or Park et al. (2021) were not reviewed in this study. Two practices were not reviewed because their implementation relies heavily on the teacher's judgment and would be difficult to observe in the textbooks: supporting productive struggle in learning mathematics (e.g., being prepared to support student productively through the struggle, praising students for their efforts in making sense of mathematical ideas) and building precise mathematical vocabulary (using the same terms in subsequent lessons). Three practices were not reviewed because they pertain more to assessment of instruction (i.e., elicit and use evidence of student thinking, conduct error analysis, provide constructive feedback).

Clear Goals

Two items in the coding scheme assessed this practice. First, goals should articulate the mathematics the students should learn in the lesson. Second, goals should be connected to learning standards in the CCSS-M or the relevant state standards (NGA & CCSSO, 2010). I coded the CCSS (2010) addressed in the lessons in the domain of operation and algebraic (i.e., "Interpret products of whole numbers" [3.OA.1] ; "Interpret whole-number quotients of whole numbers" [3.OA.2]; "Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities" [3.OA.3]; "Determine the unknown whole number in a multiplication or division equation relating three whole numbers" [3.OA.4]; "Apply properties of operations as strategies to multiply and divide; OA6: Understand division as an unknown-factor problem" [3.OA.5]; "Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations" [3.OA.7]; "Solve two-step word problems using the four operations" [3.OA.8]; "Identify arithmetic patterns and explain them using properties of operations" [3.OA.9]). The coding sheet also reported information related to the mathematics practices standards addressed in the lessons ("Make sense of problems and persevere in solving them" [MP1]; "Reason abstractly and quantitatively" [MP2]; "Construct viable arguments and critique the reasoning of others" [MP3]; "Model with mathematics" [MP4]; "Use appropriate tools strategically" [MP5]; "Attend to precision" [MP6]; "Look for and make use of structure" [MP7]; "Look for and express regularity in repeated reasoning" [MP8]). Further, I coded the mathematical strands addressed in the goals of the lessons (conceptual understanding [e.g., understand the meaning of the unknown as the size of the group in division], procedural fluency [e.g., multiply and divide with familiar facts using a letter to represent the unknown], strategic competence [e.g., children use drawings and number models to represent and solve multiplication number stories], adaptive reasoning [e.g., children critique the reasoning of others by asking questions and identifying mistakes and providing suggestions for improvements], and productive disposition [children connect mathematics to real-life situations]). In cases where the goals addressed multiple mathematical strands, I coded the strand that is more aligned with content of the lesson. For example, the following goals was coded

Reasoning and Problem Solving

Tasks that promote reasoning and problem-solving tap one or more of three items: high-cognitive demand, flexibility, and generalization. High-cognitive demand tasks included reversibility tasks, real-world problems, and multiple-step problems. Flexibility tasks involved justifying the answer using different tools or in different way or explaining why different procedures work for a given problem. Generalization questions involved questions focusing on specific patterns that are identified in classes of problems or creating a specific example of a problem from a generalization

Table 2

Effective Mathematics Teaching Practices

Category	NCTM (2014)	Park et al. (2021)	Woodward et al. 2018
1	**Establish mathematics goals to focus learning**. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	**Implementing Explicit Instruction**. Teaching using a series of instructional steps including (a) opening, (b) modeling, (c) prompted practice, (d) unprompted practice, (e) closing, (f) pacing, (g) check for understanding, and (h) reflection. "Teachers can help students to build conceptual understanding and procedural fluency through the use of clear objectives, modeling, and practice (Doabler et al., 2017 cited in Park et al., 149)."	
2	**Implement tasks that promote reasoning and problem solving**. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	**Reversibility**. Having students solve problems presented in different ways for a given solution. **Flexibility**. Teaching students ways to apply different strategies for solving problems. **Generalization**. Teaching mathematical patterns and structures that can be applied to similar core concepts across similar problems.	**Prepare problems and use them in whole-class instruction.** Include non-routine problem in problem solving activities. **Expose student to multiple problem-solving strategies.** Ask students to generate and share multiple strategies for solving the problem.
3	**Use and connect mathematical representations**. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.	**Presenting Multiple Representations**. Teaching students ways to apply multiple representations for understanding concepts. **Promoting Contextual Teaching**. *(Concept development)* Connecting real-world situations and problems to the concepts being studied.	**Teach student how to use visual representations.** Use think-alouds and discussions to teach studens how to represent problems visually.

4	**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. **Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. **Build procedural fluency from conceptual understanding**. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	**Encouraging Mathematical Discourse**. Planning for student discussion as an integral part of the lesson.	**Help Student recognize and articulate mathematical concepts.** Ask student to explain each step used to solve a problem.
5			**Strategy instruction.** Provide students with a list of prompts to help them monitor and reflect during the problem-solving process. Model how to monitor and reflect on the problem-solving process.

Visual Representations

Four items assessed the use of visual representations in WPS tasks. The first item was introducing multiple forms of representation (e.g., concrete and abstract). The second item was connecting the mathematical ideas to real-world situations. The third item assessed whether tasks engage students to generate their own representations. The fourth item involved having students make choices between representations to use as tools for solving problems. Additional information was recorded including the level of the representation (concrete, semi-concrete, or abstract) and the generator of the representation (students, teacher, both).

Discourse and Conceptual Understanding

Four items assessed the extent to which the textbook included prompts intended to facilitate mathematical discourse to build conceptual understanding. The first item was providing students with opportunities to use their own reasoning strategies for solving problems. The second item assessed mathematical discourse, which involved facilitating discourse among students or having students explain and defend their approaches and procedures. The third item was asking questions that probe thinking and require explanation and justification. The fourth item involved making explicit connections between student reasoning and key mathematical ideas or more efficient procedures.

Strategies Instruction

Two items assessed the strategy instruction in the WPS tasks. The first item was incorporating and modeling the use of cognitive strategies. The second item was incorporating and modeling the use of meta-cognitive strategies.

Coding Scheme Alignment

I assessed the extent to which the coding scheme is aligned with practices and indicators it is intended to assess. I assessed the alignment of the coding scheme through eliciting the opinion of a panel of experts regarding the coding scheme and its coverage and alignment with effective teaching practices. The panel consisted of two experts in mathematics instruction from special education and mathematics education.

Coder Training and Inter-Coder Agreement

Two coders (doctoral students in special education) were trained on coding WPS tasks using the coding scheme. Coder training was conducted in several rounds that involved coding, discussion, consensus building, revisions, and coder training on revisions. Following the training, a subsample of the WPS tasks from all textbooks was coded by the two coders to assess reliability and revise or clarify items with high disagreements. To assess intercoder reliability, the two coders independently coded 30% of the tasks distributed throughout coding to account for coder drifting. Cohen's Kappa was used to estimate the inter-coder agreement and is preferred over simple agreement as it accounts for agreement due to chance. Cohen's Kappa was calculated using the following formula:

$$kappa = \frac{p_o - p_e}{1 - p_e}$$

In this formula p_o refers to proportion agreement observed between coders and p_e refers to proportion of agreement between coders expected by chance. The averaged Cohen's Kappa for WPS tasks coding was 0.74 (95% confidence interval [CI]: [0.71, 0.79]) indicating substantial agreement (McHugh, 2012). The most frequent disagreements included the following indicators generalization questions, discourse, and posing purposeful questions. The two coders discussed and resolved the disagreements.

Data Collection and Analysis

Data was collected at the WPS task level. I chose WPS task as the level for data collection over collecting data at the lesson level because it facilitated consistent and systematic data collection. Using the coding scheme, WPS tasks were coded for presence of indicators (e.g., high-cognitive demand, flexibility, generalization) that tap each practice (promoting reasoning and problem solving). A WPS task received a score of one if it tapped one or more of the indicators for the practice. A WPS task received a score of zero for the practice if it did not tap any of the indicators for the practice. For example, a WPS task would receive a score of 1 in reasoning and problem solving if it tapped one or more of the indicators tapping this practice (i.e., high-cognitive demand, flexibility, generalization) and a score of 0 if it did not include any of the three indicators.

Although data was collected at WPS task level, data was analyzed at the lesson level (as determined by the textbook) given that lesson is the holistic level where all the indicators can be reasonably incorporate. Practices and indicators data from WPS tasks were aggregated to the lesson level to calculate the practice and indicator score for the lesson. The practice score for the lesson was calculated as the sum of the practice scores for WPS tasks in the lesson divided by the total number of WPS tasks in the lesson. The indicator score for the lesson was calculated as the sum of the indicator scores for WPS tasks in the lesson divided by the total number of WPS tasks in the lesson. The practice score for a textbook was calculated as sum of the practice scores for the lessons divided by the total number of lessons in the textbook. The indicator score for a textbook was calculated as sum of the indicator scores for the lessons divided by the total number of lessons in the textbook.

In addition, I calculated a score for the incorporation of the three forms of representations (concrete, semi-concrete, abstract). Word-problem solving tasks were coded for presence of each form of representation. Then, a form of representation score (abstract) for a lesson was calculated as the number of WPS tasks in the lesson incorporating the form representation divided by the total number for the WPS tasks in the lesson. The form of representation score for a textbook was calculated as sum of the form of representation scores for the lessons divided by total number the lessons in the textbook.

Further, I calculated a score for the incorporation of forms of representation (concrete, semi-concrete, abstract, multiple forms) by generator (student, teacher, student and teacher). WPS tasks were coded for presence of each form of representation by generator. Then, a form of representation by generator score (abstract-teacher) for a lesson was calculated as the number of WPS tasks in the lesson incorporating the form representation by the generator divided by the total number for the WPS tasks in the lesson. The form of representation by generator score for a textbook was calculated as the sum of the form of representation by generator scores for the lessons divided by total number the lessons in the textbook.

The research suggests that effective teaching practices should be incorporated in every task (e.g., reasoning and problem solving and visual representations) (NCTM, 2014). However, it is not expected of every task to incorporate all of the indicators for the practices. I averaged the lessons' practice and indicator scores to account for differences between lessons in their incorporation of the practices and indicators. I selected to average the lesson score over averaging task score for the textbook, because averaging the task score does not account for lessons that had a large number of tasks in a few lessons tapping the practice or the indicator providing a limited opportunity for learning over time. I conducted a correlation analysis between the indicators for each practice to assess whether the indicators were highly correlated. The Pearson correlation coefficients (r = -0.09-0.26) were low for most indicators except multiple representations and sketching diagrams as well as cognitive strategies and meta-cognitive strategies which were moderately correlated (r = 0.44 and 0.75 respectively). Further, the correlational analysis showed similar results (low to moderate correlations) when examining the correlations between the indicators for each textbook.

To address Research Question 1 (To what extent are mathematics teaching practices incorporated in WPS tasks in third-grade mathematics textbooks?), I used descriptive analysis. I calculated the mean proportion for practices in a lesson for the textbook as well as the standard deviation.

To address Research Question 2 (Are there significant differences between textbooks in incorporating effective teaching practices?), I conducted a series of one-way analyses of variance (ANOVA) of mean practice scores using textbook title (*Everyday Mathematics, Eureka Math, My Math, Math Expression, Go Math, and enVision Math*) as a factor in each model.

$$\gamma_{ij} = \mu + Practice_j + + \varepsilon_{ij}$$

The γ_{ij} represents the j-th observation of the i-th textbook. μ is the general mean effect, $Practice_i$ is the effect of Textbook_j. I performed tests of all possible pairwise comparisons between textbooks when the main effect was significant using Tukey's HSD. Alpha level was maintained at 0.05 given that Tukey's HSD allowed for testing all possible pairwise comparisons and maintaining alpha level of 0.05 (Maxwell et al., 2018)

Chapter 4

Results

The mathematics textbooks included a total of 309 lessons addressing the domain of operation and algebraic thinking (see Table 3 for lessons and tasks information). However, only 249 lessons contained WPS tasks and were included in this analysis. The 249 lessons included a total of 1457 WPS tasks that were reviewed and coded. As can be seen in Table 2, textbooks differed in the number of lessons that included WPS tasks addressing the domain of operation and algebraic thinking, with *Math Expressions* having the lowest number (37 lessons) and *My Math* having the highest number (48 lessons). In comparison, *Math Expressions* included the largest number of WPS tasks (338 tasks) and *Eureka Math* had the lowest number (167 tasks).

Table 3

Number of Lessons and Coded Tasks in the Textbooks

Textbook	Number of lessons	Number of WPS tasks		
		Total	Average per lesson	Range per lesson
Everyday Mathematics	45	197	4	1-16
Eureka Math	40	167	4	1-11
EnVisionmath	41	246	6	1-13
Go Math	38	205	5	1-10
Math Expressions	37	338	9	1-23
My Math	48	304	6	3-12

Research Question 1: To what extent are mathematics teaching practices incorporated in WPS tasks in third-grade mathematics textbooks?

A summary of the findings with respect to effective mathematical teaching practices in the textbooks is presented in Table 4. All lessons included mathematical goals and stated the addressed content standards; therefore, establishing mathematical goals to focus learning is not included in this table. As can be seen in Table 4, discourse and conceptual understanding was the highest incorporated practice (Mean =0.57-0.93) and strategy instruction was the lowest incorporated practice (*M*= 0.03- 0.53). A description of the findings organized by mathematical teaching practices and their indicators follows.

Table 4

Average Proportion for the Incorporation of Mathematics Teaching Practice

Practice	Items	EM	EUM	ENM	GM	ME	MM
				Mean (SD)			
Reasoning and Problem Solving		0.60 (0.36)	0.55 (0.39)	0.57 (0.31)	0.60 (0.22)	0.47 (0.41)	0.41 (0.26)
	Cognitive demand	0.46 (0.41)	0.34 (0.43)	0.38 (0.38)	0.50 (0.24)	0.39 (041)	0.28 (0.25)
	Flexibility	0.15 (0.27)	0.27 (0.35)	0.10 (0.16)	0.05 (0.09)	0.06 (0.17)	0.05 (0.10)
	Generalization	0.18 (0.31)	0.13 (0.30)	0.13 (0.18)	0.10 (0.16)	0.08 (0.21)	0.11 (0.17)
Visual Representations		0.62 (0.43)	0.60 (0.38)	0.30 (0.29)	0.26 (0.19)	0.29 (0.39)	0.29 (0.26)
	Multiple representation	0.55 (0.42)	0.52 (0.41)	0.24 (0.27)	0.21 (0.17)	0.20 (0.35)	0.25 (0.24)
	Real-world connections	0.24 (0.35)	0.14 (0.26)	0.04 (0.09)	0.09 (0.12)	0.06 (0.15)	0.04 (0.10)
	Create diagrams	0.34 (0.40)	0.40 (0.37)	0.07 (0.13)	0.07 (0.14))	0.14 (0.31)	0.06 (0.17)
	Representation Selection	0.09 (0.22)	0.00 (0.00)	0.06 (0.12)	0.03 (0.07)	0.00 (0.00)	0.03 (0.12)
Discourse and Conceptual Understanding		0.93 (0.19)	0.98 (0.09)	0.82 (0.22)	0.90 (0.17)	0.98 (0.06)	0.57 (0.28)
	Reasoning strategies	0.83 (0.27)	0.63 (0.38)	0.59 (0.25)	0.63 (0.24)	0.91 (0.20)	0.44 (0.30)
	Facilitating discourse	0.54 (0.40)	0.75 (0.30)	0.01 (0.04)	0.12 (0.15)	0.76 (0.32)	0.10 (0.14)
	Purposeful questions	0.41 (0.38)	0.35 (0.36)	0.37 (0.29)	0.31 (0.19)	0.22 (0.30)	0.19 (0.17)
	Explicit connections	0.29 (0.35)	0.11 (0.28)	0.10 (0.13)	0.05 (0.11)	0.09 (0.22)	0.01 (0.05)
Strategies Instruction		0.18 (0.33)	0.53 (0.37)	0.02 (0.09)	0.07 (0.15)	0.04 (0.14)	0.03 (0.09)
	Cognitive strategies	0.05 (0.15)	0.27 (0.37)	0.01 (0.04)	0.05 (0.12)	0.02 (0.01)	0.02 (0.07)
	Meta cognitive strategies	0.02 (0.08)	0.26 (0.36)	0.01 (0.04)	0.01 (0.04)	0.00 (0.00)	0.00 (0.00)

Note. EM: *Everyday Mathematics*; EUM: *Eureka Math*; ENM: *enVision Math*; GM: *Go Math*; ME: *Math Expressions*; MM: *My Math*

Clear Goals and Objectives

Lessons in the textbooks included mathematical goals that stated what students should learn. Table 5 depicts the mathematical proficiency strands targeted in the goals of the lessons. Overall, strategic competence was the highest addressed strand in all of the goals of the lessons in the textbooks (41%- 61%) followed by conceptual understanding (24% to 38%) and procedural fluency (10% to 26%). Productive disposition was lowest addressed mathematical strand in the goals of lessons in the textbooks (0%). With an exception of *Everyday Mathematics* (24%) the goals of the remaining textbooks addressed adaptive reasoning at low rate (<11%).

Table 5

Mathematical Proficiency Standards in the Textbooks

Textbook	Procedural fluency	Conceptual understanding	Strategic competence	Adaptive reasoning	Productive Disposition
Everyday Mathematics	6 (13%)	11 (24%)	17 (38%)	11 (24%)	0 (0%)
Eureka Math	4 (10%)	15 (38%)	20 (50%)	1 (3%)	0 (0%)
EnvisionMath	11 (26%)	12 (29%)	16 (38%)	3 (7%)	0 (0%)
Go Math	4 (11%)	11(29%)	23 (61%)	0 (0%)	0 (0%)
Math Expressions	5 (14%)	13 (35%)	15 (41%)	4 (11%)	0 (0%)
My Math	6 (13%)	12 (25%)	28 (58%)	2 (4%)	0 (0%)

Table 6 shows the number and percentage CCSS (2010) addressed in the lessons as reported by the textbooks. Lessons in the textbooks addressed multiple content standards, hence the discrepancy between the number of lessons included and the number of the standards addressed. Textbooks varied with the CCSS (2010) they emphasized. "Use multiplication and division within 100 to solve word problems " (3.OA.3) was the highest addressed CCSS (2010) in *enVision Math, Math Expressions,* and *My Math*. "Apply properties of operations as strategies to multiply and divide" (3.OA.5) was the highest addressed CCSS (2010) in *Eureka Math* and *Go Math* and "Fluently multiply and divide within 100" (3.OA.7) was the highest addressed CCSS (2010) in *Everyday Mathematics* and *My Math*.

Textbooks also differed with CCSS (2010) they emphasized less. "Interpret products of whole numbers" (3.OA.1) was the least addressed CCSS (2010) in *Eureka Math*. "Determine the unknown whole number in a multiplication or division equation relating three whole numbers" (3.OA.4) was the least addressed CCSS (2010) in *enVision Math* and *Go Math* and "Apply properties of operations as strategies to multiply and divide" (3.OA.5) was the least addressed CCSS (2010) in *Go Math*. "Solve two-step word problems using the four operations" (3.OA.8) was the least addressed CCSS (2010) in *My Math* and "Identify arithmetic patterns and explain them using properties of operation" was the least addressed CCSS (2010) in *Everyday Mathematics* and *Math Expressions*.

Table 6

Number and Percentage of Lessons Addressing Operation and Algebraic Thinking Standards in the Textbooks

	OA1	OA2	OA3	OA4	OA5	OA6	OA7	OA8	OA9
Everyday Mathematics	14(9%)	13(9%)	25(17%)	14(9%)	10(7%)	10(7%)	40(27%)	15(10%)	8(5%)
Eureka Math	6(5%)	7(6%)	15(12%)	15(12%)	26(21%)	7(6%)	25(20%)	11(9%)	10(8%)
EnvisionMath	9(11%)	3(4%)	30(36%)	2(2%)	9(11%)	7(8%)	8(10%)	5(6%)	11(13%)
Go Math	2(4%)	2(4%)	11(23%)	1(2%)	14(29%)	1(2%)	10(21%)	5(10%)	2(4%)
Math Expressions	24(13%)	23(13%)	33(19%)	23(13%)	9(5%)	21(12%)	25(14%)	12(7%)	8(4%)
My Math	19(12%)	12(7%)	28(17%)	24(15%)	19(12%)	10(6%)	27(17%)	7(4%)	17(10%)

Note. OA1: Interpret products of whole numbers ; OA2: Interpret whole-number quotients of whole numbers; OA3; Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities; OA4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers;

OA5: Apply properties of operations as strategies to multiply and divide; OA6: Understand division as an unknown-factor problem; OA7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations; OA8: Solve two-step word problems using the four operations; OA9: Identify arithmetic patterns and explain them using properties of operations.

Table 7 shows the number and percentage mathematics practice standards addressed in the lessons as reported by textbooks. Textbooks varied with the mathematics practice standards they emphasized. "Make sense of problems and persevere in solving them" (MP1) was the highest addressed mathematics practice standard in *Everyday Mathematics* and "Construct viable arguments and critique the reasoning of others" "MP3" was the highest addressed mathematics practice standard in *enVision Math* and *Math Expressions*. "Model with mathematics" (MP4) was the highest addressed mathematics practice standard in *Go Math* and *My Math*. "Attend to precision" (MP.6) was the highest addressed mathematics practice standard in *Math Expressions* and "Look for and make use of structure" (MP7) was the highest addressed mathematics practice standard in *Everyday Mathematics.*

Textbooks varied with the mathematics practice standards they emphasized less. "Construct viable arguments and critique the reasoning of others" (MP3) was the least addressed mathematics practice standard in *Go Math* and "Use appropriate tools strategically" was the least addressed mathematics practice standard in *Everyday Mathematics*, *enVision Math* and *Go Math.* "Attend to precision" (MP6) was the least addressed mathematics practice standard in *Eureka Math* and *enVision Math* and "Look for and express regularity in repeated reasoning" (MP8) was the least addressed mathematics practice standard in *Eureka Math*, *Math Expressions*, and *My Math*.

Table 7

Number and Percentage of Lessons Addressing Mathematic Practice Standards in the Textbooks

Textbooks	MP1	MP2	MP3	MP4	MP5	MP6	MP7	MP8
Everyday Mathematics	24(21%)	18(16%)	12(10%)	15(13%)	3(3%)	17(15%)	19(16%)	8(7%)
Eureka Math	4(15%)	3(12%)	3(12%)	4(15%)	3(12%)	0(0%)	9(35%)	0(0%)
EnvisionMath	31(16%)	31(16%)	34(18%)	28(15%)	13(7%)	14(7%)	22(12%)	15(8%)
Go Math	22(18%)	16(13%)	7(6%)	25(20%)	7(6%)	16(13%)	20(16%)	12(10%)
Math Expressions	33(16%)	16(8%)	35(17%)	33(16%)	26(12%)	37(17%)	19(9%)	13(6%)
My Math	33(14%)	38(16%)	34(14%)	40(17%)	27(11%)	30(13%)	23(10%)	14(6%)

Note. MP1: Make sense of problems and persevere in solving them; MP2: Reason abstractly and quantitatively; MP3: Construct viable arguments and critique the reasoning of others; MP4: Model with mathematics; MP5: Use appropriate tools strategically; MP6: Attend to precision; MP7: Look for and make use of structure; MP8: Look for and express regularity in repeated reasoning.

Reasoning and Problem Solving

Table 3 reports means and standard deviations for the incorporation of mathematics teaching practices and their indicators in the textbooks. The mean proportion for incorporating reasoning and problem solving in *Everyday Mathematics* was 0.60 tasks per lesson (*SD*=0.36). In *Everyday Mathematics*, high-cognitive demand task was the highest incorporated indicator for reasoning and problem solving (*M* =0.46; *SD* = 0.41) and flexibility was the lowest (*M*= 0.15; *SD*= 0.27). The mean proportion for incorporating reasoning and problem solving in *Eureka Math* was 0.55 tasks per lesson (*SD*=0.39). High-cognitive demand task was the highest incorporated indicator for reasoning and problem solving in *Eureka Math* (*Mean* =0.34; *SD* = 0.43) and generalization was the lowest (*M*= 0.13; *SD*= 0.30). The mean proportion for incorporating reasoning and problem solving in *enVision Math* was 0.57 tasks per lesson (*SD*=0.31). In *enVision Math*, high-cognitive demand task was the highest incorporated indicator for reasoning and problem solving (*M* =0.38; *SD* = 0.38) and flexibility was the lowest (*M*= 0.10; *SD*= 0.16).

The mean proportion for incorporating reasoning and problem solving in *Go Math* was 0.60 tasks per lesson (*SD*=0.22). High-cognitive demand task was the highest incorporated indicator for reasoning and problem solving in *Go Math* (*M* =0.50; *SD* = 0.24) and flexibility was the lowest (*M*= 0.05; *SD*= 0.09). The mean proportion for incorporating reasoning and problem solving in *Math Expressions* was 0.47 tasks per lesson (*SD*=0.41). In *Math Expressions,* high-cognitive demand task was the highest incorporated indicator for reasoning and problem solving (*M* =0.39; *SD* = 0.41) and flexibility was the lowest (*M*= 0.06; *SD*= 0.17). The mean proportion for incorporating reasoning and problem solving in *My Math* was 0.41 tasks per lesson (*SD*=0.26). In *My Math*, high-cognitive demand task was the highest incorporated indicator for reasoning and problem solving (*M* =0.28; *SD* = 0.25) and flexibility was the lowest (*M*= 0.05; *SD*= 0.10).

Visual Representations

The mean proportion for incorporating visual representations in *Everyday Mathematics* was 0.62 tasks per lesson (*SD*=0.43). In *Everyday Mathematics*, multiple representation was the highest incorporated indicator for visual representations (*Mean* =0.55; *SD* = 0.42) and selecting from multiple representation was the lowest (*M*= 0.09; *SD*= 0.22). The mean proportion for incorporating visual representations in *Eureka Math* was 0.60 tasks per lesson (*SD*=0.38). Multiple representation was the highest incorporated indicator for visual representations in *Eureka Math* (*Mean* =0.52; *SD* = 0.41) and selecting from multiple representation was the lowest (*M*= 0.00; *SD*= 0.00). The mean proportion for incorporating visual representations in *enVision Math* was 0.30 tasks per lesson (*SD*=0.29). In *enVision Math*, multiple representation task was the highest incorporated indicator for visual representations (*M* =0.24; *SD* = 0.27) and selecting from multiple representation was the lowest (*M*= 0.06; *SD*= 0.12).

The mean proportion for incorporating visual representations in *Go Math* was 0.26 tasks per lesson (*SD*=0.19). Multiple representation was the highest incorporated indicator for visual representations in *Go Math* (*M* =0.21; *SD* = 0.17) and selecting from multiple representation was the lowest (*M*= 0.03; *SD*= 0.07). The mean proportion for incorporating visual representations in *Math Expressions* was 0.29 tasks per lesson (*SD*=0.39). In *Math Expressions*, multiple representation was the highest incorporated indicator for visual representations (*M* =0.20; *SD* = 0.35) and selecting from multiple representation was the lowest (*M*= 0.00; *SD*= 0.00). The mean proportion for incorporating visual representations in *My Math* was 0.29 tasks per lesson (*SD*=0.26). In *My Math*, multiple representation task was the highest incorporated indicator for visual representations (*M* =0.25; *SD* = 0.24) and selecting from multiple representation was the lowest (*M*= 0.03; *SD*= 0.12).

Table 8 reports the average score for the incorporation of each form of representations. Abstract representations had the highest incorporation in most of textbooks (*M*=0.34-0.78) followed by semi-concrete (*M*=0.28-0.60). Concrete representations were the lowest form of representation incorporated in most of the textbooks (*M*= 0.00-0.36). Only *Eureka Math* had a higher incorporation of semi-concrete than abstract representation and *GO Math* had a higher incorporation of concrete representations than semi-concrete representations. Examining the forms of representations by textbooks revealed that lessons in *Everyday Mathematics* (0.78) and had the highest incorporation of abstract representations and *Go Math* (0.34) had the lowest incorporation. For semi-concrete representations, *Eureka Math* (0.60) had the highest incorporation and *enVision Math* (0.32) had the lowest incorporation. For concrete representation, *Go Math* (0.36) had the highest incorporation and *Math Expressions* (0.0) had the lowest incorporation.

Table 8

Average Proportion of Representation by Representation Type

	Abstract	Semi-concrete	Concrete
Everyday Mathematics	0.78	0.49	0.18
Eureka Math	0.55	0.60	0.01
enVision Math	0.50	0.32	0.11
Go Math	0.34	0.28	0.36
Math Expressions	0.61	0.28	0.00
My Math	0.63	0.35	0.10

Table 9 reports the average proportion for the form of representations (abstract, semi-concrete, concrete, multiple forms) by the generator of the representation (teacher, student, both). For the three categories of generator of representation, the average proportion student-generated representation was the highest in all textbooks (*M*= 0.16-0.62). The mean proportion for teacher-generated representation ranged from 0.02 to 0.023 and for student-teacher-generated representation from 0.01 to 0.21.

For *Everyday Mathematics* student-generated multiple forms of representations (*M* = 0.38) and abstract representations (*M*=0.23) were the highest and the remaining forms of representations for all generators were low (*M* < 0.13). Student-generated (*M*= 0.27) and student-teacher-generated (*M*=0.21) multiple forms of representations were the highest in *Eureka Math* and the remaining forms of representations for all generators were low (*M* < 0.08). For *enVision Math*, student-generated (*M*= 0.13) and teacher-generated (*M*=0.12) abstract representations were the highest and in the remaining forms of representations in all generators were low (*M* < 0.08). Student-teacher-generated multiple forms of representations (*M* = 0.17) was the highest in *Go Math* and the remaining forms of representations for all generators were low (*M* < 0.08). For

Math Expressions, student-generated multiple forms representations (*M* = 0.20) and abstract representations (*M*=0.38) were the highest and the remaining forms of representations for all generators were low (*M* < 0.06). Student-generated abstract representations (*M*= 0.27) and student-teacher-generated multiple forms of representations (*M*=0.19) were the highest in *My Math* and the remaining forms of representations for all generators were low (*M* < 0.09).

Table 9

The Average Proportion of Representation by Generator and Representation Type

		Everyday Mathematics	*Eureka Math*	*enVision Math*	*Go Math*	*Math Expressions*	*My Math*
Student							
	Concrete	0	0	0.04	0.01	0	0.01
	Semi-concrete	0.03	0.08	0.04	0.03	0.06	0.06
	abstract	0.23	0.07	0.13	0.08	0.38	0.26
	Multiple forms	0.38	0.27	0.04	0.04	0.20	0.07
	Total	0.64	0.42	0.25	0.16	0.64	0.40
Teacher							
	Concrete	0	0	0	0	0	0
	Semi-concrete	0.01	0.05	0.03	0	0	0.01
	abstract	0.03	0	0.12	0.01	0.01	0.03
	Multiple forms	0.03	0.03	0.08	0.02	0.01	0
	Total	0.07	0.08	0.23	0.03	0.02	0.04
Student-teacher							
	Concrete	0	0	0	0	0	0
	Semi-concrete	0.01	0.01	0.02	0.05	0	0.02
	abstract	0.01	0	0.01	0.03	0.0	0.09
	Multiple forms	0.13	0.21	0.08	0.17	0.01	0.19
	Total	0.13	0.21	0.08	0.17	0.01	0.19

Discourse and Conceptual Understanding

The mean proportion for incorporating discourse and conceptual understanding in *Everyday Mathematics* was 0.93 tasks per lesson (*SD*=0.19). In *Everyday Mathematics,* opportunity for reasoning strategies was the highest incorporated indicator for discourse and conceptual understanding (*M* =0.83; *SD* = 0.27) and making explicit connections was the lowest (*M*= 0.29 *SD*= 0.35). The mean proportion for incorporating discourse and conceptual understanding in *Eureka Math* was 0.98 tasks per lesson (*SD*=0.09). Discourse was the highest incorporated indicator for discourse and conceptual understanding in *Eureka Math* (*Mean* =0.75;

SD = 0.30) and making explicit connection was the lowest (*M*= 0.11; *SD*= 0.21). The mean proportion for incorporating discourse and conceptual understanding in *enVision Math* was 0.82 tasks per lesson (*SD*=0.22). In *enVision Math*, opportunity for reasoning strategies was the highest incorporated indicator for discourse and conceptual understanding (*M* =0.59; *SD* = 0.25) and discourse was the lowest (*M*= 0.01; *SD*= 0.04).

The mean proportion for incorporating discourse and conceptual understanding in *Go Math* was 0.90 tasks per lesson (*SD*=0.17). Opportunity for reasoning strategies was the highest incorporated indicator for discourse and conceptual understanding in *Go Math* (*M* =0.63; *SD* = 0.24) and making explicit connection was the lowest (*M*= 0.05; *SD*= 0.11). The mean proportion for incorporating discourse and conceptual understanding in *Math Expressions* was 0.98 tasks per lesson (*SD*=0.06). In *Math Expressions*, opportunity for reasoning strategies was the highest incorporated indicator for discourse and conceptual understanding (*M* =0.91; *SD* = 0.20) and making explicit connection was the lowest (*M*= 0.09; *SD*= 0.22). The mean proportion for incorporating discourse and conceptual understanding in *My Math* was 0.57 tasks per lesson (*SD*=0.28). In *My Math*, opportunity for reasoning strategies was the highest incorporated indicator for discourse and conceptual understanding (*M* =0.44; *SD* = 0.30) and making explicit connection was the lowest (*M*= 0.01; *SD*= 0.05).

Strategy Instruction

The mean proportion for incorporating strategy instruction in *Everyday Mathematics* was 0.18 tasks per lesson (*SD*=0.33). In *Everyday Mathematics*, incorporation of cognitive and meta-cognitive strategies were similarly low (*M* =0.05 and 0.02; *SD* = 0.18 and 0.08 respectively). The mean proportion for incorporating strategy instruction in *Eureka Math* was 0.53 tasks per lesson (*SD*=0.37). The incorporation of cognitive and meta-cognitive strategies were similar in *Eureka*

Math (*M* =0.27 and 0.26; *SD* = 0.37 and 0.36 respectively). For the remaining textbooks, the mean proportion for incorporating strategy instruction was similarly low (*M* = 0.02-0.07) tasks per lesson (*SD*=0.09-0.15). In addition, the mean proportion for incorporating cognitive strategies was similarly low in the remaining textbooks (*M* = 0.01-0.05) tasks per lesson (*SD*=0.01-0.12). The mean proportion for incorporating mata-cognitive strategies was similarly low in the remaining textbooks (*M* = 0.00-0.01) tasks per lesson (*SD*=0.00-0.04).

Summary

The results showed that all textbooks had mathematical goals and objectives in their lessons. However, textbooks varied in the mathematical strands, practices, and content standards they addressed in their lessons. Mean proportion for reasoning and problem solving in the textbooks ranged from 0.41 to 0.60. High-cognitive demand tasks was the highest incorporated reasoning and problem solving's indicator in the textbooks and flexibility tasks was the lowest in most textbooks. With regard to visual representations, the mean proportion in the textbooks ranged from 0.26 to 0.62. Multiple forms of representation was the highest incorporated indicator in all textbooks and selection from multiple representation was the lowest. For discourse and conceptual understanding, the mean proportion ranged from 0.53 to 0.98. Student reasoning strategies was the highest incorporated indicator and making explicit connection was the lowest in most textbooks. The mean proportion for strategy instruction ranged from 0.03 to 0.53. Except for *Eureka Math*, the incorporation of cognitive strategy instruction and its indicators was very low in the textbooks.

Research Question 2: Are there significant differences between textbooks in incorporating effective teaching practices?

Reasoning and Problem Solving

Results of the One-Way ANOVA revealed a significant difference between textbooks in implementing tasks that promote reasoning and problem solving, $F(5, 244) = 2.498$, $p = 0.0315$. Table 10 reports the results of post hoc analysis for all pairs of the textbooks in incorporating tasks that promote reasoning and problem solving. As can be seen in the table, no significant difference was found between all pairs of textbooks in incorporating this practice.

Table 10

Post hoc Comparison between Textbooks in Incorporating Reasoning and Problem Solving

Textbook Comparison	Mean Difference	95% CI		Adjusted *p*-value
		Lower	Upper	
Everyday Mathematics - Eureka Math	0.046	- 0.255	0.161	0.987
Everyday Mathematics – enVision Math	0.046	- 0.252	0.159	0.987
Everyday Mathematics - Go Math	- 0.007	- 0.203	0.219	0.999
Everyday Mathematics - Math Expressions	0.134	- 0.347	0.079	0.462
Everyday Mathematics - My Math	0.188	- 0.388	0.010	0.075
Eureka Math - enVision Math	-0.001	- 0.211	0.212	1.000
Eureka Math - Go Math	-0.054	- 0.163	0.272	0.979
Eureka Math - Math Expressions	0.087	- 0.306	0.131	0.862
Eureka Math - My Math	0.141	- 0.347	0.063	0.356
enVision Math - Go Math	-0.054	- 0.161	0.269	0.979
enVision Math - Math Expressions	0.087	- 0.304	0.128	0.852
enVision Math - My Math	0.142	- 0.345	0.060	0.337
Go Math - Math Expressions	0.142	- 0.364	0.079	0.443
Go Math - My Math	0.196	- 0.405	0.012	0.077
Math Expressions - My Math	0.054	- 0.264	0.155	0.976

Visual Representations

Results of the One-Way ANOVA revealed a significant difference between textbooks in implementing tasks that use visual representations, $F(5, 244) = 10.52$, $p = 0.000$. Table 10 reports the results of post hoc analysis for all pairs of the textbooks in incorporating tasks that use visual representations. As can be seen in Table 11, the average proportion for using visual representations in *Everyday Mathematics* and *Eureka Math* was significantly higher than *enVision Math*, *Go Math*, *Math Expressions*, and *My Math*.

Table 11

Post hoc Comparison between Textbooks in Incorporating Visual Representations

Textbook Comparison	Mean Difference	CI		Adjusted *p*-value
		Lower	Upper	
Everyday Mathematics - Eureka Math	0.015	- 0.224	0.193	0.999
Everyday Mathematics - enVision Math	0.322	- 0.528	- 0.116	0.000*
Everyday Mathematics - Go Math	0.357	- 0.568	- 0.145	0.000*
Everyday Mathematics - Math Expressions	0.329	- 0.543	- 0.116	0.000*
Everyday Mathematics - My Math	0.333	- 0.533	- 0.134	0.000*
Eureka Math - enVision Math	0.307	- 0.519	- 0.094	0.000*
Eureka Math - Go Math	0.341	- 0.559	- 0.123	0.000*
Eureka Math - Math Expressions	0.314	- 0.533	- 0.094	0.000*
Eureka Math - My Math	0.318	- 0.523	- 0.112	0.000*
enVision Math - Go Math	0.034	- 0.249	0.180	0.997
enVision Math - Math Expressions	0.007	- 0.223	0.209	0.999
enVision Math - My Math	0.011	- 0.214	0.192	0.999
Go Math - Math Expressions	- 0.027	- 0.194	0.249	0.999
Go Math - My Math	- 0.023	- 0.185	0.232	0.999
Math Expressions - My Math	0.003	- 0.214	0.206	0.999

Discourse and Conceptual Understanding

Results of the One-Way ANOVA revealed a significant difference between textbooks in implementing tasks intended to facilitate discourse and conceptual understanding, $F(5, 244) = 26.62$, $p = 0.00$. Table 12 reports the results of post hoc analysis for all pairs of textbooks in incorporating tasks intended to facilitate discourse and conceptual understanding. The average proportion for discourse and conceptual understanding in all of the textbooks was significantly higher than *My Math*. In addition, the average proportion for discourse and conceptual understanding in *Everyday Mathematics*, *Eureka Math*, and *Math Expressions was* significantly higher than *enVision Math*.

Table 12

Post hoc Comparison between Textbooks in Incorporating Discourse and Conceptual Understanding

Textbook Comparison	Mean Difference	CI		Adjusted *p*-value
		Lower	Upper	
Everyday Mathematics - Eureka Math	- 0.045	- 0.077	0.168	0.895
Everyday Mathematics - enVision Math	0.132	- 0.254	- 0.010	0.023*
Everyday Mathematics - Go Math	0.041	- 0.166	0.082	0.928
Everyday Mathematics - Math Expressions	- 0.048	- 0.077	0.174	0.876
Everyday Mathematics - My Math	0.353	- 0.470	- 0.235	0.000*
Eureka Math - enVision Math	0.178	- 0.303	- 0.052	0.000*
Eureka Math - Go Math	0.087	- 0.215	0.040	0.371
Eureka Math - Math Expressions	- 0.003	- 0.126	0.132	0.999
Eureka Math - My Math	0.398	- 0.520	- 0.277	0.000*
enVision Math - Go Math	- 0.090	- 0.036	0.217	0.317
enVision Math - Math Expressions	- 0.181	0.053	0.308	0.000*
enVision Math - My Math	0.220	- 0.340	- 0.100	0.000*
Go Math - Math Expressions	- 0.090	- 0.040	0.221	0.352
Go Math - My Math	0.311	- 0.434	- 0.188	0.000*
Math Expressions - My Math	0.401	- 0.525	- 0.277	0.000*

Strategy Instruction

Results of the One-Way ANOVA revealed a significant difference between textbooks in implementing tasks that use cognitive and meta-cognitive strategies, $F(5, 244) = 31.6$, $p = 0.000$. Table 13 reports the results of post hoc analysis for all pairs of the textbooks in incorporating tasks that promote the use of cognitive. The average proportion for cognitive strategies in *Eureka Math* was significantly higher than the other textbooks. The average proportion for cognitive strategies *in Everyday Mathematics* was significantly higher than *enVision Math* and *My Math*.

Table 13

Post hoc Comparison between Textbooks in Incorporating Strategy instruction

Textbook Comparison	Mean Difference	CI		Adjusted *p*-value
		Lower	Upper	
Everyday Mathematics - Eureka Math	- 0.367	0.229	0.505	0.000*
Everyday Mathematics - enVision Math	0.139	- 0.275	- 0.003	0.040*
Everyday Mathematics - Go Math	0.089	- 0.228	0.050	0.446
Everyday Mathematics - Math Expressions	0.122	- 0.263	0.017	0.125
Everyday Mathematics - My Math	0.134	- 0.267	- 0.002	0.042*
Eureka Math - enVision Math	0.506	- 0.646	- 0.366	0.000*
Eureka Math - Go Math	0.456	- 0.600	- 0.312	0.000*
Eureka Math - Math Expressions	0.490	- 0.634	- 0.345	0.000*
Eureka Math - My Math	0.502	- 0.638	- 0.365	0.000*
enVision Math - Go Math	- 0.050	- 0.091	0.192	0.911
enVision Math - Math Expressions	- 0.016	- 0.126	0.159	0.999
enVision Math - My Math	- 0.004	- 0.129	0.139	0.999
Go Math - Math Expressions	0.033	- 0.180	0.112	0.985
Go Math - My Math	0.045	- 0.183	0.092	0.933
Math Expressions - My Math	0.011	- 0.151	0.127	0.999

Summary

The one-way ANOVA tests were significant for three practices use of visual representations, discourse and conceptual understanding, and strategy instruction. For use of visual representations, the mean proportion for *Everyday Mathematics* and *Eureka Math* was significantly higher than the other textbooks. The mean proportion for discourse and conceptual understanding in all of the textbooks was significantly higher than *My Math.* Further, the mean proportion for discourse and conceptual in *Everyday Mathematics*, *Eureka Math*, and *Math Expressions* was significantly higher than *enVision Math*. For strategy instruction, the mean proportion in *Eureka Math* was significantly higher than the other textbooks and the mean proportion in *Everyday Mathematics* was significantly higher than *enVision Math* and *My Math.*

Chapter 5

Discussion

This study examined the incorporation of effective teaching practices in mathematics textbooks in the U.S. Results revealed variations in the incorporation of effective teaching practices: Some practices had high incorporation (goals and discourse and purposeful questions), while others had moderate incorporation (promote reasoning and visual representation) or low incorporation (strategy instruction). In addition, textbooks differed in the extent to which they incorporated effective teaching practices. While *Everyday Mathematics* and *Eureka Math* had relatively high incorporation of most effective teaching practices, *enVision Math* and *My Math* had relatively low incorporation of most effective teaching practices. *Go Math* and *Math Expressions* had mixed profiles with relatively high incorporation of some practices and low incorporation of other practices.

Clear Goals

Lessons in the textbooks included clear mathematical goals and content standards which results in high scores in this practice. Overall, most of the goals focused on two mathematical proficiency strands (conceptual understanding and strategic competence) and rarely focused on adaptive reasoning or productive disposition, with an observed between-textbook variations. For example, *Everyday Mathematics* excelled in focusing on adaptive reasoning, and *enVision Math* excelled in addressing procedural fluency. Although three content standards (3.OA.3, 3.OA.5, and 3.OA.7) were the most addressed in the textbooks, some differences between textbooks can be observed with regard to content standards. For example, while 15% of the lessons in *My Math* addressed 3.OA.4, only 2% of the lessons in *enVision Math* and *Go Math* address 3.OA.4. Further, 10% of the lessons in *enVision* Math addressed 3.OA.7 and 27% *of Everyday*

Mathematics addressed the same standards. The observed difference between textbooks also extends to the standards for mathematical practices. For example, 0% of the lessons in *Eureka Math* addressed “Attend to precision”; in comparison, 17% of the lessons in Math Expressions addressed this standard. Another example is “Look for and make use of structure” which was addressed in 35% of the lessons in *Eureka Math* and only in 9% of lessons in *Math Expressions*.

The research suggests substantial misalignments between tasks in mathematics textbooks and the CCSS (2010) with textbooks overemphasizing low-order thinking tasks compared to the standards (Polikoff, 2015). In this study, differences were found in the proficiency strands and content and mathematical standards addressed in the observed textbooks and lessons which call into question whether the CCSS (2010) resulted in a more coherent mathematics curriculum, one of the main goals of the CCSS (2010).

Goals and standards guide the design of all aspects of lessons (e.g., tasks and activities, assessment) and it is imperative for goals to engage various standards of mathematical proficiency and practice. For example, the low level of adaptive-reasoning-focused goals in the textbooks was reflected as low incorporation of indicators of adaptive reasoning in the textbooks (e.g., explaining why a specific procedure works for a given problem, defending their approach). Further, only 7% of the standards of mathematical practice addressed “Look for and express regularity in repeated reasoning,” which translated in a low incorporation of tasks promoting generalization in the textbooks. Thus, it is insufficient to write clear mathematical goals and list content standards and it is critical that the goals engage a range of mathematical proficiency and practice and reflect them in the tasks and activities in the lessons.

Reasoning and Problem Solving

Differences in mean incorporation of tasks promoting reasoning and problem solving between the textbooks was not significant. An examination of the indicators of reasoning and problem solving revealed that high-cognitive demand tasks was the highest used indicator across textbooks and flexibility was the lowest in all textbook except *Eureka Math*. A qualitative examination of high-cognitive demand tasks showed that 75% of the high-cognitive demand tasks were multiple-steps word problems and the other forms of high-cognitive demand tasks (e.g., real-world context or reversibility) were rarely included in the textbooks. In fact, real-world word problems were not observed in two textbooks (*Eureka Math* and *Go Math*) indicating a major limitation in these textbooks given the importance of real-world problems in developing critical thinking, communication, and creativity skills that students can use in other areas of their lives.

Further, the mean incorporation flexibility and generalization tasks was lower compared to high-cognitive demand tasks. Tasks promoting flexibility were observed in a variety of contexts (e.g., whole classrooms, partner, independent practice) and forms (e.g., embedded questions in the tasks [e.g., solve the problem in different ways or how can you use multiplication and division to solve the problem], a follow-up question or discussion [find a different way to solve the problem; how would explain this child strategy? what is similar and different about these strategies]). Further, generalization tasks engaged students with a variety of generalizable concepts (e.g., the inverse relation between multiplication and division, even and odd numbers, multiplicative identity property of one, zero property of multiplication, commutative property, and distributive property of multiplication).

Previous studies reported inconsistencies among mathematics textbooks in the U.S. regarding the incorporation of high-cognitive demand tasks, with some textbooks including as high as 59% (Jones & Tarr, 2007) and as low as 15% high-cognitive demand tasks (Jones & Tarr, 2007; Son, 2012). In this study, less variation was observed regarding the incorporation of high-cognitive demand tasks (28%-50%). This study focused only on word-problem solving tasks compared to previous studies that focused on all tasks targeting fractions (Son, 2012) or probability (Jones & Tarr, 2007). As such, this finding should be interpreted with caution as the incorporation of high-cognitive demand tasks might be different if all contextual and non-contextual tasks in the textbooks were included. It is difficult to assess trends in the textbooks with regard to flexibility and generalization as I did not find studies that examine these teaching practices in mathematics textbooks. Although textbooks showed that flexibility and generalizability can be promoted in various contexts and forms or for different concepts, it is disappointing that textbooks rarely promoted them despite the long-standing recommendation of these teaching practices (CCSS, 2010; NCTM, 2000).

Visual Representations

In *Everyday Mathematics* and *Eureka Math* the incorporation of visual representation (~60% tasks per lesson) was significantly higher than the remaining textbooks (~30% task per lesson). When examining the indicators for visual representations, multiple representations had the highest incorporation across textbooks (20%-55% tasks per lesson) and selecting from multiple representations had the lowest incorporation (0%-9% tasks per lessons. With a few exceptions (e.g., *Everyday Mathematics* and *Eureka Math* for creating diagrams and *Everyday Mathematics* for making real world connections), the incorporation of creating diagrams making real world connections was very low (0%-14%).

The result of this study was consistent with Jitendra (2005; 2007) that textbooks promote making real-world connections and creating diagrams at a low level overall with between-textbook variations. This finding indicates minimal changes in textbooks regarding prompting students to make real-world connections and create representation. However, the observed low incorporation of tasks prompting students to select among multiple forms of representation in this study (0%-9%) is inconsistent with findings in Jitendra (34%; 2005, 2007). It is unclear whether the difference between the finding of this study and Jitendra (2005, 2007) is due to coding differences or to a real decrease in the incorporation of the practice in the textbooks. It is disappointing that textbooks provided a low opportunity for using and connecting representation in the observed lessons depriving students of opportunities to conceptualize, communicate and observe mathematical ideas in different forms and developing a deep and flexible understating of the concepts (Hiebert & Carpenter, 1992; Piez & Voxman, 1997).

Overall, abstract representation was the highest included representation in this study (57%) followed by semi-concrete representation (39%) and concrete representation (13%). This finding is inconsistent with Jitendra (2007), who found concrete representation (80%) to be the most included followed by abstract representation (73%) and semi-concrete representation (52%). This discrepancy between the findings of this study and Jitendra (2007) is likely due to difference in the grade level of the textbooks examined. It is reasonable that textbooks included more concrete representation in Jitendra (2007) given the grade level examined (first grade) that requires an increased level of concrete representation to foster students' understanding of mathematical concepts. Students develop more sophisticated and abstract thinking reflected in the use of abstract representations as they progress through elementary school (Baroody, 2017; Clements, 2000). The incorporation of concrete representation (13% in a lesson on average) was low with two textbooks (*Eureka Math and Math Expressions*) rarely including concrete representation. The low incorporation of concrete representation in the observed lessons suggest that these lessons are ill-prepared to address the needs of some students with MLD who might need concrete representation to build a conceptual understanding of foundational mathematical ideas (Gersten et al., 2009).

With regard to the generator of the representation, the results indicated that representations tended to be generated by students more often and were rarely given to students. This is inconsistent with Garderen et al. (2012) who found representations tended to be given to or generated by students at a similar rate in the textbooks. It is encouraging that textbooks provided opportunities to learn and practice with representations more often in this study. Despite the overall improvement in providing opportunities for students to construct representation, variations between textbooks existed. For example, representations were generated by students and given to students at a similar level in *enVision Math*. In addition, *enVision Math* and *My Math* provided substantially more opportunity for student-generated abstract representation and less opportunity for student-generated semi-concrete representation. These variations suggest textbooks may differ in the opportunity for practicing and constructing representations and the level of scaffolding provided.

Discourse and Conceptual Understanding

There were significant difference between textbooks in incorporating discourse and conceptual understanding as *My Math* and *enVision Math* incorporated this practice significantly lower than most of the other textbooks. The difference between textbooks in discourse and conceptual understanding might be explained in part by the textbook's orientation. *Everyday Mathematic*s and *Math Expression* used more student-centered approach allowing more opportunity for reasoning and discourse. In comparison, *enVision Math* and *My Math* emphasizes teachers' modeling to develop conceptual understanding.

Providing students with opportunities to use their own reasoning strategies and methods fosters the conceptual understanding of mathematical ideas (Carpenter et al., 1989). Only *Everyday Mathematics* and *Math Expression* provided the highest opportunities for reasoning strategies (83% and 91%) compared to remaining textbooks (44% to 63%). A qualitative examination of the tasks in the textbooks revealed that tasks invite reasoning strategies through posing WPS tasks with minimal or generic guidance from the teacher or with follow-up discussion. In comparison, tasks limited students' reasoning strategies through teacher modeling with minimal student input or requiring the use of a certain solution method or representation providing little opportunity for students to grapple with mathematical concepts and use their own strategies.

Although 44% to 91% task per lesson provided opportunities for reasoning strategies, with an exception of *Everyday Mathematics* (29%), textbooks rarely prompted teachers to make explicit connections between students' thinking and key mathematical ideas or efficient procedures (1% to 11%). This finding suggests textbooks rarely prompt teachers to build on student current understanding, a critical feature of effective teaching in mathematics.

Actively engaging students in mathematical discourse develops students' mathematical competencies and identity (Walshaw & Anthony, 2008). Textbooks were divided with regard to intention to facilitate discourse evident in the tasks. Over 50% of the tasks per lesson *Everyday Mathematics, Eureka Math,* and *Math Expression* incorporated prompts that guide teachers to facilitate discourse. In contrast, less than 10% of the task per lesson in *enVision Math, Go Math,* and *My Math* provided opportunity for discourse. A qualitative examination showed salient differences between textbooks in approaching discourse. For example, in *Everyday Mathematics,* mathematical discourse was incorporated in a variety of ways that is tailored specifically to the tasks (e.g., have children share different ways they decomposed the array). In comparison, *Eureka Math* and *Math Expression* used a generic phrase or process that promotes discourse in most lessons with little task-specific discourse direction (e.g., invite students to review their solutions for the problem set or Solve and Discuss). As such, *Everyday Mathematics* provided more task-specific guidance with regard to discourse that novice teachers might find helpful, and *Eureka Math* and *Math Expression* provided generic guidance that is more suitable for expert teachers who can rely on their experience and knowledge to tailor it to the tasks. Further, posing purposeful questions that engage high-order thinking was generally low in all textbooks (less than 41% of tasks in a lesson limiting students' opportunity to develop a deeper understating of concepts.

Strategy Instruction

Only *Eureka Math* incorporated this practice at a level that was significantly higher than the remaining textbooks (53% of tasks per lesson incorporated this practice in *Eureka Math*). Examination of the indicators for this practice revealed similar patterns as *Eureka Math* incorporated meta-cognitive and cognitive strategies more than the other textbooks did. Qualitative examination showed that *Eureka Math* prompted students to use cognitive strategies in almost every lesson but rarely guided students in how to use them. In comparison, the incorporation of cognitive and meta-cognitive strategies in the other textbooks was condensed to a few lessons limiting students' opportunity to practice using the strategies. The low incorporation of cognitive instruction is consistent with Bryant et al. (2008), indicating little change in the U.S. textbooks in promoting this practice.

Implication for Practice

This study found the incorporation of some practices was modest at best (e.g., visual representation, strategy instruction, flexibility tasks). Some of the practices are evidence-based for students with MLD (use of visual representations, cognitive strategies, explicit connection) or effective teaching in general (e.g., flexibility tasks, generalization). If the goal is to use scientifically based core curricula to better support students with MLD, textbook developers should ensure the incorporation of these practices at an acceptable level. It is also important to distribute the incorporation of these teaching practices across lessons to offer students with MLD ample opportunity to develop deep and flexible conceptual and strategic knowledge as well as reasoning in mathematics. In addition, the observed variations between textbooks underscore that teachers should carefully examine mathematics textbooks used and whether they promote effective teaching practices. Teachers might need to rely on their professional knowledge to supplement the textbooks with other material or adjust some of the textbook-presented tasks to make them more effective.

Future Research

The results indicate some practices are incorporated more often than others in general and that some textbooks are better than others in incorporating some of the effective teaching practices. Within the different frameworks of mathematics curricula, textbooks could be placed either at the written curricula or the potentially implemented curricula, which influence intended and implemented curricula by teachers. This study and previous textbook analysis studies showed differences between textbooks in incorporating effective teaching practices. However, there is a dearth of studies exploring whether the increased or decreased incorporation of the practices transfer to an increased or decreased implementation by teachers. As such future studies should attempt to establish the influence of textbooks on teachers' instructional decisions.

Although this study provides valuable insight regarding the incorporation of effective teaching practice quantitively, important qualitative information about the incorporation of effective teaching practices was not captured. For example, research suggests that the cognitive demand of a task is susceptible to change as the task goes through different phases of development (Stein et al., 1996). That is, a task might be written as high-cognitive demand but get watered down during implementation. Further, representation should build on what is familiar to students to extend students' learning and understanding (Baroody, 2017). This study provides little information on whether textbooks promoted these important qualities. Future research should delve deeper in the tasks proposed by the textbooks to better understand how textbooks promote important qualities within each practice.

Limitations

There are several limitations to this study. The focus of this study was on WPS tasks in lessons addressing operation and algebraic thinking. Although limiting the focus to WPS as a task and operation and algebraic thinking as a domain ensured the observed tasks were coherent in terms of nature and domain, it is unclear whether the results would generalize to non-contextual tasks and other domains. In addition, the literature in mathematics education and special education recommended the effective teaching practices that were included, but it is unclear how often these practices should be included (e.g., every task, every lesson, 50% of the tasks in a lesson). As such, it is difficult to judge the extent the difference between textbooks in the use of the practices is practically important which could be different from on practice to another. Although the incorporations of some of the practices was significantly high or low in some of the textbooks, it is unclear whether the level of incorporation of the practice was meaningful relative to student learning needs.

Conclusion

This study found some practices are incorporated more than others and some textbooks promoted effective practices at different rates. If the goal is to provide scientifically based core curricula to all students in general (ESSA, 2015) and students with MLD in particular (IDEA, 2004), there is a need to align mathematics textbooks with the recommended teaching practices to better support all students. This study found that some textbooks incorporated several effective teaching practices at a high level underscoring the importance of reviewing the textbooks' alignment with effective teaching during the textbook adoption process by districts or states and instructional decisions by teachers.

References

Andersson, U. (2008). Mathematical competencies in children with different types of learning difficulties. *Journal of Educational Psychology*, *100*(1), 48–66. https://doi.org/10.1037/0022-0663.100.1.48

Baroody, A. J. (2017). The use of concrete experiences in early childhood mathematics instruction. In *Advances in child development and Behavior* (Vol. 53, pp. 43–94). Academic Press Inc. https://doi.org/10.1016/bs.acdb.2017.03.001

Baxter, J. A., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction on low achievers in five third-grade classrooms. *The Elementary School Journal*, *101*(5). https://doi.org/10.1086/499686

Baxter, J., Woodward, J., Voorhies, J., & Wong, J. (2002). We talk about it, but do they get it? *Learning Disabilities Research and Practice*, *17*(3). https://doi.org/10.1111/1540-5826.00043

Blazar, D., Heller, B., Kane, T. J., Polikoff, M., Staiger, D. O., Carrell, S., Goldhaber, D., Harris, D. N., Hitch, R., Holden, K. L., & Kurlaender, M. (2020). Curriculum reform in the common core era: Evaluating elementary math textbooks across six U.S. states. *Journal of Policy Analysis and Management*, *39*(4), 966–1019. https://doi.org/10.1002/pam.22257

Blöte, A. W., van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology*, *93*(3), 627–638. https://doi.org/10.1037/0022-0663.93.3.627

Boonen, A. J. H., van der Schoot, M., van Wesel, F., de Vries, M. H., & Jolles, J. (2013). What underlies successful word problem solving? A path analysis in sixth grade students.

Contemporary Educational Psychology, *38*(3), 271–279. https://doi.org/10.1016/j.cedpsych.2013.05.001

Bostic, J. D., Pape, S. J., & Jacobbe, T. (2016). Encouraging sixth-grade performance by teaching through problem solving. *Investigations in Mathematics Learning*, *8*(3), 30–59. https://doi.org/10.1080/24727466.2016.11790353

Bransford, J. (1994). Who ya gonna call? Thoughts about teaching problem solving. In P. Hallinger, K. Leithwood, & J. Murphy (Eds.), *Cognitive perspectives on educational leadership* (pp. 171–191). Teacher College Press.

Bransford, J., Brown, A., & Cocking, R. (2000). *How people learn*. National Academies Press. https://doi.org/10.17226/9853

Bryant, B. R., Bryant, D. P., Kethley, C., Kim, S. A., Pool, C., & Seo, Y. J. (2008). Preventing mathematics difficulties in the primary grades: The critical features of instruction in textbooks as part of the equation. *Learning Disability Quarterly*, *31*(1), 21–35. https://doi.org/10.2307/30035523

Buzza, D. C., & Dol, M. (2015). Goal-setting support, motivation, and engagement in alternative math classes. *Exceptionality Education International*, *25*(1), 35–66. https://doi.org/10.5206/eei.v25i1.7716

Carpenter, T., Fennema, E., Franke, M., Linda Levi, & Empson, S. (2015). *Children's mathematics: Cognitively guided instruction* (K. Bryant, Ed.; 2nd ed.). Heinemann.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, *26*(4), 499-531. https://doi.org/10.2307/1162862

Chapin, S., O'Connor, C., & Anderson, N. (2009). *Classroom discussions: Using math talk to help students learn* (2nd ed.). Math Solutions.

Charles, R., & Carmel, C. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education*.

Cirino, P. T., Tolar, T. D., Fuchs, L. S., & Huston-Warren, E. (2016). Cognitive and numerosity predictors of mathematical skills in middle school. *Journal of Experimental Child Psychology, 145*, 95–119. https://doi.org/10.1016/j.jecp.2015.12.010

Clements, D. H. (2000). 'Concrete' manipulatives, concrete ideas. *Contemporary Issues in Early Childhood, 1*(1), 45–60. https://doi.org/10.2304/ciec.2000.1.1.7

Dinsmore, D. L. (2017). Strategic Processing in Education. In *Strategic Processing in Education*. Routledge. https://doi.org/10.4324/9781315505732

Doabler, C. T., Fien, H., Nelson-Walker, N. J., & Baker, S. K. (2012). Evaluating three elementary mathematics programs for presence of eight research-based instructional design principles. *Learning Disability Quarterly, 35*(4), 200–211. 10.1177/0731948712438557

Dougherty, B., Bryant, D. P., Bryant, B. R., Darrough, R. L., & Pfannenstiel, K. H. (2015). Developing concepts and generalizations to build algebraic thinking: The reversibility, flexibility, and generalization approach. *Intervention in School and Clinic*, *50*(5), 273–281. http://dx.doi.org/10.1177/1053451214560892

Dougherty, C. (2003). Numeracy, literacy and earnings: Evidence from the National Longitudinal Survey of Youth. *Economics of Education Review, 22*(5), 511–521. https://doi.org/10.1016/S0272-7757(03)00040-2

Elementary and Secondary Education Act of 1965, H.R. 2362, 89th Cong., Pub. L. No. 89-10 (1965).

enVision Math. (2014). New York, NY: Pearson/Scott Foresman– Addison Wesley.

Eureka Math. (2015). Washington, D.C. VA: Great Minds.

Every Student Succeeds Act, Pub. L. No. 114-95 (2015).

Everyday Mathematics. (2015). New York, NY: McGraw Hill.

Fan, L. (2000). The influence of textbooks on teaching strategies: An empirical study. *Mid-Western Educational Researcher*, *13*(4), 2–9.

Fan, L., Cheng, J., Xie, S., Luo, J., Wang, Y., & Sun, Y. (2021). Are textbooks facilitators or barriers for teachers' teaching and instructional change? An investigation of secondary mathematics teachers in Shanghai, China. *ZDM - Mathematics Education*, *53*(6), 1313–1330. https://doi.org/10.1007/s11858-021-01306-6

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics*, *66*(1), 61–75. https://doi.org/10.1007/s10649-006-9069-6

Forman, E. A., Mccormick, D. E., & Donato, R. (1997). Learning what counts as a mathematical explanation. *Linguistics and Education*, *9*(4). https://doi.org/10.1016/S0898-5898(97)90004-8

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., Schatschneider, C., & Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, *98*(1), 29–43. https://doi.org/10.1037/0022-0663.98.1.29

Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M., Bryant, J. D., & Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology*, *46*(6), 1731–1746. https://doi.org/10.1037/a0020662

Fuchs, L. S., Malone, A. S., Schumacher, R. F., Namkung, J., Hamlett, C. L., Jordan, N. C., Siegler, R. S., Gersten, R., & Changas, P. (2016). Supported self-explaining during fraction intervention. *Journal of Educational Psychology*, *108*(4), 493–508. 10.1037/edu0000073

Fuchs, L. S., Powell, S. R., Seethaler, P. M., Cirino, P. T., Fletcher, J. M., Fuchs, D., Hamlett, C. L., & Zumeta, R. O. (2009). Remediating number combination and word problem deficits among students with mathematics difficulties: A randomized control trial. *Journal of Educational Psychology*, *101*(3), 561–576. https://doi.org/10.1037/a0014701

Fuson, K., Kalchman, M., & Bransford, J. (2005). Mathematical understanding: An introduction. In S. Donovan & J. Bransford (Eds.), *How students learn*. National Academies Press. https://doi.org/10.17226/10126

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities*, *37*(1), 4–15. https://doi.org/10.1177/00222194040370010201

Geary, D. C. (2013). Early foundations for mathematics learning and their relations to learning disabilities. *Current Directions in Psychological Science*, *22*(1), 23–27. https://doi.org/10.1177/0963721412469398

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, *78*(4), 1343–1359. https://doi.org/10.1111/j.1467-8624.2007.01069.x

Gersten, R., Chard, D. J., Jayanthi, M., Baker, S. K., Morphy, P., & Flojo, J. (2009). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research*, *79*(3), 1202–1242. https://doi.org/10.3102/0034654309334431

Gersten, R., Clarke, B., & Mazzocco, M. (2007). Historical and contemporary perspectives on mathematical learning disabilities. In D. Berch & M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 7–27). Paul H. Brookes Publishing Co.

Go Math! Teacher Edition. (2016). Boston, MA: Houghton Mifflin Harcourt.

Goldin, G., & Kaput, J. (1996). A Joint Perspective on the Idea of Representation in Learning and Doing Mathematics. In L. Steffe, P. Nesher, P. Cobb, B. Sriraman, & B. Greer (Eds.), *Theories of Mathematical Learning* (pp. 397–430). Routledge.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in english, french and german classrooms: Who gets an opportunity to learn what? *British Educational Research Journal*, *28*(4). https://doi.org/10.1080/0141192022000005832

Haggarty, L., & Pepin, B. (2017). Learning opportunities offered to pupils in England: A cause of concern. In B. Grevholm (Ed.), *Mathematics textbooks, their content, use and influences: Research in Nordic and Baltic countries* (pp. 117–128). Cappelen Damm Akademisk.

Hansen, N., Jordan, N. C., Fernandez, E., Siegler, R. S., Fuchs, L., Gersten, R., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development*, *35*, 34–49. https://doi.org/10.1016/j.cogdev.2015.02.001

Hatano, G. (1988). Social and motivational bases for mathematical understanding. *New Directions for Child and Adolescent Development*, *1988*(41), 55–70. https://doi.org/10.1002/cd.23219884105

Hattie, J. (2008). Visible learning: A synthesis of over 800 meta-analyses relating to achievement. In *Visible Learning: A Synthesis of Over 800 Meta-Analyses Relating to Achievement*. https://doi.org/10.4324/9780203887332

Haystead, M. W., & Marzano, R. J. (2009). *Meta-Analytic Synthesis of Studies Conducted at Marzano Research Laboratory on Instructional Strategies*. Marzano Research Laboratory.

Hegarty, M., Mayer, R. E., & Monk, C. a. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology*, *87*(1), 18–32. https://doi.org/10.1037/0022-0663.87.1.18

Hekimoglu, S., & Sloan, M. (2005). A Compendium of Views on the NCTM Standards. *The Mathematics Educator*, *15*(1), 35–43.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, *30*(1), 3–19. https://doi.org/10.2307/749627

Hiebert, J., & Carpenter, T. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). Macmillan.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and earning*. Information Age.

Hiebert, J., & Wearne, D. (1993). Instructional Tasks, Classroom Discourse, and Students' Learning in Second-Grade Arithmetic. *Source: American Educational Research Journal*, *30*(2), 393–425.

Hunt, J. H., & Empson, S. B. (2015). Exploratory study of informal strategies for equal sharing problems of students with learning disabilities. *Learning Disability Quarterly*, *38*(4). https://doi.org/10.1177/0731948714551418

Hwang, J., & Riccomini, P. J. (2016). Enhancing mathematical problem solving for secondary students with or at risk of learning disabilities: A literature review. *Learning Disabilities Research & Practice*, *31*(3), 169–181. https://doi.org/10.1111/ldrp.12105

Individuals with Disabilities Education Improvement Act of 2004, 20 U.S.C. § 614 *et seq.*

Jayanthi, M., Gersten, R., Schumacher, R. F., Dimino, J., Smolkowski, K., & Spallone, S. (2021). Improving struggling fifth-grade students' understanding of fractions: A randomized controlled trial of an intervention that stresses both concepts and procedures. *Exceptional Children*, *88*(1), 81–100. https://doi.org/10.1177/00144029211008851

Jitendra, A. K., Deatline-Buchman, A., & Sczesniak, E. (2005a). A Comparative analysis of third-grade mathematics textbooks before and after the 2000 NCTM standards. *Assessment for Effective Intervention*, *30*(2), 47–62. https://doi.org/10.1177/073724770503000205

Jitendra, A. K., Dupuis, D. N., Rodriguez, M. C., Zaslofsky, A. F., Slater, S., Cozine-Corroy, K., & Church, C. (2013). A randomized controlled trial of the impact of schema-based instruction on mathematical outcomes for third-grade students with mathematics difficulties. *The Elementary School Journal*, *114*(2), 252–276. https://doi.org/10.1086/673199

Jitendra, A. K., Dupuis, D. N., Star, J. R., & Rodriguez, M. C. (2016). The effects of schema-based instruction on the proportional thinking of students with mathematics difficulties with and without Reading difficulties. *Journal of Learning Disabilities*, *49*(4), 354–367. https://doi.org/10.1177/0022219414554228

Jitendra, A. K., Griffin, C., Deatline-Buchman, A., Dipipi-Hoy, C., Sczesniak, E., Sokol, N. G., & Van Xin, P. (2005b). Adherence to mathematics professional standards and instructional design criteria for problem-solving in mathematics. *Exceptional Children*, *71*(3), 319–337. https://doi.org/10.1177/001440290507100307

Jitendra, A. K., Harwell, M. R., Dupuis, D. N., & Karl, S. R. (2017a). A randomized trial of the effects of schema-based instruction on proportional problem-solving for students with mathematics problem-solving difficulties. *Journal of Learning Disabilities*, *50*(3), 322–336. 10.1177/0022219416629646

Jitendra, A. K., Harwell, M. R., Karl, S. R., Simonson, G. R., & Slater, S. C. (2017b). Investigating a tier 1 intervention focused on proportional reasoning: A follow-up study. *Exceptional Children*, *83*(4), 340–358. https://doi.org/10.1177/0014402917691017

Jitendra, A. K., Petersen-Brown, S., Lein, A. E., Zaslofsky, A. F., Kunkel, A. K., Jung, P., & Egan, A. M. (2015). Teaching mathematical word problem solving: The quality of evidence for strategy instruction priming the problem structure. *Journal of Learning Disabilities*, *48*(1), 51–72. https://doi.org/10.1177/0022219413487408

Jonassen, D. (1997). Instructional design models for well-structured and ill-structured problem-solving learning outcomes. *Educational Technology Research and Development*, *45*(1), 65–94. http://www1.folha.uol.com.br/ciencia/880408-bahia-inicia-uso-de-inseto-transgenico-contra-dengue.shtml

Jonassen, D. (2000). Toward a design theory of problem solving. *Educational Technology Research and Development*, *48*(4), 63–85. https://doi.org/10.1007/BF02300500

Jones, D. L., & Tarr, J. E. (2007). An examination of the levels of cognitive demand required by probability tasks in middle grades mathematics textbooks. *Statistics Education Research*, *6*(2), 4–27. https://doi.org/10.52041/serj.v6i2.482

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology*, *116*(1), 45–58. https://doi.org/10.1016/j.jecp.2013.02.001

Kaufman, J., Doan, S., Prado Tuma, A., Woo, A., Henry, D., & Lawrence, R. (2020). How Instructional Materials Are Used and Supported in U.S. K-12 Classrooms: Findings from the 2019 American Instructional Resources Survey. In *RAND Corporation*. https://doi.org/10.7249/rra134-1

Kilpatrick, J. (2014). *Handbook on the history of mathematics education* (A. Karp & G. Schubring, Eds.). Springer. https://doi.org/10.1007/978-1-4614-9155-2

Kilpatrick, Jeremy., Swafford, Jane., Findell, Bradford., & National Research Council (U.S.). Mathematics Learning Study Committee. (2001). *Adding it up : helping children learn mathematics*. National Academy Press.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review*, *92*(1). https://doi.org/10.1037/0033-295X.92.1.109

Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and metacognitive training. *American Educational Research Journal*, *40*(1), 281–310. https://doi.org/10.3102/00028312040001281

Krammer, H. P. (1985). The textbook as classroom context variable. *Teaching and Teacher Education*, *1*(4), 273–278. https://doi.org/https://doi.org/10.1016/0742-051X(85)90015-0

Lein, A. E., Jitendra, A. K., & Harwell, M. R. (2020). Effectiveness of mathematical word problem solving interventions for students with learning disabilities and/or mathematics difficulties: A meta-analysis. *Journal of Educational Psychology*, *112*(7), 1388–1408. https://doi.org/10.1037/edu0000453

Leuwerke, W. C., Robbins, S., Sawyer, R., & Hovland, M. (2004). Predicting engineering major status from mathematics achievement and interest congruence. *Journal of Career Assessment*, *12*(2). https://doi.org/10.1177/1069072703257756

Marzano, R. J. (2003). What works in schools: Translating research into action. Association for Supervision and Curriculum Development.

Math Expressions Teacher Edition. (2018). Boston, MA: Houghton Mifflin Company.

Maxwell, S., Delaney, H., & Kelley, K. (2018). *Designing experiments and analyzing data: A model comparison perspective* (3rd ed.). Routledge.

Mayer, R. E. (2002). Rote versus meaningful learning. *Theory Into Practice*, *41*(4), 226–232. https://doi.org/10.1207/s15430421tip4104_4

Mazzocco, M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. Berch & M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 29–47). Brookes Publishing Company.

McCloskey, M. (2007). Quantitative literacy and developmental dyscalculias. In D. Berch & M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 215–429). Paul H. Brookes Publishing Co.

McHugh, M. L. (2012). Interrater reliability: the kappa statistic. *Biochemia Medica*, *22*(3), 276–282.

McTighe, J., & Wiggins, G. (2013). What makes a question essential? In *Essential Questions: Opening Doors to Student Understanding*. Association for Supervision and Curriculum Development.

Mevarech, Z. R., & Kramarski, B. (1997). IMPROVE: A multidimensional method for teaching mathematics in heterogeneous classrooms. *American Educational Research Journal*. https://doi.org/10.3102/00028312034002365

Montague, M., & Applegate, B. (1993). Mathematical problem-solving characteristics of middle school students with learning disabilities. *The Journal of Special Education*, *27*(2), 175–201. https://doi.org/10.1177/002246699302700203

Montague, M., Enders, C., & Dietz, S. (2011). Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. *Learning Disability Quarterly*, *34*(4), 262–272. https://doi.org/10.1177/0731948711421762

Mullis, I. V., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. International Association for the Evaluation of Educational Achievement. Herengracht 487, Amsterdam, 1017 BT, The Netherlands.

My Math. (2013). New York, NY: McGraw Hill.

National Center for Education Statistics. (2019). Nation's report card. National Assessment of Educational Progress.

National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematics success for all. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (1989). Professional standards for school mathematics. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

National Mathematics Advisory Panel. *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*, U.S. Department of Education: Washington, DC, 2008.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Nelson, G., Hughes Pfannenstiel, K., & Zumeta Edmonds, R. (2020). Examining the alignment of mathematics instructional practices and mathematics vocabulary between core and intervention materials. *Learning Disabilities Research & Practice, 35*(1), 14–24. https://doi.org/10.1111/ldrp.12210

Neuendorf, K. A. (2020). The content analysis guidebook. In *The Content Analysis Guidebook* (2nd ed.). SAGE Publications, Inc. https://doi.org/10.4135/9781071802878

Newell, A., & Simon, H. (1972). *Human problem solving* (9th ed., Vol. 104). Prentice-hall.

No Child Left Behind Act of 2001, 20 U.S.C. § 6301 *et seq.*

Ostad, S. A., & Sorensen, P. M. (2007). Private speech and strategy-use patterns. *Journal of Learning Disabilities*, *40*(1). https://doi.org/10.1177/00222194070400010101

Park, S., Bryant, D. P., & Dougherty, B. (2021). Actionable 10: A checklist to boost mathematics teaching for students with learning disabilities. *Intervention in School and Clinic*, *56*(3), 148–154. https://doi.org/10.1177/1053451220942189

Peake, C., Jiménez, J. E., Rodríguez, C., Bisschop, E., & Villarroel, R. (2015). Syntactic awareness and arithmetic word problem solving in children with and without learning disabilities. *Journal of Learning Disabilities*, *48*(6), 593–601. https://doi.org/10.1177/0022219413520183

Pierson, J. (2008). *The relationship between patterns of classroom discourse and mathematics learning*. The University of Texas at Austin.

Piez, C. M., & Voxman, M. H. (1997). Multiple representations—using different perspectives to form a clearer picture. *The Mathematics Teacher*, *90*(2), 164–166. https://doi.org/10.5951/MT.90.2.0164

Polikoff, M. S. (2015). How well aligned are textbooks to the common core standards in mathematics? *American Educational Research Journal*, *52*(6), 1185–1211. https://doi.org/10.3102/0002831215584435

Powell, S. R., Berry, K. A., & Benz, S. A. (2020). Analyzing the word-problem performance and strategies of students experiencing mathematics difficulty. *Journal of Mathematical Behavior*, *58*(100759), 1–17. https://doi.org/10.1016/j.jmathb.2020.100759

Powell, S. R., Fuchs, L. S., Cirino, P. T., Fuchs, D., Compton, D. L., & Changas, P. C. (2015). Effects of a multitier support system on calculation, word problem, and prealgebraic performance among at-risk learners. *Exceptional Children*, *81*(4), 443–470. https://doi.org/10.1177/0014402914563702

Remillard, J. T., Harris, B., & Agodini, R. (2014). The influence of curriculum material design on opportunities for student learning. *ZDM*, *46*(5), 735–749. https://doi.org/10.1007/s11858-014-0585-z

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM - Mathematics Education*, *53*(6), 1189–1206. https://doi.org/10.1007/s11858-021-01309-3

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology*, *99*(3). https://doi.org/10.1037/0022-0663.99.3.561

Schmidt, W. H., Wang, H. C., & McKnight, C. C. (2005). Curriculum coherence: An examination of US mathematics and science content standards from an international perspective. *Journal of Curriculum Studies*. https://doi.org/10.1080/0022027042000294682

Schoen, H. L., & Charles, R. I. (2003). *Teaching Mathematics Through Problem Solving*. National Council of Teachers of Mathematics.

Schoenfeld, A. H. (2004). The Math Wars. *Educational Policy*, *18*(1). https://doi.org/10.1177/0895904803260042

Seethaler, P. M., Fuchs, L. S., Fuchs, D., & Compton, D. L. (2016). Does the value of dynamic assessment in predicting end-of-first-grade mathematics performance differ as a function of english language proficiency? *The Elementary School Journal*, *117*(2), 171–191. https://doi.org/10.1086/688870

Sherin, M. G., & Drake, C. (2009). Curriculum strategy framework: investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies*, *41*(4). https://doi.org/10.1080/00220270802696115

Siemon, D., Breed, M., & Virgona, J. (2006). From additive to multiplicative thinking–The big challenge of the middle years. *42nd Conference of the Mathematical Association of Victoria*.

Sievert, H., van den Ham, A. K., & Heinze, A. (2021). The role of textbook quality in first graders' ability to solve quantitative comparisons: a multilevel analysis. *ZDM - Mathematics Education*, *53*(6), 1417–1431. https://doi.org/10.1007/s11858-021-01266-x

Sievert, H., van den Ham, A.-K., Niedermeyer, I., & Heinze, A. (2019). Effects of mathematics textbooks on the development of primary school children's adaptive expertise in arithmetic. *Learning and Individual Differences*, *74*. https://doi.org/10.1016/j.lindif.2019.02.006

Son, J. W. (2012). A cross-national comparison of reform curricula in Korea and the US in terms of cognitive complexity: The case of fraction addition and subtraction. *ZDM - International Journal on Mathematics Education*, *44*(2), 161–174. https://doi.org/10.1007/s11858-012-0386-1

Son, J. W., & Kim, O. K. (2015). Teachers' selection and enactment of mathematical problems from textbooks. *Mathematics Education Research Journal*, *27*(4), 491–518. https://doi.org/10.1007/s13394-015-0148-9

Sood, S., & Jitendra, A. K. (2007). A comparative analysis of number sense instruction in reform-based and traditional mathematics textbooks. *Journal of Special Education*, *41*(3), 145–157.

Star, J. R., & Seifert, C. (2006). The development of flexibility in equation solving. *Contemporary Educational Psychology*, *31*(3). https://doi.org/10.1016/j.cedpsych.2005.08.001

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, *33*(2). https://doi.org/10.2307/1163292

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation*, *2*(1), 50–80. https://doi.org/10.1080/1380361960020103

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In *Second handbook of research on mathematics teaching and learning*.

Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction*, *4*(4). https://doi.org/10.1016/0959-4752(94)90003-5

Sweller, J., Chandler, P., Tierney, P., & Cooper, M. (1990). Cognitive load as a factor in the structuring of technical material. *Journal of Experimental Psychology: General*, *119*(2), 176–192. https://doi.org/10.1037/0096-3445.119.2.176

Terwel, J., van Oers, B., van Dijk, I., & van den Eeden, P. (2009). Are representations to be provided or generated in primary mathematics education? Effects on transfer. *Educational Research and Evaluation*, *15*(1), 25–44. https://doi.org/10.1080/13803610802481265

Van Garderen, D. (2006). Spatial Visualization, Visual Imagery, and Mathematical Problem Solving of Students With Varying Abilities. *Journal of Learning Disabilities*, *39*(6), 496–506. https://doi.org/10.1177/00222194060390060201

Van Garderen, D., & Montague, M. (2003). Visual-spatial representation, mathematical problem solving, and students of varying abilities. *Learning Disabilities Research and Practice*. https://doi.org/10.1111/1540-5826.00079

Van Garderen, D., Scheuermann, A., & Jackson, C. (2012). Developing representational ability in mathematics for students with learning disabilities: A content analysis of grades 6 and 7 textbooks. *Learning Disability Quarterly*, *35*(1), 24–38. https://doi.org/10.1177/0731948711429726

Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Learning to solve mathematical application problems: A design experiment with fifth graders. *Mathematical Thinking and Learning*, *1*(3), 195–229. https://doi.org/10.1207/s15327833mtl0103_2

Verschaffel, L., Greer, B., & de Corte, E. (2007). Whole number concepts and operations. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557–628). Information Age Publishing.

Verschaffel, L., Schukajlow, S., Star, J., & Van Dooren, W. (2020). Word problems in mathematics education: a survey. *ZDM*, *52*(1), 1–16. https://doi.org/10.1007/s11858-020-01130-4

Vukovic, R. K., & Siegel, L. S. (2010). Academic and cognitive characteristics of persistent mathematics difficulty from first through fourth grade. *Learning Disabilities Research & Practice*. https://doi.org/10.1111/j.1540-5826.2009.00298.x

Vygotsky, L. (1986). *Thought and Language*. The MIT Press.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research*, *78*(3), 516–551. https://doi.org/10.3102/0034654308320292

Wang, A. Y., Fuchs, L. S., Fuchs, D., Gilbert, J. K., Krowka, S., & Abramson, R. (2019). Embedding self-regulation instruction within fractions intervention for third graders with mathematics difficulties. *Journal of Learning Disabilities*, *52*(4), 337–348. https://doi.org/10.1177/0022219419851750

Watts, T. W. (2020). Academic achievement and economic attainment: Reexamining associations between test scores and long-run earnings. *AERA Open, 6*(2), 233285842092898. https://doi.org/10.1177/2332858420928985

Wiliam, D. (2007). Content then process: Teacher learning communities in the service of formative assessment. In D. Reeves (Ed.), *Ahead of the curve: The power of assessment to transform teaching and learning* (pp. 183–204). Solution Tree Press.

Witzel, B. S. (2005). Using CRA to teach algebra to students with math difficulties in inclusive settings. *Learning Disabilities: A Contemporary Journal*, *3*(2), 49–60.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research and Practice*, *18*(2). https://doi.org/10.1111/1540-5826.00068

Woodward, J. (2004). Mathematics education in the United States. *Journal of Learning Disabilities*, *37*(1). https://doi.org/10.1177/00222194040370010301

Woodward, J., & Baxter, J. (1997). The effects of an innovative approach to mathematics on academically low-achieving students in inclusive settings. *Exceptional Children*, *63*(3), 373–388. https://doi.org/10.1177/001440299706300306

Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2018). *Improving mathematical problem solving in grades 4 through 8: A practice guide* (NCEE 2012-4055). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http:// ies.ed.gov/ncee/wwc/publications_reviews.aspx#pubsearch/.

Woodward, J., & Brown, C. (2006). Meeting the curricular needs of academically low-achieving students in middle grade mathematics. *Journal of Special Education*, *40*(3), 151–159.

Xin, Y. P., Chiu, M. M., Tzur, R., Ma, X., Park, J. Y., & Yang, X. (2020). Linking teacher–learner discourse with mathematical reasoning of students with learning disabilities: An exploratory study. *Learning Disability Quarterly*. https://doi.org/10.1177/0731948719858707

Yang, X., & Xin, Y. P. (2021). Teaching problem posing to students with learning disabilities. *Learning Disability Quarterly*. https://doi.org/10.1177/0731948721993117

Zahner, D., & Corter, J. E. (2010). The process of probability problem solving: Use of external visual representations. *Mathematical Thinking and Learning*, *12*(2), 177–204. https://doi.org/10.1080/10986061003654240

Zhang, D., & Xin, Y. P. (2012). A follow-up meta-analysis for word-problem-solving interventions for students with mathematics difficulties. *The Journal of Educational Research*, *105*(5), 303–318. https://doi.org/10.1080/00220671.2011.627397

Zhang, D., Xin, Y. P., Harris, K., & Ding, Y. (2014). Improving multiplication strategic development in children with math difficulties. *Learning Disability Quarterly*, *37*(1), 15–30. https://doi.org/10.1177/0731948713500146

Zhang, D., Xin, Y. P., & Si, L. (2013). Transition from intuitive to advanced strategies in multiplicative reasoning for students with math difficulties. *Journal of Special Education*, *47*(1), 50–64. https://doi.org/10.1177/0022466911399098

Zheng, X., Swanson, H. L., & Marcoulides, G. A. (2011). Working memory components as predictors of children's mathematical word problem solving. *Journal of Experimental Child Psychology*. https://doi.org/10.1016/j.jecp.2011.06.001

APPENDICES

Appendix A

Coding Sheet

Lesson level information					
Name of Coder :				Date:	
Name of Textbook:		Lesson # and Focus:		Page #:	
Select the section of the lesson:	Warm-up/Review	Concept Development/Build It	Try It/Practice it	Application Problem/ Apply it	Lesson Closure
Clear Goals	**Establishes clear goals that articulate the mathematics that students are learning**. Example: Students will recognize equal groups of counters as units and count units using the language of groups and unit form: “6 equal groups of 2 counters make 12 counters,” or “6 twos make 12.” By the end of Lesson 1, students use the multiplication symbol to represent these descriptions as more efficient multiplication equations.	**Select the CCSS addressed in the lesson:** 1. 3.OA.1 2. 3.OA.2 3. 3.OA.3 4. 3.OA.4 5. 3.OA.5 6. 3.OA.6 7. 3.OA.7 8. 3.OA.8 9. 3.OA.9	**Select the strand/s addressed in the goal/s of the lesson:** 1. Conceptual understanding 2. Procedural fluency 3. Strategic competence 4. Adaptive reasoning 5. Productive disposition	**Select the mathematical practices in the lesson:** 1. MP.1 2. MP.2 3. MP.3 4. MP.4 5. MP.5 6. MP.6 7. MP.7 8. MP.8	

Task Level Coding			
Effective teaching practice	**Codes**	**Indicator**	**Example**
Reasoning and Problem Solving	**High cognitive Demand tasks**	Reversibility tasks	(a) Write a real-world problem for the number model below and write a multiplication sentence to find the total. Number Model: 4 * ----- = 8 (b) Choose a division equation. Write a story problem that goes with it. Use a bar model to show your problem. (c) Write and solve a word problem for the bar model.
		Tasks with real-world context.	(a) Tonya is making a button dolls for the school fair. On each doll, she uses 2 buttons for the eyes, 1 button for the nose, and 3 buttons for the clothes. There are 8 buttons in each package. Tonya needs to buy packages for buttons so that all of the buttons will be used without any left over. • How many packages could she buy? • How many dolls will that make? (b) The animal keepers at zoos feed and care for the animals. The animal keepers consult a zoo nutritionist to decide what and how much to feed the animals. In the zoo kitchens there are recipes posted for each type of animal such as the one shown below. **Gorilla's Zoo Stew** 33 carrots 33 oranges 24 apples 1 yam • How much of each ingredient is in 1 gorilla serving? • How much of each ingredient is needed to serve 6 gorillas?
		Multiple steps word problems.	(a) Ada buys 9 packs of highlighters with 4 in each pack. After giving 1 highlighter to each classmate, she has 17 left. • How many highlighters does Ada give away? (b) Mrs. Tomar buys 2 packs of vanilla yogurt and 3 packs of strawberry yogurt. Each pack has 4 yogurts. • How many yogurts does Mrs. Tomar buy?

	<u>Flexibility</u>	Solve a problem in multiple ways or using tools	(a) There are 20 children in at class. If 4 children can set at each table, How many tables do they need? • When finished, ask children to solve the problem using different strategy or representation. (b) Jameisha has 16 wheels to make toy cars. She uses 4 wheels for each car. • How many toy cars can Jameisha make? • Solve this problem using count by. Write division sentence.
		Explain why different procedures work for a given problem.	(a) Lila makes a chart that has 9 rows and 4 columns. • How many spaces are in her chart? • Explain why Lila can use 9s facts or 4s facts to solve. (b) Justin And Dolores made a dragon float for a parade. They connected 9 equal sections to make the dragon's body. If each section is 3 feet long, what is the total length of the dragon's body in feet? • Use known facts to find 9*3. • What two other facts can you use to find 9 *3? Explain?
	<u>Generalization:</u>	Questions focus on specific patterns that are identified in classes of problems	(a) Students set up 6 rows of seats for a concert. They put 6 seats in each row. • What is the total number of seats? • How can distributive property help you solve the problem? (b) Pam went to the fair. She went on the same ride 6 times and used the same number of tickets each time. She used 18 tickets. • How many tickets did she use each time she went on the ride? • Why can you use multiplication to solve a division problem?
		Create a specific example of a problem from a generalization.	(a) Write two different word problems about 12 birds to show 2 × 6 and 6 × 2. Solve each problem. (b) Write a word problem using the zero property of multiplication. • Explain one way to solve it.

Effective teaching practice	**<u>Codes</u>**	**<u>Indicators</u>**	**<u>Example</u>**
Visual Representations	**Use multiple forms of representations**	Using representations from at least two of the following levels: 1. Concrete (counters, blocks) 2. Semi-concrete (diagram, drawing) 3. Abstract (equation, number model)	(a) Lind had $45 in her bank account. Each week for 4 weeks she deposited $10 she earned from doing chores. • How much money does have now? • Use diagrams to help organize the information and write the number models. (b) Marc bought some bags of limes. There were 5 limes in each bag. He bought 15 limes altogether. • How many bags did he buy. • You may use math drawings or equal shares drawings to represent the problem. • Write an equation and solve the problem.
	Connect mathematical ideas to real-world situations.	Connecting mathematical ideas (e.g., equal groups, factors, array) to problem situation.	(a) 3 boxes of crayons, there are 5 crayons in each box. • How many crayons are there in all? • Draw an array to solve the problem. • How does the array represent the problem? (b) There are 6 teams on the field for a tournament. There are 54 players in all. Each team has the same number of players. • How many players are on each team? • Review the meaning of the product, factors, and missing factor and relate them to the problem situation. (c) Sandra bought 4 bags of lemons. There were 6 lemons in each bag. • How many lemons did she buy in all. • Does this problem involve groups? • What are the groups? • How do you know all the groups are the same size? (d) Eight vans with the same number of students in each van took 40 students to the science center for a field trip. • How many students were in each van. • Write a situation equation and a solution equation.

			• Explain the difference between the situation equation and a the solution equation.
	Sketch diagram	Have students sketch diagrams to make sense of problem situations.	(a) A bus can carry 40 passengers. How many passengers can 6 buses carry? • Model with a tape diagram. (b) Mali put some crackers on a tray. She put the crackers in 3 rows with 5 crackers per row. How many crackers did she put on the array? • Make a math drawing to solve the problem.
	Make choice between forms of representation	Has students make choices about which forms of representations to use as tools for solving problems.	(a) Leah and Mathew share 14 pennies equally. How many pennies does each child get. • Use drawing, numbers, or words to solve the problem.
Effective teaching practice	**<u>Codes</u>**	**<u>Indicators</u>**	**<u>Example</u>**
Discourse and conceptual understanding	**Opportunities for reasoning**	Providing students with opportunities to use their own reasoning strategies and methods for solving problems	(a) Eric Saw 4 stop signs on the way to school. Each stop sign had 8 sides. • How many sides were on all 4 stop signs? • Allow students time to solve the problem. (b) Jamal buys 4 sets of animal postcards and 5 sets of nature postcards. Each set has 6 cards. • How many postcards does Jamal buy? • Solve the problem any way you want.
	Facilitate discourse	Facilitate discourse among students and having them explain and defend why their approaches and the procedures they are using work to solve particular problems.	(a) Tonya is making a button dolls for the school fair. On each doll, she uses 2 buttons for the eyes, 1 button for the nose, and 3 buttons for the clothes. There are 8 buttons in each package. Tonya needs to buy packages for buttons so that all of the buttons will be used without any left over. • How many packages could she buy? • How many dolls will that make? • Observe children’s strategies and representation, which may include drawings of dolls and groups of buttons or number sentences. Remind children that they may need to explain their drawings and number sentences using words so that their solutions are clear to others.

			(b) Ms. Santor divides 32 students into 8 equal groups for a field trip. • Draw a tape diagram, and label the number of students in each group as *n*. • Write an equation, and solve for *n*. • Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. (c) Galen has 20 pictures to place in his book. If he puts 4 pictures on each page, how many pages will he fill? • After the Children have worked on the problem for a few minutes, have them discuss it with a partner and agree on solution.
	Purposeful Questions	Ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.	(a) Thirty third-graders go on a field trip. They are equally divided into 3 vans. • How many students are in each van? • Some friends spend $24 altogether on frozen yogurt. Each person pays $3. How many people buy frozen yogurt? • Draw and label a tape diagram to solve. • Compare Problems 4 and 5. How did your approach to drawing the tape diagram change? Why? (b) The Park District has canoes stored in 3 rows. There are 6 canoes in each row. • What is the total number of canoes stored? • Why is it ok to break apart the array? • How is breaking apart the array helpful? (c) Chad bought 4 packs of T-shirts. He gave 5 T-shirts to his brother. Now Chad has 19 shirts. • How many T-shirts were in each pack? • Suppose you used counters to solve the problem. How many counters would you start with and why? • What would you do next, and why? • Why did you divide by 4? • How might you use mental math to find if your answer is reasonable?

	Explicit connections	Make explicit connections between student approaches and reasoning and key mathematical ideas or to more efficient procedures.	(a) Ellie bought 3 packs of stickers. There are 6 stickers in each pack. How many stickers did Ellie buy in all? Solve the problem and include sketches to show your thinking. • Ask: what do these representations have in common? • Explain: groups with the same number of objects are called equal groups. • Ask: share number a model that matches the sketches you make. • Explain: 3 * 6 = 18 is a multiplication number model. Multiplication is an operation involves finding the number of objects in equal groups. (b) Greg wants to train for a race that is 10 weeks away. He runs 10 miles a week to train for the race. How many miles will Greg run to train for the race? • You can use 10 patterns to solve the problem.
Effective teaching practice	**<u>Codes</u>**	**<u>Indicators</u>**	**<u>Example</u>**
Strategy Instruction	***Problem solving strategies are provided.***		a. There are 4 spoons in each row. How many spoons are in 2 rows? Students should solve this problems using the Read-Draw-Write approach.
	Problem solving strategies are modeled (Learn-Practice-Apply-Review)		(a) There are 10 hummingbirds and 2 feeders in Marissa's backyard. If there are an equal number of birds at each feeder, how many birds are at each one? The teacher solve the problem using Model Think Record.
	Meta-cognitive strategies (e.g., self-questioning) are provided.		(a) Chad bought 4 packs of T-shirts. He gave 5 T-shirts to his brother. Now Chad has 19 shirts. How many T-shirts were in each pack? Ask yourself the following questions: What do I need to find?; What information do I need to use?; How will I use the information?
	Meta-cognitive strategies are modeled **(**Learn-Practice-Apply-Review).		(b) Madilyn bought 2 packs of pens and a notebook for $11. The notebook cost $3. Each pack of pens cost the same amount. What is the price of 1 pack of pens? The teacher ask themselves the following questions as they solve the problem: What do I need to find?; What information do I need to use?; How will I use the information?