The homology of the kernel space of the Thom spectrum in low degrees

A Dissertation
SUBMITTED TO THE FACULTY OF THE
UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Advisor: Tyler Lawson

2024

Acknowledgements

I wish to thank the members of my thesis committee, Sasha Voronov, Craig Westerland and Alex Kamenev, for all of their guidance and support. I especially want to thank my advisor, Tyler Lawson, for his help in the conception and completion of this project and for always having answers to my many questions along the way.

I also wish to thank all my friends and family, especially my wife, Jenny, for her many sacrifices during the writing of this thesis; my parents for their steadfast encouragement in all my ambitions; and my brother, Jeremy, for always being a source of intellectual stimulation.

Finally, I wish to thank all of my past professors and teachers for their help in getting to this place. In particular, I thank my undergraduate research mentors, Bree Cummins and Tomas Gedeon, for giving me such a firm foundation on which to build. I thank Konstantin Mischaikow and Ilse-Mari Lee for their significant roles in my admission to graduate school. I also thank my former instructor Rob Malo for inspiring me to pursue a degree in mathematics.

In loving memory of my father

David Huttinga

1963–2018

Abstract

I present an investigation of the multiplicative structure of $H_*(SL_1MU;\mathbb{F}_2)$ as an algebra over $\mathbb{F}_2$. This includes a list of generators in low degrees and the multiplication table for these generators. This ring has applications in topology related to orientability of vector bundles, as it is closely tied to the homology of the spectrum sl_1MU. The ring $H_*(SL_1MU)$ is closely tied to the Hopf ring $H_*(\underline{MU}_{2\star})$ introduced by Ravenel and Wilson in 1977 [17]. All of the available multiplicative information on $H_*(SL_1MU)$ descends from $H_*(\underline{MU}_{2\star})$, and accordingly, I present a complete algebraic description of this Hopf ring.

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Introduction

In recent history, there has been considerable interest in the spectra sl_1R and gl_1R for a ring spectrum R [3, 10, 11, 18]. However, much remains unknown; in particular, the homology of sl_1R is largely a mystery.

In [13, Section 1], Miller constructs a spectral squence that provides an algebraic connection between SL_1R and sl_1R or GL_1R and gl_1R. In particular, the Miller spectral sequence provides a method to derive information about $H_*(sl_1R)$ from information about $H_*(SL_1R)$ and the Dyer–Lashof operations on the latter. This motivates us to study $H_*(SL_1R)$, as information on this ring is also very limited. This is the central topic of this thesis.

I will briefly recall the relevant definitions and motivation for these constructions. Given a ring spectrum R, the spaces GL_1R and SL_1R are defined by the following pullback squares (see, for example, [3, p. 3]):

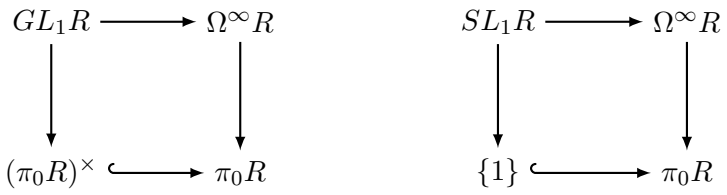

Figure 1: Pullback squares defining GL_1R and SL_1R for a ring spectrum R

We refer to GL_1R and SL_1R as the *units space* and *kernel space* of R, respectively. From these spaces, we obtain the *units spectrum* gl_1R and the *kernel spectrum* sl_1R: they are the (-1)-connective spectra satisfying $\Omega^\infty gl_1R = GL_1R$ and $\Omega^\infty sl_1R = SL_1R$.

Historically, much of the motivation for these concepts related to investigations of vector bundles. Given a vector bundle $\xi \to X$, one of the most basic questions one might ask is how far this bundle is from being trivial; that is, how "twisted" it is. One of the primary obstacles to triviality for a vector bundle is orientability; certainly if $\xi \to X$ is not orientable, it cannot be trivial. To address this question, we typically consider the characteristic classes

of $\xi \to X$. In particular, if we assume this is a real vector bundle, then we find an obstruction to orientability in the form of the first Stiefel–Whitney class, $w_1(\xi)$. This can be constructed as a composite

$$X \to BO(n) \to BGL_1\mathbb{S} \to BGL_1H\mathbb{Z}$$

of maps which I will define here.

First, recall that the bundle $\xi \to X$ is determined by a classifying map $X \to BO(n)$, where $BO(n)$ is the classifying space for the orthogonal group $O(n)$. Let $\mathbb{S} = \Sigma^\infty S^0$ be the sphere spectrum and $H\mathbb{Z}$ the Eilenberg-MacLane spectrum representing integral homology. Both are ring spectra, which in the latter case means there is a canonical unit map $\mathbb{S} \to H\mathbb{Z}$. This map is then a ring map with respect to the ring structures on both spectra.

If R is an E_∞ ring spectrum, then GL_1R is a group object in the category of spaces, as it inherits a product from the multiplicative structure of R. Therefore, in this case there is a classifying space BGL_1R, and the unit map $\mathbb{S} \to R$ induces a map $BGL_1\mathbb{S} \to BGL_1R$ [10, p. 333]. In particular, we have a map $BGL_1\mathbb{S} \to BGL_1H\mathbb{Z}$.

We can further define a map $O(n) \to BGL_1\mathbb{S}$ as follows. $GL_1\mathbb{S}$ includes into the space $\mathrm{Map}(\mathbb{S},\mathbb{S}) \simeq \Omega^\infty\mathbb{S}$ as the subspace of self-homotopy equivalences. One way to obtain a self-homotopy equivalence of $\mathbb{S}$ is to consider the composite

$$O(n) \to \mathrm{Map}_*(S^n, S^n) \xrightarrow{\Sigma^\infty} \mathrm{Map}(\mathbb{S},\mathbb{S})$$

where the second map is induced by the suspension functor and the first is given by the action of $O(n)$ on $\mathbb{R}^n \cup \{\infty\} \cong S^n$. The image of the composite $O(n) \to \mathrm{Map}(\mathbb{S},\mathbb{S})$ lies inside $GL_1\mathbb{S}$ since the elements of $O(n)$ are invertible. Therefore, we have a map $O(n) \to GL_1\mathbb{S}$, which induces a map $BO(n) \to BGL_1\mathbb{S}$.

These three maps then define the first Stiefel–Whitney class

$$X \to BO(n) \to BGL_1\mathbb{S} \to BGL_1H\mathbb{Z}$$

as noted above. We generalize this construction by replacing $H\mathbb{Z}$ with an arbitrary E_∞ ring spectrum E. In this case, if the map $X \to BGL_1E$ is trivial (that is, nullhomotopic), then there is a Thom isomorphism for E-cohomology. That is,

$$E^*(X) \cong E^{*+d}(Th(\xi))$$

where $Th(\xi)$ is the Thom space for $\xi \to X$.

Thus we obtain an avenue to study the orientability of $\xi \to X$. However, the result

is of limited practicality because in general, computing a map of spaces $X \to BGL_1E$ is difficult. One way we can improve the situation is by transporting this result from the category of spaces to the category of spectra, where there is much more structure on maps available. Provided that E has a strictly commutative structure, then we may replace the map $X \to BGL_1E$ with a map $\Sigma^\infty X \to \Sigma gl_1E$. This spectrum map $\Sigma^\infty X \to \Sigma gl_1E$ motivates us to study the object gl_1R in general.

Now I recount the construction of the Miller spectral sequence. This is heavily based on [13, Section 1]. Given an Ω-spectrum E and $X = \Omega^\infty E$, the natural map $\Sigma^\infty X \to E$ induces a split epimorphism $\Omega^\infty\Sigma^\infty X \to X$. Therefore, there is a fibration sequence

$$E(1) \to \Sigma^\infty X \to E$$

in the category of Ω-spectra. Let $X(0) = X$ and $E(0) = E$. Then iterating the process above, we obtain a resolution

$$E(0) \leftarrow \Sigma^\infty X(0) \leftarrow E(1) \leftarrow \Sigma^\infty X(1) \leftarrow E(2) \leftarrow \cdots$$

of E in which, for each $n \geq 0$, the sequence $E(n+1) \to \Sigma^\infty X(n) \to E(n)$ is a fibration sequence of Ω-spectra, and each map $\Sigma^\infty X(n) \to E(n)$ induces a split epimorphism on infinite loop spaces. If we fix a prime p, then applying mod-p homology $H_*(-)$ to this resolution gives a spectral sequence converging to the associated graded module of a certain filtration of $H_*(E)$ [13, p. 1]. We compute the E^2 page of this spectral sequence as follows.

If we apply Ω^∞ to each sequence $E(n+1) \to \Sigma^\infty X(n) \to E(n)$, then we obtain fibration sequences of spaces

$$X(n+1) \to \Omega^\infty\Sigma^\infty X(n) \to X(n)$$

in which the second map has a section. This implies that $\Omega^\infty\Sigma^\infty X(n)$ is weakly homotopy equivalent to $X(n+1) \times X(n)$. Therefore, applying $H_*(-)$, we obtain a split short exact sequence of Hopf algebras over the Dyer–Lashof algebra:

$$\mathbb{F}_p \to H_*(X(n+1)) \to H_*(\Omega^\infty\Sigma^\infty X(n)) \to H_*(X(n)) \to \mathbb{F}_p$$

These then concatenate into a resolution

$$\mathbb{F}_p \leftarrow H_*(X) \leftarrow H_*(\Omega^\infty\Sigma^\infty X(0)) \leftarrow H_*(\Omega^\infty\Sigma^\infty X(1)) \leftarrow \cdots$$

of $H_*(X)$ of Hopf algebras over the Dyer–Lashof algebra.

Now, using this resolution, we can construct the E^2 page. Letting QA be the module

of indecomposables for an algebra A and letting $L_\bullet(\mathbb{F}_p \otimes Q)(-)$ be the left-derived functor of $(\mathbb{F}_p \otimes Q)(-)$, we have

$$E_s^2 = L_s(\mathbb{F}_p \otimes Q)(H_*(X))$$

The E^∞ page is not $H_*(E)$, but the associated graded module of a natural filtration on $H_*(E)$ (see [13, Section 4] for details on this filtration). That is, the extension problem remains. Regardless, this spectral sequence does provide a connection between $H_*(E)$ and $H_*(X)$, and thus in particular, $H_*(gl_1R)$ and $H_*(GL_1R)$ or $H_*(sl_1R)$ and $H_*(SL_1R)$. Because the resolution of $H_*(X)$ is in the category of Hopf algebras over the Dyer–Lashof algebra, it is also necessary to understand the Dyer–Lashof algebra itself. In particular, we would like to understand how this algebra acts on Hopf algebras of the form $H_*(X)$ for a space X. Note that the homology of a space X does not necessarily have a natural Hopf algebra structure, but it does when X is an infinite loop space; see [4, p. 35] and [7, p. 2]. See Definition 1.10(1)–(3) for an example relevant to this thesis. Given a Hopf algebra $H_*(X)$, the action of the Dyer–Lashof algebra on $H_*(X)$ takes the form of Dyer–Lashof operations $Q^s : H_*(X) \to H_*(X)$. Explicit formulas for these operations are not known in general, though Turner computes some of those on the homology of complex bordism in [19, Theorem 3.15]. Further details can be found in [7, 12, 8, 14, 2, 6].

In order to study $H_*(sl_1R)$ via the Miller spectral sequence, we must first understand the structure of $H_*(SL_1R)$. In this thesis, I present a description of the low-dimensional multiplicative structure of $H_*(SL_1MU; \mathbb{F}_2)$. Given the universal role of MU in stable homotopy theory and the beneficial properties of $\mathbb{F}_2$ coefficients in the subject, this is a natural starting point for the problem.

In order to understand $H_*(SL_1MU; \mathbb{F}_2)$, we take advantage of a certain isomorphism of $\mathbb{F}_2$-vector spaces between $H_*(SL_1MU; \mathbb{F}_2)$ and a subspace of $H_*(\underline{MU}_{2\star}; \mathbb{F}_2)$, where $\underline{MU}_{2\star}$ is the Ω-spectrum associated with MU. In particular, in [17, Section 4], Ravenel and Wilson show that $H_*(\underline{MU}_{2\star})$ has the structure of a Hopf ring with additive product $\#$ and multiplicative product $\circ$ on generators $[z]$, $z \in MU^{2\star}(\text{pt})$, and b_n, $n \geq 0$. These will be defined in Chapter 1.

The element $[0] \in H_*(\underline{MU}_{2\star})$ is represented by an element in a certain path component of $\underline{MU}_0$. Following Ravenel and Wilson, we denote this path component by $\underline{MU}_0'$ [17, p. 28]. We then have the following result, to be proved in Chapter 2.

Proposition 2.2. *The map*

$$H_*(\underline{MU}_0') \xrightarrow{[1]\#(-)} H_*(SL_1MU)$$

is an isomorphism of abelian groups.

This provides our computational approach: certain parts of the algebraic structure on $H_*(\underline{MU}_{2\star})$ pass to $H_*(\underline{MU}'_0)$ and then in turn to $H_*(SL_1MU)$.

In Chapter 1, I present the Hopf ring structure of $H_*(\underline{MU}_{2\star})$. This is largely based on [17] but includes several new calculations that are necessary for the rest of the thesis. In Chapter 2, I use the content of Chapter 1 and the isomorphism above to compute the main result, the low-degree multiplicative structure of $H_*(SL_1MU)$. Chapter 2 is followed by an appendix with more explicit information on the Hopf ring structure of $H_*(\underline{MU}_{2\star})$. The generators $[z]$ and b_n above are subject to many very complicated relations, and the appendix contains an explicit list of those relations in low degrees.

Chapter 1

The Hopf ring $H_*(\underline{MU}_{2\star})$

1.1 Definition

1.1.1 Hopf rings in the abstract

Definition 1.1. Let R be a $\mathbb{Z}$-graded commutative ring with 1. A ($\mathbb{Z}$-graded, cocommutative, coassociative) coalgebra (with counit) over R is a $\mathbb{Z}$-graded R-module $C = \bigoplus_{n\in\mathbb{Z}} C_n$ with maps

$$\begin{aligned} \varepsilon &: C \to R && \text{(counit)} \\ \Delta &: C \to C \otimes C && \text{(coproduct)} \end{aligned}$$

satisfying $\Delta : C_n \to \bigoplus_{i+j=n} C_i \otimes C_j$ and making the diagrams in Figure 1.1 commute.

A *morphism of coalgebras* is an R-module morphism $f : C \to D$ that respects the counit and coproduct; that is, f makes the diagrams in Figure 1.2 commute. From this point on, a *coalgebra* will always mean a graded, coassociative, cocommutative coalgebra with counit. Unless otherwise stated, the grading will be over $\mathbb{Z}$. We denote the category of such objects over R and accompanying morphisms by GCoAlg_R.

Remark 1.2. Much of the focus of this thesis is actually on *evenly graded* coalgebras over R, the definition of which is completely analogous to the definition of $\mathbb{Z}$-graded coalgebras. We denote the category of evenly graded coalgebras over R by $\mathsf{G_2CoAlg}_R$. Different gradings, such as nonnegative grading, are also possible (and the definitions are again analogous).

Proposition 1.3. *R is naturally an object in GCoAlg_R, with the identity $\mathrm{id}_R : R \to R$ and the canonical isomorphism $R \to R \otimes R$ acting as the counit and coproduct, respectively. R is a terminal object in the category, with $\varepsilon : C \to R$ being the unique morphism in $\mathrm{Hom}(C, R)$.*

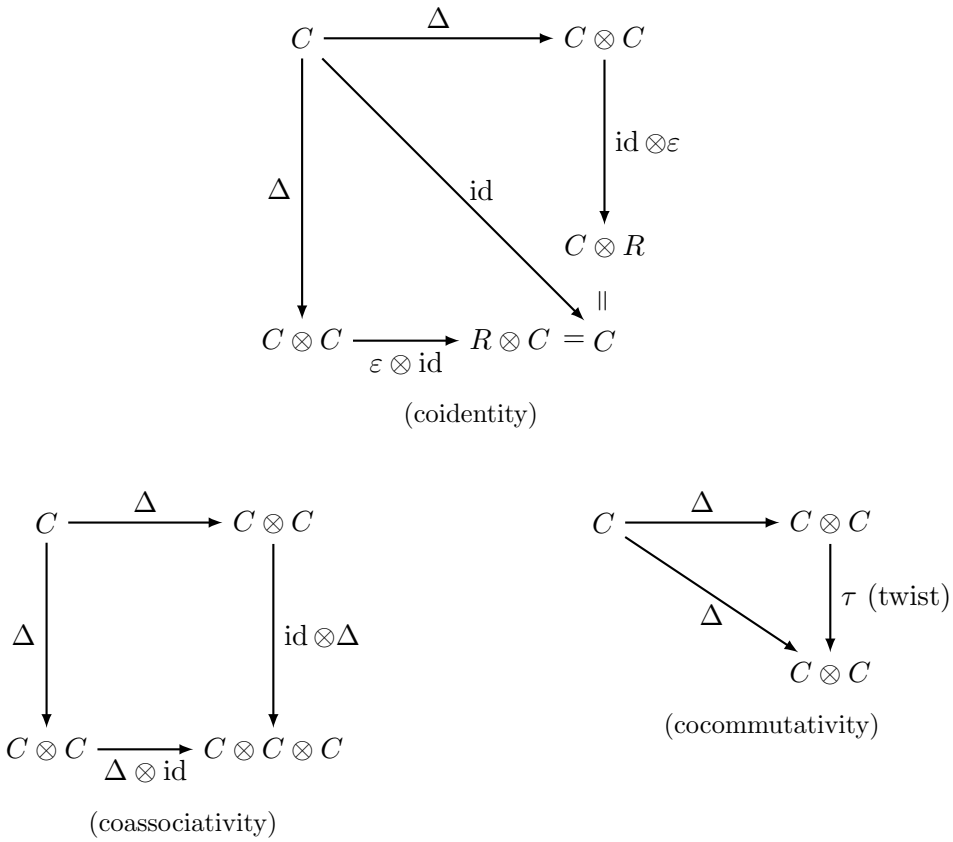

Figure 1.1: Conditions on an R-coalgebra C

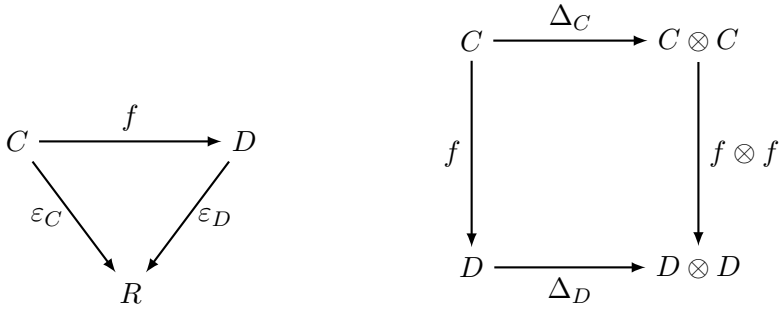

Figure 1.2: Conditions on an R-coalgebra morphism $f : C \to D$

The category GCoAlg_R *has finite products given by* $C \prod D = C \otimes D$.

Proof. R and the proposed counit and coproduct trivially satisfy Definition 1.1. For any R-coalgebra C with counit ε, any morphism $f : C \to R$ must satisfy $\mathrm{id}_R f = \varepsilon$, proving that R is terminal.

For the pairwise product, take as projections the maps $\mathrm{id}_C \otimes \varepsilon_D : C \otimes D \to C \otimes R = C$ and $\varepsilon_C \otimes \mathrm{id}_D : C \otimes D \to R \otimes D = D$. Given another coalgebra B and maps $f : B \to C$ and $g : B \to D$, we note $(f \otimes g) : B \otimes B \to C \otimes D$ satisfies $(\mathrm{id}_C \otimes \varepsilon_D)(f \otimes g) = (\mathrm{id}_C \otimes \varepsilon_C)(f \otimes f)$. Then precomposing by Δ_B gives $(\mathrm{id}_C \otimes \varepsilon_D)(f \otimes g)\Delta_B = (\mathrm{id}_C \otimes \varepsilon_C)\Delta_C f = f$. The other projection is analogous, so $(f \otimes g)\Delta_B$ satisfies the universal property. □

Definition 1.4. Let R be a commutative ring with 1. A ($\mathbb{Z}$-graded) *Hopf ring* over R is a graded ring object $H_*(\star)$ in GCoAlg_R. That is, for each $n \in \mathbb{Z}$, $H_*(n)$ is a coalgebra over R with counit and coproduct

$$\varepsilon : H_*(n) \to R$$
$$\Delta : H_*(n) \to H_*(n) \otimes H_*(n)$$

and there are coalgebra maps

$$\begin{aligned}
&\# : H_*(n) \otimes H_*(n) \to H_*(n) && \text{(additive product)}\\
&\eta : R \to H_*(n) && \text{(additive identity)}\\
&c : H_*(n) \to H_*(n) && \text{(additive inverse)}\\
&\circ : H_*(m) \otimes H_*(n) \to H_*(m+n) && \text{(multiplicative product)}\\
&e : R \to H_*(0) && \text{(multipicative identity)}
\end{aligned}$$

making the diagrams in Figures 1.3 and 1.4 commute. As with coalgebras generally, we define Hopf rings with other gradings analogously.

There are various basic properties of Hopf rings presented in [17, Lemma 1.12]. I reproduce most in the following lemma (others are inherent in Definition 1.4, such as the fact that the Hopf ring products, identities and the additive inverse are maps of coalgebras). All of these properties follow directly from the definition of a ring object.

Lemma 1.5. *Let* $x \in H_i(m)$, $y \in H_j(n)$ *and* $z \in H_k(n)$. *For an arbitrary* w, *let* $|w|$ *denote the degree of* w *with respect to the* $*$*-grading; that is, for* $w \in H_p(q)$, *we define* $|w| = p$. *I will use the notational convention* $\Delta(w) = \sum w' \otimes w''$.

(1) Properties of the coalgebra structure:

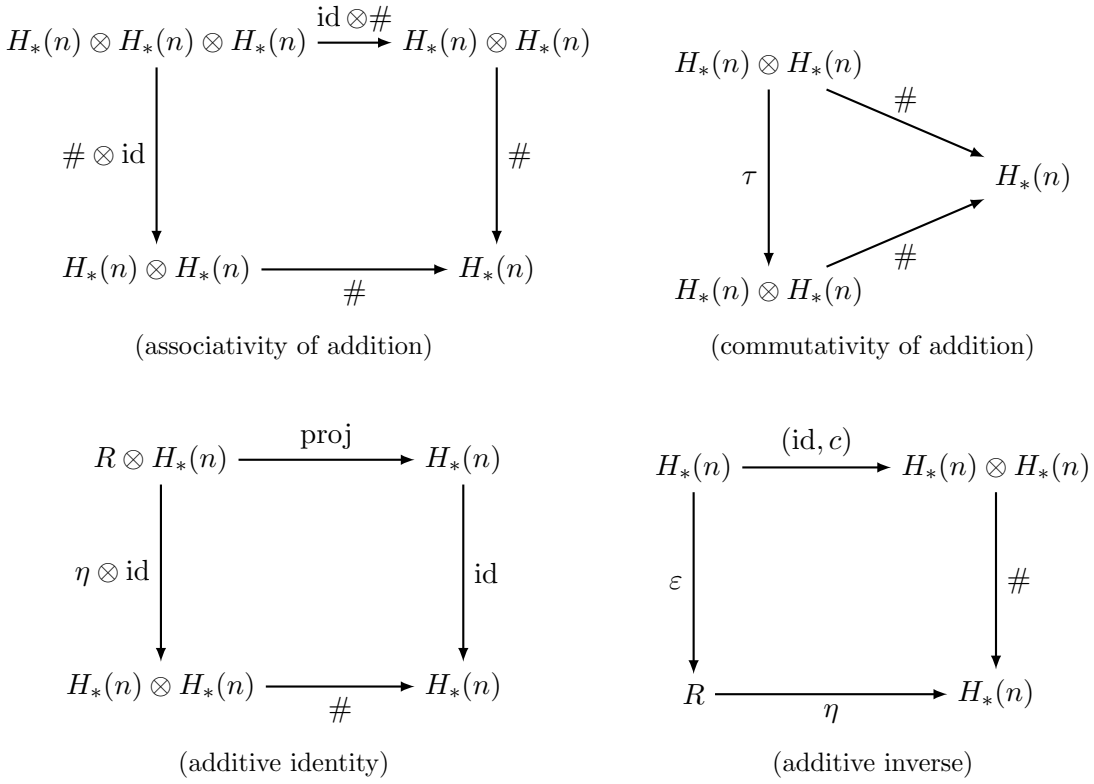

Figure 1.3: Additive conditions on a Hopf ring over R

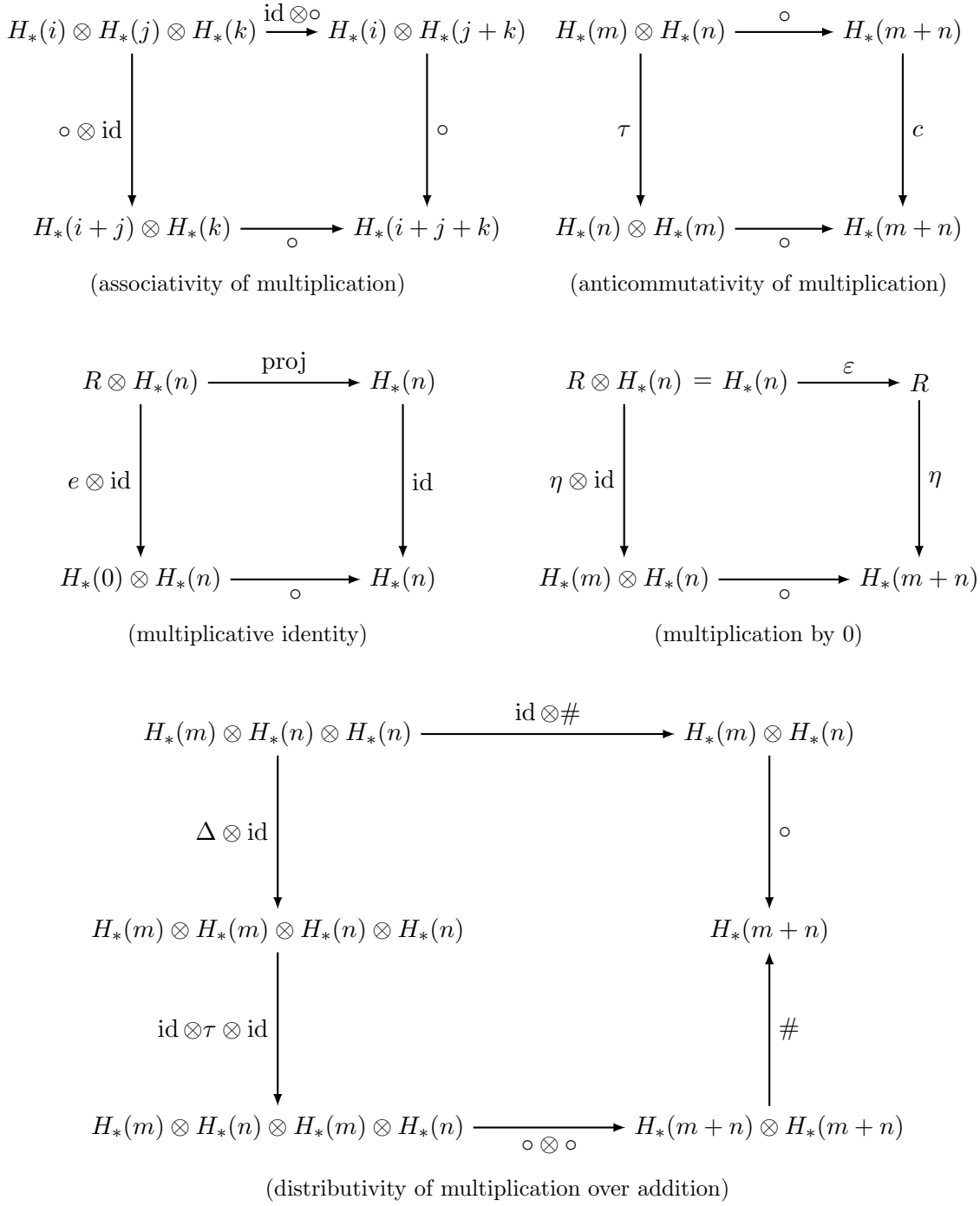

Figure 1.4: Multiplicative conditions on a Hopf ring over R

(a) $\Delta(x) = \sum(-1)^{|x'||x''|}x'' \otimes x'$

(b) $x = \sum x'\varepsilon(x'')$

(2) *Properties of the abelian group object structure:*

(a) $y\#z = (-1)^{jk}z\#y \in H_{j+k}(n)$

(b) $\Delta(y\#z) = \sum(-1)^{|z'||y''|}(y'\#z') \otimes (y''\#z'')$

(c) *Let* $[0_n] = \eta(1)$ *for* $\eta : R \to H_*(n)$. *Then* $[0_n]\#y = y$

(d) $cc = \mathrm{id}$

(e) $\eta\varepsilon(y) = \sum y'\#c(y'')$

(3) *Properties of the multiplicative structure:*

(a) $\Delta(x \circ y) = \sum(x' \otimes x'') \circ (y' \otimes y'') = \sum(-1)^{|x''||y'|}(x' \circ y') \otimes (x'' \circ y'')$

(b) $[0_n] \circ y = \eta\varepsilon(y)$

(c) *Let* $[1] = e(1) \in H_0(0)$. *Then* $[1] \circ y = y$

(d) *Let* $[-1] = c([1]) \in H_0(0)$. *Then* $c(x) = [-1] \circ x$

(e) $c(x \circ y) = c(x) \circ y = x \circ c(y)$

(f) $x \circ y = (-1)^{ij}[-1]^{\circ mn} \circ y \circ x = (-1)^{ij}c(y \circ x) \in H_{i+j}(m+n)$

(g) $x \circ (y\#z) = \sum(-1)^{|x''||y|}(x' \circ y)\#(x'' \circ z)$

(h) *Let* $[n] = [1 + 1 + \cdots + 1] = e(1 + 1 + \cdots + 1) \in H_0(0)$. *By the coassociativity of* Δ, *we can unambiguously write*

$$\Delta^n(y) = \sum y^{(1)} \otimes y^{(2)} \otimes \cdots \otimes y^{(n+1)}$$

Then we have $[n] \circ y = \sum y^{(1)}\#y^{(2)}\# \cdots \#y^{(n+1)}$.

I will also make extensive use of the following property, albeit indirectly via Proposition 1.20 and Lemma 2.9 in Section 1.2.

Proposition 1.6. *In any Hopf ring* $H_*(\star)$ *and any* $x \in H_*(\star)$,

$$0\#x = 0 = x\#0$$
$$0 \circ x = 0 = x \circ 0$$

Proof. Both follow from the fact $\#$ and $\circ$ are defined on a tensor product of R-modules, and $0 \otimes x = 0 = x \otimes 0$. □

1.1.2 The Hopf ring for complex bordism

Definition 1.7. Let MU be the Thom spectrum, representing complex bordism. In particular, for $n \geq 1$, $MU(2n)$ is the Thom space for the universal bundle $\gamma \to BU(n)$ with base space $BU(n)$, the classifying space of the unitary group $U(n)$. The structure maps $\Sigma^2 MU(2n) \to MU(2n+2)$ are induced by the inclusion $U(n) \to U(n+1)$.

We denote the Ω-spectrum associated to MU by $\underline{MU}_{2\star}$. In particular, for $n \in \mathbb{Z}$, we define $\underline{MU}_{2n}$ as follows. The structure map $\Sigma^2 MU(2n) \to MU(2n+2)$ for MU is adjoint to a map $MU(2n) \to \Omega^2 MU(2n+2)$. By iterating the loop space functor, we obtain another map $\Omega^k MU(2n) \to \Omega^{k+2} MU(2n+2)$ for all $k \geq 2$. We then have a sequence

$$MU(2n) \to \Omega^2 MU(2n+2) \to \Omega^4 MU(2n+4) \to \Omega^6 MU(2n+6) \to \cdots$$

from which we define

$$\underline{MU}_{2n} = \varinjlim \Omega^{2k} MU(2n+2k)$$

Observe that $\underline{MU}_{2n} = \Omega^2 \underline{MU}_{2n+2}$. Furthermore, by construction, there is a natural isomorphism $MU^{2n}(X) = [X, \underline{MU}_{2n}]$ where $MU^{2\star}(X)$ is the MU-cohomology of X.

For many common choices of coefficients, the homology of $\underline{MU}_{2\star}$ has the structure of a Hopf ring [17, Lemma 1.13]. This thesis will mostly focus on the case of $\mathbb{F}_2$ coefficients; however, many of the necessary results occur in more general cases, so I will retain some generality while defining the Hopf ring structure.

Proposition 1.8. *Let R be a commutative ring with 1 such that the Künneth isomorphism $H_*(\underline{MU}_{2m}; R) \otimes H_*(\underline{MU}_{2n}; R) \xrightarrow{\cong} H_*(\underline{MU}_{2m} \times \underline{MU}_{2n}; R)$ holds for any m and n. Then for each $n \in \mathbb{Z}$, $H_*(\underline{MU}_{2n}; R)$ is a coalgebra over R with counit $\varepsilon : H_*(\underline{MU}_{2n}; R) \to R$ and coproduct $\Delta : H_*(\underline{MU}_{2n}; R) \to H_*(\underline{MU}_{2n}; R) \otimes H_*(\underline{MU}_{2n}; R)$ induced by the unique map $\underline{MU}_{2n} \to \mathrm{pt}$ and by the diagonal map $\underline{MU}_{2n} \to \underline{MU}_{2n} \times \underline{MU}_{2n}$, respectively.*

Proof. The maps $\underline{MU}_{2n} \to \mathrm{pt}$ and $\underline{MU}_{2n} \to \underline{MU}_{2n} \times \underline{MU}_{2n}$ trivially satisfy the coidentity, coassociativity and cocommutativity conditions at the level of spaces. Then taking singular homology with R coefficients, we immediately obtain the necessary commutative diagrams in Definition 1.1 by the Künneth isomorphism

$$H_*(\underline{MU}_{2n} \times \underline{MU}_{2n}; R) \cong H_*(\underline{MU}_{2n}; R) \otimes H_*(\underline{MU}_{2n}; R)$$

and the functoriality of $H_*(-; R)$. □

Remark 1.9. The Künneth isomorphism holds for R a field k or $\mathbb{Z}$. Therefore, there are coalgebra structures as in Proposition 1.8 on $H_*(\underline{MU}_{2n}; k)$ and $H_*(\underline{MU}_{2n}; \mathbb{Z})$.

Now that we have a collection of R-coalgebras $H_*(\underline{MU}_{2n}; R)$, $n \in \mathbb{Z}$, we can construct the Hopf ring structure on the graded object $H_*(\underline{MU}_{2\star}; R)$. I will do this initially using only the properties of the spectrum $\underline{MU}_{2\star}$ and singular homology. However, we will often require a more explicit treatment of the elements and properties of the Hopf ring $H_*(\underline{MU}_{2\star}; R)$. Therefore, I will also give an alternate construction using the abstract properties of Hopf rings, originally due to Ravenel and Wilson [17, p. 242, pp. 250–251, Theorem 3.8, p. 262].

Definition 1.10. Let R be a commutative ring with 1 such that the Künneth isomorphism $H_*(\underline{MU}_{2m}; R) \otimes H_*(\underline{MU}_{2n}; R) \xrightarrow{\cong} H_*(\underline{MU}_{2m} \times \underline{MU}_{2n}; R)$ holds for any m and n. We define the necessary maps on $H_*(\underline{MU}_{2\star}; R)$ as follows.

(1) Because $\underline{MU}_{2n}$ is a loop space, there is a natural product $\underline{MU}_{2n} \times \underline{MU}_{2n} \to \underline{MU}_{2n}$ defined by concatenation of loops. We denote the induced map on homology by

$$\# : H_*(\underline{MU}_{2n}; R) \otimes H_*(\underline{MU}_{2n}; R) \to H_*(\underline{MU}_{2n}; R)$$

(2) The unique based map $\mathrm{pt} \to \underline{MU}_{2n}$ induces a map

$$\eta : R = H_*(\mathrm{pt}; R) \to H_*(\underline{MU}_{2n}; R)$$

(3) There is a map $\underline{MU}_{2n} \to \underline{MU}_{2n}$ defined by reversing the direction of loops. This induces a map

$$c : H_*(\underline{MU}_{2n}; R) \to H_*(\underline{MU}_{2n}; R)$$

(4) Since MU is a ring spectrum, there is a product on cohomology

$$MU^{2m}(X) \times MU^{2n}(Y) \to MU^{2n+2m}(X \times Y)$$

Recall that for each k, $MU^{2k}(X) = [X, \underline{MU}_{2k}]$. Therefore, if we choose $X = \underline{MU}_{2m}$ and $Y = \underline{MU}_{2n}$, this product becomes a map

$$[\underline{MU}_{2m}, \underline{MU}_{2m}] \times [\underline{MU}_{2n}, \underline{MU}_{2n}] \to [\underline{MU}_{2m} \times \underline{MU}_{2n}, \underline{MU}_{2m+2n}]$$

Then the element $(\mathrm{id}, \mathrm{id})$ on the left hand side is sent to a particular map

$$\underline{MU}_{2m} \times \underline{MU}_{2n} \to \underline{MU}_{2m+2n}$$

on the right. This map in turn induces a map

$$\circ : H_*(\underline{MU}_{2m}; R) \otimes H_*(\underline{MU}_{2n}; R) \to H_*(\underline{MU}_{2m+2n}; R)$$

(5) Given a space X, consider the element $1 \in MU^0(X)$. Since $MU^0(X) = [X, \underline{MU}_0]$, we can identify the inclusion $\{1\} \hookrightarrow MU^0(X)$ with a map $[X, \mathrm{pt}] \to [X, \underline{MU}_0]$. This defines a natural transformation of functors $[-, \mathrm{pt}] \to [-, \underline{MU}_0]$, which is determined by a particular element of $[\mathrm{pt}, \underline{MU}_0]$. The latter induces a map

$$e : R = H_*(\mathrm{pt}; R) \to H_*(\underline{MU}_0; R)$$

Lemma 1.11. *The maps of $\underline{MU}_{2\star}$ from Definition 1.10 make $\underline{MU}_{2\star}$ into a graded ring object in the (unbased) homotopy category of spaces,* $\mathsf{Ho}(\mathsf{Top})$.

Proof. The commutativity (up to homotopy) of the diagrams defining the associativity, commutativity, identity, inverse and multiplication by 0 conditions is immediate. Distributivity follows readily as well. Let

$$+ : \underline{MU}_{2n} \times \underline{MU}_{2n} \to \underline{MU}_{2n} \text{ and } \bullet : \underline{MU}_{2m} \times \underline{MU}_{2n} \to \underline{MU}_{2m+2n}$$

be the addition and multiplication from Definition 1.10 (1) and (4). For any spaces X and Y, the cohomology product $MU^{2m}(X) \times MU^{2n}(Y) \to MU^{2m+2n}(X \times Y)$ is distributive; that is, for $\alpha, \beta \in MU^{2m}(X)$ and $\gamma \in MU^{2n}(Y)$, we have $(\alpha + \beta)\gamma = \alpha\gamma + \beta\gamma$. Consider (α, β) and γ as elements of $[X, \underline{MU}_{2m}] \times [X, \underline{MU}_{2m}] = [X, \underline{MU}_{2m} \times \underline{MU}_{2m}]$ and $[Y, \underline{MU}_{2n}]$, respectively. If we set $X = \underline{MU}_{2m} \times \underline{MU}_{2m}$ and $Y = \underline{MU}_{2n}$, then in particular we can take α and β to be the projection maps $\pi_1, \pi_2 : \underline{MU}_{2m} \times \underline{MU}_{2m} \to \underline{MU}_{2m}$, respectively, and γ to be the identity $\mathrm{id} : \underline{MU}_{2n} \to \underline{MU}_{2n}$. Then

$$(\pi_1 + \pi_2)\,\mathrm{id} = \pi_1\,\mathrm{id} + \pi_2\,\mathrm{id}$$

in $[\underline{MU}_{2m} \times \underline{MU}_{2m} \times \underline{MU}_{2n}, \underline{MU}_{2m+2n}]$, which is precisely what it means for $\bullet$ to distribute over $+$. □

Theorem 1.12. *Let R be a commutative ring with 1 such that the Künneth isomorphism*

$$H_*(\underline{MU}_{2m}; R) \otimes H_*(\underline{MU}_{2n}; R) \xrightarrow{\cong} H_*(\underline{MU}_{2m} \times \underline{MU}_{2n}; R)$$

holds for any m and n. With the maps $\#, \eta, c, \circ, e$ of Definition 1.10, $H_*(\underline{MU}_{2\star}; R)$ is a

Hopf ring over R.

Proof. Because the Künneth isomorphism holds and the tensor product is a product in GCoAlg_R, and because pt and $H_*(\mathrm{pt}; R) = R$ are terminal objects in $\mathsf{Ho}(\mathsf{Top})$ and GCoAlg_R, respectively, we see that $H_*(-; R)$ is a product-preserving functor $\mathsf{Ho}(\mathsf{Top}) \to \mathsf{GCoAlg}_R$. Therefore, if we apply $H_*(-; R)$ to the commutative diagrams defining the associativity, commutativity, identity, inverse, anticommutativity, multiplication by 0 and distributivity properties of the graded ring object structure for $\underline{MU}_{2\star}$, we obtain the corresponding diagrams for $H_*(\underline{MU}_{2\star}; R)$. Therefore $H_*(\underline{MU}_{2\star}; R)$ is a ring object in GCoAlg_R. □

1.1.3 An algebraic construction of the Hopf ring $H_*(\underline{MU}_{2\star}; R)$

In this section, I will provide an alternate construction of the Hopf ring for complex bordism, which Ravenel and Wilson showed is isomorphic to the Hopf ring of Theorem 1.12 [17, Theorem 4.2(a)].

Throughout, I will assume that R is a commutative unital ring such that the Künneth isomorphism $H_*(\underline{MU}_{2m}; R) \otimes H_*(\underline{MU}_{2n}; R) \xrightarrow{\cong} H_*(\underline{MU}_{2m} \times \underline{MU}_{2n}; R)$ holds for any m and n. Recall that by Proposition 1.8, this means $H_*(\underline{MU}_{2n})$ is a coaglebra over R for each $n \in \mathbb{Z}$.

We obtain the formal group law F for for MU cohomology, which we recall is complex-oriented, as follows. Denote the orientation by $x^{MU} \in \widetilde{MU}^2(\mathbb{CP}^\infty)$. Recall that $\mathbb{CP}^\infty$ has an H-space product $\mathbb{CP}^\infty \times \mathbb{CP}^\infty \to \mathbb{CP}^\infty$. This induces a map

$$MU^{2\star}(\mathbb{CP}^\infty; R) \to MU^{2\star}(\mathbb{CP}^\infty; R) \otimes MU^{2\star}(\mathbb{CP}^\infty; R)$$

under which

$$x^{MU} \mapsto \sum_{i,j \geq 0} a_{i,j} (x^{MU})^i \otimes (x^{MU})^j$$

for some coefficients $a_{i,j} \in MU^{-2(i+j-1)}(\mathrm{pt}) = MU_{2(i+j-1)}(\mathrm{pt})$. We define the formal group law F to be

$$F(y, z) = y +_F z = \sum_{i,j \geq 0} a_{i,j} y^i z^j$$

$H^*(-; R)$ is also a complex-oriented multiplicative cohomology theory, so for a particular generator $x \in \widetilde{H}^2(\mathbb{CP}^\infty; R)$, we have $H^*(\mathbb{CP}^\infty; R) = R[\![x]\!]$. Let $\beta_n \in H_{2n}(\mathbb{CP}^\infty; R)$ be dual to x^n. By Proposition 1.8, the diagonal $\mathbb{CP}^\infty \to \mathbb{CP}^\infty \times \mathbb{CP}^\infty$ induces a coproduct on $H_*(\mathbb{CP}^\infty; R)$; in this case, we have $\beta_n \mapsto \sum_{i=0}^n \beta_{n-i} \otimes \beta_i$. We can consider the orientation for MU, x^{MU}, as a map $\mathbb{CP}^\infty \to \underline{MU}_2$. For each n, define $b_n := x^{MU}_*(\beta_n) \in H_{2n}(\underline{MU}_2; R)$.

Furthermore, any element $z \in MU^{2\star}(\mathrm{pt})$ yields an element $[z] \in H_0(\underline{MU}_{2\star}; R)$ as follows. Such a $z \in MU^{2\star}$ is representable as a map $z : \mathrm{pt} \to \underline{MU}_{2\star}$. This induces a map on homology, $R = H_0(\mathrm{pt}; R) \to H_0(\underline{MU}_{2\star}; R)$, and we define $[z]$ to be the image of $1 \in R$ under this map.

Define $\beta(s) = \sum_{i\geq 0} \beta_i s^i \in H_*(\mathbb{CP}^\infty; R)[\![s]\!]$ and $b(s) = \sum_{i\geq 0} b_i s^i \in H_*(\underline{MU}_{2\star}; R)[\![t]\!]$. We define another formal group law

$$y +_{[F]} z = \#_{i,j\geq 0} [a_{i,j}] \circ y^{\circ i} \circ z^{\circ j}$$

In particular, we have

$$b(s) +_{[F]} b(t) = \#_{i,j\geq 0} [a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$$

In [17, Theorem 3.4], Ravenel and Wilson prove that in $H_*(\mathbb{CP}^\infty; R)[\![s,t]\!]$,

$$\beta(s)\beta(t) = \beta(s +_f t) \tag{1.1}$$

where f is the formal group law of HR. Because HR is a Eilenberg–MacLane spectrum, $f(X,Y)$ can be taken to be the additive formal group law. Then in [17, Theorem 3.8], the authors use (1.1) to obtain

$$b(s +_f t) = b(s) +_{[F]} b(t) \tag{1.2}$$

in $H_*(\underline{MU}_{2\star}; R)[\![s,t]\!]$.

Definition 1.13. Let $\mathcal{H}$ be the free Hopf ring generated by the coefficients b_i of $b(s)$ and the elements $[z]$ (for $z \in MU^{2\star}(\mathrm{pt})$), modulo the relations implied by (1.2).

The following is Ravenel and Wilson's main result [17, Theorem 4.7(a), Corollary 4.7(a)] and provides us with an algebraic description of the Hopf ring $H_*(\underline{MU}_{2\star}; R)$ of Theorem 1.12.

Theorem 1.14. *There is an isomorphism of Hopf rings $\mathcal{H} \cong H_*(\underline{MU}_{2\star}; R)$.*

Remark 1.15. The statement that $H_*(\underline{MU}_{2\star}; R)$ is generated by the $[z]$ and b_i means that any $x \in H_*(\underline{MU}_{2\star}; R)$ can be expressed by combining these elements using the three operations $+$, $\#$ and $\circ$. In particular, because $\#$ and $\circ$ distribute over $+$ and because $\circ$ distributes over $\#$, any such x is of the form

$$\sum \left(\# \left(\bigcirc([z] \circ b_i) \right) \right)$$

For this reason, we often only need to consider $\circ$-products of the generators, and at most $\#$-sums of such $\circ$-products.

1.2 Some simple computations

The following is an immediate consequence of the definitions of $[z]$ for $z \in MU^{2\star}(\text{pt})$ and $\#$ and $\circ$.

Proposition 1.16. *Let $u, v \in MU^{2m}(\text{pt})$ and $w \in MU^{2n}(\text{pt})$. Then*

(a) $[u]\#[v] = [u+v]$

(b) $[u] \circ [w] = [uw]$

Proposition 1.17. $b_0 = [0]$

Proof. By definition, $b_0 = x_*^{MU}(\beta_0)$, and $\beta_0 \in H_0(\mathbb{CP}^\infty; R)$ is dual to the constant 1 in $H^*(\mathbb{CP}^\infty; R) = R[\![x]\!]$. On the other hand, $[0] \in H_*(\underline{MU}_2; R)$ is the image of $1 \in R$ under the map $\text{pt} \to \underline{MU}_2$ representing $0 \in MU^2(\text{pt})$. □

Lemma 1.18. *For any $x \in H_m(\underline{MU}_{2n}; R)$,*

$$[0] \circ x = \begin{cases} [0] & m = 0 \\ 0 & m \neq 0 \end{cases}$$

Proof. By Lemma 1.5(3)(b), $[0] \circ x = \eta\varepsilon(x)$. We have

$$H_*(\underline{MU}_{2n}; R) \xrightarrow{\varepsilon} R \xrightarrow{\eta} H_*(\underline{MU}_{2n}; R)$$

so the structure outside of homology degree 0 collapses. When x is in homology degree 0, $[0] \circ x = [0]$ by Proposition 1.16. □

Proposition 1.19. *For any $n \geq 0$ and any $z \in MU^{2\star}(\text{pt})$,*

(a) $\Delta^k[z] = \underbrace{[z] \otimes [z] \otimes \cdots \otimes [z]}_{k \text{ copies}}$

(b) $\Delta^k b_n = \sum_{i_1 + \cdots + i_k = n} \bigotimes_{m=1}^{k} b_{i_m}$

Proof. It is enough to show the two-fold cases, as the general results follow by induction.

(a) This is immediate from the fact that Δ is induced on homology by the diagonal map on $\underline{MU}_{2\star}$.

(b) As noted above, for any $n \geq 0$, the diagonal map $\mathbb{CP}^\infty \to \mathbb{CP}^\infty \otimes \mathbb{CP}^\infty$ induces a coproduct

$$\Delta : \beta_n \mapsto \sum_{i=0}^{n} \beta_i \otimes \beta_{n-i}$$

on $H_*(\mathbb{CP}^\infty; R)$. Because $b_n = x_*^{MU}(\beta_n) \in H_{2n}(\underline{MU}_2; R)$ and x_*^{MU} is a map of R-modules, we have the commutative diagram in Figure 1.5.

$$\begin{array}{ccc} H_*(\mathbb{CP}^\infty) & \xrightarrow{\Delta} & H_*(\mathbb{CP}^\infty) \otimes H_*(\mathbb{CP}^\infty) \\ \Big\downarrow{\scriptstyle x_*^{MU}} & & \Big\downarrow{\scriptstyle x_*^{MU} \otimes x_*^{MU}} \\ H_*(\underline{MU}_{2\star}; R) & \xrightarrow[\Delta]{} & H_*(\underline{MU}_{2\star}; R) \otimes H_*(\underline{MU}_{2\star}; R) \end{array}$$

Figure 1.5: The effect of Δ on b_n

It follows that $\Delta b_n = \sum_{i=0}^{n} b_i \otimes b_{n-i}$.

□

From now on, unless otherwise stated, I restrict to the case of $\mathbb{F}_2$ coefficients (that is, $R = \mathbb{F}_2$). I will simply write $H_*(\underline{MU}_{2\star})$ to refer to $H_*(\underline{MU}_{2\star}; \mathbb{F}_2)$.

Proposition 1.20. *Let $u \in MU^{2\star}(\text{pt})$ and $v, w \in H_*(\underline{MU}_{2l})$ for some $l \geq 0$. Let $x, z_1, z_2, \ldots, z_n \in H_*(\underline{MU}_{2\star})$ be homogeneous in both degrees and not in homology degree 0. Then for any $m \neq 0$,*

(a) $[u] \circ (v \# w) = ([u] \circ v) \# ([u] \circ w)$

(b) $b_{2m} \circ (x \# x) = (b_m \circ x)^{\#2}$

(c) $b_{2m+1} \circ (x \# x) = 0$

(d) $b_m \circ (z_1 \# z_2 \# \cdots \# z_n) = 0$ *if* $m < n$

Proof.

(a) This follows immediately from the distributive law since $\Delta[u] = [u] \otimes [u]$.

(b) In the case $m = 0$, because x and $x\#x$ are not in homology degree 0, we have

$$b_0 \circ (x\#x) = 0 = 0\#0 = (b_0 \circ x)^{\#2}$$

by Proposition 1.17 and Lemma 1.18. Now let $m > 0$. By the distributive law and Proposition 1.20,

$$b_{2m} \circ (x\#x) = \sum_{i=0}^{2m} (b_i \circ x)\#(b_{2m-i} \circ x)$$

Then both $(b_i \circ x)\#(b_{2m-i} \circ x)$ and $(b_{2m-i} \circ x)\#(b_i \circ x)$ are $+$-summands, and they are distinct except when $i = m$. Therefore, all $+$-summands except $(b_m \circ x)^{\#2}$ cancel over $\mathbb{F}_2$.

(c) If we replace $2m$ with $2m + 1$, then all $+$-summands cancel over $\mathbb{F}_2$.

(d) Again, if $m = 0$, the result follows by Proposition 1.17 and Lemma 1.18. If $m > 0$, then again by the distributive law and Proposition 1.20, we have

$$b_m \circ (z_1\#z_2\#\cdots\#z_n) = \sum_{i_1+\cdots+i_n=m} \#_{k=1}^{n} (b_{i_k} \circ z_k)$$

Since $n > m$, in each $+$-summand at least one i_k is 0, and so there is a $\#$-summand $[0] \circ z_k = 0$. Therefore, the $+$-summand vanishes.

□

1.3 Polynomial generators of $\pi_\star(MU)$

It is well known that $\pi_\star(MU)$ is a polynomial algebra $\mathbb{Z}[x_1, x_2, x_3, \ldots]$ with x_i in degree $2i$ (see, for instance, [1, Theorem 8.1]). Because these generate the coefficient ring for MU, and because $[ax + by] = ([a] \circ [x])\#([b] \circ [y])$ in $H_*(\underline{MU}_{2\star})$, the generators $[z]$ for the Hopf ring coming from $z \in MU^{2\star}(\text{pt})$ are expressible in terms of $[x_1], [x_2], [x_3], \ldots$. Therefore, in mind of Remark 1.15, we typically only need to consider $\circ$-products of the $[x_i]$ and the b_n or at most $\#$-sums of such $\circ$-products.

On the other hand, the Ravenel–Wilson relations are all in terms of the b_n and the $[a_{i,j}]$ (see Table A.6). The formal group law coefficients $a_{i,j}$ are unfortunately not polynomial generators of $\pi_\star(MU)$ in general, which significantly complicates computations in the Hopf ring and in turn in $H_*(SL_1MU)$. However, in [5], Hazewinkel provides a connection

between the two. In particular, in [5, Section I.5], Hazewinkel constructs the universal one-dimensional formal group law $F_U(X,Y)$ over the Lazard ring L. As Lazard proved in [9, Theorem II], L is isomorphic to a polynomial ring over $\mathbb{Z}$ in infinitely many variables. In Hazewinkel's notation, $L = \mathbb{Z}[U] = \mathbb{Z}[U_2, U_3, U_4, \ldots]$, with U_i in degree $2(i-1)$. In [5, p. xxi, p. 24], Hazewinkel provides the following expression for $F_U(X,Y)$ in low degrees:

$$\begin{aligned} F_U(X,Y) = X + Y &+ XY(-U_2) + (XY^2 + X^2Y)(-U_3 + U_2^2) \\ &+ (XY^3 + X^3Y)(-2U_4 + 2U_2U_3 + 2U_2^3) \\ &+ X^2Y^2(-3U_4 + 4U_2U_3 - 4U_2^3) \\ &+ (XY^4 + X^4Y)(-U_5 + 4U_2U_4 - 3U_2^2U_3 + 3U_2^4 + U_3^2) \\ &+ (X^2Y^3 + X^3Y^2)(-2U_5 + 11U_2U_4 - 11U_2^2U_3 + 10U_2^4 + 3U_3^2) \\ &+ \cdots \end{aligned}$$

Remark 1.21. There appears to be a typo in this equation in [5]: in the last two lines, U_2^4 is written as U_2^3, which is not in the correct degree.

Since, as Quillen showed in [15], the homotopy ring of MU is isomorphic to L, the formal group law for MU is a universal one-dimensional formal group law over $\pi_\star(MU)$ [5, Theorem 34.2.16][16, Theorem 1.3.4]. We can therefore treat the coefficient on X^iY^j above as $a_{i,j}$. Note that there are no constant term and no monomials of the form X^i or Y^j for $i, j \geq 2$. Therefore,

$$a_{0,0} = a_{i,0} = a_{0,j} = 0 \text{ for } i, j \geq 2$$

and

$$a_{1,0} = a_{0,1} = 1$$

Furthermore, $F_U(X,Y)$ is commutative (note that X^iY^j and X^jY^i have the same coefficient above), so

$$a_{i,j} = a_{j,i} \text{ for all } i, j$$

As an abelian group, $\pi_\star(MU)$ is generated in degree n by the monomials of degree n in the x_i. Note also that there are multiple isomorphisms $\pi_\star(MU) \to \mathbb{Z}[x_1, x_2, x_3, \ldots]$, which gives us a certain amount of freedom to choose convenient generators. For example, degree 4 is generated by x_2 and x_1^2, but also by x_2 and $x_2 - x_1^2$ or x_2 and $x_2 + x_1^2$. We can readily see the consequences of this in low degrees:

First, $a_{1,1} = -U_2$, so $a_{1,1}$ generates degree 2. Therefore, we can choose $x_1 = a_{1,1}$. Similarly, $a_{2,1} = -U_3 + U_2^2 = -U_3 + a_{1,1}^2$. Therefore, $U_3 = a_{1,1}^2 - a_{2,1}$. If we take $a_{1,1}^2 = x_1^2$ as a generator for degree 4, then $a_{2,1}$ is also a generator. Hence we can choose $x_2 = a_{2,1}$.

The situation gets more complicated in degree 6: we have $a_{3,1} = -2U_4 + 2U_2U_3 + 2U_2^3$ and $a_{2,2} = -3U_4 + 4U_2U_3 - 4U_2^3$, so

$$\begin{aligned} U_4 &= 3U_4 - 2U_4 \\ &= (4U_2U_3 - 4U_2^3 - a_{2,2}) - (2U_2U_3 + 2U_2^3 - a_{3,1}) \\ &= 2U_2U_3 - 6U_2^3 + a_{3,1} - a_{2,2} \\ &= 2(-a_{1,1})(a_{1,1}^2 - a_{2,1}) - 6(-a_{1,1})^3 + a_{3,1} - a_{2,2} \\ &= -4a_{1,1}^3 + 2a_{1,1}a_{2,1} + a_{3,1} - a_{2,2} \end{aligned}$$

Now, taking $a_{1,1}^3 = x_1^3$ and $a_{1,1}a_{2,1} = x_1x_2$ as generators for degree 6, we see that $a_{3,1} - a_{2,2}$ is also a generator. However, $a_{2,2} - a_{3,1}$ is as well, and as we will see in Section 2.1, this is a more convenient choice.

We address degree 8 in a similar way: we have $a_{4,1} = -U_5 + 4U_2U_4 - 3U_2^2U_3 + 3U_2^4 + U_3^2$ and $a_{3,2} = -2U_5 + 11U_2U_4 - 11U_2^2U_3 + 10U_2^4 + 3U_3^2$, so

$$\begin{aligned} U_5 &= 2U_5 - U_5 \\ &= (11U_2U_4 - 11U_2^2U_3 + 10U_2^4 + 3U_3^2 - a_{3,2}) - (4U_2U_4 - 3U_2^2U_3 + 3U_2^4 + U_3^2 - a_{4,1}) \\ &= 7U_2U_4 - 8U_2^2U_3 + 7U_2^4 + 2U_3^2 + a_{4,1} - a_{3,2} \\ &= 7(-a_{1,1})(-4a_{1,1}^3 + 2a_{1,1}a_{2,1} + a_{3,1} - a_{2,2}) - 8(-a_{1,1})^2(a_{1,1}^2 - a_{2,1}) \\ &\qquad + 7(-a_{1,1})^4 + 2(a_{1,1}^2 - a_{2,1})^2 + a_{4,1} - a_{3,2} \\ &= 29a_{1,1}^4 - 10a_{1,1}^2a_{2,1} - 7a_{1,1}a_{3,1} + 7a_{1,1}a_{2,2} + 2a_{2,1}^2 + a_{4,1} - a_{3,2} \end{aligned}$$

Finally, this simplifies to

$$29a_{1,1}^4 - 10a_{1,1}^2a_{2,1} + 7a_{1,1}(a_{2,2} - a_{3,1}) + 2a_{2,1}^2 + a_{4,1} - a_{3,2}$$

Taking $a_{1,1}^2 = x_1^4$, $a_{1,1}^2a_{2,1} = x_1^2x_2$, $a_{1,1}(a_{2,2} - a_{3,1}) = x_1x_3$ and $a_{2,1}^2 = x_2^2$ as generators for degree 8, we see that we can choose $x_4 = a_{4,1} - a_{3,2}$ or $x_4 = a_{3,2} - a_{4,1}$. For convenience in Section 2.1, we will choose the latter.

Therefore, in summary, we have

Proposition 1.22. *There is an isomorphism of rings $\pi_\star(MU) = \mathbb{Z}[x_1, x_2, x_3, \ldots]$ under which $x_1 = a_{1,1}$, $x_2 = a_{2,1}$, $x_3 = a_{2,2} - a_{3,1}$ and $x_4 = a_{3,2} - a_{4,1}$.*

1.4 The general $\circ$-product formulas

Computation of the multiplication table in Section 2.3 involves evaluating $\circ$-products in $H_*(\underline{MU}_{2\star})$ of the form

$$([1]\#x) \circ ([1]\#y)$$

where x and y are certain elements of the Hopf ring. In the simplest case, x and y are $\circ$-products of the generators $[x_i]$ and b_n. In particular, we write

$$x = \bigcirc_{i=1}^{K} [x_{k_i}] \circ \bigcirc_{j=1}^{M} b_{m_j}$$

By Proposition 1.16, we can simplify this to

$$x = [x_{k_1} x_{k_2} \cdots x_{k_K}] \circ \bigcirc_{j=1}^{M} b_{m_j} \tag{1.3}$$

Similarly, let

$$y = [x_{l_1} x_{l_2} \cdots x_{l_L}] \circ \bigcirc_{j=1}^{N} b_{n_j} \tag{1.4}$$

By Lemma 1.5(3)(a) and Proposition 1.19, because the generators b_n and $[z]$ for the Hopf ring are in even degrees, we have

$$\Delta\left(\bigcirc_{j=1}^{M} b_{m_j}\right) = \sum_{\substack{0\le i_j\le m_j \\ 1\le j\le M}} (b_{i_1} \circ \cdots \circ b_{i_M}) \otimes (b_{m_1-i_1} \circ \cdots \circ b_{m_M-i_M}) \tag{1.5}$$

and

$$\Delta[x_{n_1} x_{n_2} \cdots x_{n_N}] = [x_{n_1} x_{n_2} \cdots x_{n_N}] \otimes [x_{n_1} x_{n_2} \cdots x_{n_N}]$$

Now, combining this with and (1.5) and applying Lemma 1.5(3)(a) again, we obtain the following.

$$\begin{aligned}&\Delta\left([x_{k_1} \cdots x_{k_K}] \circ \bigcirc_{j=1}^{M} b_{m_j}\right) \\ &\quad = \sum_{\substack{0\le i_j\le m_j \\ 1\le j\le M}} ([x_{k_1} \cdots x_{k_K}] \circ b_{i_1} \circ \cdots \circ b_{i_M}) \otimes ([x_{k_1} \cdots x_{k_K}] \circ b_{m_1-i_1} \circ \cdots \circ b_{m_M-i_M})\end{aligned}$$

We now proceed to the result, which is an expanded formula for $([1]\#x)\circ([1]\#y)$ where x and y are as in (1.3) and (1.4), respectively. I will write $\Delta x = \sum x' \otimes x''$ and $\Delta y = \sum y' \otimes y''$,

and analogously for other elements, to keep the notation simple until the end.

Lemma 1.5(3)(a) implies that $\Delta([1]\#x) = \sum([1]'\#x') \otimes ([1]''\#x'')$, which we can immediately simplify to $\Delta([1]\#x) = \sum([1]\#x') \otimes ([1]\#x'')$. Continuing the convention, let $([1]\#x)' = ([1]\#x')$ and $([1]\#x)'' = ([1]\#x'')$.

Now, applying the distributive law (Lemma 1.5(3)(g)), we have

$$([1]\#x) \circ ([1]\#y) = \sum(([1]\#x)' \circ [1])\#(([1]\#x)'' \circ y) \tag{1.6}$$
$$= \sum([1]\#x')\#(([1]\#x'') \circ y) \tag{1.7}$$

We can expand the second #-summand in this formula, $(([1]\#x'') \circ y)$, using a modified version of the distributive law. Since for homogeneous elements a, b and c we have

$$a \circ (b\#c) = \sum(-1)^{|a''||b|}(a' \circ b)\#(a'' \circ c)$$

and

$$b \circ a = (-1)^{|a||b|} a \circ b$$

we obtain

$$\begin{aligned}(a\#b) \circ c &= c \circ (a\#b) \\ &= \sum(1)^{|c''||a|}(c' \circ a)\#(c'' \circ b)\end{aligned}$$

When a, b and c are in even degrees, this of course simplifies to $(a\#b)\circ c = \sum(a\circ c')\#(b\circ c'')$.

Now, returning to (1.7), we have $(([1]\#x'') \circ y) = \sum([1] \circ y')\#(x'' \circ y'') = \sum y'\#(x'' \circ y'')$. Plugging this back into (1.7), we find

$$\begin{aligned}([1]\#x) \circ ([1]\#y) &= \sum([1]\#x')\#\left(\sum y'\#(x'' \circ y'')\right) \\ &= \sum\left(\sum([1]\#x')\#(y'\#(x'' \circ y''))\right) \\ &= \sum([1]\#x'\#y'\#(x'' \circ y'')) \\ &= [1]\#\left(\sum(x'\#y'\#(x'' \circ y''))\right)\end{aligned}$$

Finally, in the notation of (1.3) and (1.4), this yields the following result.

Lemma 1.23. *Let*

$$x = [x_{k_1}x_{k_2}\cdots x_{k_K}] \circ \bigcirc_{j=1}^{M} b_{m_j}$$

and

$$y = [x_{l_1}x_{l_2}\cdots x_{l_L}] \circ \bigcirc_{j=1}^{N} b_{n_j}$$

Then $([1]\#x) \circ ([1]\#y) =$

$$[1]\#\Bigg(\sum ([x_{k_1}\cdots x_{k_K}] \circ b_{i_1} \circ \cdots \circ b_{i_M}) \#([x_{l_1}\cdots x_{l_L}] \circ b_{j_1} \circ \cdots \circ b_{j_N}) \\ \#([x_{k_1}\cdots x_{k_K}x_{l_1}\cdots x_{l_L}] \circ b_{m_1-i_1} \circ \cdots \circ b_{m_M-i_M} \circ b_{n_1-j_1} \circ \cdots \circ b_{n_N-j_N})\Bigg)$$

where the +-sum is over $0 \le i_p \le m_p$, $1 \le p \le M$ *and* $0 \le j_q \le n_q$, $1 \le q \le N$.

Note that in practice, this +-sum is much smaller than this formula implies. Many of the b_{i_p} and b_{j_q} terms are [0] (whenever $i_p = 0$ or $j_q = 0$), in which case either the entire #-summand containing that term is 0, or every b_{i_p} and b_{j-q} in the #-summand is [0], in which case the #-summand is [0]. Note that a #-summand of [0] does not affect the #-sum at all since [0] is the identity element for #, while a #-summand of 0 makes the entire +-summand vanish by Proposition 1.6.

We will need a more general result than this as well. Here we allow x and y to be #-sums of elements of the form

$$[x_{k_1}x_{k_2}\cdots x_{k_K}] \circ \bigcirc_{i=1}^{M} b_{m_i}$$

Lemma 1.24. *Let* $a_1, a_2, \ldots, a_m$ *and* $c_1, c_2, \ldots, c_n$ *be elements of* $H_*(\underline{MU}_0)$ *of the form*

$$[x_{k_1}x_{k_2}\cdots x_{k_K}] \circ \bigcirc_{i=1}^{M} b_{m_i}$$

Then in the notation of Lemma 1.5(3)(h),

$$([1]\#a_1\#\cdots a_m) \circ ([1]\#c_1\#\cdots\#c_n) = \\ \sum_{\substack{1\le i\le m\\ 1\le j\le n}} [1]\# \left(\#_{i=1}^{m} a_i^{(1)} \right) \# \left(\#_{j=1}^{n} c_j^{(1)} \right) \# \left(\#_{\substack{1\le i\le m\\ 1\le j\le n}} \left(a_i^{(j+1)} \circ c_j^{(i+1)} \right) \right)$$

where the elements $a_i^{(k)}$ *and* $c_j^{(l)}$ *can be evaluated by Proposition 1.19.*

Proof. This follows easily enough from Lemma 1.23 when we examine the coproduct of a #-sum (Lemma 1.5(2)(b)) and the distributive law (Lemma 1.5(3)(g)). Starting in the simplest case, if $a, b, c, d \in H_*(\underline{MU}_{2\star})$ are homogeneous in both degrees and lie in even

degrees, then we have

$$\begin{aligned}(a\#b)\circ(c\#d) &= \sum((a\#b)'\circ c)\#((a\#b)''\circ d)\\ &= \sum((a'\#b')\circ c)\#((a''\#b'')\circ d)\\ &= \sum(a'\circ c')\#(b'\circ c'')\#(a''\circ d')\#(b''\circ d'')\end{aligned}$$

Iterations Δ^n of the coproduct Δ are uniquely defined by coassociativity (as in Lemma 1.5(3)(h)). By induction, the distributive law of Lemma 1.5(3)(g) implies

$$a\circ(c_1\#c_2\#\cdots\#c_n) = \sum(a^{(1)}\circ c_1)\#(a^{(2)}\circ c_2)\#\cdots\#(a^{(n+1)}\circ c_{n+1})$$

where a and the c_i are homogenous in both degrees and lie in even degrees. Replacing a with $a_1\#\cdots\#a_{m+1}$, and distributing from the left, we have

$$(a_1\#\cdots\#a_m)\circ(c_1\#\cdots\#c_n) = \sum_{\substack{1\le i\le m\\ 1\le j\le n}}\left(a_i^{(j)}\circ c_j^{(i)}\right)$$

The result now follows because $[1]^{(k)} = [1]$ for all k by Proposition 1.19 and $[1]$ is the identity for $\circ$. □

Chapter 2

The multiplicative structure of $H_*(SL_1MU)$

In this chapter, I will present the central result of this thesis, the multiplication table in low degrees for $H_*(SL_1MU)$ as an $\mathbb{F}_2$-algebra under $\circ$. The procedure for its construction is based around a certain sequence of maps of abelian groups:

$$K_* \hookrightarrow P_* \twoheadrightarrow H_*(\underline{MU}'_0) \xrightarrow{[1]\#(-)} H_*(SL_1MU)$$

I will define each component of the sequence in this opening section. The final map is the isomorphism of abelian groups discussed in the Introduction. I provide the missing details here.

Definition 2.1. Following [17, p. 28], I will use $\underline{MU}'_{2\star}$ to denote the sequence of path components of $\underline{MU}_{2\star}$ containing the representatives of the homology class [0]. In particular, $\underline{MU}'_0$ is that path component in degree 0.

Proposition 2.2. *The map*

$$H_*(\underline{MU}'_0) \xrightarrow{[1]\#(-)} H_*(SL_1MU)$$

is an isomorphism of abelian groups.

Proof. First, note that the image of $H_*(\underline{MU}'_0)$ under the map $[1]\#(-)$ lies in the homology of the path component of [1]. By definition, SL_1MU consists of those $x \in \Omega^\infty MU$ whose images in $\pi_0(MU)$ are 1. Since $\pi_0(MU) = [\mathrm{pt}, \underline{MU}_0]$, this is equivalent to being in the homology class [1] in $H_*(\underline{MU}_0)$. Therefore, $H_*(SL_1MU)$ is exactly the homology of the path component of [1] in $\underline{MU}_0$.

Now, to see that $[1]\#(-)$ is an isomorphism, we simply observe that $[-1]\#(-)$ is an inverse since $[1]\#[-1] = [0]$, and $[0]\#(-)$ is the identity map. □

Ravenel and Wilson [17, Induction 4.18] show that $H_*(\underline{MU}'_{2\star})$ is a polynomial algebra over $\mathbb{F}_2$ and provide a formula for the dimension of the subspace of #-indecomposables, $Q^{\#}H_*(\underline{MU}'_{2\star})$, which I will discuss in detail in Section 2.1.1. First, let us formally define the #-indecomposables, following [17, p. 26].

Definition 2.3. For a Hopf ring $H_*(\star)$, let $IH_*(n)$ be the *augmentation ideal*:

$$IH_*(n) = \ker \varepsilon$$

Then we define the submodule of indecomposables (with respect to #) to be

$$Q^{\#}H_*(n) = IH_*(n)/(IH_*(n)\#IH_*(n))$$

$Q^{\#}H_*(\underline{MU}'_{2\star})$ is generated as an abelian group by $\circ$-products of the Hopf ring generators $[x_i]$, $i \geq 1$, and b_n, $n \geq 0$ [17, Induction 4.18]. Degrees are inherited from the Hopf ring structure: $[x_i] \in H_0(\underline{MU}_{-2i})$ and $b_n \in H_{2n}(\underline{MU}_2)$, so

$$\bigcirc_{j=1}^{J} [x_{i_j}] \circ \bigcirc_{k=1}^{K} b_{n_k} \in H_{2(n_1+\cdots+n_K)}(\underline{MU}_{2K-2(i_1+\cdots+i_J)})$$

Remark 2.4. In fact, Ravenel and Wilson provide a more strict set of generators for the $\mathbb{F}_2$-vector space $Q^{\#}H_*(\underline{MU}'_{2\star})$, using only those b_n of the form b_{2^m}, $m \geq 0$, but for the purposes of this chapter it is more convenient to start with a larger set, which will reduce to a somewhat different set of generators from that of Ravenel and Wilson.

$H_*(\underline{MU}'_{2\star})$ is the polynomial algebra over $\mathbb{F}_2$ in these $\circ$-products with $+$ and $\#$ as the addition and multiplication, respectively. $H_*(\underline{MU}'_0)$ is the subalgebra generated by those $\circ$-products $\bigcirc_{j=1}^{J}[x_{i_j}] \circ \bigcirc_{k=1}^{K} b_{n_k}$ such that $K = i_1 + \cdots + i_J$; that is, the number of $\circ$-factors b_n equals the sum of the indices of the $\circ$-factors $[x_i]$, and this $\circ$-product lies in degree $2(n_1 + \cdots + n_K)$. For example, the generators in degrees 0 through 6 are

Degree 0: $[0]$

Degree 1: $\emptyset$

Degree 2: $[x_1] \circ b_1$

Degree 3: $\emptyset$

Degree 4: $[x_1] \circ b_2,\ [x_1^2] \circ b_1 \circ b_1,\ [x_2] \circ b_1 \circ b_1$

Degree 5: $\emptyset$

Degree 6: $[x_1] \circ b_3,\ [x_1^2] \circ b_1 \circ b_2,\ [x_2] \circ b_1 \circ b_2,$
$[x_1^3] \circ b_1 \circ b_1 \circ b_1,\ [x_1x_2] \circ b_1 \circ b_1 \circ b_1,$
$[x_3] \circ b_1 \circ b_1 \circ b_1$

These $\circ$-products are generators of $Q^{\#}H_*(\underline{MU}'_0)$ as an abelian group under $+$, and $H_*(\underline{MU}'_0)$ is generated as an abelian group by $\#$-monomials in these $\circ$-products. Therefore, we can construct $H_*(\underline{MU}'_0)$ as a quotient group of the graded free abelian group P_* generated by these $\#$-monomials. P_* is then the free algebra over $\mathbb{F}_2$ generated by these $\circ$-products with $+$ and $\#$ as the addition and multiplication, respectively. We define K_* to be the kernel of the quotient map $P_* \to H_*(\underline{MU}'_0)$.

Restricting our discussion to the indecomposable elements, we see that in degree $2n$, we have an exact sequence of abelian groups

$$0 \to Q^{\#}K_{2n} \to Q^{\#}P_{2n} \to Q^{\#}H_*(\underline{MU}'_0) \to 0$$

This shows us that $Q^{\#}H_*(\underline{MU}'_0)$ has a presentation with generators $\bigcirc_{j=1}^{J}[x_{i_j}] \circ \bigcirc_{k=1}^{K} b_{n_k}$ such that $n_1 + \cdots + n_k = n$ and relations coming from K_{2n}. Since these are $\mathbb{F}_2$-vector spaces, we can determine the exact number of linearly independent relations in degree $2n$ by counting dimensions of $Q^{\#}P_{2n}$ and $Q^{\#}H_*(\underline{MU}'_0)$. Specifically, the number of relations is

$$\dim_{\mathbb{F}_2} Q^{\#}P_{2n} - \dim_{\mathbb{F}_2} Q^{\#}H_{2n}(\underline{MU}'_0)$$

The computation of these numbers will be done in Section 2.1.1. These relations allow us to reduce very significantly our set of generators for $Q^{\#}H_*(\underline{MU}'_0)$ as an abelian group and in turn reduce our set of generators for $H_*(\underline{MU}'_0)$ as an algebra over $\mathbb{F}_2$.

Now we turn to the final map in the sequence, $[1]\#(-) : H_*(\underline{MU}'_0) \to H_*(SL_1MU)$. Since this is a group isomorphism with respect to the $+$, we immediately obtain a set of generators for $H_*(SL_1MU)$ as an abelian group, simply by taking the $\#$-sum of $[1]$ with each group generator for $H_*(\underline{MU}'_0)$. These are $\#$-sums of $[1]$ and the $\circ$-products in $Q^{\#}H_*(\underline{MU}'_0)$ that remain after modding out the relations in $Q^{\#}K_*$.

2.1 The polynomial algebra structure of $H_*(\underline{MU}'_0)$

In this section, I will construct a minimal set of generators in low degrees for $H_*(\underline{MU}'_0)$ as a polynomial algebra over $\mathbb{F}_2$.

2.1.1 Counting relations

It is relatively straightforward to compute $\dim_{\mathbb{F}_2} Q^{\#}P_{2n}$. We need only count the distinct $\circ$-products $\bigcirc_{j=1}^{J}[x_{i_j}] \circ \bigcirc_{k=1}^{K} b_{n_k}$ with $K = i_1 + \cdots + i_J$ and $n_1 + \cdots + n_K = n$. The second condition indicates that we obtain a set of $\circ$-products for each integer partition of n, and the first condition shows that for each such partition of n, we obtain one $\circ$-product for each partition of K, the number of summands in the partition of n.

For example, consider the generators in degree 6:

$$[x_1] \circ b_3,\ [x_1^2] \circ b_1 \circ b_2,\ [x_2] \circ b_1 \circ b_2,\ [x_1^3] \circ b_1 \circ b_1 \circ b_1,\ [x_1x_2] \circ b_1 \circ b_1 \circ b_1,\ [x_3] \circ b_1 \circ b_1 \circ b_1$$

The partitions of $n = 3$ are 3, $1 + 2$ and $1 + 1 + 1$, corresponding to the $\circ$-factors b_3, $b_1 \circ b_2$ and $b_1 \circ b_1 \circ b_1$, respectively. The partition 3 has one summand, and there is one partition of 1, namely 1 itself. This corresponds to the $\circ$-factor $[x_1]$ and tells us that $[x_1] \circ b_3$ is the only generator in degree 6 with a $\circ$-factor of b_3. The partition $1+2$ has two summands, and there are two partitions of 2: $1 + 1$ and 2. These correspond to $[x_1^2] = [x_1] \circ [x_1]$ and $[x_2]$, respectively, from which we obtain the generators $[x_1^2] \circ b_1 \circ b_2$ and $[x_2] \circ b_1 \circ b_2$. Finally, the partition $1 + 1 + 1$ of 3 has three summands, and there are three partitions of 3: $1 + 1 + 1$, $1+2$ and 3. These correspond to $[x_1^3]$, $[x_1x_2]$ and $[x_3]$, so we obtain the last three generators, $[x_1^3] \circ b_1 \circ b_1 \circ b_1$, $[x_1x_2] \circ b_1 \circ b_1 \circ b_1$ and $[x_3] \circ b_1 \circ b_1 \circ b_1$. More generally, we have the following formula:

Lemma 2.5. *For $n \geq 0$, let $p(n)$ be the number of integer partitions of n. For $1 \leq k \leq n$, let $p(n|k)$ be the number of partitions of n with k summands. Then*

$$\dim_{\mathbb{F}_2} Q^{\#}P_{2n} = \sum_{k=1}^{n} p(n|k)p(k) \tag{2.1}$$

Now we turn to $\dim_{\mathbb{F}_2} Q^{\#}H_{2n}(\underline{MU}'_0)$. In [17], Ravenel and Wilson provide a formula for $\dim_{\mathbb{F}_2} Q^{\#}H_i(\underline{MU}'_{2j})$, which I recount here. $Q^{\#}H_*(\underline{MU}'_{2\star})$ has a natural bigrading $(*, \star)$ coming from the Hopf ring structure of $H_*(\underline{MU}_{2\star})$, which determines a bigrading on the polynomial algebra $\mathbb{F}_2[b_1, [x_1], [x_2], [x_3], \ldots]$. Specifically, since

$$b_1 \in H_2(\underline{MU}_2) \text{ and } [x_i] \in H_0(\underline{MU}_{-2i})$$

we have

$$b_1 \in \mathbb{F}_2[b_1, [x_1], [x_2], [x_3], \ldots]_{(2,2)} \text{ and } [x_i] \in \mathbb{F}_2[b_1, [x_1], [x_2], [x_3], \ldots]_{(0,-2i)} \text{ for each } i$$

The dimensions of this polynomial algebra and $Q^{\#}H_*(\underline{MU}'_{2\star})$ are related by the following result [17, p. 28].

Lemma 2.6.

$$\dim_{\mathbb{F}_2} Q^{\#}H_i(\underline{MU}'_{2j}) = \dim_{\mathbb{F}_2} \mathbb{F}_2[b_1, [x_1], [x_2], [x_3], \ldots]_{(i,j)}$$

Immediately from Lemma 2.6, we can compute the number of indecomposable elements in $H_{2n}(\underline{MU}'_0)$. In particular, since

$$\dim_{\mathbb{F}_2} Q^{\#}H_{2n}(\underline{MU}'_0) = \dim_{\mathbb{F}_2} \mathbb{F}_2[b_1, [x_1], [x_2], [x_3], \ldots]_{(2n,0)}$$

we simply count the products of the form $[x_{i_1}][x_{i_2}]\cdots[x_{i_J}]b_1^n$ in bidegree $(2n, 0)$. Note that

$$[x_{i_1}][x_{i_2}]\cdots[x_{i_J}]b_1^n \in \mathbb{F}_2[b_1, [x_1], [x_2], [x_3], \ldots]_{(2n+(0+\cdots+0),2n-2(i_1+\cdots+i_J))}$$

so this monomial is in bidegree $(2n, 0)$ if and only if $i_1 + i_2 + \cdots + i_J = n$. Therefore,

$$\dim_{\mathbb{F}_2} Q^{\#}H_{2n}(\underline{MU}'_0) = p(n),$$

the number of integer partitions of n.

With these two computations, we have the key result of this section:

Proposition 2.7. *$Q^{\#}H_{2n}(\underline{MU}'_0)$ is generated as an abelian group by $\circ$-products*

$$\bigcirc_{j=1}^{J} [x_{i_j}] \circ \bigcirc_{k=1}^{K} b_{n_k}$$

with $K = i_1 + \cdots + i_J$ and $n_1 + \cdots + n_K = n$, modulo

$$\left(\sum_{k=1}^{n} p(n|K)p(K)\right) - p(n)$$

linearly independent relations over $\mathbb{F}_2$.

The results of this computation in low degrees are shown in Table 2.1.

10	345	42	303
9	210	30	180
8	129	22	107
7	74	15	59
6	44	11	33
5	23	7	16
4	13	5	8
3	6	3	3
2	3	2	1
1	1	1	0
0	1	1	0
n	$\dim_{\mathbb{F}_2} Q^{\#}P_{2n}$	$\dim_{\mathbb{F}_2} Q^{\#}H_{2n}(\underline{MU}'_0)$	Relations

Table 2.1: The number of linearly independent relations on $H_{2n}(\underline{MU}'_0)$ for $n \leq 10$

2.1.2 Generators and relations

Here I will present a set of generators for $Q^{\#}H_{2n}(\underline{MU}'_0)$ for small n and a set of relations. I will then identify generators that can be removed from the set. For ease of use, I will not justify any of the relations until Section 2.1.3. The presentation is in Tables 2.2 and 2.3.

The relations in the tables allow us to eliminate the majority of our generators. Since many of the $\circ$-products $\bigcirc_{j=1}^{J}[x_{i_j}] \circ \bigcirc_{k=1}^{K} b_{n_k}$ appear in the set of relations, we can remove them immediately. Furthermore, since the $\circ$-products are generators of $Q^{\#}H_*(\underline{MU}'_0)$, any such $\circ$-products expressible as #-sums of lower-degree $\circ$-products are redundant. Finally, for any $\circ$-product x we retain in a given degree, we can remove any other $\circ$-products in the same degree that are expressable as a $+$-sum of x with decomposable elements. On the other hand, as shown above, there must be exactly $p(n)$ linearly independent generators left in $H_{2n}(\underline{MU}'_0)$.

In degrees 0 and 2, we lose no elements since there are no relations. In degree 4, we can remove $[x_1^2]\circ b_1 \circ b_1$ since it is decomposable. In degree 6, $[x_1^3]\circ b_1 \circ b_1 \circ b_1$ and $[x_1x_2]\circ b_1 \circ b_1 \circ b_1$ vanish, so we remove them. We can remove either $[x_1] \circ b_3$ or $[x_1^2] \circ b_1 \circ b_2$ since their sum is decomposable. We will arbitrarily remove the former and retain $[x_1] \circ b_3$.

In degree 8, we remove $[x_1^2] \circ b_1 \circ b_3$, $[x_1x_2] \circ b_1 \circ b_2 \circ b_2$, $[x_1^2] \circ b_2 \circ b_2$, $[x_1^3] \circ b_1 \circ b_1 \circ b_2$, $[x_1^4] \circ b_1 \circ b_1 \circ b_1 \circ b_1$, $[x_1^2x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ and $[x_1x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ because they either vanish or are decomposable. We can further eliminate $[x_2] \circ b_1 \circ b_3$ or $[x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ because their sum is decomposable. We will keep $[x_2]\circ b_1 \circ b_3$. I leave it as an exercise to the reader to verify that the list of generators below is consistent with the relations in higher

Generators	Relations
Degree 0	
$[0]$	$\emptyset$
Degree 2	
$[x_1] \circ b_1$	$\emptyset$
Degree 4	
$[x_1] \circ b_2$ $[x_1^2] \circ b_1 \circ b_1$ $[x_2] \circ b_1 \circ b_1$	$[x_1^2] \circ b_1 \circ b_1 + ([x_1] \circ b_1)^{\#2}$
Degree 6	
$[x_1] \circ b_3$ $[x_1^2] \circ b_1 \circ b_2$ $[x_2] \circ b_1 \circ b_2$ $[x_1^3] \circ b_1 \circ b_1 \circ b_1$ $[x_1x_2] \circ b_1 \circ b_1 \circ b_1$ $[x_3] \circ b_1 \circ b_1 \circ b_1$	$[x_1] \circ b_3 + [x_1^2] \circ b_1 \circ b_2$ $+([x_1] \circ b_1)\#([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}$ $[x_1^3] \circ b_1 \circ b_1 \circ b_1$ $[x_1x_2] \circ b_1 \circ b_1 \circ b_1$
Degree 8	
$[x_1] \circ b_4$ $[x_1^2] \circ b_1 \circ b_3$ $[x_2] \circ b_1 \circ b_3$ $[x_1^2] \circ b_2 \circ b_2$ $[x_2] \circ b_2 \circ b_2$ $[x_1^3] \circ b_1 \circ b_1 \circ b_2$ $[x_1x_2] \circ b_1 \circ b_1 \circ b_2$ $[x_3] \circ b_1 \circ b_1 \circ b_2$ $[x_1^4] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ $[x_1^2x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ $[x_1x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ $[x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ $[x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_1$	$[x_1^2] \circ b_1 \circ b_3 + ([x_1] \circ b_1)^{\#4}$ $[x_2] \circ b_1 \circ b_3 + [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 + ([x_2] \circ b_1 \circ b_1)^{\#2}$ $[x_1x_2] \circ b_1 \circ b_1 \circ b_2 + ([x_2] \circ b_1 \circ b_1)^{\#2}$ $[x_1^2] \circ b_2 \circ b_2 + ([x_1] \circ b_2)^{\#2} + ([x_1] \circ b_1)^{\#4}$ $[x_1^3] \circ b_1 \circ b_1 \circ b_2 + ([x_1] \circ b_1)^{\#4}$ $[x_1^4] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ $[x_1^2x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_1$ $[x_1x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1$

Table 2.2: Presentation of $H_{2n}(\underline{MU}'_0)$ for $n = 0, 1, 2, 3, 4$

Generators	Relations
Degree 10	
$[x_1]\circ b_5$ $[x_1^2]\circ b_1\circ b_4$ $[x_2]\circ b_1\circ b_4$ $[x_1^2]\circ b_2\circ b_3$ $[x_2]\circ b_2\circ b_3$ $[x_1^3]\circ b_1\circ b_1\circ b_3$ $[x_1x_2]\circ b_1\circ b_1\circ b_3$ $[x_3]\circ b_1\circ b_1\circ b_3$ $[x_1^3]\circ b_1\circ b_2\circ b_2$ $[x_1x_2]\circ b_1\circ b_2\circ b_2$ $[x_3]\circ b_1\circ b_2\circ b_2$ $[x_1^4]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_1^2x_2]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_1x_3]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_4]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_1^5]\circ b_1^{\circ 5}$ $[x_1^3x_2]\circ b_1^{\circ 5}$ $[x_1^2x_3]\circ b_1^{\circ 5}$ $[x_1x_2^2]\circ b_1^{\circ 5}$ $[x_1x_4]\circ b_1^{\circ 5}$ $[x_2x_3]\circ b_1^{\circ 5}$ $[x_5]\circ b_1^{\circ 5}$	$[x_1]\circ b_5+[x_3]\circ b_1\circ b_1\circ b_3+[x_1^2]\circ b_1\circ b_4$ $+([x_1]\circ b_1)\#([x_1]\circ b_4)+([x_1]\circ b_2)\#([x_1]\circ b_3)$ $+([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3)+([x_1]\circ b_1)^{\#5}$ $[x_1^2]\circ b_2\circ b_3+([x_1]\circ b_2)\#([x_1]\circ b_3)$ $+([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3)+([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2}$ $+([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)$ $[x_2]\circ b_2\circ b_3+[x_3]\circ b_1\circ b_1\circ b_3$ $+[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_2+([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_2)$ $[x_1^3]\circ b_1\circ b_1\circ b_3$ $[x_1^3]\circ b_1\circ b_2\circ b_2+([x_1]\circ b_2)\#([x_1]\circ b_3)$ $+([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2}+([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)$ $[x_1x_2]\circ b_1\circ b_2\circ b_2+[x_3]\circ b_1\circ b_1\circ b_3$ $[x_3]\circ b_1\circ b_1\circ b_3+[x_2x_3]\circ b_1^{\circ 5}$ $[x_1x_2]\circ b_1\circ b_1\circ b_3$ $[x_1^4]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_1^2x_2]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_1x_3]\circ b_1\circ b_1\circ b_1\circ b_2$ $[x_1^5]\circ b_1^{\circ 5}$ $[x_1^3x_2]\circ b_1^{\circ 5}$ $[x_1^2x_3]\circ b_1^{\circ 5}$ $[x_1x_2^2]\circ b_1^{\circ 5}$ $[x_1x_4]\circ b_1^{\circ 5}$

Table 2.3: Presentation of $H_{2n}(\underline{MU}'_0)$ for $n=5$

Generators	Relations
Degree 12	
$[x_1]\circ b_6$ $[x_1^2]\circ b_1\circ b_5$ $[x_2]\circ b_1\circ b_5$ $[x_1^2]\circ b_2\circ b_4$ $[x_2]\circ b_2\circ b_4$ $[x_1^2]\circ b_3\circ b_3$ $[x_2]\circ b_3\circ b_3$ $[x_1^3]\circ b_1\circ b_1\circ b_4$ $[x_1x_2]\circ b_1\circ b_1\circ b_4$ $[x_3]\circ b_1\circ b_1\circ b_4$ $[x_1^3]\circ b_1\circ b_2\circ b_3$ $[x_1x_2]\circ b_1\circ b_2\circ b_3$ $[x_3]\circ b_1\circ b_2\circ b_3$ $[x_1^3]\circ b_2\circ b_2\circ b_2$ $[x_1x_2]\circ b_2\circ b_2\circ b_2$ $[x_3]\circ b_2\circ b_2\circ b_2$ $[x_1^4]\circ b_1\circ b_1\circ b_1\circ b_3$ $[x_1^2x_2]\circ b_1\circ b_1\circ b_1\circ b_3$ $[x_1x_3]\circ b_1\circ b_1\circ b_1\circ b_3$ $[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3$ $[x_4]\circ b_1\circ b_1\circ b_1\circ b_3$	$[x_1]\circ b_6+[x_1^2]\circ b_2\circ b_4+([x_1]\circ b_1)\#([x_1]\circ b_5)$ $+([x_1]\circ b_2)\#([x_1]\circ b_4)+([x_1]\circ b_3)^{\#2}$ $+([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_4)+([x_1]\circ b_2)^{\#3}$ $+([x_1]\circ b_1)^{\#4}\#([x_1]\circ b_2)$ $[x_1^2]\circ b_1\circ b_5+[x_1^3]\circ b_1\circ b_1\circ b_4$ $[x_1^2]\circ b_3\circ b_3+([x_1]\circ b_3)^{\#2}+([x_1]\circ b_1)^{\#6}$ $[x_1^3]\circ b_1\circ b_1\circ b_4+([x_1]\circ b_3)^{\#2}$ $+([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2}+([x_1]\circ b_1)^{\#6}$ $[x_1x_2]\circ b_1\circ b_1\circ b_4+([x_2]\circ b_1\circ b_2)^{\#2}$ $[x_1^3]\circ b_1\circ b_2\circ b_3$ $[x_1x_2]\circ b_1\circ b_2\circ b_3$ $[x_1^3]\circ b_2\circ b_2\circ b_2+([x_1]\circ b_3)^{\#2}$ $+([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2}+([x_1]\circ b_1)^{\#6}$ $[x_1x_2]\circ b_2\circ b_2\circ b_2+([x_2]\circ b_1\circ b_2)^{\#2}$ $[x_1^4]\circ b_1\circ b_1\circ b_1\circ b_3$ $[x_1^2x_2]\circ b_1\circ b_1\circ b_1\circ b_3$ $[x_1x_3]\circ b_1\circ b_1\circ b_1\circ b_3$ $[x_2]\circ b_3\circ b_3+[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3$ $+([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_2\circ b_2)+([x_2]\circ b_1\circ b_2)^{\#2}$ $+([x_2]\circ b_1\circ b_1)^{\#3}$ $[x_1^4]\circ b_1\circ b_1\circ b_2\circ b_2$ $[x_1^2x_2]\circ b_1\circ b_1\circ b_2\circ b_2$

Table 2.4: Presentation of $H_{2n}(\underline{MU}'_0)$ for $n=6$ (continued in Table 2.5)

Generators	Relations
Degree 12 continued	
	$[x_1x_3]\circ b_1\circ b_1\circ b_2\circ b_2+([x_3]\circ b_1\circ b_1\circ b_1)^{\#2}$
	$[x_2]\circ b_3\circ b_3+[x_4]\circ b_1\circ b_1\circ b_1\circ b_3$ $+[x_2^2]\circ b_1\circ b_1\circ b_2\circ b_2$ $+([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_2\circ b_2)$ $+([x_2]\circ b_1\circ b_1)^{\#3}$
$[x_1^4]\circ b_1\circ b_1\circ b_2\circ b_2$	$[x_1^5]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$
$[x_1^2x_2]\circ b_1\circ b_1\circ b_2\circ b_2$	$[x_1^3x_2]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$
$[x_1x_3]\circ b_1\circ b_1\circ b_2\circ b_2$	$[x_1^2x_3]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$
$[x_2^2]\circ b_1\circ b_1\circ b_2\circ b_2$	$[x_1x_2^2]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$
$[x_4]\circ b_1\circ b_1\circ b_2\circ b_2$	$[x_1x_4]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$
$[x_1^5]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$	$[x_3]\circ b_1\circ b_2\circ b_3+[x_2x_3]\circ b_1^{\circ 4}\circ b_2$ $+([x_3]\circ b_1\circ b_1\circ b_1)^{\#2}$
$[x_1^3x_2]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$	$[x_1^6]\circ b_1^{\circ 6}$
$[x_1^2x_3]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$	$[x_1^4x_2]\circ b_1^{\circ 6}$
$[x_1x_2^2]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$	$[x_1^3x_3]\circ b_1^{\circ 6}$
$[x_1x_4]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$	$[x_1^2x_2^2]\circ b_1^{\circ 6}$
$[x_2x_3]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$	$[x_1^2x_4]\circ b_1^{\circ 6}$
$[x_5]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2$	$[x_1x_2x_3]\circ b_1^{\circ 6}$
$[x_1^6]\circ b_1^{\circ 6}$	$[x_1x_5]\circ b_1^{\circ 6}$
$[x_1^4x_2]\circ b_1^{\circ 6}$	$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3+[x_2^3]\circ b_1^{\circ 6}$
$[x_1^3x_3]\circ b_1^{\circ 6}$	$[x_4]\circ b_1\circ b_1\circ b_1\circ b_3+[x_2x_4]\circ b_1^{\circ 6}$
$[x_1^2x_2^2]\circ b_1^{\circ 6}$	$[x_1x_3]\circ b_1\circ b_1\circ b_2\circ b_2+[x_3^2]\circ b_1^{\circ 6}$
$[x_1^2x_4]\circ b_1^{\circ 6}$	
$[x_1x_2x_3]\circ b_1^{\circ 6}$	
$[x_1x_5]\circ b_1^{\circ 6}$	
$[x_2^3]\circ b_1^{\circ 6}$	
$[x_2x_4]\circ b_1^{\circ 6}$	
$[x_3^2]\circ b_1^{\circ 6}$	
$[x_6]\circ b_1^{\circ 6}$	

Table 2.5: Presentation of $H_{2n}(\underline{MU}'_0)$ for $n=6$ (continued from Table 2.4)

degrees. This leaves us with the following polynomial generators:

$$\begin{aligned}
&\text{Degree 0: } [0]\\
&\text{Degree 2: } [x_1]\circ b_1\\
&\text{Degree 4: } [x_1]\circ b_2,\ [x_2]\circ b_1\circ b_1\\
&\text{Degree 6: } [x_1]\circ b_3,\ [x_2]\circ b_1\circ b_2,\ [x_3]\circ b_1\circ b_1\circ b_1\\
&\text{Degree 8: } [x_1]\circ b_4,\ [x_2]\circ b_1\circ b_3,\ [x_2]\circ b_2\circ b_2,\ [x_3]\circ b_1\circ b_1\circ b_2,\ [x_4]\circ b_1\circ b_1\circ b_1\circ b_1\\
&\text{Degree 10: } [x_1]\circ b_5,\ [x_2]\circ b_1\circ b_4,\ [x_2]\circ b_2\circ b_3,\ [x_3]\circ b_1\circ b_1\circ b_3,\ [x_3]\circ b_1\circ b_2\circ b_2,\\
&\qquad [x_4]\circ b_1\circ b_1\circ b_1\circ b_2,\ [x_5]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\\
&\text{Degree 12: } [x_1]\circ b_6,\ [x_2]\circ b_1\circ b_5,\ [x_2]\circ b_2\circ b_4,\ [x_2]\circ b_3\circ b_3,\ [x_3]\circ b_1\circ b_1\circ b_4,\\
&\qquad [x_3]\circ b_1\circ b_2\circ b_3,\ [x_3]\circ b_2\circ b_2\circ b_2,\ [x_4]\circ b_1\circ b_1\circ b_1\circ b_3,\ [x_4]\circ b_1\circ b_1\circ b_2\circ b_2,\\
&\qquad [x_5]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2,\ [x_6]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1
\end{aligned} \tag{2.2}$$

As an aside, though we will not need it here, the list of generators in (2.2) seems to indicate the result in Conjecture 2.8.

Conjecture 2.8. $H_*(\underline{MU}'_0)$ is generated as a polynomial algebra over $\mathbb{F}_2$ by $\circ$-products of the form

$$[x_n]\circ \bigcirc_{k=1}^{K} b_{n_k}$$

with $n_1+\cdots n_k = n$, and this element lies in degree $2n$.

2.1.3 Proofs of the relations

Here I provide the justification for the relations in Section 2.1.2. This section relies heavily on the Ravenel–Wilson relations in Appendix A. For instance, in Table A.6, we see that

$$[x_1]\circ b_1\circ b_1 = b_1\#b_1$$

which implies the following result

Lemma 2.9. *For any $m\geq 0$,*

(a) $[x_1]\circ b_1\circ b_1\circ b_{2m} = (b_1\circ b_m)^{\#2}$

(b) $[x_1]\circ b_1\circ b_1\circ b_{2m+1} = 0$

Proof. These follow immediately from Proposition 1.20 and the Ravenel–Wilson relation $[x_1] \circ b_1 \circ b_1 = b_1 \# b_1$. □

Many elements vaish by Lemma 2.9(b), which I collect here.

Computation 2.10. The following vanish:

- $[x_1^3] \circ b_1 \circ b_1 \circ b_1$
- $[x_1 x_2] \circ b_1 \circ b_1 \circ b_1$
- $[x_1^4] \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^2 x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1 x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^3] \circ b_1 \circ b_1 \circ b_3$
- $[x_1 x_2] \circ b_1 \circ b_1 \circ b_3$
- $[x_1^4] \circ b_1 \circ b_1 \circ b_1 \circ b_2$
- $[x_1^2 x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_2$
- $[x_1 x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_2$
- $[x_1^5] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^3 x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^2 x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1 x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1 x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^4] \circ b_1 \circ b_1 \circ b_1 \circ b_3$
- $[x_1^2 x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_3$
- $[x_1 x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_3$
- $[x_1^5] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2$
- $[x_1^3 x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2$

- $[x_1^2x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2$
- $[x_1x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2$
- $[x_1x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2$
- $[x_1^6] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^4x_2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^3x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^2x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1^2x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1x_2x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$
- $[x_1x_5] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1$

Computation 2.11. $[x_1^2] \circ b_1 \circ b_1 + ([x_1] \circ b_1)^{\#2} = 0$

Proof. From the Ravenel–Wilson coefficient equation for st, we have $[x_1] \circ b_1 \circ b_1 + b_1 \# b_1 = 0$, and the result follows immediately by applying $[x_1] \circ (-)$ to both sides. □

Computation 2.12. $[x_1] \circ b_3 + [x_1^2] \circ b_1 \circ b_2 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3} = 0$

Proof. The Ravenel–Wilson coefficient equation for s^2t implies

$$\begin{aligned} 0 &= b_3 + [x_1] \circ b_1 \circ b_2 + [x_2] \circ b_1^{\circ 3} + b_1 \# b_2 + b_1 \# ([x_1] \circ b_1 \circ b_1) \\ &= b_3 + [x_1] \circ b_1 \circ b_2 + [x_2] \circ b_1^{\circ 3} + b_1 \# b_2 + b_1^{\#3} \end{aligned}$$

Now the result follows by applying $[x_1] \circ (-)$ to both sides. Note that the middle term, $[x_1] \circ [x_2] \circ b_1^{\circ 3}$, vanishes by Lemma 2.9(b). □

Computation 2.13. $[x_1^3] \circ b_1 \circ b_1 \circ b_2 + ([x_1] \circ b_1)^{\#4}$

Proof. By Lemma 2.9,

$$[x_1^3] \circ b_1 \circ b_1 \circ b_2 = [x_1^2] \circ (b_1 \# b_1)^{\circ 2} = ([x_1^2] \circ b_1 \circ b_1)^{\#2}.$$

Applying Lemma 2.9 again, we see that the right hand side equals $([x_1] \circ b_1)^{\#4}$. □

Computation 2.14. $[x_1^2] \circ b_1 \circ b_3 + ([x_1] \circ b_1)^{\#4} = 0$

Proof. By Computation 2.13, it suffices to show that $[x_1^2] \circ b_1 \circ b_3 + [x_1^3] \circ b_1 \circ b_1 \circ b_2 = 0$. From the Ravenel–Wilson coefficient equation for s^2t, we see that

$$b_3 + [x_1] \circ b_1 \circ b_2 = [x_2] \circ b_1^{\circ 3} + b_1 \# b_2 + b_1 \#([x_1] \circ b_1 \circ b_1)$$

We apply $[x_1^2] \circ b_1 \circ (-)$ to both sides. On the right hand side, the first summand vanishes by Lemma 2.9, and the other two vanish by Proposition 1.20(d). □

Computation 2.15. $[x_1x_2] \circ b_1 \circ b_1 \circ b_2 + ([x_2] \circ b_1 \circ b_1)^{\#2} = 0$

Proof. This follows immediately from Lemma 2.9(a) by applying $[x_2] \circ (-)$ to both sides of the equation. □

Computation 2.16. $[x_2] \circ b_1 \circ b_3 + [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 + ([x_2] \circ b_1 \circ b_1)^{\#2} = 0$.

Proof. By Computation 2.15, it suffices to show that

$$[x_2] \circ b_1 \circ b_3 + [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 + [x_1x_2] \circ b_1 \circ b_1 \circ b_2 = 0$$

Applying $[x_2] \circ b_1 \circ (-)$ to Ravenel–Wilson coefficient equation for s^2t, we obtain

$$[x_2] \circ b_1 \circ b_3 + [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 + [x_1x_2] \circ b_1 \circ b_1 \circ b_2 = [x_2] \circ b_1 \circ (b_1 \# b_2 + b_1 \#([x_1] \circ b_1 \circ b_1))$$

and the right hand side vanishes by Proposition 1.20(d). □

Computation 2.17. $[x_1^2] \circ b_2 \circ b_2 + ([x_1] \circ b_2)^{\#2} + ([x_1] \circ b_1)^{\#4} = 0$

Proof. The Ravenel–Wilson coefficient equation for s^2t^2, after canceling repeated terms, yields

$$[x_1] \circ b_2 \circ b_2 + [a_{2,2}] \circ b_1^{\circ 4} + b_2 \# b_2 + b_1 \# b_1 \#([x_1] \circ b_1 \circ b_1) = 0$$

Note that the last +-summand on the left equals $b_1^{\#4}$. Then applying $[x_1] \circ (-)$ to both sides, the second +summand on the left vanishes by Lemma 2.9(b), and the result follows. □

Computation 2.18. $[x_3] \circ b_1 \circ b_1 \circ b_3 + [x_2x_3] \circ b_1^{\circ 5} = 0$

Proof. By the Ravenel–Wilson coefficient equation for s^2t, we have

$$\begin{aligned} [x_2x_3] \circ b_1^{\circ 5} &= [x_3] \circ b_1 \circ b_1 \circ ([x_2] \circ b_1 \circ b_1) \\ &= [x_3] \circ b_1 \circ b_1 \circ (b_3 + [x_1] \circ b_1 \circ b_2 + b_1 \# b_2 + b_1 \#([x_1] \circ b_1 \circ b_1)) \end{aligned}$$

All but the first +-summand vanish by Proposition 1.20(d) and Lemma 2.9(b), and the remaining term is $[x_3] \circ b_1 \circ b_1 \circ b_3$. □

Computation 2.19. $[x_1x_2] \circ b_1 \circ b_2 \circ b_2 + [x_3] \circ b_1 \circ b_1 \circ b_3 = 0$

Proof. Using the Ravenel–Wilson coefficient equation for s^2t^2, we have

$$\begin{aligned}[x_1x_2] \circ b_1 \circ b_2 \circ b_2 &= [x_2] \circ b_1 \circ ([x_1] \circ b_2 \circ b_2) \\ &= [x_2] \circ b_1 \circ \left([a_{2,2}] \circ b_1^{\circ 4} + b_2 \# b_2 + b_1 \# b_1 \# ([x_1] \circ b_1 \circ b_1)\right)\end{aligned}$$

The last two +-summands vanish when we apply $b_1 \circ (-)$ by Proposition 1.20(d). Furthermore, from Section 1.3, we know $a_{2,2} = x_3 + a_{3,1}$. Therefore,

$$\begin{aligned}[x_1x_2] \circ b_1 \circ b_2 \circ b_2 &= [x_2] \circ b_1 \circ [x_3 + a_{3,1}] \circ b_1^{\circ 4} \\ &= [x_2] \circ b_1 \circ ([x_3] \# [a_{3,1}]) \circ b_1^{\circ 4} \\ &= [x_2] \circ (([x_3] \circ b_0) \# ([a_{3,1}] \circ b_1) + ([x_3] \circ b_1) \# ([a_{3,1}] \circ b_0)) \circ b_1^{\circ 4} \\ &= [x_2] \circ ([a_{3,1}] \circ b_1 + [x_3] \circ b_1) \circ b_1^{\circ 4}\end{aligned}$$

By the Ravenel–Wilson coefficient equation for s^3t, we have

$$[a_{3,1}] \circ b_1^{\circ 4} = [x_1] \circ b_1 \circ b_3 + b_1 \# b_3 + b_1 \# ([x_1] \circ b_1 \circ b_2) + b_1 \# ([x_2] \circ b_1 \circ b_1)$$

Applying $b_1 \circ (-)$, the right hand side vanishes by Proposition 1.20(d) and Lemma 2.9(b). Therefore,

$$[x_1x_2] \circ b_1 \circ b_2 \circ b_2 = [x_2x_3] \circ b_1^{\circ 5}$$

The right hand side is equal to $[x_3] \circ b_1 \circ b_1 \circ b_3$ by Computation 2.18. □

Computation 2.20.

$$\begin{aligned}0 = {} & [x_1] \circ b_5 + [x_3] \circ b_1 \circ b_1 \circ b_3 + [x_1^2] \circ b_1 \circ b_4 + ([x_1] \circ b_1) \# ([x_1] \circ b_4) \\ & + ([x_1] \circ b_2) \# ([x_1] \circ b_3) + ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_3) + ([x_1] \circ b_1)^{\#5}\end{aligned}$$

Proof. By the Ravenel–Wilson coefficient equation for s^4t, we have

$$\begin{aligned}[x_1^2]\circ b_1\circ b_4 = [x_1]\circ(&b_5 + [x_2]\circ b_1\circ b_2\circ b_2 + [a_{3,1}]\circ b_1\circ b_1\circ b_1\circ b_2 + [a_{4,1}]\circ b_1^{\circ 5} + b_1\#b_4\\ &+ b_1\#([x_1]\circ b_1\circ b_3) + b_1\#([a_{3,1}]\circ b_1^{\circ 4}) + b_2\#([x_1]\circ b_1\circ b_2)\\ &+ b_3\#([x_1]\circ b_1\circ b_1))\end{aligned}$$

Distributing the $[x_1]\circ(-)$, by Lemma 2.9(b), we have

$$\begin{aligned}[x_1^2]\circ b_1\circ b_4 &= b_5 + [x_2]\circ b_1\circ b_2\circ b_2 + b_1\#b_4 + b_1\#([x_1]\circ b_1\circ b_3) + b_2\#([x_1]\circ b_1\circ b_2)\\ &\quad + b_3\#([x_1]\circ b_1\circ b_1)\\ &= [x_1]\circ b_5 + [x_1x_2]\circ b_1\circ b_2\circ b_2 + ([x_1]\circ b_1)\#([x_1]\circ b_4)\\ &\quad + ([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_3) + ([x_1]\circ b_2)\#([x_1^2]\circ b_1\circ b_2)\\ &\quad + ([x_1]\circ b_3)\#([x_1^2]\circ b_1\circ b_1)\end{aligned}$$

$[x_1x_2] = b_1\circ b_2\circ b_2 = [x_3]\circ b_1\circ b_1\circ b_2$ by Computation 2.19, and the result then follows by applying the lower-degree relations

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2}\\ [x_1^2]\circ b_1\circ b_2 &= [x_1]\circ b_3\\ [x_1^2]\circ b_1\circ b_3 &= ([x_1]\circ b_1)^{\#4}\end{aligned}$$

□

Computation 2.21.

$$\begin{aligned}0 = [x_1^2]\circ b_2\circ b_3 &+ ([x_1]\circ b_2)\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3)\\ &+ ([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2} + ([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)\end{aligned}$$

Proof. We will use the Ravenel–Wilson coefficient equation for s^3t^2. We omit those $+$-summands with a $\circ$-factor of $b_1\circ b_1\circ b_{2k+1}$ since these will vanish after applying $[x_1]\circ(-)$. Therefore, we have

$$\begin{aligned}[x_1^2]\circ b_2\circ b_3 = [x_1]\circ(&b_1\#([x_1]\circ b_1\circ b_3) + b_1\circ([x_1]\circ b_2\circ b_2) + b_2\#b_3 + b_2\#([x_1]\circ b_1\circ b_2)\\ &+ b_1\#b_1\#([x_1]\circ b_1\circ b_2) + b_1\#b_1\#([x_1]\circ b_1\circ b_1))\end{aligned}$$

We distribute the $[x_1] \circ (-)$ and apply the lower-degree relations

$$\begin{aligned}
[x_1^2] \circ b_1 \circ b_1 &= ([x_1] \circ b_1)^{\#2} \\
[x_1^2] \circ b_1 \circ b_2 &= [x_1] \circ b_3 \\
[x_1^2] \circ b_1 \circ b_3 &= ([x_1] \circ b_1)^{\#4} \\
[x_1^2] \circ b_2 \circ b_2 &= ([x_1] \circ b_1)^{\#4} + ([x_1] \circ b_2)^{\#2}
\end{aligned}$$

Then canceling terms over $\mathbb{F}_2$, the result follows. □

Computation 2.22.

$$[x_2] \circ b_2 \circ b_3 + [x_3] \circ b_1 \circ b_1 \circ b_3 + [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_2 + ([x_2] \circ b_1 \circ b_1) \# ([x_2] \circ b_1 \circ b_2) = 0$$

Proof. Applying the Ravenel–Wilson coefficient equation for s^2t, we have

$$\begin{aligned}
[x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_2 &= [x_2] \circ b_2 \circ ([x_2] \circ b_1 \circ b_1 \circ b_1) \\
&= [x_2] \circ b_2 \circ (b_3 + [x_1] \circ b_1 \circ b_2 + b_1 \# b_2 + b_1 \# ([x_1] \circ b_1 \circ b_1))
\end{aligned}$$

Rewriting the last $+$-summand as $b_1 \# b_1 \# b_1$, we see that it vanishes when we apply $b_2 \circ (-)$ by Proposition 1.20(d). Furthermore, by Lemma 2.9(a), we obtain

$$[x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_2 = [x_2] \circ b_2 \circ b_3 + [x_1 x_2] \circ b_1 \circ b_2 \circ b_2 + ([x_2] \circ b_1 \circ b_1) \# ([x_2] \circ b_1 \circ b_2)$$

The result then follows since $[x_1 x_2] \circ b_1 \circ b_2 \circ b_2 = [x_3] \circ b_1 \circ b_1 \circ b_3$ by Computation 2.19. □

Computation 2.23.

$$\begin{aligned}
0 = {} & [x_1^3] \circ b_1 \circ b_2 \circ b_2 + ([x_1] \circ b_2) \# ([x_1] \circ b_3) + ([x_1] \circ b_1) \# ([x_1] \circ b_2)^{\#2} \\
& + ([x_1] \circ b_1)^{\#3} \# ([x_1] \circ b_2)
\end{aligned}$$

Proof. Again applying the Ravenel–Wilson coefficient equation for s^2t gives

$$[x_1^3] \circ b_1 \circ b_2 \circ b_2 = [x_1^2] \circ b_2 \circ (b_3 + [x_2] \circ b_1^{\circ 3} + b_1 \# b_2 + b_1 \# ([x_1] \circ b_1 \circ b_1))$$

Rewriting the last $+$-summand as $b_1 \# b_1 \# b_1$ and applying Lemma 2.9, we have

$$[x_1^3] \circ b_1 \circ b_2 \circ b_2 = [x_1^2] \circ b_2 \circ b_3 + 0 + ([x_1^2] \circ b_1 \circ b_1) \# ([x_1^2] \circ b_1 \circ b_2) + 0$$

The result follows by applying Computation 2.21 and the lower-degree relations

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$
$$[x_1^2] \circ b_1 \circ b_2 = [x_1] \circ b_3$$

and canceling terms. □

Computation 2.24. $[x_1^2] \circ b_3 \circ b_3 + ([x_1] \circ b_3)^{\#2} + ([x_1] \circ b_1)^{\#6} = 0$

Proof. We apply $[x_1] \circ (-)$ to the Ravenel–Wilson coefficient equation for s^3t^3. After canceling terms over $\mathbb{F}_2$ and omitting terms that will vanish by Lemma 2.9(b), this yields

$$\begin{aligned} 0 &= [x_1] \circ ([x_1] \circ b_3 \circ b_3 + b_1 \# b_1 \# ([x_1] \circ b_2 \circ b_2) + b_3 \# b_3 + b_1 \# b_1 \# b_2 \# b_2 \\ &= [x_1^2] \circ b_3 \circ b_3 + ([x_1] \circ b_1)^{\#2} \# ([x_1^2] \circ b_2 \circ b_2) + ([x_1] \circ b_3)^{\#2} + ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_2)^{\#2} \end{aligned}$$

From the relations in degree 8, we know $[x_1^2] \circ b_2 \circ b_2 = ([x_1] \circ b_2)^{\#2} + ([x_1] \circ b_1)^{\#4}$. Plugging this into the previous equation and canceling terms yields the result. □

Computation 2.25. $[x_1x_3] \circ b_1 \circ b_1 \circ b_2 \circ b_2 + ([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}$

Proof. Applying the Ravenel–Wilson relation $[x_1] \circ b_1 \circ b_1 = b_1 \# b_1$ and Proposition 1.20(a) and (b), we obtain

$$\begin{aligned} [x_1x_3] \circ b_1 \circ b_1 \circ b_2 \circ b_2 &= [x_3] \circ b_2 \circ b_2 \circ (b_1 \# b_1) \\ &= [x_3] \circ b_2 \circ ((b_1 \circ b_1) \# (b_1 \circ b_1)) \\ &= [x_3] \circ ((b_1 \circ b_1 \circ b_1) \# (b_1 \circ b_1 \circ b_1)) \\ &= ([x_3] \circ b_1 \circ b_1 \circ b_1) \# ([x_3] \circ b_1 \circ b_1 \circ b_1) \end{aligned}$$

□

Computation 2.26. $[x_1^2x_2] \circ b_1 \circ b_1 \circ b_2 \circ b_2 = 0$

Proof. Applying the Ravenel–Wilson relation $[x_1] \circ b_1 \circ b_1 = b_1 \# b_1$ and Proposition 1.20(a)

and (b), we obtain

$$\begin{aligned}[x_1^2x_2]\circ b_1\circ b_1\circ b_2\circ b_2 &= [x_1x_2]\circ b_2\circ b_2\circ (b_1\#b_1)\\ &= [x_1x_2]\circ b_2\circ ((b_1\circ b_1)\#(b_1\circ b_1))\\ &= [x_1x_2]\circ ((b_1\circ b_1\circ b_1)\#(b_1\circ b_1\circ b_1))\\ &= [x_2]\circ (([x_1]\circ b_1\circ b_1\circ b_1)\#([x_1]\circ b_1\circ b_1\circ b_1))\end{aligned}$$

The right hand side then vanishes by Lemma 2.9(b). □

Computation 2.27. $[x_1^4]\circ b_1\circ b_1\circ b_2\circ b_2 = 0$

Proof. Applying the Ravenel–Wilson relation $[x_1]\circ b_1\circ b_1 = b_1\#b_1$ and Proposition 1.20(a) and (b), we obtain

$$\begin{aligned}[x_1^4]\circ b_1\circ b_1\circ b_2\circ b_2 &= [x_1^3]\circ b_2\circ b_2\circ (b_1\#b_1)\\ &= [x_1^3]\circ b_2\circ ((b_1\circ b_1)\#(b_1\circ b_1))\\ &= [x_1^3]\circ ((b_1\circ b_1\circ b_1)\#(b_1\circ b_1\circ b_1))\\ &= [x_1^2]\circ (([x_1]\circ b_1\circ b_1\circ b_1)\#([x_1]\circ b_1\circ b_1\circ b_1))\end{aligned}$$

The right hand side then vanishes by Lemma 2.9(b). □

Computation 2.28. $[x_1x_2]\circ b_1\circ b_1\circ b_4 + ([x_2]\circ b_1\circ b_2)^{\#2} = 0$

Proof. By the Ravenel–Wilson relation $[x_1]\circ b_1\circ b_1 = b_1\#b_1$, Lemma 2.9(a) and Proposition 1.20(a), we have

$$\begin{aligned}[x_1x_2]\circ b_1\circ b_1\circ b_4 &= [x_2]\circ b_4\circ (b_1\#b_1)\\ &= [x_2]\circ ((b_1\circ b_2)\#(b_1\#b_2))\\ &= ([x_2]\circ b_1\circ b_2)\#([x_2]\circ b_1\circ b_2)\end{aligned}$$

□

Computation 2.29.

$$[x_1^3]\circ b_1\circ b_1\circ b_4 + ([x_1]\circ b_3)^{\#2} + ([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2} + ([x_1]\circ b_1)^{\#6} = 0$$

Proof. By the Ravenel–Wilson relation $[x_1]\circ b_1\circ b_1 = b_1\#b_1$, Lemma 2.9(a) and Proposition 1.20(a), we have

$$\begin{aligned}[x_1^3]\circ b_1\circ b_1\circ b_4 &= [x_1^2]\circ b_4\circ(b_1\#b_1)\\ &= [x_1^2]\circ((b_1\circ b_2)\#(b_1\circ b_2))\\ &= ([x_1^2]\circ b_1\circ b_2)^{\#2}\end{aligned}$$

From the relations in degree 6, we know

$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$

Taking the 2-fold #-sum of the right hand side, expanding and canceling terms yields the result. □

Computation 2.30. $[x_1^3]\circ b_1\circ b_2\circ b_3 = 0$

Proof. We have $[x_1^3]\circ b_1\circ b_2\circ b_3 = [x_1]\circ b_3\circ[x_1^2]\circ b_1\circ b_2$, and from the relations in degree 6, we know

$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$

Distributing $[x_1]\circ b_3\circ(-)$ over the +-sum and applying the distributive law in Lemma 1.5(3)(g) and Proposition 1.19, we have

$$[x_1^2]\circ b_3\circ b_3 + ([x_1^2]\circ b_1\circ b_1)\#([x_1^2]\circ b_2\circ b_2) + ([x_1^2]\circ b_1\circ b_2)^{\#2} + ([x_1^2]\circ b_1\circ b_1)^{\#3}$$

We apply the formula for $[x_1^2]\circ b_3\circ b_3$ from Computation 2.24, the formula for $[x_1^2]\circ b_1\circ b_2$ from the proof of Computation 2.29 and the relation $[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$ from degree 4 and see that all terms cancel. □

Computation 2.31. $[x_1x_2]\circ b_1\circ b_2\circ b_3 = 0$

Proof. By the Ravenel–Wilson coefficient equation for s^2t, we have

$$\begin{aligned}&[x_1x_2]\circ b_1\circ b_2\circ b_3 =\\ &\qquad [x_1x_2]\circ b_1\circ b_2\circ([x_1]\circ b_1\circ b_2 + [x_2]\circ b_1^{\circ 3} + b_1\#b_2 + ([x_1]\circ b_1\circ b_1)\#b_1)\end{aligned}$$

If we distribute $b_1\circ(-)$, the last two +-summands in the parentheses on the right hand

side will vanish by Proposition 1.20(d). Then

$$[x_1x_2] \circ b_1 \circ b_2 \circ b_3 = [x_1^2x_2] \circ b_1 \circ b_1 \circ b_2 \circ b_2 + [x_1x_2^2] \circ b_1^{\circ 4} \circ b_2$$

The first +-summand on the right hand side vanishes by Computation 2.26, and the second vanishes by Lemma 2.9(b). □

Computation 2.32.

$$\begin{aligned} 0 = {} & [x_2] \circ b_3 \circ b_3 + [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_3 \\ & + ([x_2] \circ b_1 \circ b_1)\#([x_2] \circ b_2 \circ b_2) + ([x_2] \circ b_1 \circ b_2)^{\#2} + ([x_2] \circ b_1 \circ b_1)^{\#3} \end{aligned}$$

Proof. By Computation 2.31,

$$0 = [x_1x_2] \circ b_1 \circ b_2 \circ b_3 = [x_2] \circ b_3 \circ ([x_1] \circ b_1 \circ b_2)$$

By the Ravenel–Wilson coefficient equation for s^2t, we can rewrite the right hand side to obtain

$$0 = [x_2] \circ b_3 \circ (b_3 + [x_2] \circ b_1^{\circ 3} + b_1\#b_2 + ([x_1] \circ b_1 \circ b_1)\#b_1)$$

By the Ravenel–Wilson relation $[x_1] \circ b_1 \circ b_1 = b_1\#b_1$, the last +-summand on the right hand side can be rewritten as $b_1^{\#3}$. Distributing $b_3 \circ (-)$ to the last two +-summands on the right hand side yields

$$(b_1 \circ b_1)\#(b_2 \circ b_2) + (b_1 \circ b_2)\#(b_1 \circ b_2) + (b_1 \circ b_1)^{\#3}$$

Therefore, distributing $b_3 \circ (-)$ to the entire +-sum in parentheses yields the result. □

Computation 2.33.

$$[x_1^3] \circ b_2 \circ b_2 \circ b_2 + ([x_1] \circ b_3)^{\#2} + ([x_1] \circ b_1)^{\#2}\#([x_1] \circ b_2)^{\#2} + ([x_1] \circ b_1)^{\#6} = 0$$

Proof. Applying the Ravenel–Wilson coefficient equation for s^2t^2, we have

$$[x_1^3] \circ b_2 \circ b_2 \circ b_2 = [x_1^2] \circ b_2 \circ ([x_1] \circ b_2 \circ b_2) = [x_1^2] \circ b_2 \circ (b_2\#b_2 + ([x_1] \circ b_1 \circ b_1)\#b_1\#b_1)$$

By Proposition 1.20, distributing $b_2 \circ (-)$ to the first +-summand in parentheses yields $([x_1] \circ b_1 \circ b_2)^{\#2}$ and distributing to the second +-summand yields 0. The result then follows exactly as in Computation 2.29. □

Computation 2.34. $[x_1x_2] \circ b_2 \circ b_2 \circ b_2 + ([x_2] \circ b_1 \circ b_2)^{\#2} = 0$

Proof. Replace $[x_1^2]$ with $[x_2]$ in the proof of Computation 2.33. □

Computation 2.35. $[x_3] \circ b_1 \circ b_2 \circ b_3 + [x_2x_3] \circ b_1^{\circ 4} \circ b_2 + ([x_1] \circ b_1 \circ b_1 \circ b_1)^{\#2} = 0$

Proof. By the Ravenel–Wilson coefficient equation for s^2t,

$$\begin{aligned}[x_2x_3] \circ b_1^{\circ 4} \circ b_2 &= [x_3] \circ b_2 \circ b_1 \circ ([x_2] \circ b_1 \circ b_1 \circ b_1) \\ &= [x_3] \circ b_2 \circ b_1 \circ (b_3 + [x_1] \circ b_1 \circ b_2 + b_1 \# b_2 + b_1 \# ([x_1] \circ b_1 \circ b_1))\end{aligned}$$

Distributing $b_1 \circ (-)$ to the last two $+$-summands in parentheses yields 0 by Proposition 1.20. Therefore,

$$[x_2x_3] \circ b_1^{\circ 4} \circ b_2 = [x_3] \circ b_1 \circ b_2 \circ b_3 + [x_1x_3] \circ b_1 \circ b_1 \circ b_2 \circ b_2$$

and then the result follows by Computation 2.25. □

Computation 2.36. $[x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_3 + [x_2^3] \circ b_1^{\circ 6} = 0$

Proof. By a relation in degree 8, we have

$$\begin{aligned}[x_2^3] \circ b_1^{\circ 6} &= [x_2] \circ b_1 \circ b_1 \circ ([x_2^2] \circ b_1^{\circ 4}) \\ &= [x_2] \circ b_1 \circ b_1 \circ ([x_2] \circ b_1 \circ b_3 + ([x_2] \circ b_1 \circ b_1)^{\#2})\end{aligned}$$

and the result follows because distributing $b_1 \circ (-)$ to the second $+$-summand on the right hand side yields 0 by Proposition 1.20. □

Computation 2.37. $[x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_3 + [x_2x_4] \circ b_1^{\circ 6} = 0$

Proof. By the Ravenel–Wilson coefficient equation for s^2t, we have

$$\begin{aligned}[x_2x_4] \circ b_1^{\circ 6} &= [x_4] \circ b_1 \circ b_1 \circ b_1 \circ ([x_2] \circ b_1 \circ b_1 \circ b_1) \\ &= [x_4] \circ b_1 \circ b_1 \circ b_1 \circ (b_3 + [x_1] \circ b_1 \circ b_2 + b_1 \# b_2 + b_1 \# ([x_1] \circ b_1 \circ b_1))\end{aligned}$$

Distributing $b_1 \circ (-)$ to the last three $+$-summands on the right hand side yields 0 by Lemma 2.9 and Proposition 1.20. The result follows. □

Computation 2.38. $[x_1x_3] \circ b_1 \circ b_1 \circ b_2 \circ b_2 + [x_3^2] \circ b_1^{\circ 6} = 0$

Proof. Since $x_3 = a_{2,2} - a_{3,1}$, we have

$$[x_3^2] \circ b_1^{\circ 6} = [x_3] \circ b_1 \circ b_1 \circ ([a_{2,2} - a_{3,1}] \circ b_1^{\circ 4}) = [x_3] \circ b_1 \circ b_1 \circ (([a_{2,2}] \# [-1] \circ [a_{3,1}]) \circ b_1^{\circ 4})$$

By the distributive law, since $[a_{2,2}]$ and $[-1] \circ [a_{3,1}]$ are in homology degree 0, we have

$$([a_{2,2}] \# [-1] \circ [a_{3,1}]) \circ b_1^{\circ 4} = [a_{2,2}] \circ b_1^{\circ 4} + [-1] \circ [a_{3,1}] \circ b_1^{\circ 4}$$

Applying the Ravenel–Wilson coefficient equations for s^2t and s^3t, this yields

$$\begin{aligned} &([x_1] \circ b_1 \circ b_2 + b_2 \# b_2 + b_1 \# b_1 \# ([x_1] \circ b_1 \circ b_1)) \\ +[-1] \circ \Big(&([x_1] \circ b_1 \circ b_3) + b_1 \# b_3 + b_1 \# ([x_1] \circ b_1 \circ b_2) \\ &+ b_1 \# ([x_2] \circ b_1 \circ b_1 \circ b_1) + b_2 \# ([x_1] \circ b_1 \circ b_2) \Big) \end{aligned}$$

By Proposition 1.20 and Lemma 2.9, the $\circ$-product of b_1 with this $+$-sum simplifies to $[x_1] \circ b_1 \circ b_1 \circ b_2$ since all other $+$-summands vanish. Therefore,

$$[x_3^2] \circ b_1^{\circ 6} = [x_3] \circ b_1 \circ ([x_1] \circ b_1 \circ b_1 \circ b_2)$$

and the result follows. □

Computation 2.39.

$$\begin{aligned} 0 = &[x_1] \circ b_6 + [x_1^2] \circ b_2 \circ b_4 + ([x_1] \circ b_1) \# ([x_1] \circ b_5) + ([x_1] \circ b_2) \# ([x_1] \circ b_4) + ([x_1] \circ b_3)^{\#2} \\ &+ ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_4) + ([x_1] \circ b_2)^{\#3} + ([x_1] \circ b_1)^{\#4} \# ([x_1] \circ b_2) \end{aligned}$$

Proof. Adding the Ravenel–Wilson coefficient equations for s^3t and s^2t^2 yields

$$\begin{aligned} [a_{3,1}] \circ b_1^{\circ 4} + [a_{2,2}] \circ b_1^{\circ 4} = &[x_1] \circ b_1 \circ b_3 + [x_1] \circ b_2 \circ b_2 + b_1 \# b_3 + b_2 \# b_2 + b_1^{\#4} \\ &+ b_1 \# ([x_1] \circ b_1 \circ b_2) + b_1 \# ([x_2] \circ b_1 \circ b_1 \circ b_1) + b_1 \# b_1 \# b_2 \end{aligned}$$

Then, by the Ravenel–Wilson coefficient equation for s^4t^2, we have

$$\begin{aligned}[x_1^2]\circ b_2\circ b_4 = [x_1]\circ\Big(&b_6+[x_2]\circ b_2\circ b_2\circ b_2+[x_2]\circ b_1\circ b_1\circ b_4+\\
&+[x_1]\circ b_1\circ b_3+[x_1]\circ b_2\circ b_2+b_1\#b_3+b_2\#b_2+b_1^{\#4}\\
&+b_1\#([x_1]\circ b_1\circ b_2)+b_1\#([x_2]\circ b_1\circ b_1\circ b_1)+b_1\#b_1\#b_2\\
&+b_1\#([x_1]\circ b_2\circ b_3)+b_1\#([x_1]\circ b_1\circ b_4)+b_1\#([x_2]\circ b_1\circ b_2\circ b_2)\\
&+b_2\#b_4+b_2\#([x_1]\circ b_2\circ b_2)+b_2\#([x_1]\circ b_1\circ b_3)\\
&+b_1\#b_1\#([x_1]\circ b_1\circ b_3)+b_3\#([x_1]\circ b_1\circ b_2)+b_1^{\#3}\#b_3\\
&+b_1\#b_2\#([x_1]\circ b_1\circ b_2)\Big)\end{aligned}$$

The result then follows by applying Computations 2.28 and 2.34, Lemma 2.9 and various lower-degree relations and canceling terms. □

Computation 2.40.

$$\begin{aligned}0 = &[x_2]\circ b_3\circ b_3+[x_4]\circ b_1\circ b_1\circ b_1\circ b_3+[x_2^2]\circ b_1\circ b_1\circ b_2\circ b_2\\
&+([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_2\circ b_2)+([x_2]\circ b_1\circ b_1)^{\#3}\end{aligned}$$

Proof. By the Ravenel–Wilson coefficient equation for s^3t^2 (omitting terms with two or more #-summands by Proposition 1.20), we have

$$\begin{aligned}[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3 &= [x_2]\circ b_1\circ([x_2]\circ b_1\circ b_1\circ b_3)\\
&= [x_2]\circ b_1\circ([x_1]\circ b_2\circ b_3+[a_{3,1}]\circ b_1\circ b_1\circ b_1\circ b_2+[a_{3,2}]\circ b_1^{\circ 5})\end{aligned}$$

Then Computation 2.31 implies

$$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3 = [x_2a_{3,1}]\circ b_1^{\circ 4}\circ b_2+[x_2a_{3,2}]\circ b_1^{\circ 6}$$

We see that

$$\begin{aligned}[x_2a_{3,1}]\circ b_1^{\circ 4}\circ b_2 &= [x_2(a_{2,2}-x_3)]\circ b_1^{\circ 4}\circ b_2\\
&= [x_2a_{2,2}]\circ b_1^{\circ 4}\circ b_2+[-1]\circ[x_2x_3]\circ b_1^{\circ 4}\circ b_2\end{aligned}$$

By the Ravenel–Wilson coefficient equation for s^2t and Computation 2.35, the right hand

side equals

$$([x_1x_2]\circ b_2\circ b_2\circ b_2+[x_2]\circ(b_2\#b_2+b_1^{\#4})\\ +[-1]\circ([x_3]\circ b_1\circ b_2\circ b_3+([x_3]\circ b_1\circ b_1\circ b_1)^{\#2})$$

By Computation 2.34 and Proposition 1.20,

$$([x_1x_2]\circ b_2\circ b_2\circ b_2+[x_2]\circ(b_2\#b_2+b_1^{\#4})=0$$

so

$$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3=[-1]\circ([x_3]\circ b_1\circ b_2\circ b_3+([x_3]\circ b_1\circ b_1\circ b_1)^{\#2})+[x_2a_{3,2}]\circ b_1^{\circ 6}$$

Now, note that

$$\begin{aligned}[x_2a_{3,2}]\circ b_1^{\circ 6}&=[x_2(x_4+a_{4,1})]\circ b_1^{\circ 6}\\&=([x_2x_4]\#[x_2a_{4,1}])\circ b_1^{\circ 6}\\&=[x_2x_4]\circ b_1^{\circ 6}+[x_2a_{4,1}]\circ b_1^{\circ 6}\end{aligned}$$

By Computation 2.37, this implies

$$[x_2a_{3,2}]\circ b_1^{\circ 6}=[x_4]\circ b_1\circ b_1\circ b_1\circ b_3+[x_2a_{4,1}]\circ b_1^{\circ 6}$$

Therefore,

$$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3=[x_4]\circ b_1\circ b_1\circ b_1\circ b_3+[x_2]\circ b_1\circ([a_{3,1}]\circ b_1^{\circ 3}\circ b_2+[a_{4,1}]\circ b_1^{\circ 5})$$

Then, by the Ravenel–Wilson coefficient equation for s^4t (omitting terms with two or more #-summands by Proposition 1.20), we have

$$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_3=[x_4]\circ b_1\circ b_1\circ b_1\circ b_3+[x_1x_2]\circ b_1\circ b_1\circ b_4+[x_2^2]\circ b_1\circ b_1\circ b_2\circ b_2$$

Finally, by Computations 2.28 and 2.32, this implies

$$[x_2]\circ b_3\circ b_3+([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_2\circ b_2)+([x_2]\circ b_1\circ b_2)^{\#2}+([x_2]\circ b_1\circ b_1)^{\#3}=\\ [x_4]\circ b_1\circ b_1\circ b_1\circ b_3+([x_2]\circ b_1\circ b_2)^{\#2}+[x_2^2]\circ b_1\circ b_1\circ b_2\circ b_2$$

and thus

$$[x_2] \circ b_3 \circ b_3 + ([x_2] \circ b_1 \circ b_1) \# ([x_2] \circ b_2 \circ b_2) + ([x_2] \circ b_1 \circ b_1)^{\#3} = [x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_3 + [x_2^2] \circ b_1 \circ b_1 \circ b_2 \circ b_2$$

□

Computation 2.41. $[x_1^2] \circ b_1 \circ b_5 + [x_1^3] \circ b_1 \circ b_1 \circ b_4 = 0$

Proof. By the Ravenel–Wilson coefficient equation for s^4t (omitting terms with two or more #-summands or a ∘-factor of the form $b_1 \circ b_{2k+1}$ by Proposition 1.20 and Lemma 2.9), we have

$$[x_1^2] \circ b_1 \circ b_5 = [x_1^2] \circ b_1 \circ ([x_1] \circ b_1 \circ b_4 + [x_2] \circ b_1 \circ b_2 \circ b_2)$$

Then the result follows because distributing $b_1 \circ (-)$ to the second +-summand in parentheses on the right hand side yields 0 by Computation 2.26. □

2.2 The abelian group structure of $H_*(SL_1MU)$

As outlined above, $H_*(SL_1MU)$ is generated as an abelian group or $\mathbb{F}_2$-module by applying $[1]\#(-)$ to #-monomials in the generators of $Q^{\#}H_*(\underline{MU}'_0)$. Using the polynomial generators from 2.2, we obtain the abelian group generators for $H_*(SL_1MU)$ in Table 2.6.

2.3 The multiplication table

2.3.1 SageMath code

The computations in degree 4 have been done entirely by hand. Most computations in degrees 6 and higher have been done with the assistance of a SageMath worksheet, which can be found at `https://github.com/ZaneHuttinga/PhD-thesis-files`. The only exceptions are those that follow immediately from a previous result in the multiplication table.

When using the worksheet, note that there are two sets of inputs, each with its own output. The first implements Lemma 1.23, and the second implements Lemma 1.24. Since the latter is a more general version of the former, the second part of the worksheet can implement either lemma, but the first output is cleaner, and the computation is less costly. In both cases, the outputs omit +-summands that vanish because of a #-summand with a ∘-factor of b_0.

Degree	Generators
0	$[1]$
2	$[1]\#([x_1]\circ b_1)$
4	$[1]\#([x_1]\circ b_2)$, $[1]\#([x_2]\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)^{\#2}$
6	$[1]\#([x_1]\circ b_3)$, $[1]\#([x_2]\circ b_1\circ b_2)$, $[1]\#([x_3]\circ b_1\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)$, $[1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)^{\#3}$
8	$[1]\#([x_1]\circ b_4)$, $[1]\#([x_2]\circ b_1\circ b_3)$, $[1]\#([x_2]\circ b_2\circ b_2)$, $[1]\#([x_3]\circ b_1\circ b_1\circ b_2)$, $[1]\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)\#([x_1]\circ b_3)$, $[1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_2)$, $[1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_2)^{\#2}$, $[1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)$, $[1]\#([x_2]\circ b_1\circ b_1)^{\#2}$, $[1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)$, $[1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)^{\#4}$
10	$[1]\#([x_1]\circ b_5)$, $[1]\#([x_2]\circ b_1\circ b_4)$, $[1]\#([x_2]\circ b_2\circ b_3)$, $[1]\#([x_3]\circ b_1\circ b_1\circ b_3)$, $[1]\#([x_3]\circ b_1\circ b_2\circ b_2)$, $[1]\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_2)$, $[1]\#([x_5]\circ b_1^{\circ 5})$, $[1]\#([x_1]\circ b_1)\#([x_1]\circ b_4)$, $[1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_3)$, $[1]\#([x_1]\circ b_1)\#([x_2]\circ b_2\circ b_2)$, $[1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_2)$, $[1]\#([x_1]\circ b_1)\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_2)\#([x_1]\circ b_3)$, $[1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_2)$, $[1]\#([x_1]\circ b_2)\#([x_3]\circ b_1\circ b_1\circ b_1)$, $[1]\#([x_2]\circ b_1\circ b_1)\#([x_1]\circ b_3)$, $[1]\#([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_2)$, $[1]\#([x_2]\circ b_1\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3)$, $[1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_2)$, $[1]\#([x_1]\circ b_1)^{\#2}\#([x_3]\circ b_1\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2}$, $[1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)^{\#2}$, $[1]\#([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)$, $[1]\#([x_1]\circ b_1)^{\#3}\#([x_2]\circ b_1\circ b_1)$, $[1]\#([x_1]\circ b_1)^{\#5}$

Table 2.6: Generators for $H_*(SL_1MU)$ as an abelian group or $\mathbb{F}_2$-vector space

To implement Lemma 1.23, note that the inputs are vectors whose components are the indices of the $\circ$-factors b_n. Note that the $[x_i]$ $\circ$-factor is omitted, but there is no ambiguity because the index i is always the sum of the indices of the b_n (as in Conjecture 2.8). Similarly, in the formula of Lemma 1.23 the [1] #-summands in the $\circ$-factors only have the effect of a single #-summand of [1] in the $\circ$-product. Therefore, these are also ignored in the code. Consider the following example.

Example 2.42. To compute

$$([1]\#([x_1]\circ b_2))\circ([1]\#([x_2]\circ b_1\circ b_3))$$

we enter

```
a = vector(ZZ,[2])
b = vector(ZZ,[1,3])
```

The output is

```
[(2), (1, 3), (0, 0, 0)]
[(1), (0, 0), (1, 1, 3)]
[(0), (0, 0), (2, 1, 3)]
```

Here each list (in square brackets []) is a +-summand, and the vectors inside a list (in parentheses ()) are #-summands. Again, the entries in the vectors are indices of the b_n. We apply a #-sum with [1] to the entire +-sum as in Lemma 1.23. Also note that the first two #-summands have the $[x_i]$ $\circ$-factor of the inputs, and the third has the $\circ$-product of the two. Therefore, the output represents

$$\begin{aligned}[1]\#\Big(&([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_3)\#([x_1x_2]\circ b_0\circ b_0\circ b_0)\\ &+([x_1]\circ b_1)\#([x_2]\circ b_0\circ b_0)\#([x_1x_2]\circ b_1\circ b_1\circ b_3)\\ &+([x_1]\circ b_0)\#([x_2]\circ b_0\circ b_0)\#([x_1x_2]\circ b_2\circ b_1\circ b_3)\Big)\end{aligned}$$

From here, we simplify by hand by removing the [0] #-summands and applying results such as Lemma 2.9 and relations as in Tables 2.2 and 2.3.

To implement Lemma 1.24, the inputs are lists of lists. Again, the entries in the inner lists are indices of the b_n generators. In this case, one must be careful when reading the output. Consider the following example.

Example 2.43. To compute

$$([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1))$$

we enter

```
M = [[1],[2]]
N = [[1],[1,1]]
```

The output is

```
plus
[[], [1], [2], [1], [0, 0], [0, 0], [1, 1], [0, 0, 0], [0, 0, 0]]
plus
[[], [1], [0], [0], [0, 0], [2, 1], [1, 1], [0, 0, 0], [0, 0, 0]]
plus
[[], [1], [0], [1], [0, 0], [0, 0], [0, 0], [0, 0, 0], [2, 1, 1]]
plus
[[], [1], [1], [0], [0, 0], [1, 1], [1, 1], [0, 0, 0], [0, 0, 0]]
plus
[[], [1], [1], [1], [0, 0], [0, 0], [0, 0], [0, 0, 0], [1, 1, 1]]
plus
[[], [1], [0], [0], [0, 0], [1, 1], [0, 0], [0, 0, 0], [1, 1, 1]]
plus
[[], [0], [2], [0], [1, 1], [0, 0], [1, 1], [0, 0, 0], [0, 0, 0]]
plus
[[], [0], [0], [0], [1, 1], [0, 0], [0, 0], [0, 0, 0], [2, 1, 1]]
plus
[[], [0], [1], [0], [1, 1], [0, 0], [0, 0], [0, 0, 0], [1, 1, 1]]
plus
[[], [0], [2], [1], [0, 0], [0, 0], [0, 0], [1, 1, 1], [0, 0, 0]]
plus
[[], [0], [0], [0], [0, 0], [2, 1], [0, 0], [1, 1, 1], [0, 0, 0]]
plus
[[], [0], [1], [0], [0, 0], [1, 1], [0, 0], [1, 1, 1], [0, 0, 0]]
row length:  3
```

Each list of lists represents a +-summand, and the lists inside a given list represent #-summands within that +-summand. The entries in the inner lists are indices of the b_n. The

empty list at the start represents the #-summand [1]. In order to interpret this, first note the "row length"; this refers to a matrix representation of each +-summand at which the indexing of Lemma 1.24 hints.

$$([1]\#a\#b)\circ([1]\#c\#d)=\sum\begin{pmatrix} & [1] & \# & a' & \# & b' \\ \# & c' & \# & (a''\circ c'') & \# & (b''\circ c''') \\ \# & d' & \# & (a'''\circ d'') & \# & (b'''\circ d''') \end{pmatrix}$$

The rows are printed in sequence from left to right. So, for instance, the fourth line

```
[[], [1], [1], [0], [0, 0], [1, 1], [1, 1], [0, 0, 0], [0, 0, 0]]
```

Indicates the #-sum

$$\begin{array}{cccccc} & [1] & \# & ([x_1]\circ b_1) & \# & ([x_1]\circ b_1) \\ \# & ([x_1]\circ b_0) & \# & ([x_1^2]\circ b_0\circ b_0) & \# & ([x_1^2]\circ b_1\circ b_1) \\ \# & ([x_2]\circ b_1\circ b_1) & \# & ([x_1x_2]\circ b_0\circ b_0\circ b_0) & \# & ([x_1x_2]\circ b_0\circ b_0\circ b_0) \end{array}$$

This is important because of the ambiguity in, for instance, the two entries $[1,1]$, which represent different $\circ$-products.

2.3.2 Computations

Computation 2.44. $([1]\#([x_1]\circ b_1))^{\circ 2}=0$

Proof. By Lemma 1.23,

$$\begin{aligned}([1]\#([x_1]\circ b_1))^{\circ 2} &= [1]\#\left(([x_1^2]\circ b_1\circ b_1)+([x_1]\circ b_1)\#([x_1]\circ b_1)\right)\\ &= [1]\#\left([x_1]\circ([x_1]\circ b_1\circ b_1+b_1\#b_1)\right)\end{aligned}$$

By the Ravenel–Wilson coefficient equation for st, the inner +-sum equals 0, so the result follows. □

Computation 2.45. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_2))=[1]\#([x_1]\circ b_3)$

Proof. Again by Lemma 1.23, we have

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_2)) &= [1]\#\big(([x_1^2]\circ b_1\circ b_2)+([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_1)\\ &\quad +([x_1]\circ b_1)\#([x_1]\circ b_2)\big)\\ &= [1]\#\big([x_1]\circ\big([x_1]\circ b_1\circ b_2+b_1\#([x_1]\circ b_1\circ b_1)\\ &\quad +b_1\#b_2\big)\big)\end{aligned}$$

By the Ravenel–Wilson coefficient equation for st, the right hand side equals

$$[1]\#\left([x_1]\circ(b_3+[a_{2,1}]\circ b_1\circ b_1\circ b_1)\right)$$

and the second +-summand vanishes by Lemma 2.9(b). □

Computation 2.46. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)$

Proof. The left-hand side equals

$$[1]\#\left([x_1x_2]\circ b_1\circ b_1\circ b_1+([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)\right)$$

and the first +-summand vanishes by Lemma 2.9(b). □

Computation 2.47. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#2}) = [1]\#([x_1]\circ b_1)^{\#3}$

Proof. This follows immediately from Lemma 1.24. □

Computation 2.48. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_3)) = 0$

Proof. This follows immediately from Computations 2.44 and 2.45. □

Computation 2.49.

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_2)) &= [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_2)\\ &\quad +[1]\#([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

Proof. This follows immediately from Lemma 1.23. □

Computation 2.50.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_3]\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof. Lemma 1.23 yields the +-sum of right hand side and $[1]\#([x_1x_3]\circ b_1^{\circ 4})$, which vanishes by Lemma 2.9(b). □

Computation 2.51.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_1]\circ b_3) + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)$$

Proof. Lemma 1.24 yields

$$[1]\#\big(([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2) + ([x_1]^2\circ b_1\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_2) + ([x_1]\circ b_1)^{\#2}\#([x_1^2]\circ b_1\circ b_1)\big)$$

The result follows by applying the relations

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

and

$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$

from Table 2.2 and canceling terms. □

Computation 2.52. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)) = 0$

Proof. Lemma 1.24 yields

$$[1]\#\big(([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1) + ([x_1^2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_1) + ([x_1]\circ b_1)\#([x_1x_2]\circ b_1\circ b_1\circ b_1)\big)$$

The first two +-summands cancel by the relation

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

from Table 2.2, and the last one vanishes by Lemma 2.9(b). □

Computation 2.53. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#3}) = 0$

Proof. Lemma 1.24 yields

$$\begin{aligned}[1]\#\big(([x_1]\circ b_1)^{\#4} &+ ([x_1^2]\circ b_1\circ b_1)\#([x_1]\circ b_1)^{\#2}\\ &+ ([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_1)\#([x_1]\circ b_1)\\ &+ ([x_1]\circ b_1)^{\#2}\#([x_1^2]\circ b_1\circ b_1)\end{aligned}$$

Since $[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$, all four $+$-summands are equal and thus cancel. □

From this point on, unless there is a simpler proof using calculations from earlier in the multiplication table, the process is always the same: compute the $\circ$-product using either Lemma 1.23 or Lemma 1.24 and the SageMath worksheet, and then simplify by applying relations from Tables 2.2 and 2.3, applying Lemma 2.9 and canceling $+$-summands over $\mathbb{F}_2$. Therefore, for simplicity, I will generally only list the necessary relations, and only those that are not immediate consequences of Lemma 2.9(b).

Computation 2.54.

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_4)) = [1]\#([x_1]\circ b_5)&\\ + [1]\#([x_3]\circ b_1\circ b_1\circ b_3)&\\ + [1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2}&\\ + [1]\#([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)&\\ + [1]\#([x_1]\circ b_1)^{\#5}&\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2}\\ [x_1^2]\circ b_1\circ b_2 &= [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}\\ [x_1^2]\circ b_1\circ b_4 &= [x_1]\circ b_5 + [x_3]\circ b_1\circ b_1\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_4)\\ &\quad + ([x_1]\circ b_2)\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#5}\end{aligned}$$

□

Computation 2.55. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_3)) = [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_3)$

Proof sketch. No additional relations are needed. □

Computation 2.56.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_2\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_2]\circ b_2\circ b_2) + [1]\#([x_3]\circ b_1\circ b_1\circ b_3)$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_2\circ b_2 = [x_3]\circ b_1\circ b_1\circ b_3$$

□

Computation 2.57.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_3]\circ b_1\circ b_1\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_2)$$

Proof sketch. No additional relations are needed. □

Computation 2.58.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.59.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_3)) = [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3)$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$
$$[x_1^2]\circ b_1\circ b_3 = ([x_1]\circ b_1)^{\#4}$$

□

Computation 2.60.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)^{\#2}$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

□

Computation 2.61. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)) = 0$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.62. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_2)^{\#2}) = [1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2}$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.63.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)) = [1]\#([x_2]\circ b_1\circ b_1)\#([x_1]\circ b_3)$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$
$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$

□

Computation 2.64.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_1)^{\#2}) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_2]\circ b_1\circ b_1)$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.65.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)) = [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3) + [1]\#([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$

□

Computation 2.66.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_2]\circ b_1\circ b_1)$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.67. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#4}) = [1]\#([x_1]\circ b_1)^{\#5}$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.68.

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_5)) = {} & [1]\#([x_1]\circ b_1)^{\#4}\#([x_1]\circ b_2)\\ & + [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_3)\\ & + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2}\end{aligned}$$

Proof sketch.

$$\begin{aligned}&[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}\\ &[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}\\ &[x_1^2]\circ b_1\circ b_3 = ([x_1]\circ b_1)^{\#4}\\ &[x_1^2]\circ b_1\circ b_4 = [x_1]\circ b_5 + [x_3]\circ b_1\circ b_1\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_4)\\ &\qquad + ([x_1]\circ b_2)\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#5}\end{aligned}$$

□

Computation 2.69.

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_4)) = {} & [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_4)\\ & + [1]\#([x_2]\circ b_1\circ b_2)^{\#2}\end{aligned}$$

Proof sketch.

$$[x_1x_2] \circ b_1 \circ b_1 \circ b_4 = ([x_2] \circ b_1 \circ b_2)^{\#2}$$

□

Computation 2.70.

$$([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_2] \circ b_2 \circ b_3)) = [1]\#([x_1] \circ b_1)\#([x_2] \circ b_2 \circ b_3) + [1]\#([x_2] \circ b_1 \circ b_1)^{\#3}$$

Proof sketch.

$$[x_1x_2] \circ b_1 \circ b_1 \circ b_2 = ([x_2] \circ b_1 \circ b_1)^{\#2}$$
$$[x_1x_2] \circ b_1 \circ b_2 \circ b_3 = 0$$

□

Computation 2.71.

$$([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_3] \circ b_1 \circ b_1 \circ b_3)) = [1]\#([x_1] \circ b_1)\#([x_3] \circ b_1 \circ b_1 \circ b_3)$$

Proof sketch. No additional relations are needed. □

Computation 2.72.

$$([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_3] \circ b_1 \circ b_2 \circ b_2)) = [1]\#([x_1] \circ b_1)\#([x_3] \circ b_1 \circ b_2 \circ b_2) + [1]\#([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}$$

Proof sketch.

$$[x_1x_3] \circ b_1 \circ b_1 \circ b_2 \circ b_2 = ([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}$$

□

Computation 2.73.

$$([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_2)) = [1]\#([x_1] \circ b_1)\#([x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_2)$$

Proof sketch. No additional relations are needed. □

Computation 2.74.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_5]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)\#([x_5]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.75.

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_4) = {} & [1]\#([x_1]\circ b_1)\#([x_1]\circ b_5)\\ & + [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_3)\\ & + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_4)\\ & + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2}\\ & + [1]\#([x_1]\circ b_1)^{\#4}\#([x_1]\circ b_2)\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2}\\ [x_1^2]\circ b_1\circ b_2 &= [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}\\ [x_1^2]\circ b_1\circ b_3 &= ([x_1]\circ b_1)^{\#4}\\ [x_1^2]\circ b_1\circ b_4 &= [x_1]\circ b_5 + [x_3]\circ b_1\circ b_1\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_4)\\ &\quad + ([x_1]\circ b_2)\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#5}\end{aligned}$$

□

Computation 2.76. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_3)) = 0$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.77.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_2\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_3)$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_2\circ b_2 = [x_3]\circ b_1\circ b_1\circ b_3$$

□

Computation 2.78. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_2))=0$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1=([x_1]\circ b_1)^{\#2}$$

□

Computation 2.79. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1))=0$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1=([x_1]\circ b_1)^{\#2}$$

□

Computation 2.80.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_2)\#([x_1]\circ b_3))=[1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)\#([x_1]\circ b_3)+[1]\#([x_1]\circ b_3)^{\#2}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1&=([x_1]\circ b_1)^{\#2}\\ [x_1^2]\circ b_1\circ b_2&=[x_1]\circ b_3+([x_1]\circ b_1)\#([x_1]\circ b_2)+([x_1]\circ b_1)^{\#3}\\ [x_1^2]\circ b_1\circ b_3&=([x_1]\circ b_1)^{\#4}\end{aligned}$$

□

Computation 2.81.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_2))=[1]\#([x_1]\circ b_3)\#([x_2]\circ b_1\circ b_2)+[1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)^{\#2}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1&=([x_1]\circ b_1)^{\#2}\\ [x_1x_2]\circ b_1\circ b_1\circ b_2&=([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

□

Computation 2.82.

$$([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_2)\#([x_3]\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_3)\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.83. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_1)\#([x_1]\circ b_3)) = 0$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.84.

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_2)) = \\ [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_2) \\ + [1]\#([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2} \\ [x_1x_2]\circ b_1\circ b_1\circ b_2 &= ([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

□

Computation 2.85.

$$\begin{aligned}([1]\#([x_1]\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)) = \\ [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)\end{aligned}$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.86. $([1]\#([x_1]\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3)) = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_2 = [x_1] \circ b_3 + ([x_1] \circ b_1)\#([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}$$
$$[x_1^2] \circ b_1 \circ b_3 = ([x_1] \circ b_1)^{\#4}$$

□

Computation 2.87.

$$([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_2)) = [1]\#([x_1] \circ b_1)^{\#3}\#([x_2] \circ b_1 \circ b_2) + [1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_1)^{\#2}$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$
$$[x_1x_2] \circ b_1 \circ b_1 \circ b_2 = ([x_2] \circ b_1 \circ b_1)^{\#2}$$

□

Computation 2.88.

$$([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_1] \circ b_1)^{\#2}\#([x_3] \circ b_1 \circ b_1 \circ b_1)) = [1]\#([x_1] \circ b_1)^{\#3}\#([x_3] \circ b_1 \circ b_1 \circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.89. $([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_1] \circ b_1)\#([x_1] \circ b_2)^{\#2}) = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.90.

$$([1]\#([x_1] \circ b_1)) \circ ([1]\#([x_1] \circ b_2)\#([x_2] \circ b_1 \circ b_1)) = [1]\#([x_1] \circ b_1)\#([x_1] \circ b_3)\#([x_2] \circ b_1 \circ b_1) + [1]\#([x_1] \circ b_1)^{\#2}\#([x_1] \circ b_2)\#([x_2] \circ b_1 \circ b_1)$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$
$$[x_1^2] \circ b_1 \circ b_2 = [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}$$

□

Computation 2.91. $([1] \# ([x_1] \circ b_1)) \circ ([1] \# ([x_1] \circ b_1) \# ([x_2] \circ b_1 \circ b_1)^{\#2}) = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.92.

$$([1] \# ([x_1] \circ b_1)) \circ ([1] \# ([x_1] \circ b_1)^{\#3} \# ([x_1] \circ b_2)) = [1] \# ([x_1] \circ b_1)^{\#3} \# ([x_1] \circ b_3) + [1] \# ([x_1] \circ b_1)^{\#4} \# ([x_1] \circ b_2)$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$
$$[x_1^2] \circ b_1 \circ b_2 = [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}$$

□

Computation 2.93. $([1] \# ([x_1] \circ b_1)) \circ ([1] \# ([x_1] \circ b_1)^{\#3} \# ([x_2] \circ b_1 \circ b_1)) = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.94. $([1] \# ([x_1] \circ b_1)) \circ ([1] \# ([x_1] \circ b_1)^{\#5}) = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.95. $([1] \# ([x_1] \circ b_2))^{\circ 2} = 0$

Proof. We have

$$\begin{aligned}([1]\#([x_1]\circ b_2))^{\circ 2} &= [1]\#\big([x_1^2]\circ b_2\circ b_2 + ([x_1]\circ b_1)\#([x_1^2]\circ b_2\circ b_1)\\ &\quad + ([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_2) + ([x_1]\circ b_1)\#([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_1)\\ &\quad + ([x_1]\circ b_2)\#([x_1]\circ b_2)\big)\\ &= [1]\#\big([x_1^2]\circ b_2\circ b_2 + ([x_1]\circ b_1)\#([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_1)\\ &\quad + ([x_1]\circ b_2)\#([x_1]\circ b_2)\big)\end{aligned}$$

Factoring out a $\circ$-factor of $[x_1]$ on the right hand side, we see that the remaining factor appears in the coefficient equation for s^2t^2. We replace it with the remaining terms from said equation (ignoring those that vanish or cancel over $\mathbb{F}_2$):

$$([1]\#([x_1]\circ b_2))^{\circ 2} = [1]\#\big([x_1]\circ[a_{2,2}]\circ b_1\circ b_1\circ b_1\circ b_1\big)$$

This vanishes by Lemma 2.9(b). □

Computation 2.96.

$$\begin{aligned}([1]\#([x_1]\circ b_2))\circ([1]\#([x_2]\circ b_1\circ b_1)) &= [1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)\\ &\quad + [1]\#([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

Proof. This follows from Lemma 1.23 and the relations

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

and

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

as well as Lemma 2.9(b). □

Computation 2.97.

$$([1]\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_1)^{\#2}) = [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2) + [1]\#([x_1]\circ b_1)^{\#4}$$

Proof. This follows from Lemma 1.24 and canceling terms. □

Computation 2.98. $([1]\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_3)) = 0$

Proof. This follows from Computations 2.45 and 2.95. □

Computation 2.99.

$$\begin{aligned}([1]\#([x_1]\circ b_2))\circ([1]\#([x_2]\circ b_1\circ b_2)) = {}& [1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_2)\\ &+ [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)^{\#2}\\ &+ [1]\#([x_3]\circ b_1\circ b_1\circ b_3)\end{aligned}$$

Proof. Lemma 1.23 yields

$$\begin{aligned}[1]\#\big(&([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_2) + ([x_1]\circ b_1)\#([x_1x_2]\circ b_1\circ b_1\circ b_2)\\ &+ ([x_1x_2]\circ b_1\circ b_2\circ b_2)\big)\end{aligned}$$

and the result follows by the relations

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

from Table 2.2 and

$$[x_1x_2]\circ b_1\circ b_2\circ b_2 = [x_3]\circ b_1\circ b_1\circ b_3$$

from Table 2.3. □

Computation 2.100.

$$([1]\#([x_1]\circ b_2))\circ([1]\#([x_3]\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_2)\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof. Lemma 1.23 yields

$$\begin{aligned}[1]\#\big(&([x_1]\circ b_2)\#([x_3]\circ b_1\circ b_1\circ b_1) + ([x_1]\circ b_1)\#([x_1x_3]\circ b_1\circ b_1\circ b_1)\\ &+ ([x_1x_3]\circ b_1\circ b_1\circ b_1\circ b_2)\big)\end{aligned}$$

and the last two +-summands vanish by Lemma 2.9(b). □

Computation 2.101.

$$\begin{aligned}([1]\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)) = {}& [1]\#([x_1]\circ b_2)\#([x_1]\circ b_3)\\ &+ [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3)\\ &+ [1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2}\\ &+ [1]\#([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)\end{aligned}$$

Proof. Lemma 1.24 yields

$$\begin{aligned}[1]\#\big(([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2} &+ ([x_1^2]\circ b_1\circ b_2)\#([x_1]\circ b_2)\\ &+ ([x_1]\circ b_1)\#([x_1^2]\circ b_2\circ b_2)\\ &+ ([x_1]\circ b_1)^{\#2}\#([x_1^2]\circ b_1\circ b_2)\\ &+ ([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_1)\#([x_1]\circ b_2)\\ &+ ([x_1]\circ b_1)^{\#2}\#([x_1^2]\circ b_1\circ b_2)\\ &+ ([x_1]\circ b_1)^{\#3}\#([x_1^2]\circ b_1\circ b_1)\\ &+ ([x_1^2]\circ b_1\circ b_1)\#([x_1^2]\circ b_1\circ b_2)\\ &+ ([x_1^2]\circ b_1\circ b_1)\#([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_1)\big)\end{aligned}$$

The result follows by applying the relations

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

and

$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$

from Table 2.2 and canceling terms. □

Computation 2.102.

$$\begin{aligned}([1]\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)) &= [1]\#([x_1]\circ b_3)\#([x_2]\circ b_1\circ b_1)\\ &\quad + ([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

Proof. Lemma 1.24 yields

$$\begin{aligned}[1]\#\big(([x_1]\circ b_2)\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1) &+ ([x_1^2]\circ b_1\circ b_2)\#([x_2]\circ b_1\circ b_1)\\ &+ ([x_1]\circ b_1)\#([x_1x_2]\circ b_1\circ b_1\circ b_2)\\ &+ ([x_1]\circ b_1)\#([x_1^2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_1)\\ &+ ([x_1]\circ b_1)^{\#2}\#([x_1x_2]\circ b_1\circ b_1\circ b_1)\\ &+ ([x_1^2]\circ b_1\circ b_1)\#([x_1x_2]\circ b_1\circ b_1\circ b_1)\big)\end{aligned}$$

The last two +-summands vanish by Lemma 2.9(b). Then applying the relations

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

$$[x_1^2] \circ b_1 \circ b_2 = [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}$$

and

$$[x_1 x_2] \circ b_1 \circ b_1 \circ b_2 = ([x_2] \circ b_1 \circ b_1)^{\#2}$$

from Table 2.2 and canceling terms gives the result. □

Computation 2.103.

$$([1] \# ([x_1] \circ b_2)) \circ ([1] \# ([x_1] \circ b_1)^{\#3}) = [1] \# ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_3) + [1] \# ([x_1] \circ b_1)^{\#5}$$

Proof. Lemma 1.24 yields

$$\begin{aligned}
[1] \# \big(([x_1] \circ b_2) \# ([x_1] \circ b_1)^{\#3} &+ ([x_1^2] \circ b_1 \circ b_2) \# ([x_1] \circ b_1)^{\#2} \\
&+ ([x_1] \circ b_1) \# ([x_1^2] \circ b_1 \circ b_2) \# ([x_1] \circ b_1) \\
&+ ([x_1] \circ b_1)^{\#2} \# ([x_1^2] \circ b_1 \circ b_2) \\
&+ ([x_1] \circ b_1) \# ([x_1^2] \circ b_1 \circ b_1) \# ([x_1] \circ b_1)^{\#2} \\
&+ ([x_1] \circ b_1)^{\#2} \# ([x_1^2] \circ b_1 \circ b_1) \# ([x_1] \circ b_1) \\
&+ ([x_1] \circ b_1)^{\#3} \# ([x_1^2] \circ b_1 \circ b_1) \\
&+ ([x_1^2] \circ b_1 \circ b_1)^{\#2} \# ([x_1] \circ b_1) \\
&+ ([x_1^2] \circ b_1 \circ b_1) \# ([x_1] \circ b_1) \# ([x_1^2] \circ b_1 \circ b_1) \\
&+ ([x_1] \circ b_1) \# ([x_1^2] \circ b_1 \circ b_1)^{\#2} \big)
\end{aligned}$$

The result follows by applying the relations

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

and

$$[x_1^2] \circ b_1 \circ b_2 = [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}$$

from Table 2.2 and canceling terms. □

Computation 2.104.

$$([1] \# ([x_1] \circ b_2)) \circ ([1] \# ([x_1] \circ b_4)) = [1] \# ([x_1] \circ b_6) + [1] \# ([x_1] \circ b_1) \# ([x_3] \circ b_1 \circ b_1 \circ b_3)$$

Proof sketch.

$$
\begin{aligned}
[x_1^2] \circ b_1 \circ b_1 &= ([x_1] \circ b_1)^{\#2} \\
[x_1^2] \circ b_1 \circ b_2 &= [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3} \\
[x_1^2] \circ b_1 \circ b_3 &= ([x_1] \circ b_1)^{\#4} \\
[x_1^2] \circ b_2 \circ b_2 &= ([x_1] \circ b_2)^{\#2} + ([x_1] \circ b_1)^{\#4} \\
[x_1^2] \circ b_1 \circ b_4 &= [x_1] \circ b_5 + [x_3] \circ b_1 \circ b_1 \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_4) \\
&\quad + ([x_1] \circ b_2) \# ([x_1] \circ b_3) + ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_3) + ([x_1] \circ b_1)^{\#5}
\end{aligned}
$$

$$
\begin{aligned}
[x_1^2] \circ b_2 \circ b_3 &= ([x_1] \circ b_2) \# ([x_1] \circ b_3) + ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_3) + ([x_1] \circ b_1) \# ([x_1] \circ b_2)^{\#2} \\
&\quad + ([x_1] \circ b_1)^{\#3} \# ([x_1] \circ b_2) \\
[x_1^2] \circ b_2 \circ b_4 &= [x_1] \circ b_6 + ([x_1] \circ b_1) \# ([x_1] \circ b_5) + ([x_1] \circ b_2) \# ([x_1] \circ b_4) + ([x_1] \circ b_3)^{\#2} \\
&\quad + ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_4) + ([x_1] \circ b_2)^{\#3} + ([x_1] \circ b_1)^{\#4} \# ([x_1] \circ b_2)
\end{aligned}
$$

□

Computation 2.105.

$$([1] \# ([x_1] \circ b_2)) \circ ([1] \# ([x_2] \circ b_1 \circ b_3)) = [1] \# ([x_1] \circ b_2) \# ([x_2] \circ b_1 \circ b_3)$$

Proof sketch.

$$[x_1 x_2] \circ b_1 \circ b_2 \circ b_3 = 0$$

□

Computation 2.106.

$$
\begin{aligned}
([1] \# ([x_1] \circ b_2)) \circ ([1] \# ([x_2] \circ b_2 \circ b_2)) &= [1] \# ([x_1] \circ b_1) \# ([x_3] \circ b_1 \circ b_1 \circ b_3) \\
&\quad + [1] \# ([x_1] \circ b_2) \# ([x_2] \circ b_2 \circ b_2) \\
&\quad + [1] \# ([x_2] \circ b_1 \circ b_2)^{\#2} \\
&\quad + [1] \# ([x_2] \circ b_1 \circ b_1)^{\#3}
\end{aligned}
$$

Proof sketch.

$$[x_1x_2] \circ b_1 \circ b_1 \circ b_2 = ([x_2] \circ b_1 \circ b_1)^{\#2}$$
$$[x_1x_2] \circ b_1 \circ b_2 \circ b_2 = [x_3] \circ b_1 \circ b_1 \circ b_3$$
$$[x_1x_2] \circ b_2 \circ b_2 \circ b_2 = ([x_2] \circ b_1 \circ b_2)^{\#2}$$

□

Computation 2.107.

$$([1]\#([x_1] \circ b_2)) \circ ([1]\#([x_3] \circ b_1 \circ b_1 \circ b_2)) = [1]\#([x_1] \circ b_2)\#([x_3] \circ b_1 \circ b_1 \circ b_2) + [1]\#([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}$$

Proof sketch.

$$[x_1x_3] \circ b_1 \circ b_1 \circ b_2 \circ b_2 = ([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}$$

□

Computation 2.108.

$$([1]\#([x_1] \circ b_2)) \circ ([1]\#([x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_1)) = [1]\#([x_1] \circ b_2)\#([x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.109.

$$\begin{aligned}([1]\#([x_1] \circ b_2)) \circ ([1]\#([x_1] \circ b_1)\#([x_1] \circ b_3)) = {} & [1]\#([x_1] \circ b_3)^{\#2} \\ & + [1]\#([x_1] \circ b_1)\#([x_1] \circ b_2)\#([x_1] \circ b_3) \\ & + [1]\#([x_1] \circ b_1)^{\#3}\#([x_1] \circ b_3) \\ & + [1]\#([x_1] \circ b_1)^{\#2}\#([x_1] \circ b_2)^{\#2} \\ & + [1]\#([x_1] \circ b_1)^{\#4}\#([x_1] \circ b_2) \\ & + [1]\#([x_1] \circ b_1)^{\#6}\end{aligned}$$

Proof sketch.

$$
\begin{aligned}
[x_1^2] \circ b_1 \circ b_1 &= ([x_1] \circ b_1)^{\#2} \\
[x_1^2] \circ b_1 \circ b_2 &= [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3} \\
[x_1^2] \circ b_2 \circ b_2 &= ([x_1] \circ b_2)^{\#2} + ([x_1] \circ b_1)^{\#4} \\
[x_1^2] \circ b_2 \circ b_3 &= ([x_1] \circ b_2) \# ([x_1] \circ b_3) + ([x_1] \circ b_1)^{\#2} \# ([x_1] \circ b_3) + ([x_1] \circ b_1) \# ([x_1] \circ b_2)^{\#2} \\
&\quad + ([x_1] \circ b_1)^{\#3} \# ([x_1] \circ b_2)
\end{aligned}
$$

□

Computation 2.110.

$$
\begin{aligned}
([1] \# ([x_1] \circ b_2)) \circ ([1] \# ([x_1] \circ b_1) \# ([x_2] \circ b_1 \circ b_2)) &= [1] \# ([x_1] \circ b_1) \# ([x_3] \circ b_1 \circ b_1 \circ b_3) \\
&\quad + [1] \# ([x_1] \circ b_3) \# ([x_2] \circ b_1 \circ b_2)
\end{aligned}
$$

Proof sketch.

$$
\begin{aligned}
[x_1^2] \circ b_1 \circ b_1 &= ([x_1] \circ b_1)^{\#2} \\
[x_1^2] \circ b_1 \circ b_2 &= [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3} \\
[x_1 x_2] \circ b_1 \circ b_2 \circ b_2 &= [x_3] \circ b_1 \circ b_1 \circ b_3
\end{aligned}
$$

□

Computation 2.111.

$$([1] \# ([x_1] \circ b_2)) \circ ([1] \# ([x_1] \circ b_1) \# ([x_3] \circ b_1 \circ b_1 \circ b_1)) = [1] \# ([x_1] \circ b_1) \# ([x_3] \circ b_1 \circ b_1 \circ b_1))$$

Proof sketch.

$$
\begin{aligned}
[x_1^2] \circ b_1 \circ b_1 &= ([x_1] \circ b_1)^{\#2} \\
[x_1^2] \circ b_1 \circ b_2 &= [x_1] \circ b_3 + ([x_1] \circ b_1) \# ([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}
\end{aligned}
$$

□

Computation 2.112.

$$\begin{aligned}([1]\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_2)^{\#2}) = [1]\#([x_1]\circ b_3)^{\#2} \\ + [1]\#([x_1]\circ b_2)^{\#3} \\ + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2} \\ [x_1^2]\circ b_1\circ b_2 &= [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}\end{aligned}$$

□

Computation 2.113.

$$([1]\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)^{\#2}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2} \\ [x_1x_2]\circ b_1\circ b_1\circ b_2 &= ([x_2]\circ b_1\circ b_1)^{\#2} \\ [x_1^2]\circ b_2\circ b_2 &= ([x_1]\circ b_2)^{\#2} + ([x_1]\circ b_1)^{\#4}\end{aligned}$$

□

Computation 2.114.

$$([1]\#([x_1]\circ b_2))\circ([1]\#([x_2]\circ b_1\circ b_1)^{\#2}) = [1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)^{\#2}$$

Proof sketch. No additional relations are needed. □

Computation 2.115.

$$([1]\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)) = [1]\#([x_1]\circ b_1)^{\#4}\#([x_1]\circ b_2)$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$
$$[x_1^2] \circ b_2 \circ b_2 = ([x_1] \circ b_2)^{\#2} + ([x_1] \circ b_1)^{\#4}$$

□

Computation 2.116.

$$([1]\#([x_1] \circ b_2)) \circ ([1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_1)) =$$
$$[1]\#([x_1] \circ b_1)^{\#2}\#([x_1] \circ b_2)\#([x_2] \circ b_1 \circ b_1)$$
$$+ [1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_1)^{\#2}$$
$$+ [1]\#([x_1] \circ b_1)^{\#4}\#([x_2] \circ b_1 \circ b_1)$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$
$$[x_1x_2] \circ b_1 \circ b_1 \circ b_2 = ([x_2] \circ b_1 \circ b_1)^{\#2}$$

□

Computation 2.117.

$$([1]\#([x_1] \circ b_2)) \circ ([1]\#([x_1] \circ b_1)^{\#4}) = [1]\#([x_1] \circ b_1)^{\#4}\#([x_1] \circ b_2)$$

Proof sketch. No additional relations are needed. □

Computation 2.118. $([1]\#([x_2] \circ b_1 \circ b_1))^{\circ 2} = [x_2] \circ b_1 \circ b_3$

Proof. The follows from Lemma 1.23 and the relation

$$[x_2] \circ b_1 \circ b_3 = [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 + ([x_2] \circ b_1 \circ b_1)^{\#2}$$

from Table 2.2. □

Computation 2.119.

$$([1]\#([x_2] \circ b_1 \circ b_1)) \circ ([1]\#([x_1] \circ b_1)^{\#2}) = [1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_1)$$

Proof. This follows from Lemma 1.23 and canceling terms. □

Computation 2.120.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_3)) = [1]\#([x_2]\circ b_1\circ b_1)\#([x_1]\circ b_3) + [1]\#([x_1]\circ b_1)\#([x_1]\circ b_1\circ b_1)^{\#2}$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

□

Computation 2.121.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1\circ b_2)) = [1]\#([x_2]\circ b_2\circ b_3) + [1]\#([x_3]\circ b_1\circ b_1\circ b_3)$$

Proof sketch.

$$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_2 = [x_2]\circ b_2\circ b_3 + [x_3]\circ b_1\circ b_1\circ b_3 + ([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_2)$$

□

Computation 2.122.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_3]\circ b_1\circ b_1\circ b_1)) = [1]\#([x_2]\circ b_1\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1) + [1]\#([x_3]\circ b_1\circ b_1\circ b_3)$$

Proof sketch.

$$[x_2x_3]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1 = [x_3]\circ b_1\circ b_1\circ b_3$$

□

Computation 2.123.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1) + [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)^{\#2}$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

□

Computation 2.124.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_3)$$

Proof sketch.

$$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_1 = ([x_2]\circ b_1\circ b_1)^{\#2} + ([x_2]\circ b_1\circ b_3)$$

□

Computation 2.125.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#3}) = [1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1)$$

Proof sketch. Only Lemma 1.24 and Lemma 2.9 are needed. □

Computation 2.126.

$$\begin{aligned}([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_4)) = {} & [1]\#([x_2]\circ b_1\circ b_1)\#([x_1]\circ b_4)\\ & + [1]\#([x_2]\circ b_1\circ b_2)^{\#2}\\ & + [1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1x_2]\circ b_1\circ b_1\circ b_2 &= ([x_2]\circ b_1\circ b_1)^{\#2}\\ [x_1x_2]\circ b_1\circ b_1\circ b_4 &= ([x_2]\circ b_1\circ b_2)^{\#2}\end{aligned}$$

□

Computation 2.127.

$$\begin{aligned}([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_3)) = {} & [1]\#([x_2]\circ b_3\circ b_3)\\ & + [1]\#([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_1\circ b_3)\\ & + [1]\#([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_2\circ b_2)\\ & + [1]\#([x_1]\circ b_1\circ b_2)^{\#2}\\ & + [1]\#([x_2]\circ b_1\circ b_1)^{\#3}\end{aligned}$$

Proof sketch.

$$[x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_3 = [x_2] \circ b_3 \circ b_3 + ([x_2] \circ b_1 \circ b_1) \# ([x_2] \circ b_2 \circ b_2) + ([x_2] \circ b_1 \circ b_2)^{\#2} + ([x_2] \circ b_1 \circ b_1)^{\#3}$$

□

Computation 2.128.

$$\begin{aligned}([1] \# ([x_2] \circ b_1 \circ b_1)) \circ ([1] \# ([x_2] \circ b_2 \circ b_2)) = {} & [1] \# ([x_2] \circ b_3 \circ b_3) \\ & + [1] \# ([x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_3) \\ & + [1] \# ([x_2] \circ b_1 \circ b_1) \# ([x_2] \circ b_1 \circ b_3)\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_1 &= [x_2] \circ b_1 \circ b_3 + ([x_2] \circ b_1 \circ b_1)^{\#2} \\ [x_2^2] \circ b_1 \circ b_1 \circ b_2 \circ b_2 &= [x_2] \circ b_3 \circ b_3 + [x_4] \circ b_1 \circ b_1 \circ b_1 \circ b_3 \\ &\quad + ([x_2] \circ b_1 \circ b_1) \# ([x_2] \circ b_2 \circ b_2) + ([x_2] \circ b_1 \circ b_1)^{\#3}\end{aligned}$$

□

Computation 2.129.

$$\begin{aligned}([1] \# ([x_2] \circ b_1 \circ b_1)) \circ ([1] \# ([x_3] \circ b_1 \circ b_1 \circ b_2)) = {} & \\ & [1] \# ([x_3] \circ b_1 \circ b_2 \circ b_3) \\ & + [1] \# ([x_2] \circ b_1 \circ b_1) \# ([x_3] \circ b_1 \circ b_1 \circ b_2) \\ & + [1] \# ([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}\end{aligned}$$

Proof sketch.

$$[x_2 x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 = [x_3] \circ b_1 \circ b_2 \circ b_3 + ([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}$$

□

Computation 2.130.

$$\begin{aligned}([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1)) = \\ [1]\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_3) \\ + [1]\#([x_2]\circ b_1\circ b_1)\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1))\end{aligned}$$

Proof sketch.

$$[x_2x_4]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1 = [x_4]\circ b_1\circ b_1\circ b_1\circ b_3$$

□

Computation 2.131.

$$\begin{aligned}([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_3)) = \\ [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)\#([x_1]\circ b_3) \\ + [1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

□

Computation 2.132.

$$\begin{aligned}([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_2)) = \\ [1]\#([x_1]\circ b_1)\#([x_2]\circ b_2\circ b_3) \\ + [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_3)\end{aligned}$$

Proof sketch.

$$[x_1^2]\circ b_3\circ b_3 = ([x_1]\circ b_3)^{\#2} + ([x_1]\circ b_1)^{\#6}$$

□

Computation 2.133.

$$\begin{aligned}([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)) = \\ [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_3) \\ + [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)\end{aligned}$$

Proof sketch.

$$[x_2x_3]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1 = [x_3]\circ b_1\circ b_1\circ b_3$$

□

Computation 2.134.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_2)^{\#2}) = [1]\#([x_1]\circ b_2)^{\#2}\#([x_2]\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.135.

$$\begin{aligned}([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_3) \\ + [1]\#([x_2]\circ b_1\circ b_1)^{\#3}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_1 = [x_2]\circ b_1\circ b_3 + ([x_2]\circ b_1\circ b_1)^{\#2} \\ [x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

□

Computation 2.136.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_2]\circ b_1\circ b_1)^{\#2}) = [1]\#([x_2]\circ b_1\circ b_1)^{\#3}$$

Proof sketch. No additional relations are needed. □

Computation 2.137.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)) = \\ [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1) \\ + [1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1)^{\#2}$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

□

Computation 2.138.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1)) = \\ [1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_3)$$

Proof sketch.

$$[x_2^2]\circ b_1\circ b_1\circ b_1\circ b_1 = [x_2]\circ b_1\circ b_3 + ([x_2]\circ b_1\circ b_1)^{\#2}$$

□

Computation 2.139.

$$([1]\#([x_2]\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#4}) = [1]\#([x_1]\circ b_1)^{\#4}\#([x_2]\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.140. $([1]\#([x_1]\circ b_1)^{\#2})^{\circ 2} = [1]\#([x_1]\circ b_1)^{\#4}$

Proof. This follows from Lemma 1.24 and the relation

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

from Table 2.2. □

Computation 2.141. $([1]\#([x_1]\circ b_1)^{\#2})\circ([1]\#([x_1]\circ b_3)) = [1]\#([x_1]\circ b_1)^{\#5}$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$
$$[x_1^2] \circ b_1 \circ b_2 = [x_1] \circ b_3$$

□

Computation 2.142.

$$([1]\#([x_1] \circ b_1)^{\#2}) \circ ([1]\#([x_2] \circ b_1 \circ b_2)) = [1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_2)$$

Proof sketch. Only Lemma 1.24 is needed. □

Computation 2.143.

$$([1]\#([x_1] \circ b_1)^{\#2}) \circ ([1]\#([x_3] \circ b_1 \circ b_1 \circ b_1)) = [1]\#([x_1] \circ b_1)^{\#2}\#([x_3] \circ b_1 \circ b_1 \circ b_1)$$

Proof sketch. Only Lemma 1.24 is needed. □

Computation 2.144.

$$([1]\#([x_1] \circ b_1)^{\#2}) \circ ([1]\#([x_1] \circ b_1)\#([x_1] \circ b_2)) = [1]\#([x_1] \circ b_1)^{\#3}\#([x_1] \circ b_2) + [1]\#([x_1] \circ b_1)^{\#5}$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.145.

$$([1]\#([x_1] \circ b_1)^{\#2}) \circ ([1]\#([x_1] \circ b_1)\#([x_2] \circ b_1 \circ b_1)) = [1]\#([x_1] \circ b_1)^{\#3}\#([x_2] \circ b_1 \circ b_1)$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.146. $([1]\#([x_1] \circ b_1)^{\#2}) \circ ([1]\#([x_1] \circ b_1)^{\#3}) = [1]\#([x_1] \circ b_1)^{\#5}$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.147.

$$\begin{aligned}([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_1] \circ b_4)) = [1]\#([x_1] \circ b_3)^{\#2} \\ + [1]\#([x_1] \circ b_1)^{\#2}\#([x_1] \circ b_4) \\ + [1]\#([x_1] \circ b_1)^{\#2}\#([x_1] \circ b_2)^{\#2} \\ + [1]\#([x_1] \circ b_1)^{\#4}\#([x_1] \circ b_2) \\ + [1]\#([x_1] \circ b_1)^{\#6}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1^2] \circ b_1 \circ b_1 &= ([x_1] \circ b_1)^{\#2} \\ [x_1^2] \circ b_1 \circ b_2 &= [x_1] \circ b_3 + ([x_1] \circ b_1)\#([x_1] \circ b_2) + ([x_1] \circ b_1)^{\#3}\end{aligned}$$

□

Computation 2.148.

$$([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_2] \circ b_1 \circ b_3)) = [1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_3)$$

Proof sketch. No additional relations are needed. □

Computation 2.149.

$$([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_2] \circ b_2 \circ b_2)) = [1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_2 \circ b_2)$$

Proof sketch. No additional relations are needed. □

Computation 2.150.

$$([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_3] \circ b_1 \circ b_1 \circ b_2)) = [1]\#([x_1] \circ b_1)^{\#2}\#([x_3] \circ b_1 \circ b_1 \circ b_2)$$

Proof sketch. No additional relations are needed. □

Computation 2.151.

$$([1]\#([x_1]\circ b_1)^{\#2}))\circ([1]\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)^{\#2}\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.152.

$$\begin{aligned}([1]\#([x_1]\circ b_1)^{\#2}))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_3)) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_3)\\ + [1]\#([x_1]\circ b_1)^{\#6}\end{aligned}$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.153.

$$([1]\#([x_1]\circ b_1)^{\#2}))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_2)) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_2]\circ b_1\circ b_2)$$

Proof sketch. No additional relations are needed. □

Computation 2.154.

$$([1]\#([x_1]\circ b_1)^{\#2}))\circ([1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.155.

$$([1]\#([x_1]\circ b_1)^{\#2}))\circ([1]\#([x_1]\circ b_2)^{\#2}) = [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2}$$

Proof sketch. No additional relations are needed. □

Computation 2.156.

$$\begin{aligned}([1]\#([x_1]\circ b_1)^{\#2}))\circ([1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)) =\\ [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)\\ + [1]\#([x_1]\circ b_1)^{\#4}\#([x_2]\circ b_1\circ b_1)\end{aligned}$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.157.

$$([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_2] \circ b_1 \circ b_1)^{\#2}) = [1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_1)^{\#2}$$

Proof sketch. No additional relations are needed. □

Computation 2.158.

$$([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_1] \circ b_1)^{\#2}\#([x_1] \circ b_2)) = [1]\#([x_1] \circ b_1)^{\#4}\#([x_1] \circ b_2) + [1]\#([x_1] \circ b_1)^{\#6}$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.159.

$$([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_1] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_1)) = [1]\#([x_1] \circ b_1)^{\#4}\#([x_2] \circ b_1 \circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.160.

$$([1]\#([x_1] \circ b_1)^{\#2})) \circ ([1]\#([x_1] \circ b_1)^{\#4}) = [1]\#([x_1] \circ b_1)^{\#6}$$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.161. $([1]\#([x_1] \circ b_3))^{\circ 2} = 0$

Proof. This follows immediately from Computations 2.44 and 2.45. □

Computation 2.162.

$$\begin{aligned}([1]\#([x_1]\circ b_3))\circ([1]\#([x_2]\circ b_1\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_3)\\ + [1]\#([x_1]\circ b_3)\#([x_2]\circ b_1\circ b_2)\\ + [1]\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)^{\#2}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1x_2]\circ b_1\circ b_1\circ b_2 &= ([x_2]\circ b_1\circ b_1)^{\#2}\\ [x_1x_2]\circ b_1\circ b_2\circ b_2 &= [x_3]\circ b_1\circ b_1\circ b_3\\ [x_1x_2]\circ b_1\circ b_2\circ b_3 &= 0\end{aligned}$$

□

Computation 2.163.

$$([1]\#([x_1]\circ b_3))\circ([1]\#([x_3]\circ b_1\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_3)\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.164.

$$\begin{aligned}([1]\#([x_1]\circ b_3))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)) = [1]\#([x_1]\circ b_3)^{\#2}\\ + [1]\#([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_3)\\ + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2}\\ + [1]\#([x_1]\circ b_1)^{\#4}\#([x_1]\circ b_2)\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2}\\ [x_1^2]\circ b_1\circ b_2 &= [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}\\ [x_1^2]\circ b_1\circ b_3 &= ([x_1]\circ b_1)^{\#4}\\ [x_1^2]\circ b_2\circ b_3 &= ([x_1]\circ b_2)\#([x_1]\circ b_3) + ([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_3) + ([x_1]\circ b_1)\#([x_1]\circ b_2)^{\#2}\\ &\quad + ([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_2)\end{aligned}$$

□

Computation 2.165. $([1]\#([x_1]\circ b_3))\circ([1]\#([x_1]\circ b_1)\circ([x_2]\circ b_1\circ b_1))=0$

Proof sketch.

$$\begin{aligned}
[x_1^2]\circ b_1\circ b_1 &= ([x_1]\circ b_1)^{\#2}\\
[x_1^2]\circ b_1\circ b_2 &= [x_1]\circ b_3+([x_1]\circ b_1)\#([x_1]\circ b_2)+([x_1]\circ b_1)^{\#3}\\
[x_1^2]\circ b_1\circ b_3 &= ([x_1]\circ b_1)^{\#4}
\end{aligned}$$

□

Computation 2.166. $([1]\#([x_1]\circ b_3))\circ([1]\#([x_1]\circ b_1)^{\#3})=0$

Proof sketch. This follows from Computations 2.45 and 2.53. □

Computation 2.167.

$$\begin{aligned}
([1]\#([x_2]\circ b_1\circ b_2))^{\circ 2} = {}& [1]\#([x_2]\circ b_3\circ b_3)\\
&+[1]\#([x_4]\circ b_1\circ b_1\circ b_1\circ b_3)\\
&+[1]\#([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_2\circ b_2)\\
&+[1]\#([x_2]\circ b_1\circ b_2)^{\#2}\\
&+[1]\#([x_2]\circ b_1\circ b_1)^{\#3}
\end{aligned}$$

Proof sketch.

$$\begin{aligned}
[x_2^2]\circ b_1\circ b_1\circ b_2\circ b_2 = {}& [x_2]\circ b_3\circ b_3+[x_4]\circ b_1\circ b_1\circ b_1\circ b_3\\
&+([x_2]\circ b_1\circ b_1)\#([x_2]\circ b_2\circ b_2)+([x_2]\circ b_1\circ b_1)^{\#3}
\end{aligned}$$

□

Computation 2.168.

$$\begin{aligned}
([1]\#([x_2]\circ b_1\circ b_2))\circ([1]\#([x_3]\circ b_1\circ b_1\circ b_1)) = {}&\\
&[1]\#([x_3]\circ b_1\circ b_2\circ b_3)\\
&\quad+[1]\#([x_2]\circ b_1\circ b_2)\#([x_3]\circ b_1\circ b_1\circ b_1)\\
&\quad+[1]\#([x_3]\circ b_1\circ b_1\circ b_1)^{\#2}
\end{aligned}$$

Proof sketch.

$$[x_2x_3] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 = [x_3] \circ b_1 \circ b_2 \circ b_3 + ([x_3] \circ b_1 \circ b_1 \circ b_1)^{\#2}$$

□

Computation 2.169.

$$\begin{aligned}([1]\#([x_2] \circ b_1 \circ b_2)) \circ ([1]\#([x_1] \circ b_1)\#([x_1] \circ b_2)) = \\ [1]\#([x_1] \circ b_1)\#([x_3] \circ b_1 \circ b_1 \circ b_3) \\ + [1]\#([x_1] \circ b_1)\#([x_1] \circ b_2)\#([x_2] \circ b_1 \circ b_2) \\ + [1]\#([x_2] \circ b_2)\#([x_2] \circ b_1 \circ b_1)^{\#2} \\ + [1]\#([x_2] \circ b_1)^{\#2}\#([x_2] \circ b_1 \circ b_1)^{\#2}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1x_2] \circ b_1 \circ b_1 \circ b_2 &= ([x_2] \circ b_1 \circ b_1)^{\#2} \\ [x_1x_2] \circ b_1 \circ b_2 \circ b_2 &= [x_3] \circ b_1 \circ b_1 \circ b_3\end{aligned}$$

□

Computation 2.170.

$$\begin{aligned}([1]\#([x_2] \circ b_1 \circ b_2)) \circ ([1]\#([x_1] \circ b_1)\#([x_2] \circ b_1 \circ b_1)) = \\ [1]\#([x_1] \circ b_1)\#([x_2] \circ b_2 \circ b_3) \\ + [1]\#([x_1] \circ b_1)\#([x_3] \circ b_1 \circ b_1 \circ b_3) \\ + [1]\#([x_2] \circ b_1 \circ b_1)^{\#3}\end{aligned}$$

Proof sketch.

$$\begin{aligned}[x_1x_2] \circ b_1 \circ b_1 \circ b_2 &= ([x_2] \circ b_1 \circ b_1)^{\#2} \\ [x_2^2] \circ b_1 \circ b_1 \circ b_1 \circ b_2 &= [x_2] \circ b_2 \circ b_3 + [x_3] \circ b_1 \circ b_1 \circ b_3 + ([x_2] \circ b_1 \circ b_1)\#([x_2] \circ b_1 \circ b_2)\end{aligned}$$

□

Computation 2.171.

$$([1]\#([x_2]\circ b_1\circ b_2))\circ([1]\#([x_1]\circ b_1)^{\#3}) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_2]\circ b_1\circ b_2) + [1]\#([x_1]\circ b_1)^{\#2}\#([x_2]\circ b_1\circ b_1)^{\#2}$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_1\circ b_2 = ([x_2]\circ b_1\circ b_1)^{\#2}$$

□

Computation 2.172.

$$([1]\#([x_3]\circ b_1\circ b_1\circ b_1))^{\circ 2} = 0$$

Proof sketch.

$$[x_1x_2]\circ b_1\circ b_1\circ b_2\circ b_2 = ([x_3]\circ b_1\circ b_1\circ b_1)^{\#3}$$
$$[x_1x_2]\circ b_1\circ b_1\circ b_2\circ b_2 = [x_3^2]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1$$

□

Computation 2.173.

$$([1]\#([x_3]\circ b_1\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)) = [1]\#([x_1]\circ b_1)\#([x_1]\circ b_2)\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.174.

$$([1]\#([x_3]\circ b_1\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)) = [1]\#([x_1]\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_3) + [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof sketch.

$$[x_2x_3]\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1 = [x_3]\circ b_1\circ b_1\circ b_3$$

□

Computation 2.175.

$$([1]\#([x_3]\circ b_1\circ b_1\circ b_1))\circ([1]\#([x_1]\circ b_1)^{\#3}) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_3]\circ b_1\circ b_1\circ b_1)$$

Proof sketch. No additional relations are needed. □

Computation 2.176.

$$\begin{aligned}([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2))^{\circ 2} = [1]\#([x_1]\circ b_3)^{\#2} \\ + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)^{\#2} \\ + [1]\#([x_1]\circ b_1)^{\#6}\end{aligned}$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.177.

$$\begin{aligned}([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)) = \\ [1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1)\#([x_1]\circ b_3) \\ + [1]\#([x_1]\circ b_1)^{\#2}\#([x_1]\circ b_2)\#([x_2]\circ b_1\circ b_1)\end{aligned}$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_2 = [x_1]\circ b_3 + ([x_1]\circ b_1)\#([x_1]\circ b_2) + ([x_1]\circ b_1)^{\#3}$$

□

Computation 2.178.

$$\begin{aligned}([1]\#([x_1]\circ b_1)\#([x_1]\circ b_2))\circ([1]\#([x_1]\circ b_1)^{\#3}) = [1]\#([x_1]\circ b_1)^{\#3}\#([x_1]\circ b_3) \\ + [1]\#([x_1]\circ b_1)^{\#4}\#([x_1]\circ b_2)\end{aligned}$$

Proof sketch.

$$[x_1^2]\circ b_1\circ b_1 = ([x_1]\circ b_1)^{\#2}$$

□

Computation 2.179. $([1]\#([x_1]\circ b_1)\#([x_2]\circ b_1\circ b_1))^{\circ 2} = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.180. $([1]\#([x_1] \circ b_1)\#([x_2] \circ b_1 \circ b_1)) \circ ([1]\#([x_1] \circ b_1)^{\#3}) = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

Computation 2.181. $([1]\#([x_1] \circ b_1)^{\#3})^{\circ 2} = 0$

Proof sketch.

$$[x_1^2] \circ b_1 \circ b_1 = ([x_1] \circ b_1)^{\#2}$$

□

2.4 Some general conclusions

Based on the computations in $H_*(SL_1MU)$ from Section 2.3, we can begin to draw some conclusions about the structure of this ring. For instance, we see that several elements have a $\circ$-product of 0, and in fact some square to 0 under $\circ$, which at minimum tells us the following.

Theorem 2.182. *$H_*(SL_1MU)$ is not a domain.*

In particular, I single out the following consequence, since the negation would otherwise be an especially desirable result.

Corollary 2.183. *$H_*(SL_1MU)$ is not a polynomial algebra over $\mathbb{F}_2$.*

References

[1] J. F. Adams. *Stable Homotopy and Generalized Homology*. Chicago Lectures in Mathematics. The University of Chicago Press, 1974.

[2] Matthew Ando. "Isogenies of formal group laws and power operations in the cohomology theories E_n". In: *Duke Mathematical Journal* 79.2 (1995), pp. 423–485. DOI: 10.1215/S0012-7094-95-07911-3. URL: https://doi.org/10.1215/S0012-7094-95-07911-3.

[3] Matthew Ando et al. "Units of ring spectra, orientations, and Thom spectra via rigid infinite loop space theory". In: *Journal of Topology* 7.4 (July 2014), pp. 1077–1117. ISSN: 1753-8416. DOI: 10.1112/jtopol/jtu009. eprint: https://academic.oup.com/jtopol/article-pdf/7/4/1077/3532714/jtu009.pdf. URL: https://doi.org/10.1112/jtopol/jtu009.

[4] Eldon Dyer and R. K. Lashof. "Homology of Iterated Loop Spaces". In: *American Journal of Mathematics* 84.1 (1962), pp. 35–88.

[5] Michiel Hazewinkel. *Formal Groups and Applications*. Academic Press, 1978.

[6] Niles Johnson and Justin Noel. "For complex orientations preserving power operations, p-typicality is atypical". In: *Topology and its Applications* 157.14 (2010), pp. 2271–2288. ISSN: 0166-8641. DOI: https://doi.org/10.1016/j.topol.2010.06.007. URL: https://www.sciencedirect.com/science/article/pii/S016686411000177X.

[7] Frederick R. Cohen, Thomas J. Lada and J. Peter May. *The Homology of Iterated Loop Spaces*. Lecture Notes in Mathematics. Springer-Verlag Berlin Heidelberg, 1976.

[8] Tyler Lawson. "Secondary power operations and the Brown–Peterson spectrum at the prime 2". In: *Annals of Mathematics* 188.2 (2018), pp. 513–576.

[9] Michel Lazard. "Sur les groupes de Lie formels à un paramètre". In: *Bulletin de la Société Mathématique de France* 83 (1955), pp. 251–274.

[10] J P May. "What are E_∞ ring spaces good for?" In: *Geometry & Topology Monographs* 16 (2009), pp. 331–365.

[11] J.P. May and J. Sigurdsson. *Parametrized Homotopy Theory*. Mathematical surveys and monographs Bd. 10. American Mathematical Society, 2006. ISBN: 9780821839225. URL: https://books.google.de/books?id=9x7bUzDiojEC.

[12] R. R. Bruner, J. P. May, J. E. McClure and M. Steinberger. *H_∞ Ring Spectra and Their Applications*. 1st ed. Lecture Notes in Mathematics. Springer-Verlag Berlin Heidelberg, 1986.

[13] Haynes Miller. "A spectral sequence for the homology of an infinite delooping." In: *Pacific Journal of Mathematics* 79.1 (1978), pp. 139–155. DOI: pjm/1102805992. URL: https://doi.org/.

[14] Daniel Quillen. "Elementary proofs of some results of cobordism theory using Steenrod operations". In: *Advances in Mathematics* 7.1 (1971), pp. 29–56. ISSN: 0001-8708. DOI: https://doi.org/10.1016/0001-8708(71)90041-7. URL: https://www.sciencedirect.com/science/article/pii/0001870871900417.

[15] Daniel Quillen. "On the formal group laws of unoriented and complex cobordism theory". In: *Bulletin of the American Mathematical Society* 75.6 (1969), pp. 1293–1298.

[16] Douglas C. Ravenel. *Complex Cobordism and Stable Homotopy Groups of Spheres*. 2nd ed. Vol. 347. AMS Chelsea Publishing. American Mathematical Society, 2004.

[17] Douglas C. Ravenel and W. Stephen Wilson. "The Hopf ring for complex cobordism". In: *Journal of Pure and Applied Algebra* 9.2–3 (1977), pp. 241–280.

[18] Charles Rezk. "Isogenies, power operations, and homotopy theory". In: *Proceedings of the ICM, Seoul 2014*. Vol. 2. 2014.

[19] Paul R. Turner. "Dyer-Lashof operations in the Hopf ring for complex cobordism". In: *Mathematical Proceedings of the Cambridge Philosophical Society* 114.3 (1993), pp. 453–460.

Appendix A

The Ravenel–Wilson relations

In this appendix, I explicitly present the low-degree relations of (1.2):

$$b(s +_f t) = b(s) +_{[F]} b(t)$$

These are a basic building block of the multiplication table for $H_*(SL_1MU)$ since, in order to compute said table, we need to find generators and relations of $H_*(\underline{MU}'_0)$, and those relations all follow from (1.2) by Theorem 1.14.

While computing these relations explicitly is an algebraically straightforward task, as the degree increases, the number and length of the relations grows extremely quickly, as one can clearly see in the tables starting with Table A.6.

Recall that $b(s) = \sum_{k=0}^{\infty} b_k s^k$, a formal power series in s with coefficients given by the generators b_k for the Hopf ring $H_*(\underline{MU}_{2\star})$. Note also that because $H_*()$ is represented by the Eilenberg–MacLane spectrum $H\mathbb{F}_2$, the formal group law F may be taken to be the additive law $s +_f t = s + t$. Therefore, the left hand side of the general relation is

$$b(s+t) = \sum_{k=0}^{\infty} b_k (s+t)^k = \sum_{k=0}^{\infty} b_k \left(\sum_{i=0}^{k} \binom{k}{i} s^i t^{k-i} \right)$$

$$= b_0 + b_1(s+t) + b_2(s^2 + 2st + t^2) + b_3(s^3 + 3s^2t + 3st^2 + t^3) + \cdots$$

On the other hand, the right hand side is

$$b(s) +_{[F]} b(t) = \#_{i,j \geq 0} [a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j} \tag{A.1}$$

which is much more cumbersome to expand. I will show the general process for doing so in the following section.

A.1 Computation of $b(s)^{\circ n}$

To begin with, we must know how to compute a $\circ$-power of the series $b(s) = \sum_{i=0}^{\infty} b_i s^i$. Though I will still work over $\mathbb{F}_2$ in general, in some of the results of this section I work over $\mathbb{Z}$ for computational clarity.

First, note that we have the trivial results $b(s)^{\circ 0} = [1]$ (the empty $\circ$-product) and $b(s)^{\circ 1} = b(s)$. Now, fix some $n \geq 2$. For a formal power series $\sum_{i=0}^{\infty} a_i x^i$ in a variable x over a ring R, recall the standard formula for its n^{th} multiplicative power:

$$\left(\sum_{i=0}^{\infty} a_i x^i\right)^n = \sum_{i=0}^{\infty} \left(\sum_{\substack{0 \leq u_1, \ldots, u_n \leq i \\ u_1 + \cdots + u_n = i}} a_{u_1} a_{u_2} \cdots a_{u_n} \right) x^i$$

Because $\circ : H_*(\underline{MU}_{2\star}; R) \otimes H_*(\underline{MU}_{2\star}; R) \to H_*(\underline{MU}_{2\star}; R)$ is a coalgebra map, $\circ$ distributes over $+$. Therefore, it is immediate that the $\circ$-powers of a formal power series over $H_*(\underline{MU}_{2\star}; R)$ follow the analogous formula. That is,

$$b(s)^{\circ n} = \left(\sum_{i=0}^{\infty} b_i s^i\right)^{\circ n} = \sum_{i=0}^{\infty} \left(\sum_{\substack{0 \leq u_1, \ldots, u_n \leq i \\ u_1 + \cdots + u_n = i}} b_{u_1} \circ b_{u_2} \circ \cdots \circ b_{u_n} \right) s^i \tag{A.2}$$

By Proposition 1.6, if $u_k = 0$ for any k, then the $+$-summand $b_{u_1} \circ b_{u_2} \circ \cdots \circ b_{u_n}$ is 0, unless $u_j = 0$ for all j, in which case the $+$-summand is $[0]$). Note that because $u_1 + \cdots + u_n = i$, we have $u_1 = \cdots = u_n = 0$ if and only if $i = 0$. Therefore, for $i \geq 1$, we can take the sum in the coefficient over $1 \leq u_1, \ldots, u_n \leq i$:

$$b(s)^{\circ n} = [0] + \sum_{i=1}^{\infty} \left(\sum_{\substack{1 \leq u_1, \ldots, u_n \leq i \\ u_1 + \cdots + u_n = i}} b_{u_1} \circ b_{u_2} \circ \cdots \circ b_{u_n} \right) s^i \tag{A.3}$$

If we wish, we can also rewrite the coefficients on the right hand side using the fact that the generators b_i are in even degrees and therefore commute under the $\circ$-product. This means that the sum in parentheses on the right hand side will very often have repeated

summands, and so we may rewrite (A.2) as

$$b(s)^{\circ n} = [0] + \sum_{i=1}^{\infty} \left(\sum_{\substack{0 \le d_1, d_2, d_3 \ldots \\ \sum d_k = n \\ \sum k d_k = i}} \binom{n}{d_1\ d_2\ d_3\ \cdots} b_1^{\circ d_1} \circ b_2^{\circ d_2} \circ b_3^{\circ d_3} \circ \cdots \right) s^i \tag{A.4}$$

where

$$\binom{n}{d_1\ d_2\ d_3\ \cdots} = \frac{n!}{d_1!\, d_2!\, d_3!\, \cdots} \tag{A.5}$$

Note that the product in the denominator and the $\circ$-product in (A.4) are both finite since $\sum d_k = n$ and $\sum k d_k = i$. This formula is still somewhat cumbersome, but we will see that there are further possible simplifications.

Proposition A.1. *For any $n \ge 1$, $b(s)^{\circ n} = [0] + b_1^{\circ n} s^n + \mathcal{O}(s^{n+1})$, where $\mathcal{O}$ refers to the minimal power in the remaining terms.*

This follows easily from (A.2) or (A.4), but I prove it here by induction for clarity.

Proof. When $n = 1$, $b(s)^{\circ n} = b(s) = b_0 + b_1 s + \mathcal{O}(s^2) = [0] + b_1 s + \mathcal{O}(s^2)$. If the result holds for a given n, then

$$\begin{aligned} b(s)^{\circ(n+1)} &= b(s) \circ \big([0] + b_1^{\circ n} s^n + \mathcal{O}(s^{n+1})\big) \\ &= \big(b_0 \circ \big([0] + b_1^{\circ n} s^n + \mathcal{O}(s^{n+1})\big)\big) + \big(b_1 s \circ \big([0] + b_1^{\circ n} s^n + \mathcal{O}(s^{n+1})\big)\big) \\ &\qquad + \big(b_2 s^2 \circ \big([0] + b_1^{\circ n} s^n + \mathcal{O}(s^{n+1})\big)\big) + \cdots \\ &= ([0] + 0 + 0 + \cdots) + \Big(0 + b_1^{\circ(n+1)} s^{n+1} + \mathcal{O}(s^{n+2})\Big) \\ &\qquad + \big(0 + b_2 \circ b_1^{\circ n} s^{n+2} + \mathcal{O}(s^{n+3})\big) + \cdots \\ &= [0] + b_1^{\circ(n+1)} s^{n+1} + \mathcal{O}(s^{n+2}) \end{aligned}$$

□

Combining Proposition A.1 with (A.4) yields the following result.

Theorem A.2. *For any $n \geq 1$,*

$$b(s)^{\circ n} = [0] + b_1^{\circ n} s^n + \sum_{i=n+1}^{\infty} \left(\sum_{\substack{0 \leq d_1, d_2, d_3, \ldots \\ \sum d_k = n \\ \sum k d_k = i}} \binom{n}{d_1\ d_2\ d_3\ \ldots} b_1^{\circ d_1} \circ b_2^{\circ d_2} \circ b_3^{\circ b_3} \circ \cdots \right) s^i$$

over $\mathbb{Z}$.

For clarity, I now present the first several $\circ$-powers of $b(s)$ in low degrees.

Example A.3. Over $\mathbb{Z}$,

$$\begin{aligned}
b(s)^{\circ 2} = [0] &+ (b_1 \circ b_1)s^2 + (2b_1 \circ b_2)s^3 + (2b_1 \circ b_3 + b_2 \circ b_2)s^4 + (2b_1 \circ b_4 + 2b_2 \circ b_3)s^5 \\
&+ (2b_1 \circ b_5 + 2b_2 \circ b_4 + b_3 \circ b_3)s^6 + (2b_1 \circ b_6 + 2b_2 \circ b_5 + 2b_3 \circ b_4)s^7 \\
&+ (2b_1 \circ b_7 + 2b_2 \circ b_6 + 2b_3 \circ b_5 + b_4 \circ b_4)s^8 \\
&+ (2b_1 \circ b_8 + 2b_2 \circ b_7 + 2b_3 \circ b_6 + 2b_4 \circ b_5)s^9 \\
&+ (2b_1 \circ b_9 + 2b_2 \circ b_8 + 2b_3 \circ b_7 + 2b_4 \circ b_6 + b_5 \circ b_5)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 3} = [0] &+ (b_1 \circ b_1 \circ b_1)s^3 + (3b_1 \circ b_1 \circ b_2)s^4 + (3b_1 \circ b_1 \circ b_3 + 3b_1 \circ b_2 \circ b_2)s^5 \\
&+ (3b_1 \circ b_1 \circ b_4 + 6b_1 \circ b_2 \circ b_3 + b_2 \circ b_2 \circ b_2)s^6 \\
&+ (3b_1 \circ b_1 \circ b_5 + 6b_1 \circ b_2 \circ b_4 + 3b_1 \circ b_3 \circ b_3 + 3b_2 \circ b_2 \circ b_3)s^7 \\
&+ (3b_1 \circ b_1 \circ b_6 + 6b_1 \circ b_2 \circ b_5 + 6b_1 \circ b_3 \circ b_4 + 3b_2 \circ b_2 \circ b_4 + 3b_2 \circ b_3 \circ b_3)s^8 \\
&+ (3b_1 \circ b_1 \circ b_7 + 6b_1 \circ b_2 \circ b_6 + 6b_1 \circ b_3 \circ b_5 + 3b_2 \circ b_2 \circ b_5 + 3b_1 \circ b_4 \circ b_4 \\
&\qquad + 6b_2 \circ b_3 \circ b_4 + b_3 \circ b_3 \circ b_3)s^9 \\
&+ (3b_1 \circ b_1 \circ b_8 + 6b_1 \circ b_2 \circ b_7 + 6b_1 \circ b_3 \circ b_6 + 6b_1 \circ b_4 \circ b_5 + 3b_2 \circ b_2 \circ b_6 \\
&\qquad + 6b_2 \circ b_3 \circ b_5 + 3b_2 \circ b_4 \circ b_4 + 3b_3 \circ b_3 \circ b_4)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 4} = [0] &+ (b_1 \circ b_1 \circ b_1 \circ b_1)s^4 + (4b_1 \circ b_1 \circ b_1 \circ b_2)s^5 \\
&+ (4b_1 \circ b_1 \circ b_1 \circ b_3 + 6b_1 \circ b_1 \circ b_2 \circ b_2)s^6 \\
&+ (4b_1 \circ b_1 \circ b_1 \circ b_4 + 12b_1 \circ b_1 \circ b_2 \circ b_3 + 4b_1 \circ b_2 \circ b_2 \circ b_2)s^7 \\
&+ (4b_1 \circ b_1 \circ b_1 \circ b_5 + 12b_1 \circ b_1 \circ b_2 \circ b_4 + 6b_1 \circ b_1 \circ b_3 \circ b_3 + 12b_1 \circ b_2 \circ b_2 \circ b_3 \\
&\qquad + b_2 \circ b_2 \circ b_2 \circ b_2)s^8 \\
&+ (4b_1 \circ b_1 \circ b_1 \circ b_6 + 12b_1 \circ b_1 \circ b_2 \circ b_5 + 12b_1 \circ b_1 \circ b_3 \circ b_4 + 12b_1 \circ b_2 \circ b_2 \circ b_4 \\
&\qquad + 12b_1 \circ b_2 \circ b_3 \circ b_3 + 4b_2 \circ b_2 \circ b_2 \circ b_3)s^9
\end{aligned}$$

$$
\begin{aligned}
&+ (4b_1 \circ b_1 \circ b_1 \circ b_7 + 12b_1 \circ b_1 \circ b_2 \circ b_6 + 12b_1 \circ b_1 \circ b_3 \circ b_5 + 6b_1 \circ b_1 \circ b_4 \circ b_4 \\
&\quad + 12b_1 \circ b_2 \circ b_2 \circ b_5 + 24b_1 \circ b_2 \circ b_3 \circ b_4 + 4b_1 \circ b_3 \circ b_3 \circ b_3 \\
&\quad + 4b_2 \circ b_2 \circ b_2 \circ b_4 + 6b_2 \circ b_2 \circ b_3 \circ b_3)s^{10} \\
&+ \mathcal{O}(s^{11}) \\
b(s)^{\circ 5} = [0] &+ (b_1^{\circ 5})s^5 + (5b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2)s^6 \\
&+ (5b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3 + 10b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2)s^7 \\
&+ (5b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_4 + 20b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_3 + 10b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_2)s^8 \\
&+ (5b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_5 + 20b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_4 + 10b_1 \circ b_1 \circ b_1 \circ b_3 \circ b_3 \\
&\quad + 30b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_3 + 5b_1 \circ b_2 \circ b_2 \circ b_2 \circ b_2)s^9 \\
&+ (5b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_6 + 20b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_5 \\
&\quad + 20b_1 \circ b_1 \circ b_1 \circ b_3 \circ b_4 + 30b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_4 \\
&\quad + 30b_1 \circ b_1 \circ b_2 \circ b_3 \circ b_3 + 20b_1 \circ b_2 \circ b_2 \circ b_2 \circ b_3 + b_2^{\circ 5})s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 6} = [0] &+ (b_1^{\circ 6})s^6 + (6b_1^{\circ 5} \circ b_2)s^7 \\
&+ (6b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3 + 15b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2)s^8 \\
&+ (6b_1^{\circ 5} \circ b_4 + 30b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_3 + 20b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_2)s^9 \\
&+ (6b_1^{\circ 5} \circ b_5 + 30b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_4 + 15b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3 \circ b_3 \\
&\quad + 60b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_3 + 15b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_2 \circ b_2)s^{10} \\
&+ \mathcal{O}(s^{11}) \\
b(s)^{\circ 7} = [0] &+ (b_1^{\circ 7})s^7 + (7b_1^{\circ 6} \circ b_2)s^8 + (7b_1^{\circ 6} \circ b_3 + 21b_1^{\circ 5} \circ b_2 \circ b_2)s^9 \\
&+ (7b_1^{\circ 6} \circ b_4 + 42b_1^{\circ 5} \circ b_2 \circ b_3 + 35b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_2)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 8} = [0] &+ (b_1^{\circ 8})s^8 + (8b_1^{\circ 7} \circ b_2)s^9 + (8b_1^{\circ 7} \circ b_3 + 28b_1^{\circ 6} \circ b_2 \circ b_2)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 9} = [0] &+ (b_1^{\circ 9})s^9 + (9b_1^{\circ 8} \circ b_2)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 10} = [0] &+ (b_1^{\circ 10})s^{10} + \mathcal{O}(s^{11})
\end{aligned}
$$

Passing to $\mathbb{F}_2$, the coefficients become substantially more simple.

Example A.4. Over $\mathbb{F}_2$,

$$\begin{aligned}
b(s)^{\circ 2} &= [0] + (b_1 \circ b_1)s^2 + 0s^3 + (b_2 \circ b_2)s^4 + 0s^5 + (b_3 \circ b_3)s^6 + 0s^7 + (b_4 \circ b_4)s^8 + 0s^9 \\
&\quad + (b_5 \circ b_5)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 3} &= [0] + (b_1 \circ b_1 \circ b_1)s^3 + (b_1 \circ b_1 \circ b_2)s^4 + (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2)s^5 \\
&\quad + (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2)s^6 + (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3)s^7 \\
&\quad + (b_1 \circ b_1 \circ b_6 + b_2 \circ b_2 \circ b_4 + b_2 \circ b_3 \circ b_3)s^8 \\
&\quad + (b_1 \circ b_1 \circ b_7 + b_1 \circ b_4 \circ b_4 + b_2 \circ b_2 \circ b_5 + b_3 \circ b_3 \circ b_3)s^9 \\
&\quad + (b_1 \circ b_1 \circ b_8 + b_2 \circ b_2 \circ b_6 + b_2 \circ b_4 \circ b_4 + b_3 \circ b_3 \circ b_4)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 4} &= [0] + (b_1 \circ b_1 \circ b_1 \circ b_1)s^4 + 0s^5 + 0s^6 + 0s^7 + (b_2 \circ b_2 \circ b_2 \circ b_2)s^8 + 0s^9 + 0s^{10} \\
&\quad + \mathcal{O}(s^{11}) \\
b(s)^{\circ 5} &= [0] + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1)s^5 + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2)s^6 \\
&\quad + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3)s^7 + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_4)s^8 \\
&\quad + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_5 + b_1 \circ b_2 \circ b_2 \circ b_2 \circ b_2)s^9 \\
&\quad + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_6 + b_2 \circ b_2 \circ b_2 \circ b_2 \circ b_2)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 6} &= [0] + (b_1^{\circ 6})s^6 + 0s^7 + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2)s^8 + 0s^9 \\
&\quad + (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3 \circ b_3 + b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_2 \circ b_2)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 7} &= [0] + (b_1^{\circ 7})s^7 + (b_1^{\circ 6} \circ b_2)s^8 + (b_1^{\circ 6} \circ b_3 + b_1^{\circ 5} \circ b_2 \circ b_2)s^9 \\
&\quad + (b_1^{\circ 6} \circ b_4 + b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2 \circ b_2)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 8} &= [0] + (b_1^{\circ 8})s^8 + 0s^9 + 0s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 9} &= [0] + (b_1^{\circ 9})s^9 + (b_1^{\circ 8} \circ b_2)s^{10} + \mathcal{O}(s^{11}) \\
b(s)^{\circ 10} &= [0] + (b_1^{\circ 10})s^{10} + \mathcal{O}(s^{11})
\end{aligned}$$

A.2 Computing the #-sum

Now that we have explicit expansions of the $\circ$-powers of $b(s)$, we can begin to expand the #-sum

$$b(s) +_{[F]} b(t) = \#_{i,j \geq 0} [a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$$

Because the #-summands become extremely long as the degree increases, I introduce the following notation for this section.

Definition A.5. In the formal power series $b(s)^{\circ n}$, denote the coefficient of s^k by A^n_k. That

is,

$$b(s)^{\circ n} = \sum_{k=0}^{\infty} A_k^n s^k$$

Using this notation, we first identify the #-summands $[a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$. Since $\circ$ distributes over $+$, we have

$$[a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j} = [a_{i,j}] \circ \sum_{k,l \geq 0} \left(A_k^i \circ A_l^j \right) s^k t^l$$

Before we begin to take #-sums of these terms, let us eliminate as many terms as possible in order to streamline the process. First, if $i = 0$, then because $b(s)^{\circ 0} = [1]$, we have $b(s)^{\circ i} \circ b(t)^{\circ j} = b(t)^{\circ j}$, and similarly, $b(s)^{\circ i} \circ b(t)^{\circ j} = b(s)^{\circ i}$ when $j = 0$. In particular, when $i = j = 0$, the #-summand is simply $[a_{0,0}] \circ [1] \circ [1] = [0]$.

Now, for $i, j \geq 1$, when $0 \leq k < i$, Proposition A.1 and Lemma 1.18 imply that

$$A_k^i \circ A_l^j = \begin{cases} 0 \circ A_l^j & k \neq 0 \\ [0] \circ A_l^j & k = 0 \end{cases} = \begin{cases} 0 & k \neq 0 \text{ or } l \neq 0 \\ [0] & k = l = 0 \end{cases}$$

Similarly, when $0 \leq l < j$,

$$A_k^i \circ A_l^j = \begin{cases} 0 & k \neq 0 \text{ or } l \neq 0 \\ [0] & k = l = 0 \end{cases}$$

Therefore, when $i, j > 0$, $b(s)^{\circ i} \circ b(t)^{\circ j}$ has no monomial terms $s^k t^l$ with $k < i$ or $j < l$, except the constant term $[0]$.

Now we can begin to compute the #-sum in low degrees. This is the content of the tables at the end of this appendix, starting with Table A.6; however, I will compute a few examples explicitly to show the process. First, let us compute the first few #-summands $[a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$ (each through degree 4).

$$\#_{i,j\geq 0}\,[a_{i,j}]\circ b(s)^{\circ i}\circ b(t)^{\circ j} =$$

$$\begin{aligned}
&(i=j=0) && [0]\\
&(i=0,\ j=1) && \#\left([a_{0,1}]\circ\left([0]+A^1_1t+A^1_2t^2+A^1_3t^3+A^1_4t^4+\cdots\right)\right)\\
&(i=1,\ j=0) && \#\left([a_{1,0}]\circ\left([0]+A^1_1s+A^1_2s^2+A^1_3s^3+A^1_4s^4+\cdots\right)\right)\\
&(i=0,\ j=2) && \#\left([a_{2,0}]\circ\left([0]+A^2_2t^2+A^2_3t^3+A^2_4t^4+\cdots\right)\right)\\
&(i=j=1) && \#\big([a_{1,1}]\circ\big([0]+\left(A^1_1\circ A^1_1\right)st+\left(A^1_2\circ A^1_1\right)s^2t+\left(A^1_1\circ A^1_2\right)st^2\\
&&&\qquad+\left(A^1_3\circ A^1_1\right)s^3t+\left(A^1_2\circ A^1_2\right)s^2t^2+\left(A^1_1\circ A^1_3\right)st^3+\cdots\big)\big)\\
&(i=2,\ j=0) && \#\left([a_{0,2}]\circ\left([0]+A^2_2s^2+A^2_3s^3+A^2_4s^4+\cdots\right)\right)\\
&(i=0,\ j=3) && \#\left([a_{0,3}]\circ\left([0]+A^3_3t^3+A^3_4t^4+\cdots\right)\right)\\
&(i=1,\ j=2) && \#\big([a_{1,2}]\circ\big([0]+\left(A^1_1\circ A^2_2\right)st^2+\left(A^1_2\circ A^2_2\right)s^2t^2\\
&&&\qquad+\left(A^2_1\circ A^2_3\right)st^3+\cdots\big)\big)\\
&(i=2,\ j=1) && \#\big([a_{2,1}]\circ\big([0]+\left(A^2_2\circ A^1_1\right)s^2t+\left(A^2_3\circ A^1_1\right)s^3t\\
&&&\qquad+\left(A^2_2\circ A^1_2\right)s^2t^2+\cdots\big)\big)\\
&(i=3,\ j=0) && \#\left([a_{3,0}]\circ\left([0]+A^3_3s^3+A^3_4s^4+\cdots\right)\right)\\
&(i=0,\ j=4) && \#\left([a_{0,4}]\circ\left([0]+A^4_4t^4+\cdots\right)\right)\\
&(i=1,\ j=3) && \#\left([a_{1,3}]\circ\left([0]+\left(A^1_1\circ A^3_3\right)st^3+\cdots\right)\right)\\
&(i=2,\ j=2) && \#\left([a_{2,2}]\circ\left([0]+\left(A^2_2\circ A^2_2\right)s^2t^2+\cdots\right)\right)\\
&(i=3,\ j=1) && \#\left([a_{3,1}]\circ\left([0]+\left(A^3_3\circ A^1_1\right)s^3t+\cdots\right)\right)\\
&(i=4,\ j=0) && \#\left([a_{4,0}]\circ\left([0]+A^4_4s^4+\cdots\right)\right)\\
&(i+j\geq 5) && \#\cdots
\end{aligned}$$

We can clear further terms by examining the $a_{i,j}$ coefficients. Recall in particular that $a_{0,0}=0$, $a_{0,1}=a_{1,0}=1$ and $a_{0,j}=a_{i,0}=0$ for $i,j>1$. Therefore, the $\circ$-factors of $[a_{0,1}]=[1]=[a_{1,0}]$ can be dropped because $[1]$ is the $\circ$-identity. Furthermore, any $\#$-summand with a $[0]$ $\circ$-factor is equal to 0 (since the higher coefficients are in positive homology degrees, meaning their $\circ$-product with $[0]$ is 0). These $\#$-summands can therefore be dropped from the $\#$-sum. Therefore, the $\#$-sum becomes

$$\begin{aligned}
\#_{i,j\geq 0}\,[a_{i,j}]\circ b(s)^{\circ i}\circ b(t)^{\circ j} = {} & \big([0]+A^1_1t+A^1_2t^2+A^1_3t^3+A^1_4t^4+\cdots\big)\\
& \#\,\big([0]+A^1_1s+A^1_2s^2+A^1_3s^3+A^1_4s^4+\cdots\big)\\
& \#\big([a_{1,1}]\circ\big([0]+\big(A^1_1\circ A^1_1\big)\,st+\big(A^1_2\circ A^1_1\big)\,s^2t+\big(A^1_1\circ A^1_2\big)\,st^2\\
& \qquad +\big(A^1_3\circ A^1_1\big)\,s^3t+\big(A^1_2\circ A^1_2\big)\,s^2t^2+\big(A^1_1\circ A^1_3\big)\,st^3\cdots\big)\big)\\
& \#\big([a_{1,2}]\circ\big([0]+\big(A^1_1\circ A^2_2\big)\,st^2+\big(A^1_2\circ A^2_2\big)\,s^2t^2\\
& \qquad +\big(A^1_1\circ A^2_3\big)\,st^3+\cdots\big)\big)\\
& \#\big([a_{2,1}]\circ\big([0]+\big(A^2_2\circ A^1_1\big)\,s^2t+\big(A^2_3\circ A^1_1\big)\,s^3t\\
& \qquad +\big(A^2_2\circ A^1_2\big)\,s^2t^2+\cdots\big)\big)\\
& \#\,\big([a_{1,3}]\circ\big([0]+\big(A^1_1\circ A^3_3\big)\,st^3+\cdots\big)\big)\\
& \#\,\big([a_{2,2}]\circ\big([0]+\big(A^2_2\circ A^2_2\big)\,s^2t^2+\cdots\big)\big)\\
& \#\,\big([a_{3,1}]\circ\big([0]+\big(A^3_3\circ A^1_1\big)\,s^3t+\cdots\big)\big)\\
& \#\cdots
\end{aligned}$$

Now we can evaluate the #-sums. Note that because # distributes over +, the coefficient on a given monomial s^mt^n will be the +-sum of the #-sums of the coefficients of monomials above whose product is s^mt^n.

Remark A.6. There is a minor technical issue with this infinite #-sum in that, because each #-summand has a nonzero constant term, there are infinitely many constant monomials that can be #-summed with other monomials. For instance, the coefficient on s is technically $[0]\#A^1_1\#[0]\#[0]\#[0]\#\cdots$, which is not defined in the Hopf ring. However, the infinite #-sum is simply a notational convenience. Because the non-constant monomial factors of a monomial s^mt^n all lie in degrees less than or equal to $m+n$, we use the convention that the constant factors do as well. Therefore, the coefficient on s is merely $[0]\#A^1_1 = A^1_1$.

I will compute the overall #-sum termwise and match coefficients with $b(s+t)$ as computed above. I will place each coefficient from $b(s+t)$ on the left hand side and those from $b(s)+_{[F]}b(t)$ on the right. I begin with a few examples over $\mathbb{Z}$ to show the process and then pass to $\mathbb{F}_2$ in the tables starting with Table A.6.

$$2b_2 = [a_{1,1}]\circ A^1_1\circ A^1_1 + A^1_1\#A^1_1 = [a_{1,1}]\circ b_1\circ b_1 + b_1\#b_1 \tag{A.6}$$

In degree 3, for s^2t we have

$$\begin{aligned} 3b_3 &= [a_{1,1}] \circ A_2^1 \circ A_1^1 + [a_{2,1}] \circ A_2^2 \circ A_1^1 + A_1^1 \# A_2^1 + A_1^1 \#([a_{1,1}] \circ A_1^1 \circ A_1^1) \\ &= [a_{1,1}] \circ b_2 \circ b_1 + [a_{2,1}] \circ b_1 \circ b_1 \circ b_1 + b_1 \# b_2 + b_1 \#([a_{1,1}] \circ b_1 \circ b_1) \end{aligned}$$

Finally in degree 4, we have two monomials. For s^3t, we have

$$\begin{aligned} 4b_4 = [a_{1,1}] \circ A_3^1 \circ A_1^1 &+ [a_{2,1}] \circ A_3^2 \circ A_1^1 + [a_{3,1}] \circ A_3^3 \circ A_1^1 + A_1^1 \# A_3^1 \\ &+ A_2^1 \#([a_{1,1}] \circ A_1^1 \circ A_1^1) + A_1^1 \#([a_{1,1}] \circ A_2^1 \circ A_1^1) \\ &+ A_1^1 \#([a_{2,1}] \circ A_2^2 \circ A_1^1) \\ = [a_{1,1}] \circ b_3 \circ b_1 &+ [a_{2,1}] \circ 2b_1 \circ b_2 \circ b_1 + [a_{3,1}] \circ b_1 \circ b_1 \circ b_1 \circ b_1 + b_1 \# b_3 \\ &+ b_2 \#([a_{1,1}] \circ b_1 \circ b_1) + b_1 \#([a_{1,1}] \circ b_2 \circ b_1) + b_1 \#([a_{2,1}] \circ b_1 \circ b_1 \circ b_1) \end{aligned}$$

For s^2t^2, we have

$$\begin{aligned} 6b_4 = [a_{1,1}] \circ A_2^1 \circ A_2^1 &+ [a_{1,2}] \circ A_2^1 \circ A_2^2 + [a_{2,1}] \circ A_2^2 \circ A_2^1 + [a_{2,2}] \circ A_2^2 \circ A_2^2 \\ &+ A_1^1 \#([a_{1,1}] \circ A_2^1 \circ A_1^1) + A_1^1 \#([a_{1,2}] \circ A_1^1 \circ A_2^2) \\ &+ A_1^1 \# A_1^1 \#([a_{1,1}] \circ A_1^1 \circ A_1^1) + A_2^1 \# A_2^1 + A_1^1 \#([a_{1,1}] \circ A_2^1 \circ A_1^1) \\ &+ A_1^1 \#([a_{2,1}] \circ A_2^2 \circ A_1^1) \\ = [a_{1,1}] \circ b_2 \circ b_2 &+ [a_{1,2}] \circ b_2 \circ b_1 \circ b_1 + [a_{2,1}] \circ b_1 \circ b_1 \circ b_2 + [a_{2,2}] \circ b_1 \circ b_1 \circ b_1 \circ b_1 \\ &+ b_1 \#([a_{1,1}] \circ b_2 \circ b_1) + b_1 \#([a_{1,2}] \circ b_1 \circ b_1 \circ b_1) \\ &+ b_1 \# b_1 \#([a_{1,1}] \circ b_1 \circ b_1) + b_2 \# b_2 + b_1 \#([a_{1,1}] \circ b_2 \circ b_1) \\ &+ b_1 \#([a_{2,1}] \circ b_1 \circ b_1 \circ b_1) \end{aligned}$$

A.3 Coefficient equations

We are now ready to compute the explicit Ravenel–Wilson relations. These will appear in the final set of tables, beginning with Table A.6. First note that in the #-sums in Section A.2, for any $m, n \geq 0$, the coefficient on s^m is $A_m^1 = b_m$ and that on t^n is $A_n^1 = b_n$, which are exactly the coefficients on these monomials in $b(s+t)$. Therefore, these monomials will not give us any information. Furthermore, given a monomial $s^m t^n$, note that $s^n t^m$ has the same factorizations with s and t swapped. Let $s^k t^l$ be a divisor of $s^m t^n$. Then $s^l t^k$ is a divisor of $s^n t^m$, and if $s^k t^l$ appears in a #-summand $[a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$ with coefficient

α, then $s^l t^k$ appears in $[a_{j,i}] \circ b(s)^{\circ j} \circ b(t)^{\circ i}$ with coefficient α as well. Since this applies to all divisors in all factorizations of $s^m t^n$ and $s^n t^m$, the coefficients on $s^m t^n$ and $s^n t^m$ in the #-sum of (A.1) coincide. Therefore, we need only consider those monomials $s^m t^n$ with $m \geq n \geq 1$.

A.3.1 Monomial factorizations

Factors are listed in degree reverse lexicographical order. The factorizations are listed first by the partition of the degree (in descending order of the largest summand, descending order of the second largest summand in the event of a tie, and so on). Within each partition, they are in degree reverse lexicographical order of their first factors (and their second factors in the event of a tie, and so on). So, for example, the factorizations of $s^4 t^2$ are ordered as follows:

1. The partitions of the degree, 6, are ordered 6, $5+1$, $4+2$, $4+1+1$, $3+3$, $3+2+1$, $3+1+1+1$, $2+2+2$, $2+2+1+1$, $2+1+1+1+1$, $1+1+1+1+1+1$.

2. For each partition, for example $3+2+1$, the factorizations are listed by degree reverse lexicographical order of the cubic factor: $(s^3)(st)(t)$ is first, followed by $(s^2t)(s^2)(t)$ and $(s^2t)(st)(s)$. These two have the same cubic factor, so they are listed by degree reverse lexicographical order of the quadratic factor: $(s^2t)(s^2)(t)$ is first, and then $(s^2t)(st)(s)$.

Note that only factorizations that can appear as the monomial on a #-sum in the Ravenel–Wilson relations are included. This is subject to the following facts:

1. The unmixed monomials s^m and t^n respectively appear only in the #-summands $[a_{1,0}] \circ b(s)^{\circ 1} \circ b(t)^{\circ 0} = b(s)$ and $[a_{0,1}] \circ b(s)^{\circ 0} \circ b(t)^{\circ 1} = b(t)$. In particular, this means that at most one power of s and at most one power of t can appear as factors in any factorization. Additionally, this means we need only consider partitions with 2 or fewer instances of 1.

2. The mixed monomial $s^m t^n$, $m, n \geq 1$, appears only in #-summands $[a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$ with $1 \leq i \leq m$ and $1 \leq j \leq n$. This is a total of mn #-summands, so there can be at most mn factors of $s^m t^n$.

The set of factorizations of each monomial in degrees 2–10 are collected in Tables A.1, A.2, A.3, A.4 and A.5.

Monomial	Factorizations
Degree 2	
st	(st), $(s)(t)$
Degree 3	
s^2t	(s^2t), $(s^2)(t)$, $(st)(s)$
Degree 4	
s^3t	(s^3t), $(s^3)(t)$, $(s^2t)(s)$, $(s^2)(st)$,
s^2t^2	(s^2t^2), $(s^2t)(t)$, $(st^2)(s)$, $(s^2)(t^2)$, $(st)(s)(t)$
Degree 5	
s^4t	(s^4t), $(s^4)(t)$, $(s^3t)(s)$, $(s^3)(st)$, $(s^2t)(s^2)$
s^3t^2	(s^3t^2), $(s^3t)(t)$, $(s^2t^2)(s)$, $(s^3)(t^2)$, $(s^2t)(st)$, $(st^2)(s^2)$, $(s^2t)(s)(t)$, $(s^2)(st)(t)$
Degree 6	
s^5t	(s^5t), $(s^5)(t)$, $(s^4t)(s)$, $(s^4)(st)$, $(s^3t)(s^2)$, $(s^3)(s^2t)$
s^4t^2	(s^4t^2), $(s^3t^2)(s)$, $(s^4t)(t)$, $(s^4)(t^2)$, $(s^3t)(st)$, $(s^2t^2)(s^2)$, $(s^3t)(s)(t)$, $(s^3)(st^2)$, $(s^2t)(s^2t)$, $(s^3)(st)(t)$, $(s^2t)(s^2)(t)$, $(s^2t)(st)(s)$
s^3t^3	(s^3t^3), $(s^3t^2)(t)$, $(s^2t^3)(s)$, $(s^3t)(t^2)$, $(s^2t^2)(st)$, $(st^3)(s^2)$, $(s^2t^2)(s)(t)$, $(s^3)(t^3)$, $(s^2t)(st^2)$, $(s^2t)(t^2)(s)$, $(st^2)(s^2)(t)$, $(s^2)(st)(t^2)$

Table A.1: Monomial factorizations in degrees 1–6

Degree 7	
s^6t	(s^6t), $(s^6)(t)$, $(s^5t)(s)$, $(s^5)(st)$, $(s^4t)(s^2)$ $(s^4)(s^2t)$, $(s^3t)(s^3)$
s^5t^2	(s^5t^2), $(s^5t)(t)$, $(s^4t^2)(s)$, $(s^5)(t^2)$, $(s^4t)(st)$, $(s^3t^2)(s^2)$, $(s^4t)(s)(t)$, $(s^4)(st^2)$, $(s^3t)(s^2t)$, $(s^2t^2)(s^3)$, $(s^4)(st)(t)$, $(s^3t)(s^2)(t)$, $(s^3t)(st)(s)$, $(s^3)(s^2t)(t)$, $(s^2t)(s^2t)(s)$, $(s^2t)(s^2)(st)$
s^4t^3	(s^4t^3), $(s^4t^2)(t)$, $(s^3t^3)(s)$, $(s^4t)(t^2)$, $(s^3t^2)(st)$, $(s^2t^3)(s^2)$, $(s^3t^2)(s)(t)$, $(s^4)(t^3)$, $(s^3t)(st^2)$, $(s^2t^2)(s^2t)$, $(st^3)(s^3)$, $(s^3t)(st)(t)$, $(s^3t)(t^2)(s)$, $(s^2t^2)(s^2)(t)$, $(s^2t^2)(st)(s)$, $(s^3)(st^2)(t)$, $(s^2t)(s^2t)(t)$, $(s^2t)(st^2)(s)$, $(s^3)(st)(t^2)$, $(s^2t)(s^2)(t^2)$, $(st^2)(s^2)(st)$, $(s^2t)(st)(s)(t)$
Degree 8	
s^7t	(s^7t), $(s^7)(t)$, $(s^6t)(s)$, $(s^6)(st)$, $(s^5t)(s^2)$ $(s^5)(s^2t)$, $(s^4t)(s^3)$, $(s^4)(s^3t)$
s^6t^2	(s^6t^2), $(s^6t)(t)$, $(s^5t^2)(s)$, $(s^6)(t^2)$, $(s^5t)(st)$, $(s^4t^2)(s^2)$, $(s^5t)(s)(t)$, $(s^5)(st^2)$, $(s^4t)(s^2t)$, $(s^3t^2)(s^3)$, $(s^5)(st)(t)$, $(s^4t)(s^2)(t)$, $(s^4t)(st)(s)$, $(s^4)(s^2t^2)$, $(s^3t)(s^3t)$, $(s^4)(s^2t)(t)$, $(s^3t)(s^3)(t)$, $(s^3t)(s^2t)(s)$, $(s^3t)(s^2)(st)$, $(s^3)(s^2t)(st)$, $(s^2t)(s^2t)(s^2)$
s^5t^3	(s^5t^3), $(s^5t^2)(t)$, $(s^4t^3)(s)$, $(s^5t)(t^2)$, $(s^4t^2)(st)$, $(s^3t^3)(s^2)$, $(s^4t^2)(s)(t)$, $(s^5)(t^3)$, $(s^4t)(st^2)$, $(s^3t^2)(s^2t)$, $(s^2t^3)(s^3)$, $(s^4t)(st)(t)$, $(s^4t)(t^2)(s)$, $(s^3t^2)(s^2)(t)$, $(s^3t^2)(st)(s)$, $(s^4)(st^3)$, $(s^3t)(s^2t^2)$, $(s^4)(st^2)(t)$, $(s^3t)(s^2t)(t)$, $(s^3t)(st^2)(s)$, $(s^2t^2)(s^3)(t)$, $(s^2t^2)(s^2t)(s)$, $(s^4)(st)(t^2)$, $(s^3t)(s^2)(t^2)$, $(s^2t^2)(s^2)(st)$, $(s^3t)(st)(s)(t)$, $(s^3)(s^2t)(t^2)$, $(s^3)(st^2)(st)$, $(s^2t)(st^2)(s^2)$, $(s^2t)(s^2t)(s)(t)$, $(s^2t)(s^2)(st)(t)$
s^4t^4	(s^4t^4), $(s^4t^3)(t)$, $(s^3t^4)(s)$, $(s^4t^2)(t^2)$, $(s^3t^3)(st)$, $(s^2t^4)(s^2)$, $(s^3t^3)(s)(t)$, $(s^4t)(t^3)$, $(s^3t^2)(st^2)$, $(s^2t^3)(s^2t)$, $(st^4)(s^3)$, $(s^3t^2)(st)(t)$, $(s^3t^2)(t^2)(s)$, $(s^2t^3)(s^2)(t)$, $(s^2t^3)(st)(s)$, $(s^4)(t^4)$, $(s^3t)(st^3)$, $(s^2t^2)(s^2t^2)$, $(s^3t)(t^3)(s)$, $(s^3t)(st^2)(t)$, $(s^2t^2)(s^2t)(t)$, $(s^2t^2)(st^2)(s)$, $(st^3)(s^3)(t)$, $(st^3)(s^2t)(s)$, $(s^3t)(st)(t^2)$, $(s^2t^2)(s^2)(t^2)$, $(st^3)(s^2)(st)$, $(s^2t^2)(st)(s)(t)$, $(s^3)(st^2)(t^2)$, $(s^3)(t^3)(st)$, $(s^2t)(s^2t)(t^2)$, $(s^2t)(st^2)(st)$, $(s^2t)(t^3)(s^2)$, $(st^2)(st^2)(s^2)$, $(s^2t)(st^2)(s)(t)$, $(s^2t)(st)(t^2)(s)$, $(st^2)(s^2)(st)(t)$

Table A.2: Monomial factorizations in degrees 7 and 8

Degree 9	
s^8t	(s^8t), $(s^8)(t)$, $(s^7t)(s)$, $(s^7)(st)$, $(s^6t)(s^2)$, $(s^6)(s^2t)$, $(s^5t)(s^3)$, $(s^5)(s^3t)$, $(s^4t)(s^4)$
s^7t^2	(s^7t^2), $(s^6t^2)(s)$, $(s^7t)(t)$, $(s^7)(t^2)$, $(s^6t)(st)$, $(s^5t^2)(s^2)$, $(s^6t)(s)(t)$, $(s^6)(st^2)$, $(s^5t)(s^2t)$, $(s^4t^2)(s^3)$, $(s^6)(st)(t)$, $(s^5t)(st)(s)$, $(s^5t)(s^2)(t)$, $(s^5)(s^2t^2)$, $(s^4t)(s^3t)$, $(s^3t^2)(s^4)$, $(s^5)(s^2t)(t)$, $(s^4t)(s^3)(t)$, $(s^4t)(s^2t)(s)$, $(s^4t)(s^2)(st)$, $(s^4)(s^3t)(t)$, $(s^3t)(s^3t)(s)$, $(s^4)(s^2t)(st)$, $(s^3t)(s^3)(st)$, $(s^3t)(s^2t)(s^2)$, $(s^3)(s^2t)(s^2t)$
s^6t^3	(s^6t^3), $(s^6t^2)(t)$, $(s^5t^3)(s)$, $(s^6t)(t^2)$, $(s^5t^2)(st)$, $(s^4t^3)(s^2)$, $(s^5t^2)(s)(t)$, $(s^6)(t^3)$, $(s^5t)(st^2)$, $(s^4t^2)(s^2t)$, $(s^3t^3)(s^3)$, $(s^5t)(st)(t)$, $(s^5t)(t^2)(s)$, $(s^4t^2)(s^2)(t)$, $(s^4t^2)(st)(s)$, $(s^5)(st^3)$, $(s^4t)(s^2t^2)$, $(s^3t^2)(s^3t)$, $(s^2t^3)(s^4)$, $(s^5)(st^2)(t)$, $(s^4t)(s^2t)(t)$, $(s^4t)(st^2)(s)$, $(s^3t^2)(s^3)(t)$, $(s^3t^2)(s^2t)(s)$, $(s^5)(st)(t^2)$, $(s^4t)(s^2)(t^2)$, $(s^3t^2)(s^2)(st)$, $(s^4t)(st)(s)(t)$, $(s^4)(s^2t^2)(t)$, $(s^3t)(s^3t)(t)$, $(s^3t)(s^2t^2)(s)$, $(s^4)(s^2t)(t^2)$, $(s^4)(st^2)(st)$, $(s^3t)(s^3)(t^2)$, $(s^3t)(s^2t)(st)$, $(s^3t)(st^2)(s^2)$, $(s^2t^2)(s^3)(st)$, $(s^2t^2)(s^2t)(s^2)$, $(s^3t)(s^2t)(s)(t)$, $(s^3t)(s^2)(st)(t)$, $(s^3)(s^2t)(st^2)$, $(s^3)(s^2t)(st)(t)$, $(s^2t)(s^2t)(s^2)(t)$
s^5t^4	(s^5t^4), $(s^5t^3)(t)$, $(s^4t^4)(s)$, $(s^5t^2)(t^2)$, $(s^4t^3)(st)$, $(s^3t^4)(s^2)$, $(s^4t^3)(s)(t)$, $(s^5t)(t^3)$, $(s^4t^2)(st^2)$, $(s^3t^3)(s^2t)$, $(s^2t^4)(s^3)$, $(s^4t^2)(st)(t)$, $(s^4t^2)(t^2)(s)$, $(s^3t^3)(s^2)(t)$, $(s^3t^3)(st)(s)$, $(s^5)(t^4)$, $(s^4t)(st^3)$, $(s^3t^2)(s^2t^2)$, $(s^2t^3)(s^3t)$, $(st^4)(s^4)$, $(s^4t)(st^2)(t)$, $(s^4t)(t^3)(s)$, $(s^3t^2)(s^2t)(t)$, $(s^3t^2)(st^2)(s)$, $(s^2t^3)(s^3)(t)$, $(s^2t^3)(s^2t)(s)$, $(s^4t)(st)(t^2)$, $(s^3t^2)(s^2)(t^2)$, $(s^2t^3)(s^2)(st)$, $(s^3t^2)(st)(s)(t)$, $(s^4)(st^3)(t)$, $(s^3t)(s^2t^2)(t)$, $(s^3t)(st^3)(s)$, $(s^2t^2)(s^2t^2)(s)$, $(s^4)(st^2)(t^2)$, $(s^4)(t^3)(st)$, $(s^3t)(s^2t)(t^2)$, $(s^3t)(st^2)(st)$, $(s^3t)(t^3)(s^2)$, $(s^2t^2)(s^3)(t^2)$, $(s^2t^2)(s^2t)(st)$, $(s^2t^2)(st^2)(s^2)$, $(st^3)(s^3)(st)$, $(st^3)(s^2t)(s^2)$, $(s^3t)(st^2)(s)(t)$, $(s^2t^2)(s^2t)(s)(t)$, $(s^3t)(st)(t^2)(s)$, $(s^2t^2)(s^2)(st)(t)$, $(s^3)(s^2t)(t^3)$, $(s^3)(st^2)(st^2)$, $(s^2t)(s^2t)(st^2)$, $(s^3)(st^2)(st)(t)$, $(s^2t)(s^2t)(st)(t)$, $(s^2t)(s^2t)(t^2)(s)$, $(s^2t)(st^2)(s^2)(t)$, $(s^2t)(st^2)(st)(s)$, $(s^2t)(s^2)(st)(t^2)$

Table A.3: Monomial factorizations in degree 9

Degree 10	
s^9t	(s^9t), $(s^9)(t)$, $(s^8t)(t)$, $(s^8)(st)$, $(s^7t)(s^2)$, $(s^7)(s^2t)$, $(s^6t)(s^3)$, $(s^6)(s^3t)$, $(s^5t)(s^4)$, $(s^5)(s^4t)$
s^8t^2	(s^8t^2), $(s^8t)(t)$, $(s^7t^2)(s)$, $(s^8)(t^2)$, $(s^7t)(st)$, $(s^6t^2)(s^2)$, $(s^7t)(s)(t)$, $(s^7)(st^2)$, $(s^6t)(s^2t)$, $(s^5t^2)(s^3)$, $(s^7)(st)(t)$, $(s^6t)(s^2)(t)$, $(s^6t)(st)(s)$, $(s^6)(s^2t^2)$, $(s^5t)(s^3t)$, $(s^4t^2)(s^4)$, $(s^6)(s^2t)(t)$, $(s^5t)(s^3)(t)$, $(s^5t)(s^2t)(s)$, $(s^5t)(s^2)(st)$, $(s^5)(s^3t^2)$, $(s^4t)(s^4t)$, $(s^5)(s^3t)(t)$, $(s^4t)(s^4)(t)$, $(s^4t)(s^3t)(s)$, $(s^5)(s^2t)(st)$, $(s^4t)(s^3)(st)$, $(s^4t)(s^2t)(s^2)$, $(s^4)(s^3t)(st)$, $(s^3t)(s^3t)(s^2)$, $(s^4)(s^2t)(s^2t)$, $(s^3t)(s^3)(s^2t)$
s^7t^3	(s^7t^3), $(s^7t^2)(t)$, $(s^6t^3)(s)$, $(s^7t)(t^2)$, $(s^6t^2)(st)$, $(s^5t^3)(s^2)$, $(s^6t^2)(s)(t)$, $(s^7)(t^3)$, $(s^6t)(st^2)$, $(s^5t^2)(s^2t)$, $(s^4t^3)(s^3)$, $(s^6t)(st)(t)$, $(s^6t)(t^2)(s)$, $(s^5t^2)(s^2)(t)$, $(s^5t^2)(st)(s)$, $(s^6)(st^3)$, $(s^5t)(s^2t^2)$, $(s^4t^2)(s^3t)$, $(s^3t^3)(s^4)$, $(s^6)(st^2)(t)$, $(s^5t)(s^2t)(t)$, $(s^5t)(st^2)(s)$, $(s^4t^2)(s^3)(t)$, $(s^4t^2)(s^2t)(s)$, $(s^6)(st)(t^2)$, $(s^5t)(s^2)(t^2)$, $(s^4t^2)(s^2)(st)$, $(s^5t)(st)(s)(t)$, $(s^5)(s^2t^3)$, $(s^4t)(s^3t^2)$, $(s^5)(s^2t^2)(t)$, $(s^4t)(s^3t)(t)$, $(s^4t)(s^2t^2)(s)$, $(s^3t^2)(s^4)(t)$, $(s^3t^2)(s^3t)(s)$, $(s^5)(s^2t)(t^2)$, $(s^5)(st^2)(st)$, $(s^4t)(s^3)(t^2)$, $(s^4t)(s^2t)(st)$, $(s^4t)(st^2)(s^2)$, $(s^3t^2)(s^3)(st)$, $(s^3t^2)(s^2t)(s^2)$, $(s^4t)(s^2t)(s)(t)$, $(s^4t)(s^2)(st)(t)$, $(s^4)(s^3t)(t^2)$, $(s^4)(s^2t^2)(st)$, $(s^3t)(s^3t)(st)$, $(s^3t)(s^2t^2)(s^2)$, $(s^3t)(s^3t)(s)(t)$, $(s^4)(s^2t)(st^2)$, $(s^3t)(s^3)(st^2)$, $(s^3t)(s^2t)(s^2t)$, $(s^2t^2)(s^3)(s^2t)$, $(s^4)(s^2t)(st)(t)$, $(s^3t)(s^3)(st)(t)$, $(s^3t)(s^2t)(s^2)(t)$, $(s^3t)(s^2t)(st)(s)$, $(s^3)(s^2t)(s^2t)(t)$, $(s^2t)(s^2t)(s^2t)(s)$, $(s^2t)(s^2t)(s^2)(st)$
s^6t^4	(s^6t^4), $(s^6t^3)(t)$, $(s^5t^4)(s)$, $(s^6t^2)(t^2)$, $(s^5t^3)(st)$, $(s^4t^4)(s^2)$, $(s^5t^3)(s)(t)$, $(s^6t)(t^3)$, $(s^5t^2)(st^2)$, $(s^4t^3)(s^2t)$, $(s^3t^4)(s^3)$, $(s^5t^2)(st)(t)$, $(s^5t^2)(t^2)(s)$, $(s^6)(t^4)$, $(s^5t)(st^3)$, $(s^4t^2)(s^2t^2)$, $(s^3t^3)(s^3t)$, $(s^2t^4)(s^4)$, $(s^5t)(st^2)(t)$, $(s^5t)(t^3)(s)$, $(s^4t^2)(s^2t)(t)$, $(s^4t^2)(st^2)(s)$, $(s^3t^3)(s^3)(t)$, $(s^3t^3)(s^2t)(s)$, $(s^5t)(st)(t^2)$, $(s^4t^2)(s^2)(t^2)$, $(s^3t^3)(s^2)(st)$, $(s^4t^2)(st)(s)(t)$, $(s^5)(st^4)$, $(s^4t)(s^2t^3)$, $(s^3t^2)(s^3t^2)$, $(s^5)(st^3)(t)$, $(s^4t)(s^2t^2)(t)$, $(s^4t)(st^3)(s)$, $(s^3t^2)(s^3t)(t)$, $(s^3t^2)(s^2t^2)(s)$, $(s^2t^3)(s^4)(t)$, $(s^2t^3)(s^3t)(s)$, $(s^5)(st^2)(t^2)$, $(s^5)(t^3)(st)$, $(s^4t)(s^2t)(t^2)$, $(s^4t)(st^2)(st)$, $(s^4t)(t^3)(s^2)$, $(s^3t^2)(s^3)(t^2)$, $(s^3t^2)(s^2t)(st)$, $(s^3t^2)(st^2)(s^2)$, $(s^2t^3)(s^3)(st)$, $(s^2t^3)(s^2t)(s^2)$, $(s^4t)(st^2)(s)(t)$, $(s^3t^2)(s^2t)(s)(t)$, $(s^4t)(st)(t^2)(s)$, $(s^3t^2)(s^2)(st)(t)$, $(s^4)(s^2t^2)(t^2)$, $(s^4)(st^3)(st)$, $(s^3t)(s^3t)(t^2)$, $(s^3t)(s^2t^2)(st)$, $(s^3t)(st^3)(s^2)$, $(s^2t^2)(s^2t^2)(s^2)$, $(s^3t)(s^2t^2)(s)(t)$, $(s^4)(s^2t)(t^3)$, $(s^4)(st^2)(st^2)$, $(s^3t)(s^3)(t^3)$, $(s^3t)(s^2t)(st^2)$, $(s^2t^2)(s^3)(st^2)$, $(s^2t^2)(s^2t)(s^2t)$, $(st^3)(s^3)(s^2t)$, $(s^4)(st^2)(st)(t)$, $(s^3t)(s^2t)(st)(t)$, $(s^3t)(s^2t)(t^2)(s)$, $(s^3t)(st^2)(s^2)(t)$, $(s^3t)(st^2)(st)(s)$, $(s^2t^2)(s^3)(st)(t)$, $(s^2t^2)(s^2t)(s^2)(t)$, $(s^2t^2)(s^2t)(st)(s)$, $(s^3t)(s^2)(st)(t^2)$, $(s^3)(s^2t)(st^2)(t)$, $(s^2t)(s^2t)(st^2)(s)$, $(s^3)(s^2t)(st)(t^2)$, $(s^2t)(s^2t)(s^2)(t^2)$, $(s^2t)(st^2)(s^2)(st)$

Table A.4: Monomial factorizations in degree 10 (continued in Table A.5)

Degree 10 continued	
s^5t^5	(s^5t^5), $(s^5t^4)(t)$, $(s^4t^5)(s)$, $(s^5t^3)(t^2)$, $(s^4t^4)(st)$, $(s^3t^5)(s^2)$, $(s^4t^4)(s)(t)$, $(s^5t^2)(t^3)$, $(s^4t^3)(st^2)$, $(s^3t^4)(s^2t)$, $(s^2t^5)(s^3)$, $(s^4t^3)(t^2)(s)$, $(s^4t^3)(st)(t)$, $(s^3t^4)(s^2)(t)$, $(s^3t^4)(st)(s)$, $(s^5t)(t^4)$, $(s^4t^2)(st^3)$, $(s^3t^3)(s^2t^2)$, $(s^2t^4)(s^3t)$, $(st^5)(s^4)$, $(s^4t^2)(st^2)(t)$, $(s^4t^2)(t^3)(s)$, $(s^3t^3)(s^2t)(t)$, $(s^3t^3)(st^2)(s)$, $(s^2t^4)(s^3)(t)$, $(s^2t^4)(s^2t)(s)$, $(s^4t^2)(st)(t^2)$, $(s^3t^3)(s^2)(t^2)$, $(s^2t^4)(s^2)(st)$, $(s^3t^3)(st)(s)(t)$, $(s^5)(t^5)$, $(s^4t)(st^4)$, $(s^3t^2)(s^2t^3)$, $(s^4t)(st^3)(t)$, $(s^4t)(t^4)(s)$, $(s^3t^2)(s^2t^2)(t)$, $(s^3t^2)(st^3)(s)$, $(s^2t^3)(s^3t)(t)$, $(s^2t^3)(s^2t^2)(s)$, $(st^4)(s^4)(t)$, $(st^4)(s^3t)(s)$, $(s^4t)(st^2)(t^2)$, $(s^4t)(t^3)(st)$, $(s^3t^2)(s^2t)(t^2)$, $(s^3t^2)(st^2)(st)$, $(s^3t^2)(t^3)(s^2)$, $(s^2t^3)(s^3)(t^2)$, $(s^2t^3)(s^2t)(st)$, $(s^2t^3)(st^2)(s^2)$, $(st^4)(s^3)(st)$, $(st^4)(s^2t)(s^2)$, $(s^3t^2)(st^2)(s)(t)$, $(s^2t^3)(s^2t)(s)(t)$, $(s^3t^2)(st)(t^2)(s)$, $(s^2t^3)(s^2)(st)(t)$, $(s^4)(st^3)(t^2)$, $(s^4)(t^4)(st)$, $(s^3t)(s^2t^2)(t^2)$, $(s^3t)(st^3)(st)$, $(s^3t)(t^4)(s^2)$, $(s^2t^2)(s^2t^2)(st)$, $(s^2t^2)(st^3)(s^2)$, $(s^3t)(st^3)(s)(t)$, $(s^2t^2)(s^2t^2)(s)(t)$, $(s^4)(st^2)(t^3)$, $(s^3t)(s^2t)(t^3)$, $(s^3t)(st^2)(st^2)$, $(s^2t^2)(s^3)(t^3)$, $(s^2t^2)(s^2t)(st^2)$, $(st^3)(s^3)(st^2)$, $(st^3)(s^2t)(s^2t)$, $(t^4)(s^3)(s^2t)$, $(s^3t)(st^2)(st)(t)$, $(s^3t)(st^2)(t^2)(s)$, $(s^3t)(t^3)(st)(s)$, $(s^2t^2)(s^2t)(st)(t)$, $(s^2t^2)(s^2t)(t^2)(s)$, $(s^2t^2)(st^2)(s^2)(t)$, $(s^2t^2)(st^2)(st)(s)$, $(st^3)(s^3)(st)(t)$, $(st^3)(s^2t)(s^2)(t)$, $(st^3)(s^2t)(st)(s)$, $(s^2t^2)(s^2)(st)(t^2)$, $(s^3)(st^2)(st^2)(t)$, $(s^2t)(s^2t)(st^2)(t)$, $(s^2t)(s^2t)(t^3)(s)$, $(s^2t)(st^2)(st^2)(s)$, $(s^3)(st^2)(st)(t^2)$, $(s^2t)(st^2)(s^2)(t^2)$, $(s^2t)(t^3)(s^2)(st)$, $(s^2t)(st^2)(st)(s)(t)$,

Table A.5: Monomial factorizations in degree 10 (continued from Table A.4)

A.3.2 The relations

The relations are collected in the final set of tables, starting with Table A.6. In the left column of the tables, each +-summand is one coefficient that appears on the monomial in the leftmost column in the general Ravenel–Wilson relation (when expanded). Each #-summand within a +-summand is a coefficient on one of the monomial factors in the rightmost column (and the order of #-summands is written to match the order of monomial factors). Each #-summand is a $\circ$-product $[a_{i,j}] \circ A^i_k \circ A^j_l$ coming from the #-summand $[a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$ in the general Ravenel–Wilson relation. Each coefficient A^i_k or A^j_l appears in parentheses (), unless it has a single $\circ$-factor. Parentheses are also used around #-summands within +-summands if they have more that one $\circ$-factor. The $[a_{1,0}]$ and $[a_{0,1}]$ from $[a_{1,0}] \circ b(s)^{\circ 1} \circ b(t)^{\circ 0}$ and $[a_{0,1}] \circ b(s)^{\circ 0} \circ b(t)^{\circ 1}$, respectively, are dropped since they are both [1].

Note that not every possible #-sum corresponding to the given factorization in the rightmost column appears in the coefficient from the Ravenel–Wilson relations; bear in mind that the +-summands in each coefficient are #-sums of coefficients from the $\circ$-products $[a_{i,j}] \circ b(s)^{\circ i} \circ b(t)^{\circ j}$, and these coefficients must come from different #-summands in the Ravenel–Wilson relations. For example, consider the coefficient equation for s^3t^2. In the factorization $(s^2t)(st)$, the factor (s^2t) appears as $([a_{1,1}] \circ b_2 \circ b_1)s^2t$ in $[a_{1,1}] \circ b(s) \circ b(t)$ and as $([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)s^2t$ in $[a_{2,1}] \circ b(s)^{\circ 2} \circ b(t)$. However, only the latter appears in the Ravenel–Wilson coefficient equation for s^3t^2 because the factor (s^2t) is multiplied by (st), which only appears in $[a_{1,1}] \circ b(s) \circ b(t)$. The Ravenel–Wilson coefficient cannot contain a #-sum of two terms from this one #-summand.

Copies of these tables without page breaks in the middle of the equations can be found at `https://github.com/ZaneHuttinga/PhD-thesis-files`.

Coefficient equation	Factorization
Degree 2	
st	
$0 =$ $[a_{1,1}] \circ b_1 \circ b_1$ $+b_1 \# b_1$	(st) $(s)(t)$
Degree 3	
s^2t	
$b_3 =$ $[a_{1,1}] \circ b_2 \circ b_1$ $+[a_{2,1}] \circ (b_1 \circ b_1) \circ b_1$ $+b_2 \# b_1$ $+([a_{1,1}] \circ b_1 \circ b_1) \# b_1$	(s^2t) $(s^2)(t)$ $(st)(s)$
Degree 4	
s^3t	
$0 =$ $[a_{1,1}] \circ b_3 \circ b_1$ $+[a_{2,1}] \circ 0 \circ b_1$ $+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1$ $+b_3 \# b_1$ $+([a_{1,1}] \circ b_2 \circ b_1) \# b_1$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1) \# b_1$ $+b_2 \# ([a_{1,1}] \circ b_1 \circ b_1)$	(s^3t) $(s^3)(t)$ $(s^2t)(s)$ $(s^2)(st)$
s^2t^2	
$0 =$ $[a_{1,1}] \circ b_2 \circ b_2$ $+[a_{2,1}] \circ (b_1 \circ b_1) \circ b_2$ $+[a_{1,2}] \circ b_2 \circ (b_1 \circ b_1)$ $+[a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1)$ $+([a_{1,1}] \circ b_2 \circ b_1) \# b_1$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1) \# b_1$ $+([a_{1,1}] \circ b_1 \circ b_2) \# b_1$ $+([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1)) \# b_1$ $+b_2 \# b_2$ $+([a_{1,1}] \circ b_1 \circ b_1) \# b_1 \# b_1$	(s^2t^2) $(s^2t)(t)$ $(st^2)(s)$ $(s^2)(t^2)$ $(st)(s)(t)$

Table A.6: The Ravenel–Wilson relations (continued through the end of Appendix A)

Coefficient equation	Factorization
Degree 5	
s^4t	
$b_5 =$	
$[a_{1,1}] \circ b_4 \circ b_1$	
$+[a_{2,1}] \circ (b_2 \circ b_2) \circ b_1$	(s^4t)
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1$	
$+[a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1$	
$+b_4 \# b_1$	$(s^4)(t)$
$([a_{1,1}] \circ b_3 \circ b_1) \# b_1$	
$+([a_{2,1}] \circ 0 \circ b_1) \# b_1$	$(s^3t)(s)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1) \# b_1$	
$+b_3 \#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^3)(st)$
$+([a_{1,1}] \circ b_2 \circ b_1) \# b_2$	$(s^2t)(s^2)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1) \# b_2$	
s^3t^2	
$0 =$	
$[a_{1,1}] \circ b_3 \circ b_2$	
$+[a_{2,1}] \circ 0 \circ b_2$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2$	(s^3t^2)
$+[a_{1,2}] \circ b_3 \circ (b_1 \circ b_1)$	
$+[a_{2,2}] \circ 0 \circ (b_1 \circ b_1)$	
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_3 \circ b_1) \# b_1$	
$+([a_{2,1}] \circ 0 \circ b_1) \# b_1$	$(s^3t)(t)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1) \# b_1$	
$+([a_{1,1}] \circ b_2 \circ b_2) \# b_1$	
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2) \# b_1$	$(s^2t^2)(s)$
$+([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1)) \# b_1$	
$+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1)) \# b_1$	
$+b_3 \# b_2$	$(s^3)(t^2)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1) \#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^2t)(st)$
$+([a_{1,1}] \circ b_1 \circ b_2) \# b_2$	$(st^2)(s^2)$
$+([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1)) \# b_2$	
$+([a_{1,1}] \circ b_2 \circ b_1) \# b_1 \# b_1$	$(s^2t)(s)(t)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1) \# b_1 \# b_1$	
$+b_2 \#([a_{1,1}] \circ b_1 \circ b_1) \# b_1$	$(s^2)(st)(t)$

Coefficient equation	Factorization
Degree 6	
s^5t	
$0 =$	
$[a_{1,1}] \circ b_5 \circ b_1$	
$+[a_{2,1}] \circ 0 \circ b_1$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1$	(s^5t)
$+[a_{4,1}] \circ 0 \circ b_1$	
$+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1$	
$+b_5 \# b_1$	$(s^5)(t)$
$+([a_{1,1}] \circ b_4 \circ b_1) \# b_1$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1) \# b_1$	$(s^4t)(s)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1) \# b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1) \# b_1$	
$+b_4 \# ([a_{1,1}] \circ b_1 \circ b_1)$	$(s^4)(st)$
$+([a_{1,1}] \circ b_3 \circ b_1) \# b_2$	
$+([a_{2,1}] \circ 0 \circ b_1) \# b_2$	$(s^3t)(s^2)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1) \# b_2$	
$+b_3 \# ([a_{1,1}] \circ b_2 \circ b_1)$	
$+b_3 \# ([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	$(s^3)(s^2t)$

Coefficient equation	Factorization
Degree 6 continued	
s^4t^2	
$b_6 =$	
$[a_{1,1}] \circ b_4 \circ b_2$ $+[a_{2,1}] \circ (b_2 \circ b_2) \circ b_2$ $+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2$ $+[a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2$ $+[a_{1,2}] \circ b_4 \circ (b_1 \circ b_1)$ $+[a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1)$ $+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1)$ $+[a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1)$	(s^4t^2)
$+([a_{1,1}] \circ b_3 \circ b_2)\#b_1$ $+([a_{2,1}] \circ 0 \circ b_2)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1$ $+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#b_1$ $+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$ $+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1)\#b_1$	$(s^3t^2)(s)$
$+([a_{1,1}] \circ b_4 \circ b_1)\#b_1$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#b_1$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_1$	$(s^4t)(t)$
$+b_4\#b_2$	$(s^4)(t^2)$
$+([a_{2,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^3t)(st)$
$+([a_{1,1}] \circ b_2 \circ b_2)\#b_2$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2)\#b_2$ $+([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1))\#b_2$ $+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_2$	$(s^2t^2)(s^2)$
$+([a_{1,1}] \circ b_3 \circ b_1)\#b_1\#b_1$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_1\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#b_1\#b_1$	$(s^3t)(s)(t)$
$+b_3\#([a_{1,1}] \circ b_1 \circ b_2)$ $+b_3\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	$(s^3)(st^2)$
$+([a_{1,1}] \circ b_2 \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	$(s^2t)(s^2t)$
$+b_3\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^3)(st)(t)$
$+([a_{1,1}] \circ b_2 \circ b_1)\#b_2\#b_1$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#b_2\#b_1$	$(s^2t)(s^2)(t)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^2t)(st)(s)$

Coefficient equation	Factorization
Degree 6 continued	
s^3t^3	
$0 =$	
$[a_{1,1}] \circ b_3 \circ b_3$	
$+[a_{2,1}] \circ 0 \circ b_3$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_3$	
$+[a_{1,2}] \circ b_3 \circ 0$	
$+[a_{2,2}] \circ 0 \circ 0$	(s^3t^3)
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ 0$	
$+[a_{1,3}] \circ b_3 \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{3,3}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_3 \circ b_2)\#b_1$	
$+([a_{2,1}] \circ 0 \circ b_2)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1$	$(s^3t^2)(t)$
$+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#b_1$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$	
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1)\#b_1$	
$+([a_{1,1}] \circ b_2 \circ b_3)\#b_1$	
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_3)\#b_1$	
$+([a_{1,2}] \circ b_2 \circ 0)\#b_1$	$(s^2t^3)(s)$
$+([a_{2,2}] \circ (b_1 \circ b_1) \circ 0)\#b_1$	
$+([a_{1,3}] \circ b_2 \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{2,3}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{1,1}] \circ b_3 \circ b_1)\#b_2$	
$+([a_{2,1}] \circ 0 \circ b_1)\#b_2$	$(s^3t)(t^2)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2$	
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)$	
$+([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^2t^2)(st)$
$+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_1 \circ b_3)\#b_2$	
$+([a_{1,2}] \circ b_1 \circ 0)\#b_2$	$(st^3)(s^2)$
$+([a_{1,3}] \circ b_1 \circ (b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{1,1}] \circ b_2 \circ b_2)\#b_1\#b_1$	
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2)\#b_1\#b_1$	$(s^2t^2)(s)(t)$
$+([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+b_3\#b_3$	$(s^3)(t^3)$

Coefficient equation	Factorization
Degree 6 continued	
s^3t^3 continued	
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^2t)(st^2)$
$+([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^2t)(t^2)(s)$
$+([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$	$(st^2)(s^2)(t)$
$+b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^2)(st)(t^2)$
Degree 7	
s^6t	
$b_7=$	
$[a_{1,1}]\circ b_6\circ b_1$ $+[a_{2,1}]\circ(b_3\circ b_3)\circ b_1$ $+[a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1$ $+[a_{4,1}]\circ 0\circ b_1$ $+[a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1$ $+[a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1$	(s^6t)
$+b_6\#b_1$	$(s^6)(t)$
$+([a_{1,1}]\circ b_5\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^5t)(s)$
$+b_5\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^5)(st)$
$+([a_{1,1}]\circ b_4\circ b_1)\#b_2$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	$(s^4t)(s^2)$
$b_4\#([a_{1,1}]\circ b_2\circ b_1)$ $+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^4)(s^2t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_3$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3$	$(s^3t)(s^3)$

Coefficient equation	Factorization
Degree 7 continued	
s^5t^2	
$b_7 =$	
$[a_{1,1}] \circ b_5 \circ b_2$	
$+[a_{2,1}] \circ 0 \circ b_2$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_2$	
$+[a_{4,1}] \circ 0 \circ b_2$	
$+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2$	(s^5t^2)
$+[a_{1,2}] \circ b_5 \circ (b_1 \circ b_1)$	
$+[a_{2,2}] \circ 0 \circ (b_1 \circ b_1)$	
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1)$	
$+[a_{4,2}] \circ 0 \circ (b_1 \circ b_1)$	
$+[a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_5 \circ b_1)\#b_1$	
$+([a_{2,1}] \circ 0 \circ b_1)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#b_1$	$(s^5t)(t)$
$+([a_{4,1}] \circ 0 \circ b_1)\#b_1$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_1$	
$+([a_{1,1}] \circ b_4 \circ b_2)\#b_1$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1$	$(s^4t^2)(s)$
$+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1$	
$+b_5\#b_2$	$(s^5)(t^2)$
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^4t)(st)$
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_3 \circ b_2)\#b_2$	
$+([a_{2,1}] \circ 0 \circ b_2)\#b_2$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#b_2$	$(s^3t^2)(s^2)$
$+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#b_2$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_2$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_2$	

Coefficient equation	Factorization
Degree 7 continued	
s^5t^2 continued	
$+([a_{1,1}] \circ b_4 \circ b_1)\#b_1\#b_1$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#b_1\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#b_1\#b_1$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_1\#b_1$	$(s^4t)(s)(t)$
$+b_4\#([a_{1,1}] \circ b_1 \circ b_2)$ $+b_4\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	$(s^4)(st^2)$
$+([a_{1,1}] \circ b_3 \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$ $+([a_{2,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	$(s^3t)(s^2t)$
$+([a_{1,1}] \circ b_2 \circ b_2)\#b_3$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2)\#b_3$ $+([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1))\#b_3$ $+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_3$	$(s^2t^2)(s^3)$
$+b_4\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^4)(st)(t)$
$+([a_{1,1}] \circ b_3 \circ b_1)\#b_2\#b_1$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_2\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2\#b_1$	$(s^3t)(s^2)(t)$
$+([a_{2,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^3t)(st)(s)$
$+b_3\#([a_{1,1}] \circ b_2 \circ b_1)\#b_1$ $+b_3\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#b_1$	$(s^3)(s^2t)(t)$
$+([a_{1,1}] \circ b_2 \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#b_1$	$(s^2t)(s^2t)(s)$

Coefficient equation	Factorization
Degree 7 continued	
s^4t^3	
$b_7 =$	
$[a_{1,1}] \circ b_4 \circ b_3$	
$+[a_{2,1}] \circ (b_2 \circ b_2) \circ b_3$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_3$	
$+[a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3$	
$+[a_{1,2}] \circ b_4 \circ 0$	
$+[a_{2,2}] \circ (b_2 \circ b_2) \circ 0$	(s^4t^3)
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ 0$	
$+[a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0$	
$+[a_{1,3}] \circ b_4 \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{2,3}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{3,3}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{4,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_4 \circ b_2)\#b_1$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1$	$(s^4t^2)(t)$
$+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1$	
$+([a_{1,1}] \circ b_3 \circ b_3)\#b_1$	
$+([a_{2,1}] \circ 0 \circ b_3)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_3)\#b_1$	
$+([a_{1,2}] \circ b_3 \circ 0)\#b_1$	
$+([a_{2,2}] \circ 0 \circ 0)\#b_1$	$(s^3t^3)(s)$
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ 0)\#b_1$	
$+([a_{1,3}] \circ b_3 \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{1,1}] \circ b_4 \circ b_1)\#b_2$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#b_2$	$(s^4t)(t^2)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#b_2$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2$	

Coefficient equation	Factorization
Degree 7 continued	
s^4t^3 continued	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t^2)(st)$
$+([a_{1,1}]\circ b_2\circ b_3)\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_2$ $+([a_{1,2}]\circ b_2\circ 0)\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_2$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_2$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_2$	$(s^2t^3)(s^2)$
$+([a_{1,1}]\circ b_3\circ b_2)\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_2)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_1\#b_1$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_1\#b_1$	$(s^3t^2)(s)(t)$
$+b_4\#b_3$	$(s^4)(t^3)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^3t)(st^2)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^2t^2)(s^2t)$
$+([a_{1,1}]\circ b_1\circ b_3)\#b_3$ $+([a_{1,2}]\circ b_1\circ 0)\#b_3$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3$	$(st^3)(s^3)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(st)(t)$

Coefficient equation	Factorization
Degree 7 continued	
s^4t^3 continued	
$+([a_{1,1}] \circ b_3 \circ b_1)\#b_2\#b_1$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_2\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2\#b_1$	$(s^3t)(t^2)(s)$
$+([a_{1,1}] \circ b_2 \circ b_2)\#b_2\#b_1$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2)\#b_2\#b_1$ $+([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1))\#b_2\#b_1$ $+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_2\#b_1$	$(s^2t^2)(s^2)(t)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^2t^2)(st)(s)$
$+b_3\#([a_{1,1}] \circ b_1 \circ b_2)\#b_1$ $+b_3\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_1$	$(s^3)(st^2)(t)$
$+([a_{1,1}] \circ b_2 \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#b_1$	$(s^2t)(s^2t)(t)$
$+([a_{1,1}] \circ b_2 \circ b_1)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_1$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_2)\#b_1$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_1$	$(s^2t)(st^2)(s)$
$b_3\#([a_{1,1}] \circ b_1 \circ b_1)\#b_2$	$(s^3)(st)(t^2)$
$+([a_{1,1}] \circ b_2 \circ b_1)\#b_2\#b_2$ $+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#b_2\#b_2$	$(s^2t)(s^2)(t^2)$
$+([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_2\#([a_{1,1}] \circ b_1 \circ b_1)$	$(st^2)(s^2)(st)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1\#b_1$	$(s^2t)(st)(s)(t)$
Degree 8	
s^7t	
$0 =$ $[a_{1,1}] \circ b_7 \circ b_1$ $+[a_{2,1}] \circ 0 \circ b_1$ $+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ b_1$ $+[a_{4,1}] \circ 0 \circ b_1$ $+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ b_1$ $+[a_{6,1}] \circ 0 \circ b_1$ $+[a_{7,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1$ $+b_7\#b_1$	(s^7t) $(s^7)(t)$

Coefficient equation	Factorization
Degree 8 continued	
s^7t continued	
$+([a_{1,1}]\circ b_6\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_3\circ b_3)\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#b_1$ $+([a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^6t)(s)$
$+b_6\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^6)(st)$
$+([a_{1,1}]\circ b_5\circ b_1)\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_2$ $+([a_{4,1}]\circ 0\circ b_1)\#b_2$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	$(s^5t)(s^2)$
$+b_5\#([a_{1,1}]\circ b_2\circ b_1)$ $+b_5\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^5)(s^2t)$
$([a_{1,1}]\circ b_4\circ b_1)\#b_3$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#b_3$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#b_3$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3$	$(s^4t)(s^3)$
$+b_4\#([a_{1,1}]\circ b_3\circ b_1)$ $+b_4\#([a_{2,1}]\circ 0\circ b_1)$ $+b_4\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	$(s^4)(s^3t)$
s^6t^2	
$0=$ $[a_{1,1}]\circ b_6\circ b_2$ $+[a_{2,1}]\circ(b_3\circ b_3)\circ b_2$ $+[a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_2$ $+[a_{4,1}]\circ 0\circ b_2$ $+[a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_2$ $+[a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2$ $+[a_{1,2}]\circ b_6\circ(b_1\circ b_1)$ $+[a_{2,2}]\circ(b_3\circ b_3)\circ(b_1\circ b_1)$ $+[a_{3,2}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ(b_1\circ b_1)$ $+[a_{4,2}]\circ 0\circ(b_1\circ b_1)$ $+[a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ(b_1\circ b_1)$ $+[a_{6,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1)$	(s^6t^2)

Coefficient equation	Factorization
Degree 8 continued	
s^6t^2 continued	
$+([a_{1,1}] \circ b_6 \circ b_1)\#b_1$ $+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_1)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_1)\#b_1$ $+([a_{4,1}] \circ 0 \circ b_1)\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_1)\#b_1$ $+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_1$	$(s^6t)(t)$
$+([a_{1,1}] \circ b_5 \circ b_2)\#b_1$ $+([a_{2,1}] \circ 0 \circ b_2)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_2)\#b_1$ $+([a_{4,1}] \circ 0 \circ b_2)\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1$ $+([a_{1,2}] \circ b_5 \circ (b_1 \circ b_1))\#b_1$ $+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_1$ $+([a_{4,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$ $+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1$	$(s^5t^2)(s)$
$+b_6\#b_2$	$(s^6)(t^2)$
$+([a_{2,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{4,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^5t)(st)$
$+([a_{1,1}] \circ b_4 \circ b_2)\#b_2$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#b_2$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#b_2$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_2$ $+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#b_2$ $+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_2$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_2$ $+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_2$	$(s^4t^2)(s^2)$
$+([a_{1,1}] \circ b_5 \circ b_1)\#b_1\#b_1$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_1\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#b_1\#b_1$ $+([a_{4,1}] \circ 0 \circ b_1)\#b_1\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_1\#b_1$	$(s^5t)(s)(t)$
$+b_5\#([a_{1,1}] \circ b_1 \circ b_2)$ $+b_5\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	$(s^5)(st^2)$

Coefficient equation	Factorization
Degree 8 continued	
s^6t^2 continued	
$([a_{1,1}] \circ b_4 \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	$(s^4t)(s^2t)$
$+([a_{1,1}] \circ b_3 \circ b_2)\#b_3$ $+([a_{2,1}] \circ 0 \circ b_2)\#b_3$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#b_3$ $+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#b_3$ $+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_3$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_3$	$(s^3t^2)(s^3)$
$+b_5\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^5)(st)(t)$
$+([a_{1,1}] \circ b_4 \circ b_1)\#b_2\#b_1$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#b_2\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#b_2\#b_1$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2\#b_1$	$(s^4t)(s^2)(t)$
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^4t)(st)(s)$
$+b_4\#([a_{1,1}] \circ b_2 \circ b_2)$ $+b_4\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_2)$ $+b_4\#([a_{1,2}] \circ b_2 \circ (b_1 \circ b_1))$ $+b_4\#([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1))$	$(s^4)(s^2t^2)$
$+([a_{1,1}] \circ b_3 \circ b_1)\#([a_{2,1}] \circ 0 \circ b_1)$ $+([a_{1,1}] \circ b_3 \circ b_1)\#([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)$ $+([a_{2,1}] \circ 0 \circ b_1)\#([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)$	$(s^3t)(s^3t)$
$+b_4\#([a_{1,1}] \circ b_2 \circ b_1)\#b_1$ $+b_4\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#b_1$	$(s^4)(s^2t)(t)$
$+([a_{1,1}] \circ b_3 \circ b_1)\#b_3\#b_1$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_3\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)\#b_3\#b_1$	$(s^3t)(s^3)(t)$

Coefficient equation	Factorization
Degree 8 continued	
s^6t^2 continued	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t)(s^2t)(s)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t)(s^2)(st)$
$+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3)(s^2t)(st)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^2t)(s^2t)(s^2)$
s^5t^3	
$0=$ $[a_{1,1}]\circ b_5\circ b_3$ $+[a_{2,1}]\circ 0\circ b_3$ $+[a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_3$ $+[a_{4,1}]\circ 0\circ b_3$ $+[a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_3$ $+[a_{1,2}]\circ b_5\circ 0$ $+[a_{2,2}]\circ 0\circ 0$ $+[a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ 0$ $+[a_{4,2}]\circ 0\circ 0$ $+[a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ 0$ $+[a_{1,3}]\circ b_5\circ(b_1\circ b_1\circ b_1)$ $+[a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1)$ $+[a_{3,3}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1\circ b_1)$ $+[a_{4,3}]\circ 0\circ(b_1\circ b_1\circ b_1)$ $+[a_{5,3}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1)$	(s^5t^3)
$+([a_{1,1}]\circ b_5\circ b_2)\#b_1$ $+([a_{2,1}]\circ 0\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#b_1$ $+([a_{4,1}]\circ 0\circ b_2)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_1$ $+([a_{1,2}]\circ b_5\circ(b_1\circ b_1))\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#b_1$ $+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#b_1$ $+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	$(s^5t^2)(t)$

Coefficient equation	Factorization
Degree 8 continued	
s^5t^3 continued	
$+([a_{1,1}]\circ b_4\circ b_3)\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_3)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_3)\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#b_1$	
$+([a_{1,2}]\circ b_4\circ 0)\#b_1$	
$+([a_{2,2}]\circ (b_2\circ b_2)\circ 0)\#b_1$	$(s^4t^3)(s)$
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_2)\circ 0)\#b_1$	
$+([a_{4,2}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#b_1$	
$+([a_{1,3}]\circ b_4\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,3}]\circ (b_2\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{4,3}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_5\circ b_1)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#b_2$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_2$	$(s^5t)(t^2)$
$+([a_{4,1}]\circ 0\circ b_1)\#b_2$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t^2)(st)$
$+([a_{2,2}]\circ (b_2\circ b_2)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,2}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_3)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_3)\#b_2$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_3)\#b_2$	
$+([a_{1,2}]\circ b_3\circ 0)\#b_2$	
$+([a_{2,2}]\circ 0\circ 0)\#b_2$	$(s^3t^3)(s^2)$
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ 0)\#b_2$	
$+([a_{1,3}]\circ b_3\circ (b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{2,3}]\circ 0\circ (b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#b_2$	

Coefficient equation	Factorization
Degree 8 continued	
s^5t^3 continued	
$+([a_{1,1}] \circ b_4 \circ b_2)\#b_1\#b_1$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#b_1\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#b_1\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1\#b_1$	$(s^4t^2)(s)(t)$
$+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+b_5\#b_3$	$(s^5)(t^3)$
$+([a_{1,1}] \circ b_4 \circ b_1)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_2)$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_2)$	$(s^4t)(st^2)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_2)$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	
$+([a_{1,1}] \circ b_3 \circ b_2)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	
$+([a_{2,1}] \circ 0 \circ b_2)\#([a_{1,1}] \circ b_2 \circ b_1)$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{1,1}] \circ b_2 \circ b_1)$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	
$+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_2 \circ b_1)$	$(s^3t^2)(s^2t)$
$+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_2 \circ b_1)$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_2 \circ b_1)$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	
$+([a_{1,1}] \circ b_2 \circ b_3)\#b_3$	
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_3)\#b_3$	
$+([a_{1,2}] \circ b_2 \circ 0)\#b_3$	$(s^2t^3)(s^3)$
$+([a_{2,2}] \circ (b_1 \circ b_1) \circ 0)\#b_3$	
$+([a_{1,3}] \circ b_2 \circ (b_1 \circ b_1 \circ b_1))\#b_3$	
$+([a_{2,3}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_3$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^4t)(st)(t)$
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	

Coefficient equation	Factorization
Degree 8 continued	
s^5t^3 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#b_2\#b_1$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#b_2\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^4t)(t^2)(s)$
$+([a_{1,1}]\circ b_3\circ b_2)\#b_2\#b_1$ $+([a_{2,1}]\circ 0\circ b_2)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#b_1$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#b_1$	$(s^3t^2)(s^2)(t)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t^2)(st)(s)$
$+b_4\#([a_{1,1}]\circ b_1\circ b_3)$ $+b_4\#([a_{1,2}]\circ b_1\circ 0)$ $+b_4\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	$(s^4)(st^3)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	$(s^3t)(s^2t^2)$
$+b_4\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+b_4\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^4)(st^2)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t)(s^2t)(t)$

Coefficient equation	Factorization
Degree 8 continued	
s^5t^3 continued	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^3t)(st^2)(s)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_2\circ b_2)\#b_3\#b_1$	$(s^2t^2)(s^3)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#b_1$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#b_1$	
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^2t^2)(s^2t)(s)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+b_4\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^4)(st)(t^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_2\#b_2$	$(s^3t)(s^2)(t^2)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_2\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#b_2$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t^2)(s^2)(st)$
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^3t)(st)(s)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+b_3\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	$(s^3)(s^2t)(t^2)$
$+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3)(st^2)(st)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(s^2t)(st^2)(s^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	$(s^2t)(s^2t)(s)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t)(s^2)(st)(t)$

Coefficient equation	Factorization
Degree 8 continued	
s^4t^4	
$0 =$	
$[a_{1,1}] \circ b_4 \circ b_4$	
$+[a_{2,1}] \circ (b_2 \circ b_2) \circ b_4$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_4$	
$+[a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_4$	
$+[a_{1,2}] \circ b_4 \circ (b_2 \circ b_2)$	
$+[a_{2,2}] \circ (b_2 \circ b_2) \circ (b_2 \circ b_2)$	
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_2 \circ b_2)$	
$+[a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_2 \circ b_2)$	(s^4t^4)
$+[a_{1,3}] \circ b_4 \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{2,3}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{3,3}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{4,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{1,4}] \circ b_4 \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{2,4}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{3,4}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{4,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_4 \circ b_3)\#b_1$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_3)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_3)\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3)\#b_1$	
$+([a_{1,2}] \circ b_4 \circ 0)\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ 0)\#b_1$	$(s^4t^3)(t)$
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ 0)\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_1$	
$+([a_{1,3}] \circ b_4 \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{2,3}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{4,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 8 continued	
s^4t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_4)\#b_1$	$(s^3t^4)(s)$
$+([a_{2,1}]\circ 0\circ b_4)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_4)\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_2\circ b_2))\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_2\circ b_2))\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{1,4}]\circ b_3\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,4}]\circ 0\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,4}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_2)\#b_2$	$(s^4t^2)(t^2)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#b_2$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#b_2$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t^3)(st)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	

Coefficient equation	Factorization
Degree 8 continued	
s^4t^4 continued	
$+([a_{1,1}] \circ b_2 \circ b_4)\#b_2$	$(s^2t^4)(s^2)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_4)\#b_2$	
$+([a_{1,2}] \circ b_2 \circ (b_2 \circ b_2))\#b_2$	
$+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_2 \circ b_2))\#b_2$	
$+([a_{1,3}] \circ b_2 \circ (b_1 \circ b_1 \circ b_2))\#b_2$	
$+([a_{2,3}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_2))\#b_2$	
$+([a_{1,4}] \circ b_2 \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{2,4}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{1,1}] \circ b_3 \circ b_3)\#b_1\#b_1$	$(s^3t^3)(s)(t)$
$+([a_{2,1}] \circ 0 \circ b_3)\#b_1\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_3)\#b_1\#b_1$	
$+([a_{1,2}] \circ b_3 \circ 0)\#b_1\#b_1$	
$+([a_{2,2}] \circ 0 \circ 0)\#b_1\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ 0)\#b_1\#b_1$	
$+([a_{1,3}] \circ b_3 \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{1,1}] \circ b_4 \circ b_1)\#b_3$	$(s^4t)(t^3)$
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#b_3$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#b_3$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_3$	
$+([a_{1,1}] \circ b_3 \circ b_2)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	$(s^3t^2)(st^2)$
$+([a_{2,1}] \circ 0 \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_2)$	
$+([a_{2,1}] \circ 0 \circ b_2)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_2)$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	
$+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_2)$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_2)$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_2)$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	

Coefficient equation	Factorization
Degree 8 continued	
s^4t^4 continued	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^2t^3)(s^2t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_1\circ b_4)\#b_3$	$(st^4)(s^3)$
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#b_3$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#b_3$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t^2)(st)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#b_2\#b_1$	$(s^3t^2)(t^2)(s)$
$+([a_{2,1}]\circ 0\circ b_2)\#b_2\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_2\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_2\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#b_1$	
$+([a_{1,1}]\circ b_2\circ b_3)\#b_2\#b_1$	$(s^2t^3)(s^2)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_2\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#b_2\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_2\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_2\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_2\#b_1$	

Coefficient equation	Factorization
Degree 8 continued	
s^4t^4 continued	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t^3)(st)(s)$
$+b_4\#b_4$	$(s^4)(t^4)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	$(s^3t)(st^3)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	$(s^2t^2)(s^2t^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_3\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_1$	$(s^3t)(t^3)(s)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^3t)(st^2)(t)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^2t^2)(s^2t)(t)$

Coefficient equation	Factorization
Degree 8 continued	
s^4t^4 continued	
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^2t^2)(st^2)(s)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_1\circ b_3)\#b_3\#b_1$	$(st^3)(s^3)(t)$
$+([a_{1,2}]\circ b_1\circ 0)\#b_3\#b_1$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3\#b_1$	
$+([a_{1,1}]\circ b_1\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(st^3)(s^2t)(s)$
$+([a_{1,2}]\circ b_1\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_1\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^3t)(st)(t^2)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{1,1}]\circ b_2\circ b_2)\#b_2\#b_2$	$(s^2t^2)(s^2)(t^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2\#b_2$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2\#b_2$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#b_2$	
$+([a_{1,2}]\circ b_1\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(st^3)(s^2)(st)$
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^2t^2)(st)(s)(t)$
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+b_3\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	$(s^3)(st^2)(t^2)$
$+b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+b_3\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3)(t^3)(st)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^2t)(s^2t)(t^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t)(st^2)(st)$
$+([a_{1,1}]\circ b_2\circ b_1)\#b_3\#b_2$	$(s^2t)(t^3)(s^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_3\#b_2$	
$+([a_{1,1}]\circ b_1\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(st^2)(st^2)(s^2)$

Coefficient equation	Factorization
Degree 8 continued	
s^4t^4 continued	
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	$(s^2t)(st^2)(s)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	$(s^2t)(st)(t^2)(s)$
$+([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(st^2)(s^2)(st)(t)$
Degree 9	
s^8t	
$b_9 =$ $[a_{1,1}]\circ b_8\circ b_1$ $+[a_{2,1}]\circ(b_4\circ b_4)\circ b_1$ $+[a_{3,1}]\circ(b_1\circ b_1\circ b_6+b_2\circ b_2\circ b_4+b_2\circ b_3\circ b_3)\circ b_1$ $+[a_{4,1}]\circ(b_2\circ b_2\circ b_2\circ b_2)\circ b_1$ $+[a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_4)\circ b_1$ $+[a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2\circ b_2)\circ b_1$ $+[a_{7,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1$ $+[a_{8,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1$	(s^8t)
$+b_8\#b_1$	$(s^8)(t)$
$+([a_{1,1}]\circ b_7\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_5+b_1\circ b_3\circ b_3+b_2\circ b_2\circ b_3)\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_3)\circ b_1)\#b_1$ $+([a_{6,1}]\circ 0\circ b_1)\#b_1$ $+([a_{7,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^7t)(s)$
$+b_7\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^7)(st)$
$+([a_{1,1}]\circ b_6\circ b_1)\#b_2$ $+([a_{2,1}]\circ(b_3\circ b_3)\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#b_2$ $+([a_{4,1}]\circ 0\circ b_1)\#b_2$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#b_2$ $+([a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	$(s^6t)(s^2)$
$+b_6\#([a_{1,1}]\circ b_2\circ b_1)$ $+b_6\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^6)(s^2t)$

Coefficient equation	Factorization
Degree 9 continued	
s^8t continued	
$+([a_{1,1}] \circ b_5 \circ b_1)\#b_3$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_3$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#b_3$ $+([a_{4,1}] \circ 0 \circ b_1)\#b_3$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_3$	$(s^5t)(s^3)$
$+b_5\#([a_{1,1}] \circ b_3 \circ b_1)$ $+b_5\#([a_{2,1}] \circ 0 \circ b_1)$ $+b_5\#([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_1)$	$(s^5)(s^3t)$
$([a_{1,1}] \circ b_4 \circ b_1)\#b_4$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#b_4$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#b_4$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_4$	$(s^4t)(s^4)$
s^7t^2	
$0 =$ $[a_{1,1}] \circ b_7 \circ b_2$ $+[a_{2,1}] \circ 0 \circ b_2$ $+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ b_2$ $+[a_{4,1}] \circ 0 \circ b_2$ $+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ b_2$ $+[a_{6,1}] \circ 0 \circ b_2$ $+[a_{7,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2$ $+[a_{1,2}] \circ b_7 \circ (b_1 \circ b_1)$ $+[a_{2,2}] \circ 0 \circ (b_1 \circ b_1)$ $+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ (b_1 \circ b_1)$ $+[a_{4,2}] \circ 0 \circ (b_1 \circ b_1)$ $+[a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ (b_1 \circ b_1)$ $+[a_{6,2}] \circ 0 \circ (b_1 \circ b_1)$ $+[a_{7,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1)$	(s^7t^2)

Coefficient equation	Factorization
Degree 9 continued	
s^7t^2 continued	
$+([a_{1,1}]\circ b_6\circ b_2)\#b_1$ $+([a_{2,1}]\circ (b_3\circ b_3)\circ b_2)\#b_1$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_2)\#b_1$ $+([a_{4,1}]\circ 0\circ b_2)\#b_1$ $+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_2)\#b_1$ $+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_1$ $+([a_{1,2}]\circ b_6\circ (b_1\circ b_1))\#b_1$ $+([a_{2,2}]\circ (b_3\circ b_3)\circ (b_1\circ b_1))\#b_1$ $+([a_{3,2}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ (b_1\circ b_1))\#b_1$ $+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#b_1$ $+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ (b_1\circ b_1))\#b_1$ $+([a_{6,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_1$	$(s^6t^2)(s)$
$+([a_{1,1}]\circ b_7\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_5+b_1\circ b_3\circ b_3+b_2\circ b_2\circ b_3)\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#b_1$ $+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_3)\circ b_1)\#b_1$ $+([a_{6,1}]\circ 0\circ b_1)\#b_1$ $+([a_{7,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^7t)(t)$
$+b_7\#b_2$	$(s^7)(t^2)$
$+([a_{2,1}]\circ (b_3\circ b_3)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^6t)(st)$
$+([a_{1,1}]\circ b_5\circ b_2)\#b_2$ $+([a_{2,1}]\circ 0\circ b_2)\#b_2$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#b_2$ $+([a_{4,1}]\circ 0\circ b_2)\#b_2$ $+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2$ $+([a_{1,2}]\circ b_5\circ (b_1\circ b_1))\#b_2$ $+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#b_2$ $+([a_{3,2}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ (b_1\circ b_1))\#b_2$ $+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#b_2$ $+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_2$	$(s^5t^2)(s^2)$

Coefficient equation	Factorization
Degree 9 continued	
s^7t^2 continued	
$+([a_{1,1}] \circ b_6 \circ b_1)\#b_1\#b_1$ $+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_1)\#b_1\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_1)\#b_1\#b_1$ $+([a_{4,1}] \circ 0 \circ b_1)\#b_1\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_1)\#b_1\#b_1$ $+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_1\#b_1$	$(s^6t)(s)(t)$
$+b_6\#([a_{1,1}] \circ b_1 \circ b_2)$ $+b_6\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))$	$(s^6)(st^2)$
$+([a_{1,1}] \circ b_5 \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$ $+([a_{2,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$ $+([a_{4,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{4,1}] \circ 0 \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_2 \circ b_1)$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	$(s^5t)(s^2t)$
$+([a_{1,1}] \circ b_4 \circ b_2)\#b_3$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#b_3$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#b_3$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_3$ $+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#b_3$ $+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_3$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_3$ $+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_3$	$(s^4t^2)(s^3)$
$+b_6\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^6)(st)(t)$
$+([a_{2,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{4,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^5t)(st)(s)$
$+([a_{1,1}] \circ b_5 \circ b_1)\#b_2\#b_1$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_2\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#b_2\#b_1$ $+([a_{4,1}] \circ 0 \circ b_1)\#b_2\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2\#b_1$	$(s^5t)(s^2)(t)$

Coefficient equation	Factorization
Degree 9 continued	
s^7t^2	
$+b_5\#([a_{1,1}]\circ b_2\circ b_2)$ $+b_5\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_2)$ $+b_5\#([a_{1,2}]\circ b_2\circ (b_1\circ b_1))$ $+b_5\#([a_{2,2}]\circ (b_1\circ b_1)\circ (b_1\circ b_1))$	$(s^5)(s^2t^2)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)$ $+([a_{1,1}]\circ b_4\circ b_1)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)$	$(s^4t)(s^3t)$
$+([a_{1,1}]\circ b_3\circ b_2)\#b_4$ $+([a_{2,1}]\circ 0\circ b_2)\#b_4$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#b_4$ $+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#b_4$ $+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#b_4$ $+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_4$	$(s^3t^2)(s^4)$
$+b_5\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+b_5\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$	$(s^5)(s^2t)(t)$
$+([a_{1,1}]\circ b_4\circ b_1)\#b_3\#b_1$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#b_3\#b_1$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#b_3\#b_1$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_1$	$(s^4t)(s^3)(t)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$	$(s^4t)(s^2t)(s)$
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t)(s^2)(st)$

Coefficient equation	Factorization
Degree 9 continued	
s^7t^2 continued	
$+b_4\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+b_4\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+b_4\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^4)(s^3t)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t)(s^3t)(s)$
$+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4)(s^2t)(st)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t)(s^3)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^3t)(s^2t)(s^2)$
$+b_3\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^3)(s^2t)(s^2t)$
s^6t^3	
$0=$ $[a_{1,1}]\circ b_6\circ b_3$ $+[a_{2,1}]\circ(b_3\circ b_3)\circ b_3$ $+[a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_3$ $+[a_{4,1}]\circ 0\circ b_3$ $+[a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_3$ $+[a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_3$ $+[a_{1,2}]\circ b_6\circ 0$ $+[a_{2,2}]\circ(b_3\circ b_3)\circ 0$ $+[a_{3,2}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ 0$ $+[a_{4,2}]\circ 0\circ 0$ $+[a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ 0$ $+[a_{6,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ 0$ $+[a_{1,3}]\circ b_6\circ(b_1\circ b_1\circ b_1)$ $+[a_{2,3}]\circ(b_3\circ b_3)\circ(b_1\circ b_1\circ b_1)$ $+[a_{3,3}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ(b_1\circ b_1\circ b_1)$ $+[a_{4,3}]\circ 0\circ(b_1\circ b_1\circ b_1)$ $+[a_{5,3}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_1)$ $+[a_{6,3}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1)$	(s^6t^3)

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+([a_{1,1}] \circ b_6 \circ b_2)\#b_1$ $+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_2)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_2)\#b_1$ $+([a_{4,1}] \circ 0 \circ b_2)\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_2)\#b_1$ $+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1$ $+([a_{1,2}] \circ b_6 \circ (b_1 \circ b_1))\#b_1$ $+([a_{2,2}] \circ (b_3 \circ b_3) \circ (b_1 \circ b_1))\#b_1$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_1$ $+([a_{4,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$ $+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_1$ $+([a_{6,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1$	$(s^6t^2)(t)$
$+([a_{1,1}] \circ b_5 \circ b_3)\#b_1$ $+([a_{2,1}] \circ 0 \circ b_3)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_3)\#b_1$ $+([a_{4,1}] \circ 0 \circ b_3)\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3)\#b_1$ $+([a_{1,2}] \circ b_5 \circ 0)\#b_1$ $+([a_{2,2}] \circ 0 \circ 0)\#b_1$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ 0)\#b_1$ $+([a_{4,2}] \circ 0 \circ 0)\#b_1$ $+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_1$ $+([a_{1,3}] \circ b_5 \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	$(s^5t^3)(s)$
$+([a_{1,1}] \circ b_6 \circ b_1)\#b_2$ $+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_1)\#b_2$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_1)\#b_2$ $+([a_{4,1}] \circ 0 \circ b_1)\#b_2$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_1)\#b_2$ $+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2$	$(s^6t)(t^2)$

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,2}]\circ b_5\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,2}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^5t^2)(st)$
$+([a_{1,1}]\circ b_4\circ b_3)\#b_2$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_3)\#b_2$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_3)\#b_2$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#b_2$ $+([a_{1,2}]\circ b_4\circ 0)\#b_2$ $+([a_{2,2}]\circ (b_2\circ b_2)\circ 0)\#b_2$ $+([a_{3,2}]\circ (b_1\circ b_1\circ b_2)\circ 0)\#b_2$ $+([a_{4,2}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#b_2$ $+([a_{1,3}]\circ b_4\circ (b_1\circ b_1\circ b_1))\#b_2$ $+([a_{2,3}]\circ (b_2\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_2$ $+([a_{3,3}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_2$ $+([a_{4,3}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#b_2$	$(s^4t^3)(s^2)$
$+([a_{1,1}]\circ b_5\circ b_2)\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_2)\#b_1\#b_1$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#b_1\#b_1$ $+([a_{4,1}]\circ 0\circ b_2)\#b_1\#b_1$ $+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_1\#b_1$ $+([a_{1,2}]\circ b_5\circ (b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#b_1\#b_1$ $+([a_{3,2}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ (b_1\circ b_1))\#b_1\#b_1$ $+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#b_1\#b_1$ $+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_1\#b_1$	$(s^5t^2)(s)(t)$
$+b_6\#b_3$	$(s^6)(t^3)$

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^5t)(st^2)$
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	$(s^4t^2)(s^2t)$
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_3)\#b_3$	
$+([a_{2,1}]\circ 0\circ b_3)\#b_3$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#b_3$	
$+([a_{1,2}]\circ b_3\circ 0)\#b_3$	
$+([a_{2,2}]\circ 0\circ 0)\#b_3$	$(s^3t^3)(s^3)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#b_3$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_3$	

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+([a_{2,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{4,1}] \circ 0 \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^5t)(st)(t)$
$+([a_{1,1}] \circ b_5 \circ b_1)\#b_2\#b_1$ $+([a_{2,1}] \circ 0 \circ b_1)\#b_2\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_1)\#b_2\#b_1$ $+([a_{4,1}] \circ 0 \circ b_1)\#b_2\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_2\#b_1$	$(s^5t)(t^2)(s)$
$+([a_{1,1}] \circ b_4 \circ b_2)\#b_2\#b_1$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#b_2\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#b_2\#b_1$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_2\#b_1$ $+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#b_2\#b_1$ $+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_2\#b_1$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_2\#b_1$ $+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_2\#b_1$	$(s^4t^2)(s^2)(t)$
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$ $+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^4t^2)(st)(s)$
$+b_5\#([a_{1,1}] \circ b_1 \circ b_3)$ $+b_5\#([a_{1,2}] \circ b_1 \circ 0)$ $+b_5\#([a_{1,3}] \circ b_1 \circ (b_1 \circ b_1 \circ b_1))$	$(s^5)(st^3)$

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$	$(s^4t)(s^2t^2)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	$(s^3t^2)(s^3t)$
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_2\circ b_3)\#b_4$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_4$	
$+([a_{1,2}]\circ b_2\circ 0)\#b_4$	$(s^2t^3)(s^4)$
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_4$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_4$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_4$	

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+b_5\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+b_5\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^5)(st^2)(t)$
$([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^4t)(s^2t)(t)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^4t)(st^2)(s)$
$+([a_{1,1}]\circ b_3\circ b_2)\#b_3\#b_1$ $+([a_{2,1}]\circ 0\circ b_2)\#b_3\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_3\#b_1$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_3\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_3\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#b_1$	$(s^3t^2)(s^3)(t)$
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t^2)(s^2t)(s)$
$+b_5\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^5)(st)(t^2)$
$+([a_{1,1}]\circ b_4\circ b_1)\#b_2\#b_2$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#b_2\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#b_2\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#b_2$	$(s^4t)(s^2)(t^2)$

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+([a_{2,1}]\circ 0\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t^2)(s^2)(st)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^4t)(st)(s)(t)$
$+b_4\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$ $+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$ $+b_4\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+b_4\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	$(s^4)(s^2t^2)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t)(s^3t)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	$(s^3t)(s^2t^2)(s)$
$+b_4\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^4)(s^2t)(t^2)$
$+b_4\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4)(st^2)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_3\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_2$	$(s^3t)(s^3)(t^2)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t)(s^2t)(st)$

Coefficient equation	Factorization
Degree 9 continued	
s^6t^3 continued	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(s^3t)(st^2)(s^2)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t^2)(s^3)(st)$
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^2t^2)(s^2t)(s^2)$
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	$(s^3t)(s^2t)(s)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,1}]\circ 0\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(s^2)(st)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+b_3\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	$(s^3)(s^2t)(st^2)$
$+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3)(s^2t)(st)(t)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^2t)(s^2t)(s^2)(t)$

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4	
$0 =$	
$[a_{1,1}] \circ b_5 \circ b_4$	
$+[a_{2,1}] \circ 0 \circ b_4$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_4$	
$+[a_{4,1}] \circ 0 \circ b_4$	
$+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_4$	
$+[a_{1,2}] \circ b_5 \circ (b_2 \circ b_2)$	
$+[a_{2,2}] \circ 0 \circ (b_2 \circ b_2)$	
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_2 \circ b_2)$	
$+[a_{4,2}] \circ 0 \circ (b_2 \circ b_2)$	
$+[a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_2 \circ b_2)$	
$+[a_{1,3}] \circ b_5 \circ (b_1 \circ b_1 \circ b_2)$	(s^5t^4)
$+[a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{3,3}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_2)$	
$+[a_{1,4}] \circ b_5 \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{2,4}] \circ 0 \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{3,4}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{4,4}] \circ 0 \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{5,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_5 \circ b_3)\#b_1$	
$+([a_{2,1}] \circ 0 \circ b_3)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_3)\#b_1$	
$+([a_{4,1}] \circ 0 \circ b_3)\#b_1$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3)\#b_1$	
$+([a_{1,2}] \circ b_5 \circ 0)\#b_1$	
$+([a_{2,2}] \circ 0 \circ 0)\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ 0)\#b_1$	$(s^5t^3)(t)$
$+([a_{4,2}] \circ 0 \circ 0)\#b_1$	
$+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_1$	
$+([a_{1,3}] \circ b_5 \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{1,1}] \circ b_4 \circ b_4)\#b_1$	$(s^4t^4)(s)$
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_4)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_4)\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_4)\#b_1$	
$+([a_{1,2}] \circ b_4 \circ (b_2 \circ b_2))\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_2 \circ b_2))\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_2 \circ b_2))\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_2 \circ b_2))\#b_1$	
$+([a_{1,3}] \circ b_4 \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{2,3}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{4,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{1,4}] \circ b_4 \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{2,4}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{3,4}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{4,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{1,1}] \circ b_5 \circ b_2)\#b_2$	$(s^5t^2)(t^2)$
$+([a_{2,1}] \circ 0 \circ b_2)\#b_2$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_2)\#b_2$	
$+([a_{4,1}] \circ 0 \circ b_2)\#b_2$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_2$	
$+([a_{1,2}] \circ b_5 \circ (b_1 \circ b_1))\#b_2$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_2$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_2$	
$+([a_{4,2}] \circ 0 \circ (b_1 \circ b_1))\#b_2$	
$+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_2$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t^3)(st)$
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_4)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_4)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_4)\#b_2$	
$+([a_{1,2}]\circ b_3\circ(b_2\circ b_2))\#b_2$	
$+([a_{2,2}]\circ 0\circ(b_2\circ b_2))\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#b_2$	$(s^3t^4)(s^2)$
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_2))\#b_2$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_2))\#b_2$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_2$	
$+([a_{1,4}]\circ b_3\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{2,4}]\circ 0\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{3,4}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{1,1}]\circ b_4\circ b_3)\#b_1\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_3)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_3)\#b_1\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_4\circ 0)\#b_1\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ 0)\#b_1\#b_1$	$(s^4t^3)(s)(t)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ 0)\#b_1\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#b_1\#b_1$	
$+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_1))\#b_1\#b_1$	
$+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#b_1\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_1))\#b_1\#b_1$	
$+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_1\#b_1$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{1,1}]\circ b_5\circ b_1)\#b_3$	
$+([a_{2,1}]\circ 0\circ b_1)\#b_3$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_3$	$(s^5t)(t^3)$
$+([a_{4,1}]\circ 0\circ b_1)\#b_3$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3$	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^4t^2)(st^2)$
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^3t^3)(s^2t)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{1,1}] \circ b_2 \circ b_4)\#b_3$	$(s^2t^4)(s^3)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_4)\#b_3$	
$+([a_{1,2}] \circ b_2 \circ (b_2 \circ b_2))\#b_3$	
$+([a_{2,2}] \circ (b_1 \circ b_1) \circ (b_2 \circ b_2))\#b_3$	
$+([a_{1,3}] \circ b_2 \circ (b_1 \circ b_1 \circ b_2))\#b_3$	
$+([a_{2,3}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_2))\#b_3$	
$+([a_{1,4}] \circ b_2 \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_3$	
$+([a_{2,4}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_3$	
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	$(s^4t^2)(st)(t)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	
$+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_1)\#b_1$	
$+([a_{1,1}] \circ b_4 \circ b_2)\#b_2\#b_1$	$(s^4t^2)(t^2)(s)$
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_2)\#b_2\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_2)\#b_2\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_2\#b_1$	
$+([a_{1,2}] \circ b_4 \circ (b_1 \circ b_1))\#b_2\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_2\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_2\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_2\#b_1$	
$+([a_{1,1}] \circ b_3 \circ b_3)\#b_2\#b_1$	$(s^3t^3)(s^2)(t)$
$+([a_{2,1}] \circ 0 \circ b_3)\#b_2\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_3)\#b_2\#b_1$	
$+([a_{1,2}] \circ b_3 \circ 0)\#b_2\#b_1$	
$+([a_{2,2}] \circ 0 \circ 0)\#b_2\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ 0)\#b_2\#b_1$	
$+([a_{1,3}] \circ b_3 \circ (b_1 \circ b_1 \circ b_1))\#b_2\#b_1$	
$+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_2\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_2\#b_1$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t^3)(st)(s)$
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_3\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,3}]\circ 0\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+b_5\#b_4$	$(s^5)(t^4)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$	$(s^4t)(st^3)$
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	$(s^3t^2)(s^2t^2)$
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ 0\circ b_1)$	$(s^2t^3)(s^3t)$
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{1,1}]\circ b_1\circ b_4)\#b_4$	$(st^4)(s^4)$
$+([a_{1,2}]\circ b_1\circ (b_2\circ b_2))\#b_4$	
$+([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_2))\#b_4$	
$+([a_{1,4}]\circ b_1\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_4$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	$(s^4t)(st^2)(t)$
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_1)\#b_3\#b_1$	$(s^4t)(t^3)(s)$
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#b_3\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#b_3\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t^2)(s^2t)(t)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	$(s^3t^2)(st^2)(s)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{1,1}]\circ b_2\circ b_3)\#b_3\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_3\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#b_3\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_3\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_3\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_3\#b_1$	$(s^2t^3)(s^3)(t)$
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^2t^3)(s^2t)(s)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^4t)(st)(t^2)$
$+([a_{1,1}]\circ b_3\circ b_2)\#b_2\#b_2$ $+([a_{2,1}]\circ 0\circ b_2)\#b_2\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#b_2$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_2\#b_2$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_2\#b_2$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#b_2$	$(s^3t^2)(s^2)(t^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,2}]\circ b_2\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t^3)(s^2)(st)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$ $+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^3t^2)(st)(s)(t)$

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+b_4\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$ $+b_4\#([a_{1,2}]\circ b_1\circ 0)\#b_1$ $+b_4\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$	$(s^4)(st^3)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	$(s^3t)(s^2t^2)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$	$(s^3t)(st^3)(s)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	$(s^2t^2)(s^2t^2)(s)$
$+b_4\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$ $+b_4\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(s^4)(st^2)(t^2)$
$+b_4\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4)(t^3)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^3t)(s^2t)(t^2)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t)(st^2)(st)$

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_1)\#b_3\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_2$	$(s^3t)(t^3)(s^2)$
$+([a_{1,1}]\circ b_2\circ b_2)\#b_3\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#b_2$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#b_2$	$(s^2t^2)(s^3)(t^2)$
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t^2)(s^2t)(st)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(s^2t^2)(st^2)(s^2)$
$+([a_{1,2}]\circ b_1\circ 0)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(st^3)(s^3)(st)$
$+([a_{1,1}]\circ b_1\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{1,2}]\circ b_1\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{1,2}]\circ b_1\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(st^3)(s^2t)(s^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	$(s^3t)(st^2)(s)(t)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	$(s^2t^2)(s^2t)(s)(t)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	$(s^3t)(st)(t^2)(s)$

Coefficient equation	Factorization
Degree 9 continued	
s^5t^4 continued	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t^2)(s^2)(st)(t)$
$+b_3\#([a_{1,1}]\circ b_2\circ b_1)\#b_3$ $+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_3$	$(s^3)(s^2t)(t^3)$
$+b_3\#([a_{1,1}]\circ b_1\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^3)(st^2)(st^2)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^2t)(s^2t)(st^2)$
$+b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3)(st^2)(st)(t)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^2t)(s^2t)(t^2)(s)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$	$(s^2t)(st^2)(s^2)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t)(st^2)(st)(s)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^2t)(s^2)(st)(t^2)$

Coefficient equation	Factorization
Degree 10	
s^9t	
$0 =$	
$[a_{1,1}] \circ b_9 \circ b_1$	
$+[a_{2,1}] \circ 0 \circ b_1$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_7 + b_1 \circ b_4 \circ b_4 + b_2 \circ b_2 \circ b_5 + b_3 \circ b_3 \circ b_3) \circ b_1$	
$+[a_{4,1}] \circ 0 \circ b_1$	
$+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_5 + b_1 \circ b_2 \circ b_2 \circ b_2 \circ b_2) \circ b_1$	(s^9t)
$+[a_{6,1}] \circ 0 \circ b_1$	
$+[a_{7,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3 + b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2) \circ b_1$	
$+[a_{8,1}] \circ 0 \circ b_1$	
$+[a_{9,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1$	
$+b_9 \# b_1$	$(s^9)(t)$
$+([a_{1,1}] \circ b_8 \circ b_1) \# b_1$	
$+([a_{2,1}] \circ (b_4 \circ b_4) \circ b_1) \# b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_6 + b_2 \circ b_2 \circ b_4 + b_2 \circ b_3 \circ b_3) \circ b_1) \# b_1$	
$+([a_{4,1}] \circ (b_2 \circ b_2 \circ b_2 \circ b_2) \circ b_1) \# b_1$	$(s^8t)(t)$
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_4) \circ b_1) \# b_1$	
$+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2 \circ b_2) \circ b_1) \# b_1$	
$+([a_{7,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_1) \# b_1$	
$+([a_{8,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1) \# b_1$	
$+b_8 \#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^8)(st)$
$+([a_{1,1}] \circ b_7 \circ b_1) \# b_2$	
$+([a_{2,1}] \circ 0 \circ b_1) \# b_2$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ b_1) \# b_2$	
$+([a_{4,1}] \circ 0 \circ b_1) \# b_2$	$(s^7t)(s^2)$
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ b_1) \# b_2$	
$+([a_{6,1}] \circ 0 \circ b_1) \# b_2$	
$+([a_{7,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1) \# b_2$	
$+b_7 \#([a_{1,1}] \circ b_2 \circ b_1)$	$(s^7)(s^2t)$
$+b_7 \#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)$	
$+([a_{1,1}] \circ b_6 \circ b_1) \# b_3$	
$+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_1) \# b_3$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_1) \# b_3$	
$+([a_{4,1}] \circ 0 \circ b_1) \# b_3$	$(s^6t)(s^3)$
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_1) \# b_3$	
$+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1) \# b_3$	

Coefficient equation	Factorization
Degree 10 continued	
s^9t continued	
$+b_6\#([a_{1,1}]\circ b_3\circ b_1)$	
$+b_6\#([a_{2,1}]\circ 0\circ b_1)$	$(s^6)(s^3t)$
$+b_6\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_5\circ b_1)\#b_4$	
$+([a_{2,1}]\circ 0\circ b_1)\#b_4$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_4$	$(s^5t)(s^4)$
$+([a_{4,1}]\circ 0\circ b_1)\#b_4$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_4$	
$+b_5\#([a_{1,1}]\circ b_4\circ b_1)$	
$+b_5\#([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)$	$(s^5)(s^4t)$
$+b_5\#([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)$	
$+b_5\#([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)$	
s^8t^2	
$b_{10}=$	
$[a_{1,1}]\circ b_8\circ b_2$	
$+[a_{2,1}]\circ(b_4\circ b_4)\circ b_2$	
$+[a_{3,1}]\circ(b_1\circ b_1\circ b_6+b_2\circ b_2\circ b_4+b_2\circ b_3\circ b_3)\circ b_2$	
$+[a_{4,1}]\circ(b_2\circ b_2\circ b_2\circ b_2)\circ b_2$	
$+[a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_4)\circ b_2$	
$+[a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2\circ b_2)\circ b_2$	
$+[a_{7,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_2$	
$+[a_{8,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2$	(s^8t^2)
$+[a_{1,2}]\circ b_8\circ(b_1\circ b_1)$	
$+[a_{2,2}]\circ(b_4\circ b_4)\circ(b_1\circ b_1)$	
$+[a_{3,2}]\circ(b_1\circ b_1\circ b_6+b_2\circ b_2\circ b_4+b_2\circ b_3\circ b_3)\circ(b_1\circ b_1)$	
$+[a_{4,2}]\circ(b_2\circ b_2\circ b_2\circ b_2)\circ(b_1\circ b_1)$	
$+[a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_4)\circ(b_1\circ b_1)$	
$+[a_{6,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2\circ b_2)\circ(b_1\circ b_1)$	
$+[a_{7,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ(b_1\circ b_1)$	
$+[a_{8,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^8t^2 continued	
$+([a_{1,1}]\circ b_8\circ b_1)\#b_1$	
$+([a_{2,1}]\circ (b_4\circ b_4)\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_6+b_2\circ b_2\circ b_4+b_2\circ b_3\circ b_3)\circ b_1)\#b_1$	
$+([a_{4,1}]\circ (b_2\circ b_2\circ b_2\circ b_2)\circ b_1)\#b_1$	$(s^8t)(t)$
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_4)\circ b_1)\#b_1$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2\circ b_2)\circ b_1)\#b_1$	
$+([a_{7,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#b_1$	
$+([a_{8,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_7\circ b_2)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_5+b_1\circ b_3\circ b_3+b_2\circ b_2\circ b_3)\circ b_2)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_2)\#b_1$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_3)\circ b_2)\#b_1$	
$+([a_{6,1}]\circ 0\circ b_2)\#b_1$	
$+([a_{7,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_1$	$(s^7t^2)(s)$
$+([a_{1,2}]\circ b_7\circ (b_1\circ b_1))\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_5+b_1\circ b_3\circ b_3+b_2\circ b_2\circ b_3)\circ (b_1\circ b_1))\#b_1$	
$+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#b_1$	
$+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_3)\circ (b_1\circ b_1))\#b_1$	
$+([a_{6,2}]\circ 0\circ (b_1\circ b_1))\#b_1$	
$+([a_{7,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_1$	
$+b_8\#b_2$	$(s^8)(t^2)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_5+b_1\circ b_3\circ b_3+b_2\circ b_2\circ b_3)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^7t)(st)$
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_3)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{6,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{7,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^8t^2 continued	
$+([a_{1,1}]\circ b_6\circ b_2)\#b_2$	$(s^6t^2)(s^2)$
$+([a_{2,1}]\circ (b_3\circ b_3)\circ b_2)\#b_2$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_2)\#b_2$	
$+([a_{4,1}]\circ 0\circ b_2)\#b_2$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_2)\#b_2$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2$	
$+([a_{1,2}]\circ b_6\circ (b_1\circ b_1))\#b_2$	
$+([a_{2,2}]\circ (b_3\circ b_3)\circ (b_1\circ b_1))\#b_2$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ (b_1\circ b_1))\#b_2$	
$+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#b_2$	
$+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ (b_1\circ b_1))\#b_2$	
$+([a_{6,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_2$	
$+([a_{1,1}]\circ b_7\circ b_1)\#b_1\#b_1$	$(s^7t)(s)(t)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_5+b_1\circ b_3\circ b_3+b_2\circ b_2\circ b_3)\circ b_1)\#b_1\#b_1$	
$+([a_{4,1}]\circ 0\circ b_1)\#b_1\#b_1$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_3)\circ b_1)\#b_1\#b_1$	
$+([a_{6,1}]\circ 0\circ b_1)\#b_1\#b_1$	
$+([a_{7,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+b_7\#([a_{1,1}]\circ b_1\circ b_2)$	$(s^7)(st^2)$
$+b_7\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))$	
$+([a_{1,1}]\circ b_6\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)$	$(s^6t)(s^2t)$
$+([a_{2,1}]\circ (b_3\circ b_3)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^8t^2 continued	
$+([a_{1,1}]\circ b_5\circ b_2)\#b_3$	
$+([a_{2,1}]\circ 0\circ b_2)\#b_3$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#b_3$	
$+([a_{4,1}]\circ 0\circ b_2)\#b_3$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_3$	$(s^5t^2)(s^3)$
$+([a_{1,2}]\circ b_5\circ(b_1\circ b_1))\#b_3$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_3$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#b_3$	
$+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#b_3$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_3$	
$+b_7\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^7)(st)(t)$
$+([a_{1,1}]\circ b_6\circ b_1)\#b_2\#b_1$	
$+([a_{2,1}]\circ(b_3\circ b_3)\circ b_1)\#b_2\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#b_2\#b_1$	$(s^6t)(s^2)(t)$
$+([a_{4,1}]\circ 0\circ b_1)\#b_2\#b_1$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#b_2\#b_1$	
$+([a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#b_1$	
$+([a_{2,1}]\circ(b_3\circ b_3)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^6t)(st)(s)$
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+b_6\#([a_{1,1}]\circ b_2\circ b_2)$	
$+b_6\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+b_6\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	$(s^6)(s^2t^2)$
$+b_6\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^8t^2 continued	
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)$ $+([a_{1,1}]\circ b_5\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	$(s^5t)(s^3t)$
$+([a_{1,1}]\circ b_4\circ b_2)\#b_4$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#b_4$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#b_4$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_4$ $+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#b_4$ $+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#b_4$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#b_4$ $+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_4$	$(s^4t^2)(s^4)$
$+b_6\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+b_6\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^6)(s^2t)(t)$
$+([a_{1,1}]\circ b_5\circ b_1)\#b_3\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_3\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#b_3\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_1$	$(s^5t)(s^3)(t)$
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^5t)(s^2t)(s)$

Coefficient equation	Factorization
Degree 10 continued	
s^8t^2 continued	
$+([a_{2,1}]\circ 0\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,1}]\circ 0\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^5t)(s^2)(st)$
$+b_5\#([a_{1,1}]\circ b_3\circ b_2)$ $+b_5\#([a_{2,1}]\circ 0\circ b_2)$ $+b_5\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)$ $+b_5\#([a_{1,2}]\circ b_3\circ (b_1\circ b_1))$ $+b_5\#([a_{2,2}]\circ 0\circ (b_1\circ b_1))$ $+b_5\#([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))$	$(s^5)(s^3t^2)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)$ $+([a_{1,1}]\circ b_4\circ b_1)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)$ $+([a_{1,1}]\circ b_4\circ b_1)\#([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_4\circ b_1)$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_4\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_4\circ b_1)$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)$	$(s^4t)(s^4t)$
$+b_5\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+b_5\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+b_5\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^5)(s^3t)(t)$
$+([a_{1,1}]\circ b_4\circ b_1)\#b_4\#b_1$ $+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#b_4\#b_1$ $+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#b_4\#b_1$ $+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_4\#b_1$	$(s^4t)(s^4)(t)$

Coefficient equation	Factorization
Degree 10 continued	
s^8t^2 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{1,1}]\circ b_4\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^4t)(s^3t)(s)$
$+b_5\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^5)(s^2t)(st)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t)(s^3)(st)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^4t)(s^2t)(s^2)$
$+b_4\#([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$ $+b_4\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4)(s^3t)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_2$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	$(s^3t)(s^3t)(s^2)$
$+b_4\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^4)(s^2t)(s^2t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^3t)(s^3)(s^2t)$

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3	
$0 =$	
$[a_{1,1}] \circ b_7 \circ b_3$	
$+[a_{2,1}] \circ 0 \circ b_3$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ b_3$	
$+[a_{4,1}] \circ 0 \circ b_3$	
$+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ b_3$	
$+[a_{6,1}] \circ 0 \circ b_3$	
$+[a_{7,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3$	
$+[a_{1,2}] \circ b_7 \circ 0$	
$+[a_{2,2}] \circ 0 \circ 0$	
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ 0$	
$+[a_{4,2}] \circ 0 \circ 0$	(s^7t^3)
$+[a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ 0$	
$+[a_{6,2}] \circ 0 \circ 0$	
$+[a_{7,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0$	
$+[a_{1,3}] \circ b_7 \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{3,3}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{6,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1)$	
$+[a_{7,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1)$	
$+([a_{1,1}] \circ b_7 \circ b_2)\#b_1$	
$+([a_{2,1}] \circ 0 \circ b_2)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ b_2)\#b_1$	
$+([a_{4,1}] \circ 0 \circ b_2)\#b_1$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ b_2)\#b_1$	
$+([a_{6,1}] \circ 0 \circ b_2)\#b_1$	
$+([a_{7,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1$	$(s^7t^2)(t)$
$+([a_{1,2}] \circ b_7 \circ (b_1 \circ b_1))\#b_1$	
$+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_5 + b_1 \circ b_3 \circ b_3 + b_2 \circ b_2 \circ b_3) \circ (b_1 \circ b_1))\#b_1$	
$+([a_{4,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$	
$+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_3) \circ (b_1 \circ b_1))\#b_1$	
$+([a_{6,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1$	
$+([a_{7,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_6\circ b_3)\#b_1$	
$+([a_{2,1}]\circ (b_3\circ b_3)\circ b_3)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_3)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_3)\#b_1$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_3)\#b_1$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#b_1$	
$+([a_{1,2}]\circ b_6\circ 0)\#b_1$	
$+([a_{2,2}]\circ (b_3\circ b_3)\circ 0)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ 0)\#b_1$	$(s^6t^3)(s)$
$+([a_{4,2}]\circ 0\circ 0)\#b_1$	
$+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ 0)\#b_1$	
$+([a_{6,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#b_1$	
$+([a_{1,3}]\circ b_6\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,3}]\circ (b_3\circ b_3)\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{4,3}]\circ 0\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{5,3}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{6,3}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_7\circ b_1)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#b_2$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_5+b_1\circ b_3\circ b_3+b_2\circ b_2\circ b_3)\circ b_1)\#b_2$	
$+([a_{4,1}]\circ 0\circ b_1)\#b_2$	$(s^7t)(t^2)$
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_3)\circ b_1)\#b_2$	
$+([a_{6,1}]\circ 0\circ b_1)\#b_2$	
$+([a_{7,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,1}]\circ (b_3\circ b_3)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_6\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^6t^2)(st)$
$+([a_{2,2}]\circ (b_3\circ b_3)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{6,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}] \circ b_5 \circ b_3)\#b_2$	
$+([a_{2,1}] \circ 0 \circ b_3)\#b_2$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_3)\#b_2$	
$+([a_{4,1}] \circ 0 \circ b_3)\#b_2$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3)\#b_2$	
$+([a_{1,2}] \circ b_5 \circ 0)\#b_2$	
$+([a_{2,2}] \circ 0 \circ 0)\#b_2$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ 0)\#b_2$	$(s^5t^3)(s^2)$
$+([a_{4,2}] \circ 0 \circ 0)\#b_2$	
$+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_2$	
$+([a_{1,3}] \circ b_5 \circ (b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_2$	
$+([a_{1,1}] \circ b_6 \circ b_2)\#b_1\#b_1$	
$+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_2)\#b_1\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_2)\#b_1\#b_1$	
$+([a_{4,1}] \circ 0 \circ b_2)\#b_1\#b_1$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_2)\#b_1\#b_1$	
$+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_2)\#b_1\#b_1$	$(s^6t^2)(s)(t)$
$+([a_{1,2}] \circ b_6 \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{2,2}] \circ (b_3 \circ b_3) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{4,2}] \circ 0 \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{6,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_1\#b_1$	
$+b_7\#b_3$	$(s^7)(t^3)$

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_6\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_3\circ b_3)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,1}]\circ(b_3\circ b_3)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	$(s^6t)(st^2)$
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_5\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{4,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,1}]\circ 0\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_5\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,2}]\circ b_5\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^5t^2)(s^2t)$
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_4\circ b_3)\#b_3$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_3)\#b_3$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_3)\#b_3$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#b_3$	
$+([a_{1,2}]\circ b_4\circ 0)\#b_3$	
$+([a_{2,2}]\circ (b_2\circ b_2)\circ 0)\#b_3$	$(s^4t^3)(s^3)$
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_2)\circ 0)\#b_3$	
$+([a_{4,2}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#b_3$	
$+([a_{1,3}]\circ b_4\circ (b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{2,3}]\circ (b_2\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{4,3}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{2,1}]\circ (b_3\circ b_3)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^6t)(st)(t)$
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_6\circ b_1)\#b_2\#b_1$	
$+([a_{2,1}]\circ (b_3\circ b_3)\circ b_1)\#b_2\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_1)\#b_2\#b_1$	$(s^6t)(t^2)(s)$
$+([a_{4,1}]\circ 0\circ b_1)\#b_2\#b_1$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_1)\#b_2\#b_1$	
$+([a_{6,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#b_1$	
$+([a_{1,1}]\circ b_5\circ b_2)\#b_2\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#b_2\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#b_2\#b_1$	
$+([a_{4,1}]\circ 0\circ b_2)\#b_2\#b_1$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#b_1$	$(s^5t^2)(s^2)(t)$
$+([a_{1,2}]\circ b_5\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_2\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^5t^2)(st)(s)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_5\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+b_6\#([a_{1,1}]\circ b_1\circ b_3)$	$(s^6)(st^3)$
$+b_6\#([a_{1,2}]\circ b_1\circ 0)$	
$+b_6\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	$(s^5t)(s^2t^2)$
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	$(s^4t^2)(s^3t)$
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_3)\#b_4$	
$+([a_{2,1}]\circ 0\circ b_3)\#b_4$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#b_4$	
$+([a_{1,2}]\circ b_3\circ 0)\#b_4$	
$+([a_{2,2}]\circ 0\circ 0)\#b_4$	$(s^3t^3)(s^4)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#b_4$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#b_4$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#b_4$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_4$	
$+b_6\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	$(s^6)(st^2)(t)$
$+b_6\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^5t)(s^2t)(t)$
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^5t)(st^2)(s)$
$+([a_{1,1}]\circ b_4\circ b_2)\#b_3\#b_1$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#b_3\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#b_3\#b_1$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_3\#b_1$ $+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#b_3\#b_1$ $+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#b_3\#b_1$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#b_3\#b_1$ $+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#b_1$	$(s^4t^2)(s^3)(t)$

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^4t^2)(s^2t)(s)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+b_6\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^6)(st)(t^2)$
$+([a_{1,1}]\circ b_5\circ b_1)\#b_2\#b_2$	$(s^5t)(s^2)(t^2)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_2\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_2\#b_2$	
$+([a_{4,1}]\circ 0\circ b_1)\#b_2\#b_2$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#b_2$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t^2)(s^2)(st)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^5t)(st)(s)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+b_5\#([a_{1,1}]\circ b_2\circ b_3)$	$(s^5)(s^2t^3)$
$+b_5\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	
$+b_5\#([a_{1,2}]\circ b_2\circ 0)$	
$+b_5\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+b_5\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+b_5\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ 0\circ b_2)$	$(s^4t)(s^3t^2)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_3\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,2}]\circ 0\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_3\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{2,2}]\circ 0\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ 0\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_3\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,2}]\circ 0\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ 0\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_3\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ 0\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))$	
$+b_5\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	$(s^5)(s^2t^2)(t)$
$+b_5\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+b_5\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+b_5\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	$(s^4t)(s^3t)(t)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	$(s^4t)(s^2t^2)(s)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#b_4\#b_1$	$(s^3t^2)(s^4)(t)$
$+([a_{2,1}]\circ 0\circ b_2)\#b_4\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_4\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_4\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_4\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_4\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	$(s^3t^2)(s^3t)(s)$
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+b_5\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	$(s^5)(s^2t)(t^2)$
$+b_5\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#b_2$	
$+b_5\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^5)(st^2)(st)$
$+([a_{1,1}]\circ b_4\circ b_1)\#b_3\#b_2$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#b_3\#b_2$	$(s^4t)(s^3)(t^2)$
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#b_3\#b_2$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_2$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t)(s^2t)(st)$
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_2$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	$(s^4t)(st^2)(s^2)$
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_2$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ (b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ 0\circ b_2)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t^2)(s^3)(st)$
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^3t^2)(s^2t)(s^2)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	$(s^4t)(s^2t)(s)(t)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^4t)(s^2)(st)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+b_4\#([a_{1,1}]\circ b_3\circ b_1)\#b_2$	$(s^4)(s^3t)(t^2)$
$+b_4\#([a_{2,1}]\circ 0\circ b_1)\#b_2$	
$+b_4\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	
$+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4)(s^2t^2)(st)$
$+b_4\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+b_4\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t)(s^3t)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2$	$(s^3t)(s^2t^2)(s^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	

Coefficient equation	Factorization
Degree 10 continued	
s^7t^3 continued	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_1\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1\#b_1$	$(s^3t)(s^3t)(s)(t)$
$+b_4\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$ $+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^4)(s^2t)(st^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^3t)(s^3)(st^2)$
$([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^3t)(s^2t)(s^2t)$
$+([a_{1,1}]\circ b_2\circ b_2)\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^2t^2)(s^3)(s^2t)$
$+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^4)(s^2t)(st)(t)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(s^3)(st)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^3t)(s^2t)(s^2)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(s^2t)(st)(s)$
$+b_3\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^3)(s^2t)(s^2t)(t)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4	
$0 =$ $[a_{1,1}] \circ b_6 \circ b_4$ $+[a_{2,1}] \circ (b_3 \circ b_3) \circ b_4$ $+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_4$ $+[a_{4,1}] \circ 0 \circ b_4$ $+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_4$ $+[a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_4$ $+[a_{1,2}] \circ b_6 \circ (b_2 \circ b_2)$ $+[a_{2,2}] \circ (b_3 \circ b_3) \circ (b_2 \circ b_2)$ $+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ (b_2 \circ b_2)$ $+[a_{4,2}] \circ 0 \circ (b_2 \circ b_2)$ $+[a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ (b_2 \circ b_2)$ $+[a_{6,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_2 \circ b_2)$ $+[a_{1,3}] \circ b_6 \circ (b_1 \circ b_1 \circ b_2)$ $+[a_{2,3}] \circ (b_3 \circ b_3) \circ (b_1 \circ b_1 \circ b_2)$ $+[a_{3,3}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_2)$ $+[a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_2)$ $+[a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_2)$ $+[a_{6,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_2)$ $+[a_{1,4}] \circ b_6 \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$ $+[a_{2,4}] \circ (b_3 \circ b_3) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$ $+[a_{3,4}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$ $+[a_{4,4}] \circ 0 \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$ $+[a_{5,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$ $+[a_{6,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1)$	(s^6t^4)

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}] \circ b_6 \circ b_3)\#b_1$ $+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_3)\#b_1$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_3)\#b_1$ $+([a_{4,1}] \circ 0 \circ b_3)\#b_1$ $+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_3)\#b_1$ $+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3)\#b_1$ $+([a_{1,2}] \circ b_6 \circ 0)\#b_1$ $+([a_{2,2}] \circ (b_3 \circ b_3) \circ 0)\#b_1$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ 0)\#b_1$ $+([a_{4,2}] \circ 0 \circ 0)\#b_1$ $+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ 0)\#b_1$ $+([a_{6,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_1$ $+([a_{1,3}] \circ b_6 \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{2,3}] \circ (b_3 \circ b_3) \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_1$ $+([a_{6,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1$	$(s^6t^3)(t)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_5\circ b_4)\#b_1$	$(s^5t^4)(s)$
$+([a_{2,1}]\circ 0\circ b_4)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_4)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_4)\#b_1$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_4)\#b_1$	
$+([a_{1,2}]\circ b_5\circ(b_2\circ b_2))\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_2\circ b_2))\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_2\circ b_2))\#b_1$	
$+([a_{4,2}]\circ 0\circ(b_2\circ b_2))\#b_1$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#b_1$	
$+([a_{1,3}]\circ b_5\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{4,3}]\circ 0\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{5,3}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_1$	
$+([a_{1,4}]\circ b_5\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,4}]\circ 0\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,4}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{4,4}]\circ 0\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{5,4}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_6\circ b_2)\#b_2$	$(s^6t^2)(t^2)$
$+([a_{2,1}]\circ(b_3\circ b_3)\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ b_2)\#b_2$	
$+([a_{4,1}]\circ 0\circ b_2)\#b_2$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ b_2)\#b_2$	
$+([a_{6,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2$	
$+([a_{1,2}]\circ b_6\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,2}]\circ(b_3\circ b_3)\circ(b_1\circ b_1))\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_4+b_2\circ b_2\circ b_2)\circ(b_1\circ b_1))\#b_2$	
$+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#b_2$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#b_2$	
$+([a_{6,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,2}]\circ b_5\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,3}]\circ b_5\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,3}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{5,3}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^5t^3)(st)$
$+([a_{1,1}]\circ b_4\circ b_4)\#b_2$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_4)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_4)\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_4)\#b_2$ $+([a_{1,2}]\circ b_4\circ(b_2\circ b_2))\#b_2$ $+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_2\circ b_2))\#b_2$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_2\circ b_2))\#b_2$ $+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#b_2$ $+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_2))\#b_2$ $+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_2))\#b_2$ $+([a_{3,3}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_2))\#b_2$ $+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_2$ $+([a_{1,4}]\circ b_4\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2$ $+([a_{2,4}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2$ $+([a_{3,4}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2$ $+([a_{4,4}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2$	$(s^4t^4)(s^2)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}] \circ b_5 \circ b_3)\#b_1\#b_1$	
$+([a_{2,1}] \circ 0 \circ b_3)\#b_1\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_3)\#b_1\#b_1$	
$+([a_{4,1}] \circ 0 \circ b_3)\#b_1\#b_1$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_3)\#b_1\#b_1$	
$+([a_{1,2}] \circ b_5 \circ 0)\#b_1\#b_1$	
$+([a_{2,2}] \circ 0 \circ 0)\#b_1\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ 0)\#b_1\#b_1$	$(s^5t^3)(s)(t)$
$+([a_{4,2}] \circ 0 \circ 0)\#b_1\#b_1$	
$+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_1\#b_1$	
$+([a_{1,3}] \circ b_5 \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_1\#b_1$	
$+([a_{1,1}] \circ b_6 \circ b_1)\#b_3$	
$+([a_{2,1}] \circ (b_3 \circ b_3) \circ b_1)\#b_3$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_4 + b_2 \circ b_2 \circ b_2) \circ b_1)\#b_3$	$(s^6t)(t^3)$
$+([a_{4,1}] \circ 0 \circ b_1)\#b_3$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_2) \circ b_1)\#b_3$	
$+([a_{6,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_3$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_5\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{4,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{4,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{1,2}]\circ b_5\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^5t^2)(st^2)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ(b_4\circ b_3)\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^4t^3)(s^2t)$
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_4)\#b_3$	
$+([a_{2,1}]\circ 0\circ b_4)\#b_3$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_4)\#b_3$	
$+([a_{1,2}]\circ b_3\circ(b_2\circ b_2))\#b_3$	
$+([a_{2,2}]\circ 0\circ(b_2\circ b_2))\#b_3$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#b_3$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_2))\#b_3$	$(s^3t^4)(s^3)$
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_2))\#b_3$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_3$	
$+([a_{1,4}]\circ b_3\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{2,4}]\circ 0\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{3,4}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_3$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_5\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^5t^2)(st)(t)$
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_5\circ b_2)\#b_2\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#b_2\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#b_2\#b_1$	
$+([a_{4,1}]\circ 0\circ b_2)\#b_2\#b_1$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#b_1$	$(s^5t^2)(t^2)(s)$
$+([a_{1,2}]\circ b_5\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{4,2}]\circ 0\circ (b_1\circ b_1))\#b_2\#b_1$	
$+([a_{5,2}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#b_2\#b_1$	
$+b_6\#b_4$	$(s^6)(t^4)$
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	$(s^5t)(st^3)$
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	$(s^4t^2)(s^2t^2)$
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	$(s^3t^3)(s^3t)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_2\circ b_4)\#b_4$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_4)\#b_4$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#b_4$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#b_4$	$(s^2t^4)(s^4)$
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#b_4$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_4$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_4$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_4$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{1,1}]\circ b_5\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^5t)(st^2)(t)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_5\circ b_1)\#b_3\#b_1$	
$+([a_{2,1}]\circ 0\circ b_1)\#b_3\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_3\#b_1$	$(s^5t)(t^3)(s)$
$+([a_{4,1}]\circ 0\circ b_1)\#b_3\#b_1$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	$(s^4t^2)(s^2t)(t)$
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^4t^2)(st^2)(s)$
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_3)\#b_3\#b_1$	
$+([a_{2,1}]\circ 0\circ b_3)\#b_3\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#b_3\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#b_3\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#b_3\#b_1$	$(s^3t^3)(s^3)(t)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#b_3\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#b_3\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#b_3\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_3\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t^3)(s^2t)(s)$
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^5t)(st)(t^2)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{4,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{1,1}]\circ b_4\circ b_2)\#b_2\#b_2$	$(s^4t^2)(s^2)(t^2)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#b_2\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#b_2\#b_2$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#b_2$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#b_2\#b_2$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#b_2\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#b_2\#b_2$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#b_2$	
$+([a_{2,1}]\circ 0\circ b_3)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t^3)(s^2)(st)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ 0\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^4t^2)(st)(s)(t)$
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+b_5\#([a_{1,1}]\circ b_1\circ b_4)$	
$+b_5\#([a_{1,2}]\circ b_1\circ(b_2\circ b_2))$	$(s^5)(st^4)$
$+b_5\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))$	
$+b_5\#([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	$(s^4t)(s^2t^3)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ 0\circ b_2)$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_3\circ (b_1\circ b_1))$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,2}]\circ 0\circ (b_1\circ b_1))$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_3\circ (b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{2,2}]\circ 0\circ (b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ 0\circ b_2)$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_3\circ (b_1\circ b_1))$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ 0\circ (b_1\circ b_1))$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))$	$(s^3t^2)(s^3t^2)$
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_2)$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{2,2}]\circ 0\circ (b_1\circ b_1))$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_2)$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_3\circ (b_1\circ b_1))$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_2)$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_2)$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_3\circ (b_1\circ b_1))$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{2,2}]\circ 0\circ (b_1\circ b_1))$	
$+b_5\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+b_5\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	$(s^5)(st^3)(t)$
$+b_5\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	$(s^4t)(s^2t^2)(t)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	$(s^4t)(st^3)(s)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	$(s^3t^2)(s^3t)(t)$
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	$(s^3t^2)(s^2t^2)(s)$
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_2\circ b_3)\#b_4\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_4\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#b_4\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_4\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_4\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_4\#b_1$	$(s^2t^3)(s^4)(t)$
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{1,1}]\circ b_2\circ b_3)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ 0)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$ $+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	$(s^2t^3)(s^3t)(s)$
$+b_5\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$ $+b_5\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(s^5)(st^2)(t^2)$
$+b_5\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^5)(t^3)(st)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^4t)(s^2t)(t^2)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t)(st^2)(st)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}] \circ b_4 \circ b_1)\#b_3\#b_2$ $+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_1)\#b_3\#b_2$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_1)\#b_3\#b_2$ $+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_1)\#b_3\#b_2$	$(s^4t)(t^3)(s^2)$
$+([a_{1,1}] \circ b_3 \circ b_2)\#b_3\#b_2$ $+([a_{2,1}] \circ 0 \circ b_2)\#b_3\#b_2$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#b_3\#b_2$ $+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#b_3\#b_2$ $+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#b_3\#b_2$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#b_3\#b_2$	$(s^3t^2)(s^3)(t^2)$
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{2,1}] \circ (b_1 \circ b_1) \circ b_1)\#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^3t^2)(s^2t)(st)$
$+([a_{1,1}] \circ b_3 \circ b_2)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_2$ $+([a_{2,1}] \circ 0 \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_2)\#b_2$ $+([a_{2,1}] \circ 0 \circ b_2)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_2$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{1,1}] \circ b_1 \circ b_2)\#b_2$ $+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_1) \circ b_2)\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_2$ $+([a_{1,2}] \circ b_3 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_2)\#b_2$ $+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_2)\#b_2$ $+([a_{2,2}] \circ 0 \circ (b_1 \circ b_1))\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_2$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,1}] \circ b_1 \circ b_2)\#b_2$ $+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1))\#([a_{1,2}] \circ b_1 \circ (b_1 \circ b_1))\#b_2$	$(s^3t^2)(st^2)(s^2)$
$+([a_{2,1}] \circ (b_1 \circ b_1) \circ b_3)\#b_3\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{1,2}] \circ b_2 \circ 0)\#b_3\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{2,2}] \circ (b_1 \circ b_1) \circ 0)\#b_3\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{1,3}] \circ b_2 \circ (b_1 \circ b_1 \circ b_1))\#b_3\#([a_{1,1}] \circ b_1 \circ b_1)$ $+([a_{2,3}] \circ (b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1))\#b_3\#([a_{1,1}] \circ b_1 \circ b_1)$	$(s^2t^3)(s^3)(st)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^2t^3)(s^2t)(s^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	$(s^4t)(st^2)(s)(t)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	$(s^3t^2)(s^2t)(s)(t)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	$(s^4t)(st)(t^2)(s)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t^2)(s^2)(st)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+b_4\#([a_{1,1}]\circ b_2\circ b_2)\#b_2$ $+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2$ $+b_4\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$ $+b_4\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	$(s^4)(s^2t^2)(t^2)$
$+b_4\#([a_{1,2}]\circ b_1\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+b_4\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4)(st^3)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ 0\circ b_1)\#b_2$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2$	$(s^3t)(s^3t)(t^2)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t)(s^2t^2)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_2$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$	$(s^3t)(st^3)(s^2)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	$(s^2t^2)(s^2t^2)(s^2)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1\#b_1$	$(s^3t)(s^2t^2)(s)(t)$
$+b_4\#([a_{1,1}]\circ b_2\circ b_1)\#b_3$ $+b_4\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_3$	$(s^4)(s^2t)(t^3)$
$+b_4\#([a_{1,1}]\circ b_1\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^4)(st^2)(st^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_3\#b_3$ $+([a_{2,1}]\circ 0\circ b_1)\#b_3\#b_3$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#b_3$	$(s^3t)(s^3)(t^3)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^3t)(s^2t)(st^2)$
$+([a_{1,1}]\circ b_2\circ b_2)\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^2t^2)(s^3)(st^2)$
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^2t^2)(s^2t)(s^2t)$
$+([a_{1,1}]\circ b_1\circ b_3)\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{1,2}]\circ b_1\circ 0)\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{1,2}]\circ b_1\circ 0)\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(st^3)(s^3)(s^2t)$
$+b_4\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^4)(st^2)(st)(t)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(s^2t)(st)(t)$

Coefficient equation	Factorization
Degree 10 continued	
s^6t^4 continued	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^3t)(s^2t)(t^2)(s)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$	$(s^3t)(st^2)(s^2)(t)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(st^2)(st)(s)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t^2)(s^3)(st)(t)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^2t^2)(s^2t)(s^2)(t)$
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t^2)(s^2t)(st)(s)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^3t)(s^2)(st)(t^2)$
$+b_3\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$ $+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$ $+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^3)(s^2t)(st^2)(t)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^2t)(s^2t)(st^2)(s)$
$+b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3)(s^2t)(st)(t^2)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_2$	$(s^2t)(s^2t)(s^2)(t^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t)(st^2)(s^2)(st)$

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5	
$0 =$	
$[a_{1,1}] \circ b_5 \circ b_5$	
$+[a_{2,1}] \circ 0 \circ b_5$	
$+[a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_5$	
$+[a_{4,1}] \circ 0 \circ b_5$	
$+[a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_5$	
$+[a_{1,2}] \circ b_5 \circ 0$	
$+[a_{2,2}] \circ 0 \circ 0$	
$+[a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ 0$	
$+[a_{4,2}] \circ 0 \circ 0$	
$+[a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0$	
$+[a_{1,3}] \circ b_5 \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2)$	
$+[a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2)$	
$+[a_{3,3}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2)$	(s^5t^5)
$+[a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2)$	
$+[a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2)$	
$+[a_{1,4}] \circ b_5 \circ 0$	
$+[a_{2,4}] \circ 0 \circ 0$	
$+[a_{3,4}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ 0$	
$+[a_{4,4}] \circ 0 \circ 0$	
$+[a_{5,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0$	
$+[a_{1,5}] \circ b_5 \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{2,5}] \circ 0 \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{3,5}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{4,5}] \circ 0 \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1)$	
$+[a_{5,5}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}] \circ b_5 \circ b_4)\#b_1$	$(s^5t^4)(t)$
$+([a_{2,1}] \circ 0 \circ b_4)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ b_4)\#b_1$	
$+([a_{4,1}] \circ 0 \circ b_4)\#b_1$	
$+([a_{5,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_4)\#b_1$	
$+([a_{1,2}] \circ b_5 \circ (b_2 \circ b_2))\#b_1$	
$+([a_{2,2}] \circ 0 \circ (b_2 \circ b_2))\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_2 \circ b_2))\#b_1$	
$+([a_{4,2}] \circ 0 \circ (b_2 \circ b_2))\#b_1$	
$+([a_{5,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_2 \circ b_2))\#b_1$	
$+([a_{1,3}] \circ b_5 \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{2,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{4,3}] \circ 0 \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{5,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_2))\#b_1$	
$+([a_{1,4}] \circ b_5 \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{2,4}] \circ 0 \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{3,4}] \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{4,4}] \circ 0 \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{5,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}] \circ b_4 \circ b_5)\#b_1$	$(s^4t^5)(s)$
$+([a_{2,1}] \circ (b_2 \circ b_2) \circ b_5)\#b_1$	
$+([a_{3,1}] \circ (b_1 \circ b_1 \circ b_2) \circ b_5)\#b_1$	
$+([a_{4,1}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ b_5)\#b_1$	
$+([a_{1,2}] \circ b_4 \circ 0)\#b_1$	
$+([a_{2,2}] \circ (b_2 \circ b_2) \circ 0)\#b_1$	
$+([a_{3,2}] \circ (b_1 \circ b_1 \circ b_2) \circ 0)\#b_1$	
$+([a_{4,2}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_1$	
$+([a_{1,3}] \circ b_4 \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2))\#b_1$	
$+([a_{2,3}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2))\#b_1$	
$+([a_{3,3}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2))\#b_1$	
$+([a_{4,3}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_3 + b_1 \circ b_2 \circ b_2))\#b_1$	
$+([a_{1,4}] \circ b_4 \circ 0)\#b_1$	
$+([a_{2,4}] \circ (b_2 \circ b_2) \circ 0)\#b_1$	
$+([a_{3,4}] \circ (b_1 \circ b_1 \circ b_2) \circ 0)\#b_1$	
$+([a_{4,4}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ 0)\#b_1$	
$+([a_{1,5}] \circ b_4 \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{2,5}] \circ (b_2 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{3,5}] \circ (b_1 \circ b_1 \circ b_2) \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	
$+([a_{4,5}] \circ (b_1 \circ b_1 \circ b_1 \circ b_1) \circ (b_1 \circ b_1 \circ b_1 \circ b_1 \circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_5\circ b_3)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_3)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_3)\#b_2$	
$+([a_{4,1}]\circ 0\circ b_3)\#b_2$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#b_2$	
$+([a_{1,2}]\circ b_5\circ 0)\#b_2$	
$+([a_{2,2}]\circ 0\circ 0)\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ 0)\#b_2$	$(s^5t^3)(t^2)$
$+([a_{4,2}]\circ 0\circ 0)\#b_2$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#b_2$	
$+([a_{1,3}]\circ b_5\circ(b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{4,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{5,3}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_4)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_4)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_4)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_4\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t^4)(st)$
$+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,4}]\circ b_4\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,4}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,4}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,4}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_3\circ b_5)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_5)\#b_2$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_5)\#b_2$	
$+([a_{1,2}]\circ b_3\circ 0)\#b_2$	
$+([a_{2,2}]\circ 0\circ 0)\#b_2$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ 0)\#b_2$	
$+([a_{1,3}]\circ b_3\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2))\#b_2$	
$+([a_{2,3}]\circ 0\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2))\#b_2$	$(s^3t^5)(s^2)$
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2))\#b_2$	
$+([a_{1,4}]\circ b_3\circ 0)\#b_2$	
$+([a_{2,4}]\circ 0\circ 0)\#b_2$	
$+([a_{3,4}]\circ (b_1\circ b_1\circ b_1)\circ 0)\#b_2$	
$+([a_{1,5}]\circ b_3\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{2,5}]\circ 0\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{3,5}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1))\#b_2$	
$+([a_{1,1}]\circ b_4\circ b_4)\#b_1\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_4)\#b_1\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_4)\#b_1\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_4)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_4\circ (b_2\circ b_2))\#b_1\#b_1$	
$+([a_{2,2}]\circ (b_2\circ b_2)\circ (b_2\circ b_2))\#b_1\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_2)\circ (b_2\circ b_2))\#b_1\#b_1$	
$+([a_{4,2}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_2\circ b_2))\#b_1\#b_1$	$(s^4t^4)(s)(t)$
$+([a_{1,3}]\circ b_4\circ (b_1\circ b_1\circ b_2))\#b_1\#b_1$	
$+([a_{2,3}]\circ (b_2\circ b_2)\circ (b_1\circ b_1\circ b_2))\#b_1\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_2))\#b_1\#b_1$	
$+([a_{4,3}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_2))\#b_1\#b_1$	
$+([a_{1,4}]\circ b_4\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_1\#b_1$	
$+([a_{2,4}]\circ (b_2\circ b_2)\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_1\#b_1$	
$+([a_{3,4}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_1\#b_1$	
$+([a_{4,4}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_1\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_5\circ b_2)\#b_3$	$(s^5t^2)(t^3)$
$+([a_{2,1}]\circ 0\circ b_2)\#b_3$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_2)\#b_3$	
$+([a_{4,1}]\circ 0\circ b_2)\#b_3$	
$+([a_{5,1}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_3$	
$+([a_{1,2}]\circ b_5\circ(b_1\circ b_1))\#b_3$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_3$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ(b_1\circ b_1))\#b_3$	
$+([a_{4,2}]\circ 0\circ(b_1\circ b_1))\#b_3$	
$+([a_{5,2}]\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_3$	
$+([a_{1,1}]\circ b_4\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^4t^3)(st^2)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{1,2}]\circ b_4\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ 0)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ 0)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{1,3}]\circ b_4\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{2,3}]\circ(b_2\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	
$+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)$	
$+([a_{4,3}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_3\circ b_4)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_4)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_4)\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_4)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,2}]\circ b_3\circ(b_2\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ 0\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,2}]\circ 0\circ(b_2\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(s^3t^4)(s^2t)$
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,4}]\circ b_3\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{1,4}]\circ b_3\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{2,4}]\circ 0\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{2,4}]\circ 0\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{3,4}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)$	
$+([a_{3,4}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_2\circ b_5)\#b_3$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_5)\#b_3$	
$+([a_{1,2}]\circ b_2\circ 0)\#b_3$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_3$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2))\#b_3$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2))\#b_3$	$(s^2t^5)(s^3)$
$+([a_{1,4}]\circ b_2\circ 0)\#b_3$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ 0)\#b_3$	
$+([a_{1,5}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1))\#b_3$	
$+([a_{2,5}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1))\#b_3$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_4\circ b_3)\#b_2\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_3)\#b_2\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_3)\#b_2\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#b_2\#b_1$	
$+([a_{1,2}]\circ b_4\circ 0)\#b_2\#b_1$	
$+([a_{2,2}]\circ (b_2\circ b_2)\circ 0)\#b_2\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_2)\circ 0)\#b_2\#b_1$	$(s^4t^3)(t^2)(s)$
$+([a_{4,2}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#b_2\#b_1$	
$+([a_{1,3}]\circ b_4\circ (b_1\circ b_1\circ b_1))\#b_2\#b_1$	
$+([a_{2,3}]\circ (b_2\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_2\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_1))\#b_2\#b_1$	
$+([a_{4,3}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#b_2\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_4\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,2}]\circ (b_2\circ b_2)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_2)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^4t^3)(st)(t)$
$+([a_{4,2}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_4\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,3}]\circ (b_2\circ b_2)\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_2)\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{4,3}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_4)\#b_2\#b_1$	
$+([a_{2,1}]\circ 0\circ b_4)\#b_2\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_4)\#b_2\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_2\circ b_2))\#b_2\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_2\circ b_2))\#b_2\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_2\circ b_2))\#b_2\#b_1$	$(s^3t^4)(s^2)(t)$
$+([a_{1,3}]\circ b_3\circ (b_1\circ b_1\circ b_2))\#b_2\#b_1$	
$+([a_{2,3}]\circ 0\circ (b_1\circ b_1\circ b_2))\#b_2\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_2))\#b_2\#b_1$	
$+([a_{1,4}]\circ b_3\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_2\#b_1$	
$+([a_{2,4}]\circ 0\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_2\#b_1$	
$+([a_{3,4}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1\circ b_1))\#b_2\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{2,1}]\circ 0\circ b_4)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_4)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_2\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_2\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_2\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_3\circ (b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t^4)(st)(s)$
$+([a_{2,3}]\circ 0\circ (b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,3}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,4}]\circ b_3\circ (b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,4}]\circ 0\circ (b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{3,4}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_5\circ b_1)\#b_4$	
$+([a_{2,1}]\circ 0\circ b_1)\#b_4$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2)\circ b_1)\#b_4$	$(s^5t)(t^4)$
$+([a_{4,1}]\circ 0\circ b_1)\#b_4$	
$+([a_{5,1}]\circ (b_1\circ b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_4$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)$	$(s^4t^2)(st^3)$
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ 0)$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	$(s^3t^3)(s^2t^2)$
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_2\circ b_4)\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,1}]\circ b_2\circ b_4)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_4)\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_4)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ 0\circ b_1)$	$(s^2t^4)(s^3t)$
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)$	
$+([a_{1,1}]\circ b_1\circ b_5)\#b_4$	
$+([a_{1,2}]\circ b_1\circ 0)\#b_4$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_3+b_1\circ b_2\circ b_2))\#b_4$	$(st^5)(s^4)$
$+([a_{1,4}]\circ b_1\circ 0)\#b_4$	
$+([a_{1,5}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1\circ b_1))\#b_4$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_4\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^4t^2)(st^2)(t)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_2)\#b_3\#b_1$	$(s^4t^2)(t^3)(s)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#b_3\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#b_3\#b_1$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#b_3\#b_1$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#b_3\#b_1$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#b_3\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#b_3\#b_1$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^3t^3)(s^2t)(t)$
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^3t^3)(st^2)(s)$
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_2\circ b_4)\#b_3\#b_1$	$(s^2t^4)(s^3)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_4)\#b_3\#b_1$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#b_3\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#b_3\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#b_3\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_3\#b_1$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_3\#b_1$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_3\#b_1$	
$+([a_{1,1}]\circ b_2\circ b_4)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	$(s^2t^4)(s^2t)(s)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_4)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^4t^2)(st)(t^2)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_4\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{2,2}]\circ(b_2\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_2)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	
$+([a_{4,2}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_3\circ b_3)\#b_2\#b_2$	
$+([a_{2,1}]\circ 0\circ b_3)\#b_2\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#b_2\#b_2$	
$+([a_{1,2}]\circ b_3\circ 0)\#b_2\#b_2$	
$+([a_{2,2}]\circ 0\circ 0)\#b_2\#b_2$	$(s^3t^3)(s^2)(t^2)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#b_2\#b_2$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#b_2\#b_2$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#b_2\#b_2$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_2\#b_2$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_4)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,2}]\circ b_2\circ(b_2\circ b_2))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_2\circ b_2))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_2))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t^4)(s^2)(st)$
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_2))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,4}]\circ b_2\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,4}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,1}]\circ 0\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_3\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{2,2}]\circ 0\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^3t^3)(st)(s)(t)$
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{1,3}]\circ b_3\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{2,3}]\circ 0\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+([a_{3,3}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	
$+b_5\#b_5$	$(s^5)(t^5)$

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_2\circ b_2))$	$(s^4t)(st^4)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))$	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_4)$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_2\circ b_2))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_4)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_2\circ b_2))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_4)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_2\circ b_2))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	$(s^3t^2)(s^2t^3)$
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_3)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ 0)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ 0)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	$(s^4t)(st^3)(t)$
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_4\circ b_1)\#b_4\#b_1$	$(s^4t)(t^4)(s)$
$+([a_{2,1}]\circ (b_2\circ b_2)\circ b_1)\#b_4\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_2)\circ b_1)\#b_4\#b_1$	
$+([a_{4,1}]\circ (b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_4\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_2)\#b_1$	$(s^3t^2)(s^2t^2)(t)$
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_2\circ (b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,2}]\circ (b_1\circ b_1)\circ (b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_2\circ (b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{2,2}]\circ (b_1\circ b_1)\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ (b_1\circ b_1)\circ (b_1\circ b_1))\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{2,2}]\circ (b_1\circ b_1)\circ (b_1\circ b_1))\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{2,1}]\circ (b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ (b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{2,2}]\circ (b_1\circ b_1)\circ (b_1\circ b_1))\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	$(s^3t^2)(st^3)(s)$
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{1,2}]\circ b_3\circ (b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{2,2}]\circ 0\circ (b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ 0)\#b_1$	
$+([a_{3,2}]\circ (b_1\circ b_1\circ b_1)\circ (b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ (b_1\circ b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,1}]\circ (b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{2,1}]\circ (b_1\circ b_1)\circ b_3)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,2}]\circ (b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	$(s^2t^3)(s^3t)(t)$
$+([a_{2,2}]\circ (b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{2,2}]\circ (b_1\circ b_1)\circ 0)\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_2\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_2\circ (b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_2\circ (b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{2,3}]\circ (b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{2,3}]\circ (b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{2,3}]\circ (b_1\circ b_1)\circ (b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ (b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	$(s^2t^3)(s^2t^2)(s)$
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_2)\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1$	
$+([a_{1,1}]\circ b_1\circ b_4)\#b_4\#b_1$	$(st^4)(s^4)(t)$
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#b_4\#b_1$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#b_4\#b_1$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_4\#b_1$	
$+([a_{1,1}]\circ b_1\circ b_4)\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	$(st^4)(s^3t)(s)$
$+([a_{1,1}]\circ b_1\circ b_4)\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_3\circ b_1)\#b_1$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ 0\circ b_1)\#b_1$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_4\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(s^4t)(st^2)(t^2)$
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ(b_2\circ b_2)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4t)(t^3)(st)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_2)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{4,1}]\circ(b_1\circ b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(s^3t^2)(s^2t)(t^2)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t^2)(st^2)(st)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_3\circ b_2)\#b_3\#b_2$	$(s^3t^2)(t^3)(s^2)$
$+([a_{2,1}]\circ 0\circ b_2)\#b_3\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#b_3\#b_2$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#b_3\#b_2$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#b_3\#b_2$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#b_2$	
$+([a_{1,1}]\circ b_2\circ b_3)\#b_3\#b_2$	$(s^2t^3)(s^3)(t^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_3\#b_2$	
$+([a_{1,2}]\circ b_2\circ 0)\#b_3\#b_2$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_3\#b_2$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_3\#b_2$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_3\#b_2$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t^3)(s^2t)(st)$
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	$(s^2t^3)(st^2)(s^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_2$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2$	
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	$(st^4)(s^3)(st)$
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)$	
$+([a_{1,1}]\circ b_1\circ b_4)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	$(st^4)(s^2t)(s^2)$
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,2}]\circ b_1\circ(b_2\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_2))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2$	
$+([a_{1,4}]\circ b_1\circ(b_1\circ b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2$	
$+([a_{1,1}]\circ b_3\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	$(s^3t^2)(st^2)(s)(t)$
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_1\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1\#b_1$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,1}]\circ b_2\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	$(s^2t^3)(s^2t)(s)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{1,2}]\circ b_2\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_1\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_1\#b_1$	
$+([a_{2,1}]\circ 0\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	$(s^3t^2)(st)(t^2)(s)$
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	
$+([a_{1,2}]\circ b_3\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	
$+([a_{2,2}]\circ 0\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	
$+([a_{3,2}]\circ(b_1\circ b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2\#b_1$	
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_3)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t^3)(s^2)(st)(t)$
$+([a_{1,2}]\circ b_2\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,2}]\circ(b_1\circ b_1)\circ 0)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{1,3}]\circ b_2\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+([a_{2,3}]\circ(b_1\circ b_1)\circ(b_1\circ b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	
$+b_4\#([a_{1,1}]\circ b_1\circ b_3)\#b_2$	$(s^4)(st^3)(t^2)$
$+b_4\#([a_{1,2}]\circ b_1\circ 0)\#b_2$	
$+b_4\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$	
$+b_4\#b_4\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^4)(t^4)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2$	$(s^3t)(s^2t^2)(t^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$	
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2$	
$+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2$	

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^3t)(st^3)(st)$
$+([a_{1,1}]\circ b_3\circ b_1)\#b_4\#b_2$ $+([a_{2,1}]\circ 0\circ b_1)\#b_4\#b_2$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_4\#b_2$	$(s^3t)(t^4)(s^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t^2)(s^2t^2)(st)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)\#b_2$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_3)\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ 0)\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)\#b_2$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_3)\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ 0)\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_2$	$(s^2t^2)(st^3)(s^2)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1\#b_1$ $+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_3)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ 0)\#b_1\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_1\#b_1$	$(s^3t)(st^3)(s)(t)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_1\#b_1$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_1\#b_1$	$(s^2t^2)(s^2t^2)(s)(t)$

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+b_4\#([a_{1,1}]\circ b_1\circ b_2)\#b_3$ $+b_4\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_3$	$(s^4)(st^2)(t^3)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_3$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_3$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_2\circ b_1)\#b_3$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_3$	$(s^3t)(s^2t)(t^3)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^3t)(st^2)(st^2)$
$+([a_{1,1}]\circ b_2\circ b_2)\#b_3\#b_3$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_3\#b_3$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_3\#b_3$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_3\#b_3$	$(s^2t^2)(s^3)(t^3)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(s^2t^2)(s^2t)(st^2)$
$+([a_{1,1}]\circ b_1\circ b_3)\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$ $+([a_{1,2}]\circ b_1\circ 0)\#b_3\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_2)$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))$	$(st^3)(s^3)(st^2)$
$+([a_{1,2}]\circ b_1\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(st^3)(s^2t)(s^2t)$
$+b_4\#b_3\#([a_{1,1}]\circ b_2\circ b_1)$ $+b_4\#b_3\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)$	$(t^4)(s^3)(s^2t)$
$+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(st^2)(st)(t)$
$+([a_{1,1}]\circ b_3\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{2,1}]\circ 0\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$	$(s^3t)(st^2)(t^2)(s)$
$+([a_{2,1}]\circ 0\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{3,1}]\circ(b_1\circ b_1\circ b_1)\circ b_1)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^3t)(t^3)(st)(s)$

Coefficient equation	Factorization
Degree 10 continued	
s^5t^5 continued	
$+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t^2)(s^2t)(st)(t)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(s^2t^2)(s^2t)(t^2)(s)$
$+([a_{1,1}]\circ b_2\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_1$	$(s^2t^2)(st^2)(s^2)(t)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(s^2t^2)(st^2)(st)(s)$
$+([a_{1,2}]\circ b_1\circ 0)\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#b_3\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(st^3)(s^3)(st)(t)$
$+([a_{1,1}]\circ b_1\circ b_3)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{1,2}]\circ b_1\circ 0)\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{1,2}]\circ b_1\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{1,1}]\circ b_2\circ b_1)\#b_2\#b_1$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_2\#b_1$	$(st^3)(s^2t)(s^2)(t)$
$+([a_{1,2}]\circ b_1\circ 0)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$ $+([a_{1,3}]\circ b_1\circ(b_1\circ b_1\circ b_1))\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_1)\#b_1$	$(st^3)(s^2t)(st)(s)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_2)\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$ $+([a_{1,2}]\circ b_2\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$ $+([a_{2,2}]\circ(b_1\circ b_1)\circ(b_1\circ b_1))\#b_2\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^2t^2)(s^2)(st)(t^2)$
$+b_3\#([a_{1,1}]\circ b_1\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^3)(st^2)(st^2)(t)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^2t)(s^2t)(st^2)(t)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_3\#b_1$	$(s^2t)(s^2t)(t^3)(s)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_1$	$(s^2t)(st^2)(st^2)(s)$
$+b_3\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_2$	$(s^3)(st^2)(st)(t^2)$
$+([a_{1,1}]\circ b_2\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,1}]\circ b_1\circ b_2)\#b_2\#b_2$ $+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#b_2\#b_2$	$(s^2t)(st^2)(s^2)(t^2)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#b_3\#b_2\#([a_{1,1}]\circ b_1\circ b_1)$	$(s^2t)(t^3)(s^2)(st)$
$+([a_{2,1}]\circ(b_1\circ b_1)\circ b_1)\#([a_{1,2}]\circ b_1\circ(b_1\circ b_1))\#([a_{1,1}]\circ b_1\circ b_1)\#b_1\#b_1$	$(s^2t)(st^2)(st)(s)(t)$