EXPECTATIONS EQUILIBRIUM WITH EXPECTATIONS CONDITIONED ON PAST DATA *

by

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Discussion Paper No. 77-90, September 1977

*This research was supported by a Summer Research Appointment at the University of Minnesota Graduate School.

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2.3 Initial Information: For each i, let $I_1^i = \{0,1\}$, with generic element η_1^i. If $\eta_1^i = 1$, the i^{th} agent is said to be initially informed; and if $\eta_1^i = 0$, the ith agent is said to be initially uninformed. Let

$E = \{[\pi;(\eta_1^1,e^1),\ldots,(\eta_1^N,e^N)]\varepsilon(0,1) \times \Pi_i(I_1^i \times E^i) : \text{for}$ ....

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Since $({}^1p_a, {}^1y_{a1}) = ({}^1p_{b1}, {}^1y_{b1}) = (p_1, y_1)$, the hypothesis of Lemma 4.1 is satisfied, so for each π, $(e'(\pi), 0)$ must have equilibria $({}^0p_{a1}, {}^0y_{a1}, {}^0p_{a2}, {}^0y_{a2})(\pi)$ and $({}^0p_{b1}, {}^0y_{b1}, {}^0p_{b2}, {}^0y_{b2})(\pi)$ with $f^1[({}^0p_{a1}, {}^0y_{a1})(\pi)] = f^1[({}^0p_{a1}, {}^0y_{a1}(\pi)]$. However, as $\pi \rightarrow 1$, $({}^0p_{a1}, {}^0y_{a1})(\pi) \rightarrow (p_1,y_1)$ and $({}^0p_{b1}, {}^0y_{b1})(\pi) \rightarrow ({}^0p_{b1}, {}^0y_{b1})$, ...

Introduction

The theory of economic prediction has recently been complicated by the discovery that if current predictions of future events are based on current market variables, statistically correct prediction can be inconsistent with market equilibrium, even in otherwise well-behaved (classical) economies.[1] For example, suppose that the future variable to be predicted is systematically related to a current exogenous variable which is not observed directly. Suppose that the exogenous variable can take two possible values, a and b ; and that predictions are to be based on a currently determined equilibrium price. If the price differs between the states a and b , agents will have different expectations in the two states. However, it may happen that the equilibrium prices for the two excess demand functions are the same, so that no distinction between the states can be observed. Then expectations must be the same in both states. But then the excess demand functions will not be the same as those mentioned above, and the equilibrium prices for the new excess demand functions may differ between the two states, making the distinction observable. Thus there is no equilibrium with prediction.

In response to this difficulty it is natural to suppose that predictions are based on past rather than current market data. Hellwig and Rothschild [2] have presented a securities market model in which, under the assumption of constant absolute risk-aversion, equilibrium

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[1] As far as I am aware, this problem was first mentioned by Radner [7, p.5], and the first examples were constructed by Green [1] and Kreps [5].

exists when expectations are conditioned on previous prices. Since an agent's current demand must be sensitive to current prices, it may seem artificial to suppose that his current expectations are not. However, if agents rely on published predictions, for example, some lag in the formation of expectations is inevitable. In the macroeconomic rational expectations literature, it is conventional to suppose that current expectations are not directly influenced by current endogenous variables (e.g. [9, pp. 18-19]).

Unfortunately, constraining current expectations to be insensitive to current market variables can easily worsen the existence problem. This paper develops a very simple three-period model in which agents who are not exogenously informed of the state rely on previous market data. Thus, in period 1 they have no information, and in period 2 their expectations about period 3 are based on data generated as some function of first period market variables. The main result is that, under the assumption that these data functions are continuous, the only data functions which admit the general existence of equilibrium are constant functions. Stating the conclusion somewhat differently, the general existence of an equilibrium with prediction, which will be called an expectations equilibrium, cannot be assured for any nontrivial previous market data. This result may be contrasted with the results in [3] and [4] which indicate that if current expectations are conditioned on current market data, there do exist some nonconstant data functions which admit the general existence of expectations equilibrium.

2. The Model

This section describes a model of a three-period exchange economy. In period 1 each agent receives an endowment of a current consumption good and a durable good, called money, which is not consumed until period 3. In period 2, each agent receives an endowment of current consumption but no additional endowment of money. No endowment is received in the final period. In each of the first two periods, current consumption is exchanged for money, and in the final period, each agent consumes the money he holds after trading in period 2. Utility is additively separable over time, so the final period utility of money can be interpreted as an indirect utility. The model can be viewed as a two-period slice of an ongoing monetary exchange economy, since the final period utility of money is introduced only in order to model trading behavior in periods 1 and 2. Nonmonetary interpretations are also possible.

There are two possible states of the world, and each state determines a three-period sequence of characteristics for each agent. More specifically, the state may influence each agent's consumption endowments and final period utility function. For simplicity, each agent's endowment of money and utility of current consumption in periods 1 and 2 will be assumed to be state-independent.

<u>2.1 Definitions</u>: There are three periods, indexed by the subscript t , $t=1,2,3,$; and N agents, indexed by the superscript i . In each of the first two periods there are markets for two commodities, a current consumption good, c_t , and money, m_t . In the final period, each agent consumes his holding of money.

For each i, t, let C_t^i denote the consumption set of agent i in period t, with $C_t^i = \text{int } R_+$; and for each i let M^i denote the space of money holdings for agent i, with $M^i = R_+$. Since the i^{th} agent's consumption in period 3 is equal to his holding of money at the end of period 2, elements of C_3^i will sometimes be denoted m_2^i .

The preferences of the i^{th} agent are determined in part by a three period utility function which is additive over time. The utility of consumption in periods 1 and 2 is state-independent but the utility of consumption in the final period depends on the state. There are two states, indexed by the subscript s , $s = a,b$. For each i, t, let U_t^i denote the set of utility functions u_t^i on C_t^i to R such that

i) u_t^i is C^∞ and for each $c_t^i \in C_t^i$, $Du_t^i(c_t^i) > 0$ and $D^2u_t^i(c_t^i) < 0$, and

ii) $\lim_{c_t^i \to 0} Du_t^i(c_t^i) = \infty$.

In period 1, the i^{th} agent receives an endowment of money and consumption, in period 2 he receives an endowment of consumption only, and in period 3 he receives no endowment. Endowments of consumption are state-dependent, while the endowment of money is state-independent. For each i, let $E^i = (C_1^i)^2 \times M^i \times (C_2^i)^2 \times U_1^i \times U_2^i \times (U_3^i)^2$, with generic element $e^i = (\bar{c}_{a1}^i, \bar{c}_{b1}^i; \bar{m}^i; \bar{c}_{a2}^i, \bar{c}_{b2}^i; u_1^i, u_2^i; u_{a3}^i, u_{b3}^i)$, where the first five coordinates represent endowments. For each i, the i^{th} agent is characterized in part as an element of E^i .

<u>2.2 Remarks</u>: The endowments $(\bar{c}^i_{s1}, \bar{m}^i)$ are realized before trade in period 1, so if $\bar{c}^i_{a1} \neq \bar{c}^i_{b1}$, the i^{th} agent can infer the state directly from his endowment. If $\bar{c}^i_{a1} = \bar{c}^i_{b1}$, we will assume that the i^{th} agent has no exogenous source of information, so his first period trades must be chosen in ignorance of the state. If $\bar{c}^i_{a1} = \bar{c}^i_{b1}$ and $\bar{c}^i_{a2} \neq \bar{c}^i_{b2}$, the agent would become exogenously informed at the beginning of period 2. For simplicity, we will ignore this case by assuming that for each i, if $\bar{c}^i_{a1} = \bar{c}^i_{b1}$ then $\bar{c}^i_{a2} = \bar{c}^i_{b2}$. Thus, with respect to exogenous information, each agent is either informed or uninformed in both of the first two periods. These assumptions are introduced formally in the definitions below.

<u>2.3 Initial Information</u>: For each i , let $I^i_1 = \{0,1\}$, with generic element η^i_1 . If $\eta^i_1 = 1$, the i^{th} agent is said to be <u>initially informed</u>; and i[illegible] the i^{th} agent is said to be <u>initially uninformed</u>. Let $E = \{[\pi;(\eta^1_1,e^1),\ldots,(\eta^N_1,e^N)]\varepsilon(0,1) \times \Pi_i(I^i_1 \times E^i)$: for each i , $\bar{m}^i > 0$; $\eta^i_1 = 0$ only if $\bar{c}^i_{a1} = \bar{c}^i_{b1}$ and $\bar{c}^i_{a2} = \bar{c}^i_{b2}$; and $\eta^i_1 = 1$ only if $\bar{c}^i_{a1} \neq \bar{c}^i_{b1}\}$, with generic element e . The set E is the set of environments, and the first coordinate, π , of an environment represents the probability of state a .

3. Expectations Equilibrium

However expectations are formed, an equilibrium will somehow associate prices, p_{s1} and p_{s2}, of current consumption in terms of money in periods 1 and 2 respectively, with each state s. As in [7], each agent will be assumed to know the joint distribution of states and prices. Then if $\eta_1^i = 1$, the i^{th} agent knows in period 1 the price he will face in period 2 and his final period utility function. Such an agent will thus choose his demands in periods 1 and 2 in each state as though he were participating in a three commodity static exchange environment, with the endowment $(\bar{c}_{s1}^i, \bar{c}_{s2}^i, \bar{m}^i)$ and the utility function $u_1^i(c_{s1}^i) + u_2^i(c_{s2}^i) + u_{s3}^i(m_s^i)$, subject to the additional constraint: $\bar{m}^i - p_{s1}(c_{s1}^i - \bar{c}_{s1}^i) \geq 0$. These remarks are made precise in 3.1 below.

An initially uninformed agent does not know in period 1 whether he will face the price p_{a2} or p_{b2} in period 2. Thus if $\eta_1^i = 0$, agent i is faced with a stochastic dynamic programming problem in period 1. In period 2, the i^{th} agent's demand will depend on whether he has become informed by data generated in period 1. The behavior of an initially uninformed agent is derived in section 3.2 below, where the variable η_2^i is introduced to represent second period information.

<u>3.1 The Behavior of Initially Informed Agents</u>: For each $t=1,2$, let $P_t = \text{int}R_+$ denote the space of prices for the consumption good in terms of money in period t. For each s, each $(p_{s1}, p_{s2}) \in P_1 \times P_2$, each i, and each $(\eta_1^i, e^i) \in I_1^i \times E^i$ with $\eta_1^i = 1$, the i^{th} agent chooses $(c_{s1}^i, m_{s1}^i; c_{s2}^i, m_{s2}^i) \in C_1^i \times M^i \times C_2^i \times C_3^i$ to maximize $u_1^i(c_{s1}^i) + u_2^i(c_{s2}^i) + u_{s3}^i(m_{s2}^i)$ subject to $p_{s1}c_{s1}^i + m_{s1}^i \leq p_{s1}\bar{c}_{s1}^i + \bar{m}^i$

and $p_{s2}c^i_{s2} + m^i_{s2} \leq p_{s2}\bar{c}^i_{s2} + m^i_{s1}$. The constraints can be written equivalently as $p_{s1}c^i_{s1} + p_{s2}c^i_{s2} + m^i_{s2} \leq p_{s1}\bar{c}^i_{s1} + p_{s2}\bar{c}^i_{s2} + \bar{m}^i$ and $\bar{m}^i - p_{s1}(c^i_{s1} - \bar{c}^i_{s1}) \geq 0$.

<u>3.2 The Behavior of Initially Uninformed Agents</u>: For each i , let $I^i_2 = \{0,1\}$, with generic element η^i_2 , and let $E^{0i} =$

$\{(\eta^i_1, \eta^i_2, e^i) \in I^i_1 \times I^i_2 \times E^i : \eta^i_1 = 0, \bar{c}^i_{a1} = \bar{c}^i_{b1}$, and $\bar{c}^i_{a2} = \bar{c}^i_{b2}\}$, with generic element $(\eta^i_2, e^i) = (\eta^i_2; \bar{c}^i_1, \bar{m}^i, \bar{c}^i_2; u^i_1, u^i_2; u^i_{a3}, u^i_{b3})$, suppressing redundant coordinates. For each s, each $(p_{s1}; p_{a2}, p_{b2}) \in P_1 \times (P_2)^2$, each $\pi \in (0,1)$, each i , and each $(\eta^i_2, e^i) \in E^{0i}$ with $\eta^i_2 = 1$, the i^{th} agent chooses $(c^i_{s1}, m^i_{s1}) \in C^i_1 \times M^i$ to maximize $u^i_1(c^i_{s1}) + \pi[\max\{u^i_2(c^i_{a2}) + u^i_{a3}(m^i_{a2}): p_{a2}c^i_{a2} + m^i_{a2} \leq p_{a2}\bar{c}^i_2 + m^i_{s1}\}] + (1-\pi)[\max\{u^i_2(c^i_{b2}) + u^i_{b3}(m^i_{b2}): p_{b2}c^i_{b2} + m^i_{b2} \leq p_{b2}\bar{c}^i_2 + m^i_{s1}\}]$; and chooses $(c^i_{s2}, m^i_{s2}) \in C^i_2 \times C^i_3$ to maximize $u^i_2(c^i_{s2}) + u^i_{s3}(m^i_{s2})$ subject to $p_{s2}c^i_{s2} + m^i_{s2} \leq p_{s2}\bar{c}^i_2 + m^i_{s1}$. If $\eta^i_2 = 0$, (c^i_{s1}, m^i_{s1}) is chosen to maximize $u^i_1(c^i_{s1}) + \pi[\max\{u^i_2(c^i_{a2}) + \pi u^i_{a3}(m^i_{a2}) + (1-\pi)u^i_{b3}(m^i_{a2}) : p_{a2}c^i_{a2} + m^i_{a2} \leq p_{a2}\bar{c}^i_2 + m^i_{s1}\}] + (1-\pi)[\max\{u^i_2(c^i_{b2}) + \pi u^i_{a3}(m^i_{b2}) + (1-\pi)u^i_{b3}(m^i_{b2}) : p_{b2}c^i_{b2} + m^i_{b2} \leq p_{b2}\bar{c}^i_2 + m^i_{s1}\}]$ subject to $p_{s1}c^i_{s1} + m^i_{s1} \leq p_{s1}\bar{c}^i_{s1} + \bar{m}^i_1$; and

(c^i_{s2}, m^i_{s2}) is chosen to maximize $u^i_2(c^i_{s2}) + \pi u^i_{a3}(m^i_{s2}) + (1-\pi)u^i_{b3}(m^i_{s2})$ subject to $p_{s2}c^i_{s2} + m^i_{s2} \leq p_{s2}\bar{c}^i_2 + m^i_{s1}$.

<u>3.3 Remarks</u>: Using the demands derived in 3.1 and 3.2, a two-period equilibrium can be defined in each state, for each environment $e \in E$ and each second period information structure $(\eta^1_2,\ldots,\eta^N_2)$. This definition is given in 3.4 below. To define an expectations equilibrium it only remains to relate the second period information structure to the market data generated in period 1. Of course, if $\eta^i_1 = 1$, the i^{th} agent's demand, as derived in 3.1, is independent of η^i_2 .

<u>3.4 Equilibrium</u>: Let $I_2 = \Pi_i I^i_2$, with generic element η_2 . For each $(e, \eta_2) \in E \times I_2$ and each s , an element

$$[p_{s1}, (c^i_{s1}, m^i_{s1})^N_{i=1} , \quad p_{s2}, (c^i_{s2}, m^i_{s2})^N_{i=1}] \in P_1 \times \Pi_i(C^i_1 \times M^i_1) \times P_2 \times \Pi_i(C^i_2 \times C^i_3)$$

is said to be an <u>equilibrium</u> for (e, η_2) in state s if $\Sigma_i c^i_{s1} = \Sigma_i \bar{c}^i_{s1}$. $\Sigma_i c^i_{s2} = \Sigma_i \bar{c}^i_{s2}$, $\Sigma_i m^i_{s1} = \Sigma_i m^i_{s2} = \Sigma_i \bar{m}^i_1$, and for each i , $(c^i_{s1}, m^i_{s1};$ $c^i_{s2}, m^i_{s2})$ is chosen according to 3.1 and 3.2. Equilibrium can of course be equivalently defined in terms of net trades. For each i and each $t = 1,2,$ let $Y^i_t = R$, and for each $t = 1,2,$ let

$Y_t = \{y_t = (y^1_t ,\ldots,y^N_t) \in \Pi_i Y^i_t\colon \; \Sigma_i y^i_t = 0\}$. Using the budget constraint in each period, an equilibrium $[p_{s1}, (c^i_{s1}, m^i_{s1})^N_{i=1} , \; p_{s2}, (c^i_{s2}, m^i_{s2})^N_{i=1}]$

for (e, η_2) in state s can be identified with an element $(p_{s1}, y_{s1}, p_{s2}, y_{s2}) \varepsilon P_1 \times Y_1 \times P_2 \times Y_2$, with $y^i_{st} = c^i_{st} - \bar{c}^i_{st}$ for each i and each $t = 1,2$.

For each (e, η_2) $E \times I_2$, the existence of equilibrium in each state is easily established.

<u>3.5 Data Structures</u>: For each i, let F^i denote the set of functions f^i on $P_1 \times Y_1$, and let $F = \Pi_i F^i$, with generic element f. For each i, elements of F^i are called data functions, and elements of F are called data structures. For each i, a data function f^i is said to be <u>continuous</u> if f^i is a continuous function on $P_1 \times Y_1$ to a Hausdorff space. A data structure f is continuous if f^i is continuous for each i.

<u>3.6 Expectations Equilibrium:</u> An <u>expectations equilbrium</u> for an environment $e \varepsilon E$ and a data structure f is an element $(\eta_2; p_{a1}, y_{a1}, p_{a2}, y_{a2}; p_{b1}, y_{b1}, p_{b2}, y_{b2}) \varepsilon I_2 \times (P_1 \times Y_1 \times P_2 \times Y_2)^2$ such that

i) for each s, $(p_{s1}, y_{s1}, p_{s2}, y_{s2})$ is an equilibrium for (e, η_2) in state s; and

ii) for each i, $\eta^i_2 = 1$ if and only if $f^i(p_{a1}, y_{a1}) \neq f^i(p_{b1}, y_{b1})$.

A data structure f is said to be <u>admissible</u> if for each $e \varepsilon E$, there exists an expectations equilibrium for (e,f); and f is said to be <u>trivial</u> if for each i, f^i is a constant function.

3.7 Remarks: The following theorem, which is the main result of this paper, states that the only admissible continuous data structures are the trivial ones. In [3, Theorem 3.5] admissibility is characterized without the continuity hypothesis in a model in which current expectations are based on current data. It seems likely that the continuity hypothesis can be dropped from the present theorem also, but I have not proved this.

3.8 Theorem: A continuous data structure is admissible if and only if it is trivial.

Proof: Sufficiency is immediate, and necessity follows from Proposition 4.2 below.

3.9 Remarks: The nonexistence of expectations equilibrium for nontrivial data structures can be motivated in the following way. Given an environment e and a data structure f, choose (p_{a1}, y_{a1}) and (p_{b1}, y_{b1}) in $P_1 \times Y_1$, and let η_2 be consequently determined by f. Given η_2, and the second period distribution of money determined by initial endowments and the above first period prices and trades, second period equilibrium prices p_{a2} and p_{b2} can be obtained for e. Given η_2, p_{a2}, and p_{b2}, first period equilibria (p'_{a1}, y'_{a1}), and (p'_{b2}, y'_{b2}) can be derived. An expectations equilibrium is a fixed-point of this process. However, the presence of the discontinuous variable η_2 suggests that if f is nontrivial, a fixed-point may fail to exist.

4. Proof of Necessity in Theorem 3.8

This section is devoted to proving Proposition 4.2 below, which is a generalization of the necessity assertion of the Theorem. The following Lemma states the implication of admissibility which will be used in the proof. The Lemma is an immediate consequence of the definition of expectations equilibrium (3.6).

4.1 Lemma: Let $e \varepsilon E$ with $\eta_1^i = 0$ and $\eta_1^j = 1$ for all $j \neq i$, let $\eta_2 \varepsilon I_2$ with $\eta_2^i = 1$, and let $\eta_2' \varepsilon I_2$ with $\eta'^i_2 = 0$. Suppose that (e, η_2) has unique equilibria $(p_{a1}, y_{a1}, p_{a2}, y_{a2})$ and $(p_{b1}, y_{b1}, p_{b2}, y_{b2})$ in states a and b respectively, with $(p_{a1}, y_{a1}) = (p_{b1}, y_{b1})$. If f is an admissible data structure then (e_2, η_2') has equilibria $(p'_{a1}, y'_{a1}, p'_{a2}, y'_{a2})$ and $(p'_{b1}, y'_{b1}, p'_{b2}, y'_{b2})$ in states a and b respectively with $f^i(p'_{a1}, y'_{a1}) = f^i(p'_{b1}, y'_{b1})$.

4.2 Proposition: Let $m^i > 0$ for each i, let $E_m = \{e \varepsilon E: \bar{m}^i = m^i$ for each $i\}$, and let $(P_1 \times Y_1)_m = \{(p_1, y_1) \varepsilon P_1 \times Y_1: m^i - p_1 y_1^i \geq 0$ for each $i\}$. Suppose that f is a data structure such that (e,f) has an expectations equilbirium for each $e \varepsilon E_m$. Then for each i, if f^i is continuous then f^i is constant on $(P_1 \times Y_1)_m$.

Proof: Since $(m^i)_{i=1}^N$ is arbitrary and E is symmetric with respect to agents, it suffices to prove the assertion for $i = 1$, so suppose that f^1 is continuous. Let $L = \{e \varepsilon E_m$: $\eta_1^1 = 0$ and $\eta_1^i = 1$ for each $i > 1$; and for each i, there exist positive numbers α_1^i, α_2^i, α_{a3}^i, and

α^i_{b3} such that $u^i_1(\cdot) = \alpha^i_1 \ln(\cdot)$, $u^i_2(\cdot) = \alpha^i_2 \ln(\cdot)$, $u^i_{a3}(\cdot) = \alpha^i_{a3} \ln(\cdot)$, and $u^i_{b3}(\cdot) = \alpha^i_{b3} \ln(\cdot)\}$. For each $e \in L$ and each $\eta_2 \in I_2$, the uniqueness of equilibria for (e, η_2) in each state can be established as follows. Let the price of money in each period and each state be positive and equal across periods and states but not necessarily equal to unity, and let the prices p_{st}, $s = 1,2$, $t = 1,2$ vary as above. Then (e, η_2) determines an aggregate demand for the commodities c_{st}, $s = 1,2$, $t = 1,2,3$ as a function on $\text{int } R^5_+$ which exhibits gross substitutability in the finite increment version [6, p. 305] and indecomposability [6, p. 306]. Uniqueness then follows by a well known result [6, p. 335]. If $e \in L$, since $\eta^i_1 = 1$ for each $i > 1$, only the value of η^1_2 influences the equilibria of (e, η_2) . Accordingly, notation will be saved by replacing both η_2 and η^1_2 by the 0, 1 - valued variable η . Variables which will be associated with values of η will be preceeded by a superscript.

Let $(p_1, y_1) \in \text{int } (P_1 \times Y_1)_m$, that is, $m^i - p_1 y_1{}^i > 0$ for each i . For each i , choose positive numbers $(\bar{c}^i_{a1}, \bar{c}^i_{b1}, \bar{c}^i_{a2}, \bar{c}^i_{b2};$ $\alpha^i_1, \alpha^i_2, \alpha^i_{a3}, \alpha^i_{b3})$, with $\bar{c}^1_{at} = \bar{c}^1_{bt}$, $t = 1,2$, such that

i) The three-commodity static exchange environment with the endowment $(\bar{c}^i_{a1}, \bar{c}^i_{a2}, m^i)$ and utility function v^i_a for each i , where $v^i_a(c^i_1, c^i_2, c^i_3) = \alpha^i_1 \ln c^i_1 + \alpha^i_2 \ln c^i_2 + \alpha^i_{a3} \ln c^i_3$ for each $(c^i_1, c^i_2, c^i_3) \in \text{int } R^3_+$, has a (unique) equilibrium $({}^1p_{a1}, {}^1y_{a1}, {}^1p_{a2}, {}^1y_{a2})$ with $({}^1p_{a1}, {}^1y_{a1}) = (p_1, y_1)$;

ii) the three-commodity static exchange environment with the endowment $(\bar{c}^i_{b1}, \bar{c}^i_{b2}, m^i)$ and utility function v^i_b for each i, where $v^i_b(c^i_1, c^i_2, c^i_3) = \alpha^i_1 \ln c^i_1 + \alpha^i_2 \ln c^i_2 + \alpha^i_{b3} \ln c^i_3$ for each $(c^i_1, c^i_2, c^i_3) \varepsilon \text{ int } R^3_+$, has a (unique) equilibrium $({}^1p_{b1}, {}^1y_{b1}, {}^1p_{b2}, {}^1y_{b2})$ with $({}^1p_{b1}, {}^1y_{b1}) = (p_1, y_1)$; and

iii) the three-commodity static exchange environment with the endowment $(\bar{c}^i_{b1}, \bar{c}^i_{b2}, m^i)$ for each i, and utility functions v^1_a, and v^i_b for each $i > 1$, has a (unique) equilibrium $({}^0p_{b1}, {}^0y_{b1}, {}^0p_{b2}, {}^0y_{b2})$ with $({}^0p_{b2}, {}^0y_{b2}) \varepsilon \text{ int } (P_1 \times Y_1)_m$ and ${}^0c^1_{b3} \neq {}^1c^1_{s3}$ for each s, where ${}^0c^1_{b3} = m^1 - {}^0p_{b1}\,{}^0y^1_{b1} - {}^0p_{b2}\,{}^0y^1_{b2}$ and ${}^1c^1_{s3} = m^1 - {}^1p_{s1}\,{}^1y^1_{s1} - {}^1p_{s2}\,{}^1y^1_{s2}$ for each s.

For each i, let u^i_1, u^i_2, u^i_{a3}, and u^i_{b3} denote the logarithmic utility functions parameterized by α^i_1, α^i_2, α^i_{a3}, and α^i_{b3} respectively, and for each $0 < \pi < 1$, let $e(\pi)$ denote the economy in L determined by the characteristics $(\bar{c}^i_{a1}, \bar{c}^i_{b1}, \bar{c}^i_{a2}, \bar{c}^i_{b2}; u^i_1, u^i_2, u^i_{a3}, u^i_{b3})$ for each i, and the probability π. Since ${}^1p_{a1} = {}^1p_{b1}$ and ${}^1y^1_{a1} = {}^1y^1_{b1}$, for each π, $({}^1p_{a1}, {}^1y_{a1}, {}^1p_{a2}, {}^1y_{a2})$ and $({}^1p_{b1}, {}^1y_{b1}, {}^1p_{b2}, {}^1y_{b2})$ are (unique) equilibria for $(e(\pi), 1)$ in states a and b respectively. Since ${}^0c^1_{b3} \neq {}^1c^1_{s3}$ for each s, $\alpha^1_{b3}\ln(\cdot)$ can be replaced by a different

$u^1_{b3} \varepsilon U^1_3$, which differs from $\alpha^1_{b3}\ln(\cdot)$ only near ${}^0c^1_{b3}$, so that statements (i - iii) above remain true and, in addition, ${}^0c^i_{bt} \neq {}^1c^i_{st}$ for each i, s, t, where ${}^0c^i_{bt} = \bar{c}^i_{bt} + {}^0y^i_{bt}$ and ${}^1c^i_{st} = \bar{c}^i_{st} + {}^1y^i_{st}$ for each i, s, and t = 1,2, and ${}^0c^i_{b3} = m^i - {}^0p_{b1}\,{}^0y^i_{b1} - {}^0p_{b2}\,{}^0y^i_{b2}$ and ${}^1c^i_{s3} = m^i - {}^1p_{s1}\,{}^1y^i_{s1} - {}^1p_{s2}\,{}^1y^i_{s2}$ for each i, s . For each π , let $e'(\pi)$ denote the resulting economy in E , and note that u^1_{b3} can be chosen to insure the uniqueness of the equilibria $({}^1p_{a1}, {}^1y_{a1}, {}^1p_{a2}, {}^1y_{a2})$ and $({}^1p_{b1}, {}^1y_{b1}, {}^1p_{b2}, {}^1y_{b2})$ of $(e'(\pi), 1)$ for each π . Since $({}^1p_a, {}^1y_{a1}) = ({}^1p_{b1}, {}^1y_{b1}) = (p_1, y_1)$, the hypothesis of Lemma 4.1 is satisfied, so for each π , $(e'(\pi), 0)$ must have equilibria $({}^0p_{a1}, {}^0y_{a1}, {}^0p_{a2}, {}^0y_{a2})(\pi)$ and $({}^0p_{b1}, {}^0y_{b1}, {}^0p_{b2}, {}^0y_{b2})(\pi)$ with $f^1[({}^0p_{a1}, {}^0y_{a1})(\pi)] = f^1[({}^0p_{a1}, {}^0y_{a1}(\pi))]$. However, as $\pi \to 1$, $({}^0p_{a1}, {}^0y_{a1})(\pi) \to (p_1, y_1)$ and $({}^0p_{b1}, {}^0y_{b1})(\pi) \to ({}^0p_{b1}, {}^0y_{b1})$, so since f^1 is continuous, $f^1(p_1, y_1) = f^1({}^0p_{b1}, {}^0y_{b1})$. Since ${}^0c^i_{bt} \neq {}^1c^i_{st}$ for each i, s, t, there is a neighborhood V of (p_1, y_1) such that for each $(p'_1, y'_1) \varepsilon V$, for each i there exist utility functions u'^i_1, u'^i_2, u'^i_{a3}, and u'^i_{b3} which differ, respectively, from u^i_1 only near ${}^1c^i_{a1}$ and ${}^1c^i_{b1}$, from u^i_2 only near ${}^1c^i_{a2}$ and ${}^1c^i_{b2}$, from u^i_{a3} only near ${}^1c^i_{a3}$, and from u^i_{b3} only near ${}^1c^i_{b3}$, so that if the new utility functions are substituted into statements (i - iii) above, statement (iii) remains true with $({}^0p_{b1}, {}^0y_{b1})$ unchanged, and statements (i) and (ii) remain true if (p_1, y_1) is replaced by (p'_1 , y'_1) . In addition the new utility functions can be chosen so that if $e''(\pi)$ denotes the new economy for each π, the equilibria for $(e''(\pi), 1)$ are unique in each state. It follows from Lemma 4.1 that f^1 is constant on V.

Since (p_1, y_1) was chosen arbitrarily in int $(P_1 \times Y_1)_m$ we have shown that the function f^1 induces a partition of int $(P_1 \times Y_1)_m$ into disjoint open subsets. Since int $(P_1 \times Y_1)_m$ is connected, the partition must contain only one set, so f^1 is constant on int $(P_1 \times Y_1)_m$. Since f^1 is continuous, the Proposition follows.

4.3 Remarks: In an expectations equilibrium, the first period equilibria (p_{1s}, y_{1s}) will generally differ between the two states, so that the second period distribution of money $(\bar{m}^i - p_{1s}y^i_{1s})^N_{i=1}$ will generally differ between states. Our assumption that the initial distribution of money, $(\bar{m}^i)^N_{i=1}$, is state-independent might therefore seem unnatural. Of course if the model is generalized to allow the initial distribution of money to differ across states, the resulting class of economies would include E so the Theorem would be inherited. However, under this generalization, it is natural to allow data functions to depend on the initial distribution of money as well as (p_1, y_1). For example, if the initial distribution of money can be influenced by economic policy, it should not be assumed to be completely unobservable. Therefore, suppose that data functions are redefined to have the domain

$\Gamma = \{(p,y,\bar{m}) \in P_1 \times Y_1 \times \Pi_i(\text{int } M^i): \bar{m}^i - p_1y^i_1 \geq 0 \text{ for each } i\}$.

Then we have as an immediate corollary to Proposition 4.2: If f^i is a continuous data function in an admissable data structure then for each

$\bar{m}$, $\bar{m}' \in \Pi_i(\text{int } M^i)$, either

i) $f^i(p_1, y_1, \bar{m}) = f^i(p_1', y_1', \bar{m}')$ for all $(p_1, y_1) \in (P_1 \times Y_1)_{\bar{m}}$ and all $(p_1', y_1') \in (P_1 \times Y_1)_{\bar{m}'}$, or

ii) $f^i(p_1, y_1, \bar{m}) \neq f^i(p_1', y_1', \bar{m}')$ for all $(p_1, y_1) \in (P_1 \times Y_1)_{\bar{m}}$ and all $(p_1', y_1') \in (P_1 \times Y_1)_{\bar{m}'}$.

Putting the conclusion somewhat differently, the observable data must be expressable as a function of the exogenous initial distribution of money.

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