Weakly bound diquarks and Efimov hyperons in QCD*

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PLAN:

- Motivation: Diquarks, Hadron Spectrum, Nucleon SL EM form factors
- QCD Model with color antriplet-scalar diquark
- Running coupling constant – Fixed point
- Gluon and photon in-medium masses
- Zero-binding diquarks & Thomas-Efimov physics
- Efimov hyperons and delta-like
- Conclusion

Continuous Advances in QCD, U. Minnesota, 2006

Motivation
Diquarks

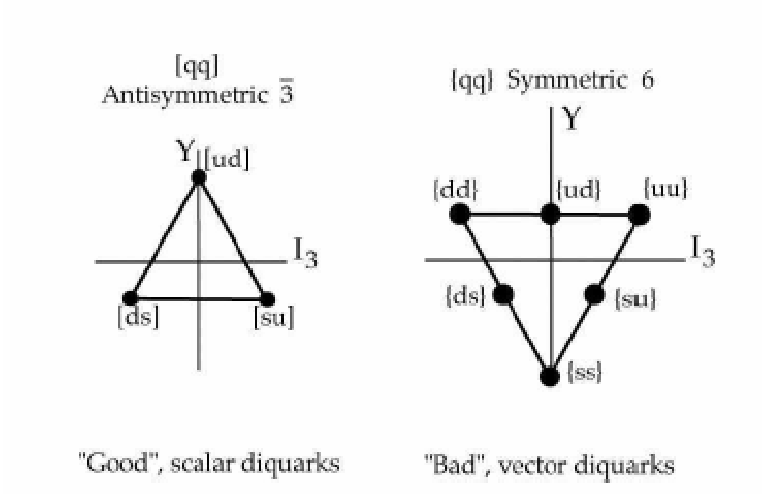
R. L. Jaffe, Nucl. Phys. Proc. Suppl **142** (2005) 343; Phys.Rep. **409** (2005) 1

$$|p\rangle = |qqq\rangle + |qqqq\bar{q}\rangle + |qqqg\rangle \ldots$$

Quark-quark color/flavor IRep. $3 \otimes 3 = 6 \oplus \overline{3}$

$$|(qq)(\overline{3})_c(A)(\overline{3})_f(A)(0^+)(A)\rangle.$$

$$|(qq)(\overline{3})_c(A)(6)_f(S)(1^+)(S)\rangle$$

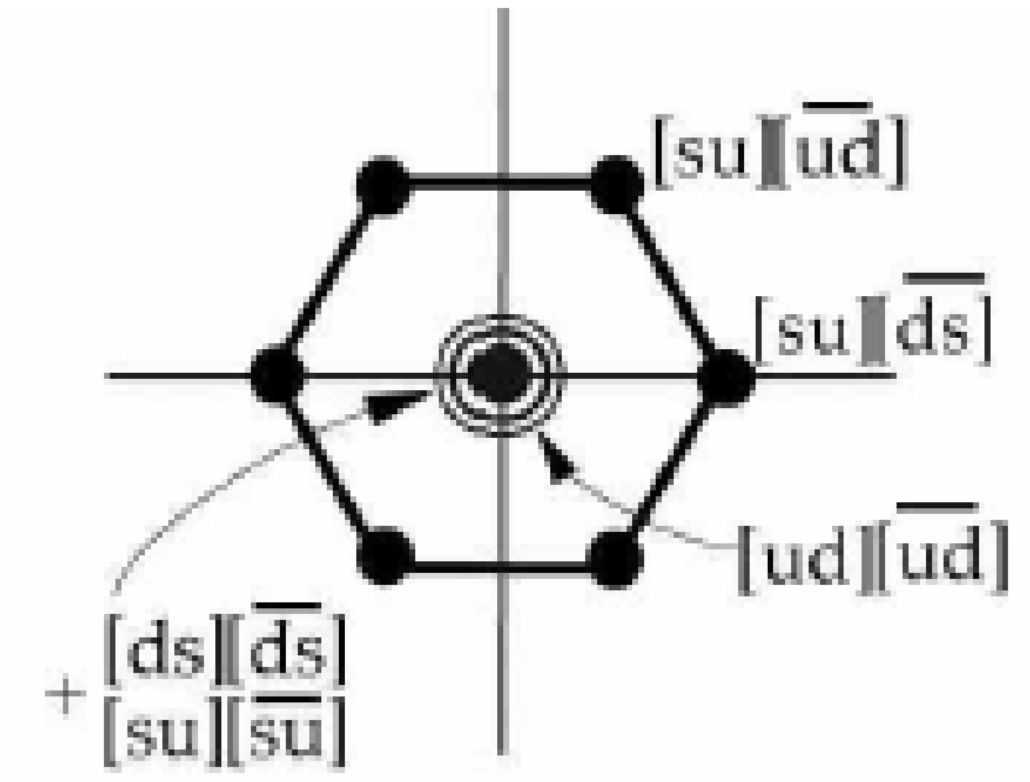


N(938) $\Delta(1232)\ I(J^P) = 3/2(3/2)^+$ **~ 300 MeV**

Scalar diquarks: diquark-antidiquark system

Singlet & octect exotic mesons

$$\overline{3}_f \otimes 3_f = 1_f \oplus 8_f$$



Tetraquark nonet

L. Maiani et.al., "New look to scalar mesons", Phys. Rev. Lett. 93 (2004) 212002;

TABLE I. Experimental values for the scalar meson masses.

Meson	Mass (MeV)	Source
σ	$478 \pm 24 \pm 17$	[7]
κ	$797 \pm 19 \pm 43$	E791 [8]
f	980 ± 10	PDG [2]
a	984.7 ± 1.2	PDG [2]

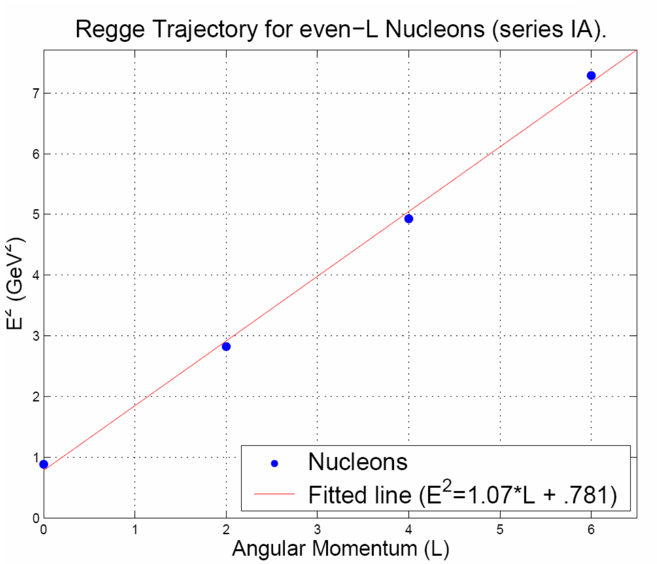
[7] KLOE Collaboration, A. Aloisio *et al.*, Phys. Lett. B 537, 21 (2002); E. M. Aitala *et al.*, Phys. Rev. Lett. 86, 770 (2001).

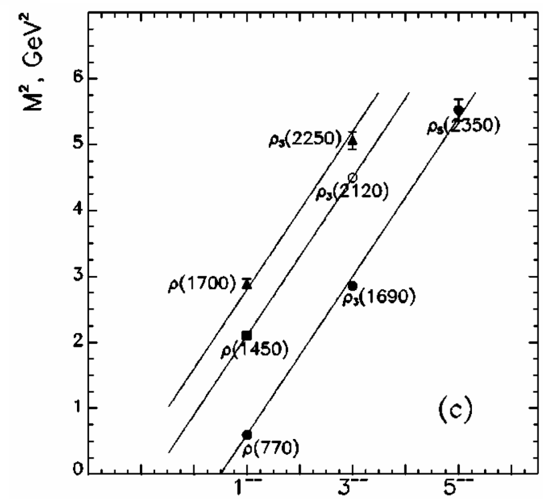
[8] E. M. Aitala *et al.*, Phys. Rev. Lett. 89, 121801 (2002).

Hadron spectrum

Chew-Frautschi formula

$$M^2 = a + \sigma L$$





slope $\simeq 1.25 \pm 0.15\ \mathrm{GeV}^2$

Diquarks as Inspiration and as Objects

Frank Wilczek*

hep-ph/0409168

PHYSICAL REVIEW D, VOLUME 62, 051502(R)

Systematics of $q\bar{q}$ states in the (n,M^2) and (J,M^2) planes

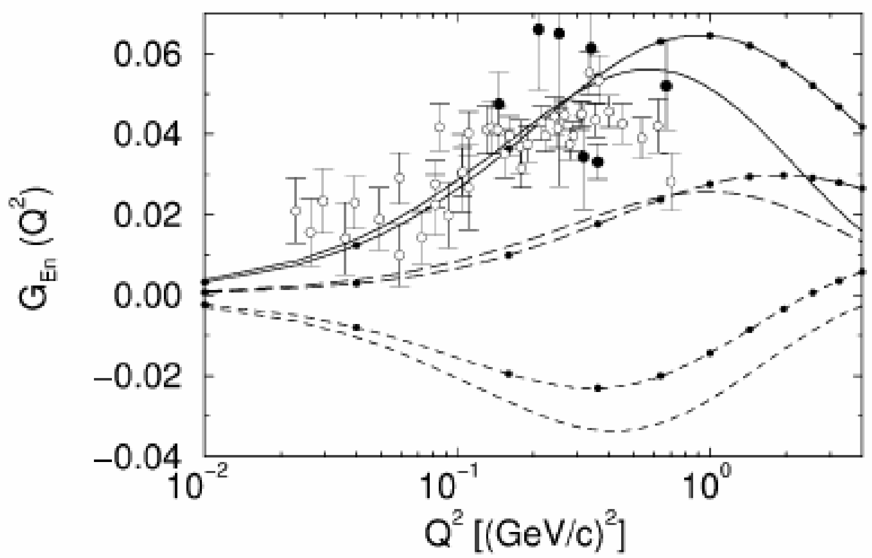
A. V. Anisovich, V. V. Anisovich, and A. V. Sarantsev

Nucleon SL EM form factors

$$\mathcal{L}_{N-3q} = \alpha m_N \epsilon^{lmn} \overline{\Psi}_{(l)} i\tau_2 \gamma^5 \Psi^C_{(m)} \overline{\Psi}_{(n)} \Psi_N + (1-\alpha)\epsilon^{lmn} \overline{\Psi}_{(l)} i\tau_2 \gamma_\mu \gamma^5 \Psi^C_{(m)} \overline{\Psi}_{(n)} i\partial^\mu \Psi_N + \text{H.C.},$$

Light-front model of the nucleon wave function

E.F. Suisso et al. / Nuclear Physics A 694 (2001) 351–371



solid α=1

dashed α=1/2

dotted α=0

Some evidences of diquark correlations:

Classification of scalar mesons

Orbital excitations in Barion X Meson Spectrum

EM structure of the nucleon

EFFECTIVE MODEL WITH DIQUARKS

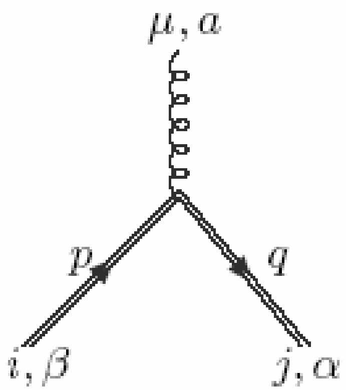
QCD Model with color antriplet-scalar diquark

Hong, Sohn, Zahed Phys. Lett. B596 (2004) 191

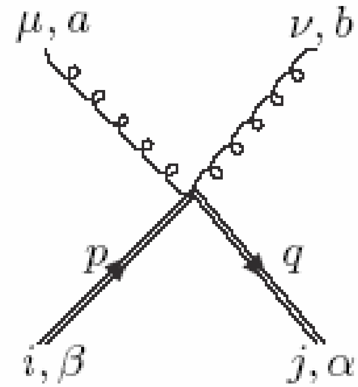
$$L = -\frac{1}{4}G^{a,\mu\nu}G^{a}_{\mu\nu} + \bar{\Psi}\left(\imath \not{D}_{(q)} - m_q\right)\Psi + \left(D^{\mu}_{(s)}\phi\right)^{\dagger}\left(D_{(s)\mu}\phi\right) - \phi^{\dagger}\hat{m}^2_s\phi$$

$$\left(D_{(s)\mu}\phi\right)_i = \partial_\mu\phi_i - igA^a_\mu (T^a_{ij})^*\phi_j \quad \left(D_{(q)\mu}\psi\right)_i = \partial_\mu\psi_i + igA^a_\mu T^a_{ij}\psi_j$$

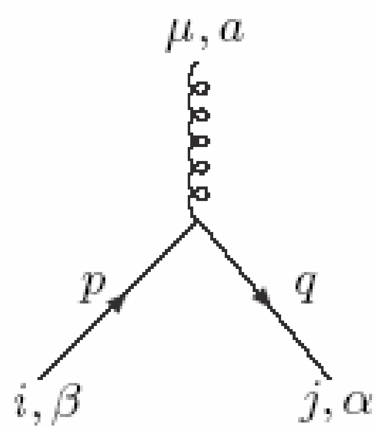
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu$$



$$= igT^a_{ij}(p+q)^\mu$$



$$= ig^2 g^{\mu\nu}\{T^a, T^b\}_{ij}$$



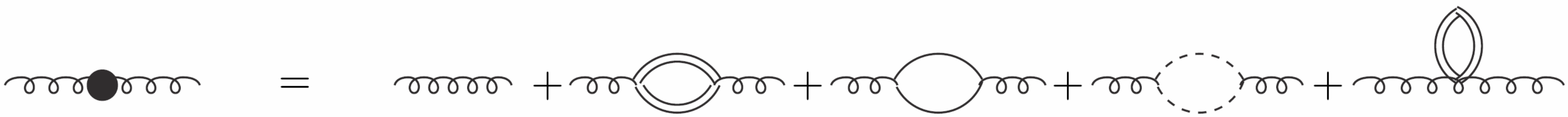
$$= -igT^a_{ji}\gamma^\mu$$

Running coupling constant

$$\Psi = \sqrt{Z_2}\Psi_B,\ A^a_{B\mu} = \sqrt{Z_3}A^a_\mu,\ \phi_B = \sqrt{Z^s_2}\phi_B$$

$$g_B = g\mu^{\frac{\epsilon}{2}}\frac{Z_1}{Z_2\sqrt{Z_3}} = g\mu^{\frac{\epsilon}{2}}\frac{Z^s_1}{Z^s_2\sqrt{Z_3}} = \ldots$$

$$\frac{dg_B}{d\mu} = 0$$



$$\Pi_{ab}^{\mu\nu}(d1+d2) = \frac{g^2}{48\pi^2\varepsilon} N_s \delta_{ab}[(p^\mu p^\nu - p^2 g^{\mu\nu})]$$

Ns number of diquark flavors

$$Z_3 = 1 + \frac{g^2}{8\pi^2\varepsilon}(5 - \frac{2}{3}N_q - \frac{1}{6}N_s)$$

$$\alpha_{QCD}^{q+s}(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}N_q - \frac{1}{6}N_s)\log\frac{Q^2}{\Lambda^2}}$$

QCD FIXED-POINT

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left(11 - \frac{2}{3}N_q - \frac{1}{6}N_s\right)$$

only $[qq]^{\bar{3}_c}$ configurations $\quad N_s = N_q(N_q - 1)/2$

$$11 - \tfrac{2}{3}N_q^* - \tfrac{1}{12}N_q^*(N_q^* - 1) = 0$$

$N_q^* \approx 8.5$ → freezing of the coupling constant near the physical flavor number!

T. Banks and A. Zaks, Nucl. Phys. **B196** (1982) 189

P. M. Stevenson, Phys. Lett. **B331** (1994) 187

Gluon and photon in-medium masses

$$L_{A^2} = g^2 A^{\mu a} A^b_\mu \phi^\dagger_j (T^a)_{ij} T^{b*}_{\ ik} \phi_k$$

$$m_g^2 = 2g^2 \langle \phi^\dagger_j (T^a)_{ij} T^{a*}_{\ ik} \phi_k \rangle$$

colorless medium $\quad \langle \phi^\dagger \phi \rangle = \frac{1}{3} \sum_{flavor,i} \langle \phi_i^{f\dagger} \phi_i^f \rangle$

$$m_g^2 = 2g^2 \langle \phi^\dagger \phi \rangle (T^a)_{ij} T^{a*}_{\ ij} = 4g^2 \langle \phi^\dagger \phi \rangle$$

Estimative of in-medium gluon mass

$$m_g^2 = 2g^2\langle\phi^\dagger\phi\rangle(T^a)_{ij}T^{a*}_{ij} = 4g^2\langle\phi^\dagger\phi\rangle$$

$$\sum_{flavor,i} m_{s,f}^2\langle\phi_i^{f\dagger}\phi_i^f\rangle \sim \varepsilon$$

$$\varepsilon_0 = 1\mathrm{GeV}/\mathrm{fm}^3$$

- $\alpha_{QCD} \sim 1 \qquad \epsilon = \epsilon_0 \qquad m_s \sim 0.6\ \mathrm{GeV}$

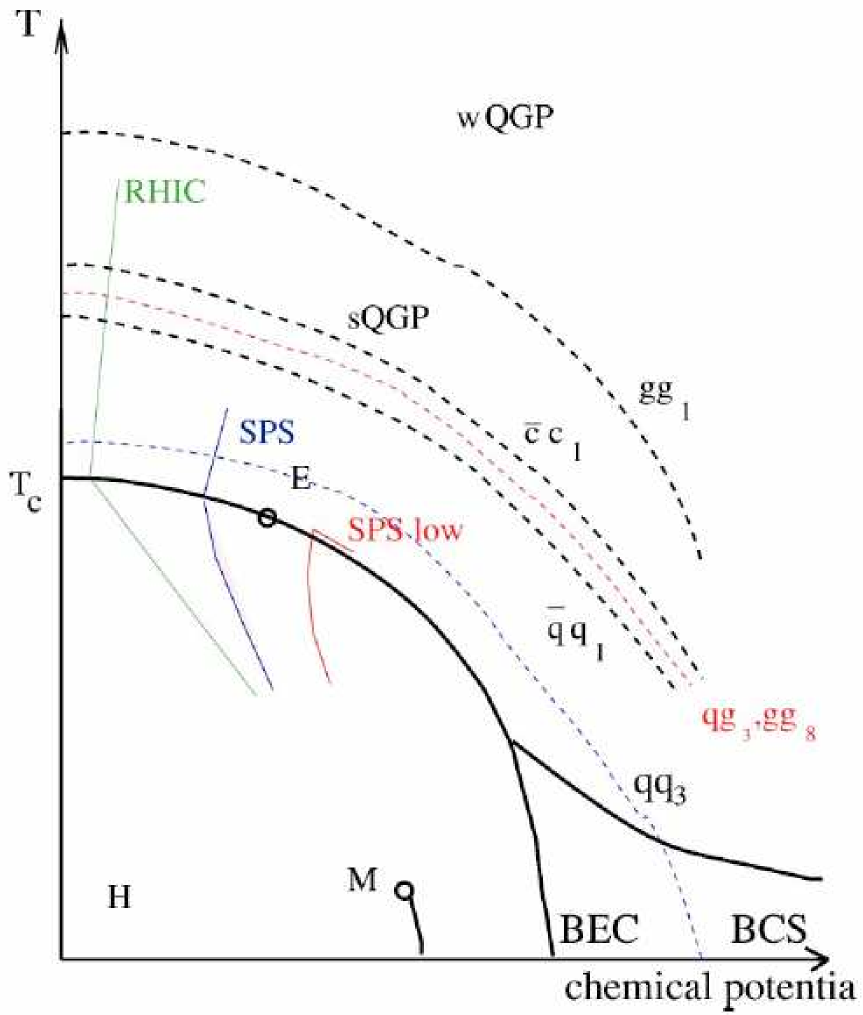
$$m_g \sim 0.6\ \mathrm{GeV}$$

Estimative of in-medium photon mass

$$m_\gamma \sim \sqrt{e^2 \varepsilon \sum_f q_f^2 / (N_s m_s^2)}.$$

few tens of MeV near the chiral phase transition

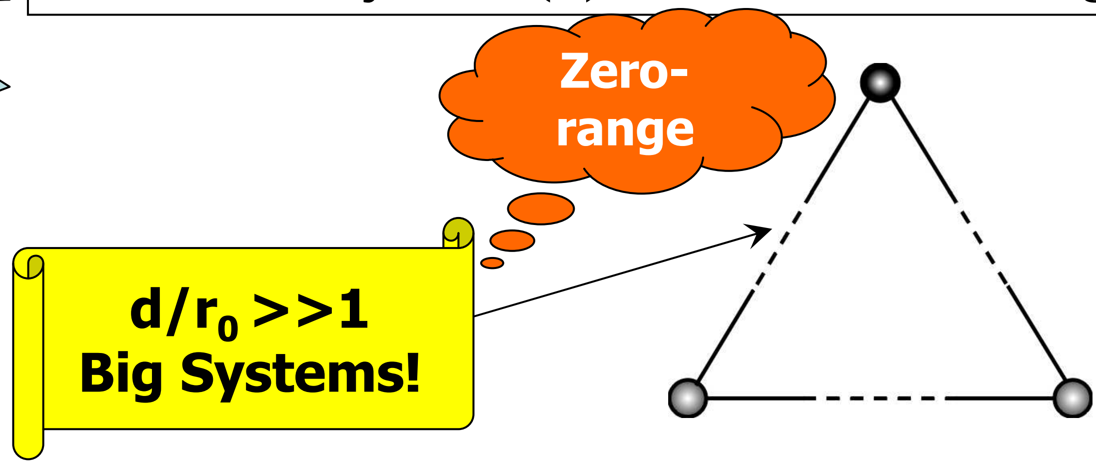
Zero-binding diquarks & Thomas-Efimov physics



E. V. SHURYAK AND I. ZAHED
PHYSICAL REVIEW C **70**, 021901(R) (2004)

Weakly bound systems & contact interaction

Size of the system (d) >> interaction range (r0)



The eigenfunction of the system satisfies a free Schrödinger equation almost everywhere for nonzero interparticle distances!

asymptotic wf behaviour & universality

A. S. Jensen, K. Riisager, D. V. Fedorov, and E. Garrido, Rev. Mod. Phys. **76**, 215 (2004).

Weakly bound systems & contact interaction

Example: Trinucleon quartet state S=3/2

$$(E - H_0)\psi = 0$$

$$\psi = \int d^3q_1 \frac{\exp\{i[E-(3/4)q_1^2]^{1/2}R_1\}}{R_1} e^{i\mathbf{q}_1\cdot\mathbf{r}_1}\chi(\mathbf{q}_1) - \int d^3q_3 \frac{\exp\{i[E-(3/4)q_3^2]^{1/2}R_3\}}{R_3} e^{i\mathbf{q}_3\cdot\mathbf{r}_3}\chi(\mathbf{q}_3)$$

R_i relative distance between particles j and k

r_i   relative distance between i and jk CM

Scale invariance in few-body systems

Spectator function for 3-bosons

$$a^{-1} = \pm\sqrt{\epsilon_2}$$

$$\chi(\vec{y}) = \frac{-\pi^{-2}}{\pm\sqrt{\epsilon_2} - \sqrt{\epsilon_3 + \frac{3}{4}\vec{y}^2}} \int d^3x \frac{1}{\epsilon_3 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} \chi(x)$$

If ϵ_2 and $\epsilon_3 = 0$

Spectator function for a triquark-system

$$\chi(q) = C_\kappa \frac{4}{\sqrt{3}\pi} \int_0^\infty dk \chi(k) \ln \frac{q^2 + qk + k^2}{q^2 - qk + k^2}$$

$$\kappa \equiv \{S, IRep_c, IRep_f\}$$

Scale invariance: $\chi(\xi\ y)$ is also solution!!!

$$\chi(q) = q^{i\pi s_0 - 1} \qquad 1 = C_\kappa \frac{8}{\sqrt{3} s_0} \frac{\sinh \frac{\pi s_0}{6}}{\cosh \frac{\pi s_0}{2}}$$

$$C_\kappa > \frac{3\sqrt{3}}{4\pi} \quad (s_0 = 0)$$

$$E^{(n+1)} \longrightarrow e^{-2\pi/s_0} E^{(n)}$$

EFIMOV STATES

V. Efimov, Phys. Lett. **B 33** (1970) 563; Nucl. Phys. **A362** (1981) 45; V. Efimov, Comm. Nucl. Part. Phys. **19** (1990) 271.

E. Braaten and H.-W. Hammer, to appear in Physics Reports (cond-mat/0410417)

Spontaneous breaking of scale invariance&Thomas-Efimov effect

Nature breaks scale invariance → momentum/energy scale

$$\chi(\vec{y}) = \frac{-\pi^{-2}}{\pm\sqrt{\epsilon_2} - \sqrt{\epsilon_3 + \frac{3}{4}\vec{y}^2}} \int d^3x \left(\frac{1}{\epsilon_3 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} - \frac{1}{1 + \vec{y}^2 + \vec{x}^2 + \vec{y}\cdot\vec{x}} \right) \chi(\vec{x})$$

$$\mu^2_{(3)} = 1$$

Adhikari,TF,Goldman, PRL74 (1995) 487

Thomas collapse: $\mu^2_{(3)} \to \infty$

Efimov effect: $E_2 \to 0$

(both) $\epsilon_2 \to 0$

S.K. Adhikari, A. Delfino, T. Frederico, I.D. Goldman, and L. Tomio, Phys. Rev. A **37**, 3666 (1988).

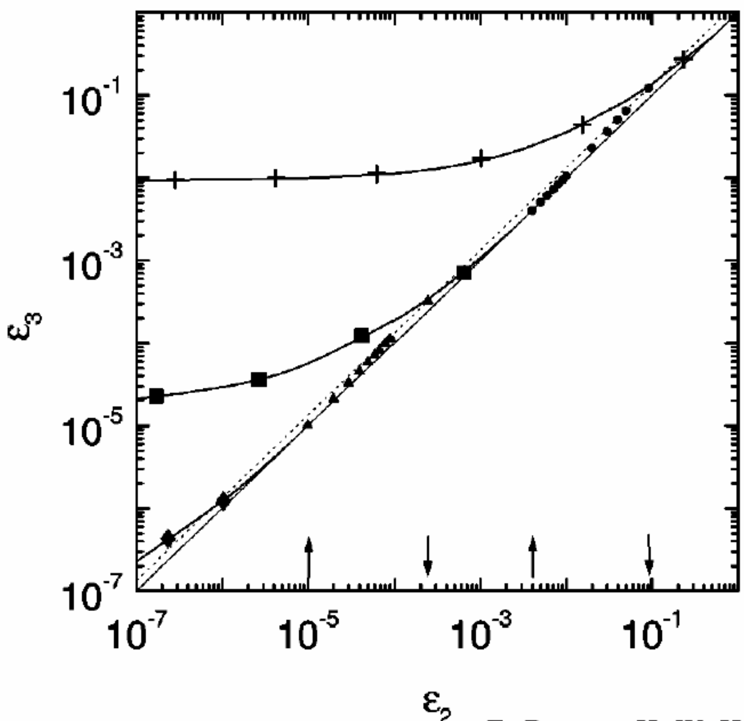
$$\epsilon_3^{(N)} \equiv \epsilon_3^{(N)}(\pm\sqrt{\epsilon_2})$$

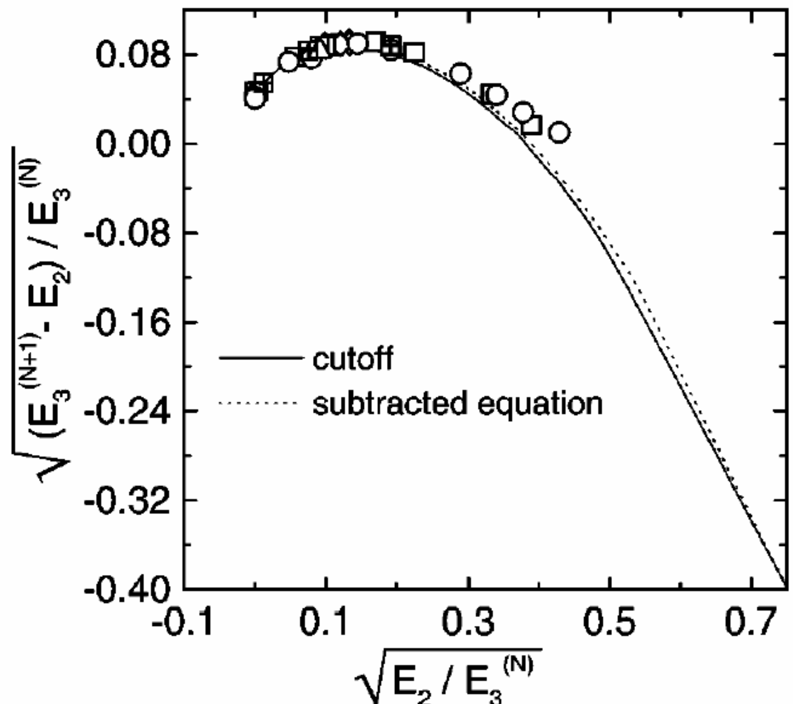
Scaling limit & limit cycle

$$\frac{E_3^{(N+1)}}{E_3^{(N)}} = \lim_{N\to\infty} \frac{\epsilon_3^{(N+1)}(\xi)}{\epsilon_3^{(N)}} = \mathcal{F}\left(\pm \sqrt{\frac{E_2}{E_3^{(N)}}} \right)$$

Scaling limit:
Frederico et al PRA60 (1999)R9
Yamashita et al PRA66(2003)052702

Limit cycle:
R.F. Mohr, R.J. Furnstahl, H.-W. Hammer, R.J. Perry, and K.G. Wilson, Ann. Phys. **321**, 225 (2006).

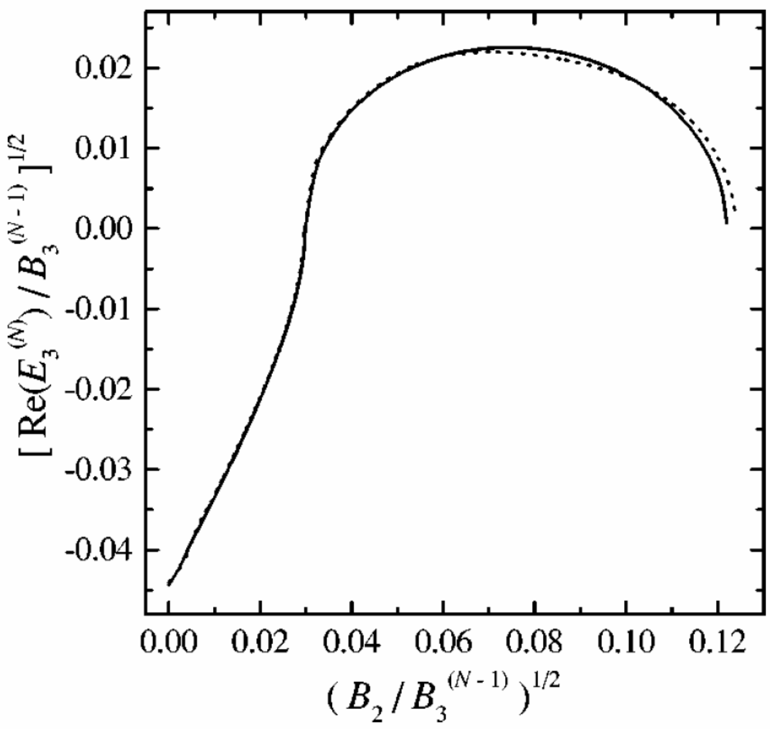


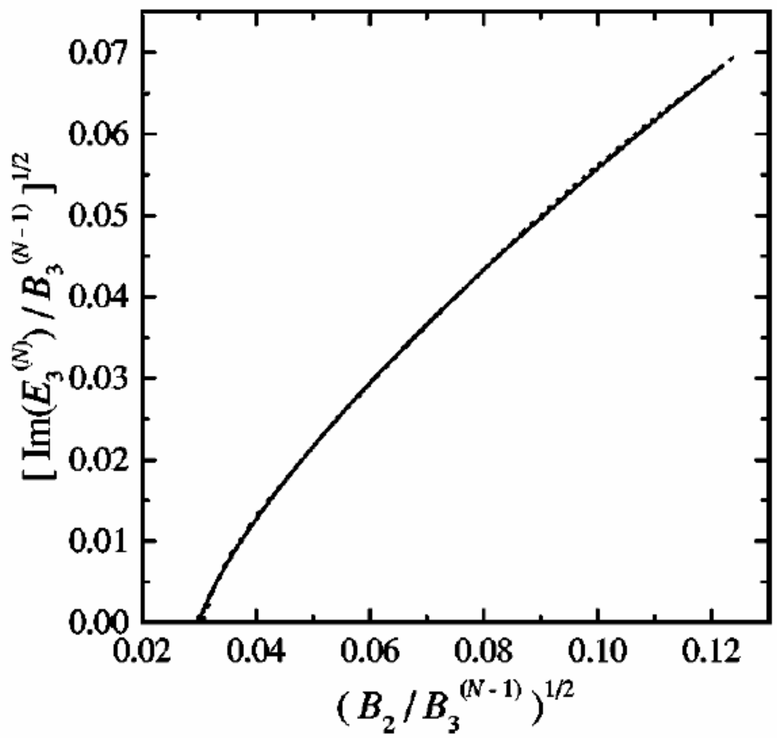


E. Braaten, H.-W. Hammer, and M. Kusunoki, Phys. Rev. A **67**, 022505 (2003).

Scaling functions: Correlation between observables

S-wave three-boson resonance





Bringas, Yamashita, TF, PRA69(2004)040702(R)

Evidence for Efimov quantum states in an ultracold gas of caesium atoms

T. Kraemer[1], M. Mark[1], P. Waldburger[1], J. G. Danzl[1], C. Chin[1,2], B. Engeser[1], A. D. Lange[1], K. Pilch[1], A. Jaakkola[1], H.-C. Nägerl[1] & R. Grimm[1,3]

NATURE|Vol 440|16 March 2006

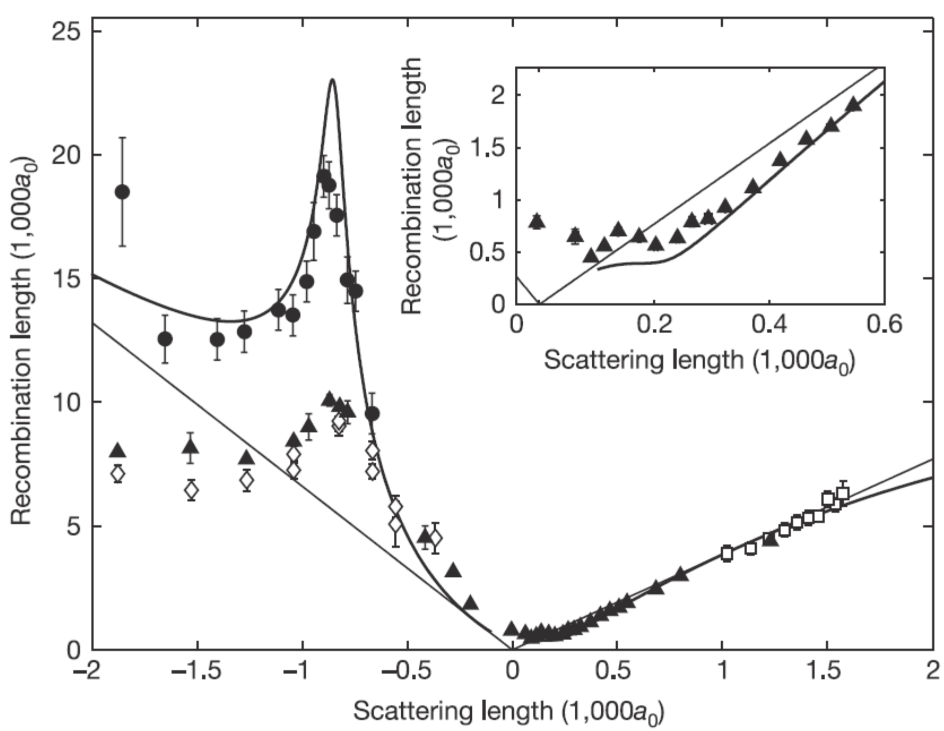


Figure 2 | Observation of the Efimov resonance in measurements of three-body recombination. The recombination length $\rho_3 \propto L_3^{1/4}$ is plotted

Efimov hyperons and delta like

- vector diquark dissolves more easily than the scalar
- condensed scalars **&** zero binding vector diquarks
- u, d and s quarks
- state with zero color, flavor symmetric and spin $3/2$

$\Rightarrow$ $C_\kappa = 1$ (analogous to a 3-boson system)

DELTA and HYPERONIC LIKE weakly bound/resonant states!!!

Conclusion

- Diquarks can be introduced in QCD as an effective degree of freedom (toy model);

- Strong correlation between the quarks – forming diquarks – approaches the fixed point to the real world;

- Near zero-binding diquarks it is possible weakly bound our resonant ***hyperons and delta-like states***;

Next : - Structure of exotic mesons dq – anti dq;
- Nucleon q – dq;
- Thomas-Efimov physics in the formation of strange and non-strange matter...
- extension to other scalar correlations $[qqqq]^{3c}$

R. L. Jaffe, Phys. Rev. **D72** (2005) 074508