WHY DON'T UNIONS CHARGE HIGH
INITIATION FEES?

by

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I.

For the purposes of this paper, an effective union is one which raises and maintains the wages of its members above the level in their next best alternative nonunion employment. It can accomplish this either through collective bargaining and the power to strike or by restricting the supply of qualified labor through some combination of apprenticeship and occupational licensing. Examples of groups relying on the latter method include the barbers, morticians, and physicians and surgeons. Groups ordinarily regarded as unions, however, rely primarily on bargaining and strikes to achieve their ends. Success generates an excess supply of labor and this, in turn, requires the rationing of employment opportunities. Rationing can be done by using either price or nonprice techniques.

A union whose effectiveness depends on its ability to organize all or a very high proportion of the employees of a firm or industry, regardless of their skill level or occupational classification, has an incentive to keep its initiation fees low in order not to discourage membership. In a survey of national and international union finances carried out by the National Industrial Conference Board in 1967 [2], for example, it was found that the Auto Workers' allowed its locals to charge a fee ranging only from $5.00 to $15.00 at their discretion, the Steelworkers' instructed its locals to charge a flat $5.00 fee, and the Rubber Workers' merely
provided that a new member's first month's dues were to be equal to twice the current dues rate of the local union which itself was restricted to the range between $4.00 and $5.75. Industrial unions, therefore, must employ nonprice techniques to ration scarce job opportunities.

A common way of accomplishing this is through the adoption of work-sharing devices that determine the maximum hours or amount of work per member per unit time (with penalty wage rates when overtime hours are required), coupled with the establishment of seniority rules to govern layoffs and recalls and a grievance system to enforce the agreement. Since an industrial union is typically not in a position to determine the volume and composition of new hires, the principal function of the union shop clause (if any) in its contract is to ensure that newly hired workers promptly join the union. The clause thus provides a measure of organizational security. Under these circumstances, high initiation fees would be counterproductive.

On the other hand, a union whose effectiveness is a function of its ability to organize all or a very high proportion of the workers possessing a given skill or employed in a given occupation, regardless of the firm or industry in which they work, must guard against employer attempts to substitute less skilled labor or nonunion labor for the labor of its members. The best way of controlling the volume and composition of new hires is by maintaining an effective closed- or preferential-shop clause in its collective agreements with employers. In a closed shop, only workers who are already union members can be hired; in a preferential shop, union members are given preference in hiring. In either case,
when vacancies occur, the union has the exclusive right to refer workers for employment. If an exclusive union work referral system exists, the simplest way of controlling new hires and rationing employment opportunities is to limit the referral list, which in practice means to limit membership in the union. Membership may be restricted by using either price or nonprice rationing.

The available evidence suggests that craft unions with closed-shop agreements, like industrial unions with union-shop agreements, rely on nonprice rationing to cope with the excess supply of qualified labor created by their success in raising the relative wages of their members. That initiation fees are not used to restrict union membership is confirmed by data collected in 1960 by the Department of Labor under the reporting and disclosure provisions of the Landrum-Griffin Act of 1959. These data revealed a typical initiation fee of $5.00, for this was the fee charged by almost 25 percent of approximately 39,000 reporting local unions. The data also showed that about 60 percent of the unions levied a fee of $10.00 or less, that only 10 percent charged $100.00 or more, and that just 17 locals charged a fee high enough to lie in the $500.00 to $1,400.00 range. As will be shown below, none of these fees can be

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1 Both the closed and the preferential shop are illegal under Section 8(b)(2) of the Taft-Hartley Act of 1947. All the same, they exist. With the tacit understanding of employers, a union-shop agreement can in practice serve as a cover for a closed- or preferential-shop agreement. For a description of exclusive union work referral systems in the building trades, see [10].

2 See [8]. In an earlier survey of local union finances, Taft [7] found that the highest fees charged by AFL (craft) and CIO (industrial) locals were $350.00 and $50.00, respectively.
considered high enough to restrict entry. Since these data embraced unions in the amusement, local delivery, and maritime industries, as well as the building and printing trades, where restriction of membership is common practice, it is clear that unions in general prefer to use nonprice rationing.

Nor is it the case that local craft unions are necessarily prevented by their parent unions from charging restrictively high initiation fees. The 1967 Conference Board study cited above reveals that the constitutions of many national and international craft unions prescribe only the minimum fees that must be charged by local unions. For example, the Carpenters' stipulated a $15.00 minimum fee plus the current month's dues which itself had a $4.00 minimum, the Lithographers-and-Photoengravers' left the fee entirely to local determination, the Longshoremen's set only a $50.00 minimum fee, the Operating Engineers' had only a $5.00 minimum, and the Teamsters' abdicated entirely in favor of local determination.

One reason why unions do not ration membership and employment opportunities by price may be that they believe it would be illegal to do so. Section 8 (b)(5) of the Taft-Hartley Act makes it an unfair labor practice for a union to charge an initiation fee that the National Labor Relations Board finds "excessive or discriminatory under all the circumstances". One of the circumstances that the Board is required to take into account, however, is "the wages currently paid to the employees affected". But a fee that is not greater than the capitalized value of the union-generated differential in wages over the expected working life of a new member can hardly be called "excessive .... under all the circumstances" (nor "discriminatory", if charged to each person in a
cohort of new members). Therefore, even the union that reported the highest fee ($1,400.00) in the 1960 Labor Department report cited above was well within the law (unless, of course, the expected annual earnings differential was substantially less than $140.00 over the expected working life of a new member or the appropriate discount rate was substantially greater than 10 percent). Moreover, Taft [7] has shown that fees were low long before the statute prohibiting "excessive" fees was enacted. It is difficult to argue, therefore, that the law prevents unions from using initiation fees as a rationing device.

A more likely reason, discussed by both Becker [1, pages 221-223] and Rees [5, page 128], for relying on nonprice rationing to restrict membership to the number and kind of persons desired may be the belief that an initiation fee high enough to be restrictive would also be high enough to call forth the wrath of the government and the nonunion public and thus direct unwanted attention to the union's closed shop and monopoly position in the labor market. As Hicks [3, page 8] once remarked in a different context, "The best of all monopoly profits is a quiet life". The union may believe that even unsubtle discrimination against persons not wanted by the membership (or employers) and open favoritism toward those desired may be passed off as selection by quality and thus be less controversial than initiation fees in the five figures. In any case, closed- and preferential-shop unions do not use initiation fees but, instead, rely on discrimination and favoritism to implement their admissions policies.

Although the reasons for this choice may include the fears (of the law and adverse publicity) mentioned above, perhaps compounded by a belief
among union members that in some sense it would be morally wrong to traffic in jobs, there is another, and probably more compelling, reason why unions do not charge restrictively high initiation fees. In the analysis which follows, it will be argued that the preference of union members for discrimination and favoritism over initiation fees is based on their belief that they will lose more than they gain if they charge newcomers a fee.

II.

Consider the labor market depicted in Figure 1. The $D_L$ curve shows the quantity of labor services demanded per unit time (say, a year) at various wage rates, given the production function, the demand function for output, and the supply functions of other inputs. The $S_L$ curve shows the quantity of labor services supplied per year at various wage rates, given the wage rates in alternative employments, nonlabor income, tastes, and other parameters. The market-clearing wage rate and level of employment are denoted by $W_0$ and $L_0$, respectively. Since employment is measured in man-hours, $L_0 = M_0H_0$, where $M_0$ represents the number of men and $H_0$ is hours per man. Let $W_1 (=kW_0)$ be the wage rate set by collective bargaining, and $k (>1)$ denote the relative wage effect of the union. The excess supply of labor at this wage rate is given by $L_2 - L_1$. To simplify the analysis, assume that the corresponding level of employment, $L_1$, represents a smaller number of men but the same hours per man as before, that is, $L_1 = M_1H_0$. Assume also that neither income nor leisure is an inferior good and that the typical worker is in equilibrium.

$^{3}$It is more likely, of course, that the new level of employment will entail both fewer men and fewer hours per man, or even the same number of men and still fewer hours per man, rather than fewer men each working the same hours as before. But assuming the latter to be the case simplifies the analysis without affecting the results. Points along the horizontal axis in Figure 1 now represent "men per year", and $L_0$ becomes $M_0$, $L_1$ becomes $M_1$, and $L_2$ becomes $M_2$. 
DEMAND FOR AND SUPPLY OF LABOR

Figure 1
in the sense that his marginal rate of indifference substitution of
market goods (income) for leisure equals the wage rate \( W_1 \), that is,
his desired hours of work at this wage rate equals his actual hours \( H_0 \).
Let \( M^* \) denote the number of union members. Then the differential, \( D \),
between annual earnings per union member and annual earnings per worker
under competition is given by:

\[
D = \frac{W_1 L_1}{M^*} - \frac{W_0 L_0}{M_0} = \frac{W_1 M_1 H_0}{M^*} - \frac{W_0 M_0 H_0}{M_0} \\
= kW_0 H_0 \left( \frac{M_1}{M^*} \right) - W_0 H_0 = W_0 H_0 \left[ k \left( \frac{M_1}{M^*} \right) - 1 \right].
\]

The first problem to consider is the case where \( M^* = M_1 \), that is,
where the only members of the union are those employed for \( H_0 \) hours at
wage rate \( kW_0 \). Let \( R \) denote the number of union members who resign,
retire, or die each year and who are replaced, and define \( t = \frac{R}{M^*} \). Any
new replacement member can expect to receive annually an income equal to
\( kW_0 H_0 \) and, in return for this right, ought to be willing to pay, at most,
a sum approximately equal\(^4\) to \( \frac{D}{r} = W_0 H_0 \left[ k - 1 \right] \left( \frac{r}{r} \right) \),
where \( r \) is the rate of interest the new member could obtain on alternative investments
involving the same degree of risk. Assume that any new replacement member
can also expect, after the first year of membership, to receive annually
(provided that all subsequent replacement members are charged a fee, \( F \))
a distribution equal to \( tF \). For this right, he ought to be willing to

\(^4\)To be exact, a sum equal to \( \frac{D}{r} \left[ 1 - \frac{1}{(1 + r)^n} \right] \), where \( n \) equals
the number of years the new member expects to be in the union. As \( n \)
approaches infinity, this sum approaches \( \frac{D}{r} \). Since the present value
of an annuity of 40 or 50 years is not greatly different from that of a
perpetuity ($9.78 or $9.91 versus $10.00, for \( D = $1.00 \) and \( r = 0.10 \)),
the familiar capitalization formula will be used throughout.
pay, at most, an additional sum approximately equal to \( \frac{tF}{r} \). Therefore, the maximum fee, \( F \), any new replacement member ought to be willing to pay is approximately given by:

\[
F = \frac{D}{r} + \frac{tF}{r} = \frac{D}{r - t}
\]

\[
= \frac{WoRo \left[ k - 1 \right]}{r - t}. \tag{2}
\]

So long as \( M^0 = M_1 \), the gain per member per year, \( g \), from charging a fee \( F \) to each of \( R \) replacement members per year is given by:

\[
g = tF, \tag{3}
\]

the capitalized value, \( G \), of which is:

\[
G = \frac{G}{r} = \frac{t}{r} F. \tag{4}
\]

Both the annual gain per member and the present value of the stream of such gains vary directly with the relative wage effect of the union and the turnover rate of the membership and inversely with the rate of interest. Both \( g \) and \( G \) can be substantial for any plausible set of values of \( k \), \( t \), and \( r \). For example, let \( WoRo = 7,200, k = 1.2, t = 0.025 \),

\[
= \left[ tF \left( \frac{1}{(1 + r)^n} \right) \right]. \tag{5}
\]

Again, as \( n \) approaches infinity, this amount approaches

\[
\frac{tF}{r}.
\]

\(^5\) To be exact, an additional sum equal to \( \frac{tF}{r} \left[ 1 - \frac{1}{(1 + r)^n} \right] \left( 1 - \frac{1}{(1 + r)} \right) \). Again, as \( n \) approaches infinity, this amount approaches \( \frac{tF}{r} \).

\(^6\) Equations (3) and (4) are only approximately correct. See the caveats in footnotes 4 and 5, which hereafter will not be repeated.
and \( r = 0.10 \). Then, \( g = $480 \), or over 5.5 percent of the typical union member's annual earnings; and \( G = $4,800 \), or 25 percent of the equilibrium initiation fee of $19,200. It was calculations similar to these that prompted Becker [1, page 221] to say: "This only deepens the mystery of why trade unions have not used initiation fees more extensively."

And, indeed, if it were the case that the only members of an effective union were those employed for \( H_0 \) hours at wage rate \( kW_0 \), that is, if it were true that \( M^* = M_1 \), then the failure to charge a fee \( F \) to each of \( R \) replacements per year would be very hard to explain.

Suppose, however, that it is necessary for the number of members to be greater than the number of workers who can be employed for \( H_0 \) hours at wage rate \( kW_0 \). One reason for this requirement might be to minimize the threat to union security (the closed- or preferential-shop) and union effectiveness \((k > 1)\) posed by the excess supply of labor \((L_2 - L_1)\) overhanging the market. If \( L_2 = M_2 H_0 \), then the excess supply is given by \( M_2 - M_1 \), that is, by the number of men seeking work at wage rate \( kW_0 \). The greater the excess supply (the greater the elasticity of \( M \) with respect to \( W \)), the greater the probability that employers will regard these men as an alternative source of labor and, therefore, the greater the threat to the security and effectiveness of the union. The danger diminishes as the number of such men admitted to, and brought under the control of, the union increases. If all employers are bound by closed- or preferential-shop agreements and if the union's power to strike is real, the remaining excess supply will clearly be too small to be a

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7 The average annual earnings of production or nonsupervisory workers on private nonagricultural payrolls were approximately $7,060 in 1972 [9, Table 18, page 90]. The weighted average of the estimates by Lewis [4, page 193] of the relative wage effect of all unions at different dates in the period 1923-1958 is about 1.14. A turnover rate of 0.025 is consistent with a uniform age distribution and an average working life of 40 years. An interest rate of 0.10 is an arbitrary, but not unreasonable, figure.
threat when the total number of members is at least one-half the total number of men willing to work $H_o$ hours at wage rate $kW_o$, that is, when the number of actual members ($M^\ast$) is at least equal to the number of potential members ($M_2 - M^\ast$). Therefore, if $M_2 > 2$, additional members will be admitted until $M^\ast / M_1$, the required coefficient of union expansion, equals $\frac{1}{2} \left( \frac{M_2}{M_1} \right)$. Given the elasticity of demand, the likelihood that $\frac{M_2}{M_1}$ will exceed two will be the greater, the greater the elasticity of supply.

But even if $\frac{M_2}{M_1} \leq 2$, there is another reason why it might be necessary for $\frac{M^\ast}{M_1} > 1$. Because the union is committed to supply employers with man-hours on demand and because it is not always possible or feasible to increase hours per man, additional members might be required to guarantee the availability of sufficient union men to meet unanticipated fluctuations in demand or unexpected absences of workers. In any case, suppose it is required, in order to preserve the equilibrium of the union qua union, that $M^\ast > M_1$. Let $\left[ \frac{M^\ast}{M_1} \right]^\ast$ denote the equilibrium coefficient of union expansion. If $\frac{M_2}{M_1} \leq 2$, then $\left[ \frac{M^\ast}{M_1} \right]^\ast = \frac{M_1 + \delta}{M_1}$, where $\delta$ represents the number of additional members required to meet unanticipated fluctuations in demand and unexpected absences of workers; if $\frac{M_2}{M_1} > 2$, then $\left[ \frac{M^\ast}{M_1} \right]^\ast = \max \left\{ \frac{M_1 + \delta}{M_1}, \frac{1}{2} \left( \frac{M_2}{M_1} \right) \right\}$. In either case, let $M^\prime = M^\ast - M_1$ represent the number of additional members admitted to the union.

Although this way of putting the matter introduces a discontinuity into the model, it does emphasize the point that the life expectancy of the union (and thus the permanence of the wage differential) cannot be taken for granted. The formulation in the text is sufficient for present purposes, but it would probably be more correct to say that life expectancy, $n$, is an increasing continuous function of $\frac{M^\ast}{M_1}$, and that the equilibrium value of the latter is positively related to the stochastic term, $\delta$, and to $\frac{M_2}{M_1}$.
Any additional member can expect to receive annually, provided he works the same hours as any original (or any replacement of an original) member, an income equal to \( kW_0H_0 \left( \frac{M_1}{M^*} \right) \). For this right, he ought to be willing to pay, at most, a sum equal to \( \frac{D}{r} \left( = W_0H_0 \left[ k \left( \frac{M_1}{M^*} \right) - 1 \right] \left( \frac{1}{r} \right) \right) \). As before, let \( R \) denote the number of members who resign, retire, or die each year and who are replaced, and define \( t = \frac{R}{M^*} \left( = \frac{R}{M_1 + M^*} \right) \).

Any new replacement member can also expect to receive annually, provided he works the same hours as any other (original or additional) member, an income equal to \( kW_0H_0 \left( \frac{M_1}{M^*} \right) \). For this right, he, too, ought to be willing to pay, at most, a sum equal to \( \frac{D}{r} \). As before, assume that any new (additional or replacement) member can expect, after the first year of membership, to receive annually (provided all subsequent members pay a fee, \( F \)) a distribution equal to \( tF \). In return for this right, he ought to be willing to pay, at most, an additional sum equal to \( \frac{tF}{r} \). Therefore, the maximum fee, \( F \), any new member ought to be willing to pay is given by:

\[
F = \frac{D}{r} + \frac{tF}{r} = \frac{D}{r - t}
\]

\[
= \frac{W_0H_0 \left[ k \left( \frac{M_1}{M^*} \right) - 1 \right]}{r - t}.
\] (5)

But if \( M^* > M_1 \), each original member enjoys only a once-and-for-all gain, \( G' \), from charging a fee \( F \) to each of \( M' \) additional members. This is given by:

\[
G' = \left( \frac{M'}{M_1} \right) F.
\] (6)
Note first that the condition for \( G' > 0 \) is \( \left( \frac{M'}{M_1} \right) < (k - 1) \) or, alternatively, \( \frac{M'^*}{M_1} < k \). The reason is that \( F \) approaches zero as \( \frac{M'^*}{M_1} \) approaches \( k \). This means that the maximum coefficient of expansion of a fee-charging union is determined by the relative wage effect of the union. Therefore, if \( \left( \frac{M'^*}{M_1} \right) \geq k \), the union clearly will not be able to employ price rationing as means of allocating scarce job opportunities. Note secondly that, since \( G' \) is a one-time gain, its value is a capital sum. Therefore, the gain per original member per year, \( g' \), from charging a fee \( F \) to each of \( M' \) additional members is given by:

\[
g' = G'r = \left( \frac{M'}{M_1} \right) F r \quad . \tag{7}
\]

To this gain \( g' \) must be added each original member's share of the annual gain, \( g'' \), from charging a fee \( F \) to the \( R \) new members who replace the \( tM'^* \) (\( = t [M_1 + M'] \)) persons who resign, retire, or die each year. This is given by:

\[
g'' = t F \quad , \tag{8}
\]

the capitalized value, \( G'' \), of which is:

\[
G'' = \frac{g''}{r} = \left( \frac{t}{r} \right) F \quad . \tag{9}
\]

Again, note that the condition for \( G'' > 0 \) is \( \left( \frac{M'}{M_1} \right) < (k - 1) \) or, alternatively, \( \frac{M'^*}{M_1} < k \). It follows that the total gain per original member per year, \( g \), is given by:

\[
g = g' + g'' = \left( \frac{M'}{M_1} \right) F r + t F
\]

\[
= \left[ r \left( \frac{M'}{M_1} \right) + t \right] \left[ \frac{W_0 H_0 [K \left( \frac{M_1}{M'^*} \right) - 1]}{r - t} \right] \quad . \tag{10}
\]
This says that \( g \) is a maximum \( (= t F) \) when \( \frac{M^\ast}{M_1} = 1 \) and that \( g \) declines as the membership expands, reaching zero when \( \frac{M^\ast}{M_1} = k \). The same thing is true, of course, of the capitalized value, \( G \), of the stream of these gains:

\[
G = G' + G'' = \left(\frac{1}{r}\right) (g' + g'')
\]

\[
= \left(\frac{M' + t}{M_1 + t} \right) F 
\]\n
(11)

So, if \( \frac{M^\ast}{M_1} \geq k \), the union clearly will not be able to charge fees to new members. The first four columns of Table 1 illustrate the effect on \( F \), \( G \), and \( g \) of an expansion in membership, when \( W_0H_0 = \$7,200 \), \( k = 1.2 \), \( t = 0.025 \), and \( r = 0.10 \).

This is not the end of the matter, however. Since none of the additional \( M' \) members will be willing to pay a fee \( F \) unless he is assured of working the same hours as any other (original or additional) member, and at the same wage rate, \( kW_0 \), as any other member, it follows that the annual wage income of all \( M^\ast \) members must be the same, viz., \( kW_0H_0 \left(\frac{M_1}{M^\ast}\right) \). This means that, by charging a fee \( F \) when admitting \( M' \) additional members, each of the original \( M_1 \) members must reduce his hours of work and thereby suffer a reduction in annual wage income. The loss per original member per year, \( \ell \), is given by:

\[
\ell = kW_0H_0 - kW_0H_0 \left(\frac{M_1}{M^\ast}\right) = kW_0H_0 \left(1 - \frac{M_1}{M^\ast}\right)
\]

\[
= kW_0H_0 \left(\frac{M'}{M^\ast}\right) 
\]\n
(12)
Table 1

Values of $F$, $G$, $g$, $\ell$, and $\Pi$ for Specified Values of $\frac{M^*}{M_1}$,  
When $W_0 R_0 = 7,200$, $k = 1.2$, $t = 0.025$, and $r = 0.10$

<table>
<thead>
<tr>
<th>$\frac{M^*}{M_1}$</th>
<th>$F$</th>
<th>$G$</th>
<th>$g$</th>
<th>$\ell$</th>
<th>$\Pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>$19,200$</td>
<td>$4,800$</td>
<td>$480$</td>
<td>$0$</td>
<td>$480$</td>
</tr>
<tr>
<td>1.01</td>
<td>18,059</td>
<td>4,695</td>
<td>470</td>
<td>86</td>
<td>384</td>
</tr>
<tr>
<td>1.02</td>
<td>16,941</td>
<td>4,574</td>
<td>457</td>
<td>169</td>
<td>288</td>
</tr>
<tr>
<td>1.03</td>
<td>15,845</td>
<td>4,436</td>
<td>444</td>
<td>252</td>
<td>192</td>
</tr>
<tr>
<td>1.04</td>
<td>14,769</td>
<td>4,283</td>
<td>428</td>
<td>332</td>
<td>96</td>
</tr>
<tr>
<td>1.05</td>
<td>13,714</td>
<td>4,114</td>
<td>411</td>
<td>411</td>
<td>0</td>
</tr>
<tr>
<td>1.10</td>
<td>8,727</td>
<td>3,054</td>
<td>305</td>
<td>785</td>
<td>-480</td>
</tr>
<tr>
<td>1.15</td>
<td>4,174</td>
<td>1,670</td>
<td>167</td>
<td>1,127</td>
<td>-960</td>
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<tr>
<td>1.20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1,440</td>
<td>-1,440</td>
</tr>
</tbody>
</table>
This says that \( \ell \) is zero when \( \frac{M^*}{M_1} = 1 \) and that \( \ell \) increases as the membership expands. When \( \frac{M^*}{M_1} = k \), the loss is complete in the sense that an original member's annual wage income is the same as it would have been in the absence of the union. The fifth column of Table 1 illustrates the effect on \( \ell \) of an expansion in membership.

The net gain per original member per year, \( \pi \), from charging a fee \( F \) when admitting \( M' \) additional members is given by:

\[
\pi = g - \ell \quad (13)
\]

Since \( g \) decreases from \( tF \) to zero as \( \frac{M^*}{M_1} \) increases from one to \( k \), and since \( \ell \) increases from zero to \( W_0H_0(k - 1) \) as \( \frac{M^*}{M_1} \) increases from one to \( k \), it is clear that \( \pi \geq 0 \) only when \( 1 \leq \frac{M^*}{M_1} \leq k \). In fact, the condition for \( \pi \geq 0 \) is much more restrictive. The last three columns of Table 1 show the effect on \( g \), \( \ell \), and \( \pi \) of an expansion in membership, when \( W_0H_0 = \$7,200 \), \( k = 1.2 \), \( t = 0.025 \), and \( r = 0.10 \). In this case, the use of initiation fees to ration scarce job opportunities will be profitable to the original members only when the expansion in membership is between zero and five percent, that is, \( \pi \geq 0 \) only when \( 1 \leq \frac{M^*}{M_1} \leq 1.05 \). If for any reason (for example, if \( \frac{M_2}{M_1} > 2.10 \)) it is necessary to increase the membership by more than five percent, that is, if \( \left[ \frac{M^*}{M_1} \right]^* > 1.05 \), it will not pay the original members to make the additional members pay the equilibrium fee. Some other (nonprice) rationing technique will have to be devised.

The underlying reason for this is that if a prospective member is asked to pay such a fee in return for full citizenship in the union and the right to work at wage rate \( kW_0 \), he will find it worth his while
to do so only if he is assured that his annual wage income will be $kW_0H_0 \left( \frac{M_1}{M^*} \right)$ and that, after his first year, his annual share of the receipts from the fees paid by replacement members will be $tF$. So, part of what the original members gain from charging fees to additional members, they lose by necessarily having to share the work and the fee receipts with them. In the preceding example, the loss exceeds the gain when the increase in membership is greater than five percent; when $\frac{M^*}{M_1} = 1.05$, even a fee of $13,714 is no better than a break-even proposition.

Let $\left[ \frac{M^*}{M_1} \right]_0$ denote the break-even ($n = 0$) value of the coefficient of expansion of a fee-charging union. Suppose that $\left[ \frac{M^*}{M_1} \right]_0 > \left[ \frac{M^*}{M_1} \right]_0$ and that the equilibrium fee is charged to additional members. Is it possible for the typical original member to be better off even though the loss in wage income exceeds the gain from fee receipts? The answer, of course, depends on the specification of the utility function. Consider the income-leisure preferences of the individual portrayed in Figure 2. Income is measured along the vertical axis and time along the horizontal. The wage rate set by collective bargaining, $kW_0$, is measured by the slope of the line $TY$. For convenience, it is assumed that the individual has no nonlabor income other than fee receipts. Initially, when $\frac{M^*}{M_1} = 1$, the individual is in equilibrium at point $A$ on line $TY$, where hours of work $= H_0$ (measured right-to-left from $T$) and income $= kW_0H_0$.

When $\frac{M^*}{M_1} = \left[ \frac{M^*}{M_1} \right]_0 ( > 1)$, the individual moves to point $B$ on line $T_0Y_0$ (parallel to line $TY$), where hours of work $= \frac{H_0}{\left[ \frac{M^*}{M_1} \right]_0} (< H_0)$.
INCOME-LEISURE PREFERENCES OF THE TYPICAL ORIGINAL MEMBER

Figure 2
and income = $\left[ \frac{M^*_1}{M_1} \right] \circ + g_0 = (kW_0H_0)$. Although he is not in equilibrium, he is clearly better off at B than he was at A. When 

\[ \frac{M^*_1}{M_1} = \left( \frac{M^*_0}{M_1} \right) \circ \left( > \frac{M^*_0}{M_1} \right) \], the individual moves to point C on line $T^*Y^*$ (also parallel to line $TY$), where hours of work = \( \frac{H_0}{k} \) and income = $\left( kW_0H_0 \right) + g^* (\leq kW_0H_0)$. Again, he is not in equilibrium and he is clearly worse off at C than he was at B. On the other hand, he might be better off or worse off (or neither) at C than he was initially at A: It all depends on whether indifference curve I lies above or below (or, as drawn, passes through) C, and this depends on the individual's utility function. Finally, when \[ \frac{M^*_1}{M_1} = k \], the individual moves down to point D on line $TY$, where hours of work = \( \frac{H_0}{k} \left( < \frac{H_0}{k} \right) \) and income = \( \frac{kW_0H_0}{k} = (W_0H_0) \), and where he is clearly worse off than he was at A.

Given \( \frac{M^*_1}{M_1} \circ < \frac{M^*_1}{M_1} \) \( < \) and given that additional members are charged the equilibrium fee, the only conclusion that can safely be drawn is that the farther \( \frac{M^*_1}{M_1} \) \( * \) is from \( \frac{M^*_1}{M_1} \circ \) (that is, the closer \( \frac{M^*_1}{M_1} \) \( * \) is to k), the more likely it is that the typical original member will be worse off than he was initially at A. This statement is true whether or not the individual has the opportunity to "moonlight,"

\[ g_0 = \left[ \left( \frac{M^*_1}{M_1} \circ - 1 \right) \frac{r \circ t}{k} \left( \frac{W_0H_0}{r \circ t} \left( \frac{k}{M^*_1} \right) \circ - 1 \right) \right] \) is the individual's share of total fee receipts when \( \frac{M^*_1}{M_1} = \left( \frac{M^*_1}{M_1} \circ \right) \circ .

\[ g^* = \left[ \left( \frac{M^*_1}{M_1} \right) * - 1 \right) \frac{r \circ t}{k} \left( \frac{W_0H_0}{r \circ t} \left( \frac{k}{M^*_1} \right) * - 1 \right) \) is the individual's share of total fee receipts when \( \frac{M^*_1}{M_1} = \left( \frac{M^*_1}{M_1} \right) * \circ .

that is, to take a second job at a wage rate less than $kW_0$. Although the convexity of the indifference curves ensures that there is some wage rate less than $kW_0$ at which, by working additional hours, the individual can move from B to a higher indifference curve, it does not guarantee this if the opportunity to "moonlight" is available only at the competitive wage rate, $W_0$, given by the slope of the line TX. Similarly, if C and A are on the same indifference curve, the convexity property does not ensure an improvement in welfare if the opportunity to work additional hours is available only at the competitive wage rate. Finally, if C lies on a lower indifference curve than A, convexity does not guarantee that, by "moonlighting" at any available wage rate less than $kW_0$, the individual can move from C to a higher indifference curve than the one passing through A. In particular, it does not ensure this if the opportunity to "moonlight" is available only at the wage rate $W_0$. Consequently, the greater the difference between $\left[\frac{M^*_k}{M_1}\right]_0$ and $\left[\frac{M^*_k}{M_1}\right]$, the more likely it is that C will be on a lower indifference curve than A, and the less likely it is that the individual can move from C to a higher indifference curve than the one that passes through A by "moonlighting" at a wage rate less than $kW_0$, especially if the opportunity to work additional hours is available only at the wage rate $W_0$.

The relationship between $\left[\frac{M^*_k}{M_1}\right]_0$, the break-even value of the coefficient of expansion of a fee-charging union, and $\left[\frac{M^*_k}{M_1}\right]$, the equilibrium coefficient of union expansion, is thus seen to be crucial. If $\left[\frac{M^*_k}{M_1}\right]_0 \geq \left[\frac{M^*_k}{M_1}\right]^*$, it will not only pay the original members to charge the equilibrium fee to additional members, but they will make
themselves better off by doing so. If \( \left[ \frac{M^*}{M_1} \right]_0 < \left[ \frac{M^*}{M_1} \right]^* \), it will not pay them to charge initiation fees, and they may make themselves worse off if they do. In this case, all that can be said is that the original members will be made worse off by charging fees to additional members if \( \alpha \left[ \frac{M^*}{M_1} \right]_0 < \left[ \frac{M^*}{M_1} \right]^* \), where \( 1 \leq \alpha < \frac{k}{\left[ \frac{M^*}{M_1} \right]_0} \). But the value of \( \alpha \) cannot be determined without knowledge of the typical original member's utility function. So, at the risk of oversimplifying the argument, set \( \alpha = 1 \) and \( \left[ \frac{M^*}{M_1} \right]^* = \frac{1}{2} \left[ \frac{M_2}{M_1} \right] \).\(^{11}\) Then, if \( \left[ \frac{M^*}{M_1} \right]_0 < \frac{1}{2} \left[ \frac{M_2}{M_1} \right] \), initiation fees will not be charged. For given values of the parameters \( k, t, \) and \( r \), the value of \( \left[ \frac{M^*}{M_1} \right]_0 \) is obtained by substituting equations (10) and (12) into equation (13), setting \( \pi = 0 \), and solving for \( \left[ \frac{M^*}{M_1} \right]_0 \):

\[
\left[ \frac{M^*}{M_1} \right]_0 = 1 + \frac{t}{r} [k - 1] .
\]

Thus, \( \left[ \frac{M^*}{M_1} \right]_0 \) varies directly with \( k \) and \( t \) and inversely with \( r \), and is independent of \( \omega \). Table 2 gives the values of \( \left[ \frac{M^*}{M_1} \right]_0 \) for several alternative specifications of these parameters. Three pairs of the underlying \( g \)- and \( \ell \)-functions are plotted in Figure 3. The three sets of parameter values chosen are those that yield low (1.008), medium (1.05), and high (1.16) values of \( \left[ \frac{M^*}{M_1} \right]_0 \).\(^{11}\)

\(^{11}\) In the case where \( k = 1.2, t = 0.025, \) and \( r = 0.10 \),

\( \left[ \frac{M^*}{M_1} \right]_0 = 1.05 \) and, therefore \( 1 \leq \alpha < 1.143 \). A reason for setting

\( \left[ \frac{M^*}{M_1} \right]^* = \frac{1}{2} \left[ \frac{M_2}{M_1} \right] \) was given on pages 10 and 11, above.
in Table 2.\textsuperscript{12}

III.

The implications of this analysis are clear: If \( \left[ \frac{M^*}{N_1} \right]_0 < \left[ \frac{M^*}{M_1} \right] \), it does not pay to charge initiation fees. But \( \left[ \frac{M^*}{N_1} \right] = \frac{1}{2} \left[ \frac{M^*}{M_1} \right] \). Therefore, it does not pay to charge fees if \( \left[ \frac{M^*}{M_1} \right]_0 < \frac{1}{2} \left[ \frac{M^*}{M_1} \right] \). For the low, medium, and high values of \( \left[ \frac{M^*}{M_1} \right]_0 \) illustrated in Figure 3, it does not pay to charge fees if \( \frac{M^*}{M_1} > 2.016, 2.10, \) and 2.32, respectively. Given the elasticity of demand for labor, the value of \( \frac{M^*}{M_1} \) is determined by the elasticity of supply. For any given set of parameter values, the greater the elasticity of demand, the smaller must be the elasticity of supply in order for \( \frac{M^*}{M_1} \) not to exceed \( 2 \left[ \frac{M^*}{M_1} \right]_0 \) and thus to permit fees to be charged. For any given elasticity of demand, the greater the value of \( \left[ \frac{M^*}{M_1} \right]_0 \), the smaller must be the elasticity of supply in order for \( \frac{M^*}{M_1} \) not to exceed \( 2 \left[ \frac{M^*}{M_1} \right]_0 \).

\textsuperscript{12}Although this paper has been concerned only with the practical question, "Why don't unions use initiation fees as a rationing device?", the model employed has implications for a more theoretical question: "What do unions try to maximize?" For the present purpose, it has been sufficient to take the relative wage effect, \( k \), as a parameter and to treat the number of union members, \( M^* \), as a variable. But, clearly, both \( k \) and \( M^* \) are subject to the control of the union. In fact, they are frequently taken to represent the goals of union policy. (For example, see \[6, pages 129-130\].) It now appears, however, that a more fruitful approach to the question of union policy might be to regard \( k \) and \( M^* \) as joint means to a further end and to develop a model in which the optimal values of \( k \) and \( M^* \) are those that maximize the present value (to the typical member) of the benefits from membership, \( G = \left[ \frac{M^*}{M_1} + \frac{t}{r} \right] F \), subject to the constraints imposed by \( r, t, \) and the demand and supply elasticities. Such an effort, however, is more properly the subject of another paper.
Table 2

Values of $\left[ \frac{M^k}{M_1} \right]_0$ for Specified Combinations of $k$, $t$, and $r$

<table>
<thead>
<tr>
<th>$k$</th>
<th>1.1</th>
<th></th>
<th>1.2</th>
<th></th>
<th>1.3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$r$</td>
<td>.075</td>
<td>.10</td>
<td>.125</td>
<td>.075</td>
<td>.10</td>
</tr>
<tr>
<td>.01</td>
<td>.0133</td>
<td>1.01</td>
<td>1.01</td>
<td>1.008</td>
<td>1.0267</td>
<td>1.02</td>
</tr>
<tr>
<td>.025</td>
<td>1.033</td>
<td>1.025</td>
<td>1.02</td>
<td>1.067</td>
<td>1.05</td>
<td>1.04</td>
</tr>
<tr>
<td>.04</td>
<td>1.0533</td>
<td>1.04</td>
<td>1.032</td>
<td>1.1067</td>
<td>1.08</td>
<td>1.064</td>
</tr>
</tbody>
</table>
g- AND l-FUNCTIONS FOR SPECIFIED COMBINATIONS
OF k, t, AND r, WHEN \( W_0 H_0 = \$7,200 \)

<table>
<thead>
<tr>
<th>g, t</th>
<th>k</th>
<th>t</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1</td>
<td>.01</td>
<td>.125</td>
</tr>
<tr>
<td>2</td>
<td>1.2</td>
<td>.025</td>
<td>.10</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>.04</td>
<td>.075</td>
</tr>
</tbody>
</table>

Figure 3
In other words, the more favorable (to charging fees) the combination of parameter values, the smaller must be the elasticity of supply in order to permit fees to be charged. (For example, with a given elasticity of demand and a given turnover rate and rate of interest, the greater the relative wage effect, the smaller must be the elasticity of supply.) In Table 3, if the elasticity of supply is greater than the values shown, the $\frac{M_2}{M_1}$ exceeds $2 \left( \frac{M^*}{M_1} \right)_0$ and it does not pay to charge fees. For example, when $k = 1.2$, $t = 0.025$, and $r = 0.10$, $\left( \frac{M^*}{M_1} \right)_0 = 1.05$; in order for $\frac{M_2}{M_1} \leq 2.10 \left( = 2 \left( \frac{M^*}{M_1} \right)_0 \right)$, the elasticity of supply cannot be greater than three and one-half, three, and two when the elasticity of demand is minus one-half, minus one, and minus two, respectively. So, if a union to which the above parameter values apply does not in fact charge fees, one may infer that the number of applicants per union member, $\frac{M_2 - M_1}{M_1} \left( = \frac{M_2}{M_1} - 1 \right)$, is greater than 1.10 and that it does not pay to charge fees.

Although it is an empirical question as to whether the value of $\left[ \frac{M^*}{M_1} \right]_0$ (which, given the relative wage effect and the elasticity of demand, depends upon the elasticity of supply) exceeds the value of $\left( \frac{M^*}{M_1} \right)_0$ (which depends upon the parameters $k$, $t$, and $r$), the

\[ \lambda_5 = \frac{M_1 - M_0}{\frac{1}{2}(M_1 + M_0)} \times \frac{\frac{1}{2}(W_1 + W_0)}{W_1 - W_0} \quad \text{and} \quad M_0 = W_0 = 1, \]

and solve for $M_1$. Determine $\left[ \frac{M^*}{M_1} \right]_0$. Let

\[ \frac{2 \left( \frac{M^*}{M_1} \right)_0 - M_0}{\frac{1}{2} \left( 2 \left( \frac{M^*}{M_1} \right)_0 + M_0 \right)} \times \frac{\frac{1}{2}(W_1 + W_0)}{W_1 - W_0} = \lambda_5, \]

and solve for $\lambda_5$.
### Table 3

Maximum Values of the Elasticity of Supply for Specified Combinations of the Elasticity of Demand and $2 \left[ \frac{M_n}{M_1} \right]_0$

<table>
<thead>
<tr>
<th>Elasticity of Demand</th>
<th>2.016</th>
<th>2.10</th>
<th>2.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 1/2</td>
<td>6 5/8</td>
<td>3 1/2</td>
<td>2 5/8</td>
</tr>
<tr>
<td>- 1</td>
<td>6 1/6</td>
<td>3</td>
<td>2 1/6</td>
</tr>
<tr>
<td>- 2</td>
<td>5 1/4</td>
<td>2</td>
<td>1 1/6</td>
</tr>
</tbody>
</table>
widespread use of nonprice rationing devices, coupled with only token initiation fees, strongly suggests that it does. If, in fact, \[ \left( \frac{N^*}{M_1} \right)^* > \left( \frac{N_0}{M_1} \right) \]
then the practice of discrimination and favoritism in the selection of new members becomes explicable. In particular, the use of discrimination and favoritism is not so much a policy that is chosen by the union as it is one that is more or less forced upon it by the underlying economic circumstances that preclude the use of more than token initiation fees.\textsuperscript{14}

When a new member has paid the equilibrium fee for the right to work at the union scale, he has done so with the full expectation of becoming a first-class citizen of the union. By virtue of this, he acquires immediate tenure and is entitled to the same annual income (from wages and fee receipts) as any other tenured member of the union. But when a new member has been more or less arbitrarily selected as one from among many in the queue, he need not be given first-class citizenship and tenure rights, nor may he claim them. What he is given is privileges, not rights. He is placed on the referral list and given the opportunity to work at the union scale when work is available, for example, when a tenured member is unexpectedly absent or the labor requirements of employers are unusually high. He is also given the prospect of acquiring tenure and earning a permanent income of \( k W_0 H_0 \) when a tenured slot becomes vacant, that is, when resignations, retirements, or deaths occur among the tenured members. In effect, he is asked to make an investment which is not unlike an investment in human capital. The investment period will vary directly with the number of nontenured members on the referral list and inversely with the turnover

\textsuperscript{14} The only other alternatives are selection by lot or according to length of time spent in the queue.
rate of the tenured members. Let $p$ denote the number of years a new member can expect to wait before acquiring tenure. Then,

$$p = \frac{\left(\frac{M^*}{M_1}\right) - 1}{r} = \frac{M^*}{R} \frac{R}{M^*} = M^* \left[\frac{M^*}{M_1}\right]$$

(15)

If the waiting time is four or five years, the decision of an applicant to accept a nontenured membership, is analogous to that of a high school (or college) graduate to invest in a college (or graduate) education. In each case, there are costs incurred in the present in the expectation of obtaining a future income stream greater than would otherwise be available. In each case, the costs and benefits can be estimated and the decision to invest rationally made.

The discounted value of the benefits, $B$, is given by:

$$B = \frac{D}{r} \left[\left(1 - \frac{1}{(1 + r)^{m+p}}\right) - \left(1 - \frac{1}{(1 + r)^{m+r}}\right)\right]$$

(16)

where $m$ is the number of years a tenured member expects to be in the union and $p$ is the expected waiting period. The discounted value of the costs (earnings foregone) is given by:

$$C = \frac{W_o H_o \left(1 - k \left[\frac{H^{'}}{H_o}\right]\right)}{r \left(1 - \frac{1}{(1 + r)^{m+r}}\right)}$$

(17)

where $H^{'}$ is the number of hours per year a nontenured member expects to work at the union scale during the waiting period. The benefits vary directly with $k$, $t$, and $m$ and inversely with $r$ and $\left[\frac{M^*}{M_1}\right]$; the costs vary directly with $\left[\frac{M^*}{M_1}\right]$ and inversely with $k$, $r$, $t$, and $H^{'}$.

If the discounted benefits are equal to or greater than the discounted costs, an applicant will find it worth his while to accept a nontenured
membership. For example, consider the case where \( k = 1.2, \ t = 0.025, \ r = 0.10, \) and \( W_0H_0 = \$7,200 (= 3.60 \times 2,000). \) We have already seen that it does not pay to charge the equilibrium initiation fee when \( \left[ \frac{M^*}{M_1} \right]^* > 1.05. \) Suppose \( \frac{M^*}{M_1} = 1.05 \) (that is, \( p = 2 \)) and \( m = 40. \) Then, \( B = C \) when \( \frac{H'}{H_0} = 0.057 \left( = \frac{114.5}{2,000} \right). \) In other words, a nontenured member need work at the union scale only 115 hours per year for two years (or 5.7 percent time, when \( H_0 = 2,000 \)) to make the investment pay off over the next 40 years. Table 4 shows how, as the waiting period increases, hours of work per year (expressed as a percentage of \( H_0 = 2,000 \)) at the union scale must increase in order to make the investment in a non-tenured membership worthwhile. The three cases illustrated are derived from the three sets of parameter values that yielded the low, medium, and high values of \( \frac{M^*}{M_1} \) given by the \( g^- \) and \( t^- \) functions plotted in Figure 3. In each case, the alternative earnings are \$7,200 per year and the number of years a person expects to remain in the union, after acquiring tenure, is 40.

In any particular case, as the waiting period increases, it becomes more and more unlikely that the union can provide sufficient hours of work at the union scale to make the investments of the nontenured members profitable. Yet equilibrium for the union requires that the number of such members, \( M', \) be such that \( \frac{M_1 + M'}{M_1} = \left[ \frac{M^*}{M_1} \right]^*. \) To obtain the required \( M' \) members, the union may have to control the value of \( p, \) and (in the short run, at least) the only feasible way it can do this is by encouraging some tenured members to retire earlier than planned, thereby increasing the value of \( t. \) But if the retirement age is permanently
Table 4

Minimum Values of $H'$ (as a percentage of $H_0$) Required with Different Values of $p$ and of $k$, $t$, and $r$, When $W_0 H_0 = $7,200 and $m = 40$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$t$</th>
<th>$r$</th>
<th>$\frac{M^k}{M_1}$</th>
<th>$\frac{H'}{H_0} \times 100$</th>
<th>$\frac{M^k}{M_1}$</th>
<th>$\frac{H'}{H_0} \times 100$</th>
<th>$\frac{M^k}{M_1}$</th>
<th>$\frac{H'}{H_0} \times 100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
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<td>5.7</td>
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<td>3</td>
<td>34.1</td>
</tr>
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<td>1.1</td>
<td>4</td>
<td>48.2</td>
<td>1.15</td>
<td>6</td>
<td>62.2</td>
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<td>62.2</td>
</tr>
<tr>
<td>1.2</td>
<td>0.04</td>
<td>0.075</td>
<td>1.16</td>
<td>11.9</td>
<td>1.28</td>
<td>7</td>
<td>43.8</td>
<td>1.32</td>
</tr>
</tbody>
</table>
reduced, this will decrease the expected value of \( m \) and, therefore, \( B \), thereby compounding the problem by raising the value of \( \frac{H'}{H_0} \) that is required to make \( B \geq C \). The only other way of controlling the value of \( p \) is to control the value of \( \left[ \frac{M_2}{M_1} \right]^* \). One way of interpreting union efforts to reduce the effective demand and supply elasticities (through featherbedding and other work rules, on the one hand, and apprenticeship and licensing requirements, on the other) is to say that such efforts are aimed (in part, at least) at reducing the value of \( \left[ \frac{M_2}{M_1} \right]^* \) in order to maintain \( p \) within sustainable limits.

In any event, when \( \left[ \frac{M_2}{M_1} \right]^* > \left[ \frac{M_2}{M_1} \right]_0 \), equilibrium initiation fees are not charged to new members. Instead, they are selected in a more or less arbitrary fashion and given a nontenured status. The tenured members continue to work at wage rate \( kW_0 \) for \( H_0 \) hours and earn annual incomes of \( kW_0 H_0 \). The nontenured members are available to work at or below wage rate \( kW_0 \), for as many hours as needed and ultimately to replace tenured members as the latter resign, retire, or die.\(^{16}\)

Since it is unlikely that the \( M_2 - M_1 \) applicants for membership will be indistinguishable from each other and thus equally acceptable to the \( M_1 \) tenured members, admission with only token fees permits the tenured members to pick and choose only acceptable (by whatever criteria) non-

\(^{15}\) There is some flexibility here. Since more nontenured members can be employed for longer periods the lower the wage rate, the union may find it convenient to have different pay scales for members in different categories.

\(^{16}\) When an old, nontenured member acquires tenure, he will be replaced by a new, nontenured member, assuming the supply and demand functions remain stable.
tenured members. Hence, discrimination against certain persons or groups and favoritism toward others will be the rule. The number of nontenured members, \( M' \), thus chosen will be such that 
\[
\frac{N_1 + M'}{M_1} = \left[ \frac{M^*}{N_1} \right]^*,
\]
the equilibrium coefficient of union expansion.