Fractionalization of Faraday lines in generalized compact quantum electrodynamics and SPT- and SET-like phases of quantum lines and particles

Olexei Motrunich and Scott Geraedts (Caltech)

with many thanks to Max Metlitski and Matthew Fisher

DMR-1206096 and

Topological phases in many-body systems and in lattice gauge systems

Exotic insulating phases in condensed matter systems (short-range interacting fermions or bosons):

* Fractionalized phases (intrinsic topolofical order)

* Symmetry-Protected Topological phases (SPT)

* Symmetry-Enriched Topological phases (fractionalization + finer symmetry distinction)

Question: Are there similar exotic gapped phases in formal lattice gauge theories, e.g. compact QED (CQED)?

- YES! Examples of CQED analogs of all of the above!

Approach: thinking in terms of monopoles, phases from multiple-monopole condensation and condensation of bound states of monopoles and charges

Such formal CQED analogs are also useful for understanding boson SPT/SET phases in (3+1)d (Chen et al, Vishwanath et al)

Bosons in (2+1)d --- perspective via vortices

Boson phase formulation: $$S = -\sum_{r\mu} \beta \cos(\nabla_\mu \phi) + \ldots$$

Boson current formulation: $$S = \sum_{r\mu} \frac{J_{r\mu}^2}{2\beta} + \ldots, \qquad \sum_\mu \nabla_\mu J_\mu = 0$$

Bosonic systems allow description in terms of vortices, formalized by duality map. In (2+1)d, vortices are quantum particles with long-range interactions.
Schematic continuum theory for vortices --- “Higgs model”:

$$\mathcal{L}_{\rm vort} = |(\vec{\nabla} - i\vec{\alpha})\Psi_v|^2 + m_v|\Psi_v|^2 + u_v|\Psi_v|^4 + \kappa(\vec{\nabla} \times \vec{\alpha})^2$$

Superfluid <---> gapped vortices

$\langle \Psi_v \rangle = 0$

Gapless Goldstone mode (SF phonon) <---> dual photon mode

Conventional Mott insulator via vortices

$$\mathcal{L}_{\rm vort} = |(\vec{\nabla} - i\vec{\alpha})\Psi_v|^2 + m_v|\Psi_v|^2 + u_v|\Psi_v|^4 + \kappa(\vec{\nabla}\times\vec{\alpha})^2$$

Mott insulator <---> condensate of vortices

"$\langle\Psi_v\rangle \neq 0$"

Excitations in the Mott insulator:

No gapless modes <---> "Higgs mechanism"

Original boson <---> vortex in the vortex field Ψ_v. N.B.: Abrikosov-Nielsen vortices in the Higgs model have short-ranged interactions

Charge quantization <---> flux quantization for A-N vortices in Ψ_v

Charge 1 <---> 2π flux of α == "unit flux" "h_{vort}""c_{vort}"/"q_{vort}"

Z_2 fractionalized Mott insulator via vortices

(Balents, Fisher, and Nayak; Senthil and Fisher)

Usefulness of dual language: simple states in terms of vortices can be non-trivial states in terms of original bosons!

Z_2 fractionalized phase <--> condensate of pairs of vortices:

$$\langle \Psi_v \rangle = 0$$

$$\langle (\Psi_v)^2 \rangle \neq 0$$

Excitations in the fractionalized Mott insulator:

* Featureless Mott insulator (no gapless modes, no order)
* Charged excitations <---> vortices in $\Psi_{pair\text{-}vort} \sim (\Psi_v)^2$
* Charge quantum <---> new flux quant. "h_{vort}""c_{vort}"/(2"q_{vort}”) = 1/2!
* Gapped "vison" <---> unpaired vortex
* Chargon and vison have mutual π statistics

Compact quantum electrodynamics (CQED)

Euclidean path integral formulation:

Compact gauge field variables:

$$S = -\sum_{r,\mu<\nu} K\cos(\nabla_\mu a_\nu - \nabla_\nu a_\mu) + \ldots$$

Integer-valued electro-magnetic tensor field variables:

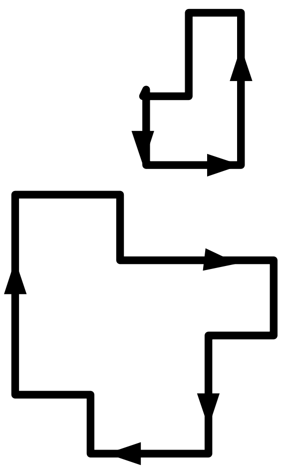
$$S = \sum_{r,\mu<\nu} F_{\mu\nu}^2/(2K) + \ldots \qquad \sum_{\nu\neq\mu} \nabla_\nu F_{\mu\nu} = 0$$

-statistical mechanics of surfaces ~ worldsheets of quantum lines
(Faraday lines; $F_{0j} = E_j$ - integer-valued electric fields)

Hamiltonian formulation:

$$[\hat{a}_{r'j'}, \hat{E}_{rj}] = i\delta_{rr'}\delta_{jj'}$$

Hilbert space: $(\nabla \cdot E)_r = 0$ <---> closed Faraday lines



$$H = \frac{U}{2}\sum_{r,j}\hat{E}_{rj}^2 - K\sum_{r,j<k}\cos(\nabla_j\hat{a}_k - \nabla_k\hat{a}_j) + \ldots$$

Compact electrodynamics in (3+1)D and duality to Higgs model

CQED allows description in terms of topological defects – monopoles, formalized by duality map. In (3+1)d, monopoles are quantum particles with long-range interactions.
Schematic continuum theory for monopoles --- “Higgs model”:

$$\mathcal{L}_{\rm monopoles} = |(\vec{\nabla} - i\vec{\alpha})\Psi_m|^2 + m|\Psi_m|^2 + u|\Psi_m|^4 + \kappa(\nabla_\mu \alpha_\nu - \nabla_\nu \alpha_\mu)^2$$

<u>Deconfined (Coulomb) phase <--> gapped monopoles</u>

$<\Psi_m> = 0$

Gapless photon <---> dual photon mode

Conventional confined phase via monopoles

$$\mathcal{L}_{\text{monopoles}} = |(\vec{\nabla} - i\vec{\alpha})\Psi_m|^2 + m|\Psi_m|^2 + u|\Psi_m|^4 + \kappa(\nabla_\mu\alpha_\nu - \nabla_\nu\alpha_\mu)^2$$

Confined phase <---> condensate of monopoles:

"$\langle\Psi_m\rangle \neq 0$"

Excitations in the confined phase:

No gapless modes <---> "Higgs mechanism"

Original Faraday line <---> vortex in the monopole field Ψ_m.
N.B.: Abrikosov-Nielsen vortices in the Higgs model have short-ranged interactions

Electric field line quantization <---> flux quantization for A-N vortices in Ψ_m
Electric field strength 1 <---> 2π flux of α == "unit flux"
"h_{monpl}""c_{monpl}"/"q_{monpl}"

Z_2 fractionalization of Faraday lines

Z_2 "fractionalization" <--> condensate of pairs of monopoles:

$<\Psi_m> = 0$

$<(\Psi_m)^2> \neq 0$

Excitations in the fractionalized gapped CQED:

* Confined phase (no gapless modes)
* Line excitations <---> vortices in $\Psi_{pair\text{-}monpl} \sim (\Psi_m)^2$
* Electric field quantum <---> new flux quant. "h_{monpl}""c_{monpl}"/(2"q_{monpl}”) = 1/2!
* Gapped "particle" <---> unpaired monopole ["m-ison” :)]
* Fractionalized Faraday line and m-ison have mutual π statistics

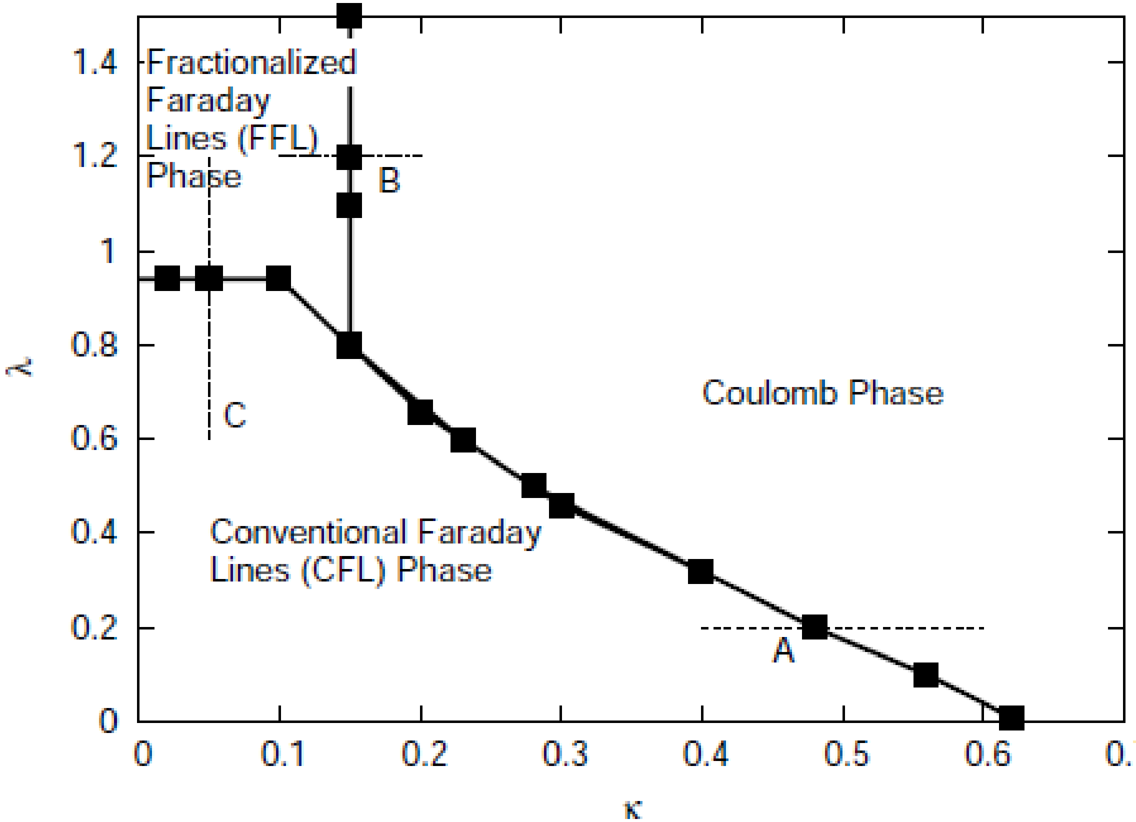
Explicit model for Z_2 fractionalization of Faraday lines

$$S = -\sum_{r,\mu<\nu} \kappa \cos(\nabla_\mu a_\nu - \nabla_\nu a_\mu) - \lambda \sum_{R\rho} \cos(\pi Q_\rho)$$

Penalty for single vs double monopole

monopole 4-current

Phase diagram from Monte Carlo:



Probing fields for quantum lines

* Bosons (quantum particles): $S = -\sum_{r\mu} \beta \cos(\nabla_\mu \phi - A_\mu^{\text{ext}}) + \ldots$

$$S = \sum_{r\mu} \frac{J_{r\mu}^2}{2\beta} + i \sum_{r\mu} J_{r\mu} A_\mu^{\text{ext}}(r) + \ldots$$

Boson condensation <-> superfluid stiffness <-> massive A_μ^{ext}

* Faraday lines (quantum lines) – probe with 2-form $h_{\mu\nu}^{\text{ext}}$:

$$S = -\sum_{r,\mu<\nu} \kappa \cos(\nabla_\mu a_\nu - \nabla_\nu a_\mu - h_{\mu\nu}^{\text{ext}}) + \ldots$$

$$S = \sum_{r,\mu<\nu} \frac{F_{\mu\nu}^2}{2\kappa} + i \sum_{r,\mu<\nu} F_{\mu\nu}(r) h_{\mu\nu}^{\text{ext}}(r) + \ldots$$

Faraday line condens. <-> "Coulomb stiffness" <-> massive $h_{\mu\nu}^{\text{ext}}$

Confined phase <-> vanishing Coulomb stiffness

By identifying quantum line excitations and how they couple to $h_{\mu\nu}^{\text{ext}}$ we can detect fractionalization of Faraday lines

SPT- and SET-like phases by binding monopoles and charges in a CQED x $U(1)_{boson}$ model in (3+1)-dim

$$S = -\sum_{R,\mu<\nu} \kappa \cos(\nabla_\mu a_\nu - \nabla_\nu a_\mu - h^{\mathrm{ext}}_{\mu\nu}) + \sum_{r\mu} \frac{\lambda}{2}(dJ_\mu - cQ_\mu)^2 + i\sum_{r\mu} J_\mu A^{\mathrm{ext}}_\mu$$

* CQED system (quantum lines, probed by $h_{\mu\nu}^{ext}$);
* Boson system (quantum particles, probed by A_μ^{ext});
* Interaction which wants to bind d monopoles and c bosons and condense the composite

-> A kind of quantum Hall state characterized by a higher-form Chern-Simons-like action for the probing fields:

$$S_{\mathrm{eff}}[h^{\mathrm{ext}}_{\mu\nu}, A^{\mathrm{ext}}_\mu] = -\frac{ic}{4\pi d}\sum h^{\mathrm{ext}}_{\mu\nu}\epsilon_{\mu\nu\rho\sigma}\nabla_\rho A^{\mathrm{ext}}_\sigma$$

- “integer” for d=1 (non-fractionalized)
- “fractional” for d>1; fractionalized Faraday lines and fractionalized bosons

Binding monopoles and charges in a CQED x $U(1)_{\text{boson}}$ model in (3+1)-dim

Monopole action, including coupling to the CQED probing field:

$$\mathcal{L}_{\text{monopoles}} = |(\vec{\nabla} - i\vec{\alpha})\Psi_m|^2 + m|\Psi_m|^2 + u|\Psi_m|^4 + \kappa(\nabla_\mu \alpha_\nu - \nabla_\nu \alpha_\mu)^2$$
$$+ih^{\text{ext}}_{\mu\nu} \frac{\epsilon_{\mu\nu\rho\sigma}\nabla_\rho \alpha_\sigma}{4\pi}$$

Boson action, including coupling to its probing field:

$$\mathcal{L}_{\text{bosons}} = |(\vec{\nabla} - i\vec{A}^{\text{ext}})\Psi_b|^2 + m_b|\Psi_b|^2 + u_b|\Psi_b|^4$$

(d,c) monopole-boson composite field: $\Phi_{\text{comp.}} \sim \Psi_m^d \Psi_b^c$

$$\mathcal{L}_{\text{comp.}} = |[\vec{\nabla} - i(d\vec{\alpha} + c\vec{A}^{\text{ext}})]\Phi_{\text{comp.}}|^2 + m_c|\Phi_{\text{comp.}}|^2 + u_c|\Psi_{\text{comp.}}|^4$$

Condense the composite ($m_c<0$) while keeping individual monopoles and bosons gapped ($m_m>0$ and $m_b>0$):

By Higgs mechanism: $\vec{\alpha} = -(c/d)\vec{A}^{\text{ext}}$

$$S_{\text{eff}}[h^{\text{ext}}_{\mu\nu}, A^{\text{ext}}_\mu] = -\frac{ic}{4\pi d}\sum h^{\text{ext}}_{\mu\nu}\epsilon_{\mu\nu\rho\sigma}\nabla_\rho A^{\text{ext}}_\sigma$$

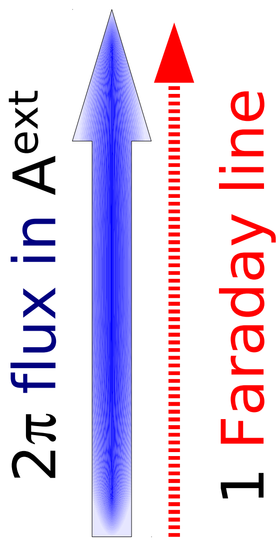
Physical meaning of the CS-like response

$$S_{\mathrm{eff}}[h^{\mathrm{ext}}_{\mu\nu}, A^{\mathrm{ext}}_{\mu}] = -\frac{ic}{4\pi d}\sum h^{\mathrm{ext}}_{\mu\nu}\epsilon_{\mu\nu\rho\sigma}\nabla_{\rho}A^{\mathrm{ext}}_{\sigma}$$

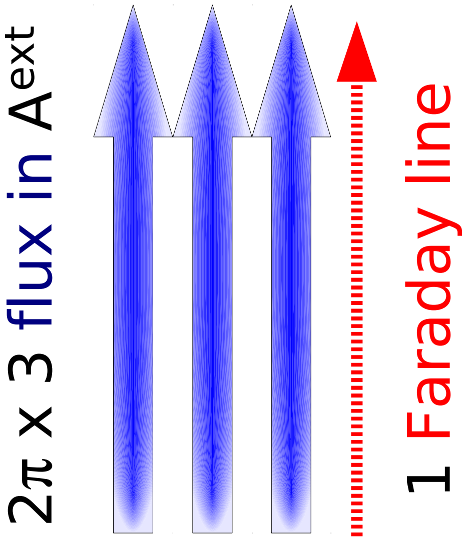
d=1: Insert a 2π flux line in A^{ext} probing the bosons -> binds c Faraday electric field lines

d>1: Insert a 2π flux line in A^{ext} probing the bosons -> binds (c/d) quantity of Faraday electric field line strength

d=1, c=1:



d=3, c=1:



- related to an equivalent description of this phase as a condensate of bound states of c Faraday lines of the CQED and d vortices of the boson system (and is a nice way for probing SPT phases)

Excitations

* Condensation of objects containing d-tupled monopoles of the CQED -> 1/d fractionalization of Faraday lines

* Condensation of objects containing d-tupled vortices of the boson system -> 1/d fractionalization of bosons

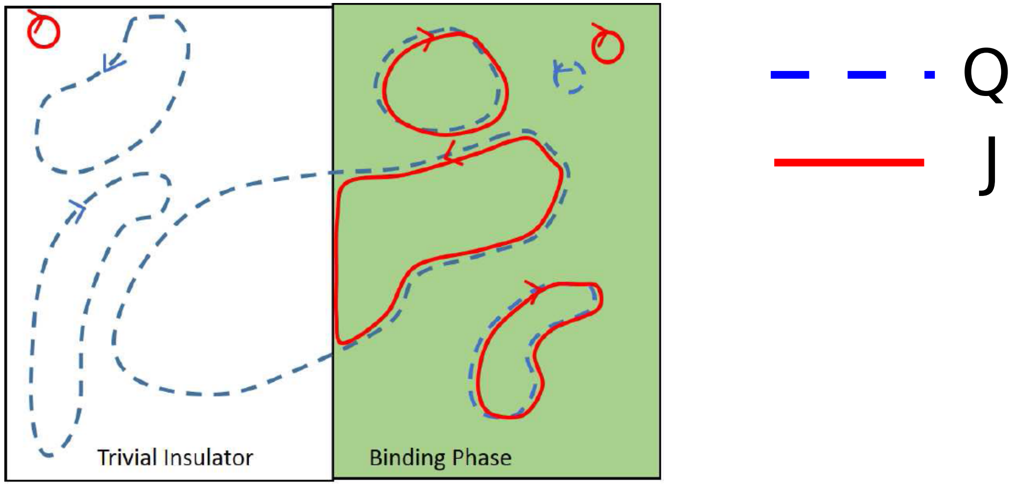
* Fractionalized particles and fractionalized Faraday lines have mutual statistics determined by c and d

Complete action in terms of gapped excitations, obtained by an exact transformations [duality and SL(2,Z)] on the original model:

$$S = \frac{1}{2\kappa d^2} \sum_{R,\mu<\nu} \left[\tilde{F}_{\mu\nu} + c \frac{\epsilon_{\mu\nu\sigma\rho} \nabla_\sigma A_\rho^{\text{ext}}}{2\pi} \right]^2 + \frac{\lambda}{2} \sum_{r,\rho} \left[\tilde{J}_\rho + c \frac{\frac{1}{2} \epsilon_{\rho\sigma\mu\nu} \nabla_\sigma h_{\mu\nu}^{\text{ext}}}{2\pi} \right]^2$$

$$+ i \sum_{R,\mu<\nu} \frac{2\pi b}{d} \tilde{F}_{\mu\nu} u_{\tilde{J},\mu\nu} - i \sum_{R,\mu<\nu} \frac{1}{d} \tilde{F}_{\mu\nu} h_{\mu\nu}^{\text{ext}} - i \sum_{r,\rho} \frac{1}{d} \tilde{J}_\rho A_\rho^{\text{ext}} - i \sum_{r,\rho} \frac{c}{4\pi d} A_\rho^{\text{ext}} \epsilon_{\rho\sigma\mu\nu} \nabla_\sigma h_{\mu\nu}^{\text{ext}}$$

Physics at the edge



<u>CQED d.o.f. perspective:</u> At the edge, we have effectively (2+1)d CQED with complete prohibition of monopoles, hence always deconfined -> propagation photon

<u>Boson d.o.f. perspective:</u> At the edge, we have effectively (2+1)d boson with complete suppression of vortices, hence always in spin-wave state -> LRO for bosons and propagating phonon

These are just dual description of the same state, photon=phonon; The CQED and boson systems are “entangled” because of the topological bulk phase

Application to boson SPT/SET phases with $U(1)_{spin}$ x $U(1)_{boson}$ & Z_2^T symmetry

* We can use a CP1-like model to faithfully represent the $U(1)_{spin}$ (Metlitski, Kane, and Fisher)

$$S^+ \sim z_\uparrow^\dagger z_\downarrow \qquad \mathcal{L}_{CP1} = \mathcal{L}_{CQED} + i[\vec{J}_\uparrow + \vec{J}_\downarrow]\cdot\vec{a} + i\frac{1}{2}[\vec{J}_\uparrow - \vec{J}_\downarrow]\cdot\vec{A}_1^{\rm ext}$$

$F_{\mu\nu}$ language: “spinons” are sources and sinks of Faraday lines:

$$\sum_\nu \nabla_\nu F_{\mu\nu} = J_{\uparrow,\mu} + J_{\downarrow,\mu}$$

* $U(1)_{boson}$ same as simply “boson” earlier

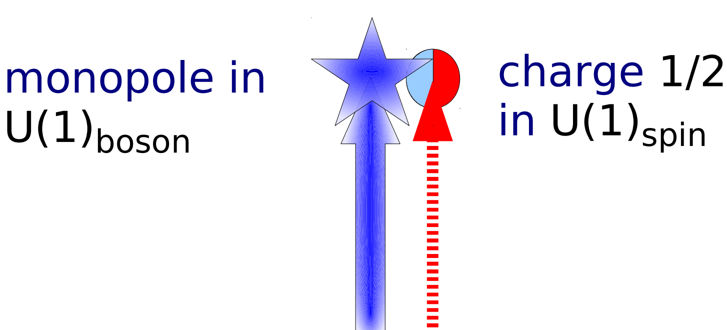
* Try same energetics binding d monopoles and c charges – what is the resulting phase? Coupling to $h^{ext}_{\mu\nu}$ is no longer defined – no CS-like characterization! In fact, in the absence of additional symmetries, all distinctions between phases other than fractionalization collapse: d=1 case is the same as trivial insulator in both $U(1)_{spin}$ and $U(1)_{boson}$, while d>1 is the same as trivial insulator in $U(1)_{spin}$ and Z_d fractionalized insulator in $U(1)_{boson}$!

Application to boson SPT/SET phases with $U(1)_{spin}$ x $U(1)_{boson}$ & Z_2^T symmetry

* Bnding d monopoles and c charges:
In the presence of additional discrete symmetries (time reversal or some other discrete symmetry interchanging the up and down spinons), the construction gives distinct phases! (Distinction by c still collapses, so need to consider only c=1.)

* d=1 – the resulting phase is bosonic topological insulator (analog of electronic TI protected by charge conservation and time reversal) – SPT phase discovered by Chen et al, Vishwanath and Senthil

To see this, can use Witten effect discussed by Metlitski et al:
Insert a monopole in A_2^{ext} probing the $U(1)_{boson}$ – binds up or down spinon - charge +1/2 or -1/2 of $U(1)_{spin}$



$$\theta = 2\pi$$

$$S = \frac{i\theta}{8\pi^2}\int d^3r d\tau \epsilon_{\mu\nu\rho\sigma}\partial_\mu A^{\rm ext}_{1\nu}\partial_\rho A^{\rm ext}_{2\sigma}$$

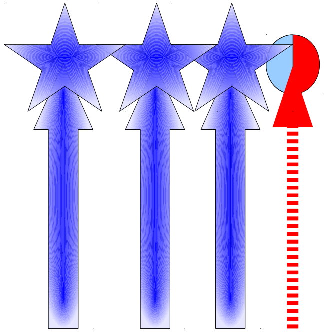
Application to boson SPT/SET phases with $U(1)_{spin}$ x $U(1)_{boson}$ & Z_2^T symmetry

* d>1 – the resulting phase is bosonic symmetry-enriched topological insulator (fractionalized, "SET"). It has particle excitations carrying charge 1/d with respect to $U(1)_{boson}$ and quantum line exciations with Z_d character (remnants of the fractionalized Faraday lines, since the spinons can source and sink these only in multiples of d); the particle and line excitations have mutual statistics.

$U(1)_{spin}$ is not fractionalized, but the two U(1)'s are "entangled" showing fractional Witten effect (Metlitski et al):

A monopole in A_2^{ext} probing $U(1)_{boson}$ binds 1/(2d) charge of $U(1)_{spin}$

d=3:
3 monopoles in $U(1)_{boson}$



charge 1/2 in $U(1)_{spin}$

$$\theta = \frac{2\pi}{d}$$

$$S = \frac{i\theta}{8\pi^2} \int d^3 r d\tau \epsilon_{\mu\nu\rho\sigma} \partial_\mu A^{\text{ext}}_{1\nu} \partial_\rho A^{\text{ext}}_{2\sigma}$$

Similar ideas (original talk abstract): CQED x CQED model in (4+1)-dim

Monopoles in (4+1)-dim are quantum lines!

Can engineer binding of d monopoles of the first CQED and c Faraday lines of the second CQED -> SPT (d=1, non-fractionalized) and SET (d>1, fractionalized) phases of this quantum line system, with quantum Hall-like response (Kravec, McGreevy, and Swingle):

$$S_{\mathrm{eff}}[h_{\mu\nu}^{(1),\mathrm{ext}}, h_{\mu\nu}^{(2),\mathrm{ext}}] = -\frac{ic}{2\pi d}\sum h_{\mu\nu}^{(1)\mathrm{ext}}\epsilon_{\mu\nu\sigma\kappa\lambda}\nabla_\sigma h_{\kappa\lambda}^{(2)\mathrm{ext}}$$

At the edge, this gives (3+1)-dim CQED with no monopoles in either CQED_1 or CQED_2 language – self-dual Maxwell (non-compact) electrodynamics.

Similar ideas:
$Z_N \times Z_N$ lattice gauge theory in (3+1)-dim

Z_N “electric fields” are quantum lines (in any dim)

Z_N magnetic fluxes are quantum lines in (3+1)-dim

Can consider binding electric field lines and magnetic field lines and condensing the composites -> SPT-like phase in this lattice gauge theory (Kapustin et al, von Keyserlingk et al, Wang et al)

Add matter with some symmetries - possible route to some bosonic SPTs in (3+1)-dim? (Wang, Naum, and Senthil)

Conclusions

* Simple example of fractionalization of Faraday lines, by analogy to fractionalization of bosons

* Simple examples of SPT- and SET-like phases in CQED systems, by analogy to SPT- and SET-like phases of bosons in (2+1)-dim

--- CQED x U(1) in (3+1)-dim via condensation of bound states of monopoles and charges

--- CQEDxCQED in (4+1)-dim via condensation of bound states of monopoles and Faraday lines

* Besides being interesting in HEP-TH, these can also be useful for understanding SPT and SET-like phases of bosons in CMP (local d.o.f., short-range interactions).

THANK YOU!

Some mathematics of deriving excitations

Schematic action (not writing short-range pieces):

$$\mathcal{L} = i\vec{J}_m \cdot \vec{\alpha} + i\vec{J}_b \cdot \vec{A}^{\rm ext} + \kappa(\nabla_\mu a_\nu - \nabla_\nu a_\mu)^2 + ih^{\rm ext}_{\mu\nu}\frac{\epsilon_{\mu\nu\rho\sigma}\nabla_\rho \alpha_\sigma}{4\pi}$$

Mathematics to describe condensation of (d, c) bound states:
SL(2, Z) change of variables (a,b,c,d integers with ad - bc = 1)

$$H = aJ_m - bJ_b \qquad \leftrightarrow \qquad J_m = dH - bG$$

$$G = cJ_m - dJ_b \qquad \leftrightarrow \qquad J_b = cH - aG$$

$$i\vec{J}_m \cdot \vec{\alpha} + i\vec{J}_b \cdot \vec{A}^{\rm ext} = i\vec{H}\cdot(d\vec{\alpha} + c\vec{A}^{\rm ext}) - i\vec{G}\cdot(b\vec{\alpha} + a\vec{A}^{\rm ext})$$

Phase with G gapped and H condensed (driven by s.r. interactions)

$$\vec{\alpha} = -(c/d)\vec{A}^{\rm ext} \rightarrow -i\vec{G}\cdot(-b\frac{c}{d} + a)\vec{A}^{\rm ext} = -i\vec{G}\frac{1}{d}\cdot\vec{A}^{\rm ext}$$

-> G carries boson charge 1/d
Precise description of H condensing is that quantum lines dual to H are gapped; we find that these carry 1/d unit (derivation not shown)
Gapped particles and gapped lines have statistical interaction 2πb/d