Domain Lines as Fractional Vortices

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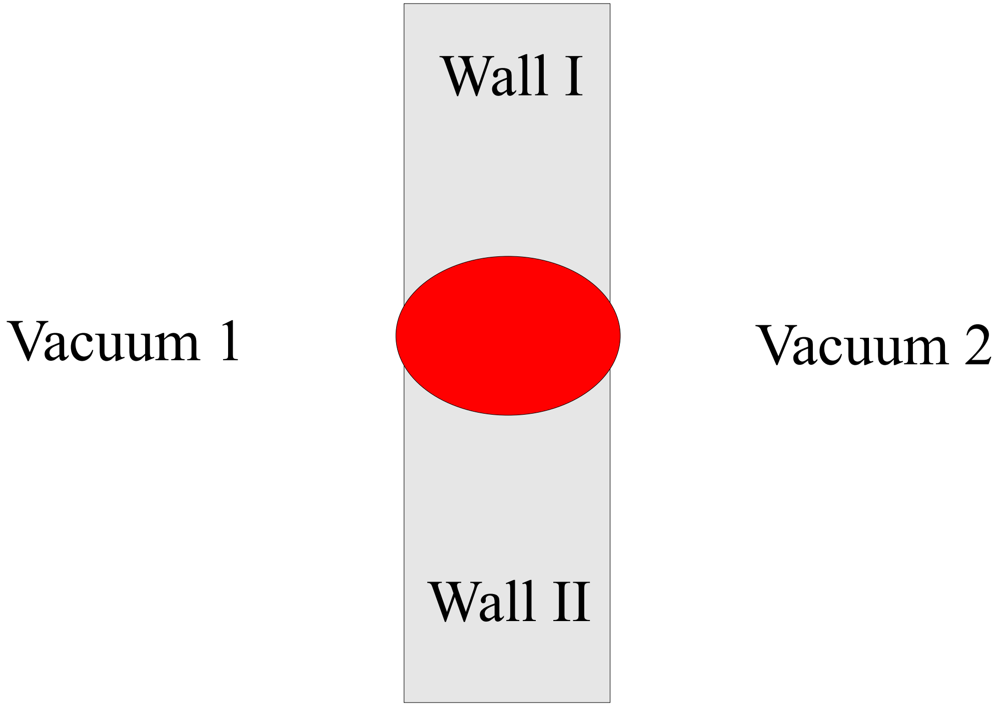
Introduction

Domain walls natural objects in supersymmetric theories

There are the following alternatives (without taking in account the trivial translational modulus):

a) A **single** domain wall interpolating two vacua.
b) A **moduli space** of solution. The modulus can be promoted to a massless field on the wall world-volume (for example $\mathcal{N}=2$ SQED)
c) A **discrete number** $\neq 1$ of solutions (expected in $\mathcal{N}=1$ Super-Yang-Mills)

Domain Lines



Domain Lines interpolating between discrete Domain Wall solutions:
an explicit solitonic example at weak coupling

Fields & Superpotential

$\mathcal{N} = 2$ U(1) gauge theory

Bosonic Fields:

* U(1) gauge vector superfield

* superfield A with zero charge

* $N_f = 2$ squark superfields Q_B,$\tilde{Q}_B^\dagger$ from hypermultiplets

$$W = \frac{1}{\sqrt{2}} A Q_B \tilde{Q}_B + m_B Q_B \tilde{Q}_B + \frac{\beta}{\sqrt{2}} (Q_1 \tilde{Q}_2 - \tilde{Q}_1 Q_2) - \frac{1}{2\sqrt{2}} \xi A,$$

$$m_1 = -m_2 = m$$

$\beta \neq 0$ we are breaking the extended supersymmetry, and also the flavor group

$$U(1) \times U(1) \rightarrow U(1)$$

Two Vacua

$$1)\ \ a = -\sqrt{2m^2 - \beta^2},\ \ q_1 = \tilde{q}_1^\dagger = \Omega,\ \ q_2 = -\tilde{q}_2^\dagger = \omega.$$

$$2)\ \ a = \sqrt{2m^2 - \beta^2},\ \ q_1 = -\tilde{q}_1^\dagger = \omega,\ \ q_2 = \tilde{q}_2^\dagger = \Omega.$$

where:

$$\Omega = \frac{1}{2}\sqrt{\xi + \frac{m\xi}{\sqrt{m^2 - \beta^2/2}}},\ \ \omega = \frac{1}{2}\sqrt{-\xi + \frac{m\xi}{\sqrt{m^2 - \beta^2/2}}}$$

$\beta < \sqrt{2}m$ in order to avoid run-away vacua

Sigma Model description

$\xi >> m, \beta$ we can integrate out a

Constraints:

$$|q_1|^2 + |q_2|^2 = |\tilde{q}_1|^2 + |\tilde{q}_2|^2, \qquad \tilde{q}_A q_A = \frac{\xi}{2}$$

Eguchi-Hanson manifold (four real dimensions)

Coordinates: $(r, \theta, \varphi, \psi)$

Potential induced by m, β

$$V = \frac{m^2(4r^2 + \xi^2 \sin^2\theta)}{\sqrt{4r^2+\xi^2}} + 2\sqrt{2} m\beta r(\cos\theta\cos\varphi\cos\psi - \sin\varphi\sin\psi) +$$

$$+ \frac{\beta^2(4r^2 + 3(\xi/2)^2 + 2\xi^2\cos(2\theta)\sin^2\varphi + \xi^2\cos(2\varphi))}{2\sqrt{4r^2+\xi^2}}$$

Wall at $\beta = 0$

Both the vacua and the wall are living in the S^2 subspace of Eguchi-Hanson manifold at $\beta = 0$ Vacua: $r = 0$, $\theta = 0, \pi$
Moduli space of BPS domain walls parameterized by an $U(1)$ phase $e^{i\sigma}$

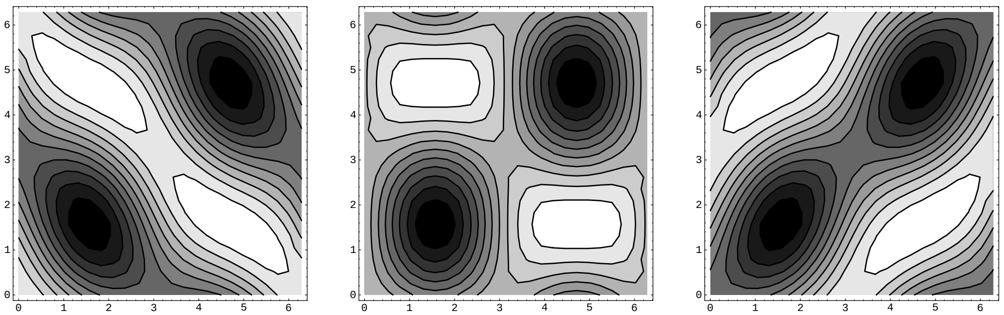
$$\phi(z) = \sigma, \;\; r(z) = 0$$

$$\theta(z) = 2\tan^{-1}(\exp(2mz))$$

Modulus promoted to a field on the wall world-volume:

$$S = \int d^3x \left(\frac{\xi}{4m} (\partial_n \sigma)^2 \right) .$$

Wall at $\beta \neq 0$



Plot of the potential at some sections at constant r and at $\theta = \pi/3, \pi/2, 2/3\pi$. There are always two symmetric minima, one at $\phi = \psi = \pi/2$ and the other at $\phi = \psi = 3\pi/2$.

BPS wall equations can be solved in correspondence of these two values.

$$q_1 = \pm i\Omega \cos(\eta/2) + \omega \sin(\eta/2), \;\; q_2 = \pm i\omega \cos(\eta/2) + \Omega \sin(\eta/2)$$

$$\tilde{q}_1 = \mp i\Omega \cos(\eta/2) - \omega \sin(\eta/2), \;\; \tilde{q}_2 = \pm i\omega \cos(\eta/2) + \Omega \sin(\eta/2)$$

$$a = -\sqrt{2m^2 - \beta^2}(\cos\eta \pm i \frac{\beta}{\sqrt{2}m} \sin\eta)$$

Metastable walls

The BPS wall solutions at $\beta \neq 0$ correspond to $\sigma = \pi/2, 3\pi/2$.
Ansatz for a "meta-solution" at generic σ:

$$q_1 = (e^{i\sigma}\Omega\cos(\eta/2) + \omega\sin(\eta/2))/A,$$

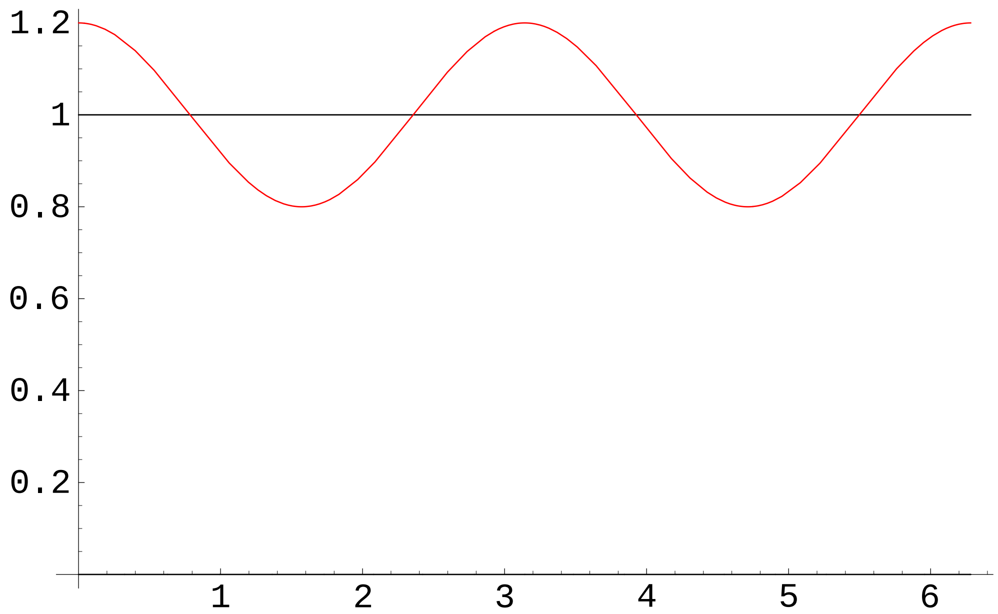
$$q_2 = (e^{i\sigma}\omega\cos(\eta/2) + \Omega\sin(\eta/2))/A,$$
$$\tilde{q}_1 = (e^{-i\sigma}\Omega\cos(\eta/2) - \omega\sin(\eta/2))A,$$
$$\tilde{q}_2 = (-e^{-i\sigma}\omega\cos(\eta/2) + \Omega\sin(\eta/2))A.$$

where $\eta(z)$ is a profile function which is calculated by minimization of the action and the factor A is introduced in order to maintain the sigma model constraints.

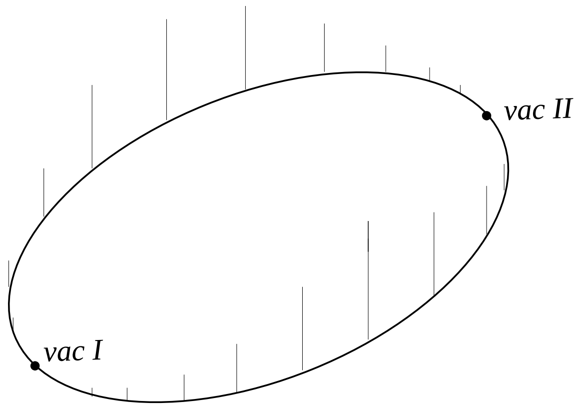
Effective world-volume potential

At the first non trivial order in β, the following potential is generated:

$$V = \frac{\beta^2 \xi}{2m} \cos(2\sigma) + O(\beta^3)$$



Domain line as a Sine Gordon kink

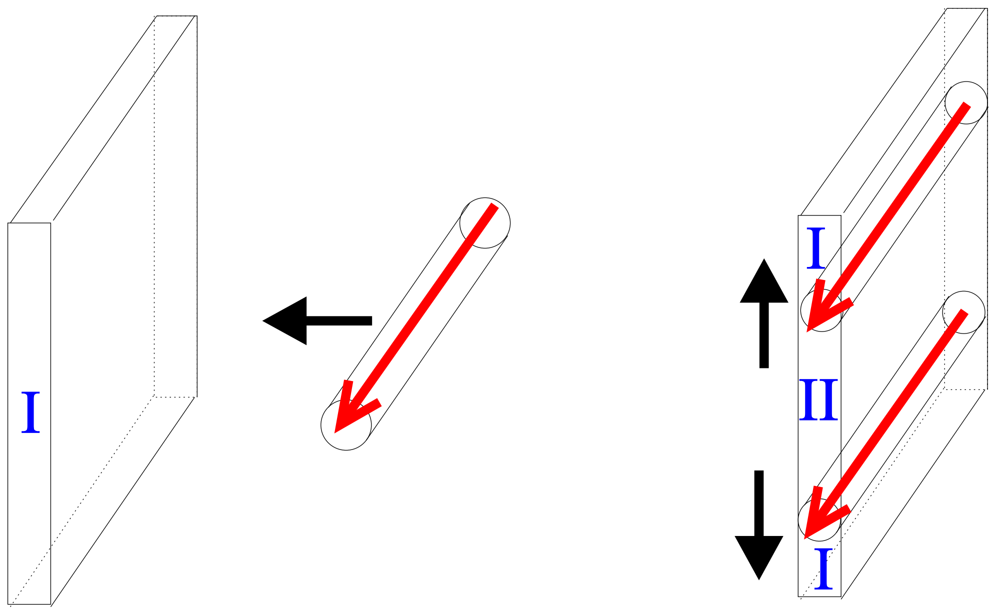


There are two different kinks interpolating the two world-volume vacua

$$\mathcal{H} = \int d^3x \left(\frac{\xi}{4m} (\partial_n \sigma)^2 + \frac{\beta^2 \xi}{2m} \cos^2 \sigma \right) =$$

$$= \int d^3x \left\{ \frac{\xi}{2m} \left(\frac{1}{\sqrt{2}} (\partial_x \sigma) \mp \beta \cos \sigma \right)^2 \mp \frac{\xi \beta}{\sqrt{2} m} \partial (\sin \sigma) \right\} .$$

The fate of a bulk vortex



Bulk: $T = 2\pi\xi$

Domain line: $T = \sqrt{2}\beta\xi/m$

The Domain Line carries a magnetic flux which is $1/2$ of the flux carried by a bulk vortex.

Sine Gordon vs. Chern-Simon

World-volume scalar field σ:

$$S = \int d^3x \left(\frac{\xi}{4m} (\partial_n \sigma)^2 \right).$$

Duality: $F^{(2+1)}_{nm} = \frac{e^2_{2+1}}{2\pi} \epsilon_{nmk} \partial^k \sigma,$

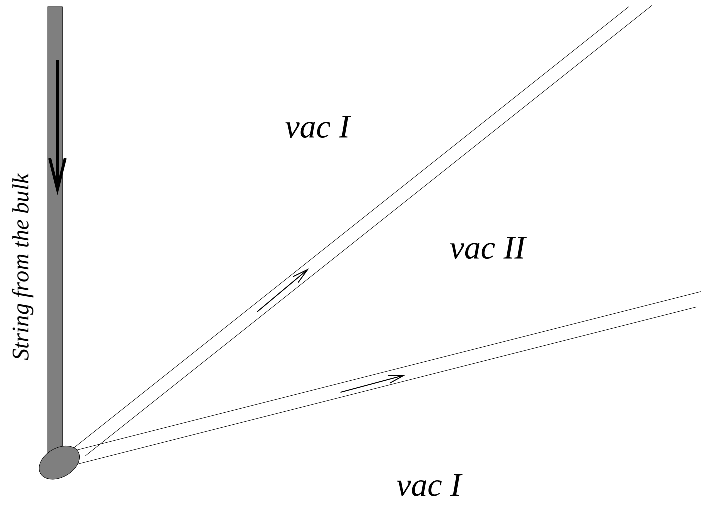
$$S = \int d^3x \left(-\frac{1}{4e^2} (F^{2+1}_{mn})^2 \right).$$

Chern-Simon term (proposed for domain walls of $\mathcal{N} = 1$ Super-Yang-Mills):

$$S_{CS} = \frac{1}{2\pi} \epsilon_{nmk} A_n \partial_m A_k.$$

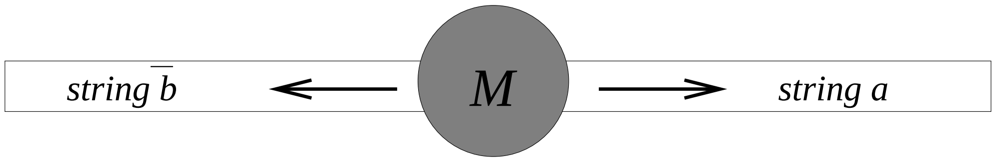
Different from the Sine-Gordon potential in our weakly coupled example!

Confined Monopoles



We can introduce external magnetic monopoles embedding the $U(1)$ theory in a $SU(2)$ one

It is possible to build a static configuration where a monopole is a junction of two domain lines (similar to what happen in the bulk for the non-abelian string)



Conclusion

* A Domain-Line soliton has been built in a weak coupled theory

* The effective world-volume description involves a Sine-Gordon theory with two vacua

* The domain line carries a magnetic flux which is $1/2$ of a bulk Abrikosov-Nielsen-Olesen vortex