Modeling, Control and Optimization of a Novel Compressed Air Energy Storage System for Off-Shore Wind Turbines

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Dedication

To my mother and father: Shahrbanoo and Gholamhossein, for their unconditional love and support, and to my sisters: Mandana, Anahita and Marjan, for their endless affection and encouragement.
Abstract

Integrating wind and solar energy into the electric power grid is challenging due to variations in wind speed and solar intensity. Moreover, to maintain the stability of electric power grid, there must be always a balance between the energy production and consumption which is not easy since both of them undergo drastic variation over time. Large scale energy storage systems can solve these issues by storing the extra energy when supply exceeds demand power, and regenerating energy and send it to the electric grid when demand power surpasses the supply.

This dissertation focuses on the optimal design and control of a new type of Compressed Air Energy Storage (CAES) system that is especially applicable to off-shore wind turbines. The system is designed such that it addresses the need for a compact and energy dense storage system with high roundtrip efficiency for large-scale energy storage applications. The contributions of this work are also beneficial for designing power dense gas compressors/expanders with high thermal efficiency. The material of this thesis can be divided into two parts: In the first part, different approaches and techniques are studied to increase the power density of a liquid piston air compressor/expander system without sacrificing its efficiency. These methods are then combined and optimized in the form of a single design to maximize the performance of an air compressor/expander unit which is the most critical component of the energy storage system under investigation. In the second part, component-level and supervisory-level controllers are designed and developed for the combined wind turbine and energy storage system such that both short-term and long-term objectives are achieved.

Improvement of thermal efficiency of an air compressor/expander is achievable by increasing heat transfer between air under compression/expansion and its surrounding solid material in the compression/expansion chamber. This will prevent heat loss by reducing air temperature rise/drop during the compression/expansion phase. Here, liquid piston (instead of conventional solid piston) is used in the compression/expansion chamber where the chamber volume is filled with porous material that increases heat transfer area by an order of magnitude, and therefore improves the thermal efficiency. Since
the liquid piston is driven by a variable displacement pump/motor, optimal compression/expansion trajectories are calculated and applied to further increase heat transfer and improve the performance of the system. This improvement is verified both analytically and experimentally. Based on numerical results, utilizing porous material in the compression/expansion chamber with optimized distribution, combined with the corresponding optimal compression/expansion trajectory has the potential to increase the power density by more than 20 folds, without reducing its thermal efficiency. An alternative method to increase heat transfer is to introduce micro-size water droplets (through water spray) in the chamber during air compression/expansion process. Since water has a high heat capacity, the generated heat during compression can be absorbed by water and therefore reduce the temperature rise of air during compression. The same phenomenon but in opposite direction happens in expansion case (heat transfer from water droplets to air) that prevents air from getting very cold which causes poor efficiency. A numerical model is developed and used to study the effect of water spray amount and timing on the thermal performance of air compressor/expander. The optimal timing of water spray is calculated to maximize the effectiveness of a given amount of water that is sprayed into the air.

The optimally designed liquid piston air compressor/expander unit is then combined with the other components of energy storage system, as well as a wind turbine. Nonlinear techniques are used to design plant-level controllers in order to coordinate different parts in the system, and to achieve both short term objectives (maintaining the frequency of electric generator while capturing maximum wind power) and long term objectives (tracking the power demanded from electric grid and regulating the pressure in the storage vessel). Finally, the combined wind turbine and energy storage system is studied for maximizing the total achievable revenue by optimizing the storage/regeneration sequence according to varying electricity price and available wind power (given storage size and its nominal power). According to the results, an increase of up to 137% in total revenue is achievable by equipping a conventional wind turbine with a CAES system while tracking the calculated optimal storage/regeneration sequence. Additionally, by incorporating the price of different components of energy storage system, a study is conducted to find the effect of system size on maximum achievable revenue that can lead to the economical size selection of the energy storage system.
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<td>Effect of CAES size on the yearly revenue (February 2012 wind speed and price profiles are used as their typical values over a year), sell/buy case. Top: Total revenue (per year) as a function of air compressor/expander rated power and storage vessel capacity (with iso-price lines); Bottom: Optimal CAES system size for a given cost, and its corresponding maximum achievable revenue (per year)</td>
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8.26 Effect of CAES size on the yearly revenue (combined February and July 2012 two-weeks wind speed and price profiles are used as their typical values over a year), sell only case. Top: Total revenue (per year) as a function of air compressor/expander rated power and storage vessel capacity (with iso-price lines); Bottom: Optimal CAES system size for a given cost, and its corresponding maximum achievable revenue (per year) 181

8.27 Effect of CAES size on the yearly revenue (combined February and July 2012 two-weeks wind speed and price profiles are used as their typical values over a year), sell/buy case. Top: Total revenue (per year) as a function of air compressor/expander rated power and storage vessel capacity (with iso-price lines); Bottom: Optimal CAES system size for a given cost, and its corresponding maximum achievable revenue (per year) 182


Chapter 1

Introduction

1.1 Energy Storage Background

The majority of total U.S. energy consumption over the past 100 years came from fossil fuel sources such as coal, natural gas and petroleum (Fig. 1.1). The electricity produced by coal-fired and natural gas-fired power plants accounts for more than 30% of the total U.S. global warming emissions [1, 2]. The greenhouse gas pollution which is the result of this high emission has affected the global climate as massive droughts, devastating tornadoes and horrific wildfires. As a result, most of the leaders across the world are now planning to take action and are looking for ways to fight this global problem. Strengthening and broadening the existing carbon taxes provides a potentially cost-effective approach to reduce greenhouse gas emissions. This will also put renewable energy sources (such as wind and solar) on a more competitive position, will quicken their growth, and will increase their shares of electricity generation in the future of power grids [3]. However, besides economical and political challenges, there are technical challenges that need to be overcome to allow integrating more and more renewable energy into the electricity grid. Renewable energy such as wind and solar energy are clean and available as long as the wind blows or the sun shines. Their inherent irregularity on both short-time scale (hourly) and long-time scale (daily, seasonally) makes them difficult to be integrated with the electric grid. This is in contrast with fossil fuels that are traditionally considered as natural storage of energy until the energy is extracted from them through a chemical reaction. Hence, the two main disadvantages
of renewable energy sources are their intermittency and that their availabilities do not often correspond to power demand. For example, wind energy tends to be more abundant at night when power demand is low. Moreover, daily solar power vanishes just before the peak demand from the consumers (usually it is between 5-8PM). In addition to variability in energy supply, there is also a large change in energy demand (Fig. 1.2).

Figure 1.1: Top: share of energy consumption in the United States (1776-2014); Bottom: estimated effect of a carbon tax on sources of United States electrical generation (Source: U.S. Energy Information Administration)

In order to have stability in any electric grid, there must be always a balance between the energy production and consumption, while both of them undergo drastic variation
over time. Currently, natural gas-fired peaker plants are used to regulate this mismatch. However, they are expensive to construct and operate, and they use fossil fuels. Instead, large scale energy storage systems can be used to solve these issues by storing the extra energy when supply exceeds demand power, and regenerating energy and send it to the grid when demand power surpasses the supply [4, 5].

![Wind energy, Solar Energy, Electricity Demand](image)

Figure 1.2: Available wind and solar powers in contrast to demand power for electricity

Large-scale cost effective energy storage technologies are receiving significant attention from researchers in recent years [6, 7]. This is due to the growing penetration of renewable sources of energy. Several studies have been done on the importance of energy storage systems for the electric grid. Assessment of potential benefits and economic market potential for energy storage used for electric-utility-related applications in [8] reveals positive impact of energy storage on the U.S. economy. In [9], a methodology for profit analysis and overall economic viability of the Battery Energy Storage System (BESS) for various applications is presented. The efficacies of batteries, flywheels and pump-hydro storage systems to mitigate the effect of wind intermittency are investigated in [7] using convex optimization techniques. The optimal power flow (OPF) problem in the presence of large-scale energy storage systems is discussed in [10]
showing significant reduction in the generation costs. Among all different types of energy storage technologies, battery, flywheel, pumped-hydro and compressed air energy storage systems are more common and have higher installed capacity. These different energy storage approaches are briefly introduced in what follows.

Perhaps, battery is one of the most well-known devices for energy storage/regeneration. The main three types of batteries used for large scale energy storage systems are: I) lead-Acid (LA); II) Nickel-Cadmium (NiCd); and III) Sodium-Sulphur (NaS) [9, 11]. Lead-Acid (LA) is the energy storage device most in use at present. Relatively low cost, long lifespan, and fast response are some of the advantages of LA batteries. It can be used for both short term (seconds) and long term (up to 8 hours) applications. However, it is extremely sensitive to temperature. Moreover, its charge to discharge time ratio is about 5 (faster rate of charge will damage the battery) that makes some limitations in practical usage. Average DC-DC efficiency of a Normal LA battery in normal operating condition is between 75% to 80%. In contrast to LA battery, NiCd battery can operate over a much wider temperature range (as high as 50°C). NiCd battery can respond very fast and get to the full power in milliseconds. Their DC-DC efficiency is 60% to 70% (in normal condition). However, the environmental effects of the NiCd batteries is one of the main concerns about this type since cadmium is a toxic material. Sodium-Sulfur (NaS) batteries have about two times more energy density of LA types. In addition, they have longer life and lower maintenance cost. Average round trip energy efficiency of this type of batteries is about 85% and due to their ability to provide power in a pulse manner, they are a good option for improving the quality of electricity power. The main disadvantage is the need of an external unit to keep the battery material at elevated temperature. Additionally, the insulator is a weak part since it slowly becomes conducting and increases the self-discharge of the battery. On average, batteries have good energy density but low power density while they need a power convertor to be connected to the electricity grid. Moreover, compared to most of the other large-scale energy storage technologies, batteries are very expensive which is the main reason why they are not as widely used.

Flywheel is another device to store/regenerate energy in large scale. The electrical energy is stored kinetically in a high angular inertia cylinder that rotates rapidly around a magnetically levitated shaft (Fig. 1.2) [12, 13]. The full storage or full generation
Figure 1.3: Two common approaches for energy storage/regeneration in large scales: flywheel (top) and pumped hydro (bottom)

can be as fast as 4 seconds. Flywheel energy storage technology is a good candidate for applications requiring high power and short storage intervals. However, high idling losses prevent it to be suitable in applications that require longer storage intervals, such as grid-based, load-following energy storage. While there is ongoing research to find new methods to reduce these idling losses (such as using an outer-rotor AC motor [14]), the use of flywheel energy storage is still limited in practice.

Probably, the most mature and largest storage method available today is pumped hydro. During the off-peak electrical power demand, water is pumped from a lower elevated reservoir (sea as an example) to a higher elevated reservoir and stored there until energy is needed (Fig. 1.3). During the generation mode, a pumped hydro system works similar to a hydroelectric power generation plant [15] [16]. Due to low energy density of pumped hydro storage (10J/lit per meter of elevation), it is required to have either a huge amount of water or a large elevation difference to maintain the efficiency. This elevation difference is usually between 200m to 300m depending on the
site availability. The round trip efficiency of these storage systems is between 70\% and 85\%. While pumped hydro energy storage system is a relatively cheap technology in large scales, it is highly dependent on the availability of a specific geological formation or man-made reservoir [17].

The conventional Compressed Air Energy Storage (CAES) system is an other type of large scale energy storage system (Fig. 1.4). In these systems electricity is used to compress air into a cavern, and energy is regenerated by burning with pre-compressed natural gas [18, 19]. Usually, the cavern cannot store the air at high pressures and therefore they have a relatively lower energy capacity. Moreover, their power density decreases as energy is depleted (during the regeneration phase). Typical plant capacities for conventional CAES are in the region of 50 MW to 300 MW. While being among the cheapest large scale energy storage technologies, the requirement for additional fuel, a combustion turbine, the extra conversion step back into electricity, and the inefficiency of the compression cycle itself (∼50\%) makes the conventional CAES process not feasible for off-shore use. Dependency on a large underground cavern is another drawback for this type of storage system.

Figure 1.4: Conventional CAES system uses off-peak electricity to compress and store air inside an underground cavern to be used further by a gas power plant
In summary, there is always a trade-off between energy density and power density in most of the conventional storage systems. Low round trip efficiency, high cost, limited number of cycles and need for a special geographical location are some other limitations of the conventional energy storage approaches which make them difficult to be used in off-shore applications. Currently, 93% of the energy storage capacity in the U.S. is provided by pumped-hydro, while the rest is shared between compressed air energy storage (3%), thermal storage (3%) and electromechanical energy storage systems (1%) [20]. Although all these technologies are applicable to wind turbines in general, a large-scale energy storage system would be more attractive (for wind farm owners) if it can provide the following benefits:

**Downsizing electrical components:** Since all the components of a wind turbine must be designed to capture the maximum power (rated power), the capacity factor of a wind turbine is usually small (less than 50% [21]). The same problem exists for all the transmission lines and the power convertors. Storing energy prior to electricity generation can downsize all the electrical elements of such a wind turbine [22, 23] and increases capacity factor up to 80-90%. This is a big advantage especially for off-shore wind turbines which can reduce the installation and maintenance costs.

**Capturing more power from wind:** by decoupling the wind turbine and generator (shaft), it would be possible to maintain the optimal tip speed ratio for the turbine such that it always captures maximum power from the wind no matter how much power is demanded at the generator side. This feature also provides the opportunity for locating some components of the wind turbine system (such as generators, power convertor) at ground level instead of up in the nacelle. This reduces maintenance and repair costs.

**Dispatchable and fast response:** Usually, the base load power plants (such as coal or nuclear) cannot change their power level fast enough for regulating any unexpected power imbalance on the electricity grid. Moreover, the power level generation of intermittent renewable energy sources (such as wind and solar power) cannot be controlled by the operator, another reason for the necessity of a dispatchable and fast-response storage system. A fast response energy storage system can balance the electrical system

---

1 Capacity factor is defined as the ratio between the average power captured by a wind turbine (over a specified time period) and its generator size (rated power of generator)  
2 Tip speed ratio is defined as the ratio between blade’s tip speed and the wind speed
(load following, frequency control), optimize the generation dispatch (merit order), provide the requested power spinning reserve and contribute to clearing grid congestions [24].

No need for a power converting system: Decoupling wind turbine and generator shafts also provides the opportunity to remove the costly and complicated power electronic unit. Since the storage system can always control and maintain the generator working condition, it is possible to generate electricity at the desired voltage and frequency demanded by the grid [25].

1.2 Motivations and Objectives of this Thesis

Despite its limitations and currently low installed storage capacity, CAES approach has attracted significant consideration from both researchers and industries. CAES approach is advantageous for energy storage in that: (1) it has a relatively high energy density (at 350 bar, ∼3MW×8hr energy requires 500m³) compared to pumped hydro (∼144,000m³ at 100m elevation); (2) it is economically scalable (energy capacity scales linearly with storage vessel volume and cost); (3) potentially cost effective and has a long life cycle relative to electric batteries [26]; (4) does not include toxic materials, which is a big concern for a battery energy storage system. In a conventional CAES system, excess electricity is used to drive an air compressor that compresses air into an underground salt cavern; and the energy is then retrieved by pre-compressing and improving the efficiency of natural gas combustion in a gas fired turbine [18, 19]. Such a system is relatively inefficient (<< 50%), requires use of hydrocarbon fuel, and depends on geological sites. An alternate novel CAES concept for wind turbines was proposed in [27] in which compressed air is stored in high pressure (200-350 bar) vessels. Increasing the pressure ratio of the compressed air can result in an appreciable increase in energy density. For example, with a pressure ratio of 350 (35 MPa), 170MJ of energy can be stored in 1m³ of volume (compared to pumped hydro where 170m³ of water is required at 100m elevation to store the same amount of energy). In addition, if the air is compressed and extracted in a near-isothermal manner, a very good efficiency (>90%) is also achievable (compare with conventional CAES systems where the average
efficiency is about 55%) \[28\]. The current costs of electrical collection and transmission for off-shore wind turbines are approximately 4 times that for on-shore deployment, accounting for 15% of the total cost (versus 6% for on-shore) \[29\]. By storing excess wind energy prior to electricity conversion, the electrical generator and the transmission lines can be sized only for demand instead of for the peak power available from the wind, thus reducing the cost and weight.

The proposed CAES is shown in Fig. 1.5. A variable displacement hydraulic pump (B) attached to the wind turbine rotor (A) in the nacelle converts wind power to hydraulic power. At down-tower (II), a variable displacement hydraulic pump/motor (C), a near-isothermal liquid piston air compressor/expander (F) and a fixed speed induction generator (G) are connected in tandem on a common shaft. They are powered by the pump (B) and exchange power with the storage vessel (E) with both liquid (hydraulic fluid) and compressed air at the same pressure in the open accumulator architecture \[35\]. This allows energy to be stored in, or extracted from, (E) either hydraulically (as in a conventional hydraulic accumulator) or pneumatically (as in a conventional air receiver). In both cases, energy is stored in the compressed air.

In general, the main challenges of compressed air energy storage approach are: (1) compressor/expanders are generally not very efficient or powerful; (2) the pressure in the storage vessel reduces as compressed air in the storage vessel depletes, making it difficult for the air compressor/expander to maintain either its efficiency or power at all energy levels.

The first challenge can be overcome by developing a liquid piston air compressor/expander \[30\] with enhanced heat transfer using porous media \[31, 32\] and droplet sprays \[33\], and reduced leakage. As shown in Fig. 1.5, the liquid piston air compressor/expander (F) consists of an air compression/expansion chamber filled with porous media (F\(_1\)), and a water pump/motor (F\(_2\)). The porous material is used to increase the heat transfer surface area. When storing energy pneumatically, the water pump/motor (F\(_2\)) pumps water into the compression/expansion chamber (F\(_1\)), compressing the air within it. Heat of compression is transferred to the porous material and to the water to maintain a near-isothermal operation. When the chamber pressure exceeds that of the storage vessel (E), the compressed air is ejected and stored in the vessel. The chamber is then refreshed by releasing the water and filling it with atmospheric air for the
next cycle. When retrieving energy, the compressed air is released into the expansion chamber. As the air expands, the liquid piston retreats, the water pump/motor (F2) is motored and work is derived. Heat is supplied from the porous material to maintain the temperature of the expanding air.

The second challenge can be solved by deploying an open accumulator configuration with a dual chamber storage vessel for both liquid and compressed air, such that energy can be stored/retrieved hydraulically and pneumatically [35]. By coordinating the hydraulic and the pneumatic power paths (see Fig. 1.5), the pressure in (E) can be maintained constant regardless of energy content, unlike a conventional closed hydraulic accumulator with only a hydraulic port or a compressed air receiver with only a pneumatic port. For example, as compressed air is being released from (E), some liquid can be added to reduce the compressed air volume to maintain pressure. The pneumatic power branch makes better use of the vessel (E) volume than the hydraulic power branch (a compressed air tank stores 20 times more energy than a hydraulic accumulator at the same peak pressure and total volume [35]) but hydraulic pump/motors
are more power dense than the pneumatic compressor/expanders. This architecture can take advantage of both by utilizing the hydraulic path to accommodate high power transient events such as wind gust or sudden generation power demand, reserving the pneumatic path for steady power. Therefore, an appropriate controller that coordinates these two power paths is essential to simultaneously achieve pressure regulation, desired generator power tracking, and maximizing wind power capture in the presence of supply or demand power variations.

Multiple faceted research is needed to achieve the objectives of the proposed CAES system. This dissertation focuses on the mathematical modeling and optimization of different components in this energy storage system, as well as the controller design for the subsystems in order to meet both the short and long term goals of the combined wind turbine and storage system. Furthermore, the most important challenges and issues for the proposed energy storage architecture are addressed, while appropriate solutions for them are introduced and investigated. Systems engineering methodology is taken for the design and operation of the compressor/expander, and for the overall energy storage system. To do so, models that can predict effects of design parameters on the performance of individual components are developed. These models are then combined to predict the system level metrics such as power and efficiency. The design and methods of operation are also determined to optimize the overall performance metric based on the combined model. In summary, the design and optimization process can be categorized in two different levels as follows:

**Compressor/Expander Design Level:** the primary performance metrics are efficiency and power density. They are, in turn, dependent on the heat transfer capabilities of the compression/expansion chamber. Three main approaches to improve the heat transfer considered in this research are solid porous material, water spray and optimal compression/expansion profile. Design choices such as geometry of the liquid piston as well as porous material, compression/expansion profile, location and timing of water spray, etc., affect both heat transfer and mechanical losses. Approximate models relating these design parameters are developed analytically and verified computationally or experimentally. A design methodology that captures the inter-relatedness of the design parameters and optimizes the overall performance (by combining all three different methods) are developed as models of the fundamental components are being assembled.
Overall Energy Storage System Design Level: for the combined wind turbine and energy storage system, it is desirable to capture available wind energy, deliver the right amount of power to the electric grid, and maximize the revenue achievable from selling electricity to the grid. It is also beneficial to minimize the cost of the system for a given energy and power capacity. Design choices include accumulator size, power and efficiency requirements for the compressor/expander and pump/motor sizes. In addition, there are choices for operating conditions (such as accumulator pressure) and operating strategy like power distribution strategy between hydraulic storage path versus pneumatic storage path. The choices are not obvious a priori especially because they are dependent on the cycle statistics of available supply and demand. Design and optimal control approaches are developed that capture these interrelations. The control strategy for the overall wind turbine with the proposed storage method must consider both short term and long term goals. For the short time scales (minutes), the storage device provides an extra degree of freedom beyond the wind power supply and electrical power demand for dynamically matching the loads that the wind or the electrical generator sees. This will maximize wind power extraction and generator efficiency. Capturing the power of wind gust is another short term objective which is achievable through the power-dense and fast-response hydraulic line connected to the open accumulator. For the longer time scales (hours), the control problem includes maximizing the wind power that can be captured and meeting demands without exceeding or depleting the storage capacity. Maximizing the total achievable revenue from the combined wind turbine and energy storage system can be considered as another long term goal. A Dynamic Programming (DP) approach is used to solve the corresponding optimal control problem that accounts for all the major losses in the CAES system as well as its nonlinear dynamics.

1.3 Thesis Outline

The main body of this thesis contains nine chapters. The need for renewable energy and the necessity of energy storage system for them was briefly presented in chapter 1. A summary of the existing grid-size energy storage technologies including conventional CAES system was also described in chapter 1. It includes a review on the relevant
literature in the field as well as discussing their advantages and disadvantages specifically for wind turbine systems. The objectives and hypotheses of this dissertation have also been presented in this chapter.

The concept of liquid piston air compressor/expander is introduced in chapter 2. A one-dimensional air dynamic model is developed to study how design parameters and control strategies affect the performance of an air compressor/expander system. This model is then used in chapter 3 to maximize the compression/expansion power density of an air compressor/expander chamber for a desired thermal efficiency. A step by step approach is used to optimize the chamber geometry, porosity distribution as well as compression/expansion rate, simultaneously, to maximize its performance. The advantage of optimal compression trajectory in increasing the power of compression (versus non-optimal compression profile) is experimentally validated on a low-pressure and high-pressure air compressor system in chapters 4 and 5 respectively. A constant heat transfer coefficient is used to calculate and implement optimal compression trajectories on the low-pressure system, while a detailed correlation is used for the same purpose in chapter 5.

Another method to improve air compressor/expander performance is to use water spray during the compression/expansion process which helps cool down/heat up the air under compression/expansion. A one-dimensional model for water spray is developed in chapter 6, which captures the main dynamic effects of the spray cooling/heating process during the air compression/expansion process. This model is then used to calculate the optimal spray timing during the compression process. Results also give some insights to the best location and direction of water spray nozzles inside the chamber.

Chapter 7 describes in detail the CAES architecture, combined with a hydraulic wind turbine, where a cycle-average dynamic model is developed for each component. These models are then combined to build a detailed dynamic system for the combined wind turbine and energy storage system. Low level controllers that achieve both short term and long term objectives of the CAES system are then designed based on the dynamics of combined system. A high level controller is also required to supervise the storage/regeneration sequence of the CAES system, and distribute power between the hydraulic and pneumatic paths in order to maximize the combined performance of the system. A supervisory controller is developed in chapter 8 which maximizes the total
achievable revenue from the combined wind turbine and CAES system that is connected to a variable electricity price grid. Dynamic Programming (DP) is used to calculate the optimal storage/regeneration sequences that maximize the revenue over a given time period, while all the losses and constraints in the combined system are considered.

Chapter 9 provides a conclusion and summary of the significant findings and contributions of this study, in addition to future works that mainly focus on the study of dynamic interaction between the designed CAES system and an electric grid. Appendix A contains detailed derivation of cycle-average efficiency for the liquid piston air compression/expansion chamber.
Chapter 2

Liquid Piston Air Compressor/Expander Modeling

2.1 Introduction

The key component of the proposed CAES system is the air compressor/expander. The efficiency of this subsystem greatly affects the entire storage/regeneration efficiency since it is responsible for the majority of the storage energy conversion. Therefore, it is required to be efficient and sufficiently powerful\(^1\). However, this is challenging because compressing/expanding air 200-300 times heats/cool the air greatly, resulting in poor efficiency, unless the process is sufficiently slow, which reduces power \[^{30}\]. To achieve a good efficiency at a certain power, the compression/expansion process must take place nearly isothermally, which means that sufficient heat transfer is needed to cool the air during compression or to heat the air during expansion. In fact, isothermal compression allows air to reach high pressure without reaching a high temperature, which means a better thermal efficiency. Inadequate heat transfer during compression causes the air temperature to be raised. This increase of internal energy will likely be lost later during storage, resulting in inefficiency. However, a good heat transfer capability during air compression will reduce the temperature rise, which, in turn, improves the compression efficiency. To illustrate the demand for heat transfer, consider the storage/regeneration

\(^1\)It means good power density in order to meet power requirement with a relatively small size compressor/expander.
power, “Power” and the heat transfer requirement “Heat” of the compressor/expander as functions of the frequency of operation “Ω” and the maximum air volume $V_{disp}$ (i.e. displacements) of the compression/expansion chamber. Based on the first law of thermodynamics, in order to achieve a near isothermal compression/expansion process, we need to have:

$$dQ \simeq -PdV$$

where $dQ$ is the differential heat transfer from the air to the environment and $-PdV$ is the differential piston work performed on the air under compression ($P$ and $V$ are air pressure and volume, respectively). Assuming that the compression/expansion takes place nearly isothermal (i.e. $P.V \simeq c$ where $c$ is a constant), the compression/expansion power and the required heat transfer rate can be calculated as (see Fig. 2.1):

$$Power = \Omega \times P_{atm} \left(\ln\left(\frac{P_{max}}{P_{min}}\right) - \left(1 - \frac{P_{min}}{P_{max}}\right)\right) V_{disp}$$

$$Heat = \Omega \times P_{atm} \ln\left(\frac{P_{max}}{P_{min}}\right) V_{disp}$$

Thus, a 3MW turbine operating at 1Hz and with $P_{max} = 35$MPa would require $\sim 6.1$m$^3$ of air displacement per cycle and the system must transfer 0.57MW of heat per m$^3$ of air volume. If the frequency increases, the needed displacement decreases proportionately,
but the heat transfer requirement per unit volume increases proportionately. If a heat transfer coefficient of $h=100 \text{ W/K.m}^2$ and a 50K temperature difference to maintain high efficiency are assumed, a surface area to volume ratio of $120\text{m}^{-1}$ would be required. This demands that the feature size (e.g. radius of cylinders or height between 2 adjacent plates) of the compressor/expansion chamber needs to be of sufficiently small scale (estimated to be about 1cm). Because of the significant heat transfer requirements, typical compressors/expanders use a small number of essentially adiabatic stages for compression/expansion and heat exchangers between stages to keep the air at reasonable temperature \cite{37}. This however reduces the efficiency of the device.

A liquid piston concept is one approach to improve the efficiency of gas compression and expansion \cite{30,39,50}. Because a liquid can conform to an irregular chamber volume, the surface area to volume ratio in the gas chamber can be maximized using a liquid piston and solid insert material added into the chamber. The porous insert can have a regular geometry (such as honeycomb structure of tiny tubes) or irregular geometry (such as metal foam)(Fig. 2.2). In both cases, the compression/expansion process will be near-isothermal, which maximizes the process thermal efficiency by minimizing the air temperature rise/drop during compression/expansion \cite{38,39,40}. Other advantages of this concept are:

**Improved sealing:** One concern with high pressure air compressor/expander is leakage. Since air viscosity is low and leakage is proportional to pressure difference, a solid piston based design would require a very small clearance that increases friction. In the liquid piston design, the liquid column will form an excellent seal for the compressed air, thus increasing volumetric efficiency.

**Infinitely variable compression/expansion rate versus time:** The compression/expansion profile affects greatly the efficiency and power trade-off. An optimal profile can potentially increase the power of a compressor/expander by 500% without sacrificing efficiency \cite{41}. Since the advance of the liquid piston is governed by the liquid flow rate and controlled by a pump/motor. This offers a very flexible approach to adapt and vary the profile as needed.

**Liquid as heat source/sink and liquid exchange:** The liquid, with its large thermal capacitance (4.2kJ/kg/K for water) in the liquid piston can be used as a heat sink or heat source during compression and expansion. If water is used to absorb all
the heat generated by compressing air from 1bar to 350bar, the temperature of water (i.e. liquid piston) will vary by about +0.13K for each compression cycle (assuming the compression heat is uniformly distributed in the water column which has the same size as the air volume at 1bar).

![Diagram of liquid piston air compressor/expanded chamber filled with porous material]

Figure 2.2: Liquid piston air compressor/expanded chamber filled with porous material

2.2 Thermal Efficiency versus Power Density

Due to limited heat transfer between air under compression/expansion and its surrounding environment, there is a trade-off between efficiency and power [41]. For compression, it is desired to compress a certain amount of air from the initial pressure $P_0$ and temperature $T_0$ to the final pressure ratio $r$ ($P_f = rP_0$) in a given compression time $t_f$. For expansion, it is desired to expand a certain amount of air from the initial pressure $rP_0$ and temperature $T_0$ to the final pressure ($P_f = P_0$) in a given expansion time $t_f$. These two processes are shown on Fig. 2.3. Mathematically, the total required work for
compression (subtracting the work done by the atmosphere) can be calculated as:

\[
W_{\text{in}} = -\int_{V_0}^{V_c} (P - P_0) dV + P_0(r - 1)V_c + \int_{0}^{t_f} W_f dt
\]  

(2.4)

the first term on the right-hand side of (2.4) is the required compression work to compress air from the initial pressure \( P_0 \) to the final desired pressure \( rP_0 \) (see Fig. 2.3). The second term in (2.4) is the summation of isobaric cooling work associated with the air temperature drops back to ambient level (occurs after the compression process) and the ejection work needed to push the compressed air out of the chamber (i.e. to storage vessel). The very last term in (2.4) includes all the losses due to mechanical and viscous friction that are summarized as friction power \( W_f \).

The stored energy \( E_{\text{st}} \) in the air after compressing to \( rP_0 \) and cooling down to ambient temperature \( T_0 \) in the storage vessel is defined as the maximum work obtainable via isothermal expansion as:

\[
E_{\text{st}} = P_0 V_0 ln(r)
\]  

(2.5)

where \( V_0 \) is the air volume after compressing to \( rP_0 \). Note that (2.5) is derived based on
ideal gas assumption. Moreover, the ejection work\(^2\) is included in (2.5) while the work done by the atmosphere is subtracted. The compression efficiency and power density are then defined as:

\[
\eta_c = \frac{E_{st}}{W_{in}} \times 100\% \quad (2.6)
\]

\[
PD_c = \frac{E_{st}}{V_{chamber}t_f^c} = \frac{P_0V_0ln(r)}{V_{chamber}t_f^c} \quad (2.7)
\]

where \(V_{chamber}\) is the compression chamber volume and \(t_f^c\) is the compression time. The same approach can be used for the expansion process. The expansion efficiency and power density are defined as:

\[
\eta_e = \frac{W_{out}}{E_{st}} \times 100\% \quad (2.8)
\]

\[
PD_e = \frac{E_{st}}{V_{chamber}t_e^e} = \frac{P_0V_0ln(r)}{V_{chamber}t_e^e} \quad (2.9)
\]

where \(t_e^e\) is the expansion time and \(W_{out}\) is (see Fig. 2.3):

\[
W_{out} = \int_{V_s}^{V_e} (P - P_0) dV + P_0(r - 1)V_s - \int_0^{t_f^e} F_f dt \quad (2.10)
\]

Compression/expansion time (and therefore power) can be found from the first law of thermodynamics:

\[
\Delta W = \Delta Q + \Delta E \quad (2.11)
\]

where \(W\) is the piston work, \(Q\) is the heat transfer from air to the walls, and \(E\) is the internal energy of air. Using real gas properties, specific internal energy and temperature of air can be found as a function of air pressure and density:

\[
e = e(P, \rho)
\]

\[
T = T(P, \rho)
\]

assuming that no leakage occurs during the compression process (i.e. \(m\) is constant) and using the differential form of (2.11), we have:

\[
-PdV = Qdt + m(\frac{\partial e}{\partial P}dP + \frac{\partial e}{\partial \rho}d\rho) \quad (2.12)
\]

\(^2\)Work needed to push the compressed air out of compression chamber into the storage vessel.
using \( Vd\rho = -\rho dV \) (since air mass is constant) and performing some mathematical manipulations, we find the total compression time as:

\[
t_c^f = \int_{V_0}^{V_c} \frac{m^2}{Y^2} \frac{\partial P}{\partial \rho} dV - \int_{P_0}^{P_f} \frac{m \partial P}{Q(t)} dP
\]  

(2.13)

where \( P, T, V, m \) and \( e \) are air lumped pressure, temperature, volume, mass and specific energy, while \( Q(t) \) is the heat transfer from air to the chamber’s wall. Using Newton’s law of cooling, \( Q \) can be expressed as:

\[
Q(t) = hA(T - T_{wall})
\]  

(2.14)

where \( h \) is the convective heat transfer coefficient and \( A \) is the total heat transfer area between air and the chamber’s wall. Generally, \( h \) is a function of air properties as well as porous material geometry \([31, 42, 43, 44]\). Therefore, if \( h \) and \( A \) correlations are known, compression time can be found from (2.13). Storage power is defined as:

\[
Power_{st} = \frac{E_{st}}{t_c^f} = \frac{P_0V_0ln(r)}{t_c^f}
\]  

(2.15)

Therefore, a faster compression results in higher storage power, which generally has a poorer thermal efficiency due to limited heat transfer. In order to investigate the effect of chamber and porous material geometry for further improvement of the efficiency versus power trade-off, a computationally-efficient thermodynamic model for air is needed that allows rapid iterations for system optimization. While a zero-dimensional (i.e. lumped properties) is used for compression/expansion trajectory optimization (chapter 4), a 1-dimensional model is developed here to study and optimize the chamber geometry. This 1-dimensional model is introduced in the next section.

### 2.3 One-Dimensional Thermodynamic Model for Air

A schematic of the liquid piston air compressor/expander is shown in Fig. 2.4. Here, we want to develop a model that describes the thermodynamic interaction between air, porous material, chamber walls and liquid piston during the compression/expansion process. In general, air compression/expansion inside a cylindrical chamber is a three-dimensional process. Due to axisymmetric geometry of the chamber, its behavior can
be studied using a two-dimensional model [45][46]. Although using a two-dimensional model makes it possible to capture all small details of the compression/expansion process, it will not be useful for studying the effect of design parameters and their optimization to find the best design. Running a two-dimensional model is relatively slow and cannot be used in an iterative procedure for optimization purposes. On the other hand, a zero-dimensional model (i.e. lumped air properties) is not effective for studying the influence of porous material location and chamber geometry. Therefore, we developed a one-dimensional thermodynamic model for air which has enough accuracy for investigating the geometric effects of compression chamber and heat transfer material. Moreover, a uniform pressure distribution in the chamber is assumed that eliminates the air velocity dynamics (i.e. the air velocity can be calculated by solving a system of algebraic equations). This assumption reduces computation time by an order of magnitude and makes the developed one-dimensional model more useful for optimization processes that involve multiple iterations.

![Figure 2.4: Schematic of the compression chamber with porous material used for one-dimensional modeling](image)

In this model, the compression chamber is divided into “n” cells (Fig. 2.5). Cell location in the chamber is described with index \(i\), where \(i = 1\) is the top cell while \(i = n\) is the bottom cell above the liquid surface. The origin of coordinate system used for this modeling is located at the top of chamber, while its positive direction is toward the
bottom. According to the real gas model, specific energy and temperature of air are functions of air pressure and density [47]:

\[ e = e(P, \rho) \] (2.16)

\[ T = T(P, \rho) \] (2.17)

To fully describe the air state in each cell, three air properties are needed: Internal energy \( E_i \), density \( \rho_i \) and cell volume \( w_i \). Air pressure can be found based on specific energy (i.e. \( e = E/m \)) and density of air using (2.16). Moreover, air temperature can be calculated by passing air pressure and density through (2.17). Conservation of mass and energy are used to find the dynamics of energy and density in each cell.

Figure 2.5: Discretization of air and solid media in the compression chamber

A uniform pressure is assumed inside the chamber. The air flow rate \( q \) at different locations is computed. Therefore, instead of applying conservation of momentum to derive air velocity dynamics, air velocity on the cells’ boundaries are directly calculated based on a system of algebraic equations that satisfy the uniform pressure constraint (will be discussed later in this chapter). Note that this is a valid assumption since the distribution of pressure inside the chamber (i.e. pressure dynamics) is orders of
magnitude faster responding than air density and temperature. The cells’ boundary locations \((Z_i)\) are not fixed in space and move with the air. Hence, as long as the inlet and outlet poppet valves are closed, the air mass in each cell remains constant. Similarly, a one-dimensional model is used for porous media inside the chamber (for evaluation of energy \(E^S\) and mass \(m^S\) in each solid cell). Note that all geometric properties, such as porosity (defined with \(\phi\)) and cross section area of the chamber (defined with \(\psi\)), at any point in the chamber are described as functions of air volume above that point (i.e. air volume between the chamber’s top and the point) which is described as \(v\) (see Fig. 2.5). Since the cell boundaries move with the same velocity as air, we have:

\[
\dot{Z}(v_i) = \frac{q(v_i)}{\phi(v_i)\psi(v_i)}
\]  

(2.18)

where \(v_i\) is the air volume between the top of the chamber and the \(i^{th}\) cell’s boundary, and \(q(v_i)\) is the air flow rate at volume \(v_i\) (see Fig. 2.5). This cumulative air volume can be related to the cell’s individual volume \(w_i\) as:

\[
v_i = \sum_{j=1}^{i} w_j
\]  

(2.19)

note that \(v_n = V(t)\) at any time during the compression/expansion process, where \(V(t)\) is the total air volume at time \(t\). Applying the conservation of mass for the \(i^{th}\) cell, we have:

\[
\rho_i = \frac{m_i}{w_i} \implies \dot{\rho}_i = \frac{\dot{m}_i}{w_i} - \frac{\rho_i}{w_i} \dot{w}_i
\]  

(2.20)

where \(\dot{w}_i\) is the rate of change of volume in the \(i^{th}\) cell and can be written in terms of air flow rate as:

\[
\dot{w}_i = q_i - q_{i-1}
\]  

(2.21)

since the cell boundaries move with the air, the mass in each cell stays constant unless additional (air) mass is added/subtracted via the low pressure (LP) or high pressure (HP) valves (see Fig. 2.5). In order to somewhat include the mixing of air (between cells) during the intake and exhaust phases (i.e. when the low and/or high pressure poppet valves are open), it is assumed that the air passing through the valves distributes among all cells (in the air medium) linearly to their instantaneous mass, as:

\[
\dot{m}_i = \frac{m_i}{\sum_{j=1}^{n} m_j} (\dot{m}_{LP} + \dot{m}_{HP})
\]  

(2.22)
where \( \sum_{j=1}^{n} m_j \) is the total mass of air in the chamber at any time. Specific internal energy of each cell can be calculated as:

\[
E_i = m_i e_{(P_i, \rho_i)}
\]

(2.23)

By applying the conservation of energy principle, the internal energy dynamics in each cell can be calculated as:

\[
\dot{E}_i = -P_i (q_i - q_{i-1}) - \tau_i - \sigma_i + \dot{E}_{i\text{valve}}
\]

(2.24)

the first term on the right hand side of (2.24) is the pressure power (due to cell boundaries movement) while the other terms are convective heat transfer between air and porous medium, conductive heat transfer in the air medium, and rate of energy added/subtracted to the cell through mass transfer via the poppet valves, respectively. These terms can be calculated as:

\[
\tau_i = \overline{hA}_i \times (T_i - T^{S}_i)
\]

(2.25)

\[
\sigma_i = 2k^{i+1} \phi_{(v_i)} v^{i+1}_i \Delta Z_i^{i+1} - 2k^{i-1} \phi_{(v_{i-1})} v^{i-1}_i \Delta Z_{i-1}^{i-1}
\]

(2.26)

\[
\dot{E}_{i\text{valve}} = \frac{m_i}{\sum_{j=1}^{n} m_j} \left[ m_{LP} \left( h_{LPS} (m_{LP}) + h_{(P_i, \rho_i)} S_{(m_{LP})} \right) 
\right.
\]

\[
+ m_{HP} \left( h_{HPS} (m_{HP}) + h_{(P_i, \rho_i)} S_{(m_{HP})} \right) 
\]

(2.27)

where \( \tau_i \) is the convective heat transfer from air to solid, \( \sigma_i \) is the conductive heat transfer in the air medium, and \( \dot{E}_{i\text{valve}} \) is rate of energy transfer by mass transfer across the valves. In (2.25), \( \overline{hA}_i \) is the product of heat transfer coefficient and heat transfer area between air and porous material, integrated over volume of the \( i^{th} \) cell. The convective heat transfer coefficient is calculated based on the correlation that is found for the specific porous material geometry used in this work, given by [48]:

\[
h_{(T, \rho, v, q)} = \frac{k(T)}{D^h} (c_1 + c_2 \cdot Re^{c_3} \cdot Pr^{c_4})
\]

(2.28)

where \( k \) is the conductivity of air (function of temperature), \( D^h \) is the hydraulic diameter at volume \( v \), \( Re \) is the Reynolds number and \( Pr \) is the Prandtl number. \( c_1, c_2, c_3 \) and \( c_4 \) are constant parameters (can be found in [48]). Defining \( \Lambda(v) \) as the heat transfer area
density (per air volume) at volume $v$, the total $hA$ over the $i^{th}$ cell can be calculated as:

$$hA_i = \int_{v_{i-1}}^{v_i} h_{(T,\rho,\vartheta)} d\nu = f_{(v_{i-1},v_i,T_i,\rho_i)} + g_{(v_{i-1},v_i,T_i,\rho_i)}u_i^m$$

(2.29)

where $f$ and $g$ are explicit functions of air temperature, density and cumulative volume ($v_{i-1}$ and $v_i$), and $c_3$ is the exponent for $Re$ in (2.28). Moreover, $u$ is the Darcian average air velocity in the $i^{th}$ cell calculated as:

$$u_i = \frac{q_{i-1} + q_i}{2} \cdot \frac{1}{\psi\left(\frac{v_{i-1} + v_i}{2}\right)}$$

(2.30)

In (2.26), $k_{i+1}^i$ is the air conductivity on the border between cell $i$ and $i+1$ (as a function of border temperature which is the average temperature of two adjacent cells) and $\Delta Z_i$ is the distance between the center of cell $i$ and $i+1$ (equal to $0.5(Z_{i+1} - Z_{i-1})$). Finally, in (2.27), $h_{LP}$, $h_{HP}$ and $h_{i(P,\rho)}$ are the enthalpies of air in the low pressure tank, the high pressure tank and the air in the $i^{th}$ cell at pressure $P_i$ and density $\rho_i$, respectively. In order to take care of flow direction, a simple direction function is defined (and used in (2.27)) as:

$$S(x) = \begin{cases} 
1 & : x > 0 \\
0 & : x \leq 0 
\end{cases}$$

Similarly, the conservation of energy principle is used to find the energy dynamics in the porous medium. The total energy of each cell in the porous medium can be written as:

$$E_i^S = c_s m_i^S T_i^S$$

(2.31)

where $m_i^S$ and $T_i^S$ are the mass of the porous material and its temperature in the $i^{th}$ cell, respectively. In addition, $c_s$ is the heat capacity of the solid material. The change of energy in the $i^{th}$ cell in the porous medium is due to heat convection between air and solid, heat conduction in the solid, and energy transfer (by mass transfer) due to boundaries movement:

$$\dot{E}_i^S = \tau_i + \epsilon_i + \zeta_i$$

(2.32)

where $\tau_i$ is the convection between air and solid material in the $i^{th}$ cell described by (2.25), $\epsilon_i$ is the rate of energy transfer by boundary movements, and $\zeta_i$ is the conduction
in the solid medium, described as:

\[ \epsilon_i = -cs\rho_s q_i(1 - \frac{1}{\phi(v_i)}) - 1) \left( T^S_i S(q_i) + T^S_{i-1} S(-q_i) \right) \]

+ \[ cs\rho_s q_i(1 - \frac{1}{\phi(v_i)}) - 1) \left( T^S_{i+1} S(q_i) + T^S_i S(-q_i) \right) \]

\[ \zeta_i = 2ks \left( \psi(v_i-1)(T^S_i - T^S_{i-1}) \right) - \psi(v_i)(1 - \phi(v_i))(T^S_i - T^S_{i+1}) \]

(2.33)

(2.34)

where \( \rho_s \) and \( k_s \) are density and heat conductivity of the solid material, and \( S \) is the direction function defined earlier. Therefore, the energy dynamics of each cell in the solid medium can be derived as:

\[ \dot{E}^S_i = \tau_i - cs\rho_s q_i(1 - \frac{1}{\phi(v_i)}) - 1) \left( T^S_i S(q_i) + T^S_{i-1} S(-q_i) \right) \]

+ \[ cs\rho_s q_i(1 - \frac{1}{\phi(v_i)}) - 1) \left( T^S_{i+1} S(q_i) + T^S_i S(-q_i) \right) \]

+ \[ 2ks \left( \psi(v_i-1)(T^S_i - T^S_{i-1}) \right) - \psi(v_i)(1 - \phi(v_i))(T^S_i - T^S_{i+1}) \]

(2.35)

since the cell boundaries inside the porous medium move, mass in each cell will not stay constant. Therefore, mass dynamics of each cell in the porous medium must be also considered, which can be found as:

\[ \dot{m}^S_i = \rho_s \left( q_i(1 - \frac{1}{\phi(v_i)}) - 1) - q_i-1(1 - \frac{1}{\phi(v_{i-1})}) - 1) \right) \]

(2.36)

Once energy and mass of each cell in the solid medium is calculated using the dynamics given by (2.35) and (2.36), the solid cell temperature can be found based on (2.31). It should be emphasized that while the air section is divided into \( n \) cells, the solid media is divided into \( n + 1 \) cells, from which \( n \) cells are exposed to air and a bottom cell that is covered by the liquid piston. During the compression process, the generated heat is transferred to the porous material via convection. This causes a nonuniform temperature distribution in the solid medium, and generates a conductive heat transfer which eventually reaches the bottom cell in the solid medium. This cell is always covered with liquid piston so heat is transferred to the liquid column and leaves the chamber with it later during the intake phase (see Fig. 2.6). A similar heat transfer mechanism exists
during the air expansion but in the opposite direction. The internal energy dynamics of the last cell \((n + 1)\) in the solid medium can be found as:

\[
\dot{E}_{S_{n+1}} = -\tau_{s2w} + 2k_s \psi(v_i)(1 - \phi(v_i)) \frac{T_{n}^S - T_{n+1}^S}{Z_{n+1} - Z_{n-1}} - c_s \rho_s q_n \left( \frac{1}{\phi(v_n)} - 1 \right) \left( T_{n+1}^S S(q_n) + T_n^S S(-q_n) \right)
\] (2.37)

where \(\tau_{s2w}\) is the convective heat transfer from the solid material (in cell \(n + 1\)) to the water column calculated as:

\[
\tau_{s2w} = h A_{n+1} \times (T_{n+1}^S - T^w)
\] (2.38)

where \(hA_{n+1}\) is the average product of heat transfer coefficient and heat transfer area over the bottom cell in the solid medium, and \(T^w\) is the bulk temperature of the water column. Mass dynamics of the last cell in the solid medium can be found as:

\[
\dot{m}_{S_{n+1}} = -\rho_s q_n \left( \frac{1}{\phi(v_n)} - 1 \right)
\] (2.39)

Figure 2.6: Heat flow during air compression (left) and expansion (right)

The last part of dynamic modeling involves the liquid column temperature and mass dynamics. Note that the liquid medium properties (i.e. liquid piston) are considered to
be lumped. Denoting liquid temperature and mass by $T^w$ and $m^w$, the internal energy of the liquid $E^w$ column can be found as:

$$E^w = c_w m^w T^w$$  \hspace{1cm} (2.40)

where $c_w$ is the heat capacity of liquid. As described earlier, the change in internal energy of the liquid column is due to heat transfer between liquid and the bottom cell in the solid medium, as well as energy transferred by the liquid mass into/out of the chamber. Therefore, the mass and energy dynamics of the liquid column can be derived as:

$$\dot{m}^w = \rho^w F(t)$$  \hspace{1cm} (2.41)

$$\dot{E}^w = \tau_{s2w} + \rho^w F(t) \left( c_w T_{amb} S_{(t)} + c_w T^w S_{(-F(t))} \right)$$  \hspace{1cm} (2.42)

where $\tau_{s2w}$ is the heat transfer from solid bottom cell to the liquid column given by (2.38), $T_{amb}$ is the ambient temperature and $F(t)$ is the liquid piston flow rate (positive if liquid enters the chamber, negative if liquid leaves the chamber). Note that it is assumed liquid enters the chamber at ambient temperature $T_{amb}$, while it leaves the chamber at temperature $T^w$. Liquid compressibility is also considered in this model, as liquid density $\rho^w$ is a function of chamber pressure given by:

$$\rho^w = \rho_0^w \exp \left( \frac{(P^w - P_{amb})}{\beta} \right)$$  \hspace{1cm} (2.43)

where $\rho_0^w$ is liquid density at ambient pressure $P_{amb}$ and $\beta$ is the bulk modulus of the liquid.

As mentioned earlier in this section, a uniform pressure distribution is assumed for air inside the chamber (i.e. $P_i = P_j$ for $(i,j) \in [0,n]$). Therefore, the rate of change of pressure in all cells must be the same. By taking the time derivative of (2.23) and substituting the appropriate terms from (2.24), we will have:

$$\dot{P}_i = \frac{\rho_i^2 e_i - P_i}{m_i e_p} \left( q_i - q_{i-1} \right) - \frac{\tau_i + \dot{E}^\text{valve}_i + (e_i + \rho_i e_p) \dot{m}_i}{m_i e_p}$$  \hspace{1cm} (2.44)
where \( e_P^i \) and \( e_\rho^i \) are partial differentials of specific energy with respects to pressure and density, defined as:

\[
e_P^i = \left. \frac{\partial e}{\partial P} \right|_{P_i, \rho_i} \quad (2.45)
\]

\[
e_\rho^i = \left. \frac{\partial e}{\partial \rho} \right|_{P_i, \rho_i} \quad (2.46)
\]

Hence, the constraint on the cell’s pressure distribution requires that:

\[
\dot{P}_{i-1} = \dot{P}_i \implies L_{i-1}(q_{i-1} - q_{i-2}) - \alpha_{i-1} = L_i(q_i - q_{i-1}) - \alpha_i \quad (2.47)
\]

This algebraic equation (between \( q_i \)’s) must hold for all the cells in the air medium. Therefore:

\[
(L_1 + L_2)q_1 - L_2q_2 = \alpha_1 - \alpha_2 \quad (2.48)
\]

\[
-L_{i-1}q_{i-2} + (L_{i-1} + L_i)q_{i-1} - L_iq_i = \alpha_{i-1} - \alpha_i \quad (i = 3, 4, ..., n) \quad (2.49)
\]

(2.48) and (2.49) define \( n - 1 \) algebraic constraints between \( q_i \)’s where \( i = 1, 2, ..., n \). In order to find a unique solution for the \( q \) vector, we need one more algebraic equation. Note that the rate of pressure change in the liquid column must be equal to that of the air. By taking the time derivative of (2.43), we have:

\[
\dot{\rho}^w = \frac{\rho^w}{\beta} \dot{\hat{P}}^w \quad (2.50)
\]

if \( V^w \) is the liquid column volume, using (2.41) and (2.50) we have:

\[
\frac{d}{dt}(m^w) = \frac{d}{dt}(\rho^w V^w) \implies \dot{\hat{P}}^w = \frac{\beta}{V^w} (F(t) + q_n) \quad (2.51)
\]

where \( -\dot{\hat{V}}^w \) is replaced by \( q_n \) (see Fig. 2.5). The last algebraic constraint can be derived as:

\[
\dot{P}_n = \dot{\hat{P}}^w \implies \left( L_n - \frac{\beta}{V^w} \right) q_n - L_nq_{n-1} = \frac{\beta}{V^w} F(t) + \alpha_n \quad (2.52)
\]
Combining (2.48), (2.49) and (2.52), the system of algebraic equations become:

\[
\begin{bmatrix}
L_1 & -L_2 & 0 & 0 \\
0 & L_2 & -L_3 & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & L_{n-1} - L_n \\
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_n \\
\end{bmatrix}
\times
\begin{bmatrix}
1 & 0 \\
-1 & 1 & 0 \\
0 & -1 & 1 & \ddots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & \cdots & \cdots & 1 \\
\end{bmatrix}
\begin{bmatrix}
\alpha_1 - \alpha_2 \\
\alpha_2 - \alpha_3 \\
\vdots \\
\alpha_{n-1} - \alpha_n \\
\end{bmatrix}
\]

where \(\Omega\) is a constant \(n\) by \(n\) matrix defined as:

\[
\begin{bmatrix}
1 & 0 \\
-1 & 1 & 0 \\
0 & -1 & 1 & \ddots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & \cdots & \cdots & 1 \\
\end{bmatrix}
\]

Therefore, the system of equations defined by (2.53) can be solved at each time step to find air flows \((q_i)\) over the cell’s boundaries. It should be mentioned that while evaluation of \(L_i\) is independent of air flow rate, evaluation of \(\alpha_i\) requires air flow to be known (see (2.25) and (2.28)). In this way, (2.53) does not have an explicit solution and therefore needs to be solved iteratively, which makes the simulation very slow and complicated. However, since the dependency of heat transfer coefficient to air flow is not very strong and the air flow will not change abruptly from one time step to the next one, we can use the air flow rate from the previous time step to evaluate \(\alpha_i\) in (2.53) and find the air flows at the new time step. Using this approach has minimal error while it makes the computational process very fast.
2.4 Sample Numerical Simulation of the One-Dimensional Chamber Model

A sample simulation for air compression has been performed here to show how the one-dimensional air-solid (in addition with the zero-dimensional water) model works. A cylinder with a constant cross sectional area ($31.75\text{cm}^2$) is assumed for the compression chamber. Somewhere in the upper portion of the chamber is filled with porous material while the rest of the chamber is empty (see Fig. 2.7). The porous material, for the present analysis, is in the form of parallel plates (with 1.8mm spacing distance) made from ABS plastic [48]. The chamber is connected to a variable-displacement pump that can provide any desired flow rate ($F(t)$) within its capabilities according to the air compression requirement. The rest of the parameters used for this simulation can be found in Table 2.1. Note that forty cells ($n=40$) are used for discretizing the air and solid medium (41 cells in solid medium), while the liquid piston properties (such as temperature and density) are assumed to be lumped. The air compression from 7bar to 200bar is numerically simulated using the dynamic model developed in section 3.

Figure 2.7: Porosity distribution and cross section area of the chamber used for sample numerical simulation. The compression flow rate is a sinusoidal trajectory.
Table 2.1: Fixed parameters used in 1D simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Air Volume</td>
<td>1.5</td>
<td>lit</td>
<td>Initial Air Temperature</td>
<td>298</td>
<td>K</td>
</tr>
<tr>
<td>Initial Water Volume</td>
<td>10</td>
<td>cc</td>
<td>Initial Solid Temperature</td>
<td>298</td>
<td>K</td>
</tr>
<tr>
<td>Porous Material Volume</td>
<td>140</td>
<td>cc</td>
<td>Initial Water Temperature</td>
<td>298</td>
<td>K</td>
</tr>
<tr>
<td>Total Chamber Volume</td>
<td>1.65</td>
<td>lit</td>
<td>Bulk Modulus of Water</td>
<td>2.1</td>
<td>GPa</td>
</tr>
<tr>
<td>Initial Air Pressure</td>
<td>7</td>
<td>bar</td>
<td>Thermal Conductivity of Water</td>
<td>0.58</td>
<td>W/m.K</td>
</tr>
<tr>
<td>Final Air Pressure</td>
<td>200</td>
<td>bar</td>
<td>Heat Capacity of Water</td>
<td>4180</td>
<td>J/Kg.K</td>
</tr>
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<td>Heat Capacity of Solid</td>
<td>1600</td>
<td>J/Kg.K</td>
<td>Thermal Conductivity of Solid</td>
<td>0.2</td>
<td>W/m.K</td>
</tr>
<tr>
<td>Prandtl Number of Air</td>
<td>0.7</td>
<td>-</td>
<td>Ambient Temperature</td>
<td>298</td>
<td>K</td>
</tr>
</tbody>
</table>

The compression flow rate and porosity distribution in the chamber are shown in Fig. 2.7. Results are shown in Figs. 2.8 and 2.9. As shown, air temperature drops when it passes through the low porosity region which is filled with porous material. This is the source of kinks in the air temperature-time history, as shown in Fig. 2.8 top-left. The air temperature starts going up again once it passes the region with porous material, and eventually reaches to more than 700K very close to the top cap (as shown in Fig. 2.9 top-right). While the air temperature goes up by more than 400K, the solid material heats up by less than 40K due to high heat capacity of the solid. In particular, note that the temperature of last cell in solid (i.e. \( i = 41 \)) starts dropping once the liquid piston reaches the low-porosity region (around \( t = 1 \text{sec} \)). Due to very high heat capacity of liquid (here water is used as liquid), the temperature change at the end of the compression process is less than 0.2K. The bulk air temperature and pressure versus air volume are shown in Fig. 2.9. The thermal efficiency of this sample compression process is about 70%, according to (2.6).

2.5 Summary

The necessity of good heat transfer to achieve a high performance in an air compression/expansion chamber was discussed in this chapter. While there are different methods and approaches to achieve this goal, the liquid piston air compressor/expander is one way to increase heat transfer rate for air under compression/expansion. By inserting porous media into the compression/expansion chamber, the available heat transfer area can be increased by an order of magnitude. This will improve the rate of heat transfer.
Figure 2.8: Results of the one-dimensional compression study. Top: air temperature in different cells (left) and different locations (right) in the chamber; Bottom: solid temperature in different cells (left) and water column (bulk) temperature (right) from air to solid (porous) medium during the compression phase, and from porous material to air during the expansion phase, which will keep the air from getting too hot/cold during each phase. This can significantly increase the thermal efficiency for a given compression/expansion time (i.e. compression/expansion power). This concept also has the benefit of good sealing, and has the potential of varying compression/expansion rate, which can be used as another approach to further increase the performance (will be discussed in detail in chapters three, four and five). Finally, a one-dimensional air compression/expansion dynamic model was developed in this chapter. While it captures
Figure 2.9: Results of the one-dimensional compression study. Air (bulk) temperature and pressure versus air volume during the compression process. Thermal efficiency of this compression is about 70%.

the effect of air property distribution (such as temperate and density) inside the chamber during the compression/expansion processes, it can be run fast enough to be used for optimization purposes, such as optimizing porous material distribution and chamber shape. These concepts are introduced and discussed in detail in the next chapter.
Chapter 3

Optimal Design of the Liquid Piston Air Compressor/expander

3.1 Introduction

As mentioned in chapter 2 (section 2.1), a critical component of this system is the high pressure (200 bar) air compressor/expander, which is responsible for the conversion between mechanical work and compressed air in storage. To be useful, the compressor/expander must be efficient and power dense. For a fixed compression/expansion ratio, increasing power density means that the compressor/expander will operate in shorter cycle time. A smaller component can therefore be used to achieve the same power capability, reducing capital expense (CAPEX). There is a natural tradeoff between efficiency and power density as the thermodynamic efficiency of air compression/expansion is highly dependent on heat transfer. When operating the compressor/expander slowly, there is more time for heat transfer to take place and efficiency increases. This is at the expense of reducing power density, and a large compressor/expander will be needed. The reverse is true when operating the compressor/expander quickly. Power density increases at the expense of efficiency since time for heat transfer becomes limited. In our approach, a liquid piston air compressor/expander with a porous medium insert is used. Here a liquid column is used to compress or expand the air above it. The interface between the liquid and air column stays stable if the maximum acceleration of the liquid (upward or downward) remains lower than a certain value [49]. Since liquid can
flow through tortuous path, adding porous media augments heat transfer through the increase in surface area and heat capacitance \cite{30, 62}. It has been shown through CFD \cite{45} and experiments \cite{39, 50} that introduction of porous media can increase the power density over the case with no porous media by more than an order of magnitude without sacrificing efficiency. Yet another approach to improving the efficiency-power density tradeoff is to optimize the compression and expansion trajectories. It has been shown that optimized trajectories can also increase power density by an order of magnitude over ad-hoc linear/sinusoidal trajectories \cite{41, 51, 52}.

The most important factor in calculating the optimal compression/expansion trajectory is the estimation of heat transfer between air and heat transfer materials inside the compression/expansion chamber. In \cite{41}, it is assumed that the product of heat transfer coefficient and heat transfer area is constant during the compression process. A more realistic assumption is made in \cite{51} where the variation of available heat transfer area during the compression/expansion process is considered, while the heat transfer coefficient is still assumed to be constant. In \cite{52}, a heat transfer correlation is considered for the convective heat transfer coefficient which is a function of instantaneous air properties in the chamber as well as chamber geometry. Moreover, practical hardware constraints (such as maximum compression/expansion rate) are also included in the calculation of optimal compression/expansion trajectories. The optimization problem is then solved by general nonlinear programming tools where there is a nonlinear cost and a nonlinear equality constraint. This method is relatively slow, and is very sensitive to the initial guess used for the optimal compression trajectory.

In this chapter, we consider a complex heat transfer correlation achieved from numerical simulations, as well as hardware constraints to find the optimal compression trajectory. In contrast to \cite{52}, the optimal control problem is formulated in pressure domain (instead of time domain) such that it can be solved by a combination of Dynamic Programming approach and Lagrange multiplier method. In particular, we consider the following questions and will try to address each one to maximize the performance of a compression/expansion chamber in this chapter:

1. If a total amount of porous media is to be introduced, how should it be distributed within the compression/expansion chamber?
2. How does the shape of the compression/expansion chamber affect the efficiency/power density tradeoff? What will be an optimal shape for optimizing the power density without sacrificing power density?

3. How should the compression/expansion trajectory be optimized in combination with the optimization of the porosity distribution and shape?

3.2 Thermodynamic Model

A schematic of the air compression/expansion chamber is shown in Fig. 3.1. Air is compressed/expanded by a liquid (water) piston that is driven by a hydraulic pump/motor [61]. The cross section area ($\psi$) of the compression/expansion chamber varies along its length. Heat exchanger material (porous media) in the form of parallel plates is inserted into the chamber (Fig. 3.2) [46]. While plate’s thickness is fixed (0.8mm), plate spacing can vary along the chamber axis. Therefore, the full geometry of the chamber can be realized by specifying the cross section area and plate spacing over the entire chamber. These two geometric properties at any point in the chamber are described as functions of air volume above that point (i.e. air volume between the chamber’s top cap and that point) specified by $v$ (i.e. $\psi(v)$ and $d(v)$ where $v \in [0, V_c]$, and $V_c$ is the total empty volume of the chamber). Note that the origin of the coordinate system used for this modeling is located at the chamber’s top, directed towards the bottom, as shown in Fig. 3.1. In this way, $v = 0$ at the top of chamber, while $v = V_c$ at it’s bottom. A zero-dimensional dynamic model is considered for air such that its temperature and density are always uniform in the chamber. Pressure and density are considered to be air dynamic states, while specific energy and temperature are dependent variables according to the real gas model [47]:

\begin{align*}
ed &= e_{(P, \rho)} \quad (3.1) \\
T &= T_{(P, \rho)} \quad (3.2)
\end{align*}
According to the first law of thermodynamics, the differential equation between piston work, heat transfer and change of internal energy can be written as:

$$-PdV = \overline{hA}_{(P,\rho,Q)}(T - T^S)dt + mde$$  \hspace{1cm} (3.3)

where $P$, $T$, $\rho$, $V$ and $e$ are air pressure, temperature, density, volume and specific energy, and $T^S$ is the porous medium temperature (assumed to remain constant during compression/expansion due to its large heat capacity relative to air). In addition, $\overline{hA}$ is the volume averaged heat transfer coefficient ($h$), heat transfer area ($A$) product that is evaluated over the air volume in the chamber. Assuming that no leakage occurs during
the compression/expansion process (i.e. air mass \( m \) is constant), we have:

\[
\frac{d}{dt}m = \frac{d}{dt}(\rho V) = 0 \implies \dot{V} = -\frac{m}{\rho^2} \dot{\rho} \tag{3.4}
\]

combining (3.3) and (3.4), a zero-dimensional air dynamic model can be derived as:

\[
m \frac{\partial e}{\partial P} \dot{P} = (P - \frac{\partial e}{\partial \rho} \rho^2)Q(t) - \overline{hA(P,\rho,Q)}(T - T^S) \tag{3.5}
\]

\[
\dot{\rho} = \frac{\rho^2}{m}Q(t) \tag{3.6}
\]

where \( m \) is the total mass of air and \( Q \) is the liquid piston flow rate into the chamber (negative if liquid leaves the chamber). Note that while \( P \) and \( \rho \) are dynamic states and independent, \( e \) and \( T \) are dependent variables calculated based on air pressure and density using (3.1) and (3.2). Liquid piston flow rate \( Q \) can be controlled during the compression/expansion process by varying the pump/motor displacement attached to the chamber. As mentioned earlier, due to high thermal capacity of the porous medium (relative to air), the heat exchanger temperature \( (T^S) \) is assumed to stay constant.

In order to evaluate \( \overline{hA} \), we need to first find a correlation that gives heat transfer coefficient \( (h) \) between air and the porous material. A specific correlation for Nusselt number \( (Nu) \) has been found for the heat exchanger geometry used in this work, given by [48]:

\[
Nu = \frac{hs_{(v)}}{K(T)} = c_1 + c_2 \cdot Re^{c_3} \cdot Pr^{c_4} \tag{3.7}
\]
where $h$ is the convective heat transfer coefficient, $K$ is the air conductivity as a function of its temperature and $s(v)$ is the hydraulic diameter of the porous medium ($s(v) = 2d(v)$). $c_1$, $c_2$, $c_3$ and $c_4$ are constant parameters ($c_1 = 9.7$, $c_2 = 0.0876$, $c_3 = 0.792$ and $c_4 = 0.33$). Finally, $Pr$ is the Prandtl number (equal to 0.7) and $Re$ is the Reynolds number of air at volume $v$ defined as:

$$Re = \frac{\rho s(v) |u(v)|}{\mu(T)}$$

(3.8)

The correlation given by (3.7) is valid for $Re < 5000$ according to [48]. In (3.8), $\mu$ is the dynamic viscosity of air (depends on air temperature) and $u(v)$ is the Darcian air velocity at volume $v$ given by:

$$u = \frac{v}{V(t) \psi(v)} Q(t)$$

(3.9)

where $V(t)$ is the total volume of air in the chamber at time $t$, $\psi(v)$ is the cross section area of chamber at volume $v$ and $Q(t)$ is the liquid piston flow rate at time $t$. Note that at any time $0 < v < V(t)$. It should be mentioned that Eqn. (3.9) is obtained assuming that the air flow rate at a cross section of the chamber is linear to the volume of air above the section $1$ (i.e. the air flow rate is equal to water flow rate at the water surface, while it is zero at the top cap). By combining (3.7), (3.8) and (3.9), the product of heat transfer coefficient and heat transfer area ($hA$) can be calculated as:

$$hA = \int_0^{V(t)} h(P,\rho,\mu,\nu,\Lambda(v))dv$$

(3.10)

where $\Lambda$ is the heat transfer area density ($m^2/m^3$) at volume $v$ and $h$ is the convective heat transfer coefficient calculated based on (3.7). After some mathematical manipulations, the final form of $hA$ can be found as:

$$hA = c_1 K(T) M(V) + c_2 K(T) \left( \frac{\rho}{\mu(T)} V \right) c_3 P_m c_4 Q c_3 N(V)$$

(3.11)

1 Although it is not as accurate as the method used to find air flows in the chamber in chapter 2, it is good enough for the zero-dimensional model to give a reasonable air flow distribution along the chamber.
where $M$ and $N$ are only functions of total air volume $V(t)$ and are given by:

\[
M(V) = \int_0^{V(t)} \frac{\Lambda(v)}{s(v)} dv,
\]
\[
N(V) = \int_0^{V(t)} \frac{\Lambda(v)}{s(v)} \left( \frac{u}{\psi(v)} \right)^{c_3} dv.
\]

(3.12)

By integrating (3.5) and (3.6) with respect to time, the history of the air states (pressure and density) can be evaluated for a given compression/expansion flow trajectory and initial condition inside the chamber ($P_0$ and $\rho_0$).

3.3 Compression Trajectory Optimization for a Given Chamber Geometry

For given chamber shape $\psi(v)$ and plate spacing $d(v)$ distributions, an optimal control problem can be formulated to maximize the power density by minimizing the required compression time while achieving the desired thermal efficiency. By manipulating Eqns. (3.5) and (3.6), it would be possible to omit the time differentiation and combine the equations as:

\[
me_p \frac{dP}{dV} = \frac{\bar{h}A}{Q(p)} (T_{(p,\rho)} - T^S) - (P - \rho^2 e_\rho)
\]

(3.13)

where $e_p$ and $e_\rho$ denote the partial derivative of the specific internal energy of air with respect to pressure and density. In addition, time can be considered as a dependent variable and calculated as:

\[
t(P) = -\int_{P_0}^{P} \frac{1}{Q(p)} \frac{dV}{dp} dp
\]

(3.14)

where $P_0$ is the air pressure at initial time ($P_{(t=0)} = P_0$). The cost function of the optimization problem is defined as the total time required for compressing air from initial pressure ($P_0$) to the final pressure ($P_f$):

\[
t_f = -\int_{P_0}^{P_f} \frac{1}{Q(p)} \frac{dV}{dp} dp
\]

(3.15)

Note that $V_0$ and $V_f$ are the total air volume in the chamber at pressure $P_0$ and $P_f$, respectively. Due to the limited heat transfer between air and its environment, there
is a trade off between compression time and compression efficiency. Since compression time is considered to be the cost, compression efficiency must be an equality constraint in this optimal control problem formulation. As described in chapter 2 (section 2.2), the total compression work has three parts as:\footnote{53}: i) work required to compress air from initial pressure $P_0$ to the final pressure $P_f$; ii) work required to push air out of the chamber after reaching the final desired pressure; and iii) isobaric cooling work due to temperature drop of air (back to ambient temperature) after storage in the pressure vessel (see Fig. 3.3). Note that work due to the ambient air pressure (i.e. $P_0(V_0 - V_f)$) is subtracted from the total compression work:

$$W_{compression} = -\int_{V_0}^{V_f} P dV + P_f(V_f - V_{iso}^f) + P_fV_{iso}^f - P_0V_0$$  \hspace{1cm} (3.16)$$

where $V_{iso}^f$ is the final volume of air if it undergoes an isothermal efficiency from $V_0$ and $P_0$ to the final pressure $P_f$ (see Fig. 3.3). According to the definition of compression efficiency, in order to achieve the desired efficiency we need to have:

$$-\int_{P_0}^{P_f} P \frac{dV}{dP} dP + P_fV_f - P_0V_0 = \frac{E_s}{\eta^*}$$  \hspace{1cm} (3.17)$$

where $\eta^*$ is the desired efficiency and $E_s$ is the energy stored in air after compression given by (2.5) (i.e. if the input work is equal to $E_s/\eta^*$ then the compression efficiency would be $\eta^*$). Therefore, (3.17) is an equality constraint for the optimal control problem defined earlier. In addition, an inequality constraint is needed to reflect the limitation on flow rate that can be provided by the hydraulic pump:

$$0 \leq Q(t) \leq Q_{max}$$ \hspace{1cm} (3.18)$$

The optimal control problem for a given $P_0$, $V_0$ and $P_f$ is then defined as:

$$Q^*(P) = \arg\min_{Q(t)} \left(-\int_{P_0}^{P_f} \frac{1}{Q(P)} \frac{dV}{dP} dP \right)$$ \hspace{1cm} (3.19)$$

such that (3.17) and (3.18) are satisfied. Note that the control variable is the flow rate ($Q$) which is assumed to be a function of air pressure (instead of time, since air
In the compression case, efficiency is defined as the stored energy divided by the total compression work whereas in the expansion case, efficiency is defined as the extracted work from expansion divided by the available energy in the air (that could be achieved through an ideal isothermal expansion) [53, 54].

Figure 3.3: Compression (left) and expansion (right) process shown on P-V diagrams.

Pressure is considered to be the independent variable). By introducing a new variable \( \lambda \) (Lagrange multiplier), the Lagrangian can be defined as [53]:

\[
L = -\int_{P_0}^{P_f} \left( \frac{1}{Q(P)} + \lambda P \right) \frac{dV}{dP} dP + \lambda \left( P_f V_f - P_0 V_0 - \frac{E_s}{\eta} \right) \tag{3.20}
\]

Hence, the optimal control problem can be summarized as:

\[
\left\{ \lambda^*, Q^*_f(P) \right\} = \arg \left( \max_{\lambda} \left( \min_{Q_f} (L(\lambda, Q_f)) \right) \right) \tag{3.21}
\]

such that:

\[
0 \leq Q \leq Q_{max} \tag{3.22}
\]

The Dynamic Programming (DP) method is used to solve the minimization problem and to find the liquid piston flow rate \( Q^*_f(P) \) for a given \( \lambda \), while a golden search algorithm\(^2\) is used to solve the maximization problem (for \( \lambda^* \)). Let \( P_0, P_1, \ldots, P_n \) be a

---

\(^2\)The golden section search is a technique for finding the extremum (minimum or maximum) of a strictly unimodal function by successively narrowing the range of values inside which the extremum is known to exist [57].
monotonically increasing sequence of pressures where \( P_0 \) is the initial pressure and \( P_n \) is the final pressure (\( P_n = P_f \)). In discrete volume-pressure domain (see Fig. 3.4), the compression trajectory can be realized as a series of volumes (as a function of pressure) such that \( V_i \) is the air volume when the pressure is at \( P_i \). Applying the forward difference method, \( \frac{dV}{dP} \) can be evaluated as:

\[
\left( \frac{dV}{dP} \right)_{i \rightarrow i+1} \approx \frac{V_{i+1} - V_i}{P_{i+1} - P_i} \tag{3.23}
\]

So, if \((P, V)\) at step \( i \) and \((P, V)\) at step \( i + 1 \) are known, \( Q_{i \rightarrow i+1} \) can be calculated from Eqn. (3.13). Once \( Q_{i \rightarrow i+1} \) and \( \Delta V_{i \rightarrow i+1} \) are found, the compression time from step \( i \) to step \( i + 1 \) (\( \Delta t_{i \rightarrow i+1} \)) can be easily calculated from Eqn. (3.14). Therefore, the total Lagrangian (\( L_{0 \rightarrow n} \)) can be evaluated through backward induction using Bellman equation [56]:

\[
L_{i \rightarrow n}^* = \min_{Q_\lambda} \left[ \left( \int_{P_i}^{P_{i+1}} \left( \frac{1}{Q(P)} + \lambda P \right) \frac{dV}{dP} dP + L_{i+1 \rightarrow n} \right) \right] \tag{3.24}
\]

where * shows the optimal sequence. As mentioned earlier, a golden section search algorithm is used to solve the corresponding maximization problem over \( \lambda \). Suppose

Figure 3.4: Discretization in volume-pressure domain. Optimal compression trajectory can be realized as a sequence of volumes over the pressure ([\( P_0, P_1, ..., P_{n-1}, P_n \)]). At each air pressure \( P_i \), air volume can be anything between \( V_i^1 \) (minimum possible volume at \( P_i \) corresponding to isothermal compression) and \( V_i^{m} \) (maximum possible volume at \( P_i \) corresponding to adiabatic compression).
that we know the maximum of $L(\lambda, Q)$ occurs in the interval $[\lambda_{\text{min}}, \lambda_{\text{max}}]$, where initially $\lambda_{\text{min}} = 0$ and $\lambda_{\text{max}} = \overline{\lambda}$, with $\overline{\lambda}$ a sufficiently large value. The algorithm works as follows:

1. $\lambda_1 = c\lambda_{\text{min}} + (1 - c)\lambda_{\text{max}}$ and $\lambda_2 = (1 - c)\lambda_{\text{min}} + c\lambda_{\text{max}}$

2. Evaluate $L(\lambda_1, Q)$ and $L(\lambda_2, Q)$

3. If $L(\lambda_1, Q) < L(\lambda_2, Q)$, then $[\lambda_{\text{min}}, \lambda_{\text{max}}] := [\lambda_1, \lambda_{\text{max}}]$, $\lambda_1 = \lambda_2$ and $\lambda_2 = (1 - c)\lambda_{\text{min}} + c\lambda_{\text{max}}$

4. If $L(\lambda_1, Q) > L(\lambda_2, Q)$, then $[\lambda_{\text{min}}, \lambda_{\text{max}}] := [\lambda_{\text{min}}, \lambda_2]$, $\lambda_2 = \lambda_1$ and $\lambda_1 = c\lambda_{\text{min}} + (1 - c)\lambda_{\text{max}}$

5. Go to step 2 until $|\lambda_{\text{max}} - \lambda_{\text{min}}| < \text{eps}$

where $c$ is approximately equal to 0.618.

It should be emphasized that in [52], nonlinear programming solvers are used to solve the corresponding optimization problem where both cost function and nonlinear constraint are complex integrations in time domain. This method is relatively slow and is sensitive to the initial guess for the optimal compression trajectory. However, the formulation of the same problem in pressure domain (as was described in this chapter) allows using Dynamic Programming in addition with Lagrange Multiplier method, which is much faster and doesn’t require an initial guess for the optimal solution.

A sample case study is used to illustrate how the compression trajectory is optimized to maximize the power density of a given chamber geometry and desired compression efficiency. A constant flow rate compression is also simulated which results in the same thermal efficiency but needs longer time to achieve the final desired pressure. A cylindrical chamber with a uniform cross section area ($\psi$) of 50cm$^2$ and volume of 1875cc is chosen. A total volume of 375cc of heat exchanger in the form of parallel plates is used inside the chamber. The heat exchanger is assumed to be made from ABS plastic with a density of 1200Kg/m$^3$, specific heat capacity of 1600J/Kg.K and heat conductivity of 0.2W/m.K. As mentioned in section 2, the temperature of heat exchanger (i.e. plastic parallel plates) is considered constant at ambient temperature $T^* \approx 293K$ during the entire compression cycle. The plate spacing is assumed to be 3mm (uniform throughout the chamber) with a thickness of 0.8mm. Note that the
chamber length is equal to 37.5cm based on these values. The initial air pressure is 7bar, the final pressure is 200bar and the desired compression efficiency is 85%. The optimal compression trajectory for these given parameters is shown in Fig. 3.5. The minimum compression time required to achieve the final pressure as well as the desired compression efficiency is found to be 3.66 seconds \((\lambda^* = 13.5)\). The stored energy per cycle is about 3.53kJ. Considering the size of the chamber (1.875lit), the maximum power density is 510kW/m\(^3\). It should be mentioned that in the case of a non-optimal compression trajectory, a longer compression process is required to achieve the same efficiency. For example, a constant flow rate of 0.2 lit/sec will result in the same efficiency for the given chamber geometry and final pressure. However, the required time to achieve the final pressure with this constant flow rate compression is about 7 seconds, which gives almost half the power density (250kW/m\(^3\)) compared to the optimal case (see Fig. 3.5). Note that the calculation of power density does not consider time to eject the compressed air to the storage tank and the time to intake fresh air at \(P_0\). However, these processes are not limited by thermodynamics and so, can, in principle, be performed at infinitely short time, and in practice, in much shorter time than the compression process itself.
Figure 3.5: Results of compression trajectory optimization for a given chamber geometry and porous material distribution. As shown, the optimal compression trajectory can achieve the same efficiency as the constant flow rate compression (85%) but in almost half the time.
3.4 Chamber Shape and Porosity Distribution Optimization

Optimization of the compression trajectory for a given geometry (described in the previous section) is the sub (inner) optimization of the combined shape, porosity distribution and flow rate optimization problem. Therefore, an outer optimization problem needs to be defined and solved to find the optimal geometry of the chamber in order to maximize its power density (Fig. 3.6). In other words, we want to find the optimal cross sectional area ($\psi^*$) and porosity distribution ($\phi^*$) of the chamber at each air volume that maximize the power density for a given chamber and porous material total volume (while achieving the desired compression efficiency). Note that the porosity at each location is a function of the chamber cross sectional area and the plate spacing. Define $\Omega$ as the open area (i.e. total cross section area minus the area occupied by heat transfer material):

$$\Omega(v) = \psi(v) \times \phi(v)$$ (3.25)

where porosity $\phi \in [0, 1]$. If the open area and porosity are known, the chamber cross sectional area $\psi$ can be found from Eqn. (3.25). Therefore, we can optimize the open area and porosity distributions in order to calculate the optimal chamber geometry.

Figure 3.6: Iterative optimization approach to find the optimal chamber shape (outer optimization) as well as optimal compression trajectory (inner optimization)
While the total chamber volume (empty volume plus volume occupied by porous material) is a fixed (given) parameter, the total volume of the heat exchanger material that is allowed to be used must also be fixed in order to have a meaningful optimization problem. Moreover, from the manufacturing point of view, the aspect ratio of the chamber must remain in a reasonable range. This constraint can be included by adding an equality constraint on the chamber length. Therefore, the optimization of the open area and porosity distribution to minimize the compression time can be summarized as:

\[
\left\{ \Omega^*_v, \phi^*_v \right\} = \text{arg.min}_{\Omega_{(v)}, \phi_{(v)}} \left( - \int_{P_0}^{P_f} \frac{1}{Q^*_{(P, \Omega_{(v)}, \phi_{(v)})}} \frac{dV}{dP}dP \right)
\]

such that:

\[
\int_0^{V_0} \frac{dV}{\phi(v)} - \frac{V_0}{\Phi} = 0
\]

\[
l_{\text{min}} \leq \int_0^{V_0} \frac{dV}{\Omega(v)} \leq l_{\text{max}}
\]

where \( \Phi \) is the allowable total porosity of the chamber and \( l_{\text{min}} \) and \( l_{\text{max}} \) are the minimum and maximum possible lengths for it. Note that \( Q^*_{(P, \Omega_{(v)}, \phi_{(v)})} \) in (3.26) is the optimal flow rate calculated from the previous iteration on flow optimization (see Fig. 3.6).

To capture the true effect of porosity distribution and cross sectional area of the chamber on compression performance, the one-dimensional air compression model that was developed in chapter 2 is used here (i.e. the outer optimization is based on one-dimensional air model). The chamber open area and porosity distribution are discretized with piecewise linear functions to transform the optimization problem to a parameter optimization problem (each has 10 pieces over the entire range of chamber volume). The interior point method is then implemented (in the \textit{fmincon} function in Matlab) to solve the corresponding optimization problem and find the optimal chamber shape to minimize the required compression time (i.e. maximize the power density). Note that the inner optimization (to find the optimal flow rate for a given chamber geometry) is based on the lumped air properties, as described in section 3.2. This procedure will

\footnote{Without this constraint, the optimal porosity distribution problem will have a trivial solution which is a chamber where all its volume is occupied by heat exchanger material, with zero space for air.}
be repeated in an iterative manner until the chamber geometry as well as porosity distribution converge to a final shape and distribution which would be their optimal forms. The corresponding optimal flow rate found for this final shape and porosity distribution is therefore the optimal flow rate for the optimal geometry design.

3.5 Case Studies

A systematic approach is introduced and developed in the previous sections to maximize the power density of a compression chamber by combined optimization of its geometry and flow rate. In what follows, this method is utilized to improve the power density of a sample chamber step-by-step. The total chamber volume is assumed to be 1875cc where 375cc of this volume occupied by the heat exchanger material (parallel plates with thickness of 0.8mm). In this case, the total porosity would be 80%. A thermal efficiency of 92% is desired for the compression process. Initial air temperature and pressure are assumed to be 293K and 7bar, while final pressure of 200bar is expected. For all the cases, a constant temperature of 293K is assumed for the heat exchanger during the compression process. Additionally, in order to prevent water trapping between the parallel plates, the minimum local porosity in the chamber is set to be 70% (while the maximum is 100%, which means no porous material). This means that the distance between parallel plates cannot be less than 1.8mm.

Case 1:
As the base case, uniform porosity (80%) and uniform open area (21.4cm$^2$) are used for the chamber. In this case, the chamber length will be 70cm. Without flow rate optimization, a constant flow rate of 43cc/s is required to achieve the desired efficiency (92%). The total compression process takes about 33 seconds in order to reach the desired final pressure while satisfying the thermal efficiency requirement (Fig. 3.7).

Case 2:
The compression (flow) rate is optimized for the chamber geometry given in case 1. As it can be seen in Fig. 3.7, the optimal compression trajectory starts with a large flow rate, followed by a slow compression rate that takes the main portion of the whole process. At
the end, a second fast compression results in the desired final pressure. Optimizing the compression trajectory without changing the chamber geometry reduces the required compression time by more than 3 times (from 33 seconds (case 1) to about 10 seconds) which enhances the power density by the same ratio.

Figure 3.7: Chamber geometry (cross section area, open area and porosity distribution) and compression flow rate. Case 1 (left) and Case 2 (right).

Case 3:

To demonstrate the effect of heat exchanger material location in the chamber, the porosity distribution is optimized over the entire chamber volume, while the open area is considered to be uniform (i.e. no optimization on shape yet). Without flow rate optimization, a constant flow rate of 149cc/s is required, which results in the final pressure after 9.6 seconds. As shown in Fig. 3.8, according to the optimal porosity distribution, all the heat exchanger material must be located in the upper region of the chamber (close to the top cap). In other words, the upper portion has the minimum allowable porosity (70%) while the lower portion is empty (100% porosity). This result is not surprising and is consistent with the physics of the problem. Note that maximum temperature of
air under compression happens at the end of process where the compressed air is very close to the top cap. Therefore, more heat exchanger material is required close to the top cap in order to prevent air from getting hot (reducing thermal efficiency). This phenomenon is experimentally tested and validated in [50].

Case 4:

Now, by optimizing the compression rate for the optimal porosity found in the previous case (3), the required compression time is reduced to 3.5 seconds. As a result, the power density of the chamber will be 669kW/m³. Note that the open area distribution (i.e. chamber shape) is not optimized yet.

Figure 3.8: Chamber geometry (cross section area, open area and porosity distribution) and compression flow rate. Case 3 (left) and Case 4 (right).

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4Also, the porous medium is used throughout the process; whereas if it is placed lower, it gets submerged into liquid piston and will not be effective throughout the entire compression process.
Case 5:

In this case, chamber geometry is optimized in addition to its porosity distribution (Fig. 3.9). Combined geometry and flow rate optimization results in power density of 1.5MW/m$^2$ which is 20 times larger than the non-optimal (base) condition (see Table 3.1). For the shape optimization, it has been assumed that the maximum allowable chamber length is 70cm (i.e. $l_{max}=70cm$). As shown in Fig. 3.11 the optimal shape has a larger area at the bottom (where water enters the chamber), while the rest of the chamber has a smaller area.

![Figure 3.9: Chamber geometry (cross section area, open area and porosity distribution) and compression flow rate. Case 5 (left) and narrowest Case (right).](image)

Minimum and Maximum Length Cases:

To clarify the effect of shape optimization on power density, two additional cases have been studied here. If we relax the length constraint in the shape optimization, the optimal shape for the chamber would be a narrow tube with an open area equal to 7cm$^2$ with a total length of 214cm. On average, a narrower chamber results in higher air velocity during compression. This will improve the heat transfer between air and...
its environment by increasing the convective heat transfer coefficient (see (3.7), (3.8) and (3.9)). If the porosity distribution as well as the compression trajectory are both optimized for this narrow chamber, power density of 1.6MW/m$^3$ will be achieved. On the other hand, a fat chamber with a uniform open area of 78cm$^2$ and total length of 19cm results in a poor power density (423kW/m$^3$) even though the porosity distribution and compression trajectory are both optimized for it. In reality, fabricating a long and narrow compression chamber is challenging. Difference in power density between the optimal geometry and the narrowest shape is small, while manufacturing the former is more cost effective than the latter geometry. Fig. 3.10 shows the air temperature versus air volume during the compression process for various situations that have been studied here. It should be mentioned that the power density optimization for the expansion mode can be done in the same manner, where the initial pressure $P_0$ is 200 bar while the final pressure $P_f$ is 7bar.

<table>
<thead>
<tr>
<th>Step</th>
<th>Porosity</th>
<th>Flow Rate</th>
<th>Shape</th>
<th>Chamber Length</th>
<th>Efficiency</th>
<th>Compression Time</th>
<th>Power Density (kW/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>uniform</td>
<td>constant</td>
<td>uniform (21.5cm$^2$)</td>
<td>70cm</td>
<td>92%</td>
<td>33s</td>
<td>71.2</td>
</tr>
<tr>
<td>2</td>
<td>uniform</td>
<td>optimal</td>
<td>uniform (21.5cm$^2$)</td>
<td>70cm</td>
<td>92%</td>
<td>10.8s</td>
<td>217.3</td>
</tr>
<tr>
<td>3</td>
<td>optimal</td>
<td>constant</td>
<td>uniform (21.5cm$^2$)</td>
<td>70cm</td>
<td>92%</td>
<td>9.6s</td>
<td>245.6</td>
</tr>
<tr>
<td>4</td>
<td>optimal</td>
<td>optimal</td>
<td>uniform (21.5cm$^2$)</td>
<td>70cm</td>
<td>92%</td>
<td>3.5s</td>
<td>669.3</td>
</tr>
<tr>
<td>5</td>
<td>optimal</td>
<td>optimal</td>
<td>optimal narrowest (7cm$^2$)</td>
<td>70cm</td>
<td>92%</td>
<td>1.6s</td>
<td>1470</td>
</tr>
<tr>
<td>N</td>
<td>optimal</td>
<td>optimal</td>
<td>fattest (78.5cm$^2$)</td>
<td>214cm</td>
<td>92%</td>
<td>1.47s</td>
<td>1600</td>
</tr>
<tr>
<td>F</td>
<td>optimal</td>
<td>optimal</td>
<td>optimal</td>
<td>19cm</td>
<td>92%</td>
<td>5.5s</td>
<td>423.6</td>
</tr>
</tbody>
</table>

Table 3.1: Effect of optimization of porosity distribution, chamber shape and compression trajectory on compression time and power density for the given thermal efficiency (92%) and total porosity (80%). For all the cases, total chamber volume is 1875cc where 375cc is occupied by the heat exchanger material. The initial and final pressure are 7bar and 200bar, respectively.
Figure 3.10: Temperature versus volume ratio for air under compression for different scenarios (steps) studied in this work.

Figure 3.11: Optimal geometry (left) resulted by combined optimization algorithm to maximize the chamber power density for a given thermal efficiency (compared with uniform geometry (right))
3.6 Including Solid Material Temperature Rise during Compression

The approach used in the previous section can overestimate the thermal efficiency since the temperature rise of the solid material is not considered in the optimal compression trajectory design process. While the temperature rise of solid is not significant (relative to air temperature rise, due to large heat capacity of solid), it can reduce the thermal efficiency of the compression process by 1-2% (for the same compression time). Including solid temperature as another dynamic state in dynamic programming will make the solution process much slower due to the additional dimension of the search domain. Instead, here we used an iterative approach to consider the effect of solid temperature rise in the optimization process.

In this method, first we assume that the solid material temperature remains constant (at the ambient temperature) and solve the compression trajectory optimization problem based on the zero-dimensional air dynamic model just like the previous section. The calculated optimal compression trajectory is then used to simulate air compression where the solid material temperature (as lumped property) and mass dynamics are also included. This zero-dimensional air-solid dynamic model can be found by combining (3.5) and (3.6) with the additional dynamics for solid material temperature and mass:

\[ \dot{T}_S = \frac{hA(P, \rho, Q)}{c_S m_S} \left( T(t) - T^S(t) \right) \]
\[ \dot{m}_S = \left( 1 - \frac{1}{\phi(V(t))} \right) \rho_S Q(t) \]

where \( T^S \) is the solid material temperature and \( m^S \) is the mass of solid material that is above the liquid piston surface. As it can be realized, this is a conservative model since we assumed that only the solid material (i.e. heat exchanger material) that is above the liquid surface (i.e. exposed to air) absorbs the heat from air. Note that \( c^S \) and \( \rho^S \) are the solid material specific heat capacity and density, respectively. Once this simulation is performed (using air-solid dynamic model), the resulted solid material temperature is saved as a function of air pressure. This function is then used in the zero-dimensional air dynamic model to calculate a new optimal compression trajectory where the solid material temperature is not constant, but instead of being a dynamic
model it is simply a function of air pressure resulted from the previous compression simulation. The new optimal compression trajectory is then used in the combined air-solid dynamic model, and the iteration continues until the solid temperature versus air pressure profile converges. Fig. 3.12 shows how the convergence occurs for one sample case study (similar to case 2 where the porosity distribution and chamber cross section are uniform). Similar to section 5, ABS plastic is used as the material for the heat exchanger in the compression chamber (density of 1200Kg/m$^3$, specific heat capacity of 1600J/Kg.K and heat conductivity of 0.2W/m.K). As shown, while the minimum compression time is about 11 sec without considering the solid material temperature rise, it increases to 20 sec (for 92% efficiency) when the solid temperature rise is included in the optimization algorithm. This is equivalent to downgrading power density by about half, which is due to reduction in heat transfer between air and porous media because of smaller temperature difference between them.

The choice of heat exchanger material can influence the power density drop. As it can be imagined, a heat exchanger with larger heat capacity (for the same unit volume of heat exchanger) achieves a smaller temperature rise for the same amount of absorbed heat. Moreover, if the material used in the heat exchanger has larger heat conductivity, its temperature distribution would be more uniform, which would also help to absorb more heat from air during the compression process. In this sense, choosing a metallic heat exchanger (such as stainless steel) is a better option rather than ABS plastic, since the heat capacity of steel in unit volume is larger than ABS plastic.

---

$^5$Heat transfer between air and solid material is equal to $hA(T_{air} - T_{solid})$, so if $T_{solid}$ rises, the temperature difference and therefore heat transfer from air to solid material will drop.
Figure 3.12: Compression trajectory optimization while considering the solid material temperature rise; Left: temperature versus pressure function used in dynamic programming to calculate the optimal trajectory; Middle: resulted optimal compression flow rate; Right: comparison between the solid temperature versus air pressure function used in the optimization process as well as the numerical result from simulation of the optimal flow rate in the zero-dimensional air-solid model.

This approach has been applied to calculate the optimal compression profile for all the cases in section 3.5, and to evaluate the optimal (maximum) achievable compression power by optimizing the compression trajectory while solid material temperature rise is taken into account. Note that for all cases, the same thermal efficiency (92%) is assumed/demanded. The result is summarized in table 3.2. As it can be seen, including the temperature rise of porous material significantly reduces the power density. However, by simultaneous optimization of chamber shape, porosity distribution and
<table>
<thead>
<tr>
<th>Porosity</th>
<th>Flow Rate</th>
<th>Shape</th>
<th>Power Density (kW/m$^3$)</th>
<th>Efficiency (0-D)</th>
<th>Efficiency (1-D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>uniform</td>
<td>constant</td>
<td>uniform area</td>
<td>39.0</td>
<td>92%</td>
<td>91.9%</td>
</tr>
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<td>uniform area</td>
<td>115.7</td>
<td>92%</td>
<td>91.8%</td>
</tr>
<tr>
<td>optimal</td>
<td>constant</td>
<td>uniform area</td>
<td>181.5</td>
<td>92%</td>
<td>91.5%</td>
</tr>
<tr>
<td>optimal</td>
<td>optimal</td>
<td>uniform area</td>
<td>511.4</td>
<td>92%</td>
<td>91.3%</td>
</tr>
<tr>
<td>optimal</td>
<td>optimal</td>
<td></td>
<td>983.1</td>
<td>92%</td>
<td>90.8%</td>
</tr>
</tbody>
</table>

Table 3.2: Effect of optimization of porosity distribution, chamber shape and compression trajectory on compression power density when solid temperature rise is considered in the flow profile optimization. While the storage power density is reduced due to including solid temperature rise, the same improvement can be achieved by simultaneous chamber and profile optimization.

...compression profile, it is still possible to enhance the performance of the system by more than 25 fold (from 39 kW/m$^3$ to 983 kW/m$^3$). Finally, a one-dimensional simulation has been performed using the model developed in chapter 2. The optimal flow rate found for each case is used to simulate the air compression using a one-dimensional air-solid model. While the compression time is very close to the zero-dimensional study (i.e. the storage power densities are equal), the thermal efficiency is a little smaller (see the last column of table 3.2).

3.7 Summary

A systematic approach was introduced and used to maximize power density of a liquid piston air compression/expansion chamber by optimizing its geometry as well as its compression/expansion rate. The combined shape and flow rate optimization problem is divided into two levels: In the inner optimization, the compression trajectory is optimized for a given chamber geometry to maximize the power density by minimizing the required compression time while achieving the desired efficiency. The Dynamic Programming technique is used for solving the optimal control problem associated with this level. Then, in the outer optimization, open area and porosity distributions of the chamber are optimized to maximize the power density. An iterative method is used to find the combined optimal geometry and flow rate for the chamber. According to the results, the optimal chamber shape in addition to its corresponding optimal flow rate have the potential to increase the power density from 71kW/m$^3$ to 1.4MW/m$^3$ which is an improvement of more than 20 times (see Table 3.1). The optimal geometry shows that the best place for locating the heat exchanger material is at the top of the chamber...
where most of the hot air accumulates at the end of the compression cycle. Results also show that the chamber is better to have a larger diameter at the bottom (where water enters the chamber) and a smaller diameter for the rest. However, these findings are based on a zero-dimensional air dynamic model where the temperature rise of the solid material is neglected. This may lead to an overestimate of the performance of the compression chamber. Therefore, an iterative procedure was used to incorporate the temperature rise of solid (porous) material during the compression process, which reduces the power density compared to the results of a zero-dimensional study. However, according to the numerical calculations, there is still a huge performance improvement (up to 25 times, see Table 3.2) by simultaneous optimization of chamber geometry and compression trajectory.
Chapter 4

Optimal Compression Trajectory Experimentation on a Low-Pressure Liquid Piston Air Compressor

4.1 Introduction

While the theoretical improvement in power/efficiency with the optimal trajectories for air compression over conventional linear and sinusoidal trajectories are validated analytically and numerically [41, 51, 58], this chapter and next chapter are devoted to present some experimental validation of the usefulness of such optimal trajectories [52]. One issue with implementing the optimal trajectories is the uncertainty of the heat transfer model. The transient heat transfer coefficient depends on many factors. Even if a constant heat transfer coefficient $h$ is assumed, its value is often difficult to obtain a-priori. The inaccuracy in estimating $h$ has two consequences: 1) The optimal compression trajectory calculation is sensitive to heat transfer coefficient $h$, therefore any error in $h$ can lead the optimal compression profile to deviate from the actual optimal trajectory; 2) A good tracking of the calculated temperature-volume profile (achieved by optimization) during the air compression process would be difficult (in a combined
feedback and feedforward controller scheme) since the expected heat transfer will be different than its actual value. A nonlinear controller equipped with adaptation for unknown heat transfer coefficient $h$ can be used to overcome the second issue and track the desired temperature-volume trajectory with a reasonable accuracy. Moreover, after each compression test, a new average heat transfer coefficient can be calculated and used to generate a new optimal compression trajectory. With this adaptive-iterative optimal control strategy, the compression efficiency successively improves for a given power-density (or compression time). Here, we assume a constant $h$ during the compression process to design the optimal trajectories. This assumption is observed to be close to reality when compared to actual heat transfer coefficient profiles, particularly when the compression ratio is not very large (here the compression ratio is 10). The iterative procedure can be extended to volume-dependent $h(V)$ profiles to result in an improved tracking of the trajectory specially at the end of the process.

4.2 Experimental Setup

An experimental setup was built to investigate the performance of the optimal trajectories for air compression as shown in Figure [4.1]. The compression chamber is an acrylic tube that has a pressure sensor on its top cap. Air compression is performed via a column of water (inside the chamber) that is driven by a water pump which circulates water inside the circuit and provides the required flow rate to compress air inside the compressor chamber. Two pressure transducers are mounted in the setup to measure the water pressure (at the bottom of the water column in the compression chamber) and air pressure (in the chamber). An Omega FTB-1412 turbine flow-meter and an Endress+Hauser Promass 80 Coriolis flow-meter are used to measure the water flow rate into the compression chamber. A relief valve is mounted in the circuit to keep the pressure of the circuit below 11 bar. A 16-bit PCI-DAS1602/16 multifunction data acquisition (DAQ) board is used to acquire the system data. This board is linked with MATLAB/Simulink via xPC Target to send and receive the control and measured signals. The sampling frequency is chosen to be 4000 Hz. The main reason for the high sampling frequency is to correctly capture the output signals of the Coriolis and turbine
meters. A Burkert 6223 servo-assisted proportional control valve is employed to manipulate the flow rate to the chamber. This valve was found to be unstable in its open-loop operation. Therefore, a Proportional-Integrator (PI) inner control loop is employed to stabilize the control valve based on the upstream pressure of the water passing through the valve. More details on this controller will be discussed in Section 4.4. An op-amp current driver circuit is also used to control the valve by an input voltage from the computer.

1. Pump
2. Stainless steel water filter
3. Relief valve
4. Coriolis flow meter
5. Turbine flow meter
6. Upstream pressure transducer
7. Proportional control valve
8. Compression/expansion chamber
9. Kulite pressure transducer
10. Ball valve
11. Low pressure filter
12. Water reservoir

Figure 4.1: Experimental setup for low pressure experimentation (taken from [62])
The Coriolis meter measurement is relatively accurate but has a time lag of $\Delta t = 71\text{ms}$. The turbine meter is not accurate at small flow rates but has a faster response. Here, we combine the flow information of the two meters to obtain a more accurate measurement of the water volume inside the chamber and consequently the volume of the air under compression. The total water passed through the flow meters that entered the chamber from time 0 until $t$ can be estimated as:

$$V_w(t) = \int_0^t F_{Cor}(\tau)d\tau + \int_{t-\Delta t}^t F_{TM}(\tau)d\tau$$

(4.1)

where $V_w(t)$ is the total water that entered the compression chamber from initial time ($t = 0$) until $t$, while $F_{Cor}$ and $F_{TM}$ are flow rates measured by the Coriolis meter and turbine meter, respectively. Note that the second integral in (4.1) is the total water volume that entered the chamber but is not seen (or measured) by the Coriolis meter from $t - \Delta t$ to $t$ due to its time-lag. If the initial air volume in the chamber ($V_0$) is known, the air volume $V$ at time $t$ can be estimated as:

$$V(t) = V_0 - V_w(t)$$

(4.2)

The temperature of air inside the compressor is obtained from the ideal gas law using the pressure and volume measurements as follows.

$$T = \frac{PV}{P_0V_0}T_0$$

(4.3)

where $P$ and $V$ are the measured air pressure and volume, and $P_0$ and $T_0$ are initial pressure and temperature of the air, respectively. In each experiment, a ball valve connected to the chamber top is opened to make the initial air pressure equal to the ambient. Then, water level in the chamber is adjusted by adding/removing water to ensure that the initial air volume (i.e. $V_0$) is set at 300cc. The maximum available flow-rate in the setup is 306cc/s. The initial volume is selected to be 300cc in order to conduct fast compressions in 1-3 seconds.

### 4.3 Theoretical Background on Optimal Trajectories

In this section, a brief background on optimal compression trajectories is presented. The approach used to define the optimization and to solve the corresponding optimal
control problem is slightly different than described in chapter 3. Here, the total input work required to compress the air from initial pressure $P_0$ to final pressure $P_f$ ($= rP_0$) is considered to be the cost function, whereas the compression time is a nonlinear equality constraint that must be satisfied. Note that, based on the results in [58], the energy loss due to viscous friction is negligible (compared with the compression work) unless the chamber is filled with porous material with very small hydraulic diameter ($< 1 mm$ in the case of parallel plates or the small tube porous inserts). Therefore, the energy required to overcome the viscous friction is neglected in the total compression work. According to Fig. 3.3 in chapter 3, the cost function can be formulated as:

$$J = \int_0^{t_c} \left( \frac{mRT(t)}{V(t)} - P_0 \right) \dot{V}(t) \, dt + \frac{(r - 1)P_0V_f}{r - 1}$$

(4.4)

where $m$, $T$ and $V$ are the mass, temperature and volume of air under compression, $R$ is the specific gas constant of air, $r$ is the final pressure ratio, and $t_c$ and $V_f$ are the compression time and air volume at which the desired pressure ratio is obtained. In Eq. (4.4), $T(t)$ (air temperature) and $V(t)$ (air volume) are dynamic states for which, the dynamic equations come from the first law of thermodynamics and water flow rate into the chamber as:

$$\dot{T}(t) = -\frac{h(t)A(t)}{mC_v}(T(t) - T^S) + (\gamma - 1)\frac{T(t)U(t)}{V(t)}$$

(4.5)

$$\dot{V}(t) = -U(t)$$

(4.6)

where $U(t)$ is the water flow rate into the compression chamber, considered to be the control input, while $C_v$ and $\gamma$ are the heat capacity of air at constant volume and the heat capacity ratio of air, respectively. The heat transfer coefficient between air and solid boundary is described by $h$, which, in general, varies with time during compression based on air properties ($h(t) = h(T(t), V(t), U(t))$). Moreover, $T^S$ is the wall temperature, which is assumed to stay constant at the ambient temperature during the compression process. Finally, $A(t)$ is the active heat transfer area between the air and cylinder wall, tube’s cap and liquid piston.

$$A(t) = \frac{4V(t)}{d} + \frac{\pi d^2}{2}$$

(4.7)
The initial conditions of air in the chamber (at $t = 0$) are $T_0$, $V_0$ and $P_0$ for temperature, volume and pressure, respectively. It is desired to compress it to the final compression ratio of $r$ while the compression time must be equal to $t_c$. Therefore, there exists an algebraic constraint between the final air temperature and the final volume described as:

$$P(t=t_c) = rP_0 \implies \frac{T(t_c)}{V(t_c)} - r\frac{T_0}{V_0} = 0$$

(4.8)

An additional inequality constraint exists due to hardware limitation on maximum water flow rate:

$$|\dot{V}(t)| \leq \dot{V}_{max}$$

(4.9)

In summary, the optimal compression profile $U^*$ which maximize the compression efficiency by minimizing the required input work (for a given compression time and compression ratio) can be defined as:

$$U^*_{(t)} = \arg \min_{U_{(t)}} (J(U))$$

(4.10)

where $J(U)$ is defined in (4.4), subject to dynamic constraints given by (4.5) and (4.6), such that it satisfies (4.8) and (4.9).

The continuous optimal control problem is then parameterized as a finite dimensional problem and solved numerically by standard algorithms for a constrained parameter optimization [58]. Figure 4.2 illustrates sample flow rate, temperature-volume and pressure-volume trajectories obtained from the optimization for $r = 10$, $t_c = 2s$, $h = 16\frac{W}{m^2K}$, $V_0 = 300cc$ and $\dot{V}_{max} = 280cc/s$. The efficiency of the trajectory is obtained from dividing the isothermal process work by the input compression work for the same pressure ratio $r$, as follows.

$$W_{iso} = P_0V_0\ln(r)$$

(4.11)

$$W_{in} = -\int_{V_0}^{V_f} (P - P_0)dV + (rP_0 - P_0)V_f$$

(4.12)

$$\eta_c = \frac{W_{iso}}{W_{in}}$$

(4.13)

where $W_{iso}$ is the isothermal compression work, $W_{in}$ is the required input work for compression and $\eta_c$ is the compression efficiency.
Figure 4.2: Sample optimal compression profile (flow rate-time, temperature-volume, pressure-volume and temperature-time) for $r = 10$, $t_c = 2s$ and $h = 16 \frac{W}{m^2 \cdot K}$

As mentioned earlier, the heat transfer coefficient $h$ is a complex function of air properties, flow regime (laminar or turbulent) and the compression chamber geometry. As the result, it is usually difficult to have an accurate model of $h$ during the compression process. Instead, we use an iterative procedure to estimate a reasonable average value for $h$ during the compression test in order to track the desired temperature-volume trajectory obtained from compression trajectory optimization.
4.4 Nonlinear Controller Design for Temperature-Volume Trajectory Tracking

In the experiment, the optimal trajectory is tracked using the controller configuration shown in Figure 4.3. An inner-loop PI controller is employed to command the control valve. This controller manipulates the upstream pressure of the control valve to provide the desired flow rate. The output of the inner loop controller is a voltage between 0 and 2V \((0 \leq v \leq 2V)\) that is sent to the power amplifier circuit. The outer nonlinear control loop determines the pressure threshold that is used as a reference for controlling the valve. This controller is responsible for tracking the desired temperature-volume trajectory \(T_d(V)\). The input flow-rate is determined by the nonlinear controller using the trajectory information \(T_d(V)\) and \(\frac{dT_d}{dV}\) and then it is converted to the desired pressure threshold. This conversion is based on a mapping obtained from the steady-state flow-rate of the valve to its upstream pressure.

![Nonlinear Controller Diagram](image)

Figure 4.3: Optimal trajectory control configuration

The nonlinear controller is adapted by estimating the heat transfer coefficient \(h\) during the compression process. From now on, we drop the dependency of states on time, for simplicity. Referring back to the first law of thermodynamics for the air volume inside the compression chamber, we have

\[
mC_v \frac{dT}{dt} = -hA(V)(T - T^S) + PU
\]

(4.14)

where \(U\) is the water flow rate to the chamber and negative of the air volume derivative (i.e. \(U = \dot{V}_w = -\dot{V}\)). The feed-forward control input is obtained from substituting
desired temperature $T = T_d$ as a function of air volume $V$ into (4.14) as:

$$mC_v \dot{T}_d = -\hat{h}A(V)(T_d - T^S) + PU_d$$

(4.15)

where $\hat{h}$ is the estimated heat transfer coefficient. By substituting $\dot{T}_d = -\frac{\partial T_d}{\partial V} U_d$ and solving for $U_d$, the feed-forward term is determined as follows.

$$U_d = \frac{\hat{h}A(V)(T_d - T^S)}{P + mC_v \frac{\partial T_d}{\partial V}}$$

(4.16)

A feedback control law is also introduced to guarantee the closed-loop exponential stability.

$$U = U_d + \lambda(T_d - T)$$

(4.17)

where $\lambda$ is the feedback gain. The temperature-volume tracking error can be defined as $T_d - T$. Therefore, the tracking error dynamics can be found as:

$$e = T_d - T \implies \dot{e} = -\left[\frac{\partial T_d}{\partial V} + \frac{P}{mC_v}\right] U + \frac{\hat{h}A(T - T^S)}{mC_v}$$

(4.18)

where an appropriate substitution for $\dot{T}$ is done based on (4.14). Now, by substituting $U$ from (4.16) and (4.17) into (4.18), the error dynamics can be found as:

$$\dot{e} = -\left[\frac{\hat{h}A}{mC_v} + \lambda\left(\frac{P}{mC_v} + \frac{\partial T_d}{\partial V}\right)\right] e$$

(4.19)

Hence, the convergence of temperature tracking error $e$ to zero can be guaranteed by an appropriate selection of $\lambda$ value in the control law. Figure 4.4 illustrates the tracking of the temperature-volume optimal trajectory for compression times of 1.5s and 2.5s. The actual temperature-volume tracks the desired T-V optimal trajectory closely. Therefore, the efficiency of each process is obtained to be approximately the same as its theoretical value. In the experiment, at the end of the process the flow rate drops because of counteracting pressure inside the column leading to decreasing temperatures. The experimental temperature profiles for compression times of 1.5s and 2s are also shown for optimal and constant flow rate trajectories in Figure 4.5. It can be seen that constant flow rate trajectories result in higher temperatures at the second half of the process that make an important effect on decreasing the compression efficiency compared to those of optimal trajectories.
4.4.1 Adaptation of Heat Transfer Coefficient

The actual value of the heat transfer coefficient $h$ is not known. Therefore, an estimation approach should be employed to adapt the nonlinear controller with the current value of $\hat{h}$. Consider the following error dynamics obtained from \(4.14\), \(4.15\) and \(4.17\): \[ mC_v e = - [hA(V) + P\lambda] e + \hat{h}A(V)(T_d - T_S) \] (4.20)

where $e = T_d - T$ and $\hat{h} = h - \hat{h}$ are the temperature tracking error and heat transfer coefficient estimation error, respectively. Then, a positive Lyapunov function is defined as follows.

\[ V = mC_v e^2 + 1 \frac{\hat{h}^2}{\sigma} \] (4.21)

where $\sigma > 0$. The derivative of this Lyapunov function\(^1\) is obtained to be as follows

\[ \dot{V} = mC_v \dot{e} e + \frac{\dot{\hat{h}}}{\sigma} \frac{\hat{h}}{\sigma} = - [hA(V) + P\lambda] e^2 + \hat{h}A(V)(T_d - T_S)e + \frac{\hat{h}}{\sigma} \] (4.22)

The estimation error dynamics are chosen such that the derivative of the Lyapunov function becomes negative and the closed-loop system stability is guaranteed. Therefore,

\(^1\)In the theory of ordinary differential equations (ODEs), Lyapunov function is a scalar function that is used to prove the stability of an equilibrium of an ODE (see [59] for more details).
the following adaptation law can be chosen:

$$\dot{h} = -\sigma A(V)(T_d - T^S) e \implies \dot{V} = -[hA(V) + P\lambda] e^2 < 0 \quad (4.23)$$

which implies that $e$ converges to zero, according to Barbalat’s lemma [59].

### 4.5 Iterative Calculation of Optimal Trajectories

In the above, the optimal trajectories were designed for constant values of $h$. Here, we consider an iterative algorithm that takes advantage of both experimental data and theory to obtain optimal compression trajectories for the experimental setup. In this algorithm, for a given compression time $t_c$ and a maximum flow rate $\dot{V}_{max}$, an initial value of heat transfer coefficient $h_0$ is assumed and the optimal compression trajectory, i.e. temperature versus volume $T_i(V)$ is determined using the optimization code [60]. This trajectory is used in the experiment for tracking. The experimental data are acquired and analyzed to obtain the corresponding efficiency $\eta_i$, compression time $t_{ci}$ and a mean value of actual $h_i$. The heat transfer coefficient during the compression process is calculated using a moving window averaging method applied on the data.
If we assume that the window length (in time) is \( \delta t \) \((<< t_c)\), then it would be possible to move the starting point of this window from \( t = 0 \) to \( t = t_c - \delta t \). According to the first law of thermodynamics, we have:

\[
-PdV = hA(T - T^S)dt + mC_vdT
\]  

By integrating (4.24) over the window interval and assuming the heat transfer coefficient is constant over the interval, we can find an average heat transfer coefficient for each window as:

\[
h_{(t_j+\delta t/2)} = -\frac{\int_{V_j}^{V_{j+1}} PdV + mC_v(T_{j+1} - T_j)}{\int_{t_j}^{t_{j+1}} A(V)(T - T^S)dt}
\]  

(4.25)

where \( P_j, V_j \) and \( T_j \) denote the air pressure, volume and temperature at the beginning of the \( j^{th} \) window, while \( P_{j+1}, V_{j+1} \) and \( T_{j+1} \) denote the air pressure, volume and temperature at the end of the \( j^{th} \) window. Note that the \( j^{th} \) window starts at \( t_j \) and ends at \( t_j + \delta t \). Therefore, the time corresponding to heat transfer coefficient value found over the \( j^{th} \) interval is \( t_j + \delta t/2 \). By moving the window over time and combining all the calculated heat transfer coefficient values (where \( j = 1, 2, ..., n \)), a smoothed time history of heat transfer coefficient can be found for each compression test. Note that at the beginning of the process, both heat transfer rate and temperature difference between air and compression chamber wall (i.e. solid wall) are small and their ratio is not meaningful. Therefore, we do not take into consideration the initial values of the heat transfer coefficient. Since the compression rate optimization is based on an average value for heat transfer coefficient over the entire process, a mean heat transfer coefficient \( h_i \) is calculated by taking the mean of values found from (4.25). Note that the subscript “\( i \)” denotes the \( i^{th} \) compression test in the iterative procedure discussed earlier. This calculated \( h_i \) is then inserted into the optimization code to obtain the next optimal trajectory to be tracked. This iterative procedure is carried on until the efficiency is not increased anymore and a final value of \( h_N \) is obtained, where \( N \) is number of iterations. Notice that since the compression time \( t_c \) is fixed as a constraint, the power density should be the same for all cases.

Fig. 4.6 shows a sample convergence curve of \( h \). Following the procedure explained above, an initial guess of \( h_0 = 8 \frac{W}{m^2K} \) converges to \( h_4 = 12 \frac{W}{m^2K} \) after 4 iterations that results in the same experimental compression time. The final results of the iterative
procedure are listed in Table 4.1 for 4 different compression times. The efficiencies are obtained for the optimal trajectory corresponding to the converged $h$ value.

![Image of a graph showing the convergence of guess and actual value of $h_i$ for $t_c = 1.5s$ after 4 iterations obtained from experiments.]

Figure 4.6: Convergence of guess and actual value of $h_i$ for $t_c = 1.5s$ after 4 iterations obtained from experiments

<table>
<thead>
<tr>
<th>Compression time (s)</th>
<th>$h(\frac{W}{m^2K})$</th>
<th>$\eta$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.990</td>
<td>18</td>
<td>82.90</td>
</tr>
<tr>
<td>2.340</td>
<td>18</td>
<td>81.91</td>
</tr>
<tr>
<td>1.999</td>
<td>16</td>
<td>79.37</td>
</tr>
<tr>
<td>1.504</td>
<td>12</td>
<td>76.72</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of results for optimal trajectories

### 4.6 Experimental Results

As explained in chapters 2 and 3, the trade-off between efficiency and power density of air compression plays an important role in CAES systems. In order to obtain a baseline for power-efficiency characteristics of our experimental setup, we first conducted a series of experiments with different constant-flow rate trajectories. Table 4.2 summarizes the compression time and efficiency values for the constant flow rate cases tested. The flow rates are not completely constant during compression and the reported values
are the averaged flow rates calculated from $U_{\text{avg}} = \frac{V_0 - V_f}{t_c}$. Fig. 4.7 shows the comparison of pressure-volume curves for optimal, iso-thermal, adiabatic and constant-flow-rate trajectories for a compression times of 2s and 3s (and compression ratio of 10). Theoretically, the isothermal curve corresponds to an infinitely slow, 100% efficient process while the lowest efficiency of 70.4% is obtained from the adiabatic infinitely fast compression (both of them are not practically doable). Tracking the optimal compression trajectory results in 2% higher efficiency for the same power density (or the same compression time). The experimental efficiency versus power density of optimal and constant flow rate trajectories are shown in Fig. 4.8. The simulation results are also presented for the optimal trajectories. It is observed that for the same value of power density (i.e. compression time), the efficiency can be improved by 2% if the constant flow rate compression is replaced by the optimal compression rate. This difference is more significant at higher efficiencies, which is consistent with the theoretical results discussed in Section 4.3. The performance improvement by deploying optimal compression flow rate can be also seen in terms of power density improvement for a fixed efficiency. For example, as shown in Fig. 4.8 for an efficiency of 80%, the power density is increased by about 30% (from 87kW/m$^3$ to 113kW/m$^3$) by using the optimal compression trajectory instead of the constant compression rate.

<table>
<thead>
<tr>
<th>$V_{\text{avg}}$ (cc/s)</th>
<th>$t_c$(s)</th>
<th>$\eta$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>84.90</td>
<td>3.015</td>
<td>81.46</td>
</tr>
<tr>
<td>92.67</td>
<td>2.751</td>
<td>79.89</td>
</tr>
<tr>
<td>108.00</td>
<td>2.359</td>
<td>79.45</td>
</tr>
<tr>
<td>132.89</td>
<td>1.904</td>
<td>77.58</td>
</tr>
<tr>
<td>157.63</td>
<td>1.599</td>
<td>76.64</td>
</tr>
<tr>
<td>169.91</td>
<td>1.487</td>
<td>76.16</td>
</tr>
<tr>
<td>179.32</td>
<td>1.407</td>
<td>75.86</td>
</tr>
<tr>
<td>207.17</td>
<td>1.209</td>
<td>75.04</td>
</tr>
<tr>
<td>264.09</td>
<td>0.935</td>
<td>72.36</td>
</tr>
</tbody>
</table>

Table 4.2: Constant flow rate compression results. Left column: flow rate; middle column: compression time; right column: compression efficiency
Figure 4.7: Comparison of pressure-volume curves for optimal, iso-thermal, adiabatic and constant flow-rate trajectories. The compression ratio is 10 for all the cases (Left: for $t_c = 2s$; Right: for $t_c = 3s$)

Figure 4.8: Comparison of efficiency vs power density curves of optimal and constant flow rate compression trajectories for $r = 10$
4.7 Summary

This chapter presented the experimental investigation of optimal trajectories for a low-pressure air compressor. The experimental setup and procedure were discussed in detail. An adaptive nonlinear controller was designed to command the desired flow rate to the compressor chamber and track the temperature-volume optimal trajectories. This controller was adapted based on the real-time estimation of heat transfer coefficient $h$. Since the actual value of $h$ is not known, an iterative procedure was introduced to estimate an average value of $h$ based on the experimental data achieved from the previous run to calculate the optimal compression flow rate for the next run. For a given compression time and compression ratio, the final optimal trajectory was determined based on the converged average value of $h$. The experimental trajectory tracking results and efficiency-power density calculations were presented. It was shown that for the same compression time and a relatively low pressure ratio of 10, the optimal trajectories can improve the efficiency up to 2% at the same power density (or they can improve power density over 20% at the same compression efficiency) compared to linear constant flow rate trajectories. The design of optimal trajectories using the iterative procedure can be suited for the actual CAES system where the compression/expansion trajectories are designed and optimized for the desired operating conditions. However, a more sophisticated heat transfer correlation can be found and used for the trajectory optimization procedure to further improve the air compression performance. In particular, this is very important for air compression with high compression ratio since air properties experience a large variation during the compression. Hence, assuming an average heat transfer coefficient over the entire compression and design the optimal compression rate based on it may cause poor performance (i.e. low efficiency and low power density) for a high-pressure-ratio air compressor. This is the topic of the next chapter where the optimal compression trajectories are found and experimentally validated for a high pressure ratio air compressor system.
Chapter 5

Optimal Compression Trajectory Experimentation on a High-Pressure Liquid Piston Air Compressor

5.1 Introduction

As introduced in chapter 3, optimizing and controlling the rate of compression/expansion is an active way to improve the air compressor/expander performance. Optimizing the compression/expansion trajectory allows the process to better match the heat transfer capability. In fact, utilizing the optimal compression/expansion trajectory optimizes the trade-off between thermal efficiency and power density, and maximizes the efficiency for a desired power (or power density). Analytical and numerical studies have shown that use of optimal compression/expansion trajectories can significantly increase power density (by 2 to 3 fold for high pressure) over ad-hoc linear or sinusoidal trajectories [41, 51, 58, 61] for both simple and complex heat transfer models. Experimental validation of the efficacy of this approach for low pressure ratio (1bar to 10bar) was presented in chapter 4, where the benefit was relatively minor. Since the advantage of optimal

\[ \text{Or minimizes the compression/expansion time for a desired thermal efficiency.} \]
trajectory is more important for high pressure ratios, the goal of this chapter is to experimentally validate this concept in high pressure operation (7bar to 200bar). According to the theoretical studies, design of optimal compression/expansion profile is very sensitive to the heat transfer model. In the other words, if the compression/expansion profile is optimized by assuming a wrong heat transfer model, there is a possibility of downgrading the system performance in terms of efficiency and power density. The enhancement achieved in low pressure system (in chapter 4) is not too significant since a constant convective heat transfer coefficient was considered in designing the optimal compression trajectories, which causes the calculated profile not to be close to the actual optimal. In addition, the total pressure ratio for that system was 10 (from 1bar to 10bar), another limitation for demonstrating the full capability of the optimal trajectories in enhancing the efficiency-power trade-off.

In this chapter, the optimal compression trajectories for a high pressure liquid piston air compressor testbed are designed and implemented. The experimental setup is introduced and presented in section 5.2. The heat transfer coefficient correlation obtained empirically from extensive CFD experiments is presented in Section 5.3. This sophisticated heat transfer correlation is then used to design the optimal flow rates for a range of desired compression efficiencies (from 62% to 82%). Calibration of the critical air volume measurement is presented in Section 5.4. Design and control of the optimal trajectories are given in Section 5.5. Experimental results are given in Section 5.6 followed by CFD numerical simulations in Section 5.7 for comparison. The optimal performance (efficiency versus power density) is compared with the performance of ad-hoc compression trajectory (constant flow rate) in Section 5.8.

5.2 Experimental Setup

The schematic and picture of the liquid piston air compressor experimental setup are shown in Figs. 5.1 and 5.2. The setup was designed to study the compression/expansion processes during single shot experiments [50]. In this system, a double-acting hydraulic cylinder (4) is coupled with a single-acting water cylinder (5) so that extension of the hydraulic piston will cause the water piston to be retracted and vice versa. The hydraulic cylinder is connected to a hydraulic power supply (at 200bar) via a solenoid-actuated...
servo-valve (3). This valve is used to control the oil flow rate to the hydraulic cylinder and to regulate its extension speed at a desired value. A magnetic incremental encoder is connected to the tandem rod (between the hydraulic cylinder and water cylinder) in order to measure the displacement of water piston, which will be used to calculate the volume of water displaced into the compression chamber. The compression chamber (11) is a vertical cylinder made of stainless steel and is connected to the water cylinder via a combination of hoses and ball valves (8) (9).

Figure 5.1: Detailed schematic of liquid piston air compressor experimental setup [39, 50]

Retracting the water piston causes water to be pushed into the compression chamber and raises the water column level inside it. This will compress the air inside the compression chamber. A pressure transducer (15) is located at the top of compression chamber to measure the air pressure during compression process. A transparent plastic side tube (21) is used to estimate the initial level of the liquid column in the compression chamber and to calculate the initial air volume in it. By knowing the initial air volume and the amount of water that is displaced into the compression chamber (from water cylinder), it would be possible to estimate the air volume inside the compression chamber during the compression process. A combination of ball valves and a single poppet valve (16) (mounted on top of the compression chamber) are used to control the filling of the chamber with fresh air. While the liquid piston air compressor is considered for
compressing air from 7bar to 200bar, a conventional off-the-shelf air compressor is used to compress air from ambient pressure to 7bar. By opening the poppet valve, the compression chamber is filled with fresh air at 7bar provided by the solid-piston air compressor. After the chamber is filled with air, the poppet valve closes and the system becomes ready for the next compression process. By regulating the flow rate through the hydraulic servo-valve, it would be possible to control the extension speed of hydraulic piston which in turn defines the retraction speed of water piston and consequently the water flow rate into the compression chamber. Therefore, a previously defined flow rate (as a function of compression time) can be tracked using an appropriate closed-loop controller. More details and information regarding this experimental facility can be found in [50] where the same setup was used to study the effect of porous media.

Figure 5.2: Liquid piston air compressor; Left: water hydraulic cylinder and connections; Right: compression chamber [39, 50]

It should be noted that the experimental validation of air compression performance improvement via optimal compression trajectories can be done with or without using the porous media inside the compression chamber. However, in case of a chamber filled with porous media, a much higher maximum compression rate is required to clearly show the performance improvement achievable by implementing optimal compression trajectory,
in compared with an add-hoc trajectory such as constant flow rate. This is mainly due
to the increase of heat transfer through the extra heat transfer area provided by the
porous media. In this condition, for low compression rates (i.e. small power densities),
the performance of optimal compression trajectory and constant compression trajectory
would be close. Therefore, a much higher maximum compression rate is needed to show
the real difference between the performance of optimal and non-optimal compression
trajectories. Due to the limited maximum compression rate that is obtainable by the
experimental facility used in this work, the validation of the air compression performance
enhancement presented in this chapter is only for the case of empty chamber.

5.3 Heat Transfer Modeling

Computing the optimal compression trajectory is sensitive to the model used for heat
transfer prediction between air under compression and compression chamber walls. Ei-
ther underestimating or overestimating the heat transfer between air and heat exchanger
material (in our case, the chamber’s walls since porous media is not used) results in a
wrong optimal compression profile, which in turn reduces the improvement of power
density (for a fixed thermal efficiency). Therefore, the first step in calculating the op-
timal compression profile is to find a reasonably accurate heat transfer model for the
chamber.

Assuming lumped properties for air (i.e. zero-dimensional temperature and pres-
sure), the heat transfer from air to its surrounding environment (described by $Q$) can
be written as:

$$Q(t) = hA(T_{air} - T_{wall})$$  \hspace{1cm} (5.1)

where $h$ is the convective heat transfer coefficient, $A$ is the available heat transfer area,
$T_{air}$ is the air temperature and $T_{wall}$ is the wall temperature that is assumed to remain
constant during the compression process ($T_{wall} = 295K$). While calculating the total
heat transfer area is easy (since it is only a function of air volume at any time), evaluating
heat transfer coefficient is relatively complex since it is an instantaneous function of air
properties, piston speed and chamber geometry. To find this dependency, a series of
numerical simulations is performed in COMSOL Multiphysics (COMSOL, Inc, MA)
software to investigate the correlation between convective heat transfer coefficient and
air properties, piston speed and chamber geometry.

While there are many parameters that affect the heat transfer coefficient, a comprehensive study is performed by changing some parameters while keeping the rest of them constant, in order to study their effect on heat transfer coefficient. It should be emphasized that such a flexibility is only available in numerical analysis since the experimental investigation for revealing the dependency of heat transfer to different parameters is very difficult and time consuming. According to the comprehensive numerical analysis that is done in COMSOL, a correlation between $h$, aspect ratio of air column $L/D$ (ratio between length of air column $L$ and its diameter $D$), piston speed $U$, heat conductivity $k$, density $\rho$ and viscosity $\mu$ of air is suggested as:

$$Y = c_1X^2 + c_2X + c_3$$

where $X$ and $Y$ are:

$$X = \frac{\rho U^a}{\mu} \left( \frac{\mu}{k} \right)^b$$

$$Y = \frac{h}{k} \left( \frac{L}{D} \right)^d$$

To find the best combination for $a, b$ and $d$, an optimization problem is defined and solved. Here, we are looking for the best set of parameters that results in minimum difference between the numerical value of $h$ (shown in Fig. 5.3-top) and the value calculated by the suggested correlation defined by (5.2), (5.3) and (5.4). Therefore, the optimization problem is formulated as:

$$\{a^*, b^*, d^*\} = \min_{a,b,d} \| h_{COMSOL} - \hat{h}_{(k,\rho,\mu,U,L,a,b,d)} \|_2$$

where $\hat{h}$ is the heat transfer coefficient according to the correlation, and calculated as:

$$\hat{h} = k \left( \frac{D}{L} \right)^d \times (c_1X^2 + c_2X + c_3)$$

All the data points found from the numerical simulation (in COMSOL) are used here to find the best set of parameters (note that both $h$ and $\hat{h}$ are column vectors). The best combination is found as: $a = 0.35851$, $b = 0.88792$ and $d = 0.35404$. The comparison between numerical $h$ and the correlation (using the optimal coefficients) is shown in Fig. 5.3.
Figure 5.3: Comparison between the numerical $h$ (from COMSOL) and the value calculated by the correlation for the optimized set of $a$, $b$ and $d$. For these optimal powers, a second order polynomial is used to fit $Y$ versus $X$ as: $Y = 3.3921e^{-0.05}X^2 + 1.5628X + 1025.4175$

5.4 Estimation of Air Volume in the Compression Chamber

Direct measurement of the air volume in the compression chamber is not available in the experimental setup. Instead, air volume is estimated from the initial air volume and
the change in water volume. Because water is slightly compressible and the components such as hoses expand, the change in water volume in the chamber consists of the volume of water injected and the volume change due to pressure. Thus, the air volume in the compression chamber can be expressed as:

$$V(t) = V_0 - V_{\text{Displaced}}(t) + V_P^C$$

(5.7)

where $V_0$ is the initial volume of air in the chamber at the beginning of compression (i.e. $t = 0$), $V_P^C$ is the pressure dependent volume adjustment due to water compressibility and system expansion, and $V_{\text{Displaced}}$ is the volume of water pushed into the compression chamber from the water cylinder. It is important to consider $V_P^C$ since it can account for 30% of the volume at the end of compression. The pressure dependent volume adjustment term $V_P^C$ is obtained by filling the chamber completely with water and compressing it. The result is shown in Fig. 5.4. $V_P^C$ has a larger slope at lower pressures which is due to soft components such as hoses and entrained air in the water. At higher pressures ($> 10$ bar), it has a constant slope which is slightly greater than that due pure water compressibility.

Figure 5.4: Summation of water compression and system expansion (such as hoses, cylinder and connections) due to pressure rise. Pure water compressibility is also plotted for comparison (bulk modulus of 2.2 GPa is assumed).

The displaced water volume term $V_{\text{Displaced}}$ should ideally be proportional to the
movement of the water hydraulic cylinder as reflected by the linear magnetic encoder measurement \( C \). In order to account for any slight nonlinearity, a quadratic relation is used. (5.7) becomes:

\[
V(t) = V_0 - K_1 C(t) - K_2 C(t)^2 + V_{C(P)}
\]  

(5.8)

To account for any small changes in each experiment, the parameters \( V_0 \), \( K_1 \) and \( K_2 \) are calibrated for each experiment. To do this, at the end of each experiment, the air in the chamber is allowed to return to ambient temperature at successive volumes (the liquid piston is withdrawn in each step). A sample pressure trace in shown Fig. 5.5. Notice the step decreases in pressures at the end of the experiment. Assuming ideal gas behavior (the same approach can be done with real gas model), air volume and pressure after the air has returned to ambient temperature (i.e. at the end of each step) must satisfy:

\[
T_1 = T_2 = \ldots = T_n
\]  

(5.9)

\[
\Rightarrow \quad P_1 V_1 = P_2 V_2 = \ldots = P_m V_m
\]  

(5.10)

where \( V_i \) can be expressed using (5.8). The coefficients \( V_0 \), \( K_1 \) and \( K_2 \) are then optimized to minimize the relative error in (5.10), specifically,

\[
\{V_0^*, K_1^*, K_2^*\} = \min_{V_0, K_1, K_2} VAR\left(P_i(V_0 - K_1 C_i - K_2 C_i^2 + V_{C(P)})\right)
\]  

(5.11)

where \( VAR \) denotes the variance of the \( m \) air pressure and volume products. The approach described above is used for each compression test since the initial air volume for each run can be slightly different than the other tests. For the sample case shown in Fig. 5.5 these parameters are found as follows:

\[
V_0 = 2.292 \times 10^{-3} m^3
\]

\[
K_1 = 1.196 \times 10^{-7} m^3/count,
\]

\[
K_2 = 2.316 \times 10^{-14} m^3/count^2
\]
Figure 5.5: Sample test shows the method for evaluating the constant parameters in air volume estimation. Air compression is from $t = 60s$ to $t = 100s$. The liquid column is maintained at its position from $t = 100s$ until the air pressure reaches its steady-state value. This means that air is cooled down to ambient temperature. To achieve more isothermal points, the liquid column is retracted by small steps to let air pressure drops and then stayed there for a while until air pressure reaches its new steady-state value.

5.5 Design and Implementation of the Optimal Compression Trajectories

The heat transfer correlation found based on COMSOL simulations is used to calculate a series of optimal compression trajectories for the given chamber geometry and desired initial and final pressures. The optimization problem is formulated such that the compression time is the cost function while the compression efficiency is an equality constraint that needs to be satisfied. The flow rate must also be below the pressure dependent flow capability of the system. Dynamic Programming (DP) approach is then used to solve the optimal control problem [61]. A combined feedback and feedforward controller is used to track the optimal flow trajectory in the compression chamber. According to (5.8), the air volume rate of change can be calculated as:

$$\dot{V} = -F(t) = -K_1 \dot{C} - 2K_2 C \dot{C} + \frac{dV^C}{dP} \dot{P}$$  \hspace{1cm} (5.12)
An open loop calibration test is first performed on the system (in terms of different voltages on hydraulic servo-valve) to evaluate the required command signal for a given flow rate at a given pressure. This map is found as shown in Fig. 5.6 Top. By inverting the results, it would be possible to find the required servo valve voltage for a desired piston speed at a given pressure, which is used in the feedforward controller. The feedback part of the controller is simply a PI controller on air volume error (difference between the actual air volume and the desired air volume calculated by time integral of optimal flow rate). The controller block diagram used for this experiment is shown in Fig. 5.6 Bottom.

Figure 5.6: Top: Calibration of hydraulic servo valve that is used in feedforward controller; Bottom: Control strategy for tracking the optimal flow rate
5.6 Experimental Results

The experimental results of implementing the optimal trajectories are shown in Fig. 5.7. The optimal compression profile starts with the maximum available flow rate \(Q_{\text{max}} = 800 \text{cc/s}\), which is followed by a much lower flow that continues for nearly the rest of the compression process. A short fast compression concludes the process to achieve the final desired pressure (200bar) at the end. Such fast-slow-fast trajectories are consistent with optimal trajectories from the previous studies [41, 51, 58, 61].

In order to evaluate the performance improvement achieved by applying the optimal compression trajectories, a series of constant flow rate compressions is also conducted on the experimental setup. Result of this experiment is shown in Fig. 5.8. The closed loop controller maintains the flow rate at the desired value, while the flow rate drops at the end of compression process due to limited flow rate at high pressures. To compare the performance of optimal and non-optimal (constant flow) compression trajectories, the compression efficiencies are calculated from [52, 61]:

\[
\eta = \frac{\text{Stored Energy}}{\text{Input Work}} = \frac{E}{W} \quad (5.13)
\]

\[
E = - \int_{V_0}^{V_{\text{iso}}} P_{\text{iso}} \, dV_{\text{iso}} + P_{f}^{\text{iso}} V_{\text{iso}} - P_0 V_0 \quad (5.14)
\]

\[
W = - \int_{V_0}^{V_f} P \, dV + P_f V_f - P_0 V_0 \quad (5.15)
\]

note that the integration for \(E\) in (5.14) is taken over an isothermal compression process\(^2\) which starts at \((P_0, V_0)\) and ends at \((V_{\text{iso}}^f, P_{\text{iso}}^f)\) (see [53, 61] for more details). The compression power density is defined as the ratio between the storage power and the total volume of compression chamber:

\[
\text{Power Density} = PD = \frac{E}{t_{\text{end}} V_0} \quad (5.16)
\]

Compression efficiency and power density for each test are calculated based on (5.13), (5.14), (5.15) and (5.16). These results combined with their corresponding expected values from the theoretical model (used for calculating the optimal trajectories) are shown in Fig. 5.9. Also, the optimal compression trajectories as well as constant flow

\(^2\)Real gas model is used instead of ideal gas model for better accuracy at high pressures [47]
rate cases are numerically simulated in COMSOL Multiphysics where a two-dimensional axisymmetric model is used to study the performance of air compression and compare it with both experimental and analytical results. The detailed description of this model is provided in the following section.

Figure 5.7: Optimal compression flow rate results; Top: optimal flow rate versus time ratio \( (t/t_{\text{end}}) \) where \( t_{\text{end}} \) is the total compression time; Middle: air pressure versus air volume ratio \( (V/V_0) \) where \( V_0 \) is the initial air volume; Bottom: air pressure versus compression time
Figure 5.8: Constant flow rate compression results; Top: flow rate versus time ratio ($t/t_{end}$ where $t_{end}$ is the total compression time); Middle: air pressure versus air volume ratio ($V/V_0$ where $V_0$ is the initial air volume); Bottom: air pressure versus compression time
5.7 CFD Simulation (Performed in COMSOL Multiphysics)

As described earlier, the optimal compression trajectories are calculated based on a zero-dimensional model for air dynamics (i.e. uniform air pressure, temperature and density in the compression chamber). Here, we want to numerically investigate the performance
of the designed optimal compression trajectories in a 3-dimensional study. To do so, a model is built in COMSOL where the geometry and initial conditions are identical to the experimental setup described in section 5.2. While the air compression is a 3-dimensional process in reality, due to the cylindrical geometry of the chamber, it can be studied as a two-dimensional axisymmetric problem. All the walls (including chamber wall, top cap and liquid piston) are assumed to be isothermal, which means that their temperature remains constant (at room temperature 293K) during the compression process. For all the cases, the initial air pressure and temperature are 7bar and 293K, while the desired final pressure to be achieved is 200bar. Real air properties are used for the air under compression. All the optimal compression trajectory (8 cases) as well as constant flow rate compressions (7 cases) are numerically simulated in COMSOL. As a sample case, the velocity magnitude, pressure and temperature distributions of air in the compression chamber at different snapshots in time are shown in Figs. 5.10 and 5.11 (for constant compression rate of 20cc/s). As shown, the air pressure distribution is very uniform in the chamber. This is expected since pressure change propagates with the speed of sound inside the chamber. However, the air temperature distribution has large variation over the chamber volume. As shown in Figs. 5.10 and 5.11, the maximum air temperature occurs close to the top cap, while the minimum air temperature exists on the moving wall (i.e. liquid piston surface). This is primarily due to the air velocity and air circulation that occurs on the liquid piston which increases the convective heat transfer coefficient and therefore enhances the heat transfer between air and isothermal walls. The compression efficiency and power density of these numerical simulations are calculated similar to the experimental cases based on (5.13), (5.14), (5.15) and (5.16). While the air volume can be evaluated based on location of the piston (i.e. moving wall), the air pressure is a distributed property over the entire air volume at each time step. Here, a volume-average pressure is calculated (at each moment) and used in (5.15) in order to calculate the total input work for air compression.
Figure 5.10: Distribution of air velocity magnitude, pressure and temperature inside the compression chamber at three different snapshots in time: 0s (top); 10s (middle); and 50s (bottom). Compression rate is constant (20cc/s).
Figure 5.11: Distribution of air velocity magnitude, pressure and temperature inside the compression chamber at three different snapshots in time: 70s (top); 90s (middle); and 92.3s (bottom). Compression rate is constant (20cc/s).
5.8 Discussions on the Results

All the experimental, numerical and analytical results achieved in this study are summarized and shown in Fig. 5.9. Results are presented in terms of efficiency versus compression time, and efficiency versus power density defined in (5.16). In general, the optimal compression flow rate results in a smaller compression time, therefore a larger storage power density for the same thermal efficiency\(^3\). According to the experimental results, for the given chamber geometry and initial and final pressures, this improvement can be as high as 100% for thermal efficiencies around 80% (from 55\(kW/m^3\) to 110\(kW/m^3\)). Hence, a compression chamber that uses constant flow rate to compress air can be downsized to its half size and maintains its performance (compression power and thermal efficiency) if it uses the optimal compression rate to compress air. The performance improvement can be also interpreted as a higher thermal efficiency for the same storage power density. According to the results, this raise in thermal efficiency can be as high as 5% for storage powers around 80\(kW/m^3\) (from 75% to 80%).

For comparison purposes, the thermal efficiency and compression time resulted by the zero-dimensional air model (used for optimal trajectory calculation) and the two-dimensional numerical simulations (performed in COMSOL) are also plotted in Fig. 5.9. In particular, note that the zero-dimensional results match well with the outcomes of two-dimensional (COMSOL) study. This is mainly due to the fact that the heat transfer model used in the zero-dimensional model is obtained through several CFD simulations performed in COMSOL, as described in section 5.3. Moreover, it can be noticed that for most of the cases, the obtained efficiency from the experiment (for both optimal and constant flow rate compressions) is higher than the expected value from the zero-dimensional model (for the same compression time). This shows that the actual heat transfer in the experimental setup is larger than what expected from correlation given by (5.6) in section 5.3. Therefore, if a more accurate heat transfer correlation can be found and used to design optimal compression trajectories, the compression performance improvement can be even better than what is shown here.

\(^3\)Note that the performances of optimal and non-optimal compression trajectory are close for very small efficiencies (i.e. the adiabatic compression efficiency) as well as for very large efficiencies (i.e. almost 100% efficiency). The adiabatic efficiency for the experimental test condition presented in this chapter is about 57%.
Again, it should be mentioned that a similar improvement is obtainable by implementing the optimal compression trajectories in case of a compression chamber filled with porous material (see sections 3.5 and 3.6). However, the optimal compression trajectories designed for this case will have a much higher average flow rate (i.e., larger power density), which cannot be experimentally verified on the experimental facility used in this study.

5.9 Summary

Theoretical and numerical studies in chapter 3 showed that applying optimal compression/expansion trajectory is an effective approach to improve the performance of an air compressor/expander machine by optimizing the trade-off between efficiency and power density. It is also known that an accurate heat transfer model for the compression/expansion chamber is critical in order to design optimal flow profiles and improve the system performance. In this chapter, a systematic approach was used to find a correlation that models the convective heat transfer coefficient between air and compression chamber wall. This correlation was obtained from extensive CFD experiments performed in COMSOL. This correlation was then used to calculate the optimal compression trajectories that minimize compression time for a given (desired) compression efficiency. Dynamic programming approach was applied to determine a family of optimal compression flow profiles. The optimal performance of the system was then compared with non-optimal performance that was generated by using ad-hoc compression trajectories (here constant flow rate compression). According to the results, a 5% thermal efficiency improvement is achievable at 80kW/m$^3$ storage power density. Likewise, the storage power can be doubled at 80% efficiency if the constant flow rate is replaced by the corresponding optimal compression trajectory.
Chapter 6

Water Spray Cooling

6.1 Introduction

As discussed in chapter 2, filling the compression/expansion chamber with porous material while using liquid piston to compress/expand air is one method to improve performance by increasing heat transfer between air and its surrounding environment. Another approach to increase the air compression efficiency (for the same power density) is to employ water spray. Large number of small size droplets with a high heat capacity can provide a high total surface area for heat transfer [63, 64, 65]. However, the presence of significant liquid volume in the piston chamber must also be accommodated. A simple theoretical analysis of a single droplet transport phenomena in humid air and the prediction of the life time of a freely-falling droplet is investigated in [66]. A descriptive mathematical model for energy and exergy analysis is presented in [67] for a co-current gas spray cooling system. One-dimensional simulations of liquid piston compression with droplet heat transfer has been investigated in [68] to determine the effect of water spray on compression performance. Two different methods of water spray have been discussed and compared: pre-mixed injection\(^1\) and direct injection\(^2\). The compression performance improvement for different mass loading (defined as ratio between the total mass of air under compression and total mass of water added to the air as water droplets

---

\(^1\) Water spray mixes with inlet gas upstream of the compression chamber during the intake phase

\(^2\) Water is directly sprayed into the compression chamber during both intake and compression phases
during compression) and droplet size is studied in [34]. It shows that a large spray discharge (as measured in terms of the mass loading) provides a more efficient compression cycle, while reducing droplet size (for a fixed mass loading) allows additional benefits since it allows for a larger total surface for heat transfer. However, for a fixed mass loading and droplet size, the timing of water spray during the compression cycle (in direct injection case) can also affect its performance. If water is sprayed too early or too late, it may not be as useful as it can be if it is added at the right time. In this chapter, a dynamic model of the water spray in a liquid piston air compressor is developed based on [34]. This model allows us to investigate the effect of spray flow rate profile (i.e. spray timing) on the air compression efficiency and optimize that profile for a given set of desired parameters. The rest of this chapter is organized as follows: dynamic model of the system describing the compression cycle including water spray is derived based on an Eulerian approach. Finite volume method is then used to transform the partial differential equations (PDE) into a system of ordinary differential equations (ODE). Next, an optimal control problem is formulated and solved, where the compression efficiency is considered as the profit function while the mass loading (i.e. total water mass to be sprayed into the air during compression) is an equality constraint. Comparison between the optimal and non-optimal spray flow profile is given in the last part of this chapter.

6.2 Modeling

Compression Chamber:

A liquid piston air compressor consists of a vertical chamber in which the conventional solid piston is replaced by a column of liquid (see Fig. 6.1). This liquid column is driven into the chamber by a variable displacement pump connected to the chamber inlet flow [30]. The chambers length and diameter are shown by $L$ and $D$, respectively. It is assumed that initially, the chamber is filled with air which means the initial liquid column height is zero. Since the heat capacity of the chamber walls and the liquid column is much larger than the air, it is assumed that the walls and liquid piston temperature maintain at ambient temperature over the compression cycle. In addition, due to good sealing property of the liquid column, no leakage is considered for the air inside the chamber.
Water Droplets:

Micro-sized water droplets can be generated by a spray nozzle, which makes use of the pressure energy of water to increase its speed through an orifice and break it into drops. Analysis of interaction between water droplets and air inside a compression chamber is naturally a complicated phenomena. While the droplets can collide and make bigger droplets, they may also touch the chamber walls as well as the liquid surface (piston) and get vanished. Moreover, droplet size can change due to mass transfer between the liquid phase to the gas phase. This interphase mass transfer is a complicated function of several properties such as droplet temperature, air temperature, pressure and humidity. Therefore, a precise dynamic model of such a process is difficult to obtain. Here, a one-dimensional distribution for water droplet’s properties (i.e. droplet density and temperature) is considered in a lumped air model. Air thermodynamic properties (i.e. density, pressure and temperature) are assumed to be uniform over the chamber volume. However, air velocity is considered to vary linearly with respect to the distance from
the top cap, given by (see Fig. 6.2):

\[ U(x,t) = -\frac{\dot{Y}(t)}{L - Y(t)} x \]  

(6.1)

where \( U(x,t) \) is the air velocity at location \( x \) (measured from the top cap) and time \( t \), and \( Y(t) \) is the liquid piston height inside the chamber at time \( t \) (measured from the bottom of chamber). Note that the origin of coordinate system for \( x \) is located at the top of the chamber and is directed toward its bottom (liquid piston surface). According to (6.1), air velocity is zero at the top \((x = 0)\) while it is maximum at the liquid surface \((x^* = L - Y(t))\).

![Diagram showing air properties, spray flow, and water droplet properties](image)

Figure 6.2: Water spray inside liquid piston air compressor

In reality, there is a mass transfer between liquid phase (water droplets) and gas phase (air). However, no mass transfer (evaporation) is assumed between these phases here. Since the maximum temperature rise of droplets during the compression is less than the saturated boiling temperature (see Fig. 6.3), the evaporation of droplets would be minor. By using this assumption, no variation in droplet size and mass takes place during the compression cycle. In summary, the droplets leave the spray nozzle (at top
of the chamber), move inside the air toward the chamber’s bottom and collide into the liquid piston surface and get accumulated into it (no droplet to droplet collision is considered).

![Figure 6.3: Saturated temperature of water as a function of pressure](image)

Let’s define $r(x,t)$ to be the number of droplets per unit length of the chamber (drop/m) at location $x$ and time $t$, and $v(x,t)$ to be the droplet velocity with respect to the top cap. By applying the conservation of mass principle on the control volume shown in Fig. 6.4 (only considering the liquid phase in the control volume), we will have:

$$
\frac{d}{dt} \left( \int_{x}^{x+\Delta x} r(\tau,t)d\tau \right) = r(x,t)v(x,t) - r(x+\Delta x,t)v(x+\Delta x,t) \tag{6.2}
$$

hence, if $\Delta x \to 0$, we have:

$$
\frac{\partial r}{\partial t} + \frac{\partial}{\partial x}(rv) = 0 \tag{6.3}
$$

While a droplet is traveling in air, two different forces act on it due to i) gravity and ii) drag. The gravity force is always constant and directed toward the bottom of the chamber. However, the drag force is a function of the relative speed between droplet and air as well as the air density. Here, the drag force (on one droplet) is modeled as:

$$
f_{\text{drag}}(x,t) = -\frac{1}{2} C_d A \rho(t)(v(x,t) - U(x,t))v(x,t) - U(x,t) \tag{6.4}
$$
where \( C_d \) is the drag coefficient and \( \overline{A} \) is the projected area. For a spherical droplet moving in air, \( C_d \) is about 0.47 and \( \overline{A} \) is \( 0.25 \pi \overline{d}^2 \) in which \( \overline{d} \) is the droplet diameter assumed to be constant over the whole process. According to (6.4), if \( v - U \) is positive, the drag force is negative (i.e. toward the top of chamber) which means that the drag force is decelerating droplets.

![Figure 6.4: Control volume along the chamber vertical axis used for deriving the governing dynamic equations](image)

By applying the conservation of momentum principle on the same control volume (shown in Fig. 6.4), we will have:

\[
\frac{d}{dt} \left( \overline{m}_d \int_{x}^{x+\Delta x} r(\tau, t) v(\tau, t) d\tau \right) = \overline{m}_d \left( r(x, t) v(x, t)^2 - r(x+\Delta x, t) v(x+\Delta x, t)^2 \right) + \overline{m}_d g \int_{x}^{x+\Delta x} r(\tau, t) d\tau - \int_{x}^{x+\Delta x} f_{\text{drag}}(\tau, t) r(\tau, t) d\tau \quad (6.5)
\]

where \( g \) is the acceleration of gravity and \( \overline{m}_d \) is the mass of each droplet (equal to \( \frac{\pi}{6} \overline{d}^3 \rho_w \) where \( \rho_w \) is the water density). Now, by letting \( \Delta x \to 0 \) and substituting \( \frac{\partial}{\partial \tau} r \) from (6.3), the second PDE describing the droplet’s velocity dynamic obtained as:

\[
\frac{\partial v}{\partial t} + \frac{\partial}{\partial x} \left( \frac{v^2}{2} \right) - g - \frac{f_{\text{drag}}}{\overline{m}_d} = 0 \quad (6.6)
\]

Finally, by applying the conservation of energy on the control volume shown in Fig. 6.4
(considering the liquid phase in it) we have:

\[
\frac{d}{dt} \left( m_d C_w \int_{x}^{x+\Delta x} r(\tau,t) \theta(\tau,t) \, d\tau \right) = m_d C_w \left( r(\tau,t) v(\tau,t) \theta(\tau,t) - r(\tau,\Delta x,t) v(\tau,\Delta x,t) \theta(\tau,\Delta x,t) \right)
\]

\[+ \bar{A}_h \int_{x}^{x+\Delta x} h(\tau,t) \left( T(t) - \theta(\tau,t) \right) \, d\tau \quad (6.7)\]

where \( \theta(x,t) \) is the temperature of the droplet at location \( x \) and time \( t \) while \( T(t) \) is the air temperature which is assumed to be uniform over the chamber volume. In addition, \( C_w \) is the specific heat capacity of water and \( \bar{A}_h \) is the total heat transfer surface area for one droplet given by:

\[ \bar{A}_h = \pi d^2 \]

In (6.7) \( h(x,t) \) is the convective heat transfer coefficient between water droplet and air, which is a function of Reynolds number as well as air temperature. Based on Ranz-Marshall correlation [68], the heat transfer coefficient of a spherical droplet can be calculated as:

\[ Nu(x,t) = \frac{h(x,t)d}{K(T(t))} = 2 + 0.6Re_{(x,t)}^{\frac{1}{2}} Pr^{\frac{1}{3}} \implies h(x,t) = \frac{K(T(t))}{d} \left( 2 + 0.6Re_{(x,t)}^{\frac{1}{2}} Pr^{\frac{1}{3}} \right) \]

\[ (6.8) \]

where \( Nu \) is the Nusselt number, \( Pr \) is the Prandtl number (assumed to be constant) and \( Re \) is the Reynolds number defined based on relative speed between droplet and air as:

\[ Re_{(x,t)} = \frac{\rho(t)\overline{d}|v(x,t) - U(x,t)|}{\mu(t)} \]

\[ (6.9) \]

where \( \rho(t) \) is the air density and \( \mu(t) \) is the dynamic viscosity of air as a function of its temperature given by (based on Sutherland’s formula [69]):

\[ \mu(t) = \mu_r \frac{T_r + C_r \left( \frac{T(t)}{T_r} \right)^{\frac{2}{3}}}{T(t) + C_r \left( \frac{T(t)}{T_r} \right)^{\frac{2}{3}}} \]

\[ (6.10) \]

where \( \mu_r \) is the reference dynamic viscosity of air (equal to \( 1.83 \times 10^{-5} \text{Pa.s} \)) at reference temperature \( T_r \) (291K) and \( C_r \) is the Sutherland’s constant for air (equal to 120K). Note that \( K(T) \) is the conductivity of air as a function of its temperature that is given by:

\[ K(T) = c_3 T^3 + c_2 T^2 + c_1 T + c_0 \]

\[ (6.11) \]
Now, by letting $\Delta x \to 0$ and substituting $\frac{\partial r}{\partial t}$ from (6.3) in (6.7) we can derive the PDE describing temperature dynamic of droplet as:

$$\frac{\partial \theta}{\partial t} + v \frac{\partial \theta}{\partial x} - \frac{6}{C_w \rho_w d} h(x,t)(T(t) - \theta(x,t)) = 0 \quad (6.12)$$

Therefore, the droplet density, velocity and temperature can be calculated by integrating (6.3), (6.6) and (6.12) over space and time.

**Liquid Piston Height Dynamics:**

While the liquid piston level is mainly affected by the liquid flow rate provided by the hydraulic pump, the accumulation of water droplets into the liquid column can also increase its level inside the chamber. Such a consideration becomes more important when the liquid piston is close to chamber’s top and the air pressure is high. In this condition, addition of a small amount of water (as water spray) can cause a large change in air pressure due to its low volume. To find the liquid piston height dynamics, consider a control volume located at the piston surface (shown in Fig. 6.5). This control volume is chosen to contain both liquid piston and water droplet. By applying the conservation of mass principle for the total water inside this control volume, we will have:

$$\frac{d}{dt} \left( A_p \delta_w + \mathbf{V}_d \int_{x^*-\delta_d}^{x^*} r dx \right) = F_p + r(x^*-\delta_d,t) \mathbf{v}(x^*-\delta_d,t) \mathbf{V}_d \quad (6.13)$$

Figure 6.5: Control volume at liquid piston surface

where $A_p$ is the cross section area of the chamber, $\mathbf{V}_d$ is the volume of each droplet (equal to $\frac{\pi}{6} d^3$), and $F_p$ is the flow rate of liquid driven into the chamber by the hydraulic
pump. In addition, \( x^* \) is the location of piston surface with respect to chamber cap (i.e. \( x^*_t = L - Y_t \)). Now, if we let both \( \delta_d \) and \( \delta_w \) approach to zero \( (\delta_d \to 0 \text{ and } \delta_w \to 0) \) in (6.13), we will have:

\[
A_p \dot{\delta}_w + \nabla_a r^* \dot{\delta}_d = F_p + r^* v^* V_d \tag{6.14}
\]

where \( r^* = r(x^*, t) \) and \( v^* = v(x^*, t) \). Considering the fact that \( \dot{\delta}_w = \dot{Y} \) and \( \dot{\delta}_d = -\dot{Y} \), the piston height dynamics can be finally determined as:

\[
\dot{Y}(t) = \frac{F_p(t) + r(x^*, t) v(x^*, t) V_d}{A_p - r(x^*, t) V} \tag{6.15}
\]

**Air Dynamics:**

The air temperature dynamics can be calculated based on the ideal gas law and the total heat transfer between air, water droplets and surrounding environment (such as chamber wall). As shown in Fig. 6.6, the air inside the chamber has heat transfer to both water droplets and the surrounding material. The heat transfer coefficient between air and solid walls as well as liquid piston surface is assumed to be constant \( (\overline{h}) \). However, the heat transfer coefficient between the air and droplets is a function of local Reynolds number given by (6.8). Combining these facts and assumptions, air temperature dynamic is:

\[
-P(t) \dot{V}(t) = Q_{air\rightarrow droplet} + Q_{air\rightarrow wall} + m_{air} C_v \dot{T}(t) \tag{6.16}
\]

where \( P \), \( T \), \( V \) and \( m_{air} \) are the air pressure, temperature, volume and total mass, respectively. Moreover, \( C_v \) is the specific heat capacity of air. In (6.16), \( Q_{air\rightarrow droplet} \) is the heat transfer from air to water droplets, while \( Q_{air\rightarrow wall} \) is the heat transfer from air to chamber’s wall. By using the ideal gas relationship and substituting the appropriate
terms for heat transfer, the air temperature dynamics can be found as:

\[
\frac{dT}{dt} = (1 - \gamma) \frac{T(t)}{V(t)} \dot{V}(t) - \frac{\pi}{m_{\text{air}} C_v} \left( \bar{d}^2 \int_0^{x^*} r_{(\tau,t)} h_{(\tau,t)} (T(t) - \theta_{(\tau,t)}) \, d\tau \right) \tag{6.17}
\]

Heat transfer to droplets \( (H_2) \)

\[
+ \left( D_x^* + \frac{D^2}{2} \right) \bar{K} (T(t) - T_{\text{wall}}) \left( \text{heat transfer to walls (H}_1 \right) \]

where \( \gamma \) is the heat capacity ratio of air and \( \bar{K} \) is the convective heat transfer coefficient between air and its surrounding walls as well as liquid surface (assumed to be constant).

Finally, by applying the conservation of mass principle for the compression chamber, the air volume dynamic can be determined as:

\[
\frac{dV}{dt} = - \left( F_{(t)}^p + F_{(t)}^s \right) \tag{6.18}
\]

where \( F_{(t)}^p \) is the liquid piston flow rate into the chamber, and \( F_{(t)}^s \) is the water spray flow rate. Note that the water droplets leave the spray nozzle with a speed calculated
as:

$$v_{(0,t)} = \frac{F^s_t}{V_d(0,t)}$$  \hspace{1cm} (6.19)

where $V_d$ is the volume of one droplet given by:

$$V_d = \frac{\pi}{6} d^3$$  \hspace{1cm} (6.20)

**Solution Method:**

Complete dynamics of this system is determined by (6.10), (6.12), (6.15), (6.17) and (6.18). The first three equations are PDE with respect to time and space. Finite Volume Method (FVM) is used to transform PDE system into an ODE system. Resulted ODE system in addition to (6.15), (6.17) and (6.18) describe the complete dynamic behavior of the whole system. This ODE system (including $3n + 3$ differential equations, $n$ is the number of finite volumes used in FVM) is then solved in MATLAB® using available ODE solvers.

### 6.3 Sample Case Study

A numerical simulation is performed for a sample case to show how the dynamic states evolve over the compression cycle. Here, a constant flow rate is assumed for the liquid piston ($F^n$). While initially there is no water droplet in the chamber, a constant flow rate spray injects water droplet into the chamber, starting at $t = 0.4$ sec and ends at $t = 0.8$ sec. Here, the droplet diameter is chosen to be $50 \mu m$. The compression ends when the desired compression ratio is achieved ($r_d = 50$). The liquid piston flow rate is chosen for a total compression time of about 1 sec. The rest of the constant parameters used in this simulation are given in Table 6.1. Results of the simulation are shown in Fig. 6.7. Due to extra heat transfer area provided by water droplets after injection, the air is cooled down and its temperature drops for a while. However, after the spray stops, the air temperature rises again until the final desired pressure ratio is achieved.

---

\[^3\] Reducing droplet size (for a fixed mass loading) increases the benefits of using water droplets since it allows for a larger heat transfer area. However, this is limited to a certain size, since droplet diameter less than $20 \mu m$ (for the current system parameters) corresponds to conditions where the droplet thermal response time becomes negligible compared to the compression cycle.
Table 6.1: Constant parameters used for numerical simulations

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</tr>
</tbody>
</table>

Figure 6.7: Air temperature and water spray flow rate vs. time (top), air pressure vs. air volume (bottom)

Droplet density, velocity and temperature distributions over the compression chamber
volume during compression cycle are shown in Fig. 6.8. While droplets leave the spray nozzle with a relatively large velocity (given by (6.19)), they decelerate fast and traverse the rest of their trip between the nozzle and liquid piston with a much smaller velocity. Consequently, the droplets are accumulated in a region between the nozzle and liquid piston surface. The temperature of droplets is equal to the ambient temperature ($T_0$) when they leave the spray nozzle. Due to heat absorption from the compressing air, they heat up and reach the liquid piston surface with a larger temperature. Once the injection stops, this temperature rise gets even larger due to vanishing number of water droplets.

Figure 6.8: Distribution of droplet density $r_{x,t}$ (top-left), velocity $v_{x,t}$ (top-right) and temperature $\theta_{x,t}$ (bottom) over the space and time during the compression cycle
6.4 Optimization of Spray Flow Rate for a Given Mass Loading

In general, increasing the relative amount of water droplets compared to air increases the compression efficiency by improving heat transfer. As defined in chapter 2, compression efficiency is the ratio between stored energy in air (after compression) and the required input work. The stored energy in the air at volume \(V\), pressure \(rP_0\) and ambient temperature is defined as the maximum work obtainable via an isothermal expansion as \([41, 58]\):

\[
W_{\text{stored}} = \int_{V_0}^{rV_0} (P - P_0) dv + (r - 1)P_0V_0 = rP_0V\ln(r) \tag{6.21}
\]

Note that the isobaric injection work (i.e. when compressed air enters the expansion chamber, before the expansion starts) is included in \((6.21)\) while the expansion work due to ambient pressure is subtracted. The input work is the summation of liquid piston work and the water spray work (to inject water droplets into the high pressure air). In addition, the energy loss due to pressure drop across the spray nozzle is also a part of required input work. This pressure drop can be expressed as a function of spray flow rate:

\[
\Delta P_{nz}^{(t)} = \left( \frac{F^s(t)}{K_{nz}} \right)^2 \tag{6.22}
\]

where \(K_{nz}\) is the discharge coefficient of the spray nozzle and depends on droplet size as well as spray nozzle type \([70]\). Thus, the input work can be calculated as:

\[
W_{\text{input}} = -\int_{V_0}^{V_f} (P(t) - P_0) \, dV + P_0(r - 1)V_f + \int_0^{t_f} F^s(t) \Delta P_{nz}^{(t)} \, dt \tag{6.23}
\]

where \(V_f\) is the final air volume at the end of compression \((t = t_f)\). The compression efficiency is then defined as:

\[
\eta_c = \frac{W_{\text{stored}}}{W_{\text{input}}} \times 100\% \tag{6.24}
\]

The baseline compression efficiency is determined according to the adiabatic compression. For a compression ratio of \(r_d = 50\), the adiabatic compression efficiency is about
54.4%. Considering the heat transfer from the surrounding walls (with the same boundary conditions and constant parameters used for the previous case study) the compression efficiency increases to 57%. However, by injecting the water droplets into the compressing air as shown in Fig. 6.7 the compression efficiency increases to about 70.8% which is much higher than the case without spray. To quantify the amount of water added to the air (in terms of water droplets) during the compression cycle, the spray mass loading (ML) is defined as follows:

$$ML = \frac{m_w}{m_{air}} = \rho_w \int_0^T \frac{F_s(t)}{m_{air}} dt$$  \hspace{1cm} (6.25)

For the sample case study that resulted in 70.8% efficiency, ML is obtained to be about 0.5. Although increasing mass loading is assumed to always improve the compression efficiency, looking at (6.23) reveals that it may have negative effect on efficiency due to energy loss across the spray nozzle. In other words, spraying too much water into air may reduce the compression efficiency due to large work required to push the water across the spray nozzle into the compression chamber. Additionally, because of dynamic behavior of droplets inside the air, the timing of water spray is also important in improving the efficiency. For example, spraying water into the air very early or late in time can be useless. Therefore, it is important to find the best spray profile (over time) for a given mass loading and liquid piston profile. This can be formulated as an optimal control problem for which the profit function is given by (6.24), while the dynamic constraint is given by the air compression model including water spray. Note that the algebraic constraints (linear and non-linear) are given desired parameters such as compression ratio, compression time and spray mass loading. Here, we assume that the liquid piston flow profile $F_p$ is specified before hand. Based on these definitions and assumptions, for a given compression time, compression pressure and spray mass loading, the optimal spray profile is:

$$F^*_s(t) = \arg\max_{F_s(t)} (\eta_c)$$  \hspace{1cm} (6.26)

The continuous optimal control problem is parameterized as a finite dimensional problem and then solved numerically by standard algorithms for constrained parameter
optimization. The control input can be parameterized as:

\[ F^*_s(t) = \sum_{i=1}^{T} f_i U_i(t) \quad 0 \leq t \leq t_f \quad (6.27) \]

where \( f_i \)'s are some constant parameters and \( U_i \)'s are a set of basis functions. Here, we used linear function and Gaussian function for \( U_i \) in different case studies. Once the control input defined over the time interval, the dynamic states (i.e. droplet and air properties) can be calculated over the time and space.

### 6.5 Optimal Spray Profile for Constant Piston Flow Rate

Optimal spray flow rate for different mass loadings are found while the liquid piston flow rate is chosen to be constant. Desired final pressure ratio \( r_d \) is 50 and the compression time \( t_f \) is about 1sec. Other constant parameters describing the compression chamber geometry, spray nozzle as well as initial and boundary conditions are given in Table 6.1. Nine equally spaced points over the time range are used to discretize the control input \( F^*_s \). The optimal spray flow rate for different mass loadings are shown in Fig. 6.9. Note that each flow profile is normalized based on its own mass loading. The thick blue curve represents the time average of all optimal spray flow rates resulted for different mass loadings.
Figure 6.9: Comparison between optimal spray flow rate and constant spray flow rate, normalized optimal spray flow rate (top), temperature vs. volume (middle), and efficiency vs. mass loading (bottom). Note that there is a maximum for compression efficiency as a function of spray mass loading, since spraying too much water requires large work to overcome the spray nozzle pressure drop (see (6.23)).

The trend of these optimal spray profiles (Fig. 6.9, top) are expected considering the fact that at the first half of the compression, there is enough heat transfer area provided by the surrounding walls while the air temperature is still not high. Thus, additional
cooling with water droplet is not necessary in this phase. On the other hand, injecting droplets into the air when the liquid surface is close to the chamber’s top cannot be very effective due to rapid transition of droplets from the spray nozzle into the liquid piston. In this situation, injected droplets will not have enough time to capture heat from air before touching the liquid piston surface. As shown in Fig. 6.9-middle, the air temperature of the optimal spray profile is higher than the constant flow spray in the first half of the compression process. However, the optimal spray profile does a better job and reduces the air temperature more in the second half (since some droplets are saved from the first half). Hence, as expected, the overall compression efficiency of optimal profile is higher than the constant flow rate case. Such an improvement is shown in Fig. 6.9-bottom where the compression efficiency for different mass loadings is shown for both optimal and constant spray flow rate. While for small and large mass loadings the optimal and constant spray result in similar efficiencies, their difference can go up to 2% for a mass loading of 0.5. Note that the compression ratio and compression time are the same for all cases. In particular, note that the compression efficiency decreases for very large mass loadings since the energy loss across the spray nozzle becomes a dominant term in the input work.

6.6 Optimal water spray profile for the optimal compression flow trajectory

Although the optimal spray profile improves the compression efficiency, it is not still satisfactory for the application of CAES system. For such a compressor, a minimum thermal efficiency of 90% is required to achieve a reasonable round-trip efficiency for the storage system. As discussed earlier, one effective way to improve compression efficiency is to increase heat transfer area inside the compression chamber by inserting some porous materials into the chamber. This will also increase the convective heat transfer coefficient between air and solid wall by reducing hydraulic diameter. Additionally, the piston flow rate can be optimized to improve the efficiency through a better use of available heat transfer capacity (as discussed in chapter 3). Let’s consider the design of an air compressor for the second stage compression in a CAES system, where the initial
(inlet) pressure is 5bar\(^4\) and the desired compression ratio is 40 (the final pressure is 200bar). The required power density for this system is about 1.8MW/m\(^3\). According to the definition of power density given by (2.7) and geometry of the chamber given in Table 6.1, the compression time must be around 1sec. If the chamber is filled with porous medium (parallel plates, with uniform porosity of 90%), the product of heat transfer coefficient and heat transfer area ($\overline{h}A$) can be increased by a factor of 50. For this geometry, a constant piston flow rate results in a compression efficiency of 74.4% (while the compression time is 1sec). By optimizing the piston flow rate, compression efficiency increased to 77.2% (Fig. 6.10-top). Introducing water spray at constant flow rate during the compression (for the optimal piston flow rate) improves the efficiency to 90.7% (for 50µm droplet diameter and a spray mass loading of 5). This efficiency can be further improved if the constant flow spray is replaced by the optimal spray flow rate. Here, in order to have a smoother optimal spray profile, a combination of Gaussian functions are used as the basis functions in (6.27) to parameterize the spray profile over the compression time. In this way, the optimization will be summarized as finding the optimal set of amplitudes for these functions. For the same mass loading (ML=5), the optimal spray flow rate is found as shown in Fig. 6.10-top. Applying this spray profile, the compression efficiency can be increased to 94.5% which has a noticeable difference compared to the constant spray flow. This step-by-step improvement of compression efficiency can be summarized as:

* Constant compression flow rate (0.6lit/sec), no water spray $\rightarrow$ efficiency: 74.4%

* Optimal compression flow rate, no water spray $\rightarrow$ efficiency: 77.2%

* Optimal compression flow rate with constant water spray flow rate (17.8cc/sec) $\rightarrow$ efficiency: 90.7%

* Optimal compression flow rate with optimal water spray flow rate $\rightarrow$ efficiency: 94.5%

Note that for all these cases, the compression time is 1sec, and initial and final pressures are 5bar and 200bar, respectively. Moreover, the compression chamber volume is about 0.59liter, filled with porous material with uniform porosity of 90%. Fig. 6.10-bottom

\(^4\)In the first stage, air is compressed from the ambient pressure to the final pressure of 5bar.
shows the air temperature versus volume for these four different compression cases, in addition with the adiabatic compression (as the baseline case). As shown, by reducing the air temperature rise over the compression process, the compression efficiency will be improved.

Figure 6.10: Optimal compression piston profile for the given compression ratio and compression time with the corresponding optimal spray profile for the given mass loading of 5 (top); temperature versus volume for five different cases (bottom)

6.7 Summary

Equipping a liquid piston air compressor with a water droplet spray can improve the compression efficiency significantly. However, for a given compression chamber geometry and liquid piston flow profile, the optimal spray profile can improve the compression efficiency even more than constant flow spray with the same mass loading. In this chapter, a general numerical optimization approach was proposed to optimize the spray
profile for different mass loadings and liquid piston profiles. For a constant liquid piston flow rate and compression ratio of 50, up to 2% improvement in efficiency was obtained by optimizing the spray profile. Similarly, the spray profile was optimized for the optimal liquid piston profile in a compression chamber with porous inserts. Combination of these heat transfer enhancement methods allows us to design an efficient and power dense air compressor where the compression efficiency is boosted up from 74.2% to 94.5%. Potentially, this improvement can be increased by simultaneous optimization of liquid piston and spray profiles instead of individual optimizations (similar to what presented in chapter 3). In addition, it is observed that the water spray is more useful at the end of compression process where the air temperature is high. However, due to small transition time of droplets between the nozzle and the liquid piston surface, it would be better to change the direction and/or location of spray nozzles. For example, spraying from the sides of the compression chamber (and close to the top) in a radial direction can be more effective as a result of longer lifetime of water droplets.
Chapter 7

Modeling and Controller Design for the Combined Wind Turbine and Energy Storage System

7.1 Introduction

So far in this dissertation, the main challenges corresponding to the design of an efficient and power dense air compressor/expander were addressed, while practical solutions were introduced and studied to overcome them. While storing/regenerating energy in an efficient way is the long-term (i.e. large time-scale) objective of the CAES system under this study, there are many short-term (i.e. small time-scale) benefits that are potentially achievable through the CAES system. For example, if an appropriate coordination exists between the subsystems in the combined wind turbine and energy storage system, it would be possible to maintain system pressure as well as electric generator frequency regardless of the stored energy level in the storage vessel. In addition, it is potentially possible to capture the maximum wind power by the wind turbine while delivering the demanded power to the electric grid (since the wind turbine and electric generator shafts are decoupled). Obtaining all these tasks simultaneously requires a good control and cooperation between different elements in the energy storage system.
To better understand how different components in the combined system affect each other, the overall system behavior is described briefly here again (see Fig. 7.1). A variable displacement hydraulic pump (B) attached to the wind turbine rotor (A) in the nacelle converts wind power to hydraulic power. At down-tower (H), a variable displacement hydraulic pump/motor (C), a near-isothermal liquid piston air compressor/expander (F) and a fixed-speed induction generator (G) are connected in tandem on a common shaft. They are powered by the pump (B) and exchange power with the storage vessel (E) with both liquid (hydraulic fluid) and compressed air at the same pressure. This allows energy to be stored in, or extracted from, (E) either hydraulically (as in a conventional hydraulic accumulator) or pneumatically (as in a conventional air receiver). In both cases, energy is stored in the compressed air. By coordinating the hydraulic and the pneumatic power paths, the pressure in (E) can be maintained constant regardless of energy content\textsuperscript{1}. For example, as compressed air is being released from (E), some liquid can be added to reduce the compressed air volume to maintain the pressure. The pneumatic power branch makes better use of the vessel (E) volume than the hydraulic power branch (a compressed air tank stores 20 times more energy than a hydraulic accumulator at the same peak pressure and total volume \cite{35}) but hydraulic pump/motors are more power dense than the pneumatic compressor/expanders. This architecture can take advantage of both by utilizing the hydraulic path to accommodate high power transient events such as a wind gust or a sudden power demand, reserving the pneumatic path for steady power. As described in the previous chapters, the liquid piston air compressor/expander (F) consists of an air compression/expansion chamber filled with porous media (F\textsubscript{1}), and a water pump/motor (F\textsubscript{2}). When storing energy pneumatically, the liquid piston pump/motor (F\textsubscript{2}) pumps water into the compression/expansion chamber, compressing the air within it. When the chamber pressure exceeds that of the storage vessel (E), the compressed air is ejected and stored in the vessel. The chamber is then refreshed by releasing the water and filling it with atmospheric air for the next cycle. When retrieving energy, the compressed air is released into the expansion chamber. As the air expands, the liquid piston retreats, the liquid piston pump/motor (F\textsubscript{2}) is motored and work is derived.

\textsuperscript{1}Unlike a conventional closed hydraulic accumulator with only a hydraulic port, or a compressed air receiver with only a pneumatic port.
The first step to study the dynamic interaction between system components is to model the dynamic behavior between inputs and outputs for each subsystem. In this chapter, a cycle-by-cycle average modeling is used to derive the governing dynamic equations of energy storage and wind turbine system in Section 7.2. The dynamic model is then used in Section 7.3 to design an appropriate non-linear controller in order to achieve the short-term objectives (such as regulating the system pressure at its desired value and tracking the electric generator desired power). The controller is then modified to meet the bandwidth requirements of each component in the energy storage system. Finally, the efficacy of this controller in obtaining the pre-defined goals is studied in Section 7.5 through a series of long-term and short-term numerical simulations.

Figure 7.1: Schematic of the proposed “open accumulator” Compressed Air Energy Storage (CAES) system

7.2 Dynamic Modeling

7.2.1 Wind Turbine

A wind turbine extracts wind kinetic energy from the swept area of its rotor blades. The aerodynamic torque for a given wind speed $V_w$ and rotor speed $\omega_r$ on the wind
turbine shaft is given by [29, 72]:

\[ T_w = \frac{1}{2} \rho_0 \pi R_r^2 C_P(\beta, \lambda) \frac{V_w^3}{\omega_r} \]  \hspace{1cm} (7.1)

where \( \rho_0 \) is the air density, \( R_r \) is the radius of the turbine, \( C_P \) is the power coefficient, \( \beta \) is the collective pitch angle and

\[ \lambda(\omega_r, V_w) := \frac{R_r \omega_r}{V_w} \] \hspace{1cm} (7.2)

is the tip speed ratio. For maximum power capture within the turbine capability (i.e. in region 2\textsuperscript{2}), the turbine tip speed ratio should be at its optimal setting \( \lambda_{opt} = 8.1 \) and \( \beta = 0 \) (see Fig. 7.2). At high wind speeds (region 3), \( \beta \) is adjusted for power curtailment.

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\textsuperscript{2}It is an operational mode of a wind turbine over which it is desirable that the turbine captures as much power as possible.
7.2.2 Nacelle (B) and down-tower (C) Pump/Motors

The nacelle pump (B) is directly connected to the wind turbine with speed $\omega_r$ and the down-tower pump/motor (C) is in series with the generator and air compressor/expander having speed $\omega_g$. Both units are connected to the accumulator (E) at pressure $P_{acc}$. Thus, the hydraulic flow $Q$ and torque $T$ for the nacelle (B) and down-tower (C) pump/motors are given by:

\[ Q_p = \frac{D_p(t)}{2\pi} \omega_r - L_p(D_p, \omega_r, r) \]  
\[ T_p = -\frac{D_p(t)}{2\pi} P_0(r - 1) - \Gamma_p(D_p, \omega_r, r) \]  
\[ Q_{pm} = \frac{D_{pm}(t)}{2\pi} \omega_g - L_{pm}(D_{pm}, \omega_g, r) \]  
\[ T_{pm} = -\frac{D_{pm}(t)}{2\pi} P_0(r - 1) - \Gamma_{pm}(D_{pm}, \omega_g, r) \]

where the subscripts “p” and “pm” refer to the nacelle pump and the down-tower pump/motor respectively, $D_{p/pm}(t)$ is the input displacement of the pump/motor and $L_{p/pm}(\cdot)$ and $\Gamma_{p/pm}(\cdot)$ are the flow and torque losses, $r := P_{acc}/P_0$ is the ratio of the accumulator (E) pressure $P_{acc}$ to the ambient pressure $P_0$. The nacelle pump displacement is restricted to $D_p(t) \in [0, D_{p,max}]$ while the down-tower pump/motor displacement can go over center: $D_{pm}(t) \in [-D_{pm,max}, D_{pm,max}]$. In our notation, $D \cdot \omega > 0$ implies that the pump/motor is pumping. The overall efficiency of a hydraulic machine is a function of its displacement, pressure and speed [73]. A sample pump efficiency plot for a fixed line pressure is shown in Fig. 7.3. Note that efficiency is reduced as displacement is low.

7.2.3 Storage Vessel (E)

The storage vessel is a fixed volume ($V_{acc}$) container with both compressed air (volume $V(t)$ and mass $m(t)$) and liquid (volume $V_{oil}(t) = V_{acc} - V(t)$) at the same pressure $P_{acc} = r P_0$. For the current model, it is assumed that the compressed air from the compressor has been cooled down and enters the vessel at ambient temperature $T_0$ (Fig. 7.4). This is a conservative assumption for estimating energy content as in reality, the liquid can either be hydraulic oil or clean water. A barrier or bladder may be used to separate the liquid and compressed air if air absorption into the liquid becomes problematic.
Figure 7.3: Overall efficiency of a typical hydraulic pump as a function of displacement and speed (@200 bar).

Compressed air entering the storage vessel will be at an elevated temperature resulting in more available energy. The compressed air volume and pressure (ratio) dynamics are given by:

\[
\dot{V} = -Q_{acc} = -(Q_p + Q_{pm}) \quad (7.7)
\]
\[
V\dot{r} = \left(\frac{1}{\rho_0}\right) \dot{m} + rQ_{acc} \quad (7.8)
\]

where \(Q_{acc}\) and \(\dot{m}\) are the liquid volumetric flow rate and mean air mass flow rate into the storage vessel and \(\rho_0 = P_0/(RT_0)\) is the air density at the ambient condition. The stored energy in the compressed air with volume \(V\) and pressure ratio \(r\) is defined to be the maximum work achievable through isothermal expansion at \(T_0\) [41]:

\[
E_{st}(r, V) = P_0V (r \ln(r) - r + 1) \quad (7.9)
\]

The actual compression work and the work that can be extracted from the compressed air differ from \(E_{st}(r, V)\). The compressor and expander efficiencies are, respectively, the ratios of the stored energy to the actual work input, and of the actual work output to stored energy.
7.2.4 Liquid Piston Air Compressor/Expander

The air compressor/expander uses mechanical energy from the rotational shaft to compress air for storage, and extracts energy from the compressed air through expansion to drive the shaft. In the liquid piston air compressor/expander, the transmission is via a water column and the liquid piston pump/motor (F₂). The compression/expansion chamber (F₁) is filled with a porous material for enhanced heat transfer, as described in chapter 2. When storing energy, i) the compression chamber is filled with atmospheric air; ii) water is pumped into the compression chamber, compressing the air; iii) when the pressure reaches \( P_{\text{acc}} \) of the storage vessel (E), air is injected into (E); iv) the liquid piston retreats, a valve opens to the atmosphere, the chamber is filled with atmospheric air and the cycle repeats. When extracting energy, i) the chamber is filled with water; ii) some amount of compressed air is injected from the storage vessel to the chamber; iii) air valve closes and the compressed air expands pushing the liquid piston and motoring the hydraulic pump/motor; iv) a valve opens, ejecting the expanded air to the atmosphere as the liquid piston refills the chamber and the cycle repeats. The liquid piston pump/motor operates cyclically with a period of \( \sim 1-2\text{sec} \).

The tradeoff between work input/output (hence efficiency) and compression/expansion times (hence power) are heavily dependent on the compression/expansion trajectory [38, 41]. To capture this tradeoff in an overall system model, a continuous cycle-average model for the compressor/expander is developed here based on a family of Adiabatic-Isothermal-Adiabatic (AIA) compression/expansion trajectories for the liquid piston, as
proposed in [41] (Fig. 7.5). This assumption is made because in the case when the \( hA \) product (where \( h \) is the convective heat transfer coefficient and \( A \) is the heat transfer area) is constant, AIA trajectories result in optimal Pareto tradeoff between efficiency and power density [41]. In implementation, the actual optimal trajectories would also have similar characteristics as AIA trajectories. Moreover, viscous liquid friction losses inside the chamber have been shown to be small compared to thermodynamic losses, with proper implementation [58]. Leakages can be lumped into the flow loss of the liquid piston pump/motor (\( P_2 \)).

![Diagram of Adiabatic-Isothermal-Adiabatic (AIA) compression/expansion trajectories](image)

Figure 7.5: Adiabatic-Isothermal-Adiabatic (AIA) compression/expansion trajectories minimize time (for the given efficiency), or efficiency (for a given time), assuming a constant \( hA \) during the compression/expansion. The isothermal temperature \( T_1 \) defines the Pareto optimal tradeoff between efficiency and power [41].

The control input to the liquid piston air compressor/expander is the cycle average displacement \( D_{lp} \). Together with shaft speed \( \omega_g \) and pressure ratio \( r \), the air mass flow rate and liquid piston pump/motor torques are:

\[
\dot{m} = \rho_0 \left[ \frac{D_{lp}(t)}{2\pi} \omega_g - L_{lp}(D_{lp}, \omega_g, r) \right] \quad (7.10)
\]

\[
T_{lp} = -\frac{D_{lp}}{2\pi} (P_w - P_0) - \Gamma_{lp}(D_{lp}, \omega_g, r) \quad (7.11)
\]
where $\overline{P}_w(D_{lp}, \omega_g, r)$ is the cycle average in chamber pressure, $L_{lp}(\cdot)$ and $\Gamma_{lp}(\cdot)$ are the volumetric and mechanical losses of the liquid piston compressor/expander. Detail derivation of these expressions as well as the thermal efficiencies of the compressor/expander are given in Appendix A. Since both the generator shaft speed $\omega_g$ and the accumulator pressure ratio $r$ will be maintained at their desired values ($r = 200$, $\omega = 1800\text{rpm}$), the cycle-average pressure $\overline{P}_w$ and thermal efficiency of the chamber are mainly functions of cycle average displacement $D_{lp}$. Figure 7.6 shows that efficiency decreases as $|D_{lp}|$ increases. Also, cycle average pressure $\overline{P}_w$ is small compared to accumulator pressure $rP_0$. This is the reason why air compressors/expanders are much less power dense than hydraulic pumps/motors.

Figure 7.6: Cycle-average thermal efficiency $\eta_{tm}$ and cycle-average water pressure $\overline{P}_w$ of compression/expansion chamber at various mean liquid piston pump/motor displacement (for the compression/expansion ratio of 200 and shaft speed of 1800rpm).

### 7.2.5 Induction Generator

A three-phase induction generator is assumed. Its static torque-speed relationship is given by [72]:

$$T_g(\omega_g) = \frac{3\rho_f r_r}{2(\omega_a - \omega_g)} \times \frac{V_s^2}{(r_s + \frac{r_r}{\omega_a - \omega_g})^2 + (X_s + X_r)^2}$$

(7.12)
where $\omega_g$ is the generator speed, $\omega_s = \frac{2\pi f_s}{p_f}$ is the synchronous speed with $f_s = 60$ Hz is the grid frequency, $r_r$ and $r_s$ are the resistances and $X_r$ and $X_s$ are the leakage reactances of the rotor/stator, $V_s$ is the voltage and $p_f$ is the number of poles. The machine is a generator when $\omega_s - \omega_g < 0$ and is a motor otherwise. From (7.12), at a given grid frequency $f_s$, $\omega_g$ is the only free variable, therefore controlling $\omega_g$ is equivalent to controlling the generator power:

$$\text{GeneratorPower} = T_g(\omega_g) \cdot \omega_g = f(\omega_g) \quad (7.13)$$

### 7.2.6 Inertia Dynamics

The inertia dynamics of the turbine rotor is:

$$J_r \ddot{\omega}_r = T_p + T_w \quad (7.14)$$

where $J_r$ is the moment of inertia of the rotor, $T_p$ is the hydraulic pump (B) torque and $T_w$ is the torque applied by wind (given by (7.1) and (7.4)). The inertia dynamics of the generator shaft is given by:

$$J_g \dot{\omega}_g = T_{lp} + T_{pm} + T_g \quad (7.15)$$

where $J_g$ is the moment of inertia of the generator shaft and $T_{pm}$, $T_{lp}$ and $T_g$ are, respectively, the down-tower pump/motor (C) torque in (7.6), the liquid piston air compressor/expander (F$_2$) torque in (7.11) and the generator torque (G) in (7.12).
Summary of Dynamic Model

In summary, the overall dynamics of the combined wind turbine and the storage system are:

\[
J_r \ddot{\omega}_r = -\frac{D_p}{2\pi} P_0 (r - 1) - \Gamma_p (D_p, \omega_r, r) + \frac{1}{2} \rho_0 \pi R_s^2 C_P (\beta, \lambda) \frac{V_w^3}{\omega_r} \tag{7.16}
\]
\[
J_g \ddot{\omega}_g = -\frac{D_{pm}}{2\pi} P_0 (r - 1) - \frac{D_{lp}}{2\pi} (P_w - P_0) - T_g (\omega_g) - \Gamma_{pm} (D_{pm}, \omega_g, r) - \Gamma_{lp} (D_{lp}, \omega_g, r) \tag{7.17}
\]
\[
V_r = \frac{1}{2\pi} \left( D_{lp} \omega_g + D_p \omega_r, r + D_{pm} \omega_r r \right) - r L_p (D_p, \omega_r, r) - r L_{pm} (D_{pm}, \omega_g, r) \tag{7.18}
\]
\[
V = -\frac{1}{2\pi} \left( D_p \omega_r + D_{pm} \omega_g \right) + L_p (D_p, \omega_r, r) + L_{pm} (D_{pm}, \omega_g, r) \tag{7.19}
\]

where “r” and “g” are subscripts standing for the turbine rotor and generator. Note that \(D_p, D_{pm}\) and \(D_{lp}\) are control inputs to the dynamic system.

7.3 Controller Design

The overall control objectives of the combined wind turbine and the storage system can be summarized as: i) capturing maximum available power from wind by controlling the wind turbine angular speed; ii) maintaining the accumulator pressure ratio during the energy storage or regeneration mode; and iii) providing the required power demanded by the electrical grid. Since the wind turbine shaft and the generator shaft are not coupled in the proposed architecture, it is possible to achieve all these goals at the same time.

Wind turbine control is performed by utilizing standard torque control approach through the hydraulic pump (B) located in the nacelle. The accumulator pressure and generator power will be controlled by the down-tower variable displacement pump/motors (C) as well as the liquid piston air compressor/expander unit (F_2). Notice that while the controller relies on the generator shaft speed error, the actual error is determined by calculating the generator power (from the measurement of its phase voltage and current) instead of measuring its shaft speed.
7.3.1 Storage System and Generator

In conventional CAES systems, the pressure in the storage vessel drops as compressed air in the storage vessel depletes, making it difficult for the air compressor/expander to maintain either its efficiency or power at all energy levels. In the open accumulator design, it is possible to maintain the pressure no matter how much compressed air is inside the vessel. Therefore, the controller should maintain the desired power on the generator shaft and the desired pressure ratio in the accumulator. Because the storage vessel is assumed to have enough space for the compressed air, the air volume dynamics given by (7.19) is not controlled at this level but will be controlled by a high-level supervisory controller instead.

To design the nonlinear controller, a generator shaft speed error $\tilde{\omega}_g$ and accumulator pressure ratio tracking error $\tilde{r}$ dependent Lyapunov function is first defined. Consider a Lyapunov function $E(\tilde{r}, \tilde{\omega}_g)$ motivated by the kinetic and potential energies of the system defined to be:

$$ E := \frac{1}{2} J_g \tilde{\omega}_g^2 + \int_{V} P_0 \tilde{r} dv $$

where $J_g$ is the generator shaft inertia, and the errors are defined as:

$$ \tilde{r} = r - r_d $$
$$ \tilde{\omega}_g = \omega_g - \omega_g^d $$

with $r_d$ and $\omega_g^d$ being the desired pressure ratio and generator speed. Note that the shaft speed error dependent part of $E$ takes the form of a kinetic energy, and the pressure dependent part is the required energy to compress-expand a fixed mass of air at pressure $rP_0$ and volume $V$ to the desired pressure $r_dP_0$, isothermally. Then the Lyapunov function at the current pressure ratio $r$ and volume $V$ becomes:

$$ E = \frac{1}{2} J_g \tilde{\omega}_g^2 + P_0 V \left( r \ln \left( \frac{r}{r_d} \right) - \tilde{r} \right) $$

By taking time derivative of (7.23) and substituting equivalent terms from (7.17), (7.18) and (7.19), we have:

$$ \frac{1}{P_0} \dot{E} = - F \tilde{\omega}_g - G \tilde{r} - H \gamma(r) \tilde{r}^2 $$

(7.24)
where $F$, $G$ and $H$ are defined as:

$$F = \frac{D_{pm}}{2\pi} (r - 1) + \frac{D_{lp}}{2\pi} \left( \frac{P_w}{P_0} - 1 \right) + \frac{\Gamma_{pm} + \Gamma_{lp} + T_g}{P_0}$$  \hspace{1cm} (7.25)

$$G = -\frac{1}{2\pi} \left( D_p \dot{\omega}_r + D_{pm} \omega_g + \frac{D_{lp}}{r_d} \omega_g \right) + \left( L_p + L_{pm} + \frac{L_{lp}}{r_d} \right)$$  \hspace{1cm} (7.26)

$$H = \frac{D_{lp}}{2\pi} \omega_g - L_{lp}$$  \hspace{1cm} (7.27)

since $\ln\left( \frac{r}{r_d} \right)$ is a monotonically increasing function of $\tilde{r}$, $\ln\left( \frac{r}{r_d} \right)$ is replaced by:

$$\ln \left( \frac{r}{r_d} \right) = \tilde{r} - \gamma(\tilde{r}) \tilde{r}^2$$  \hspace{1cm} (7.28)

where $\gamma$ is a strictly positive function of $\tilde{r}$. Now, if $D_{pm}$ and $D_{lp}$ are controlled such that $F = K_1 \tilde{\omega}_g$ and $G = K_2 \tilde{r}$, where $K_1$ and $K_2$ are two positive gains, then $\dot{E}$ becomes:

$$\dot{E} = -P_0 K_1 \tilde{\omega}_g^2 - P_0 (K_2 + H \gamma(\tilde{r})) \tilde{r}^2$$  \hspace{1cm} (7.29)

Therefore, if $K_2 > 0$ is chosen to be large enough, $K_2 + H \gamma(\tilde{r})$ is always positive and $\dot{E}$ is negative definite. Note that this analogy is valid over a large range of pressure ratios since:

$$\text{Max} \left( \gamma(\tilde{r}) \right) = 10^{-4}, \hspace{1cm} \tilde{r} \in [0, 200]$$  \hspace{1cm} (7.30)

For example, if $\text{min}(H) = -4m^3/s$, then $K_2 > 4 \times 10^{-4}$ will satisfy this requirement. Since $E$ is positive definite and radially unbounded, based on Lyapunov stability criteria, the system is asymptotically stable at the desired pressure ratio and generator speed \[74\]. Therefore, according to (7.25) and (7.26), the displacement commands for the storage system can be found by solving the system of equations given by:

$$(r - 1) \frac{D_{pm}}{2\pi} + \left( \frac{P_w}{P_0} - 1 \right) \frac{D_{lp}}{2\pi} = K_1 \tilde{\omega}_g - \frac{(\Gamma_{pm} + \Gamma_{lp} + T_g)}{P_0}$$  \hspace{1cm} (7.31)

$$\omega_g \frac{D_{pm}}{2\pi} + \frac{\omega_g D_{lp}}{r_d} = -K_2 \tilde{r} - \omega_r \frac{D_p}{2\pi} + \left( L_p + L_{pm} + \frac{L_{lp}}{r_d} \right)$$  \hspace{1cm} (7.32)

Note that $P_w$, $\Gamma$ and $L$ are dependent on the control inputs $D_{pm}$ and $D_{lp}$. In the implementation, they are calculated using the $D_{pm}$ and $D_{lp}$ from the previous time step to simplify the computation of the control inputs.
Although the convergence of states is guaranteed by this controller, a practical drawback is that the high-frequency terms appear in the control command of the compressor/expander ($D_{lp}$). These high-frequency signals, either from a wind gust or a sudden variation in power demand, can adversely affect the liquid piston air compressor/expander performance and may reduce its operational life. In the case of a transient, high-power wind, it will be more efficient to store its energy via the hydraulic path directly into the accumulator, by-passing the down-tower hydraulic pump/motor or pneumatic components. Furthermore, storing this transient high power through the hydraulic path can allow the down-tower hydraulic pump/motor and liquid piston air compressor/expander to be sized for mean wind power instead of peak power.

### 7.3.2 Control Effort Distribution Based on Subsystems’ Bandwidths

The bandwidth related concern above can be solved by using a unique feature of the storage system. The pressure and the generator shaft dynamics in (7.17) and (7.18) show that, although the generator shaft speed has fast dynamics in response to the actuator displacements, the accumulator pressure dynamics is relatively slow (except for the case when the air volume inside the storage vessel is too small). This property is used here to modify the nonlinear control commands to relax the high frequency concern. The idea is to use a low-pass filter to remove the high-frequency components of the liquid piston pump/motor displacement command signal. Let $\tilde{D}_{lp}$ be the solution to (7.31) and (7.32). Then the dynamics for the low-pass filtered liquid piston pump/motor displacement control input are:

$$\dot{\tilde{D}}_{lp} = \frac{1}{\tau} (\tilde{D}_{lp} - D_{lp})$$

(7.33)

where $\tau$ is a time constant that depends on the available bandwidth of the pump/motor in the liquid piston air compressor/expander. To ensure that generator speed control is not compromised, $D_{pm}$ is obtained by solving (7.31) again with $\tilde{D}_{lp}$ given by the solution to (7.33). The stability of both states (pressure and speed) can be proved even

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4Note that in order to maximize the efficiency of air compression/expansion process in the chamber, the liquid piston pump/motor displacement must be controlled precisely over a time range of 1 second to track the optimal AIA trajectory. Therefore, any abrupt change in its displacement can result in the compression/expansion profile to deviate from the optimal trajectory, which will reduce efficiency.
by such a modification similar to [70]. In this way, the liquid piston pump/motor (F_2) will loosely control the accumulator pressure ratio around its desired value during the storage or regeneration phase. On the other hand, the hydraulic pump/motor (C) will precisely control the generator shaft speed to maintain the desired generation power (Fig. 7.7).

This control scheme has an additional advantage in that the accumulator acts as a damper by absorbing the high-frequency flow fluctuations from the pump in the nacelle. These high-frequency power flows that are mainly caused by wind gusts will be stored in the accumulator through the power-dense hydraulic path. The nonlinear control law given in (7.31) and (7.32) uses pressure and generator shaft speed as feedback. A small error in generator speed can lead to a large error in estimated generator power. In practice, it is more reasonable and realistic to measure the instantaneous generator power based on its phase, voltage and current. The generator speed can then be inferred based on the generator’s electrical and mechanical characteristics given in (7.12)-(7.13) and Fig. 7.8. Note that the one-to-one relationship between power and shaft speed is valid over the stable region of power-speed correlation where the generator power increases by increasing the speed. If the generator is assumed to operate below its maximum power, this condition is satisfied.
Figure 7.8: Typical generator power vs. generator shaft speed for an induction generator with 2 pairs of poles. The synchronous speed is 60 Hz. Note that negative power refers to the generation mode.

7.3.3 Wind Turbine Torque Control

The nacelle pump (B) is used to control the turbine speed via the standard torque controller to maximize power capturing [77]:

\[ T_p = -K_s \omega_r^2 \quad (7.34) \]

\[ K_s = \frac{1}{2} \rho_0 \pi r^5 \frac{C_{p \text{max}}}{\lambda^3} \quad (7.35) \]

Pump displacement \( D_p \) is then set according to (7.16). When the measurements and turbine model are perfect, the rotor speed converges to the optimal value (Fig. 7.9 and (7.36)). In region 2 of wind speed, \( \beta=0 \) in order to capture maximum wind power. In this case, \( \lambda^* = 8.1 \) gives the maximum power factor \( (C_{p \text{max}} = 0.48) \).

\[ \dot{\omega}_r = \frac{1}{2J_r} \rho_0 \pi r^5 \omega_r^2 \left( \frac{C_p}{\lambda^3} - \frac{C_{p \text{max}}}{\lambda^*} \right) \quad (7.36) \]

\[ \Rightarrow \begin{cases} 
    C_p < \frac{C_{p \text{max}}}{\lambda^*} \lambda^3 \Rightarrow \dot{\omega}_r < 0 \\
    C_p > \frac{C_{p \text{max}}}{\lambda^*} \lambda^3 \Rightarrow \dot{\omega}_r > 0 
\end{cases} \]
Speed regulation of the rotor at higher wind speeds (region 3) is typically achieved using a PID controller:

\[
\beta(t) = K_p \dot{\omega}_r(t) + K_I \int_0^t \dot{\omega}_r(\tau)d\tau + K_D \frac{d\dot{\omega}_r(t)}{dt} \tag{7.37}
\]

where \( \dot{\omega}_r = \omega_r - \omega_d \) is the rotor speed error, the difference between the desired rotor speed and the measured rotor speed. In this region, the primary objective is to limit the turbine power so that safe mechanical loads are not exceeded. Power limitation can be achieved by pitching the blades or by yawing the turbine out of the wind (not considered here), both will reduce the aerodynamic torque below that which is theoretically available from an increase in wind speed [77].

![Graph](image)

Figure 7.9: Power factor versus tip speed ratio for zero pitch angle.

### 7.4 System Sizing Study

As a case study, the storage system is designed for an offshore wind turbine with a rotor radius of 45m (power rated at 3MW for wind speed rated at 12m/s). The wind profile is generated by superimposing a 10-minute average wind speed profile and the corresponding turbulent wind (Fig. 7.10). Suppose a 600kW constant power is demanded by the grid. Then, the storage system must be able to store about 16MWhr of energy.
At a pressure ratio of 200, from (7.9), the storage vessel size of 670m$^3$ is required. This size is quite compatible with turbines of this capacity and is smaller than the volume in the tower. To capture the maximum power at the rated wind speed, if directly coupled to the rotor, at pressure ratio of 200, a nacelle hydraulic pump (B) displacement $D_{p}^{\text{rated}} = 440\text{lit/rev}$ is needed at maximum speed of 20.5rpm. Correspondingly, the hydraulic pump/motor (C) rotating at the generator speed ($\sim 1800\text{rpm}$) will need a displacement of $D_{\text{pm}}^{\text{rated}} = 5\text{lit/rev}$.

To size the liquid piston air compressor/expander, we prescribe the maximum power to be 2MW and the minimum thermal efficiency to be 90%. At this rated power, an atmospheric air flow rate of 3.75m$^3$/s is needed. At 1Hz, this corresponds to an air compressor/expander displacement of 3.75m$^3$. From (A.2) and (A.8), a heat transfer capability of $h_A = 59\text{kW/K}$ would be sufficient. Since the cycle-average water pressure of the compression/expansion chamber is around 6.3bar, the rated cycle-average displacement for the pump/motor connected to the chamber (F$_2$) will be about 125lit/rev (at 1800rpm). Note that the instantaneous flow rate and displacement for the liquid piston pump/motor would be larger to meet the requirement of the optimal compression/expansion profile. The bandwidth ($\Omega$) of each actuator is also considered in the simulation. The rest of parameters and values used for this simulation are given in Table 7.1.
7.5 Simulation Results

7.5.1 Long-Term Simulation: Overall System Behavior

A long-term simulation (72 hours) has been done to show how the overall storage system works and interacts with the wind turbine. The mean wind speed is a recorded series of data at 60m elevation [79], while the turbulent wind speed is generated by Turbsim software based on the mean wind speed profile [80]. Figure 7.10 shows the captured wind power through the pump located in the nacelle as well as accumulator energy level. Energy is stored during high wind speed conditions, and regenerated during low wind speed conditions. The desired electrical power is generated and the storage vessel pressure is maintained at a constant value by adding/removing liquid to/from the accumulator. In particular, notice that wind power as high as 3MW is being captured while the generator is only sized at 600kW (i.e. generator is downsized to 20% of wind turbine capacity, while all the available wind energy is captured and delivered to the electrical grid). In this scenario, the generator is operating at a capacity factor of almost unity. As shown in Figure 7.11 (top), the accumulator pressure ratio is always close to its desired value (200) no matter how much energy is stored. Since the accumulator plays the role of a damper for the storage system, the pressure deviation is directly related to the fluctuations in the turbine speed which affects the pump displacement (in the nacelle). Figure 7.11 (bottom) shows the angular speeds of the turbine and generator. While the turbine speed changes to track the optimal tip speed ratio, generator shaft speed is maintained in order to produce a constant amount of power.

Figure 7.12 (top) shows the displacements corresponding to the down-tower hydraulic pump/motor ($D_{pm}$) and the liquid piston air compressor/expander unit ($D_{lp}$). The displacement $D_{lp}$ is responsible for controlling the energy storage/regeneration for longer time scales (i.e. 10-minutes) and $D_{pm}$ is utilized to compensate for the low-pass filtering effect and to track the generator power demand (via controlling $\omega_g$). In particular, notice that the down-tower pump/motor operates mostly as a motor, and on occasions when it operates as a pump, it works at low displacement. This condition may cause low efficiency in the pumping mode. This suggests that by dividing the pump/motor into a large motor and a relatively smaller pump/motor (with a higher bandwidth), better performance can be obtained in both pumping and motoring modes.
Figure 7.10: Wind speed profile used for simulation (top); captured power by the pump in nacelle, generator power and energy level in the accumulator (bottom).

Figure 7.12 (bottom) shows the air and liquid flows to the accumulator (negative values mean flow is from the accumulator). While being in the opposite direction, the air mass flow rate and liquid mass flow rate have about the same ratio almost all the time. This shows how the controller tries to maintain the pressure inside the vessel by adding/subtracting enough volume of liquid through the hydraulic path. Figure 7.13 shows the performance of the standard torque controller applied to the wind turbine shaft through the hydraulic pump in the nacelle. As shown, the controller tracks the desired wind turbine power versus wind speed curve in region 2 of wind speed ($V_{\text{wind}} \leq 12$ m/s), while the blade pitch angle is changed from zero to cut the extra wind power in region 3 of wind speed ($V_{\text{wind}} > 12$ m/s). The overall efficiency for the entire system (wind turbine and storage system) over the 72-hour simulation is found to be 74.8%.

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Reflects the amount of energy delivered to the grid by the generator relative to the amount of energy captured by the wind turbine.
Figure 7.11: Air pressure ratio in the accumulator and displacement of the pump in the nacelle (top); turbine and generator angular speeds (bottom)

Figure 7.13: Wind turbine captured power vs. wind speed

A breakdown of the various pump/motors as well as the thermal loss inside the compression/expansion chamber is shown in Fig. 7.14-right. Figure 7.14-left shows a sample
power flow over different components inside the combined wind turbine and storage system. Two different time snapshots are used here to show how the system operates over high and low wind power conditions. As it can be seen, for the case of high wind power, the accumulator is charging with the rate of 1.23MW, while in the low wind power situation, the accumulator is discharging at the rate of 0.73MW. From this figure, it is also possible to find the instantaneous efficiency of the pump/motors as well as the compression/expansion chamber.
Figure 7.14: Left: power flows in the combined system for two different time snapshots: High wind power case (black), Low wind power case (gray); Right: breakdown of the losses for the combined system based on percentage of total loss

7.5.2 Short-Term Simulation: Control Task Distribution

As discussed earlier, accumulator pressure regulation and generator power tracking tasks are shared between the hydraulic pump/motor and the liquid piston air compressor/expander based on their bandwidth capabilities. To see how it works, consider the wind speed and generation power demand profile shown in Fig. 7.15-1, with rising and falling step changes (at times “a” and “c”) in power demand (e.g. from unexpected generator or transmission line failure). Moreover, a transient, high-speed wind (at “b”) that causes a large hydraulic flow from the nacelle to the CAES system is assumed. The pump/motor bandwidths are considered to be 0.5Hz (in the nacelle and down-tower) while the maximum allowable frequency of the liquid piston air compressor/expander actuation command is 0.005Hz. In response to the step changes in the power demand, the pump/motor displacement changes quickly to inject/absorb the required power to the generator shaft (Fig. 7.15-2). This action is done through the power-dense hydraulic path of the accumulator, which causes the air pressure to drop/increase quickly (Fig. 7.15-3). The mean pressure variation in the accumulator is, however, compensated by the air compressor/expander. The precise generator power tracking is robust to wind power fluctuations. In fact, the high-frequency part of the captured power (in the form
of fluctuating hydraulic flow) is stored in the accumulator through the power-dense and efficient hydraulic path, while the low-frequency part is stored through the energy-dense pneumatic path after transforming into compressed air (Fig. 7.15-5). During all these events, the wind turbine is operating close to its optimal condition with the tip speed ratio of 8.1, which results in maximum power capturing (Fig. 7.15-4).

7.6 Summary

While storing/regenerating energy in an efficient way is the main (and large time-scale) objective of the CAES system under this study, there are many small time-scale features that are potentially obtainable through the CAES system. For example, by coordinating different components in the CAES system, it would be possible to regulate the air pressure in the storage vessel at its desired value while capturing the maximum available power from wind and delivering the demanded electric power to the grid. In order to simultaneously achieve all these, a comprehensive dynamic model of the combined wind turbine and energy storage system is required. In this chapter, a dynamic model was developed for the combined system to study the interaction between various subsystems and components. A continuous cycle-average model for a liquid piston air compressor/expander was extracted from the cycle-by-cycle operation with an optimal compression or expansion profile. A nonlinear controller was then designed for the storage system according to an energy-based Lyapunov function to meet power demand, to regulate storage vessel pressure and maximize wind energy capture. A modification of the control allows the hydraulic and pneumatic elements to operate advantageously according to their bandwidth and power density characteristics. Therefore, the slower air compressor/expander is sized for steady power and used to store/regenerate energy in longer time scales (steady), and the faster hydraulic components are sized and used to accommodate high transient power supply/demand. The accumulator also damps out capturing all the power fluctuations coming from the wind turbine pump (due to wind gusts). Agility of the smaller down-tower pump/motor is used to track the desired generator power precisely. The controller design methodology used in this work matches both the architecture of the storage system and the physical constraints of different subsystems in the CAES system under this study. Case studies demonstrated
that the storage system allows the electrical component to be downsized to $1/5$th of the turbine’s capacity while capturing all the available wind energy and delivering a constant mean electrical power.

The controllers developed in this chapter are low-level (i.e. component-level) controllers that are designed to ensure achievement of individual task defined for each subsystem (such as regulating the air pressure at its desired value, delivering the desired electric power to the grid and ...). A high-level (i.e. supervisory-level) controller is required to determine the appropriate values for air pressure and volume in the storage vessel (at each time), as well as to determine when to store energy and when to regenerate it and deliver it to the electric grid. Design and development of such a supervisory controller is discussed in the next chapter.
Figure 7.15: Short-term simulation results
Chapter 8

Revenue Maximization for a Wind Turbine Equipped with a CAES System

8.1 Introduction

In chapter 7, low level controllers were designed to achieve an individual task for each component in the combined system: regulating the accumulator (i.e. storage vessel) pressure at its desired value, tracking the desired electric power delivered to (or taken from) the grid by controlling the generator shaft speed (i.e. frequency), and capturing the maximum available wind power by tracking the optimal tip speed ratio of the wind turbine (when the wind speed is in region 2). However, the accumulator energy level and the desired air pressure are values that are still need to be determined based on some sort of overall system performance policy. Therefore, a high-level supervisory controller is necessary to coordinate the combined system and determines the desired value of energy level and air pressure inside the storage system according to a predefined strategy. These desired values will then be sent to the low-level controllers (i.e. component level controllers) where they are achieved by using the available control commands (such as pump/motor displacement). The high-level control strategy can be defined as an optimal control problem to optimize a certain objective over a given time horizon for
the combined wind turbine and CAES system. Here, the total achievable revenue from the combined system is considered to be maximized. Price of electricity varies hourly during the day. They tend to peak in the morning (6-9AM) and afternoon (5-8PM) when demand is high. Price can vary during the day by several folds as illustrated in Fig. 8.1 which shows price variations over two 7-day periods, between 18-120 $/MWhr in February 2012 and between 30-280 $/MWhr in July 2012. Occasionally, electricity price can peak as high as 2500$/MWhr on a hot day in Texas! A wind power plant can use large scale storage to shift the time of energy production to increase profit. Wind energy is stored during off-peak hours, energy is released to generate electricity when the price is high. Deterministic or stochastic approaches can be taken to formulate the problem to optimize (i.e. maximize) the revenue.

![Electricity Price Variation](image)

Figure 8.1: Electricity price variation during a 7-day time period: February (blue) and July (red) of 2012

The optimal storage/regeneration sequence is trivial in the case when the pump/motors are perfectly efficient and there are no size and capacity constraints: store energy when the electricity price is low, regenerate it when the electricity price is high. The optimal sequence in the presence of size and capacity constraints on the system components is much more challenging. Moreover, the flexibility of storage/regeneration through the hydraulic and/or pneumatic paths also raises the level of complexity of this optimal control problem. In this chapter, we will address the deterministic version of this optimal control problem.

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1Profit = Revenue − Total Cost
control problem by assuming that the wind speed and electricity price are two known
time-series over a given time interval. In addition, a study is conducted to investigate
how the CAES size and power capability (i.e. storage vessel size and nominal power of
the air compressor/expander system) can influence the economic operation of a wind
turbine connected to an electric grid when the electricity price varies over time.

8.2 Modeling

As derived in chapter 7, the full dynamics of the combined wind turbine and CAES sys-
tem can be represented by four dynamic states: 1) turbine rotor speed $\omega_r$, 2) generator
shaft speed $\omega_g$, 3) pressure ratio $r$ inside the storage vessel, and 4) air volume inside
the storage vessel $V$ (see Fig. 8.2). Low-level (i.e. component level) controllers were
designed in chapter 7 such that the generator shaft speed is maintained at its desired
value (to produce electric power at desired frequency) while the maximum wind power
is captured by tracking the optimal tip speed ratio of the turbine rotor (when the wind
speed is in region 2 $\left[77\right]$). The remaining dynamics of the combined system can be
represented by the air pressure ratio and the air volume in the storage vessel. Using
the ideal gas law for air and assuming isothermal compression/expansion in the storage
vessel, the air pressure ratio and air volume dynamics are determined as follows (see
$\left[53, 54\right]$ for detailed derivation):

$$\dot{r} = \frac{-1}{P_0 V(t)} \left( \frac{u_1(t)}{\ln(r(t))} + \frac{r(t)u_2(t)}{r(t) - 1} \right) \tag{8.1}$$

$$\dot{V} = \frac{u_2(t)}{P_0 (r(t) - 1)} \tag{8.2}$$

where $u_1$ and $u_2$ are the pneumatic and hydraulic powers coming out of the storage vessel
and $P_0$ is the ambient pressure (Fig. 8.2). Note that air pressure ratio and volume can be
controlled independently with the two storage/regeneration power paths: i) pneumatic
and ii) hydraulic.

In order to maintain the generator frequency, the net power to the generator shaft
must be zero at all times. Therefore, the generator power ($W_g$) delivered to the electric

\footnote{The ratio between air pressure in the accumulator and ambient air pressure ($P/P_0$)
Figure 8.2: Schematic of power flow inside the combined wind turbine and CAES system

Grid can be calculated as:

\[ W_g(t) = \zeta_4(t) \left[ \zeta_3(t)u_1(t) + \zeta_2(t) \left( \eta_1(t)W_r(t) + u_2(t) \right) \right] \]  

(8.3)

where \( W_r \) is the wind power captured by the wind turbine rotor. \( \eta_1 \) is the efficiency of the pump in the nacelle, \( \zeta_2 \) is the input/output power ratio of pump/motor, \( \zeta_3 \) is the input/output power ratio of the liquid piston air compressor/expander unit (consisting of a hydraulic pump/motor and a liquid piston compression/expansion chamber\(^3\)) and \( \zeta_4 \) is the input/output power ratio of the generator. Note that (8.3) is valid regardless of the power flow directions. Therefore, the input/output power ratios of the pump/motor, air compressor/expander and electric generator are defined as:

\[ \zeta_2 = \begin{cases} \eta_2 & \text{if } \eta_1W_r + u_2 > 0 \\ \frac{1}{\eta_2} & \text{if } \eta_1W_r + u_2 \leq 0 \end{cases} \]

\[ \zeta_3 = \begin{cases} \eta_3 & \text{if } u_1 > 0 \\ \frac{1}{\eta_3} & \text{if } u_1 \leq 0 \end{cases} \]

\(^3\)In reality, there is a first stage air compressor/expander which compresses air from 1 bar to 7 bar, or expands it from 7 bar to 1 bar. The efficiency of this compression/expansion stage is assumed to be 100% for simplicity.
\[ \zeta_4 = \begin{cases} \eta_4 & \text{if } (\eta_1 W_r + u_2 + \zeta_3 u_1) > 0 \\ \frac{1}{\eta_4} & \text{if } (\eta_1 W_r + u_2 + \zeta_3 u_1) \leq 0 \end{cases} \]

where \( \eta_2, \eta_3 \) and \( \eta_4 \) are the efficiencies of pump/motor, air compressor/expander and electric generator, respectively. These conditions simply express the fact that the presence of losses reduces the output power and increases the input power of the of the pump/motor, air compressor/expander and generator. In general, efficiency of a variable displacement pump/motor is a function of pressure, displacement and shaft speed. However, for the pump/motors in the proposed CAES system, the dependence on speed can be neglected since the generator shaft speed is under precise control to maintain the frequency of the generated electricity. A sample efficiency map for a variable displacement pump/motor is shown in Fig. 8.3.

Assuming that the electricity price (\$/MWhr) is a function of time given by \( S(t) \), the total revenue achieved by selling electricity to the grid over the time interval of \( [0, t_f] \) can be calculated as

\[ J = \int_0^{t_f} W_g(t) S(t) dt \quad (8.4) \]

The corresponding optimal control problem is now defined as the maximization of the revenue function given by (8.4) using the control inputs \( u_1 \) and \( u_2 \) while the system dynamics are given by (8.1) and (8.2). Using overline and underline to denote the allowable maximum and minimum of state variables, the physical constraints for this optimal control problem can be summarized as follows:

\[ \underline{r} \leq r(t) \leq \bar{r} \quad (8.5) \]
\[ \underline{V} \leq V(t) \leq \bar{V} \quad (8.6) \]
\[ |\eta_1(t) W_r(t) + u_2(t)| \leq \overline{W}_{pm}(r) \quad (8.7) \]
\[ |u_1(t)| \leq \overline{W}_{ce}(r) \quad (8.8) \]
\[ |W_g(t)| \leq \overline{W}_g \quad (8.9) \]

where (8.5) and (8.6) are due to allowable pressure range of the CAES system and the maximum capacity of the storage vessel. (8.7), (8.8) and (8.9) show the maximum power

\footnote{Note that the efficiencies of pump/motor and air compressor/expander are dependent on displacement, shaft speed and system pressure.}

\footnote{Bid-ask spread is assumed to be zero as well.}
capability of the pump/motor, air compressor/expander and electrical generator/motor, where $W_{pm}$, $W_{ce}$ and $W_g$ are the rated powers for each component, respectively.

One additional constraint is required to guarantee that the initial and final energy level in the storage vessel are the same (i.e. total energy sold to the electric grid is obtained from the given time interval not from any prior storage). This constraint can be written as:

$$\int_0^{t_f} (u_1(t) + u_2(t))dt = 0 \quad (8.10)$$

8.3 Solution Approach

Due to the nonlinear system dynamics and complex efficiency mappings for different components in the integrated wind turbine and CAES system, analytical approaches to solve the optimal control problem are difficult to apply. Here instead, deterministic Dynamic Programming (DP) approach is used to find the optimal storage/regeneration strategy and power path (pneumatic and/or hydraulic). By discretizing the system
dynamics in time and rearranging (8.1) and (8.2), $u_1$ and $u_2$ can be written as:

$$u_1(t_{i+1}^i) = -P_0 \ln \left( \frac{r(t_{i+1}^i)}{r(t_i)} \right) \frac{t_{i+1}^i - t_i}{t_{i+1}^i - t_i}$$

and

$$u_2(t_{i+1}^i) = P_0 \left( r(t_{i+1}^i) - 1 \right) \frac{V(t_{i+1}^i) - V(t_i)}{t_{i+1}^i - t_i}$$

(8.11)

(8.12)

where $i$ is the discrete time index and $t_{i+1}^i$ denotes the time interval between $t_i$ and $t_{i+1}$. Note that $r(t_{i+1}^i)$, $V(t_{i+1}^i)$ and $u_1(t_{i+1}^i)$ are the average air pressure ratio, air volume and pneumatic power over the time interval $t_{i+1}^i$ and defined as (see Fig. 8.4):

$$r(t_{i+1}^i) = \frac{r(t_i) + r(t_{i+1})}{2}$$

$$V(t_{i+1}^i) = \frac{V(t_i) + V(t_{i+1})}{2}$$

$$u_1(t_{i+1}^i) = \frac{u_1(t_i) + u_1(t_{i+1})}{2}$$

By discretizing the state space ($r$-$V$) over the given time range, the optimal sequence of $r$ and $V$ that maximizes the total revenue given by (8.4) and satisfies all the constraints given by (8.5), (8.6), (8.7), (8.8), (8.9) and (8.10) can be found by the DP search method.

Figure 8.4: Discretization of state-space (pressure ratio and volume) over time.

According to Bellman’s equation, it is possible to split the optimization problem in
sequence of optimizations, as:

\[ J_{t_i \rightarrow t_n}^* (r(t_i), V(t_i)) = \]

\[ \max_{u_1(.), u_2(.)} \left( W_g(t_{i+1}^+)(S(t_{i+1}^+)(t_{i+1} - t_i) + J_{t_{i+1} \rightarrow t_n} (r(t_{i+1}), V(t_{i+1}))) \right) \]

where \( W_g(t_{i+1}^+) \) and \( S(t_{i+1}^+) \) denote the generator power and electricity price over the time interval \([t_i, t_{i+1}]\) and can be calculated as (based on (8.3)):

\[ S(t_{i+1}^+) = \frac{S(t_i) + S(t_{i+1})}{2} \quad (8.13) \]

\[ W_g(t_{i+1}^+) = \zeta_4(t_{i+1}^+) \left[ \zeta_3(t_{i+1}^+)u_1(t_{i+1}^+) + \zeta_2(t_{i+1}^+) \left( \eta_1(t_{i+1}^+)W_r(t_{i+1}^+) + u_2(t_{i+1}^+) \right) \right] \quad (8.14) \]

By knowing the air volume and pressure ratio change in the storage vessel as well as the average air volume and pressure between time steps, \( u_1(t_{i+1}^+) \) and \( u_2(t_{i+1}^+) \) can be evaluated according to (8.11) and (8.12). Similarly, all the component efficiencies (i.e. \( \eta \) or \( \zeta \)) can be evaluated if the air pressure ratio and the power level of that component are known over each time interval. Note that the inequality constraints for this optimization problem are given by (8.5), (8.6), (8.7), (8.8) and (8.9). In addition, the equality constraint given by (8.10) also needs to be satisfied.

### 8.4 Ideal Case (100% Efficient Components)

In order to check the performance and accuracy of the developed DP code, a benchmark case study is first defined and solved. In this benchmark case, we assume that one degree of control freedom is utilized to maintain the system pressure ratio at the desired value \( r_d \) all the time. To do so, the down-tower pump/motor and the air compressor/expander need to cooperate with each other to maintain the accumulator pressure by adjusting the liquid and air flow rate to/from the storage vessel. According to (8.1), to obtain pressure regulation in the storage vessel (i.e. \( \dot{r} = 0 \)), \( u_1 \) and \( u_2 \) must satisfy:

\[ u_2(t) = \alpha u_1(t) \quad (8.15) \]

where \( \alpha \) is:

\[ \alpha = \frac{1 - r_d}{r_d \ln(r_d)} \quad (8.16) \]
where \( r_d \) is the desired pressure ratio of the storage vessel. The second assumption for the benchmark case is that all the efficiencies are equal to 100% (ideal system with no loss). In summary, the optimal control problem for this benchmark case can be mathematically formulated as:

\[
 u_1^* = \arg \max_{u_1(.)} \left( \int_0^{t_f} u_1(t) S(t) dt \right) \quad (8.17)
\]
such that:

\[
0 \leq e(t) \leq \bar{e} \quad (8.18)
\]
\[
|u_1(t)| \leq W_{ce}(r_d) \quad (8.19)
\]
\[
|W_r(t) + (\alpha + 1)u_1(t)| \leq W_g \quad (8.20)
\]
\[
\int_0^{t_f} u_1(t) dt = 0 \quad (8.21)
\]

where \( e \) is the energy stored in the pressure vessel and can be calculated as [35]:

\[
e(t) = P_0 V(t) \left( r_d \ln(r_d) - r_d + 1 \right) \quad (8.22)
\]

Note that since the pressure ratio is always constant, the constraints given by (8.5) and (8.6) are described in terms of a constraint on energy level given by (8.18), where it is assumed that \( V = 0 \). According to (8.22), the maximum energy level \( \bar{e} \) and maximum storage capacity \( \bar{V} \) are related as:

\[
\bar{e} = P_0 \bar{V} \left( r_d \ln(r_d) - r_d + 1 \right)
\]

The significance of the benchmark case is that the optimal control problem can be formulated as a convex optimization problem, with a linear cost function and a number of linear matrix inequalities (LMI) describing all the constraints. Therefore, it would be possible to solve the corresponding optimal control problem with the well-matured convex optimization toolboxes. This is particularly useful to evaluate the accuracy and performance of the DP approach in solving the corresponding optimization problem for a non-ideal system where all the component losses are included (will be discussed in the next section). The rate of change of energy stored in the air (in storage vessel) can be calculated by taking time derivative from (8.22) as:

\[
\dot{e}(t) = P_0 \left( r_d \ln(r_d) - r_d + 1 \right) \dot{V}(t) \quad (8.23)
\]
where it is assumed that \( u_1 \) and \( u_2 \) satisfy (8.15) so the air pressure remains constant at \( r_d \). Using (8.2) and (8.15), \( \dot{e} \) can be rewritten as:

\[
\dot{e}(t) = -\left( \alpha + 1 \right) u_1(t) \tag{8.24}
\]

In discrete time domain, using the forward difference method to evaluate time derivatives, we have:

\[
e(t_i) - e(t_{i-1}) = -\left( \alpha + 1 \right) \Delta t \times u_i \tag{8.25}
\]

where the total time interval is discretized uniformly such that \( \Delta t = t_i - t_{i-1} \) for any \( i \in (1, 2, ..., n) \) while \( t_0 = 0 \) and \( t_n = t_{\text{end}} \). Note that \( u_i \) is the pneumatic storage power between \( t_{i-1} \) and \( t_i \) (i.e. \( u_i = u_1(t_{i-1} \rightarrow t_i) \)) and is assumed to remain constant during each time step. Let’s define \( X \) and \( U \) as:

\[
U = \text{Diag}\left( [u_1(t_1), u_1(t_2), \ldots, u_1(t_n)] \right) \tag{8.26}
\]

\[
X = \text{Diag}\left( [e(t_1) - e(t_0), e(t_2) - e(t_0), \ldots, e(t_n) - e(t_0)] \right) \tag{8.27}
\]

where both \( U \) and \( X \) are \( n \)-by-\( n \) diagonal matrices. Note that \( i^{th} \) element on the diagonal of matrix \( X \) represents the energy level difference between \( t_i \) and \( t_0 \) (i.e. energy level compared to initial value \( e(t_0) \)). Hence, the matrix representation of (8.25) can be calculated as:

\[
U = \frac{1}{(\alpha + 1)\Delta t} (DXD^T - X) \tag{8.28}
\]

where \( D \) is a \( n \)-by-\( n \) lower shift matrix defined as:

\[
D = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}_{n \times n}
\]

We assume that the state of charge (i.e. energy level) of the storage vessel at \( t_0 \) is known and is equal to \( e(t_0) \). In discrete time domain, the total revenue given by (8.4) can be written as:

\[
J = \Delta t \left( (\alpha + 1) \sum_{i=1}^{n} u_1(t_i)S(t_i) + \sum_{i=1}^{n} W_r(t_i)S(t_i) \right) \tag{8.29}
\]
note that the second summation in (8.29) depends only on predefined captured power by the turbine \( W_r \) and the electricity price \( S \). It is desired to maximize the total revenue by maximizing the first summation in (8.29), while satisfying all the corresponding constraints. Based on (8.28), the first summation in (8.29) can be written in matrix form in terms of \( X \) and \( D \) as:

\[
\Delta t(\alpha + 1) \sum_{i=1}^{n} u_1(t_i)S(t_i) = \Delta t(\alpha + 1)S^T UF = S^T (DXD^T - X)F
\]

(8.30)

where \( F \) is a \( n \)-by-1 vector of ones (i.e. \( F = [1 \ 1 \ \ldots \ 1]^T \)) and \( S \) is a \( n \)-by-1 vector consists of electricity prices as:

\[
S = \begin{bmatrix} S(t_1) & \cdots & S(t_f) \end{bmatrix}^T
\]

It can be easily shown that \((DXD^T - X)F = (D - I)XF\). Hence the optimal control problem in discrete time domain can be formulated as:

\[
X^* = \arg \max_X \left( S^T (D - I)XF \right)
\]

(8.31)

where \( I \) is the \( n \)-by-\( n \) identity matrix. In order to be able to solve the corresponding optimal control problem with available convex optimization toolboxes, all the constraints given by (8.18), (8.19), (8.20) and (8.21) need to be reformulated in form of linear matrix inequalities. The first constraint given by (8.18) can be written as:

\[
0 \leq e(t_i) \leq \bar{e} \implies -e(t_0) \leq e(t_i) - e(t_0) \leq \bar{e} - e(t_0)
\]

(8.32)

which must hold for \( i \in (1, 2, \ldots, n) \). Hence, in matrix form, \( X \) needs to satisfy:

\[
-e(t_0)I \leq X \leq (\bar{e} - e(t_0))I_{n \times n}
\]

(8.33)

The second constraint given by (8.19) can be rewritten as:

\[
-W_{ce} \leq u_1(t_i) \leq W_{ce} \implies -(\alpha + 1)\Delta tW_{ce} \leq DXD^T - X \leq (\alpha + 1)\Delta tW_{ce}
\]

(8.34)

Where (8.28) is used to represent this constraint in matrix form. Similarly, the third constraint given by (8.20) can be written as:

\[
-(W_g + W_r(t_i)) \Delta t \leq DXD^T - X \leq (W_g + W_r(t_i)) \Delta t
\]

(8.35)
The constraints given by (8.34) and (8.35) can be combined into one constraint as:

\[-\min \left( W_g I + \Lambda, (\alpha + 1) W_{ce} I \right) \Delta t \leq DXD^T - X \leq \min \left( W_g I - \Lambda, (\alpha + 1) W_{ce} I \right) \Delta t\]

(8.36)

where \( I \) is the \( n \)-by-\( n \) identity matrix and \( \Lambda \) is the rotor power matrix defined as:

\[ \Lambda = \text{Diag}[W_r(t_1), \cdots, W_r(t_n)] \]

The last constraint given by (8.21) can be described in discrete form as:

\[ \sum_{i=1}^{n} u_1(t_i) = 0 \quad (8.37) \]

which is equal to the summation of all diagonal elements in \( DXD^T - X \). By using \( F \) as defined earlier, this constraint can be written as:

\[ -\epsilon \leq (D^T F)^T X D^T F - F^T XF \leq \epsilon \quad (8.38) \]

where \( \epsilon \) is a small positive value that determines the strictness of the equality constraint given by (8.37). In summary, the optimal control problem for the ideal system is defined by the profit function given by (8.31) (to be maximized) while all the constraints are in the form of Linear Matrix Inequalities (LMI) given by (8.33), (8.36) and (8.38). Here, \texttt{mincx} function is used in \texttt{Matlab} to solve this convex optimization problem with its corresponding LMI constraints. This solver uses Nesterov and Nemirovski’s polynomial-time projective algorithm described in [78].

To compare the results of DP and convex optimization methods, the optimal control problem is solved for the wind power and electricity price profiles shown in Figs. 8.5 and 8.6 (top). The first 7-day period (Fig. 8.5) belongs to February 2012, where the electricity price is relatively smooth and does not have large fluctuations. However, the second 7-day period (Fig. 8.6) is from July 2012 where the electricity price has large variations between 30$/MWhr and 280$/MWhr, mainly due to the hot summer. These two months are specially chosen to study the performance and usefulness of an energy storage system during the extreme price variation in the electric grid. Let’s consider a wind turbine with a rotor radius of 65m and rated power of 2.5MW. The wind turbine rotor power can be calculated as:

\[ P_r(t) = \frac{1}{2} \rho_{\text{air}} \pi R_r^2 C_p V_{\text{wind}}^3 \quad (8.39) \]
where $\rho_{\text{air}} (=1.18\text{Kg/m}^3)$ is the air density at ambient condition, $C_p^* \simeq 0.48$ is the power coefficient at the optimal tip speed ratio ($\simeq 8.1$) [77], and $V_{\text{wind}}$ is the wind speed. The numerical values used for wind speed are based on gathered wind data by the Wind Energy Center at University of Massachusetts [79] (a series of 10-min average wind speed recorded at the elevation of 100m). The electricity price time series is based on hourly aggregate real-time Locational Marginal Pricing (LMP) data provided by PJM regional transmission organization [82]. It should be emphasized that the electricity price and wind speed data belong to the same location and time period (as well as date). Moreover, a total volume of 150m$^3$ is considered for the pressure vessel.

At pressure ratio of 200, this volume is equivalent to about 3.6MWhr of energy. In addition, the generator and the air compressor/expander rated powers are assumed to be 1.5MW and 1.3MW, respectively. Results of this benchmark case study are shown in Figs. 8.5 and 8.6 (middle and bottom). As can be observed, the DP and convex optimization solutions match well with each other. Note that for both approaches, a total number of 300 points is used to discretize time domain ($\Delta t = 24\text{min}$). The small difference between the DP and convex optimization solution is due to the discretization resolution of the dynamic state (here the air volume or energy level in the storage vessel) used in DP approach.

In the case of no energy storage, the total revenue that can be achieved by selling the captured wind power to the electric grid is $1350$ (February) and $1800$ (July) over the given 7-day time horizon. However, according to the optimal solution (DP or convex optimization), by providing a 3.6MWhr energy storage capacity for the wind turbine, the total revenue can be increased to $1800$ (February) and $4000$ over the same time range which is about 33% (February) and 122% (July) increase in the total revenue. Here, it is assumed that the electric generator is also capable of functioning as an electric motor.

In the benchmark case an ideal system is assumed where all the components are lossless. The optimal storage/regeneration problem cannot be formulated as a convex

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6Hence, not only the wind energy can be stored but also the electric energy (provided by electric grid) can be stored and regenerated later.
optimization in the presence of component losses. In this case, the energy storage component efficiencies ($\eta_2$ and $\eta_3$) are nonlinear mappings of pump/motor or air compressor/expander displacement, shaft speed and pressure. Therefore, the profit function given by (8.4) cannot be evaluated explicitly and need to be calculated numerically through pre-defined tables. In addition, the governing dynamic equations of the energy storage system would be nonlinear if we consider the air pressure to vary (i.e. using full flexibility of the energy storage system to store/regenerate energy by changing compressed air volume or pressure). Hence, neither the profit function nor the constraint equations would be linear in the case of actual system considering the pressure dynamics and component losses. As a result, DP is the only technique that can be used to study the more realistic cases presented in the next section.
Figure 8.5: Comparison between DP and convex optimization approaches for February 2012 data. From top to bottom: Captured wind power and electricity price; Power through each component; Energy level inside the storage vessel; Total revenue.
Figure 8.6: Comparison between DP and convex optimization approaches for July 2012 data. From top to bottom: Captured wind power and electricity price; Power through each component; Energy level inside the storage vessel; Total revenue
8.5 Non-Ideal Case (with Component losses)

As mentioned before, due to nonlinear system dynamics and complex efficiency mappings, DP is the only approach that will be used in this section to solve the optimization problem. Since the focus of this work is on the CAES system, constant efficiencies are assumed for the pump connected to the wind turbine in the nacelle ($\eta_1 = 90\%$) as well as the electric generator ($\eta_4 = 95\%$). The remaining efficiencies correspond to the pump/motor ($\eta_2$) and the air compressor/expander unit ($\eta_3$), both located inside the CAES system. Since the low level controller maintains the generator shaft speed, these efficiencies in the CAES system ($\eta_2$ and $\eta_3$) are functions of the system pressure ratio ($r$) and displacement ($x$). Note that for a given pressure and generator speed, there is a one-to-one mapping between displacement and power. Hence, the pump/motor efficiency can be plotted in the pressure-power domain (instead of pressure-displacement domain). For the given efficiency map shown in Fig. 8.3 and a maximum displacement of 2.5lit/rev for the pump/motor, the resulted efficiency in pressure-power domain is shown in Fig. 8.7 (left). Note that the generator frequency is assumed to be 60Hz. Here, we define the nominal power of the pump/motor to be the maximum hydraulic power flowing out of the machine (i.e. pumping mode) at the full displacement ($x = +1$) while the pressure is 200bar. In this way, the corresponding nominal power of the pump/motor will be about 1.5MW.

![Diagram](image)

Figure 8.7: Pump/motor (left) and air compressor/expander (right) efficiency map as a function of system pressure ratio and power.
Calculating the efficiency map for the air compressor/expander would be more complex since it contains three different components (see [81] for more details): i) hydraulic pump/motor; ii) flow intensifier; and iii) compression/expansion chamber. For each pair of final/initial pressure and air storage/regeneration power, the optimal compression/expansion trajectory is found considering the efficiency map of the hydraulic pump/motor (shown in Fig. 8.3) and flow intensification ratio of 5. This approach has been used to fill all the air compressor/expander efficiency map shown in Fig. 8.7 (right). The hydraulic pump/motor displacement is 2.5 lit/rev. Compression/expansion chamber volume is 293 liters that has been filled with porous materials with heat transfer area density equal to 40000 m$^2$/m$^3$.

The optimal power flow problem for the given time series of rotor power and electricity price is solved using DP approach. The wind speed and electricity price profiles used in this study are the same as what is used in the ideal system case (Section 4). To better understand the significance of utilizing the energy storage system, a baseline case is also simulated and compared. In this case, the capacity of the storage vessel is assumed to be zero (i.e. considering the hydraulic power transmission between the wind turbine and electric generator, but no energy storage/regeneration). Let’s define the total efficiency as the ratio between the total energy delivered to the electric grid and the total energy entered into the integrated system over a given time interval. The following figures are the results of simulations for all the case studies that have been performed in this study. Here, six different scenarios have been considered and investigated:

1. Conventional wind turbine with a gearbox (90% efficiency) and an electric generator (95% efficiency) with no energy storage.

2. Wind turbine with hydrostatic power transmission with no energy storage (Figs. 8.8 and 8.13).

3. Wind turbine with hydrostatic power transmission equipped with a CAES system which operates at constant pressure, while the electric machine is only a generator (i.e. sell only) (Figs. 8.9 and 8.14).

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The desired pressure to be achieved at the end of compression process, or the pressure at which the expansion process is desired to begin.
4. Wind turbine with hydrostatic power transmission, equipped with CAES system that operates at constant pressure, while the electric machine can function as both generator and motor (i.e. sell and buy) (Figs. 8.10 and 8.15),

5. Wind turbine with hydrostatic power transmission equipped with a CAES system that operates at variable pressure (in the range of 100 to 300 bar), while the electric machine is only a generator (i.e. sell only) (Figs. 8.11 and 8.16),

6. Wind turbine with hydrostatic power transmission, equipped with CAES system that operates at variable pressure (in the range of 100 to 300 bar), while the electric machine can function as both generator and motor (i.e. sell and buy) (Figs. 8.12 and 8.17).

All these scenarios have been simulated for a one week time period taken from two different months of year (February and July, 2012). For all these cases, the wind turbine is rated at 1.5 MW while the generator is rated at 1.3 MW. In addition, 90% and 95% efficiencies have been assumed for the hydraulic pump attached to the turbine (in the Nacelle) and the down tower electric generator (or motor), respectively. The storage capacity is equal to 3.6 MWhr (i.e. 150 m$^3$ of compressed air at 200 bar) which can increase to 5.8 MWhr if pressure rises to 300 bar. The result of the total revenue and efficiency corresponding to each case has been summarized in table 8.1.

According to the results, for both periods, hydrostatic wind-turbine (with no storage) generates less revenue than the conventional wind turbine due to a lower system efficiency. This is mainly due to the down-tower pump/motor operating at low displacements and low efficiencies. Note that for both periods, the mean powers are quite low (286kW for Feb 2012 period, and 178kW for July 2012 period) compared to the rated power. With storage, more revenues are generated relative to the conventional wind turbine despite a 5-20% decrease in efficiency and a smaller amount of net electricity sold. As expected, case 6 provides more arbitrage opportunity to increase revenue by buying cheap electricity and selling it when price is high. The time shifting effect can be seen in Figs. 8.12 and 8.17 where electricity generation is highly correlated with price (case 5 is similar except that electricity generation cannot be negative). However, the

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8i.e. $\eta_1 = 90\%$ and $\eta_4 = 95\%$ (shown in Fig. 8.2).
improvement in revenue is highly dependent on the price profiles. For the July 2012 period, revenue is increased by +124% over conventional case; whereas for the February 2012 period, which has a much more modest price variation, the improvement is only +15%. Finally, in order to evaluate the average performance of the combined system, the same calculation has been performed on a 14-day period where the February and July wind speed and electricity price profiles are combined to build a two-week period case. According to the results summarized in table 8.2, the maximum improvement of +85.9% is achievable in the total revenue, while the system efficiency is obtained as 71.6% (variable pressure, sell/buy case).

According to the optimal storage/regeneration sequence, it is more beneficial to store energy using the storage vessel volume capacity instead of increasing air pressure. In the other words, if the air volume inside the storage vessel is not at its maximum allowable value, energy is better to be saved by adding compressed air at low pressure rather than higher pressures. This is consistent with the pump/motor (or air compressor/expander) efficiency trend shown in Fig. 8.3. Note that the same hydraulic (or pneumatic) power is achievable either by a low pressure and high displacement, or by high pressure and low displacement. However, the latter is less efficient since the hydraulic pump/motor has a poor efficiency at low displacements. Therefore, the storage system tries to maintain the system pressure at low values to let all the hydraulic components work at higher displacements. If the compressed air volume in the storage vessel reaches its maximum, additional energy is stored by increasing the air pressure. In particular, notice that the total energy that is stored/regenerated in the case of variable pressure (cases 5 and 6) is more than the constant pressure (cases 3 and 4). However, the total system efficiency is higher in the case of variable pressure compared to the constant pressure condition (compare case 3 with case 5, and compare case 4 with case 6).
<table>
<thead>
<tr>
<th>Case</th>
<th>February 16-23, 2012</th>
<th>July 5-12, 2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wind captured (MWh)</td>
<td>Electricity sold (MWh)</td>
</tr>
<tr>
<td>1</td>
<td>48</td>
<td>41</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
<td>35.7</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
<td>34.4</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>40.8&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
<td>35.3</td>
</tr>
<tr>
<td>6</td>
<td>48</td>
<td>47.1&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

* at 200bar.
<sup>a</sup> 8.5 MWh energy is bought
<sup>b</sup> 26 MWh energy is bought
<sup>c</sup> 14.5 MWh energy is bought
<sup>d</sup> 42.5 MWh energy is bought

Table 8.1: Energy captured, efficiency and revenues for a 1.5 MW wind turbine over two 7-day periods in February and July, 2012. Price and available wind power are shown in Fig. 8.5 (for February) and Fig. 8.6 (for July); **Case 1**: Conventional wind turbine; **Case 2**: Hydrostatic wind turbine with no storage; **Case 3**: Proposed system with 3.6* MWh storage (sell, constant pressure); **Case 4**: Proposed system with 3.6MWh storage (sell/buy, constant pressure); **Case 5**: Proposed system with 3.6MWh storage (sell, varying pressure); **Case 6**: Proposed system with 3.6MWh storage (sell/buy, varying pressure).
Table 8.2: Energy captured, efficiency and revenues for a 1.5 MW wind turbine over 14-day period (combined February and July, 2012). Case 1: Conventional wind turbine; Case 2: Hydrostatic wind turbine with no storage; Case 3: Proposed system with 3.6\(^*\) MWh storage (sell, constant pressure); Case 4: Proposed system with 3.6MWh storage (sell/buy, constant pressure); Case 5: Proposed system with 3.6MWh storage (sell, varying pressure); Case 6: Proposed system with 3.6MWh storage (sell/buy, varying pressure).
Figure 8.8: Wind turbine with hydrostatic power transmission with no energy storage, February 2012

Figure 8.9: Wind turbine with hydrostatic power transmission and CAES system, constant pressure, electric machine is only a generator, February 2012
Figure 8.10: Wind turbine with hydrostatic power transmission and CAES system, constant pressure, electric machine functions as both generator and motor, February 2012

Figure 8.11: Wind turbine with hydrostatic power transmission and CAES system, variable pressure (100-300 bar), electric machine is only a generator, February 2012
Figure 8.12: Wind turbine with hydrostatic power transmission and CAES system, variable pressure (100-300 bar), electric machine functions as both generator and motor, February 2012

Figure 8.13: Wind turbine with hydrostatic power transmission with no energy storage, July 2012
Figure 8.14: Wind turbine with hydrostatic power transmission and CAES system, constant pressure, electric machine is only a generator, July 2012

Figure 8.15: Wind turbine with hydrostatic power transmission and CAES system, constant pressure, electric machine functions as both generator and motor, July 2012
Figure 8.16: Wind turbine with hydrostatic power transmission and CAES system, variable pressure (100-300 bar), electric machine is only a generator, July 2012

Figure 8.17: Wind turbine with hydrostatic power transmission and CAES system, variable pressure (100-300 bar), electric machine functions as both generator and motor, July 2012
8.6 Effect of System Sizing on Total Revenue

After solving the optimal power flow problem, we can investigate the effect of subsystem sizing on the overall performance of the integrated system. The CAES system consists of three main components: i) down-tower pump/motor; ii) liquid piston air compressor/expander; and iii) storage vessel. Although there is no physical constraint on the size of storage vessel and air compressor/expander, the down-tower pump/motor size is restricted by the wind turbine rated power. Moreover, the main cost of the CAES system is due to the air compressor/expander and the storage vessel. Therefore, we will only investigate the sizing effect of these last two subsystems, while all the other parameters as well as the wind power and electricity price profiles are fixed. The wind turbine and electric generator sizes are kept constant at 1.5MW and 1.3MW, respectively. Different combinations of air compressor/expander rated power (in MW) and storage vessel capacity (in m$^3$) are considered to study the effect of CAES size on combined system performance (i.e. total revenue and round trip efficiency). Here, the air compressor/expander rated power is taking different values from the set [1.24, 2.85, 4.46, 6.07, 7.69, 9.30] (MW), while the storage vessel volume is taking values from the set [50, 100, 200, 350, 550, 800] (m$^3$). The size effect study has been performed for both months (February and July). In addition, two different scenarios have been assumed for each case: i) sell only (i.e. electric machine is only a generator); ii) sell and buy (i.e. electric machine functions as both generator and motor). Combined February and July months (as a two-week period) is also studied. Results of the size effect study have been shown in Figs. 8.18, 8.19, 8.20, 8.21, 8.22 and 8.23.

According to the results, the achievable revenue increases either by using a larger storage vessel or a more powerful air compressor/expander. It can be noticed that for a given air compressor/expander rated power, there exists a maximum storage capacity where the total revenue becomes saturated. This means that for the given air compressor/expander rated power, increasing the storage vessel size above the saturating value will not improve the revenue anymore (for example, compare 550m$^3$ vessel size with 600m$^3$ vessel size in Fig. 8.18). Generally, increasing the size of CAES system (increasing the storage vessel volume or air compressor/expander rated power) causes more energy to be stored/regenerated. This will reduce the roundtrip efficiency of the
combined system (see the February case results). Note that there can be some exceptions (like the July case) where enlarging the CAES size allows the components to work in a more efficient condition, therefore improves the roundtrip efficiency.

An other notable result from the size effect study is that the same revenue is achievable by choosing either a smaller storage capacity and larger air compressor/expander, or larger storage capacity and smaller air compressor/expander (see the contour lines in Figs. 8.18, 8.19, 8.20, 8.21, 8.22 and 8.23). For example, for the sell only February case (Fig. 8.18), a total revenue of $1325 is achievable (over one-week time period) by using a $1.5MW$ air compressor/expander combined with $330m^3$ of storage vessel volume. The same revenue can be achieved by a $0.88MW$ air compressor/expander and $800m^3$ of storage vessel volume. Therefore, depending on the cost of storage vessel ($$/m^3$ or $$/kWhr$) and air compressor/expander ($$/kW$), there will be an optimal size for the CAES system which maximizes the achievable revenue for a given total cost of CAES system. This aspect will be investigated in greater details in the next section.

Figure 8.18: Size Effect, February, sell only
Figure 8.19: Size Effect, February, sell/buy

Figure 8.20: Size Effect, July, sell only
Figure 8.21: Size Effect, July, sell/buy

Figure 8.22: Size Effect, Combined February and July, sell only
8.7 Iso-Price Lines for the CAES System

Considering the cost of storage vessel (per kWhr) and air compressor/expander (per kW), we can find the equation for iso-price lines as:

\[
Cost_{tot} = V_{st} \times \Gamma_{st} + W_{ce} \times \Gamma_{ce} \implies V_{st} = \frac{Cost_{tot} - W_{ce} \times \Gamma_{ce}}{\Gamma_{st}} \quad (8.40)
\]

where \(V_{st}\) is the storage vessel volume (in \(m^3\)), \(\Gamma_{st}\) is the cost of storage vessel (in \$/m^3\), \(W_{ce}\) is the air compressor/expander rated power (in kW), and \(\Gamma_{ce}\) is the cost of air compressor/expander (in \$/kW). In addition, \(Cost_{tot}\) is the total cost of the CAES system. Here, it is assumed that the cost of storage vessel is \$126/kWhr (\$3010/m^3 at 200bar) and the cost of air compressor/expander is \$1000/kW. The iso-price lines are plotted for the 6 cases of CAES system costs: 0.79M$, 1.2M$, 1.56M$, 2M$, 2.4M$ and 3M$ (Figs. [8.24] [8.25] and [8.27]). It should be noted that each contour shows the total revenue over one-year period where the wind speed and electricity price profiles of

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9About 23.9kWhr of energy can be stored in 1m$^3$ of storage vessel at 200bar.
the two-weeks period (given in Table 8.2) are considered to be typical profiles over the entire year. In this way, the total revenue achievable during one year is equal to the total revenue achieved over the typical two-weeks period multiplied by 26 (each year has 52 weeks). As it can be seen, for each CAES system price, there is an optimal combination of storage vessel size and air compressor/expander rated power that maximizes the achievable revenue. This information can be used to choose the appropriate size of a CAES system to be combined with a wind turbine. Let’s define the total revenue (achievable by tracking the optimal storage/regeneration sequence) as a function of storage vessel capacity and air compressor/expander rated power as:

\[ \text{Revenue} = R(V_{st}, W_{ce}) \]  

For a given CAES system cost (described as \( \text{Cost}^{des} \)), the optimal CAES size can be found by solving the optimization problem defined as:

\[ W_{ce}^* = \arg \max_{W_{ce}} \left( R \left( \frac{\text{Cost}^{des}}{\Gamma_{st}}, \frac{\Gamma_{ce}}{\Gamma_{st}} W_{ce}, W_{ce} \right) \right) \]  

where \( W_{ce}^* \) is the optimal size of the air compressor/expander. The optimal storage vessel size \( V_{st}^* \) and maximum achievable revenue for this optimal size \( R^* \) can be found as:

\[ V_{st}^* = \frac{\text{Cost}^{des}}{\Gamma_{st}} - \frac{\Gamma_{ce}}{\Gamma_{st}} W_{ce}^* \]  

\[ R^* = R(V_{st}^*, W_{ce}^*) \]

As an example, suppose that we want to invest 2M$ in CAES system combined with the wind turbine introduced earlier in this chapter. Let’s assume the February wind speed and electricity price data represent their typical values over a year. The optimal CAES system size can be calculated by solving the optimization problem defined in (8.42). As shown in Fig. 8.24, the best size for storage vessel is 340m³ and the best air compressor/expander rated power is 0.97MW. If this size is used for the CAES system, then $71,000 total revenue is achievable over one year. Note that in this case, the percentage of investment in the storage vessel and air compressor/expander are both 50% (340×3010=1.03M$ for storage vessel and 0.97×1=0.97M$ for air compressor/expander). On the other hand, if we use the combined February and July wind
speed and electricity price data (as their typical values over a year), the best size of the storage vessel will be found as $290m^3$ while the best size of the air compressor/expander will be $1.15MW$, as shown in Fig. 8.27. This optimal size also results in $160,000$ total revenue after one year of operation. Notice that for both cases, the optimal size (i.e., rated power) of the air compressor/expander is more than 30% smaller than the peak wind turbine power.
Figure 8.24: Effect of CAES size on the yearly revenue (February 2012 wind speed and price profiles are used as their typical values over a year), sell/buy case. Top: Total revenue (per year) as a function of air compressor/expander rated power and storage vessel capacity (with iso-price lines); Bottom: Optimal CAES system size for a given cost, and its corresponding maximum achievable revenue (per year)
Figure 8.25: Effect of CAES size on the yearly revenue (July 2012 wind speed and price profiles are used as their typical values over a year), sell/buy case. Top: Total revenue (per year) as a function of air compressor/expander rated power and storage vessel capacity (with iso-price lines); Bottom: Optimal CAES system size for a given cost, and its corresponding maximum achievable revenue (per year)
Figure 8.26: Effect of CAES size on the yearly revenue (combined February and July 2012 two-weeks wind speed and price profiles are used as their typical values over a year), sell only case. Top: Total revenue (per year) as a function of air compressor/expander rated power and storage vessel capacity (with iso-price lines); Bottom: Optimal CAES system size for a given cost, and its corresponding maximum achievable revenue (per year)
Figure 8.27: Effect of CAES size on the yearly revenue (combined February and July 2012 two-weeks wind speed and price profiles are used as their typical values over a year), sell/buy case. Top: Total revenue (per year) as a function of air compressor/expander rated power and storage vessel capacity (with iso-price lines); Bottom: Optimal CAES system size for a given cost, and its corresponding maximum achievable revenue (per year)
8.8 Summary

The optimal storage/regeneration sequence as well as the corresponding efficient power flow path were found for an integrated wind turbine and CAES system for given deterministic wind power and electricity price profiles. It was shown that the problem can be structured and solved as a convex optimization with LMI constraints in the case of ideal system with no loss and no pressure variation. However, to study the actual system, dynamic programming approach was used to solve the corresponding optimal control problem considering the losses in the CAES system as well as nonlinear volume and pressure dynamics inside the storage vessel. For the given set of parameters and component sizes, it was shown that an improvement of 14% (for February) and 137% (for July) in total revenue is achievable by equipping a conventional wind turbine with a CAES system. In this case, the total efficiency for the overall system was found to be 75.4% (for February) and 68.4% (for July) during a 7-day time period when the electric generator is also capable of functioning as an electric motor. For the combined February and July case (as a two-week time period), 86% improvement in total revenue was achieved, while the roundtrip efficiency of the combined system was calculated as 72%.

A size study was also performed to investigate the effect of the storage vessel capacity and the air compressor/expander maximum power on the total revenue. Combined with the manufacturing and maintenance costs, these results were used to determine the optimal economical size of a CAES system for a given wind turbine size and CAES system cost.

The optimal control problem introduced and solved in this chapter was a deterministic storage/regeneration problem where electricity market information as well as available wind power were assumed to be known ahead of time. Considering the volatility of electricity price as well as random nature of wind speed, the solution of the deterministic problem can be only considered as an upper bound of achievable revenue given perfect information. Although under particular market structures and very accurate forecasts it may be possible to come close to obtaining this upper bound in day ahead markets, it is not likely to be achieved in reality. To deal with the uncertainties in electricity price and wind speed, a stochastic optimal control problem needs to be defined and solved. If some information such as Probability Distribution Functions (PDF) of the wind speed
and electricity price are known (ahead of time), then Stochastic Dynamic Programming (SDP) techniques can be used to maximize the expected value of the total revenue.
Chapter 9
Conclusions

Renewable energy such as wind and solar are clean and available as long as the wind blows or sun shines. Two main disadvantages of these energy sources are their intermittency and that their availabilities do not often correspond to the electric power demand. For example, wind energy tends to be more abundant at night when power demand is low. The objective of this work was to model, optimally design and control of a new type of compressed air energy storage system for wind turbine applications. The energy storage system studied here takes the advantages of high energy density of compressed air besides the large power density of hydraulic power transmission. The excess energy from the wind turbine is stored locally, prior to electricity generation that allows electrical components to be downsized (increasing the capacity factor for the wind turbine). In addition, since the turbine and generator shafts are not coupled, it is possible to maintain the generator frequency at a desired value (60Hz) without using a power electronic system for the turbine. The focus of this work was mainly in two levels: design of a near isothermal liquid piston air compressor/expander; and overall energy storage system design and control. Gains were achieved in both levels.

The key component of the energy storage system is its air compressor/expander. The efficiency of this unit greatly affects the entire storage/regeneration efficiency since it is responsible for the majority of the energy conversion. Compressing/expanding air 200-300 times heats/cools the air greatly, resulting in poor efficiency, unless the process is sufficiently slow which reduces power. Increasing heat transfer between air and its surrounding environment in the compression/expansion chamber can improve
the thermal efficiency of process by pushing the process to be near isothermal. Different methods and approaches were used in this work to achieve this goal. Here, a liquid piston air compressor/expander with a chamber filled with porous material was considered. The main contributions in this area achieved here are as summarized below:

• A method for designing optimal compression/expansion trajectories was developed that maximize the power density of air compressor/expander without sacrificing its efficiency. Hardware constraints and limitations are considered in the optimization algorithms. Power density was improved by more than 300% compared to the conventional sinusoidal compression/expansion profiles (without sacrificing efficiency). The advantage of optimal compression trajectory over add-hoc compression rate (such as constant flow rate) was experimentally validated on a low-pressure (1bar to 10bar) and high-pressure (7bar to 200bar) liquid piston air compressor testbeds. According to the experimental results, the power density was improved by 20% on the low-pressure system, while the improvement was 100% on high-pressure setup. It was also observed that the accuracy in heat transfer prediction (in form of a correlation) has direct impact on effectiveness of the calculated optimal compression flow rate in improving the compression performance.

• A computationally efficient thermodynamic model for air compression/expansion process was developed that allows rapid iterations needed for system optimization purposes. The speed up achieved by judiciously ignoring inertia effects (in gas media) and assuming a uniform pressure distribution in the chamber. One-dimensional distributions for air temperature and air density as well as solid porous material temperature were considered. Convective and conductive heat transfer mechanisms were also included in the air thermodynamic model. While this model had enough accuracy for investigating the geometric effects of compression/expansion chamber and heat transfer porous material distribution, it resulted in 10 folds increase in computation efficiency.

• The one-dimensional model was used to find the optimal distribution of porous media in a chamber, as well as the optimal shape of chamber. A simultaneous optimization for shape, distribution and compression/expansion rate was conducted
to find the optimal design of the chamber geometry and its corresponding optimal compression/expansion rate. Based on the numerical simulations, optimal shape design combined with the optimal compression/expansion trajectory improves power density of an air compressor/expander by 2000% (i.e. 20 folds) without sacrificing efficiency.

- A one-dimensional model was developed to investigate the effect of water spray mass and timing on the compression/expansion performance. An optimal control strategy was then developed and used to maximize the system efficiency by adjusting the water spray timing. Results showed a 2% improvement in efficiency by applying optimal spray timing (compared with a constant spray rate). It was observed that the water spray is more useful at the end of compression process when the air temperature is high. In addition, in order to increase the effectiveness of water spray and extend the lifetime of water droplets in the chamber (before hitting the liquid piston surface or chamber wall), spraying in the radial direction (instead of axial direction of the chamber) was suggested based on the results.

The optimally designed liquid piston air compressor/expander system was then incorporated in the energy storage system combined with the wind turbine. For the appropriate operation of the combined system, an adequate size selection is crucial. Therefore, models that can predict the effect of design parameters on the performance of individual components as well as the entire system were developed. These models were also used to predict the systems level performance such as power and roundtrip efficiency. In addition, component-level controllers as well as supervisory-level control strategies were designed for the combined wind turbine and CAES system, to achieve both short-term and long-term objectives. In the short time scales (minutes), the storage device provided an extra degree of freedom beyond the wind power supply and electrical power demand for dynamically matching the loads that the wind or the electrical generator sees. This maximized wind power extraction and generator efficiency. In the longer time scales (hours), the storage system improved the achievable total revenue for the wind turbine by optimally using the energy storage capacity. In summary, the main contributions in this level of study were as follows:

- Continuous cycle-average models were developed for the component in the CAES
system. Nonlinear control design approach was then used to develop plant-level control strategies in order to achieve short-term objectives (such as frequency regulation of electric generator and capturing maximum wind power including wind gust power) as well as long-term objectives (such as tracking the power demanded by electricity grid and regulating air pressure in the storage vessel). A modification of the controllers was performed to allow the hydraulic and pneumatic elements to operate advantageously according to their bandwidth and power density characteristics. The combined system roundtrip efficiency was found to be around 75% for a typical wind speed and constant generator power demand over a three-day time window.

- Optimal storage/regeneration sequences (using Dynamic Programming approach) were developed that determine when to store the energy in the storage system and when to deliver the energy to the grid. This optimal sequences were found based on the variation in electricity price for a given wind speed. All the losses in different power paths in the energy storage system were considered in this optimization process. For a combined wind turbine and CAES system, acquiring the developed optimal sequences can double the achievable revenue when the fluctuations in electricity price is relatively large (during the hot seasons). In addition, the optimal economical size of the CAES system was found for the given manufacturing cost of the air compressor/expander as well as storage vessel.

**Future Works**

There is still much of interest that can be investigated on dynamic interaction between the developed CAES system and the electric grid. To obtain a good electric power quality, it is crucial to maintain the waveform of power distribution bus voltages at their desired magnitude and frequency. If the balance between the total power generation and the power consumed by system loads and electrical losses on the grid does not meet, the grid frequency will go up or down which can be very dangerous (for example, see The Northeast Blackout of 2003 [83]). Therefore, the utilities that are capable of regulating these type of fast fluctuations and unplanned events (such as sudden loss of generation) are very attractive and important to the grid operators. Potentially, the
developed CAES system is able to change its power level (both in generation phase and motoring phase) in the order of milliseconds to few seconds. This capability mainly depends on the effective bandwidth of the down-tower hydraulic pump/motor, which is directly connected to the electric generator in the CAES system. This important feature can be investigated if a detailed dynamic model is developed for the energy storage system connected to a dynamic electric grid. A detailed dynamic model of a three-phase induction generator that is capable of capturing all the fast dynamic features (in case of a sudden frequency drop) is necessary for this study. If the developed CAES system can also contribute in electric power quality improvement, its importance to the future of electric grid would be significantly increased.
References


Appendix A

Cycle-Average Model for Liquid Piston Air Compressor/Expander Unit

In this appendix, we derive the cycle average compressed air mass flow rate $\dot{m}$ and liquid piston pump/motor $(F_2)$ torque $T_{lp}$. Both quantities are averaged over a compression/expansion cycle ($\sim 1-2$ sec) and defined at a given common shaft speed $\omega_g$, storage vessel pressure ratio $r := \frac{P_{acc}}{P_0}$, and cycle average displacement $\bar{D}_{lp}$ of the liquid piston pump/motor $F_2$ which is the control input. Within the compression/expansion cycle, the liquid piston flow rate varies to achieve an Adiabatic-Isothermal-Adiabatic (AIA) trajectory as in Fig. 7.5. The family of AIA trajectories is parameterized by $T_1$, the temperature of the isothermal segment. For compression, $T_1 \geq T_0$, and for expansion $T_1 \leq T_0$. The final temperature is given by $T_f = T_1^2 / T_0$. Different choices of $T_1$ result in different power and efficiency - as $T_1$ deviates from the ambient temperature $T_0$, power increases but efficiency decreases. However, for a given heat transfer capability of the compressor/expander, specified by $hA$ [W/K] (product of heat transfer coefficient and heat transfer area), the achieved efficiency (or power) is optimal at that power (or efficiency). The time it takes to compress/expand a given mass of air with an optimal
AIA trajectory specified by $T_1$ can be calculated as [41]:

$$
\dot{m} = \frac{hA |1 - T_0/T_1|}{R \left( \frac{2\gamma}{\gamma-1} \ln \left( \frac{T_0}{T_1} \right) \right) \pm \ln(r)} \tag{A.1}
$$

where $r$ is the pressure ratio, and $+/-$ signs correspond to compression ($T_1 > T_0$) and expansion ($T_1 < T_0$). Let $Q_{lp}$ be the cycle average liquid piston flow rate. Since $\dot{m} = \rho_0 \cdot Q_{lp}$ where $\rho_0 = P_0/(RT_0)$ is the air density at $(P_0,T_0)$, we have

$$
Q_{lp} = \frac{hA |1 - T_0/T_1|}{\rho_0 R \left( \frac{2\gamma}{\gamma-1} \ln \left( \frac{T_0}{T_1} \right) \right) \pm \ln(r)} \tag{A.2}
$$

On the other hand, we can express $Q_{lp}$ in terms of the cycle-average flow rate of the liquid piston pump/motor and leakage as:

$$
Q_{lp} = \frac{\bar{D}_{lp}(t)}{2\pi} \omega_g - L_{lp}'(T_1,\omega_g,r) \tag{A.3}
$$

where $\bar{D}_{lp}$ is the cycle average displacement of the liquid piston pump/motor ($F_2$), $L_{lp}'(T_1,\omega_g,r)$ represents the volumetric losses in ($F_2$) and in the compression/expansion chamber ($F_1$). By equating (A.2)-(A.3), we can solve for $T_1$ in terms of $(\bar{D}_{lp},\omega_g,r)$. Thus, hence forth, we shall consider the liquid piston pump/motor displacement $\bar{D}_{lp}$ as the control input the compressor/expander and the corresponding AIA trajectory with isothermal temperature $T_1(\bar{D}_{lp},\omega_g,r)$ will be to be used within the cycle.

The P-V work required to compress/expand a unit mass of air to/from pressure ratio $r$ with an AIA trajectory specified by $T_1$ including the isobaric final ejection/initial injection is given by [41]:

$$
\mathcal{E}_a(T_1,r) = RT_0 \left( \frac{\pm \gamma}{\gamma-1} \left[ \left( \frac{T_1}{T_0} \right)^2 - 2 \frac{T_1}{T_0} \ln \left( \frac{T_1}{T_0} \right) - 1 \right] + \frac{T_1}{T_0} \ln(r) \right) \tag{A.4}
$$

Cycle-average chamber pressure $\mathcal{P}_w$ is defined based on the principle of virtual work:

$$(\mathcal{P}_w - P_0) \cdot Q_{lp} = \mathcal{E}_a \cdot \dot{m}$$
\[ P_w := \rho_0 E_a(T_1, r) + P_0 \quad (A.5) \]

Generally \( P_w \ll r \cdot P_0 \) since a large portion of the compression/expansion trajectory is at low pressure. For example, when \( r = 200 \), \( P_w \leq 7.5 \) in Fig. 7.6. This is the main reason air compressors/expanders are much less power dense than hydraulic pumps/motors. Given \((\overline{D}_{lp}, \omega_g, r)\), the cycle average liquid piston pump/motor torque \( T_{lp} \) and air mass flow rate are given as:

\[ T_{lp} = -\frac{D_{lp}}{2\pi} (P_w - P_0) - \Gamma_{lp}(\overline{D}_{lp}, \omega_g, r) \quad (A.6) \]

\[ \dot{m} = \rho_0 \left( \frac{D_{lp}(t)}{2\pi} \omega_g - L_{lp}(\overline{D}_{lp}, \omega_g, r) \right) \quad (A.7) \]

where \( \Gamma_{lp}(\overline{D}_{lp}, \omega_g, r) \) captures mechanical losses in the pump/motor and in the liquid piston, and \( P_w(\overline{D}_{lp}, \omega_g, r) \) is given by Eqs. (A.5) and \( L_{lp}(\overline{D}_{lp}, \omega_g, r) = L_{lp}'(T_1, \omega_g, r) \) in Eq. (A.3) with \( T_1 \) corresponding to \((\overline{D}_{lp}, \omega_g, r)\). The thermal efficiency of the compression/expansion process is defined as the ratio between the stored energy in the air (including the ejection/injection work) and the total compression/expansion work \((E_a)\):

\[ \eta_{tm} = \left( \frac{\ln(r)}{\frac{\pm \gamma}{\gamma - 1} \left( \left( \frac{T_1}{T_0} \right)^{2} - 1 - 2\frac{T_1}{T_0} \ln(\frac{T_1}{T_0}) \right) + \frac{T_1}{T_0} \ln(r)} \right)^{\pm 1} \quad (A.8) \]

where \(+/-\) signs correspond to compression and expansion modes.