Two Essays on Dynamic Learning under Information Asymmetry

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Dedication

To my parents and Lei.

Contents

Acknowledgements **i**

Dedication **ii**

List of Tables **vi**

List of Figures **vii**

1 Introduction **1**

2 Dynamic Adverse Selection and Asset Sales **3**

2.1 Introduction . . . 3

2.1.1 Related Literature . . . 7

2.2 The Model . . . 10

2.2.1 Strategies . . . 12

2.2.2 Belief . . . 13

2.2.3 Continuation Value . . . 14

2.2.4 Equilibrium Definition . . . 15

2.3 Equilibrium Analysis . . . 15

2.3.1 Optimal bidding for buyers . . . 17

2.4 Stationary Equilibrium . . . 19

2.5 Non-Stationary Equilibrium . . . 21

2.5.1 Averaging Trading Price . . . 26

2.5.2 Market liquidity . . . 26

2.6 Comparative Statics . . . 29

2.6.1 Share of sophisticated buyers . . . 29
2.6.2 Search Friction . . . 30
2.7 Extensions . . . 30
2.7.1 Equilibrium without Assumption 2.2 . . . 31
2.7.2 Quality Shocks . . . 32
2.8 Empirical Implication . . . 35
2.9 Conclusion . . . 36

3 Secret Scouting **38**
3.1 Introduction . . . 38
3.2 Baseline Setup . . . 43
3.2.1 Value Function . . . 46
3.2.2 Low Preemption Case . . . 48
3.2.3 High Preemption Case . . . 52
3.2.4 Public Disclosure . . . 53
3.2.5 Startup Welfare . . . 56
3.3 Endogenous Disclosure . . . 59
3.4 Conclusion . . . 63

References **65**

Appendix A. Proof of Results in Chapter 1 **75**
A.1 Proof of Lemma 2.1 . . . 75
A.2 Proof of Lemma 2.2 . . . 75
A.3 Proof of Lemma 2.4 . . . 76
A.4 Proof of Lemma 2.5 . . . 77
A.5 Proof of Lemma 2.6 . . . 77
A.6 Proof of Theorem 2.1 . . . 77
A.7 Proof of Lemma 2.7 . . . 78
A.8 Proof of Theorem 2.2 . . . 81
A.9 Proof of Proposition 2.2 . . . 82
A.10 Proof of Corollary 2.2 . . . 83

Appendix B. Proof of Results in Chapter 2 **84**
B.1 Proof of Lemma 3.1 84
B.2 Proof of Lemma 3.2 84
B.3 Proof of Proposition 3.1 85
B.4 Proof of Proposition 3.2 86
B.5 Proof of Lemma 3.3 88
B.6 Proof of Proposition 3.3 89
B.7 Proof of Proposition 3.4 90
B.8 Proof of Lemma 3.4 90
B.9 Proof of Proposition 3.5 91

List of Tables

List of Figures

2.1 Sequence of Events 11
2.2 Non-stationary Equilibrium 24
2.3 Trading pattern 24
2.4 Average Trading Price 27
2.5 Trading volume 28
2.6 Equilibrium without Assumption 2.2 32
3.1 a_I as Functions of π—High vs. Low Preemption 48
3.2 Equilibrium Value Function and Effort Level —Low Preemption 51
3.3 Equilibrium Waiting Value—High vs. Low Preemption 53
3.4 Value Function and Effort Level —Low Preemtpion & Public Experimentation 55
3.5 Equilibrium Value Function—High Preemtpion & Public Experimentation . . 56
3.6 Expected Discovery Time Given Initial Belief —Private & Public Experimentation 57
3.7 Expected Discovery Time Given Second Mover Payoff 58
3.8 Value Function for S —High vs. Low Preemtpion 62

Chapter 1

Introduction

In real world markets with asymmtric information, making decisions take time. In the real estate market, it takes time for the seller to find a potential buyer in the market and potential buyer can learn through the listing time; in the Venture Capital industry, VCs also spend a lot of time learning about the potential startup before make their financing decision. In these environments, timing decision features dynamic learning, and it can reveal information about the underlying house quality or the profitability of the startup. In my dissertation, I explicitly take the dynamic learning into consideration, and investigate how does the dynamic learning over time affect the agents' behavior.

In the first essay "Dynamic Adverse Selection and Asset Sales", I present a dynamic adverse selection model in the decentralized market with bilateral trading. Investors meet in decentralized market to trade heterogeneous assets under asymmetric information. The cream-skimming effect emerges due to the heterogeneous sophistication among buyers, where the low-type seller strategically forgoes trading opportunities with gains from trade in order to take advantage of the unsophisticated investors in the market. When the market is pessimistic, time to sale increases in asset quality, heterogeneous sophistication improves market liquidity; when the market is optimistic, time to sale decreases in asset quality, cream-skimming incentive endogenously occurs, which reduces the trading efficiency. The implications and predictions on initial public offerings, real estate market are discussed in the paper.

In the second essay "Secret Scouting", coathored with Xuelin Li, we consider the dynamic learning model in Venture Capital industry when there is asymmetric information about the profiability of the startups among VCs. We find that VCs prefer secrecy when searching for targets. As a result, only the investments in viable startups are disclosed, but the failed ones are discarded silently. We extend the standard preemption game to explain the efficiency loss and the individual rationale of doing so. We show that secrecy creates pessimism. Compared to the fully disclosing case, VCs will stop hunting for startups too early in an initially promising industry. This could happen even if no technology failures are observed in realization. However, hiding failures becomes a dominant strategy when the return of the VC industry is right-skewed. VCs use secret scouting to make the competitors believe that the industry is a dead end and reduce the preemption threats.

The remainder of the dissertation is organized as follows. Chapter 2 contains the first essay "Dynamic Adverse Selection and Asset Sales". Chapter 3 contains the second essay "Secret Scouting". Proofs of the results can be found in the appendix.

Chapter 2

Dynamic Adverse Selection and Asset Sales

2.1 Introduction

In financial markets with adverse selection, time to sale conveys valuable information about the asset quality. In the mortgage market, investors in mortgage-backed securities cannot observe the quality of underlying portfolios of mortgages, but they can update the quality through the time it takes between origination and securitization. In the IPO market, potential investors can infer the firm's prospect through the time it takes to go IPO. In the real estate market, potential buyers can infer the house quality through how long the house has been listed on the market. In the labor market, the employer can not observe the ability of the employee, but he can infer the employee's ability through the unemployment duration.

On the theoretical side, dynamic signaling models are widely applied to understand the role of time to sale. Many recent studies Taylor (1999); Kremer and Skrzypacz (2007); Fuchs and Skrzypacz (2013) view time to sale as a signaling device. The main logic is as follows: in a market with privately informed sellers, high type sellers, who have lower waiting cost or higher outside option of no trade, have an incentive to delay trade to signal the quality. Therefore, time to sale can serve as a signal of quality, i.e., the longer it takes for the asset to be traded, the more likely that the asset is of high quality.

On the empirical side, however, the evidence about the time to sale is mixed. In the mortgage market, Adelino et al. (2019) and Klee and Shin (2020) find that mortgage quality improves

[1] as the length of time increases between origination and securitization[2]. In the IPO market, Fuchs et al. (2016) and Gratton et al. (2018) find that high-quality firms wait longer before IPO[3]. In the labor markets, negative duration dependence is widely documented; that is, a longer unemployment spell is associated with a lower job finding rate and lower wagesClark et al. (1979); Kroft et al. (2013); Jarosch and Pilossoph (2019); in the real estate market, time-on-the-market is negatively correlated with the sale prices. Krainer (2001, 2008); Dubé and Legros (2016).This suggests that time to sale is negatively correlated with quality in these markets. The distinct predictions of time to sale in different markets are puzzles and cannot be explained by the extant models within the same framework.

The heterogeneous sophistication among buyers is nontrivial in asset sales and can play a key role in understanding the trading in the decentralized market. Many assets, such as corporate bonds and derivatives, are traded in decentralized market. Sellers of assets are heterogeneous and have private information about the quality of their assets as well as their own valuation. Buyers of assets are heterogeneously sophisticated too. Some of these buyers, such as venture capitalists, hedge funds, trusts, and investment banks, are more sophisticated so that they may know more than other buyers about the quality of assets for sale.[4] In the IPO market, institutional investors are better informed about the firm's prospectus and gain significant profits from IPO Chemmanur et al. (2010). In the real estate market, some investors are better sophisticated than others due to their expertise or familiar with the neighborhood Stroebel (2016); Levitt and Syverson (2008). Can the heterogeneity in investor sophistication explain the puzzle of time to sale in different markets? How does the share of sophisticated investors shape the trading dynamics?

To explain the empirical puzzle on the relation between time to sale and asset quality, I introduce investors' heterogeneous sophistication into a dynamic adverse selection model in the decentralized market with bilateral trading. The key result of my model is that a cream-skimming effect can emerge when the investors are heterogeneously sophisticated. When the perceived asset quality is high on the market, unsophisticated buyers are willing to pay a pooling price for the average asset. Then matching with an unsophisticated investors is more profitable since they can get a better offer. Therefore, the low-type seller has incentives to gamble on meeting with

[1]They use the default rate as the measure of mortgage performance

[2]This is also called the skimming property in the literature, which states that time to sale of an asset increases in quality

[3]Fuchs et al. (2016) use the post-IPO profitability as a measure of quality

[4]The better sophistication may be due to better information acquisition technology or larger information process capacity Kacperczyk et al. (2019)

unsophisticated investors by rejecting trading opportunities with sophisticated investors. Thus, trading between the low-type seller and sophisticated investors completely break down. This "gambling" incentive for the low-type seller can generate a negative relationship between time to sale and asset quality. These results provide a new angle to understand the negative duration dependence phenomenon in the labor market and the real estate market.

Now I will introduce the key ingredients of the model. A seller (she) has one unit of the indivisible asset to sell due to liquidity reason, and the asset quality could be high or low. Two groups of buyers are characterized by their sophistication level. Sophisticated investors possess better information about asset quality than unsophisticated investors. In a bilateral OTC meeting, a seller searches buyers sequentially, and once met, the buyer (he) makes a take-it-or-leave-it offer to the seller. The offers are private in the sense that offers history is not observable to subsequent buyers[5]. The buyer faces adverse selection because he is uninformed about the asset quality if he is unsophisticated. He makes either a pooling offer, a separating offer, or randomizes between both. The pooling offer targets sellers who own a high-quality asset. Given any price accepted by the high-type seller, under asymmetric information, owners of low-quality assets could also trade at this price and receive information rents. To avoid paying these rents, the unsophisticated buyer can instead bid a separating offers for low-quality assets, but he would forgo the gains from trade with the high-type seller. The bidding strategy for unsophisticated buyers depends on the perceived asset quality on the market, the more optimistic about the asset quality, the more likely he will bid a pooling offer.

In the model, the correlation of time to sale and asset quality relies on the market belief about the asset quality. When the market is pessimistic, i.e., the market belief is low, then time to sale is positively correlated with asset quality due to the signaling effect; when the market is optimistic, i.e., the market belief is high, then time to sale is negatively correlated with asset quality due to cream-skimming effect. Signaling effect together with the cream-skimming effect can explain the empirical puzzle on time to sale.

The first dynamic force of the model is a *signaling effect* whereby high-type seller signals her quality by turning down offer from unsophisticated investors. When the perceived asset quality is low on the market, unsophisticated buyers are reluctant to pay a pooling price for the asset, therefore, high-type seller only trades with sophisticated buyers with high valuation. On the

[5]Private offer assumption is to capture the opacity in OTC market, it also captures the widely adopted non-disclosure agreement(NDA) in financial markets

other hand, the low-type seller's waiting value depends on whether the unsophisticated investors are willing to pay a pooling offer. When the unsophisticated buyers are pessimistic, they are unwilling to pay a pooling price, therefore, waiting is too costly for the low-type and she prefers to trade earlier. The trading rates difference between high-type and low-type seller provides a signaling device for the high-type seller, the longer the asset stays on the market without trading, the more likely the asset is high quality. This signaling effect can generate positive relationship between time to sale and asset quality.

The second dynamic force in the model is a *cream-skimming* effect whereby low-type seller is only willing to trade with unsophisticated buyers and forgo trading opportunities with sophisticated buyers. When the perceived asset quality is high, the unsophisticated investors are willing to trade at pooling price. Anticipating the potential pooling offers from the unsophisticated investors under favorable market belief, the low-type seller is reluctant to trade with the sophisticated buyers at separating price as her waiting value is higher than buyers' valuation for low-quality asset. Thus, the trade between the low-type seller and the sophisticated buyers breaks down, and the low-type seller cream-skims the unsophisticated buyers on the market. This cream-skimming effect generates positive correlation between the asset quality and time to sale, in particular, it is the low-type seller who is more likely to wait in order to gamble on matching with unsophisticated investors. I also identify the condition when the cream-skimming effect can exist (Assumption 2.2). Cream-skimming effect requires that gambling on matching with unsophisticated investors for the low-type seller is not too costly. Thus, in the market with less search friction and less sophisticated investors, the cream-skimming effect is more likely to exist. The search friction and ratio of sophisticated investors difference in different market can explain the empirical puzzle on the time-to-sale.

Heterogeneous sophistication among buyers shape the trading dynamics in the following ways. First, it improves trading efficiency when the market is pessimistic. Sophisticated investors are better informed, and they can identify good assets in the market, their participation helps boost trading volume and improve trading efficiency. Unlike the static lemon model Akerlof (1970) where the market breaks down completely, in the presence of sophisticated investors, the information asymmetry between sophisticated buyers and sellers is not severe enough such that trading between sophisticated buyers and the high-type seller is still active. This mechanism is reminiscent of the mechanism in Glode and Opp (2016) which information intermediary can reduce the asymmetric information between sellers and buyers and improve trading efficiency.

Second, it provide incentives for the low-type seller to cream-skim the unsophisticated investors. The cream-skimming effect relies on the heterogeneous sophistication among buyers, where buyers with heterogeneous sophistication have different valuation for the same asset. When the valuation difference is large enough, then low-type seller is willing to take advantage of this valuation difference by cream-skimming only the unsophisticated investors. And this effect disappears when the buyers are homogeneously sophisticated.

In the model, trading volume, defined as the average trading rate, deteriorates over time regardless of the perceived asset quality. And this deterioration of trading volume is due to the learning effect from time to sale. The trading volume depends on the relative trading rates for both types of asset and market belief about asset quality. The more aggressively the high-quality assets are traded in the market, the less likely the asset remained on the market is high-quality, therefore the average trading rate for the asset remained on the market decreases over time.

In the extension, I consider the quality shock[6] to the asset where the asset quality deteriorates over time. And I find that when the quality shock is not severe, the main mechanism in the paper is still robust, and the net effect of time to sale on asset quality depends on the market belief about the asset quality. I can still get positive correlation between time to sale and asset quality when the market is pessimistic and negative correlation when the market is optimistic. However, when the quality shock is severe, then the prediction of time to sale and asset quality is purely driven by the quality shock, and only negative correlation between time to sale and asset quality can exist.

2.1.1 Related Literature

This paper builds on the large literature on adverse selection initiated by the seminal work of Akerlof (1970). Among many other papers, Swinkels (1999), Janssen and Roy1 (2002),Kremer and Skrzypacz (2007),Daley and Green (2012),Camargo and Lester (2014) analyze the dynamic versions of the lemon market models in centralized market or decentralized market where sellers are better informed than buyers. Janssen and Roy1 (2002) show that when sellers are endowed with fixed private information, equilibrium prices increase over time as owners of higher type assets delay trade in order to signal their type. While all assets are eventually sold, trade is delayed, and the market equilibrium is therefore inefficient. Camargo and Lester (2014)

[6]In real estate market and labor market, the negative correlation between time to sale and asset quality could be due to the deterioration of asset quality over time.

find related market dynamics in a decentralized market where buyers and sellers are matched randomly each period. As in the centralized market, Janssen and Roy1 (2002), trade is delayed, and prices and averages quality increase over time. introduce exogenous information into a dynamic signaling model with private offers. In their model, a grade is revealed at some fixed time, provided that trade has not already occurred. In contrast to Swinkels (1999), trade is always delayed with positive probability. A critical insight for their work is that noisy information causes an endogenous market for lemon to develop. In equilibrium, trade breaks down completely just before revelation of the information. Daley and Green (2012, 2016) consider a model in which public news affect quality of a seller's asset is released over time and construct an equilibrium that involves periods of complete market break down. My paper contributes to theoretical literature on dynamic adverse selection. In particular, I follow the line of Swinkels (1999), Zhu (2012) and Daley and Green (2012, 2016) by assuming that investors do not observe previous offers from earlier buyers. Unlike Swinkels (1999) who considers a model in which the lemon condition is not binding, and Daley and Green (2012, 2016) who consider exogenous information revelation, I primarily investigate how does the investor sophistication affects the trading efficiency and asset prices.

My paper is also related to the literature of competitive search models with asymmetric information in the OTC market Guerrieri et al. (2010); Guerrieri and Shimer (2014); Chang (2018). In this literature, the trading rate is a signaling device, where high-quality asset prefers trading in a less liquid market in order to separate from the low-quality asset. The trading rate in my model is a learning device, and different types of asset cannot be traded in different sub-markets, thus, high-type sellers cannot fully separate themselves through trading rate.

My model contributes to the literature of cream-skimming in the financial market. Fishman and Parker (2015), Bolton et al. (2016), Romanyuk and Smolin (2019), Zou (2019) and Vallée and Zeng (2019) show that sophisticated investors can obtain a large share of rent by cherry-picking good assets due to their information advantage. My paper differs with the cream-skimming literature in the following aspects. First, in my model, it is the low-type seller who chooses to cream-skim the unsophisticated buyers in the market, while in the literature, the informed buyers cream-skim the less informed buyer by cherry-picking the good asset in the market. Second, the the quality of assets remaining in the market could deteriorate or ameliorate over time depending on the market belief, while in the cream-skimming literature, they predict that the asset quality deteriorates over time.

My paper is also related to other strands of literature with developing adverse selectionMartel et al. (2020); Hwang (2018). This literature assumes that the initially there is no asymmetric information between sellers and buyers, and sellers learn from holding the asset. The endogenously developed asymmetric information can generate U-shape equilibrium where the trading price and probability drop at the beginning and then increase over time. My paper is different with literature

The most related paper with my mine is Frenkel (2020), who considers dynamic asset sales in the OTC market with a feedback effect. In his model, the seller cares about the gains from trade through the transaction, and the information revealed during the asset sales. And he finds that the market views event of no sales as a bad signal about the asset quality, which is consistent with my model under the high market belief, but the mechanism in these two papers is different. In his model, the low-type seller is reluctant to immediately sell the low-quality asset due to the negative information revealed through asset sales. In my model, the seller's outside option depends on the market belief and relative size of sophisticated investors. When the market is optimistic, the low-type seller values more about the asset than buyers' willingness to pay. Thus, low-type seller has an gambling incentive to match with an unsophisticated investors.

This chapter is also related to asset sales in the financial market. Edmans and Mann (2018) consider the firm's financing through asset sales. They distinguish between core-asset and non core-asset, and claim that by non core-asset is not information sensitive, it has less impact on the stock price due to less asymmetric information. Bond and Leitner (2015) considers the asset sales with multiple units. They show that due to the existence of multiple units of assets, sellers in the market care about the price of the asset sold and the value of their inventories. Frenkel (2020), they find that firms may choose not to trade to avoid revealing bad news about the value of the inventory. This paper provides another explanation of trade delay when the seller cares about the market price he can get and the value of their inventory. In my model, only one single indivisible asset is for sale, and there is no effect on the inventory. The trade delay occurs mainly due to two reasons. When the initial quality of the asset pools is low, unsophisticated buyers are pessimistic about the asset quality, and they are not willing to pay a pooling offer, thus, the seller with the high-type asset does not trade due to his outside option; when the initial quality of asset pools is high, unsophisticated buyers are optimistic about the asset quality, and they are willing to bid pooling price. Anticipating this, the low-type seller is reluctant to trade with sophisticated buyers since the low-type seller's waiting value is higher than the sophisticated buyer's valuation of the low-quality assets. Thus, the trade delay is mainly due to the strategic delay from the low-type seller.

The chapter is organized as follows: in Section 2.2, I describe the environment. In Section 2.3, I present preliminary analysis. In Section 2.4, I characterize the unique stationary equilibrium. In Section 2.5, I characterize the fully dynamic non-stationary equilibrium. In Section 2.6, I consider some comparative statics in the model. In Section 2.8, I present some empirical implications of the model. Section 2.9 concludes.

2.2 The Model

Environment. I consider a dynamic trading game between a long-lived seller (she) and a sequence of short-lived buyers (he), all agents are risk-neutral and discount future cash flow at same rate $r > 0$. Time, denoted by t, is continuous and runs from zero up to infinity $t \in [0, \infty)$. There is one unit of indivisible asset with quality $\theta \in \{H, L\}$. I refer to a type-H asset as "high-quality" and to a type-L as "low-quality". The common prior probability that the asset is high quality is $\pi_0 \in (0, 1)$. At $t = 0$, the seller is endowed with the single indivisible asset and the asset type is her private information.

Type-θ asset delivers cash flow $c_\theta(v_\theta)$ to the seller (buyer) if she (he) owns the asset. Define the holding value of type-θ asset as $V_\theta = v_\theta/r$ to the buyer and $C_\theta = c_\theta/r$ to the seller. C_θ can be interpreted as the outside option for type-θ seller by holding the asset forever. Throughout this paper, I assume that

Assumption 2.1. $v_H > c_H > v_L > c_L$

Assumption 2.1 says that the the assets are more productive if held by buyers rather than sellers ($v_\theta > c_\theta$), and gains from trade are positive for both type of assets. The standard lemon condition[7] holds in the model ($c_H > v_L$), that is, it is not always profitable for the buyer to make a pooling offer to the buyers.

Buyers. A continuum of buyers measures one in the market of two types $z \in \{S, U\}$, where S stands for sophisticated buyer and U stands for unsophisticated buyer. With fraction $s \in (0, 1)$ of buyers are sophisticated, and they are perfectly informed about asset quality owned by the

[7]This assumption (see also in Daley and Green (2012) is stronger than the lemon condition in static adverse selection model where the lemon condition is $\pi_0 V_H + (1 - \pi_0)V_L < C_H$, and π_0 is the initial market belief

seller[8], with fraction $1-s$ of the buyers are unsophisticated, they only observe how long the asset has been traded on the market. Buyers' types are their private information and not observable to the seller.

Trading protocol. Trading takes place in decentralized market. In a bilateral decentralized meeting, the seller searches buyers sequentially with no cost[9]. Buyers arrive at random times that correspond to the jump times of a Poisson process with intensity λ. Once meet with seller, the buyer makes a take-it-or-leave-it (TIOLI)[10] private offer to the seller and then the seller decides whether to accept it or not. The offers are private in the sense that the search history and previous rejected offers are not observable to subsequent buyers. If the seller accepts the offer, then the game ends. Otherwise, the buyer leaves the market the seller continues to search for a new buyer[11]

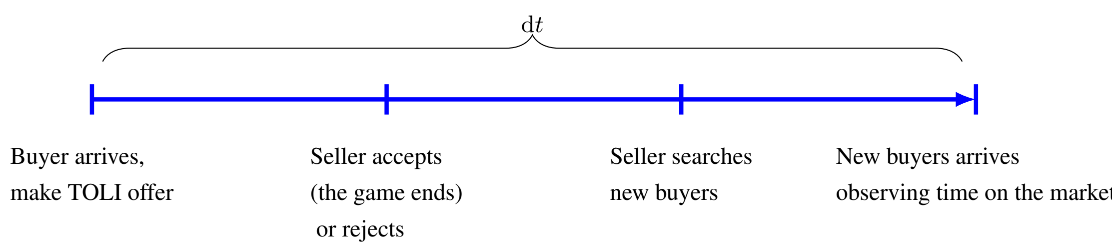


Figure 2.1: Sequence of Events

Discussion of Assumptions: The model entails a number of simplifying assumptions, which are made primarily to facilitate a tractable analysis and to keep the intuition for the main forces accessible. One of the key assumption of the model is that the trading history is private (opaque). The assumption here is that in each time t where no sale takes place, the market cannot tell whether this is because exogenous reasons (no match due to search friction) or because of the seller's decision not to sell. If however, that the seller's decision not to sell always become

[8]I model the sophisticated investors as the ones with better information. In the behavior finance literature, some papers model sophisticated investors as rational investors, and unsophisticated investors as irrational investors, see (Ljungqvist et al. (2006))

[9]This zero-cost assumption allows me to bypass the Diamond paradox (Diamond (1971)) and focus on the sequential nature of search, rather than the pecuniary cost of search.

[10]In the extension, I consider competitive buyers in the market, and the result is robust to competitive buyers in the market.

[11]Unlike Zhu (2012), repeat contacts in my model have zero probability since there is a continuum of buyers with measure 1. No repeated contacts is a key feature in search model with infinite number of buyers Duffie et al. (2005, 2007); Vayanos and Wang (2007); Vayanos and Weill (2008).

public, then in equilibrium, high-quality asset owners can signal their asset quality by turning down unfavorable offers. The private offer assumption in the model is meant to capture in a simple way trading opacity in OTC markets where assets transit through dealers before reaching end customers. Another interpretation of the private offer assumption is the widely adoption of nondisclosure agreements (NDAs) in real estate market and Venture Capital market. The NDAs are used to protect the confidential information about the seller (home owner, startups). Private offer is assumption is only relevant when the gains from trade is positive for low-quality asset. With zero gains from trade, then low-type asset can only be traded together with high-quality asset, thus, trading history does reveal any information.

Another key assumption in the model is that the seller cannot commit to when to sell the asset, which is common in the literature of dynamic adverse selection. Unlike the the static signaling models in financial market Leland and Pyle (1977), sellers cannot signal through the timing of the asset sales (or no sales) due to the search friction. Given any time t, the probability that seller has not met with the buyer is strict positive, therefore, any commitment of sale before t is not feasible. Commitment issue is the main difference between the static model and dynamic model which is first discussed by Admati and Perry (1987) in the context of education signaling.

2.2.1 Strategies

In this subsection, I will discuss the strategies and equilibrium definition. First, I will introduce some notations to describe the buyers' strategy in the full dynamic game, allowing for both pure and mixed strategy.

The strategy for the buyer is a mapping from his information set to a distribution of price offers. I allow mixed strategies for both sellers and buyers in the model. And the strategy for the seller is a mapping from her type and the price offer received to the probability of acceptance. The (Unsophisticated) buyers' belief is represented by a function $\pi : \mathbb{R}_+ \to \Delta\{H, L\}$, which maps the calendar time to the probability that the asset is good quality. For the sophisticated investors, they can observe θ perfectly, therefore, the offer strategies of type-S buyers in a matching with type-θ seller are represented by a mapping $\sigma_t^{S,\theta}$ from $\mathbb{R}_+$ to a set of probability distribution over $\mathbb{R}$, where $\sigma_t^{S,\theta}(p)$ denotes the probability that the type-S buyer's offer in a match at time t with type θ seller is lower than p. The offer strategies of type-U buyers are represented by a mapping σ_t^U from $\mathbb{R}_+$ to a set of probability distribution over $\mathbb{R}$, where $\sigma_t^U(p)$ denotes the probability of type-U buyer's offer in a match at time t is lower than p. The acceptance strategy of for type-θ

seller at time t is represented by a function $\mu_t^\theta : \mathbb{R}_+ \to [0,1]$ where $\mu_t^\theta(p)$ is the type-θ seller's acceptance decision at time t. Let $\mu_t^\theta(p) = 0$ denote the type-θ seller's decision to reject the offer p and $\mu_t^\theta(p) = 1$ denote the seller's decision to accept the offer p.

Given the strategies for buyers and sellers, denote λ_t^θ as the probability that a type-θ seller trades at time t.

$$\lambda_t^\theta = \lambda \left(\underbrace{s \int_p \mu_t^\theta(p)\, \mathrm{d}\sigma_t^{S,\theta}(p)}_{\text{prob trading with S}} + \underbrace{(1-s) \int_p \mu_t^\theta(p)\, \mathrm{d}\sigma_t^{U}(p)}_{\text{prob trading with U}} \right) \tag{2.1}$$

Trading rate for each type of asset captures the likelihood that type-θ get traded in the short interval between $[t, t+\mathrm{d}t]$ conditional no sales before t. There two components in the trading rate for type-θ asset. The first term captures the probability trading with sophisticated buyers, since sophisticated buyers are perfectly informed about the asset quality, their offers are type-dependant. The second term captures the probability trading with unsophisticated buyers. One can interpret the trading rate λ_t^θ as the market liquidity (or trading volume) for type-θ asset [12], higher trading rate means type-θ asset class is more liquid and vice verse.

In a dynamic trading game with search friction, we say the trade is *efficient* if any pair of buyer and seller with positive gains from trade trades with each other with probability one. In our game, the trading is fully efficient if and only if

$$\lambda_t^H = \lambda_t^L = \lambda \tag{2.2}$$

2.2.2 Belief

For the sophisticated investors, they can observe θ perfectly. For the unsophisticated investors, they only observe how long the asset has been traded.

Therefore, given the strategies $\{\sigma_t^{S,\theta}\}_{\theta \in \{H,L\}}$,$\sigma_t^U$, μ_t^θ their perceived quality of asset follows

$$\pi_t = \frac{\pi_0 e^{-\int_0^t \lambda_s^H \mathrm{d}s}}{\pi_0 e^{-\int_0^t \lambda_s^H \mathrm{d}s} + (1-\pi_0) e^{-\int_0^t \lambda_s^L \mathrm{d}s}} \tag{2.3}$$

where λ_t^θ is the trading rate for type-θ asset shown in (2.1). The numerator for (2.3) is the

[12] In this model there is only one unit of asset, and the trading game stops once the asset is traded, thus, we can also interpret the λ_t^θ as the trading volume for type–θ asset at time t.

probability that the high-type asset has not been traded before t, while the denominator is the probability that there is no sales before t.

Rewrite (2.3) recursively, and take $\mathrm{d}t \to 0$, I can obtain the following differential equation

$$\mathrm{d}\pi_t = \pi_t(1-\pi_t)(\lambda_t^L - \lambda_t^H)\mathrm{d}t \tag{2.4}$$

with boundary condition π_0 given.

The dynamics of the belief process is intuitive. As long as the asset has not been traded, the belief about asset quality depends on the relative trading rate of different types of assets. If buyers believe that the high-type asset is traded more aggressively on the market, then the asset remains on the market is more likely to be low-quality. On the contrary, if the low-type seller asset is trading more aggressively, then conditional on no sales, the market will revise their belief upwards. The learning speed $(\lambda_t^L - \lambda_t^H))$ is proportional to the search intensity, the higher the search intensity, the faster information is revealed to the market.

2.2.3 Continuation Value

If a seller of type-θ rejects the offer from buyer at time t and plans to accept an offer at (random) time $\tau > t$, then her expected payoff is $\mathbb{E}\left\{e^{-r\tau}\left(\tilde{p}_\tau - C_\theta\}\right) + C_\theta\right\}$, where τ is the trading time and p_τ is the trading price. Given the strategies taken by the seller and buyers, it can generate the distribution of trading time and trading price, and then I can define the continuation value to type-θ seller if she decides to wait at time t.

$$W_t^\theta = \mathbb{E}\left\{\left(1 - e^{-r(\tau-t)}\right) C_\theta + e^{-r(\tau-t)}\tilde{p}_\tau\right\} \tag{2.5}$$

There are two components in the continuation value. Before trading at time τ, type-θ seller receives the cash flow by holding the asset. $(1 - e^{-r(\tau-t})C_\theta$ captures all the cash flows received between $[t, \tau]$. The second term is the payoff from trading the asset in τ at price p_τ. The expectation is taken on all the potential trading time and trading prices generated by the strategies taken by the agents.

For the high-type seller, the continuation value is straightforward. On the one hand, the continuation value of sellers with high-quality asset is at least C_H since the sellers always have the option to hold on to their assets, and C_H is the outside option for high-type seller. On the other hand, no buyer will offer a price higher than C_H in equilibrium since the buyer has all the bargaining power. Therefore,

$$W_t^H = C_H \tag{2.6}$$

Therefore, I can focus exclusively on the continuation value for the low-type seller to characterize the equilibrium.

2.2.4 Equilibrium Definition

Definition 1. *A Perfect Bayesian Equilibrium is a quadruple* $\left\{\sigma^S, \sigma^U, \mu^\theta, \pi\right\}$ *such that*

1. *Given* $\mu^\theta, \theta \in \{H, L\}$ *type-S buyer chooses* $\sigma_t^{S,\theta}$ *to maximize*

$$\int (V_\theta - p)\mu_t^\theta(p) \mathrm{d}\sigma_t^{S,\theta}(p) \tag{2.7}$$

2. *Given* $\mu^\theta, \theta \in \{H, L\}$ *type-U buyer chooses* σ_t^U *to maximize*

$$\mathbb{E}\left\{\int (V_\theta - p)\mu_t^\theta(p) \mathrm{d}\sigma_t^U(p) \mid \pi_t\right\} \tag{2.8}$$

3. *Given* $\sigma_t^z, z \in \{S, U\}$ *, type-*θ *seller chooses* μ_t^θ *to maximize*

$$W_t^\theta + s\int_p (p - W_t^\theta)\mu_t^\theta(p) \mathrm{d}\sigma_t^{S,\theta}(p) + (1-s)\int_p (p - W_t^\theta)\mu_t^\theta(p) \mathrm{d}\sigma_t^U(p) \tag{2.9}$$

4. *Belief Consistency: Conditional on no trading, the belief follows Bayes' rule* (2.4)

2.3 Equilibrium Analysis

In this section, I will provide some preliminary analysis. First, it is easy to see that the high-quality seller always trades with informed buyers when they meet due to the gains from trade. The existence of informed buyers in the model is the key of providing information without any transaction. In the next lemma, I will show that each type of seller is taking cut-off strategy for the acceptance decision.

Lemma 2.1. *In equilibrium, the optimal acceptance decision for each type of seller $\theta \in \{H, L\}$ is given by*

$$\mu_t^\theta(p_t) = \begin{cases} 0 & \textit{if } p_t < W_t^\theta \\ [0,1] & \textit{if } p_t = W_t^\theta \\ 1. & \textit{if } p_t > W_t^\theta \end{cases} \tag{2.10}$$

Lemma 2.1 states that the optimal acceptance strategy for each seller is a threshold strategy, it depends on the price offered and the continuation value for the seller. Note that W_t^θ is continuous in t because the probability that the buyer arrives at a given time interval vanishes as the length of the time interval shrinks to zero.

Given the cut-off strategy taken by the seller, the payoff to the unsophisticated buyer if he bids p is

$$\pi_t(V_H - p)\mathbb{I}(p \geq C_H) + (1 - \pi_t)(V_L - p)\mathbb{I}(p \geq W_t^L) \tag{2.11}$$

Lemma 2.2. *In equilibrium, three types of prices $p_t^l = W_t^L$, $p_t^h = C_H$ and $p_t^n \geq V_L$ are offered from the unsophisticated buyers.*

In the remaining paper, we call p_t^l as the separating offer, which is only accepted with positive probability by low-type seller, and p_t^h as the pooling offer, which is accepted by sellers with probability 1, and p_t^n as non-serious offer, which is rejected for sure by both types of sellers.

The intuition is straightforward: in order to target the high-type seller, the optimal (lowest) offer made is the C_H, similarly in order to target the low-quality asset, the optimal (lowest) offer made is the reservation value for the low-type. However, the continuation value for the low-type seller could be higher than buyer's valuation for the low-quality asset, therefore, non-serious offer p_t^n is made by the unsophisticated buyers.

Given above lemma, we can characterize the continuation value dynamics for low-type seller. Denote $\gamma_t \equiv (1 - s)\sigma_t^U(C_H)$ as the probability that the low-type seller receives offer C_H [13]. Then the continuation value dynamics follows

$$W_t^L = rC_L\mathrm{d}t + e^{-r\mathrm{d}t}\left\{\gamma_t \mathrm{d}t C_H + (1 - \gamma_t\mathrm{d}t)W_{t+\mathrm{d}t}^L\right\} \tag{2.12}$$

[13] In Maurin (2020), γ_t is interpreted as the measure of market liquidity

We can rewrite (2.12) as the differential equation such that

$$rW_t^L \mathrm{d}t = \underbrace{rC_L \mathrm{d}t}_{\text{cash flow}} + \underbrace{\gamma_t (C_H - W_t^L)\mathrm{d}t}_{\text{trading opportunity}} + \underbrace{\mathrm{d}W_t^L}_{\text{capital gains}} \tag{2.13}$$

The LHS of (2.13) is the required return for holding the asset, and first term on the RHS of (2.13) is the cash flow by holding the asset between $[t, t + \mathrm{d}t]$, the second term captures the possibility to get $p^h = C_H$ from buyers, where γ_t is the probability of seller with low-quality asset to receive a pooling offer (C_H) from unsophisticated investors.

2.3.1 Optimal bidding for buyers

In this section, we will discuss the optimal bidding for the buyers. For the sophisticated buyers, they can observe θ, therefore the bidding strategy is given by

$$p_t^S = \begin{cases} C_H & \text{if } \theta = H \\ \min\{W_t^L, V_L\} & \text{if } \theta = L \end{cases} \tag{2.14}$$

Basically, the informed buyers bid C_H for high-quality asset, and W_t^L for low-quality asset when it is profitable, otherwise, he will bid a non-serious offer. Next, we focus on the optimal bidding for the low-quality seller.

If the unsophisticated investor is targeting both types of sellers in the market, then his payoff is

$$\pi_t(V_H - C_H) + (1 - \pi_t)(V_L - C_H) \tag{2.15}$$

if he targets only the low-type seller in the market, then his payoff is

$$(1 - \pi_t)(V_L - W_t^L) \tag{2.16}$$

and if he makes a non-serious offer, then the payoff is 0.

Lemma 2.3. *The optimal offering strategy for the uninformed buyer is characterized by a threshold belief*

$$\bar{\pi}_t = \frac{C_H - \min\{V_L, W_t^L\}}{V_H - \min\{V_L, W_t^L\}} \tag{2.17}$$

such that

1. *If $\pi_t > \bar{\pi}_t$, then unsophisticated buyers make a pooling offer C_H.*

2. *If $\pi_t < \bar{\pi}_t$ and $V_L > W_t^L$, then unsophisticated buyers make a separating offer W_t^L*

3. *if $\pi_t < \bar{\pi}_t$ and $V_L < W_t^L$, then unsophisticated buyers make a non-serious offer $p < W_t^L$.*

In Lemma 2.3, $\bar{\pi}_t$ is the cut-off belief such that pooling offer is optimal and guarantees a non-negative payoff to the buyer. The intuition for Lemma 2.3 is as follows: when the market belief about the asset quality is high enough, make pooling offer (C_H) to target both types of seller is optimal and profitable. When the market belief is below the threshold belief, and low-type seller values more by holding the asset, then separating offer is optimal and profitable. However, when market belief is below the threshold belief, and low-type seller's continuation value is above the buyer's valuation for low quality asset, then no offer can be accepted with positive probability yet still provides non-negative profit to the buyer. Thus, in this case, only non-serious offers are made by the unsophisticated buyers.

For the remaining paper, I assume that parameters in the paper satisfy

Assumption 2.2. *Assume*

$$\frac{rC_L + \lambda(1-s)C_H}{r + \lambda(1-s)} > V_L \tag{2.18}$$

One way to understand Assumption 2.2 is to consider a naive strategy from the unsophisticated buyers. In the naive strategy, the unsophisticated buyers always bid C_H regardless of his belief about the asset quality. Given the naive strategy, the continuation value to the low-type seller if only targets unsophisticated buyers is

$$\bar{W}^L = \frac{rC_L + \lambda(1-s)C_H}{r + \lambda(1-s)} \tag{2.19}$$

Assumption 2.2 states that, given the naive strategy taken by the unsophisticated investors, the low-type seller has incentive to gamble on meeting with unsophisticated buyers in the market. In the following lemma, we will show that under Assumption 2.2, fully efficient trading is not feasible.

Lemma 2.4. *If Assumption 2.2 holds, equilibrium with efficient trading is not feasible.*

Lemma 2.4 states that in any equilibrium under Assumption 2.2, the trading in the market cannot be efficient. The intuition is as follows: in order to achieve efficient trading, a pooling offer is required from the unsophisticated buyers. Otherwise, high-type seller would rather hold on to the asset. Thus, when the unsophisticated buyers are willing to offer pooling price C_H for sure, then the low-type seller has incentives to deviate and only trades with unsophisticated buyers in the market, therefore, efficient trading is not feasible. One implication of for Lemma 2.4 is the presence of sophisticated investors may deteriorate the trading efficiency.

2.4 Stationary Equilibrium

In this section, we construct a stationary equilibrium. In the stationary equilibrium, the strategies for sellers and buyers are independent of calendar time, that is $\sigma_t^z = \sigma^z$ for $z \in \{S, U\}$ and $\mu_t^\theta = \mu^\theta$ for $\theta \in \{H, L\}$. In the following lemma, we derive the necessary conditions for the stationary equilibrium.

Lemma 2.5. *Under the stationary equilibrium,* $\pi_t = \pi^* \equiv \frac{C_H - V_L}{V_H - V_L}$

Note that π^* is the threshold belief such that the unsophisticated buyers are break-even when they offer a pooling price C_H. Lemma 2.5 says that in the stationary equilibrium, the market belief is constant over time, and no trading does not reveal any information about asset quality in stationary equilibrium.

Lemma 2.6. *The continuation value for the low-type seller* $W^L = V_L$ *in the stationary equilibrium.*

Lemma 2.6 states that the continuation value for the low-type seller equals the buyer's valuation for the low-quality asset. Any continuation value different from V_L is not consistent with the definition of stationary equilibrium. First, let's consider that $W^L < V_L$, in this case, the low-type asset is traded efficiently. From the Lemma 2.4, we know that it should be the case that $\lambda^H < \lambda^L = \lambda$. Therefore, conditional on trading not occurs, the market belief improves over time, which is not consistent with Lemma 2.5, which states that in stationary equilibrium, the market belief is constant over time. Similarly, we can argue that $W^L > V_L$ is not feasible in the stationary equilibrium using the same logic. Therefore, the continuation value for the low-type seller is uniquely pinned down. Given above lemmas, we can characterize the stationary equilibrium.

Theorem 2.1. *There is an unique stationary equilibrium which is consist of* $\{\pi^*, \sigma^{U*}, \mu^{L*}\}$ *such that*

$$\pi^* = \frac{C_H - V_L}{V_H - V_L} \tag{2.20}$$

and unsophisticated offers

$$\sigma^{U*}(C_H) = \frac{\gamma^*}{\lambda(1-s)} \tag{2.21}$$

$$\sigma^{U*}(V_L) = 1 - \frac{\gamma^*}{\lambda(1-s)} \tag{2.22}$$

and low-type seller accepts

$$\mu^{L*}(V_L) = \frac{\lambda m}{\lambda - \gamma^*} \tag{2.23}$$

where γ^* *is the probability the low-type seller receives pooling offer in stationary equilibrium.*

In Theorem 2.1, I construct a stationary equilibrium, where unsophisticated buyers are randomizing between pooling offer C_H and separating offer V_L, and low-type seller is mixing between accepting and rejecting V_L. One way to think about the mixing strategy for the low-type seller is that, ideally, she can always trade with sophisticated buyer at separating price, and only accepting pooling price from the unsophisticated buyers, thus, we can reach the stationary equilibrium in which both types of asset are traded at the same rate such that market belief is freezing. But the buyer's type is not observed, therefore, she cannot tell whether the separating price is from sophisticated buyer or unsophisticated buyer. The denominator $\lambda - \gamma^*$ in (2.23) captures the probability that low-type seller receives separating offer V_L in the stationary equilibrium, and numerator λm captures the probability that sophisticated buyer offers V_L to the low-type seller. For the low-type seller, he rejects separating offer with positive probability in order to freeze the market belief.

Given Theorem 2.1, we can have the the following result about the expected trading time in stationary equilibrium.

Proposition 2.1. *The expected trading time* τ *for both types of asset in stationary equilibrium is given by*

$$\tau^* = \frac{1}{\lambda s + \gamma^*} \tag{2.24}$$

Apparently, the expected trading time in stationary equilibrium is decreasing with the share of informed buyers in market. To understand this result, I will break down the trading rate for the high quality asset $\lambda m + \gamma^*$ into two parts. First, λm in (2.24) captures the offer from the informed buyers, who are always willing to pay a high price C_H for the high-quality asset, and this term is increasing in the share of informed buyers, since the more sophisticated buyers in the market, the more likely that she will receive a good price. Second, the term γ^* captures the probability of pooling offer C_H from the uninformed buyers. This term, however, doesn't depend on the share of informed buyers in the market.[14] Therefore, the expected trading time is decreasing with the share of informed buyers in the market.

2.5 Non-Stationary Equilibrium

In the previous section, we have characterized the unique stationary equilibrium. Stationary equilibrium requires that we fix the belief at level π^* such that each agent repeats his strategy over time. In this section, I will characterize equilibrium of fully dynamic game with initial market belief about the asset quality given. To help us characterize the non-stationary equilibrium, we will first derive two lemmas about the trading dynamics.

Lemma 2.7. *(Cream-skimming) When $\pi_t < \pi^*, W_t^L < V_L$ and when $\pi_t > \pi^*, W_t^L > V_L$*

This is the key result for the equilibrium dynamics in non-stationary equilibrium. Lemma 2.7 states that when the market belief is above the stationary belief, the continuation value for low-type seller is above the buyers' valuation of low-quality asset, which leads to the trade break down between low-type seller and informed buyers in the market, even though positive gains from trade can be realized in the potential trade $(V_L - C_L > 0)$, this creates the strategic delay incentives for the low-type seller when the market condition is favorable. On the other hand, when the market belief is below π^*, then the reservation value for seller with low-quality is below V_L. Intuitively, the market belief affects the bidding strategy for the uninformed investors in the market. In particular, when the market belief is very high, the uninformed investors are more likely to bid C_H for the average asset. Anticipating that the uninformed buyers are willing to pay C_H, low-type seller has incentive to strategically reject any offer from informed buyers and waits to trade with uninformed buyers. Assumption 2.2 guarantees that the benefit from waiting outweighs the cost of waiting, therefore, the reservation value for the low-type seller is higher than buyers' valuation for low-quality asset. On the other hand, when the market belief is below

[14]The γ^* is the probability that a pooling offer is submitted by unsophisticated buyer which is pinned down by the continuation value of low-type seller in equilibrium

π^*, the uninformed investors is pessimistic about the asset quality, and they are only willing to pay a fair price to buy the low quality asset. Following Lemma 2.7, we can show that the trading rate for the two types of asset are given by

Corollary 2.1. *When the market belief $\pi_t < \pi^*$, $\lambda_t^L = \lambda$, and $\lambda_t^H = \lambda s$; when the market belief $\pi_t > \pi^*$, the $\lambda_t^L = \lambda\,(1-s)$, and $\lambda_t^H = \lambda$*

In the model, the trading rate for each type of asset can be interpreted as the trading volume. Then Corollary 2.1 says that the trading volume for each type of assets depend on the market belief about the asset quality. When market belief is very low, low-quality assets (lemon) are traded more frequently, while when the market belief is very high, assets with high-quality asset are actively traded. The first part of Corollary 2.1 is consistent with Daley and Green (2012, 2016) where when the market is very pessimistic, high-type sellers are more inclined to wait, and therefore, the low-quality assets are more actively traded.

Given the Lemma 2.7 and Corollary 2.1, we can summarize the bidding behavior for the uninformed buyers, and the probability that the low-type seller receives a pooling offer C_H is given by

$$\gamma_t = \begin{cases} 0 & \text{if } \pi_t < \pi^* \\ \gamma^* & \text{if } \pi_t = \pi^* \\ \lambda(1-s) & \text{if } \pi_t > \pi^* \end{cases}$$

we can characterize the continuation value for the low-quality seller, and therefore, characterize the full dynamics for the non-stationary equilibrium.

Recall that the continuation value dynamics follows (2.13), and from Lemma 2.7, we know the market dynamics follows

$$\mathrm{d}\pi_t = \begin{cases} \lambda\,(1-s)\,\pi_t\,(1-\pi_t)\,\mathrm{d}t & \text{if } \pi_t < \pi^* \\ -\lambda s \pi_t\,(1-\pi_t)\,\mathrm{d}t & \text{if } \pi_t > \pi^* \end{cases} \tag{2.25}$$

with π_0 is given. In the non-stationary equilibrium, the market belief about the asset quality are monotonically increasing (decreasing) depending on the initial market belief. And in either cases, the belief converges to the stationary belief level. Therefore, we can define $t_i^*(i = 1, 2)$ as the time it takes to converge to stationary equilibrium when $\pi_0 < \pi^*$ $(i = 1)$ and $\pi_0 > \pi^*$ $(i = 2)$

And then

$$t_1^* = \frac{1}{\lambda(1-s)}\left(\log(\frac{\pi^*}{1-\pi^*}) - \log(\frac{\pi_0}{1-\pi_0})\right) \tag{2.26}$$

$$t_2^* = \frac{1}{\lambda s}\left(\log\left(\frac{\pi_0}{1-\pi_0}\right) - \log\left(\frac{\pi^*}{1-\pi^*}\right)\right) \tag{2.27}$$

The time it takes to converge the stationary belief depend on the initial belief π_0, and the learning speed ($\lambda_t^L - \lambda_t^H$). And the relationship between share of informed buyers and learning speed is not monotonic

Theorem 2.2. *The equilibrium dynamics in the non-stationary game is characterized by*

1. *When $\pi_0 < \pi^*$, the equilibrium dynamics are given by*

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda(1-s)t}} & t < t_1^* \\ \pi^*, \quad t \geq t_1^* \end{cases} \tag{2.28}$$

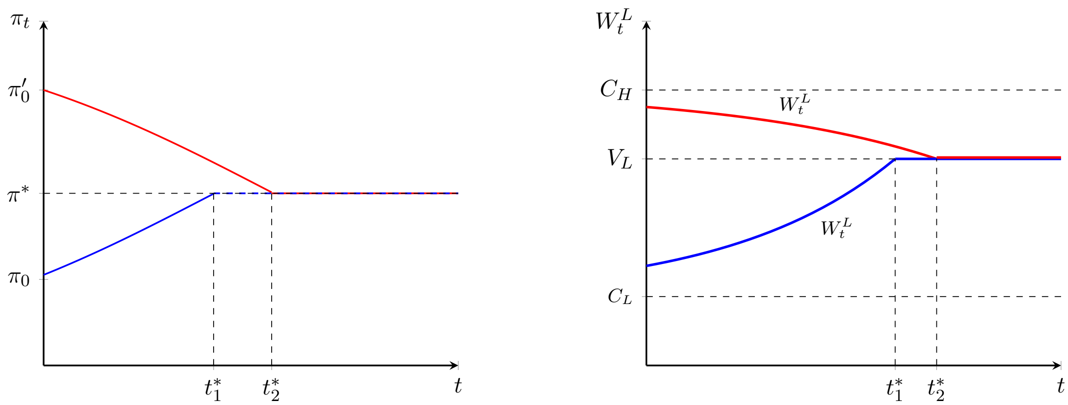
$$W_t^L = \begin{cases} C_L + (V_L - C_L)e^{-r(t_1^*-t)} & t < t_1^* \\ V_L \quad t > t_1^* \end{cases} \qquad v \tag{2.29}$$

2. *When $\pi_0 > \pi^*$, the equilibrium dynamics are given by*

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{\lambda st}} & t < t_2^* \\ \pi^*, \quad t \geq t_2^* \end{cases} \tag{2.30}$$

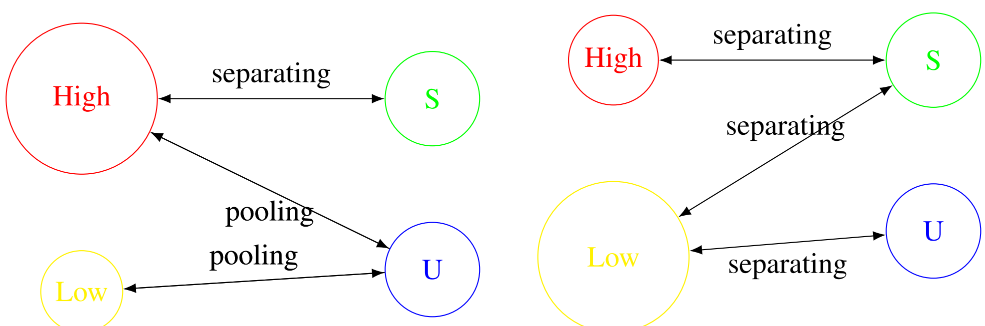
$$W_t^L = \begin{cases} V_L + \left(1 - e^{-(r+\lambda(1-s))(t_2^*-t)}\right)(C_H - V_L) & t < t_2^* \\ V_L \quad t > t_2^* \end{cases} \tag{2.31}$$

Theorem 2.2 characterize the equilibrium dynamics with different initial market belief π_0. There are two regimes in the non-stationary equilibrium (see figure 2.3): when the initial belief is low, the market is pessimistic about the asset quality sold on the market, and then unsophisticated buyers are only willing to pay a separating price for the low-quality asset. For the low-type sellers, anticipating that pooling offer is not feasible in the market, they do not have an incentive to delay, therefore, they trade efficiently. For the sophisticated buyers, since they can identify the



(a) Market Belief (b) Continuation Value

Figure 2.2: Non-stationary Equilibrium



(a) optimistic (b) pessimistic

Figure 2.3: Trading pattern

quality sold on the market, they always bid separating offer to the seller, and both types of sellers in the market are willing to take the offer from the sophisticated buyers.

When the market is optimistic about the asset quality (see figure 2.3(a) , the unsophisticated buyers are willing to pay a pooling price for both types of assets in the market. Therefore, in this situation, the low-type sellers realize that they can get a better offer by waiting for pooling offer from the unsophisticated buyers, therefore, the low-type seller's waiting value is higher than buyers' valuation of low-quality asset, therefore, the trade between low-type seller and sophisticated buyers break down completely.

Market is partially segmented in both scenarios. When the market is optimistic, low-type seller only trades with unsophisticated buyers in the market due to the strategic incentive of waiting; when the market is pessimistic, high-type seller only trades with sophisticated buyers in the market, since the asymmetric information between them is not severe to break down to forgo the gains from trade.

Theorem 2.2 can also shed some light on the signaling role of time to sale. When the market is pessimistic about the asset quality, no trade is a good news to the investors, since the low-type sellers are more eager to trade in this region; when the market is optimistic about the asset quality, no trade is a bad news to the investors, since only low-type sellers are eager to delay and wait for unsophisticated buyers. In the stationary equilibrium, however, time to sale does not transmit any information about the asset quality.

Proposition 2.2. *The average trading time for both types of seller are given by*

1. *When* $\pi_0 < \pi^*$

$$\tau_H = \frac{1}{\lambda s} + e^{-\lambda s t_1^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda s}\right) \tag{2.32}$$

$$\tau_L = \frac{1}{\lambda} + e^{-\lambda t_1^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda}\right) \tag{2.33}$$

2. *When* $\pi_0 > \pi^*$

$$\tau_H = \frac{1}{\lambda} + e^{-\lambda t_2^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda}\right) \tag{2.34}$$

$$\tau_L = \frac{1}{\lambda(1-s)} + e^{-\lambda s t_2^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda(1-s)}\right) \tag{2.35}$$

where t_1^*, t_2^* *are given by* (2.26) *and* (2.27).

Proposition 2.2 shows the average trading time for two types of sellers given different initial market belief. There are two components in the average trading time for the sellers: the first component is the how long does is take to converge to stationary equilibrium, the second component is the average trading time in stationary equilibrium. When the market belief is low, the low-type seller trades faster along the non-stationary equilibrium path, but the probability that the equilibrium converges to the stationary equilibrium is lower. Therefore, the expected trading time for the low-quality asset is lower. We can get the opposite result when the market belief about the asset quality is high.

Following the above analysis, I can compare the expected trading time for each type of asset.

Corollary 2.2. *The average trading time for both types of seller has the following relation: when* $\pi_0 < \pi^*, \tau_H < \tau_L$*; when* $\pi_0 > \pi^*, \tau_H > \tau_L$*.*

This is a direct result from Proposition 2.2. When $\pi_0 < \pi^*$, the average trading time for low-quality asset before reaching the stationary equilibrium is lower, and probability to reach the stationary equilibrium is also lower. Since the trading rate for low-quality asset in stationary equilibrium is lower than that before converging to stationary equilibrium, it is easy to find that the average trading time for low-quality asset is lower. Similarly, we can get the opposite result when $\pi_0 > \pi^*$

2.5.1 Averaging Trading Price

In this subsection, we consider the relationship between average trading price and time-on-the-market. We define the average trading price AP_t at time t as

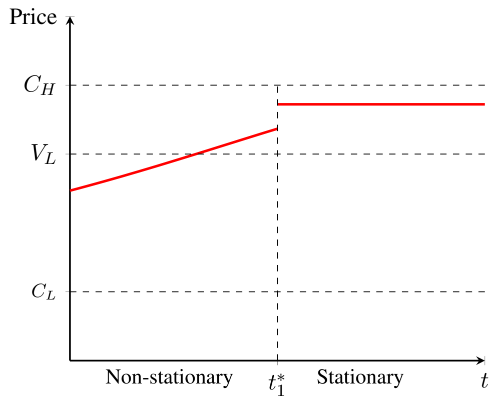
$$AP_t = \begin{cases} \frac{\pi_t s C_H + (1-\pi_t) W_t^L}{\pi_t s + 1 - \pi_t} & \pi_t < \pi^* \\ \frac{(\pi^* s + \frac{\gamma^*}{\lambda}) C_H + (1-\pi^*) s V_L}{s + \frac{\gamma^*}{\lambda}} & \pi_t = \pi^* \\ C_H & \pi_t > \pi^* \end{cases} \tag{2.36}$$

where $m\pi_t + (1 - \pi_t)$ is the total probability that the asset can be traded at time t if the seller meets a buyer. And with probability $\frac{s\pi_t}{s\pi_t + (1-\pi_t)}$ the asset is traded between high-type seller and sophisticated buyers at price C_H; with probability $\frac{(1-\pi_t)}{s\pi_t + (1-\pi_t)}$ the trade is between low-type seller and both types of buyers at reservation price W_t^L. When the initial belief about the asset quality is high, low-type seller values more by holding the asset, thus, trade only occurs at price C_H.

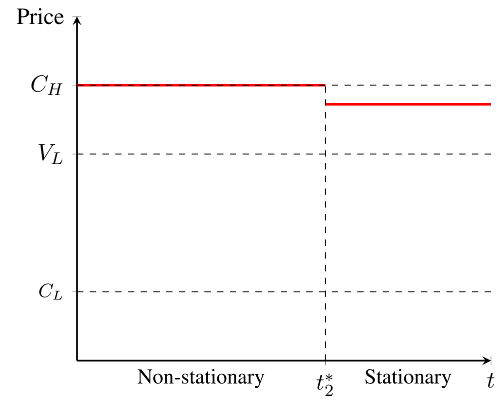
After characterize the trading price on the equilibrium path, we can show the trading price and time-on-the-market in Figure 3. In the first scenario, when the initial quality of asset pool is low, trading price is increasing with time-on-the-market until it reaches the stationary equilibrium. In stationary equilibrium, there is a jump in the asset price, and this is because that in stationary equilibrium, unsophisticated

2.5.2 Market liquidity

In this section, we consider the market liquidity dynamics. In the model, I define the market liquidity as the average trading rate for the asset (trading volume) The expected trading rate for



(a) $\pi_0 < \pi^*$



(b) $\pi_0 > \pi^*$

Figure 2.4: Average Trading Price

the asset in the market is given by

$$\eta_t = \pi_t \lambda_t^H + (1 - \pi_t)\, \lambda_t^L \tag{2.37}$$

$$= \lambda_t^L + \pi_t(\lambda_t^L - \lambda_t^H) \tag{2.38}$$

From (2.38), we can see that the market liquidity depends on the market belief (π_t) and the relative trading rate for the two types of asset ($\lambda_t^L - \lambda_t^H$). Note that higher market belief not necessarily leads to higher market belief,

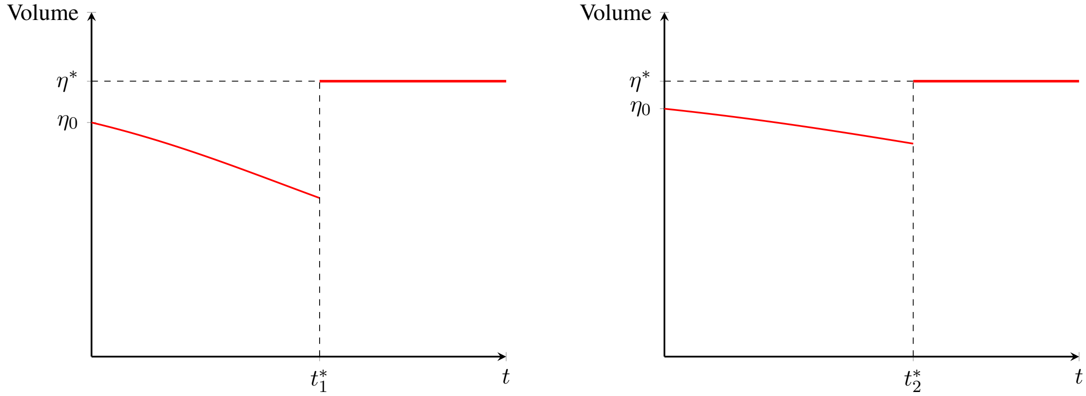
Proposition 2.3. *The expected trading rate for the asset is given by*

1. *when* $\pi_0 < \pi^*$,

$$\eta_t = \begin{cases} \lambda - \dfrac{\pi_0 \lambda (1-s)}{\pi_0 + (1-\pi_0)e^{-\lambda(1-s)t}} & \text{for } t < t_1^* \\ \gamma^* & \text{for } t \geq t_1^* \end{cases} \tag{2.39}$$

2. *When* $\pi_0 > \pi^*$,

$$\eta_t = \begin{cases} \lambda(1-s) + \dfrac{\pi_0 \lambda s}{\pi_0 + (1-\pi_0)e^{\lambda s t}} & \text{for } t < t_2^* \\ \gamma^* & \text{for } t \geq t_2^* \end{cases} \tag{2.40}$$



(a) $\pi_0 < \pi^*$ (b) $\pi_0 > \pi^*$

Figure 2.5: Trading volume

In Proposition 2.3, the market liquidity (trading volume) is decreasing over time regardless of the initial belief level. And then it bounces back once the game is reaching the stationary equilibrium. The intuition for this result is as follows: when the initial belief is very low, market infers the asset quality based on the time on the market, asset with high-quality is more likely to stay in the market, since it only trades with sophisticated buyers. But since assets with low-quality are traded faster than those of high quality in this region, therefore, the market liquidity of asset decreases due to the inference based on time-on-the-market. Once the game reaches the stationary equilibrium, the unsophisticated buyers are willing to offer pooling price C_H, which leads to a jump in the average trading rate. The underlying mechanism that drives this result is that the trading rates of both types of asset not only determines the belief updating for the uninformed buyers, but also contributes the expected trading volume.

This section, we characterize the payoff to uninformed buyers. The cream-skimming literature predicts that due to the information advantage from the sophisticated investors, which generate negative externality on the unsophisticated buyers.

The expected payoff to the unsophisticated buyer is given by

$$\Pi^U = \begin{cases} (1-\pi_t)(V_L - W_t^L) & \text{if } \pi_t < \pi^* \\ \pi_t(V_H - C_H) + (1-\pi_t)(V_L - C_H) & \text{if } \pi_t > \pi^* \end{cases} \tag{2.41}$$

And the expected payoff to the sophisticated buyer is given by

$$\Pi^S = \begin{cases} \pi_t(V_H - C_H) + (1-\pi_t)(V_L - W_t^L) & \text{if } \pi_t < \pi^* \\ \pi_t(V_H - C_H) & \text{if } \pi_t > \pi^* \end{cases} \tag{2.42}$$

2.6 Comparative Statics

I focus on two parameters of interest: the share of sophisticated investors in the market and the search friction. In the context of the model, both parameters

2.6.1 Share of sophisticated buyers

In the model, the share of sophisticated buyers plays a key role. First, it can shape the equilibrium structure. From Assumption 2.2, the share of sophisticated buyers (s) is crucial, and it affects the outside option for the low-type. When the ratio of sophisticated buyers is very high, then on average, it takes longer for the low-type seller to meet with the unsophisticated buyers, and therefore, the continuation value is lower given for any given time to sale. Thus, due to the high search cost for the low-type seller, she is willing to take separating offer from the buyer, and in this case the trade is efficient. Second, it can facilitate learning for the unsophisticated buyers. Since the unsophisticated buyers infer quality through the time-to-sale, and high-type seller is willing to trade with sophisticated buyers for sure. Then the share of sophisticated buyers affect the learning speed for the unsophisticated buyers.

In this section, I will consider how does the share of informed buyers in the market affect the market liquidity, and trading volumes.

Proposition 2.4. *When $\pi_0 < \pi^*$, $\frac{d\tau_L}{ds} < 0$, $\frac{d\tau_H}{ds}$ is undetermined; When $\pi_0 > \pi^*$, $\frac{d\tau_H}{ds} < 0$, $\frac{d\tau_L}{ds}$ is undetermined.*

Figure 2 shows the relationship between the share of informed buyer and expected trading time. There are two points need to be emphasized in this figure. First, it takes longer for the low-quality asset to be traded when the initial market belief about the asset quality is low. Second, the expected trading time is not monotone in the ratio of informed buyers in the market. We can interpret the ratio of informed buyers in the market as the market transparency, Figure 2 shows that when market belief $\pi_0 > \pi^*$, the relationship between average trading time and ratio of informed buyers is U-shape, which means that by increasing market transparency, the average trading time for low-quality seller could increase.

When there are more sophisticated buyers in the market, then it seems that the low-quality seller has less incentive to delay trade, since they are less likely to meet an uninformed buyer. But when there are more sophisticated buyers in the market, the learning effect improves, and it takes less time to reach the stationary equilibrium, moreover, the trading rate in stationary equilibrium is higher, hence the overall effect on expected trading rate is ambiguous, and this is why there are more informed buyers in the market, the expected trading time for low-quality seller could increase.

2.6.2 Search Friction

In the main model, I assume that the search intensity is independent of seller's type for simplicity, in this section, I allow type dependent search intensity. Specifically, assume that the search intensity for type-θ seller is λ_θ for $\theta \in \{H, L\}$. [15]. Without abuse of notation, I continue to use λ_t^θ as the trading rate for the type-θ asset. In order to get rid of the trivial case, we assume that

Assumption 2.3. $\lambda_L > \lambda_H m$

This assumption states that the search intensity for low-type seller is greater than the search intensity of high-type only targeting sophisticated buyers in the market. This assumption is aim to get rid of the trivial belief dynamics where the market belief and time-on-the market is purely driven by the search friction. If this assumption does not hold, then the market belief decreases regardless of the strategy taken by the sellers.

Then under Assumption 2.3, we can get the qualitatively the same result. The key insight is that when the buyers make take-it-or-leave offer, the search intensity for high-type seller does not play too much role in the equilibrium, and it only affects the learning speed for the unsophisticated buyers.

2.7 Extensions

In the main model, I make some assumptions about the ratio of sophisticated investors and the composition of asset quality. In this extension, I consider the equilibrium without Assumption

[15] I do not need to require that $\lambda_H > \lambda_L$ or $\lambda_H < \lambda_L$, either case can hold in this extension

2.2, and discuss how does it change the equilibrium dynamics. And then I introduce the asset (skill) depreciation into the model, and show that the result is still robust under mild asset depreciation.

2.7.1 Equilibrium without Assumption 2.2

In this section, we characterize the equilibrium when Assumption 2.2 does not hold. In this case, the strategic delay incentives for the low-type seller disappear due to the waiting cost is so high such that the payoff from targeting only the uninformed buyer in the market cannot compensate the cost of waiting.

Lemma 2.8. *The continuation value for the low-type seller in stationary equilibrium is*

$$W^L = \frac{rC_L + \lambda(1-s)C_H}{r + \lambda(1-s)} \tag{2.43}$$

And when Assumption 2.2 doesn't hold, then we know that $W^L < V_L$. Define $\pi^\dagger$ as the threshold belief such that uninformed buyer is indifferent between offering C_H and W^L, then

$$\pi_t(V_H - C_H) + (1-\pi_t)(V_L - C_H) = (1-\pi_t)(V_L - W^L) \tag{2.44}$$

which gives

$$\pi^\dagger = \frac{C_H - W^L}{V_H - W^L} > \pi^* \tag{2.45}$$

Then we can characterize the equilibrium in the following proposition.

Proposition 2.5. *When Assumption 2.2 does not hold, the equilibrium is characterized by*

1. *when $\pi_0 < \pi_\dagger$, the equilibrium is characterize by $\{\pi_t, W_t^L\}$ such that*

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda(1-s)t}} & t < t^\dagger \\ \pi^\dagger & t \geq t^\dagger \end{cases} \tag{2.46}$$

where $t^\dagger$ is given by

$$t^\dagger = \frac{1}{\lambda(1-s)}\left(\log(\frac{\pi^\dagger}{1-\pi^\dagger}) - \log(\frac{\pi_0}{1-\pi_0})\right) \tag{2.47}$$

2. *when* $\pi_0 < \pi_\dagger$,

$$W_t^L = W^L$$
$$\pi_t = \pi_0$$

where uninformed buyers are bidding C_H *with probability one, and informed buyers bid* C_H *for high-quality asset and* W^L *for low-quality asset.*

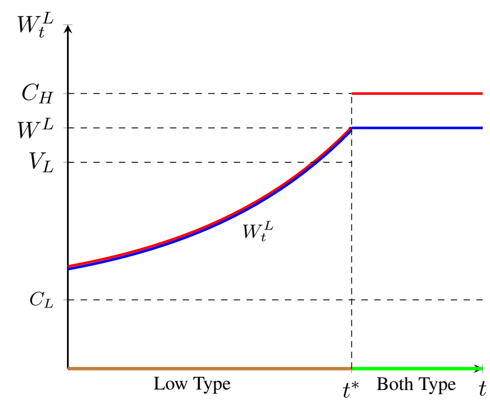


Figure 2.6: Equilibrium without Assumption 2.2

2.7.2 Quality Shocks

One argument in In the main model, I assume that the quality of the asset is fixed over time, in this extension, I introduce the quality shocks to the asset. In particular, for high type asset, it receives a quality shock with intensity β which turns the high-quality asset into a low-quality asset. Assume that the quality shock is private information to the high-quality seller, and is not observable to the buyers, regardless of sophisticated buyers or unsophisticated buyers. In this case, I can define the holding value for the high-quality asset as C'_H, which is given by

$$C'_H = \frac{rC_H + \beta C_L}{r + \beta} \tag{2.48}$$

and the higher the quality shock β, the lower the holding value for high type asset. Similarly, the holding value for type-H asset is

$$V'_H = \frac{rV_H + \beta V_L}{r + \beta} \tag{2.49}$$

Given that the asset has been sold before time t, market (unsophisticated buyers) update the belief as follows

$$\pi_{t+\mathrm{d}t} = \frac{\pi_t e^{-(\lambda_t^H+\beta)\mathrm{d}t}}{\pi_t e^{-\lambda_t^H \mathrm{d}t} + (1-\pi_t)e^{-\lambda_t^L \mathrm{d}t}} \tag{2.50}$$

where λ_t^θ is the trading rate for type-θ asset. The denominator in (2.50) is the probability that it is of high quality, conditional on that it is high quality at time t, while the denominator is the probability that no trade between $[t, t+\mathrm{d}t]$. I can rewrite the belief dynamics as follows:

$$\frac{\mathrm{d}\pi_t}{\mathrm{d}t} = \pi_t(1-\pi_t)(\lambda_t^L - \lambda_t^H) - \beta\pi_t \tag{2.51}$$

where the first term on the RHS of (2.51) captures learning through relative trading rate, and the second term captures the quality shock.

Assumption 2.4. *The quality shock intensity satisfies*

$$\beta < \lambda(1-s) \tag{2.52}$$

Assumption 2.4 provides a upper bound on the quality shock intensity to the seller. Without this assumption, the belief dynamic is strictly decreasing over time, since the quality shock dominates the learning effect.

Denote the continuation value for the high-θ seller as W_t^θ and then

$$rW_t^H = c_H + \beta(W_t^L - W_t^H) + \frac{\mathrm{d}W_t^H}{\mathrm{d}t} \tag{2.53}$$

where $\beta(W_t^L - W_t^H)$ captures the quality shock to the high-type seller. Note that the continuation value for the high-type seller could be higher than her holding value. This is the case if the unsophisticated buyers are inclined to pay a pooling price and the continuation value for the low-type is higher than unsophisticated buyer's valuation for the low-quality asset.

For the low-type seller, her continuation value is as follows

$$rW_t^L = c_L + \gamma_t(W_t^H - W_t^L) + \frac{\mathrm{d}W_t^L}{\mathrm{d}t} \tag{2.54}$$

where γ_t is the probability that pooling offer is made by the unsophisticated buyers. Since the buyer has the bargaining power, the pooling offer is the high-type seller's continuation value. We can rewrite the ODE system as follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} W_t^H \\ W_t^L \end{pmatrix} = -\begin{pmatrix} c_H \\ c_L \end{pmatrix} + \begin{pmatrix} r+\beta & -\beta \\ -\gamma_t & r+\gamma_t \end{pmatrix}\begin{pmatrix} W_t^H \\ W_t^L \end{pmatrix} \tag{2.55}$$

Stationary Equilibrium

Denote $\pi^\star$ as the stationary equilibrium belief. At the stationary belief, the unsophisticated buyers are indifferent between pooling offer and separating offer. That is

$$\pi^\star(V_H' - W^{H\star}) + (1-\pi^\star)(V_L - W^{H\star}) = 0 \tag{2.56}$$

At the stationary equilibrium, the continuation value for the low-type is

$$W^{H\star} = \frac{rC_H + \beta V_L}{r+\beta} \tag{2.57}$$

$$W^{L\star} = V_L \tag{2.58}$$

Therefore, the stationary belief is the same $\pi^\star = \pi^*$ under no quality shock. Then I can characterize the stationary equilibrium as follows.

Theorem 2.3. *When Assumption 2.2 holds, there exist a stationary equilibrium where*

1. *the unsophisticated buyers randomize between pooling offer $W^{H\star}$and separating offer $W^{L\star}$*

2. *Given the separating offer $W^{L\star}$, low-type sellers accept it with probability $\mu^L(W^{L\star})$*

3. *The market belief stays at $\pi^\star$ conditional there is no trading.*

The main difference with the benchmark model is that with quality shock, the trading rates for two types of the assets are not the same. In the benchmark model, the low-type seller turns down some separating offer to freeze the market belief where the unsophisticated buyers are willing to pay a pooling price. In the model with quality shock, the quality of the asset deteriorates over time, in order to maintain the stationary belief, the low-type seller trades faster to compensate the belief decreasing due to the quality shock.

2.8 Empirical Implication

Following the model, we can derive several empirical implications to test.

In the context of IPOs, Ritter and Welch (2002) write that "it is conventional wisdom among both academics and practitioners that the quality of firms going public deteriorates as a period of high issuing volume progresses." In terms of my model, it means that the average trading rate is positive correlated with the market belief about asset quality. First, notice that two regimes exist in the model that have distinct predictions on the role of delayed trade, and both of them are plausible, therefore, to test the result of whether trade delay is positive correlated with the post-performance or price, one need to control for the different regime. In the model, the two regimes are separated by the market belief, which is the cut-off of the average quality of asset in the market. In the IPO market, if we want to separate the two regimes, one way is to consider hot market [16] and cold market separately, and test whether in hot market, the time-to-IPO is negative correlated with post-performance. Similarly, in cold market, we can test whether time-to-IPO is positive correlated with post-performance.

Real estate market is a more suitable place to test the my model with two regimes. We can interpret the initial quality about the asset as the neighborhood house quality, and then we can test the relationship between time-on-the-market and asset prices and trading volume in different neighborhood with different quality. In the model, I predict that house in better neighbourhood corresponds to the case with optimistic belief, therefore, the longer time-on-the-market should be associated with lower trading price, On the contrary, for houses in worse neighbour quality, we should predict that the time on the market is positive correlated with sales price.

Another potential test is to consider the time to sale in normal times versus during crisis. In normal times, the perceived asset quality sold on the market is high, therefore, waiting is not too costly for the low-type seller. In crisis, buyers on the market are pessimistic about the asset quality, therefore we can test whether time-to-sale is positively correlated with asset quality during crisis, and negatively correlated with asset quality during normal times.

Another set of test is to test how does the share of sophisticated investors affect the expected trading time. We can interpret the informed buyers in the real market as dealers, and in general, they are better informed than retail investors. One way to test the relationship is to consider

[16] In the IPO literature, hot market is defined as the IPO volume and the ratio of under-pricing, in my model, the hot market is defined as the average quality about firm to go to IPO. In general, those firms who chooses to go IPO are endogenous, so the average quality for those IPO firms are also endogenous

how does the number of dealers in specific region will affect the average trading time of the real estate. In the model, we find that the relationship between the ratio of informed buyers and expected trading time is not monotonic, in this setting we can test whether number of dealers will affect the average trading time of the real estate.

Most importantly, Assumption 2.2 gives us the condition of the cream-skimming effect. One implication for Assumption 2.2 is that, in a market with less search friction (higher λ), and less fraction of sophisticated investors, the cream-skimming effect is more likely to exist. Therefore, I can test the time-to-sale in markets with different search friction, and whether in market with less search friction

2.9 Conclusion

In this paper, I build a dynamic model to reconcile the two contradicting predictions of delayed trade on performance (price). In the model, a seller with one unit asset to sell, and asset quality could be either high or low, which is his private information. Two types of buyers in the market, informed buyers know perfectly about the asset quality, uninformed buyers know nothing, and only observe how long the seller has been in the market. Seller searches buyers in the market, after they meet buyers make take-it-or-leave-it offers to sellers. There are two regimes in the model, when the market is very low, low-quality seller has no incentives to delay trading, but for high-quality seller, she is only willing to trade with informed seller. When the market belief is very high, since the uninformed buyers are willing to make a high pooling offer to both types of asset, then low-quality seller would like to wait for uninformed buyers instead of trading with informed seller.

The model is consistent with both the good signal story and bad signal story, and it depends on which type of seller wants to delay trading. When the market belief is very low, it is the high-quality seller that wants to delay trading since the offer made by the uninformed buyers is so low that he prefers to delay take the outside option, hence in this case, delayed trade is a good signal about the quality. When the market is very high, it is the low-quality seller who wants to delay trading, because they know that in good market condition, even the uninformed buyers in the market are willing to offer high price to the seller.

The model has the following predictions. First, it turns out when the market belief is below the stationary belief, low-quality asset trades fast and the expected trading time for low-quality asset is lower. Similarly, when the market belief is above the stationary belief, the high-quality asset trades at a higher rate, and hence the expected trading time for high-quality asset is lower. Another prediction of the model is that when the market belief is below the stationary belief, trading volume is decreasing with the market belief, when the market belief is above the stationary belief, the trading volume is increasing with the market belief. But when the market belief is above the stationary belief, the trading price is not sensitive to how long the seller has been on the market except that the equilibrium converges to the stationary equilibrium. This result is due to the fact that when the market belief is very high, only pooling offer could lead to transaction, hence the price is not sensitive to market belief in this region.

The model can also shed light on the relationship between market transparency and expected trading time (liquidity). In the model, the ratio of informed buyers in the market can be interpreted as market transparency. It turns out that the expected trading time of sellers is not always monotonic with market transparency. The ratio of informed buyers in the market affects the expected trading time through three channels, the first is that it affects the trading rate before the equilibrium converges to stationary equilibrium. The second is that it affects the probability that the equilibrium will converge to stationary equilibrium. The third one is that it affects the trading rate in stationary equilibrium. Due to these three effects altogether, the ratio of informed buyers and expected trading time is not always monotonic.

Chapter 3

Secret Scouting

3.1 Introduction

For venture capital firms (VCs), the role of a "scout" receives much less academic attention compared to the role of a "coach". However, fishing in a better pond can make as much, if not more, difference as being a better fisherman. Sourcing and screening projects are the first step of investments and help reduce monitoring efforts afterward.[1] Yet, hunting for good projects at premature stages faces huge uncertainties and requires a costly experimentation process(Ewens et al., 2018).

Various strategies are taken to facilitate this process. In this paper, we focus on a critical feature of them: searching activities are kept in the dark until successful. Prominent VCs, e.g., Sequoia Capital, adopt scouts networks and let individual agents invest on behalf of them. The identity of the sponsoring VC is hidden until the funded startup proves to be promising, and a large follow-up round is publicly announced. The consequence of such strategies is that searching outcomes are asymmetrically disclosed: Only the good startups are publicly financed but the failed ones are buried underground quietly.

We model a game of VC scouting competition and explain these strategies as follows. First, we take the secret scouting process as given and compare it to an alternative scenario, which is that every VC discloses both successes and failures. We prove that secrecy generates an efficiency

[1]Baum and Silverman (2004) find VCs are able to pick winning startups based on partners, patents, and human capital, which correlate with the characteristics that trigger a final exit. In the same spirit, Catalini et al. (2017) show firms achieving growth without venture capital are similar to those receiving VC funding. Once controlled for startups' initial quality, value-added by VC is much smaller than previously documented.

loss: An initially promising technology is discarded too soon since VCs become gradually more pessimistic. Then we address the follow-up question: What drives the adoption of secrecy? Our answer is the fear of preemption. If VCs could individually choose their disclosing strategy, they would only hide the failures if the first VC who finds a successful startup receives extremely more profits than runners-up. The implication is that secret scouting is an endogenous outcome as the return of the VC industry becomes right-skewed(Hall and Woodward, 2010; Kerr et al., 2014).

Imagine that there is a group of VCs continuously scouting for startups in a new industry, which all use the same uncertain technology. Search friction exists so that every period, each VC only finds a startup with a certain probability. Conditional on a match, a VC perfectly learns the quality of the technology.[2] In the secret scouting case if the startups rely on good technology, the first VC who finds it out will publicly announce an investment and receive a higher return as a first-mover advantage. But if VCs learn that technology is a bad one, they will leave the industry secretly. In the game, VCs continuously choose a costly effort to increase the periodic chance of reaching a startup.

Empirically, when new technology organizations seek VC fundings, no news is bad news(Petkova et al., 2013). Our result hinges on a similar pessimism created by concealed failures. When the VCs compete in scouting but keep observing no announcements of investments, two things can happen. On the one hand, due to the search friction, it might be the case that every individual VC has not met a startup yet. Therefore no one is aware of the technology quality. On the other hand, it is also possible that some VCs have observed a technology failure and secretly left. These two cases are indistinguishable because quit decisions are private. However, VCs consider the second case more likely as time passes by. The technology gradually becomes less promising and the searching effort declines accordingly. In the end, all VCs stop hunting in this industry even without observing any failures.

Efficiency loss comes at the cost of secrecy. We compare the searching intensity in the secret scouting case to a public disclosing alternative where the VCs will reveal both the good and bad outcomes when reaching a startup. Since now failures are also observed, no investment can be perfectly inferred as bad luck in searching: No VCs have met a startup yet. As a result, all VCs will have the same belief in the technology as they initially do and exert a constantly high effort

[2] We assume VC has knowledge advantage at interpreting different signals to forecast the industry prospect (Gompers and Lerner, 2001; Ueda, 2004). The source may be due to VC connections (Hochberg et al., 2007) and human capital (Bottazzi et al., 2008).

of searching until the first VC meets a startup. The public disclosing case is more efficient for the following two reasons. From the startups' perspective, if they use good technology, they will almost surely meet a VC investor in the end. But in the secret scouting case, there is a strictly positive probability that they are ignored in the end. From the VCs' perspective, revealing failures helps them save additional costs of searching if the technology has already been known as unviable by someone else.

We then consider the endogenous choice of secrecy and explain how it correlates with the right-skewness of returns. In the model, once the first VC announces an investment and validates the technology, all other players could make follow-up moves but only receive a substantially less return. The difference between the returns of the first and the following investors quantifies the fear of preemption. Suppose that before the searching game starts, each VC can choose and commit to disclosing failures or not, and their choices are publicly observed. Commitment to secrecy is dominant if the fear of preemption is huge. This is because VCs benefit from others' diminished efforts of hunting startups. A rational individual VC knows its externalities on others' beliefs. Regardless of others' disclosing choices, secrecy will always reduce the degree of competition.

When discussing their preference for secrecy in early-stage scouting, VC partners claim that they want to protect the startups. The argument is that if a top-tier VC later stops financing, possibly due to liquidity reasons, it generates holdup problems for follow-up rounds.[3] However, our result highlights the flip side of the story. If a startup is a potential target for investments yet no VCs reveal public interest after a while, all partners would believe this firm is of poor quality. Then no further VCs are willing to spend time on it and adverse selection endogenously occurs. This is the trick of secrecy. Each VC wants to eliminate competition by misguiding opponents into pessimism. However, while everyone does so, the adverse selection problem becomes an industry-wide phenomenon and in the end, causes the dead-weight loss.

Our rationale is also different from the alternative explanations of VC's opacity in the literature. The first argument is that VCs are afraid of attracting competitors as their activities may signal optimistic beliefs about the industry. While that may explain why VCs delay the announcement of investing in promising startups, it cannot generate a tendency to hide failures. Second, venture capitalists may delay negative information about fund performance until a new

[3]This idea is related to Khanna and Mathews (2016) where the financing terms of the current VC investors affect third-parties' interactions with the startup.

fund is raised(Chakraborty and Ewens, 2017), which is broadly in line with VCs inflating NAVs for fund-raising reasons(Barber and Yasuda, 2017; Brown et al., 2018). Thirdly and closer related is Akcigit and Liu (2015). In their model, VCs are experimenting with a sequence of projects in turns. Hiding the first project's failure will waste competitors' resources in the dead end, which buys time for the informed investor to lead in the second project. This is possible but not necessarily required. In reality, VCs usually adopt a "spray-and-pray" strategy (Ewens et al., 2018) and screen a wide range of projects simultaneously. Our model is an extreme case: The informed VCs are totally indifferent about disclosing conditional on learning technology failures, since there is only one investment opportunity. Even so, commitment to secrecy is still dominant.

Broadly, this paper contributes to the knowledge of venture financing at its earliest stage, where information asymmetry and uncertainty baffle the operation of financial intermediation. There is growing empirical evidence on the value of VC connection from seed round (Kerr et al., 2011; Gonzalez-Uribe and Leatherbee, 2017) as well as knowledge on how VCs perceive and learn about new projects(Bernstein et al., 2017; Ewens et al., 2018). Searching for projects has been theoretically shown to affect the valuation and viability of ventures(Inderst and Müller, 2004; Nanda and Rhodes-Kropf, 2013; Hellmann and Thiele, 2015). We explicitly consider the efficiency loss generated within this process, and explain how secrecy becomes an industry-wide phenomenon.

Though the focus of this paper is on early-stage VC activities, the model explains more general phenomenons on disclosing, for example, the "file drawer problem". Private research agents are reluctant to disclose failures though null or negative results are valuable as well. The model implies that if more transparent R&D outcomes are implemented, innovation efficiency will increase. Supporting evidence are available. Gross (2019) show USPTO's patent secrecy program in World War II has reduced and delayed follow-on invention. And before the passage of The American Inventors Protection Act (AIPA), only successful patent applications are announced but failed ones are kept as secret. AIPA requires mandatory publication of all patent applications 18 months after the first filing. Graham and Hegde (2015) and Hegde et al. (2018) find that AIPA does promote knowledge spillover and cumulative innovation.

The current work is closely related to the application of preemption games to disclosure. For example, Hopenhayn and Squintani (2011, 2015) and Bobtcheff et al. (2016) show the timing of revealing successful findings are impacted by competitions. Our focus is not on the timing. Akcigit and Liu (2015) consider an innovation competition in which players decide whether to pursue a risky or safe bandit privately. The risky line can yield a successful breakthrough (publicly observed) or a dead-end, after which firms switch to the safe line for a smaller payoff. Since the winner takes all, secrecy after the dead end is a dominant strategy to keep opponents wasting time as long as possible. Our model shares the same information externalities with Akcigit and Liu (2015). However, the dominance of secrecy is from an *ex ante* motivation: Players want to create pessimism and trick the opponents into giving up a potentially good target. Halac et al. (2017) consider how to optimally design communication between individual agents jointly with the prize-sharing rule. Moscarini and Squintani (2010) and Murto and Välimäki (2011) consider the opposite structure of information where only the exit after failures is publicly observable. Related with the disclosure of outcomes, Campbell et al. (2014) show that players with successful breakthroughs will hide their results in order to maintain partners' motivation as a team.

Lastly, our work is also associated with strategic experimentation literature to (Bolton and Harris, 1999). In Keller et al. (2005) a breakthrough occurs randomly at exponential times. It publicly reveals the technology's good quality and generates positive payoff spillovers. Keller and Rady (2015) analyze the opposite case in which a breakdown arrives when the technology is bad. Our paper is different in the following two ways. First, the arrival of results, which is reaching a startup in our model, does not depend on the underlying quality. Belief changes purely through the confusion about whether others have learned a bad outcome. Second, there is a first-mover advantage in the payoff conditional on success, which creates negative payoff spillovers to runners-up along with "good news". Therefore, there is no encouraging effect as common in the experimentation literature.[4] In a similar spirit, Cripps and Thomas (2019) introduce congestion cost as negative payoff spillovers joint with benefits in positive information externalities.

The paper is organized as follows. Section 2 introduces the baseline model in secret scouting. We then compare the equilibrium strategy to a case with publicly disclosing. Section 3 considers the endogenous disclosure decision. Section 4 concludes.

[4]See, for example, Bolton and Harris (1999).

3.2 Baseline Setup

Consider a game of N VCs (players), indexed by $i = 1, ...N$. Each of them is consistently searching for a startup at $t \in [0, +\infty)$ in a new industry. All startups use the same, and therefore perfectly correlated technology. Its type θ is either H (good) or L (bad). The prior probability of having good technology is $\pi_0 < 1$. The *ex-ante* distribution is common knowledge to all VCs.

Search friction exists and stops VCs from reaching the startups with certainty. The individual instantaneous rate of finding a startup is λa_i independent of the technology quality.[5]. Each VC continuously determines efforts a_{it} to exert at $t \in \mathbb{R}_+$. The effort is bounded and costly almost surely. We normalize the domain of effort to be $a_i \in [\phi,\, 1]$ and the flow cost to VC i is $c_i(a_i) = c\,(a - \phi)$.[6] $\phi < 1$ indicates the default arrival rates when VCs stay idle. Effort choice for each VC remains private and unobserved.

Once a VC reaches a startup, it perfectly and privately learns the quality of the technology. The payoff structure is as follows. If the technology is good, the first player who successfully finds a startup ("Winner") will publicly announce the investment and receive first-mover payoff $W > c/\lambda + c\phi/r$. Like in any other preemption game, the first adoption at the same time ends the game for other "losers", who receive $K < W$ as a second-mover payoff.[7] On the other hand, bad technology generates zero net profit for any VC. In the secret scouting case, we assume VCs leave the unviable startup with no public announcement. Abandoning it only terminates the game of finding VC itself. Its discovery remains unobserved to the others. So they keep their own searching process until they also find a startup.

For each VC, the game ends either if (i) it learns the quality of technology or (ii) another VC publicly announces an investment. When the game continues, there is no flow payoff for all players. They are impatient and discount future benefits and costs at a common discount rate of r. The final payoff that VC i receives depends on the scenarios. Define $\tau_i \in \mathbb{R}_+$ as the random arrival time when player i reaches the startup and $\tau = \min(\tau_1, ..., \tau_n)$. Neglecting all

[5]This rules out the possibility of learning qualities through arrival rates of results.

[6]The form of effort and the cost associated can be pecuniary, representing expenditures to hire scouts. In general, they could be also non-pecuniary such as the monitoring and advising activities (Gompers, 1995; Lerner, 1995; Hellmann and Puri, 2002).

[7]Value of W and K are exogenously given and we abstract from concerns such as hiding good results to deter competition.

higher-order terms, the expected profit for VC i is

$$
\begin{aligned}
V_t^i = \mathbb{E}_t^i \Bigg(& \mathbb{I}(\tau_i = \tau) \left(e^{-r(\tau_i - t)} W \mathbb{I}(\theta = H) - \int_t^{\tau_i} e^{-rs} c(a_t)\, ds \right) \\
& + \mathbb{I}(\tau_i > \tau) \Bigg(\mathbb{I}(\theta = H) \left(e^{-r(\tau - t)} K - \int_t^{\tau} e^{-rs} c(a_t) ds \right) \\
& - \mathbb{I}(\theta = L) \int_t^{\tau_i} e^{-rs} c(a_t) dt \Bigg) \Bigg),
\end{aligned} \tag{3.1}
$$

where $\mathbb{I}(\cdot)$ is the indicator function. In the first line, VC i finds the result before its opponents. It invests in the startup and enjoys W only if $\theta = H$. But it stops searching regardless of the technology quality. In the second line, some competing VC $-i$ meets the startup first. Transaction happens and is publicly announced if $\theta = H$. VC i stops searching immediately and takes the second mover payoff. In the last line, the competing VC $-i$ secretly walks away due to $\theta = L$. In fact, any other VC who finds out the bad technology before VC i will take the same strategy. Thus it cannot learn the fact until it meets the asset by itself.

VC's objective is to choose effort a_{it} to maximize the expected profit of V_t^i. Since an announcement ends the game automatically, the public history can be mapped to the calendar time t without any investments. We consider a pure public strategy for VC i, which is a measurable function $a_i : \mathbb{R}_+ \to [\phi, 1]$. a_{it} is the instantaneous effort exerted by VC i at t if game continues.

A symmetric public perfect Bayesian equilibrium (PPBE) is a profile $a = (a_1, ..., a_n)$, such that

1. For any player i, given a_j, $j \neq i$, a_i maximizes V_t^i for all t;

2. For any player pair (i, j), $i \neq j$, $a_i = a_j$ almost surely;

3. Players use Bayes' rule whenever possible.

If Multiple symmetric PPBE exists, we focus on the Pareto-optimal one.

Each rationally VC i updates the probability of π_t of the technology being good (hereafter the belief). When no investments are announced, a single VC does not know whether it is because (i) all others fail to find a startup or (ii) some of them have found out the bad technology and left the market permanently. However, the cumulative probability that at least one startup is reached strictly increases over time. Thus, VCs believe case (ii) to be more likely and become pessimistic about the technology quality. The equilibrium efforts chosen by others will pin down how quickly VC i's belief deteriorates and this belief, in turn, affects the effort decision of it.

Let $a_{-i} = \sum_{j \neq i} a_j$. Consider the belief changes from t to $t + dt$. Ignoring higher order terms yields

$$\pi_{t+dt} = \frac{\pi_t(1 - \lambda a_{-it} dt)}{\pi_t(1 - \lambda a_{-it} dt) + (1 - \pi_t)}.$$

This generates the first-order evolution of π_t:[8]

$$\frac{d\pi_t}{dt} = -\pi_t(1 - \pi_t)\lambda a_{-it}, \tag{3.2}$$

where π_t has two absorbing states 0 and 1. Since $\lambda a_{-it} \geq (N-1)\lambda\phi > 0$, this belief is strictly decreasing while the game continues. The speed at which the belief deteriorates depends on the total effort exerted by competing VCs.

Discussion. In standard experimentation models, agents exert private efforts to influence the arrival rate of news. The project has uncertain qualities and only one specific type can produce news. Depending on the type of news, there are models about "breakthroughs" and "breakdowns". In this model, technology type by itself does not directly influence the arrival rate of news. For any single VC i, the arrival rate only depends on the effort choice. However, the public announcement of investment by others terminates the game only if θ is H. This leads to a learning channel in the same spirit of collaborated experimentation models (breakthroughs). And this is why the belief is drifting down strictly.

The current model is different as follows. First, as the arrival rate of news is type-invariant, the individual effort has no direct effect on beliefs. Learning is purely due to the externalities of others' efforts and hidden failures. Second, the game is "winner-take-more" and there is effective no "good news". For VC i, a "breakthrough" by other players verifies that the technology is good but meanwhile indicates that VC i loses the contest and has to take $K < W$. Thus the free-riding motive is not a major concern in our model as it is hampered by the first-mover advantage.

[8] Since we consider symmetric equilibria, players share the same belief in equilibrium. Therefore we drop the subscript i to reduce clutter.

3.2.1 Value Function

π_t is the state variable. Denote VC i's equilibrium continuation value as $V(\pi_t)$. It satisfies the following equation from t to $t+dt$:

$$V(\pi_t) = \lambda a_{it}\pi_t W dt - c(a_{it}-\phi)dt + (1-\lambda a_{it}dt)\Bigg(\pi_t\lambda a_{-it}Kdt + (1-\pi_t\lambda a_{-it}dt)\frac{1}{1+rdt}V(\pi_{t+dt})\Bigg). \tag{3.3}$$

Consider what happens at t. With probability $\lambda a_{it}dt$, VC i meets a startup and receives W with probability π_t. If VC i does not find a startup, with probability $\lambda a_{-it}dt$ at least one competing VC $-i$ announces an investment given $\theta = H$. In this case VC i receives the second mover payoff K. If a transaction does not happen, game proceeds to $t+dt$. With the first order approximation,

$$V(\pi_{t+dt}) = V(\pi_t) - V'(\pi_t)\pi_t(1-\pi_t)\lambda a_{-it}dt.$$

Taking this to equation (3.3) and deleting higher-order terms generates

$$rV(\pi_t) = \lambda a_{it}(\pi_t W - V(\pi_t)) + \lambda a_{-it}\pi_t(K - V(\pi_t)) - \pi_t(1-\pi_t)\lambda a_{-it}V'(\pi_t) - c(a_{it}-\phi). \tag{3.4}$$

As equation (3.4) shows, the game can end either in VC i direct meeting the startup or any other opponents announcing investments. The first case generates more profit but also requires costly effort. The second case allows for free-riding benefits but is dominated in terms of payoff. The marginal benefit of exerting more effort is to increase searching possibility whereas the marginal cost of searching is constant. Thus, the effort decision for VC i follows a simple rule:

$$a_{it}\begin{cases} = \phi & \lambda(\pi_t W - V(\pi_t)) < c \\ \in [\phi, 1] & \lambda(\pi_t W - V(\pi_t)) = c \\ = 1 & \lambda(\pi_t W - V(\pi_t)) > c. \end{cases} \tag{3.5}$$

Equation (3.5) explains why shirking in searching is sure to come in the end. The marginal benefit of effort, $\lambda(\pi_t W - V(\pi_t))$, depending on the belief π_t. When it is sufficiently low, the benefit is approaching 0. Exerting more effort would only guarantee a higher chance of finding a bad startup. Thus players tend to stop searching as time passes.

We derive an indifferent value function $V_I(\pi) = \pi W - c/\lambda$ from equation (3.5). If $V(\pi) = V_I(\pi)$, then VC i is indifferent with any level of effort. The interpretation of V_I is a "static" version of the game without learning. It is as if VC i pays an upfront cost, immediately finds out the technology quality, and gives up all free-riding opportunities. If $V(\pi) < V_I(\pi)$, the current waiting value is relatively small and a high effort is chosen to increase the chances of preempting others. On the contrary, if $V(\pi) > V_I(\pi)$, VCs will choose $a_i = \phi$ as free-riding incentives dominate.

In equilibrium, other VCs need to exert proper level of efforts a_I to keep i being indifferent. We solve it by putting V_I back into equation (3.4) and restricting symmetric level of efforts,

$$a_I = \frac{r\left(\pi W - \frac{c}{\lambda}\right) - c\phi}{(N-1)\left(K + \frac{c}{\lambda} - W\right)\lambda\pi}. \tag{3.6}$$

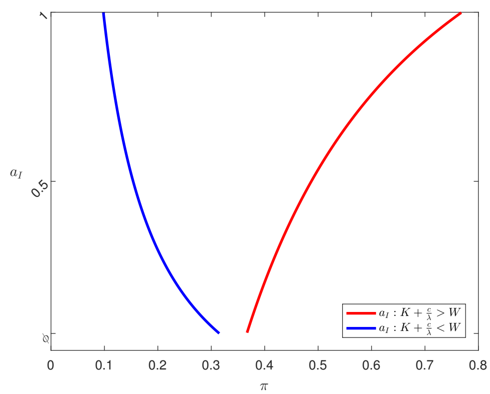
Since the efforts of players are restricted to $[\phi, 1]$, the required a_I may not be feasible when the belief π is not in a proper region. If that happens, a_i is a boundary solution ϕ or 1. Define $\Pi_F = \{\pi | a_I \in [\phi, 1]\}$ as the feasible set of π. The following lemma is important to characterize the equilibrium strategy.

Lemma 3.1. *Suppose $\pi \in \Pi_F$. $\partial a_I/\partial\pi > 0$ if and only if $K + c/\lambda > W$.*

Lemma 3.1 indicates that the monotonicity of a_I depends on the cost of being follow-up investors. K is our key parameter as it represents the fear of preemption or the return skewness in the VC industry. The following section shows the structure of equilibrium strategies changes as this monotonicity flips. Especially, we will separately solve the game given different levels of return skewness. Lemma 3.1 states that when the second mover payoff is sufficiently large, the required effort to make others indifferent is strictly decreasing as belief drifts down over time. However, when the game is "winner takes almost", VCs must backload efforts to keep their opponent indifferent.

The change in monotonicity hinges on the trade-off between preemption threats and free-riding benefits. On one hand, VCs are competing for bringing the first successful technology to the market. On the other hand, free riding successful investments from competitors save the cost. When K is large, free riding is profitable. However, this benefit is only valuable when VC i believes the technology is a good one. When it is still optimistic, competing VCs could exert high levels of effort. Even though this increases the probability of being preempted, it also

Figure 3.1: a_I as Functions of π—High vs. Low Preemption



increases the probability that VC i free rides. But when it becomes pessimistic, the marginal benefit of free-riding is lower. Therefore competing VCs must exert lower efforts to make VC i less concerned about preemption.

On the contrary, when K is small, the game is very competitive and each VC faces severe threats of being preempted. The payoff in free riding is no longer sufficient to keep individual VCs indifferent. They must promise backloading the searching effort: When the technology is more likely to be good, they reduce searching intensities so that the cumulative probability of preemption $\lambda a_{-it}\pi_t$ is constant. Thus a_I is decreasing with π in this case.

3.2.2 Low Preemption Case

We now consider the low preemption case when $K + c/\lambda > W$. This represents a scenario where the first mover informs runners-up about the viability of a new technology and followers still have enough market shares to grab. We propose and prove that the equilibrium effort follows a two-threshold strategy:

$$a_{it}\begin{cases} = \phi & \pi_t < \underline{\pi} \\ = \frac{r\left(\pi W - \frac{c}{\lambda}\right) - c\phi}{(N-1)\left(K + \frac{c}{\lambda} - W\right)\lambda\pi} & \pi_t \in [\underline{\pi}, \bar{\pi}] \\ = 1 & \pi_t > \bar{\pi}. \end{cases} \tag{3.7}$$

The solution has an intuitive structure. When the belief is high ($\pi_t > \bar{\pi}$), VCs want to be the first one to find a startup. This is a "preemption region" where all players search with the maximal efforts to become the game-winner. Then the belief goes down fast without investments until $\pi_t \in [\underline{\pi}, \bar{\pi}]$. Now the technology quality is questionable, and exerting full effort is no longer highly valuable. VCs cut off searching intensity by keeping each other indifferent with an intermediate level of effort. This requires that all players use efforts of a_I. Finally, there is a "pessimistic region". VCs shirk and accept the default arrival rate of $\lambda\phi$ when they are very pessimistic ($\pi_t < \underline{\pi}$) because the marginal payoff is too low.

To solve the equilibrium thresholds, consider the preemptive region first. Replacing all efforts with 1 in equation (3.4) generates the value function V_H:

$$rV_H = \lambda\left(\pi W - V_H\right) + \lambda(N-1)\pi\left(K - V_H\right) - (N-1)\pi\left(1-\pi\right)\lambda V_H' - c\left(1-\phi\right). \quad (3.8)$$

Similarly in the pessimistic region, replacing all efforts with ϕ in equation (3.4) generates the value function V_L:

$$rV_L = \lambda\phi\left(\pi W - V_L\right) + \lambda\left(N-1\right)\phi\pi\left(K - V_L\right) - \left(N-1\right)\phi\lambda\pi\left(1-\pi\right)V_L'. \quad (3.9)$$

V_H and V_L correspond to the waiting value of players in the region $\pi_t > \bar{\pi}$ and $\pi_t < \underline{\pi}$ respectively. They follow closed form solutions as

$$V_H = -\frac{c\left(1-\phi\right)}{\left(\lambda+r\right)} + \frac{\lambda(W + (N-1)(K + \frac{c(1-\phi)}{\lambda+r}))}{N\lambda + r}\pi + C_1\left(1-\pi\right)\left(\frac{1-\pi}{\pi}\right)^{\frac{r+\lambda}{(N-1)\lambda}},$$

$$V_L = \frac{\lambda\phi(W + K\left(N-1\right))}{N\lambda\phi + r}\pi + C_2\left(1-\pi\right)\left(\frac{1-\pi}{\pi}\right)^{\frac{r+\lambda\phi}{(N-1)\lambda\phi}}.$$

It remains to pin down the constants (C_1, C_2) and boundaries $(\underline{\pi}, \bar{\pi})$. Because V_L is the value function at the lower bound. When π approaches 0, players will have almost zero payoffs so V_L must be converging to 0. The intuition is that VCs believe the technology has zero returns almost surely and spend zero expenditure on hunting. This pins down $C_2 = 0$ and implies both V_L and V_I are linear in π.

For the lower boundary $\underline{\pi}$, notice V_L is strictly larger than V_I when π is small. Therefore it would intersect V_I from the above, which satisfies the optimal decision of effort by equation

(3.5). So the boundary condition is

$$V_L(\underline{\pi}) = V_I(\underline{\pi}). \tag{3.10}$$

The upper boundary $\bar{\pi}$ is determined by the feasibility constraint of a_I

$$a_I(\bar{\pi}) = 1,$$

and then using value matching condition to pin down C_1

$$V_H(\bar{\pi}) = V_I(\bar{\pi}).$$

Notice for the upper boundary $\bar{\pi}$, we use a feasibility constraint instead of the common smooth pasting condition. As Lemma 3.2 suggests, we do not need to explicitly do so. By design, a_I is solved to make VCs indifferent, which coincides with a full-effort decision at $\bar{\pi}$. We also do not need a smooth pasting condition at the lower boundary. This is not possible as the two value functions are both linear. The kink at $\underline{\pi}$ does not violate the optimality. VCs have a strictly higher payoff if they switch to lower efforts.

Lemma 3.2. $V_H'(\bar{\pi}) = W = V_I'(\bar{\pi})$.

As V_H is a concave function, it stays strictly below V_I when $\pi_t > \bar{\pi}$. Together this confirms the strategy in equation (3.7) consists a PPBE.

Proposition 3.1. *When $K + c/\lambda > W$, the equilibrium strategy a_i follows equation (3.7), where*

$$\underline{\pi} = \frac{c\left(\frac{r}{\lambda} + N\phi\right)}{\lambda\phi\left(N-1\right)\left(W-K\right) + rW},$$

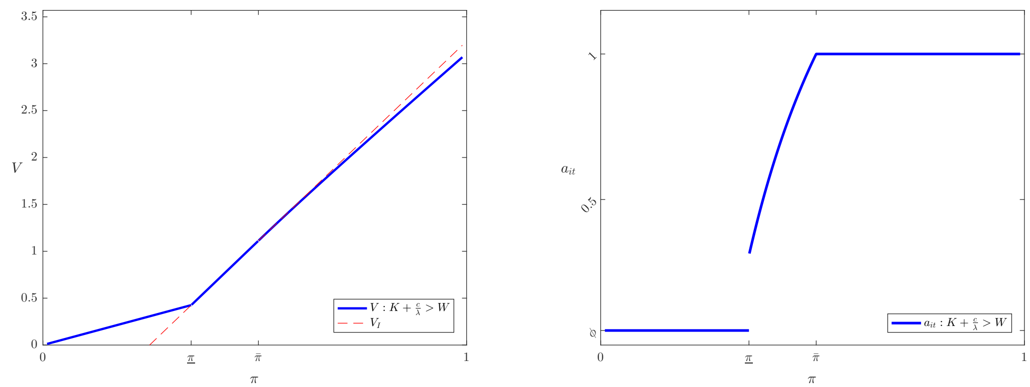
and

$$\bar{\pi} = \frac{c\left(\phi + \frac{r}{\lambda}\right)}{rW + \left(N-1\right)\left(\left(W-K\right)\lambda - c\right)}.$$

Unlike "breakthrough" models, agents do not procrastinate at the beginning due to the fear of preemption. As the belief goes down, free-riding benefits can be utilized to match the cost of preemption. This stops when the belief is very low as the marginal benefit of effort is almost 0.

The strategy in Proposition 3.1 is not the only symmetric PPBE. There is a continuum of equilibria dominated by this one. For any other $\bar{\pi}'$ with $\underline{\pi} \leq \bar{\pi}' < \bar{\pi}$, there exists a two-threshold

Figure 3.2: Equilibrium Value Function and Effort Level —Low Preemption



(a) Value Function

(b) Effort Level

equilibrium by replacing the pair $\{\underline{\pi}, \bar{\pi}\}$ with $\{\underline{\pi}, \bar{\pi}'\}$ in equation (3.7). In such equilibria, VCs keep the highest level of searching longer and has strictly lower expected payoff. As Figure 3.2 shows, once the VCs all switch to $a_i = 1$, their waiting value is strictly below the indifferent curve V_I. Individually each VC wants to preempt others by exerting more effort. But if all of them do so, the additional effort cancels out relatively. However, such actions exhaust the optimism quickly and result in earlier termination. Thus a larger upper boundary is beneficial for all players.

The strategy in Proposition 3.1 is also stable in the following sense. Suppose some VC i's action trembles by ε in $[t, t + dt]$ but no investment occurs during this period. Then after $t + dt$, the original strategy is still a PPBE. This is because VC i's deviation in efforts is private and unobserved. It will not affect the belief of competing VCs, who still assume VC i used the original strategy. And their strategies continue to hold since they are based on public information. Since there is no private learning and VC i knows its deviation would not affect others' efforts, its belief stays as if there was no deviation. This implies the original strategy is still an equilibrium from $t + dt$ for all players.

3.2.3 High Preemption Case

While the low preemption case is still reasonable in much cumulative innovation environment, it is off the trend in VC industries. For example, Myspace was once the major startup competitor to Facebook around 2006. But now it is ranked around 4,000th globally in terms of web traffic and receives very limited advertising attention while Facebook still takes the top 3. The returns of investments on these two companies are not even comparable. So in this section, we focus on the case where first-mover grabs substantially more than followers.

The high preemption condition $K + c/\lambda > W$ diminishes the free-riding benefits. The equilibrium strategy has only one threshold that corresponds to $\underline{\pi}$ in Proposition 3.1. Adopting a_I in the middle region of a two-threshold strategy is no longer an equilibrium. To see this, first, notice a_I's monotonicity flips and the new feasibility constraint is

$$a_I(\pi^\dagger) = \phi.$$

Now for a given player i, we can interpret the waiting values $V_L(\pi^\dagger)$ and $V_I(\pi^\dagger)$ respectively as follows:

1. $V_L(\pi^\dagger)$: All VCs exert $\hat{a}_i = \hat{a}_{-i} = \phi$ for $\pi \leq \pi^\dagger$.
2. $V_I(\pi^\dagger)$: VC i exerts $\tilde{a}_i = \phi$. Others exert $\tilde{a}_{-i} = \tilde{a}_I$ for $\underline{\pi} \leq \pi \leq \pi^\dagger$ and $\tilde{a}_{-i} = \phi$ for $\pi < \underline{\pi}$.

In the second case we utilize the fact that when $\tilde{a}_{-i} = \tilde{a}_I$, V_I for player i is independent of its own effort. We can show $V_L(\pi^\dagger) > V_I(\pi^\dagger)$ and by equation (3.7), shirking is the optimal choice. This is because in the second case, $\tilde{a}_{-i} > \phi = \hat{a}_{-i}$ when $\underline{\pi} \leq \pi \leq \pi^\dagger$. It hurts VC i because the *ex post* degree of competition increases, which makes it more likely to receive a substantially small second mover payoff.

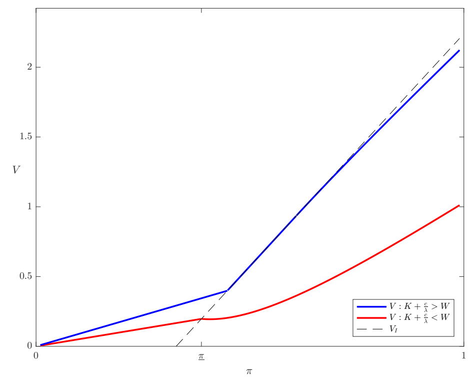
Proposition 3.2. *When $K + c/\lambda > W$,*

$$a_i = \begin{cases} 1 & \pi_t > \underline{\pi} \\ = \phi & \pi_t \leq \underline{\pi} \end{cases}$$

where

$$\underline{\pi} = \frac{c\,(r + N\lambda\phi)}{\lambda\phi\,(N-1)\,(W-K) + rW}.$$

Figure 3.3: Equilibrium Waiting Value—High vs. Low Preemption



As before, the strategy still starts with the highest searching intensities and ends up with shirking. However, when preemption fear is large, the intermediate region with moderate efforts is gone. Instead, VCs extend the preemption region. The intuition for deviating from $a_i = a_I$ to $a_i = 1$ is simple. Players no longer value the free-riding opportunities and are motivated to compete for the first place.

Figure 3.3 indicates that the expected payoff is lower when preemption is higher. The reasons are twofold. A direct effect is an "insurance effect". Players will receive less payoff when losing the games. The indirect effect comes from the change of equilibrium strategies. VCs no longer gradually reduce effort in the indifferent region. This not only increases the chance of being preempted but also expedites the termination of searching due to faster pessimism.

3.2.4 Public Disclosure

We now compare the efficiency loss in the secret scouting case to a public disclosing scenario. The latter is a case where VCs are upfront with the technology outcome and there is no hidden failures. In this case, observing no public investment could be only due to the fact that no one finds a startup. Thus the belief is constant as time passes by and VCs never become pessimistic. In other words, the game is a repeated one. Whenever a single player meets the startup, regardless of the quality, the game ends for all the others. The bellman equation becomes

$$rV_{Pub}(\pi) = \lambda a_i(\pi W - V_{Pub}(\pi)) + \lambda a_{-i}(\pi K - V_{Pub}(\pi)) - c(a_i - \phi). \tag{3.11}$$

Comparing to equation (3.4), the first-order derivative no longer enters the function since the belief is constant. Solving $V_P(\pi)$ yields

$$V_{Pub} = \frac{\lambda a_i \pi W + \lambda a_{-i} \pi K - c(a_i - \phi)}{r + \lambda(a_i + a_{-i})}. \tag{3.12}$$

The expression of equation (3.12) follows a simple interpretation with growth models. The effective discount rate is the original discount rate r with the instantaneous rate of ending $\lambda(a_i + a_{-i})$. The expected payoff at each instant is from either the first or the second mover payoffs net of the effort cost.

The first order condition of equation (3.11) indicates the effort decision rule still follows equation (3.5). With the same trick, we solve a_{Pub} such that $V_{Pub} = V_I$,

$$a_{Pub} = \frac{r\left(\pi W - \frac{c}{\lambda}\right) - c\phi}{\lambda(N-1)(\pi K - \pi W + \frac{c}{\lambda})} < a_I.$$

Define the expected payoff when all players are exerting zero efforts as

$$\underline{V}_{Pub}(\pi) = \frac{\lambda\phi(W + (N-1)K)\pi}{r + N\phi\lambda},$$

and the expected payoff when all players are exerting full efforts as

$$\overline{V}^{Pub}(\pi) = \frac{\lambda\pi W + (N-1)\lambda\pi K - c(1-\phi)}{r + N\lambda}.$$

And lastly, the two boundaries $\pi^\dagger$ and $\pi^\ddagger$ are pinned down by

$$\underline{V}_{Pub}(\pi^\dagger) = V_I(\pi^\dagger),$$
$$\overline{V}^{Pub}(\pi^\ddagger) = V_I(\pi^\ddagger).$$

$\pi^\dagger$ and $\pi^\ddagger$ coincide with the boundaries of feasibility set of a_{Pub}. The following Lemma provides useful benchmarks for analyzing the equilibrium strategies.

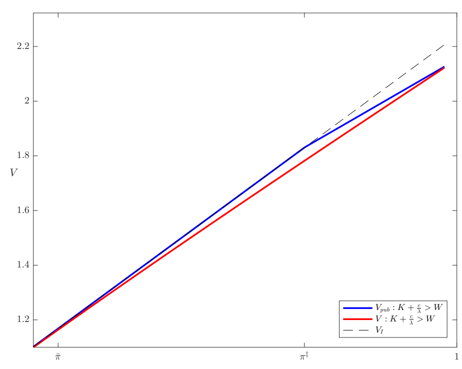
Lemma 3.3. $a_{Pub}(\pi^\dagger) = \phi$*,* $a_{Pub}(\pi^\ddagger) = 1$*.* $\partial a_{Pub}/\partial\pi > 0$ *if and only if* $\pi^\ddagger > \pi^\dagger$*.*

Suppose $\pi^\ddagger > \pi^\dagger$ so $\partial a_{Pub}/\partial\pi > 0$. This is the low preemption scenario with public disclosure. The equilibrium strategy follows a similar two-threshold structure with boundaries $\pi^\dagger$ and $\pi^\ddagger$. Compare $(\pi^\dagger, \pi^\ddagger)$ to $(\underline{\pi}, \bar{\pi})$ in the secret scouting case. Notice first $\underline{V}_{Pub}(\pi) = V_L(\pi)$.

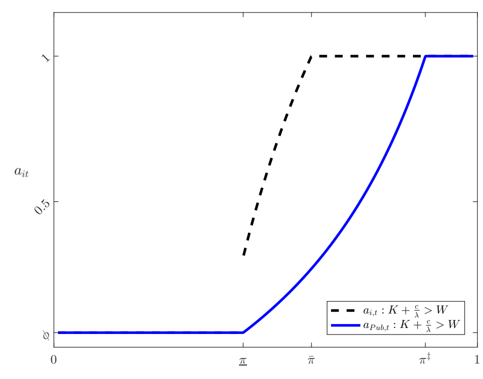
This is because in either cases VCs enter into an absorbing state of action: Exerting no more effort until game ends. Though belief is strictly drifting down with hidden failures, it does impact the actions at all. Thus the lower boundaries are the same, i.e., $\pi^{\dagger} = \underline{\pi}$.

Second the upper boundaries are both decided by $a_I(\bar{\pi}) = 1$ and $a_{Pub}(\pi^{\ddagger}) = 1$. Since $a_{Pub}(\pi) < a_I(\pi)$, it is straightforward to see $\pi^{\ddagger}$ is greater than $\bar{\pi}$. In other words, the preemption region lasts longer in the secret scouting case. This is because hidden failures generate information externalities and lead to early terminations, which reduce the waiting payoffs. In this case, VCs are motivated to find a startup sooner by exerting more effort. This is summarized by the following proposition.

Figure 3.4: Value Function and Effort Level —Low Preemtpion & Public Experimentation



(a) Value Function



(b) Effort Level

Proposition 3.3. *i) If $\partial a_P/\partial\pi > 0$, players exert constant level of efforts until any result is revealed. $a_i = 1$ if $\pi > \pi^{\ddagger}$ and $a_i = \phi$ if $\pi < \pi^{\dagger}$. If belief π is in $[\pi^{\dagger}, \pi^{\ddagger}]$, equilibrium action is pinned down by $a_{Pub}(\pi)$.*

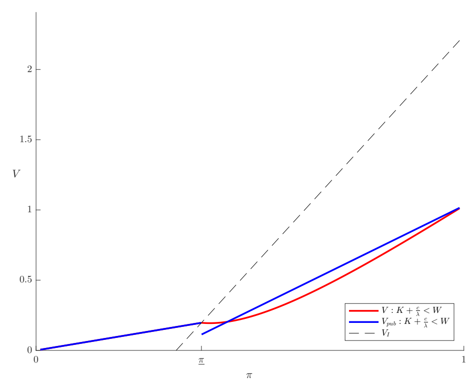
ii) Compared to the secret learning case, the upper threshold shifts rightwards $\pi^{\ddagger} > \bar{\pi}$ and the lower threshold stays the same $\pi^{\dagger} = \underline{\pi}$. Early termination drives down the continuation value of VCs given the same initially optimistic industry: $V_{Pub} > V_I$ if $\pi > \bar{\pi}$.

On the contrary, in the high preemption case players follow a simple switching rule pinned down by $\pi^{\dagger}$. Since $\underline{V}_{Pub}(\pi) = V_L(\pi)$, VCs follow the exact same strategy as they do in

Proposition 3.2. In both cases, VCs cannot align their efforts to make each other indifferent as the free-riding channel is shut down. However, the continuation value of players is still different since in the public disclosing case a constant effort is taken.

Here VCs are not necessarily better off with disclosing failures. In the secret scouting case, players suffer from the cost of early termination. This cost is larger with an initially optimistic belief. In other words, they expect to give up searching for good startups due to adverse selections. But hidden failures also reduce the degree of *ex-post* competitions. A given VC's competitors are also trapped by pessimism. This is beneficial when *ex-ante* the belief is already low enough, so the competitors would quit very soon.

Figure 3.5: Equilibrium Value Function—High Preemtpion & Public Experimentation



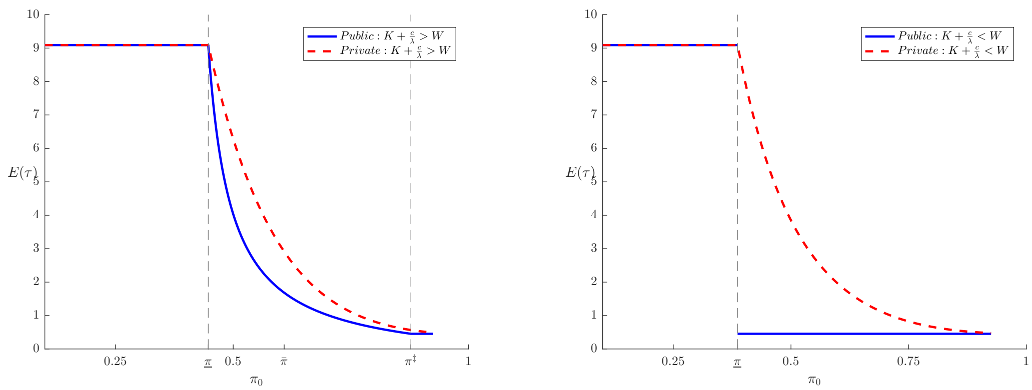
Proposition 3.4. *If $\partial a_P / \partial \pi < 0$, players exert a constant level of effort where the strategy follows the same in Proposition 3.2. There exists a $\pi^\star$ such that $V_{Pub} > V_I$ if and only if $\pi > \pi^\star$.*

As Figure 3.5 shows, the expected payoff is no longer continuous. There is a jump at $\pi^\dagger$. But we do not need to have a value matching condition since the belief is constant. Locally to the right $\pi^\dagger$, players are better off if they collectively choose $a_i = \phi$. However, exerting full search efforts is a dominant strategy and coordination failure occurs.

3.2.5 Startup Welfare

In this section we highlight the efficiency cost of secrecy on the startups' side. The utility of startups may come from different sources. We are interested in the expected waiting time before it meets a VC investor. This is of importance because it measures how fast new technology can be financed. We simulate the model solutions through backward inductions and plot the waiting time against different parameters of interest.

Figure 3.6: Expected Discovery Time Given Initial Belief —Private & Public Experimentation



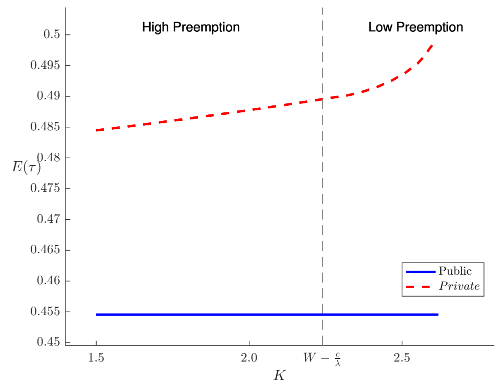
(a) Low Preemption Case (b) High Preemption Case

We start by showing given different initial beliefs about the technology quality, how long a typical startup needs before meeting an investor. Since equilibrium strategies depend on K, we separately plot low and high preemption cases. In Figure 3.6(a), players' efforts follow a two-threshold strategy. It takes strictly less time of searching when failures are publicly disclosed unless the initial belief is sufficiently low. In the latter case, all players shirk in searching. The same result holds for Figure 3.6(b), where players follow a simple one-threshold equilibrium with preemption. In either case, financing under secrecy is delayed due to pessimism.

A related question is that given an initially optimistic technology, how the second mover payoff impacts the expected financing time. In the public disclosing case, a constant highest level of effort is always optimal. In the secret scouting case, the fear of preemption forces a high level of searching to persist for a longer time. Figure 3.7 shows that startups expect to meet VCs slightly faster as industry return becomes more right-skewed.

Does this imply the right-skewness of return spurs innovation? The answer is no. Financing is globally delayed compared to the public disclosing case. We show in the following section, if VCs can *ex ante* commit to a disclosing strategy, secret scouting is an endogenous outcome only when the fear of preemption is large. The efficiency loss due to early-termination cannot be

Figure 3.7: Expected Discovery Time Given Second Mover Payoff



offset by a marginally increased effort as K decreases.

3.3 Endogenous Disclosure

In this section, we show how VCs endogenously opt for secrecy. Suppose prior to the searching game, all VCs can choose and commit to one of the following two disclosing strategies. The first is secret scouting in which a VC only announces the investment in a good startup but leaves the bad technology silently. The second is public disclosing in which a VC reveals the technology quality regardless of θ. Players make simultaneous decisions, which are observable at the beginning of the searching game. The main goal is to show how secrecy endogenously occurs due to the fear of preemption.

We analyze a case where $N = 2$ to simplify the algebra. Besides, we restrict the initial belief to be sufficiently optimistic:

$$\pi_0 > \frac{c}{\lambda} \frac{r + 2\lambda\phi}{rW + \lambda\phi(W - K)}. \tag{3.13}$$

Given equation (3.13), the technology is promising so that all VCs are willing to exert the highest searching efforts when the game starts. In the previous sections, we have solved cases when all players commit to either secret scouting or public disclosing. It remains to show the case when one player hides the failure secretly (indexed by S) and the other publicly disclose all results (indexed by P).

Asymmetric disclosing strategies generate different belief processes of VCs. VC S has a constant belief π_0 as it perfectly knows no failures are observed by VC P. On the contrary, VC P updates its belief according to

$$d\pi_{Pt} = -\lambda a_{St} \pi_{Pt} \left(1 - \pi_{Pt}\right) dt.$$

π_{Pt} is the state variable for both players. For VC S, as π_{Pt} changes, the searching effort of its opponent adjusts and therefore VC S faces a different degree of preemption threats. Its value function follows

$$\begin{aligned} rV_S\left(\pi_{Pt}\right) = &- c\left(a_{St} - \phi\right) + \lambda a_{St}\left(\pi_0 W - V_S\left(\pi_{Pt}\right)\right) + \lambda a_{Pt}\left(\pi_0 K - V_S\left(\pi_{Pt}\right)\right) \\ &- \lambda a_{St} \pi_{Pt}\left(1 - \pi_{Pt}\right) V_S'\left(\pi_{Pt}\right). \end{aligned} \tag{3.14}$$

VC S's value function consists of two parts. The first part is the payoff when VCs find a startup.

This happens either VC S does so with instantaneous probability λa_{St} or VC P finds one with λa_{Pt}. The expected payoff only depends on VC S's initial belief π_0. The second part reflects how the continuation value changes when the opponent's belief decreases. The first-order condition implies

$$a_{St} \begin{cases} = \phi & \text{if } V_S(\pi_{Pt}) > \pi_0 W - c/\lambda - \pi_{Pt}(1-\pi_{Pt}) V_S'(\pi_{Pt}) \\ \in [\phi,\, 1] & \text{if } V_S(\pi_{Pt}) = \pi_0 W - c/\lambda - \pi_{Pt}(1-\pi_{Pt}) V_S'(\pi_{Pt}) \\ 1 & \text{if } V_S(\pi_{Pt}) < \pi_0 W - c/\lambda - \pi_{Pt}(1-\pi_{Pt}) V_S'(\pi_{Pt}). \end{cases} \tag{3.15}$$

Compared to equation (3.5), VC S's marginal benefit of effort has an additional term. The logic is that it could change its searching effort to impact how fast VC P becomes pessimistic. By manipulating its opponent's belief, VC S could reduce the preemption threats. On the contrary, VC P has a value function similar with equation (3.4):

$$\begin{aligned} rV_P(\pi_{Pt}) = & -c(a_{Pt}-\phi) + \lambda a_{Pt}(\pi_{Pt}W - V_P(\pi_{Pt})) + \lambda a_{St}\pi_{Pt}(K - V_P(\pi_{Pt})) \\ & - \lambda a_{St}\pi_{Pt}(1-\pi_{Pt}) V_P'(\pi_{Pt}). \end{aligned} \tag{3.16}$$

VC P uses the same decision rule as equation (3.5). As before, once π_{Pt} becomes sufficiently small, VC P exerts 0 searching efforts. Suppose $a_{Pt} = \phi$ if $\pi_P < \breve{\pi}$. Notice when the threshold $\breve{\pi}$ is crossed, the game also becomes a repeated one for VC S. Its opponent's action enters into an absorbing state and its personal belief is a constant. Though VC P continues to become more pessimistic, it generates no additional benefit for VC S as it already faces the lowest degree of competition. Therefore $V_S'(\pi_{Pt}) = 0$ if $\pi_P < \breve{\pi}$. Define its waiting value as $\bar{V}_S$ when VC P exerts $a_{Pt} = \phi$:

$$r\bar{V}_S = \max_{a_{St}} -c(a_{St}-\phi) + \lambda a_{St}\left(\pi_0 W - \bar{V}_S\right) + \lambda\phi\left(\pi_0 K - \bar{V}_S\right),$$

and the action choice in (3.15) reduces to a simple comparison between $\bar{V}_S$ and $\pi_0 W - c/\lambda$. Lemma 3.4 implies when the initial belief is optimistic, VC S will fully utilize its opponent pessimism and exert highest searching efforts.

Lemma 3.4. *Suppose π_0 satisfies equation (3.13). Then VC S will take highest effort, i.e., $a_{St} = 1$, when VC P shirks in searching with $a_{Pt} = \phi$.*

Lemma 3.4 implies the payoff that VC S has when its opponent P stops exerting efforts follows

$$\bar{V}_S(\pi_0) = \frac{\lambda\pi_0 W + \lambda\phi\pi_0 K - c(1-\phi)}{r+\lambda+\lambda\phi}.$$

It remains to pin down $\breve{\pi}$. VC P's value function when $\pi_P < \breve{\pi}$ follows

$$V_{L,P}(\pi_{Pt}) = \frac{\lambda\phi W + \lambda K}{r+\lambda+\lambda\phi}\pi_{Pt}. \tag{3.17}$$

We can compare $V_{L,P}$ with V_L, where both players constantly exert efforts equal ϕ.[9] Now VC P faces a higher fear of preemption. This changes the effective discount rate to $r+\lambda+\phi\lambda > r+2\lambda\phi$. However, this also increases the probability that VC P receives a follower payoff K even when it is very pessimistic. In other words, the free-riding benefit is larger. These two effects work in the opposite direction and the latter dominates only when the runners-up payoff is sufficiently large. Therefore $V_{L,P}(\pi_{Pt}) > V_L(\pi_{Pt})$ if and only if

$$K > \frac{\lambda\phi}{r+\lambda\phi}W. \tag{3.18}$$

Lastly, the lower bound $\breve{\pi}$ is pinned down by

$$V_{L,P}(\breve{\pi}) = V_I(\breve{\pi}) = \breve{\pi}W - \frac{c}{\lambda},$$

which yields

$$\breve{\pi} = \frac{c(r+\lambda+\lambda\phi)}{\lambda(rW+\lambda(W-K))}.$$

Proposition 3.5 summarizes the equilibrium strategies of players. VC P follows a simple one-threshold strategy in which it exerts the highest searching efforts before $\breve{\pi}$ and shirks afterwards. The strategy of VC S depends on second place payoff. When K is sufficiently small, it benefits from its opponent's shirking so it would like to accelerate the speed at which VC P becomes more pessimistic. This requires VC S uses full efforts in searching constantly.

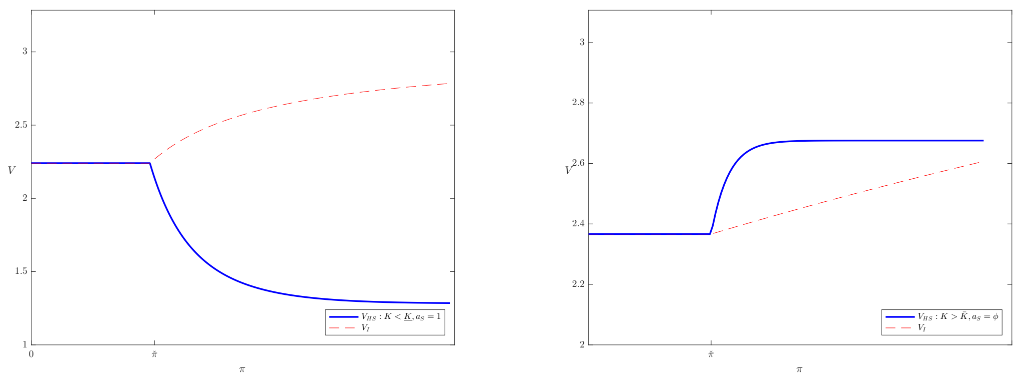
On the contrary, when K is sufficiently large, players actually take turns to make full efforts. In this case, free-riding becomes a dominant benefit. VC S wants to postpone its opponent's

[9] In the two player case,

$$V_L = \frac{\lambda\phi W + \lambda\phi K}{r+2\lambda\phi}\pi.$$

shirking as much as possible. This requires it to cut the initial searching intensity. Figure 3.8 plots the waiting value of VC S in different cases.

Figure 3.8: Value Function for S —High vs. Low Preemtpion



(a) Value Function for S, $K < \underline{K}$

(b) Value Function for S, $K > \bar{K}$

Proposition 3.5. *There exits $\underline{K}$ and $\bar{K}$, $\underline{K} < \bar{K}$, such that*

1. *Regardless of K, VC P follows a one-threshold strategy:*

$$a_P = \begin{cases} 1 & \pi_P > \check{\pi} \\ \phi & \pi_P \leq \check{\pi}. \end{cases}$$

2. *If $K < \underline{K}$, VC S always exerts full effort, $a_S(\pi_{pt}) \equiv 1$.*

3. *If $K > \bar{K}$, VC S starts with the lowest effort until VC P shirks, after which it exerts full efforts:*

$$a_S = \begin{cases} \phi & \text{if } \pi_{pt} \geq \check{\pi} \\ 1 & \text{if } \pi_{pt} < \check{\pi}. \end{cases}$$

Through backward induction, the first-stage disclosing game follows a simple simultaneous 2×2 static game format. As long as $V_{Pub}(\pi_0) < \bar{V}_S(\pi_0)$, commit to public disclosing cannot be an equilibrium. At least one player could deviate to hiding failures and therefore become a "VC S". Doing so will not change its own efforts but reduces the *ex-post* degree of competition.

At extreme case when $K < \lambda\phi W/(r+\lambda\phi)$, commitment to secrecy becomes a dominant strategy. Regardless of what the competing VC chooses, each player strictly prefers to have a pessimistic competitor. Therefore secret scouting becomes a prisoner dilemma. Our result implies the right-skewness of returns in the VC industry has an impact on the disclosing choice when searching for startups. The valuation gap between the first winner startup and a runner-up can be now as large as 100 times. This huge first-mover advantage gives rise to the dominance of secrecy.

Proposition 3.6.

1. *Public disclosing cannot be an equilibrium disclosing strategy if* $V_{Pub}(\pi_0) < \bar{V}_S(\pi_0)$*;*
2. *If* $K < \frac{\lambda\phi}{r+\lambda\phi}W$*, there exists a unique equilibrium such that both players commit to secret scouting.*

The result of Proposition 3.6 can be extended generally to other innovation settings. For example, there is a famous publication bias that researchers will not submit negative and null results. In our model, there are no reputation concerns. A failed technology, though generating no payoffs, is correctly identified. We believe this bias is instead due to increased competition in publication. If a scholar concerns that her current research has been validated as dead ends by someone else, she would spend less time on every single new project. If everyone understands the externalities of hiding failures, they could use this as a tool to reduce the fear of preemption.

3.4 Conclusion

We extend the standard preemption game to explain how VCs search for startups with uncertain technologies. In reality, hiding failed startups is widely observed. This feature of secrecy creates efficiency loss through a channel of pessimism. VCs will question the quality of the technology after observing that no investments are publicly announced. Even though secrecy is costly, hiding failures is an arms race to reduce the *ex-post* fear of preemption. If a return is extremely right-skewed, VCs rely on the secrecy to trick their opponents into a pessimistic belief so that they will stop searching. This can happen even though in realization no technology failures are observed. Therefore though full transparency about searching outcomes could improve social welfare, it cannot be sustained as an equilibrium without intervention.

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Appendix A

Proof of Results in Chapter 1

A.1 Proof of Lemma 2.1

This is direct result from (2.9). When the price is higher than the continuation value W_t^θ, the optimal strategy for the seller is to accept offer, when the price is below the continuation value W_t^θ, then the optimal strategy is to reject the offer.

A.2 Proof of Lemma 2.2

Since there are only two types of asset in our model, given any price p offered by the unsophisticated investors, the payoff is given by

$$\pi_t \left(V_H - p\right) \mu_t^H \left(p\right) + \left(1 - \pi_t\right) \left(V_L - p\right) \mu_t^L \left(p\right) \tag{A.1}$$

where $\mu_t^H = 0$ for $p < C_H$. First, we show that any price $p \in \left(p_t^l, p_t^h\right)$ is not optimal. To see this, when $p \in \left(p_t^l, p_t^h\right), \mu_t^L \left(p\right) = 1$, then by reducing the price offer from p to $p - \varepsilon$ such that $p - \varepsilon \in \left(p_t^l, p_t^h\right)$, this offer is still accepted by low-type seller for sure, but it can improve the profit to the uninformed buyer.

Second, we show that any price $p_t > C_H$ is not optimal. This is trivial, since the uninformed buyer could always lowers p_t a little bit which doesn't affect the decision for the sellers but improve the profit to the uninformed buyer.

Third, we want to show that non-serious offer could be made by the uninformed buyers. This is indeed the case when

$$\pi_t(V_H - p^h) + (1-\pi_t)(V_L - p^h) < 0 \tag{A.2}$$

and

$$W_t^L < V_L \tag{A.3}$$

where (A.2) states that offer p^h is not profitable since it generates negative profit, and (A.2) states that p_l is also not optimal, since the price paid to the low-quality asset is higher than the value to the uninformed buyers. Thus, in this case, only non-serious offers are made.

A.3 Proof of Lemma 2.4

We prove Lemma 2.4 by contradiction. Suppose that under Assumption 2.1 there exist an equilibrium where $\lambda_t^H = \lambda_t^L = \lambda$, that is both types of asset are traded at efficient rate. Then going forward, conditional on there is no trading occur, both types of sellers and buyer will repeat their strategies at time t , and this is due to the fact that the market belief stays constant, and the bidding strategy for the buyers are unaffected. For $\lambda_t^H = \lambda$, it requires that $\sigma_t^U(C_H) = 1$, otherwise, the high-type seller can always hold on to the asset instead of trading with the uninformed buyer. Similarly, in order to have $\lambda_t^L = \lambda$, it requires that $p_l = W_t^L \leq V_L$, and $\mu_t^L(p_l) = 1$. Therefore, for $t' > t, \sigma_t^U(C_H) = 1$ and $\sigma_t^I(C_H) = \mathbb{I}(\theta = H), \sigma_t^I\left(W_t^H\right) = \mathbb{I}(\theta = L)$

Under this strategy, the continuation value to the low-quality seller is

$$W_t^L \leq \frac{rC_L + \lambda\left(sV_L + (1-s)\,C_H\right)}{r+\lambda} \tag{A.4}$$

and the inequality holds when $W_t^L = V_L$. But from Assumption 2.1, we know that

$$V_L < \frac{rC_L + \lambda\left(sV_L + (1-s)\,C_H\right)}{r+\lambda} \tag{A.5}$$

which contradicts with $p_l = W_t^L \leq V_L$. Therefore, equilibrium with $\lambda_t^H = \lambda_t^L = \lambda$ is not feasible under Assumption 2.1.

A.4 Proof of Lemma 2.5

Proof. Given the market belief π_t, only two prices are offered with positive probability, either C_H or $\min\left\{V_L, W_t^L\right\}$, the payoff from offering C_H is

$$\pi_t\left(V_H - C_H\right) + \left(1-\pi_t\right)\left(V_L - C_H\right) \tag{A.6}$$

and the payoff from offering $\min\left\{V_L, W_t^L\right\}$ is given by

$$\left(1-\pi_t\right)\left(V_L - \min\left\{V_L, W_t^L\right\}\right) \tag{A.7}$$

So comparing (A.6) and (A.7), it is easy to see that when $\pi_t > \frac{C_H - \min\{W_t^L, V_L\}}{V_H - \min\{W_t^L, V_L\}}$, uninformed buyers are willing to offer C_H, and when $\pi_t < \frac{C_H - \min\{W_t^L, V_L\}}{V_H - \min\{W_t^L, V_L\}}$, the maximum price they are willing to offer is V_L □

A.5 Proof of Lemma 2.6

Proof. There are two parts in this lemma. Denote W^{L*} as the continuation value for low-type seller in stationary equilibrium. First I will show that in stationary equilibrium, the continuation value for low-quality seller is V_L.

Suppose not, and $W^{L*} > V_L$, then low-quality seller will turn down any offer from sophisticated buyers, since V_L is the maximum that the informed sellers are willing to pay for the low-quality asset. But high-quality seller always trades with the informed buyer, hence the trading rate for high quality asset is higher $\lambda_H > \lambda_L$, which contradicts with Lemma 2.2. If $W^{L*} < V_L$, then low-quality seller will trade for sure at price $p \in (W^{L*}, V_L)$ if she meets with a buyer. But this is also against the Lemma 2.2. Thus, we can conclude that in stationary equilibrium, the continuation value for low-type seller should be V_L. □

A.6 Proof of Theorem 2.1

Proof. From Lemma 2.5 we know that the continuation value for low-quality seller is V_L, thus he is indifferent between accepting and rejecting V_L. Similarly, in stationary equilibrium, the uninformed buyer is mixing between V_L and C_H. According to Lemma 2.3, we know that

$\pi^* = \frac{C_H - V_L}{V_H - V_L}$. Now we need to pin down the exact γ_t in equilibrium. Recall from (2.13), in stationary equilibrium, we have

$$rW^{L*} = rC_L + \gamma^*(C_H - W^{L*}) \tag{A.8}$$

which can uniquely pin down the γ^* as $\gamma^* = \frac{r(V_L - C_L)}{C_H - V_L}$, and therefore the probability that the uninformed buyer offers C_H is given by

$$\sigma^{U*}(C_H) = \frac{\gamma^*}{(1-s)\lambda} \tag{A.9}$$

And it is easy to check that $\sigma^{U*}(C_H) \in (0,1)$ from Assumption 2.2. The remaining part is to pin down the probability that low-quality seller is willing to accept V_L. This can be solved based on Lemma 2.4, since both types of sellers are trading at the same rate, and given γ^*, we know that high-quality asset is traded at a rate $\lambda^H = s + (1-s)\gamma^*$. Hence the law-quality seller should also trade at rate $\lambda^L = \lambda^H$. And we can express the trading rate for the low type as:

$$\lambda^L = \lambda\left(\mu^{L*}(V_L)(s + (1-s)\sigma^U(V_L)) + (1-s)\sigma^U(C_H)\right) \tag{A.10}$$

Where the first term on RHS is the probability to accept offer V_L, and second term on the RHS is the probability to trade at price C_H. Hence, we can get that the accepting rate for V_L is given by

$$\mu^{L*}(V_L) = \frac{\mu}{1-(1-\mu)\sigma^{U*}(C_H)} \tag{A.11}$$

$$= \frac{\lambda s}{\lambda - \gamma^*} \tag{A.12}$$

□

A.7 Proof of Lemma 2.7

Proof. There are two parts for this Lemma.

1. I will show that when $\pi_t > \pi^*$, then $W_t^L > V_L$. Suppose not, there exists a $\pi' > \pi^*$ such that $W_t^L(\pi') \leq V_L$. Now I consider the first case where $W_t^L < V_L$. Then the low

type will trade for sure, hence $\lambda_L = \lambda$. However, from 2.4 we know that we cannot have equilibrium where both types of sellers trade at rate λ, then it follows that $\lambda_H < \lambda_L$, so based on the belief dynamics 2.4, the market belief goes up.

Then, I claim that there exists a $\pi_1 > \pi'$ such that $W_t^L(\pi_1) = V_L$. Suppose this is not true, due to the continuity of W_t^L, we can conclude that for all $\pi_t > \pi'$, we have $W_t^L < V_L$, then applying the same logic, we can see that the trading rate for low-quality seller is always $\lambda_L = \lambda$, then the market belief goes up for all $\pi_t > \pi'$, but since the market belief is monotone and bounded above by 1, then π_t converges to $\pi_\infty < 1$, but since $\pi_\infty > \pi^*$, this cannot be as stationary belief, hence this is a contradiction.

Given that at $\pi_1 > \pi^*$, $W_t^L(\pi_1) = V_L$, and for all the $\pi_t \in (\pi', \pi_1)$, $R_t < V_L$, now consider $\pi_t'' = \pi_1 - \varepsilon$, where ε is small enough, then we know that $W_t^L(\pi_t'') \to V_L$ since reservation value is continuous in π_t, then we know that at π_t'', uninformed buyers are willing to offer C_H since by offering W_t^L, then can only get a payoff close to zero. Hence at π_t'', the both high-quality seller and low-quality seller will trade at rate λ, which is against Lemma 2.4.

Now suppose that there exists a π_t such that $W_t^L(\pi_t') = V_L$, then for all $\pi_t \in (\pi^*, \pi_t')$ we have $W_t^L > V_L$, now consider at the π_t', the dynamics of continuation value should be

$$rW_t^L = rC_L + \gamma_t(C_H - W_t^L) + \frac{\mathrm{d}W_t^L}{\mathrm{d}t} \tag{A.13}$$

Since at π_t', unsophisticated buyers offer C_H, then high-quality seller trades at maximum rate λ, then the market belief goes down, hence $\pi_{t+\mathrm{d}t}' < \pi_t'$, so we can see that $\frac{\mathrm{d}W_t^L}{\mathrm{d}t} > 0$, also $\gamma = \lambda(1-s)$ at π', then we can rewrite the continuation value of the low-quality seller at π' as

$$W_t^L = \frac{rC_L + \lambda(1-s)C_H + \frac{\mathrm{d}W_t^L}{\mathrm{d}t}}{r + \lambda(1-s)}$$

which should be greater than V_L due to the Assumption 2.1. Therefore this contradicts with the $W_t^L = V_L$ at π'.

2. Now, we need to show that when $\pi_t < \pi^*$, $W_t^L < V_L$. Suppose not, there exists t such that $W_t^L(\pi_2) \geq V_L$. Now consider the first case where $W_t^L(\pi_2) > V_L$. I claim that in

this case, there exists π'' such that $W_t^L(\pi'') = V_L$. If not, then for all $\pi_t < \pi_2$, we must have $W_t^L(\pi_t) > V_L$, then low quality asset trades at rate $\lambda_L = 0$, where high quality asset trades at rate$\lambda\mu$, hence the market belief goes down, bu t since the market belief is bounded below, then there exists π'_∞ such that $\pi_t \to \pi'_\infty > 0$, but we have shown that the stationary belief is unique, hence the this is a contradiction. Hence, we can find a market belief π''_t such that $W_t^L(\pi''_t) = V_L$, and consider the continuation dynamics at $\pi''_t + \varepsilon'$, since the market belief will go down due to the fact that the uninformed buyer will not trade with anybody. And

$$rW_t^L = rC_L + \frac{\mathrm{d}W_t^L}{\mathrm{d}t}$$

Since $W_t^L > V_L$ at $\pi''_t + \varepsilon$, then the continuation value goes up, therefore, the market belief goes down. Due to the fact that the market belief is bounded below, we can claim that $\pi_t \to \pi''_\infty$, but this is also a contradiction. Hence we cannot have π'' such that $W_t^L(\pi_t) > V_L$ for $\pi_t < \pi^*$

□

Proof of Corollary 2.1

Proof. Following Lemma 2.6, we know that when $\pi_t > \pi^*$, $W_t^L > V_L$, therefore low-quality seller will not trade with informed buyer, since the uninformed buyers will bid C_H for sure when $\pi_t > \pi^*$, then they will trade with both type of sellers in the market. Thus, high-quality seller is trading at rate λ, while low-quality seller is trading at rate $\lambda(1 - \mu)$. Similarly, suppose that $\pi_t < \pi^*$, then there is no delay trading for low-quality seller, they will accept any price that is slightly higher than their reservation value, hence they are trading at rate λ. For high-quality seller, they only trade with informed buyers, since only informed buyers are willing to pay C_H which is the reservation value for high-quality seller, hence we see that high-quality seller will only trade at rate λs. □

A.8 Proof of Theorem 2.2

1. When $\pi_t < \pi^*$,$W_t^L < V_L$, the trading rate for low-quality $\lambda_t^L = \lambda$ and $\lambda_t^H = \lambda(1-m)$, then by continuation dynamics, we have

$$\mathrm{d}\pi_t = \lambda(1-s)\pi_t(1-\pi_t)\mathrm{d}t$$
$$rW_t^L = rC_L + \frac{dW_t^L}{\mathrm{d}t}$$

with boundary condition

$$\pi_{t_1^*} = \pi^* \tag{A.14}$$
$$W_{t_1^*}^L = V_L \tag{A.15}$$

Solving these equations give the result that

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda(1-s)t}} & t < t_1^* \\ \pi^* & t > t_1^* \end{cases} \tag{A.16}$$

$$W_t^L = \begin{cases} C_L + e^{-r(t_1^*-t)}(V_L - C_L) & t < t_1^* \\ V_L & t > t_1^* \end{cases} \tag{A.17}$$

2. When $\pi_t > \pi^*$, $W_t^L > V_L$, hence the trading rate for low-quality seller $\lambda_t^L = \lambda(1-s)$ and $\lambda_t^H = \lambda$, then by continuation dynamics, we have

$$\mathrm{d}\pi_t = -\lambda\mu\pi_t(1-\pi_t)\mathrm{d}t \tag{A.18}$$
$$rW_t^L = rC_L + \lambda(1-m)(C_H - W_t^L) + \frac{dW_t^L}{\mathrm{d}t} \tag{A.19}$$

With boundary condition

$$\pi_{t_2^*} = \pi^*$$
$$W_{t_2^*}^L = V_L$$

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda\mu t}} & t < t_2^* \\ \pi^* & t > T_2^* \end{cases} \tag{A.20}$$

$$W_t^L = \begin{cases} V_L + \left(1 - e^{-(r+\lambda)(T_2^*-t)}\right)(C_H - V_L) & t < t_2^* \\ V_L & t > T_2^* \end{cases} \tag{A.21}$$

A.9 Proof of Proposition 2.2

Proof. 1. First, we consider the case where $\pi_0 > \pi^*$. For the high-type, the trading rate is λs when $t < t_1^*$ and $\lambda s + \gamma^*$. Therefore the average trading time for high-quality asset can be expressed as

$$\tau_H = \int_0^{t_1^*} u\lambda s e^{-\lambda s u} du + e^{-\lambda\mu t_1^*} \frac{1}{\lambda s + \gamma^*} \tag{A.22}$$

And we can simplify the expression and get that

$$\tau_H = (1 - e^{-\lambda\mu t_1^*})\frac{1}{\lambda\mu} + e^{-\lambda\mu t_1^*}\frac{1}{\lambda s + \gamma^*} \tag{A.23}$$

Similarly, for the low-quality asset, the trading rate is λ when$t < t_1^*$ and $\lambda\mu + \gamma^*$ when $t \geq t_1^*$. Therefore, the average trading time for low-quality asset can be expressed as

$$\tau_L = \int_0^{t_1^*} s\lambda e^{-\lambda s} ds + e^{-\lambda t_1^*}\frac{1}{\lambda s + \gamma^*}$$

which, again can be simplified as

$$\tau_L = (1 - e^{-\lambda t_1^*})\frac{1}{\lambda} + e^{-\lambda t_1^*}\frac{1}{\lambda s + \gamma^*}$$

2. When $\pi_0 > \pi^*$, following the same calculation, we can get the results.

□

A.10 Proof of Corollary 2.2

The two cases are very similar:

1. When $\pi_0 < \pi^*$, I can rewrite (2.32)(2.33) as

$$\tau_H = \frac{1}{\lambda\mu}(1 - e^{\lambda\mu T_1^*}) + e^{-\lambda\mu T_1^*}\frac{1}{\lambda\mu + \gamma^*}$$
$$\tau_L = \frac{1}{\lambda}(1 - e^{-\lambda T_1^*}) + e^{-\lambda T_1^*}\frac{1}{\lambda\mu + \gamma^*}$$

   Since $\lambda > \lambda\mu + \gamma^* > \lambda\mu$, it is easy to see that the weighted average of $\frac{1}{\lambda}$ and $\frac{1}{\lambda\mu+\gamma^*}$ should be lower than the weighted average of $\frac{1}{\lambda\mu}$ and $\frac{1}{\lambda\mu+\gamma^*}$, which is $\tau_L < \tau_H$.

2. When $\pi_0 > \pi^*$, we can applying the same logic, and using the fact that $\lambda > \gamma^* + \lambda\mu > \lambda(1 - \mu)$, then it easy to show that in this case $\tau_L > \tau_H$

Appendix B

Proof of Results in Chapter 2

B.1 Proof of Lemma 3.1

Proof. Take derivative of a_I,

$$\begin{aligned} a_I' &= (\frac{r\pi W - c(\frac{r}{\lambda} + \phi)}{(N-1)(K + \frac{c}{\lambda} - W)\lambda\pi})' \\ &= \frac{c(\frac{r}{\lambda} + \phi)(N-1)(K + \frac{c}{\lambda} - W)}{((N-1)(K + \frac{c}{\lambda} - W)\lambda\pi)^2} \end{aligned}$$

Thus $a_I' > 0$ if and only if $K + \frac{c}{\lambda} - W > 0$. □

B.2 Proof of Lemma 3.2

Proof. Use the value matching condition $V_H = \pi W - \frac{c}{\lambda}$ and smooth pasting condition $V_H' = W$ in equation (3.8)

$$
\begin{aligned}
& r(\pi W - \frac{c}{\lambda}) = c + \lambda(N-1)\pi\left(K - \pi W + \frac{c}{\lambda}\right) - (N-1)\pi\left(1-\pi\right)\lambda W - c\left(1-\phi\right) \\
\implies & r(\pi W - \frac{c}{\lambda}) = \lambda(N-1)\pi\left(K - W + \frac{c}{\lambda} + (1-\pi)W\right) - (N-1)\pi\left(1-\pi\right)\lambda W + c\phi \\
\implies & r(\pi W - \frac{c}{\lambda}) = \lambda(N-1)\pi\left(K - W + \frac{c}{\lambda}\right) + c\phi \\
\implies & \pi(rW + (N-1)\left((W-K)\lambda - c\right)) = c\left(\phi + \frac{r}{\lambda}\right) \\
\implies & \pi = \bar{\pi}
\end{aligned}
$$

□

B.3 Proof of Proposition 3.1

Proof. First notice when $\pi < \frac{c}{\lambda W}$, $V_I < 0$. The player's waiting value must be non-negaitve. This implies $V > V_I$ if π is sufficiently small. By equation (3.5), $a_t = \phi$ for any i. Consider any π that all players use $a_t = \phi$ when belief is smaller than π, then VC i's value function follows $V_L = \frac{\lambda\phi(W+K(N-1))}{N\lambda\phi+r}\pi$. Thus, $V_L \geq V_I$ if and only if $\pi \leq \underline{\pi}$.

During $[\underline{\pi}, \bar{\pi}]$, players are indifferent with regard to actions as $V = V_I$. Thus using $a_t = a_I$ is a weakly dominant strategy. If $\pi > \bar{\pi}$, a_I is no longer feasible. It remains to show V_H defined by equation (3.8) satisfies $V_H < V_I$ for all $\pi > \bar{\pi}$. Firstly, since $\hat{\pi} = \frac{c\left(\phi+\frac{r}{\lambda}\right)}{rW+(N-1)((W-K)\lambda-c)}$, this value must be strictly smaller than 1 as it is a probability, which yields

$$c(r+\lambda\phi) < \lambda[rW + (N-1)(\lambda(W-K)-c)] \tag{B.1}$$

Secondly, $V_H = -\frac{c(1-\phi)}{(\lambda+r)} + \frac{\lambda(W+(N-1)(K+\frac{c(1-\phi)}{\lambda+r}))}{N\lambda+r}\pi + C_1\left(1-\pi\right)\left(\frac{1-\pi}{\pi}\right)^{\frac{r+\lambda}{(N-1)\lambda}} = \pi W - \frac{c}{\lambda}$

at $\bar{\pi}$. Thus $C_1 < 0$ if and only if

$$\begin{aligned}
&-\frac{c}{\lambda}+\frac{c\,(1-\phi)}{(\lambda+r)}+\bar{\pi}(W-\frac{\lambda(W+(N-1)(K+\frac{c(1-\phi)}{\lambda+r}))}{N\lambda+r})<0\\
\Leftrightarrow&\bar{\pi}\frac{rW+(N-1)[\lambda(W-K)-\frac{\lambda}{r+\lambda}c(1-\phi)]}{r+N\lambda}<\frac{c(\phi+\frac{r}{\lambda})}{r+\lambda}\\
\Leftrightarrow&\bar{\pi}<\frac{r+N\lambda}{r+\lambda}\frac{c(\phi+\frac{r}{\lambda})}{rW+(N-1)[\lambda(W-K)-\frac{\lambda}{r+\lambda}c(1-\phi)]}\\
\Leftrightarrow&(\lambda+r)(rW+(N-1)\lambda(W-K))-(N-1)\lambda c(1-\phi)<\\
&(N\lambda+r)(rW+(N-1)\lambda(W-K))-(N\lambda+r)(N-1)c\\
\Leftrightarrow&-rW-(N-1)\lambda(W-K)+(N-1)c+c\phi+c\frac{r}{\lambda}<0
\end{aligned}$$

which is equivalent to equation (B.1). Take second order derivative of V_H

$$V_H''=C_1\frac{r+\lambda}{(N-1)\,\lambda}(1+\frac{r+\lambda}{(N-1)\,\lambda})(1-\pi)^{\frac{r+\lambda}{(N-1)\lambda}-1}\pi^{-(\frac{r+\lambda}{(N-1)\lambda}+2)}<0$$

Thus V_H is a concave function. Since it satisfies smooth pasting condition at $\bar{\pi}$, $V_H < V_I$ for all $\pi > \bar{\pi}$. □

B.4 Proof of Proposition 3.2

Proof. The proof for the region $\pi < \underline{\pi}$ is the same as the of Proposition 3.1. For all $\pi > \underline{\pi}$, first consider V_H' at $\underline{\pi}$ with value matching condition

$$\begin{aligned}
&r(\underline{\pi}W-\frac{c}{\lambda})=c+\lambda(N-1)\underline{\pi}\left(K-\underline{\pi}W+\frac{c}{\lambda}\right)-(N-1)\underline{\pi}\,(1-\underline{\pi})\,\lambda V_H'-c\,(1-\phi)\\
\Longrightarrow\,&r(\underline{\pi}W-\frac{c}{\lambda})=\lambda(N-1)\underline{\pi}\left(K-W+\frac{c}{\lambda}+(1-\underline{\pi})W\right)-(N-1)\underline{\pi}\,(1-\underline{\pi})\,\lambda V_H'+c\phi\\
\Longrightarrow\,&\underline{\pi}(rW+\lambda(N-1)(W-K-\frac{c}{\lambda}))-c(\frac{r}{\lambda}+\phi)=\lambda(N-1)\underline{\pi}\,(1-\underline{\pi})\,(W-V_H')
\end{aligned}$$

Thus $W < V_H'$ if and only if

$$\underline{\pi}>\frac{c(\frac{r}{\lambda}+\phi)}{rW+\lambda(N-1)(W-K-\frac{c}{\lambda})}=\bar{\pi}$$

Denote $\tilde{\pi}$ as the value of π such that $a_I(\tilde{\pi}) = \phi$. By Lemma 3.1, $a_I' < 0$ in this case and hence $\bar{\pi} < \tilde{\pi} = \frac{c(\frac{r}{\lambda}+\phi)}{rW+\lambda(N-1)\phi(W-K-\frac{c}{\lambda})}$. $\underline{\pi} > \tilde{\pi}$ is equivalent to

$$\begin{aligned}
\frac{c\left(\frac{r}{\lambda}+N\phi\right)}{\lambda\phi\left(N-1\right)\left(W-K\right)+rW} &> \frac{c(\frac{r}{\lambda}+\phi)}{rW+\lambda(N-1)\phi(W-K-\frac{c}{\lambda})} \\
\Longleftrightarrow rW+\lambda(N-1)\phi(W-K) &> c(\frac{r}{\lambda}+N\phi) \\
\Longleftrightarrow rW+\lambda(N-1)\phi(W-K) &> c(\frac{r}{\lambda}+\phi)+(N-1)\phi c
\end{aligned}$$

Last line holds as $W > \frac{c}{\lambda}+\frac{c\phi}{r}$ and $W-K-\frac{c}{\lambda} > 0$. This implies $V_H'(\underline{\pi}) < W = V_I'(\underline{\pi})$. We then show $V_H < V_I$ for all $\pi > \underline{\pi}$. Suppose not, then there exist $\pi^\star > \underline{\pi}$ such that $V_H(\pi^\star) = \pi^\star W - \frac{c}{\lambda}$ and $V_H'(\pi^\star) > W$. By equation (3.8),

$$\begin{aligned}
rV_H(\pi^\star) &< \lambda(\pi^\star W - \pi^\star W - \frac{c}{\lambda}) + \lambda(N-1)\pi^\star(K-\pi^\star W - \frac{c}{\lambda}) - (N-1)\pi^\star(1-\pi^\star)\lambda W - c(1-\phi) \\
&= \lambda(N-1)\pi^\star\left(K-W+\frac{c}{\lambda}+(1-\pi^\star)W\right) - (N-1)\pi^\star\left(1-\pi^\star\right)\lambda W + c\phi \\
&= \lambda(N-1)\pi^\star\left(K-W+\frac{c}{\lambda}\right)+c\phi
\end{aligned}$$

Since $W > K+\frac{c}{\lambda}$, the RHS is decreasing in $\pi^\star$. Thus

$$\begin{aligned}
rV_H(\pi^\star) &< \lambda(N-1)\bar{\pi}\left(K-W+\frac{c}{\lambda}\right)+c\phi \\
&= -\bar{\pi}(rW+\lambda(N-1)\left(W-K-\frac{c}{\lambda}\right)) + r\bar{\pi}W + c\phi \\
&= -c(\frac{r}{\lambda}+\phi) + r\bar{\pi}W + c\phi \\
&= r\bar{\pi}W - c\frac{r}{\lambda} \\
&< r(\pi^\star W - \frac{c}{\lambda})
\end{aligned}$$

which is contradictory to the assumption that $V_H(\pi^\star) = \pi^\star W - \frac{c}{\lambda}$. Thus $V_H < V_I$ for all $\pi > \underline{\pi}$, which implies $a_{it} = 1$ in the region. □

B.5 Proof of Lemma 3.3

Proof. i)

$$\begin{aligned}
&\overline{V}^{Pub}(\pi) = V_I(\pi) \\
\Longleftrightarrow &\frac{\lambda\pi W + (N-1)\lambda\pi K - c(1-\phi)}{r + N\lambda} = \pi W - \frac{c}{\lambda} \\
\Longleftrightarrow &\lambda(N-1)(\pi K - \pi W + \frac{c}{\lambda}) = r\left(\pi W - \frac{c}{\lambda}\right) - c\phi \\
\Longleftrightarrow &a_{pub}(\pi) = 1
\end{aligned}$$

ii)

$$\begin{aligned}
&\underline{V}_{Pub}(\pi) = V_I(\pi) \\
\Longleftrightarrow &\frac{\lambda\phi(W + (N-1)K)\pi}{r + N\phi\lambda} = \pi W - \frac{c}{\lambda} \\
\Longleftrightarrow &\lambda(N-1)(\pi K - \pi W + \frac{c}{\lambda})\phi = r(\pi W - \frac{c}{\lambda}) - c\phi \\
\Longleftrightarrow &a_{pub}(\pi) = \phi
\end{aligned}$$

iii) Take derivative of a_{pub},

$$\begin{aligned}
a'_{pub} &= (\frac{r\left(\pi W - \frac{c}{\lambda}\right) - c\phi}{\lambda(N-1)(\pi K - \pi W + \frac{c}{\lambda})})' \\
&= \frac{\lambda(N-1)(\frac{c}{\lambda}rW - (c\frac{r}{\lambda} + c\phi)(W-K))}{(\lambda(N-1)(\pi K - \pi W + \frac{c}{\lambda}))^2}
\end{aligned}$$

Thus, $a'_{pub} > 0$ if and only if $\frac{c}{\lambda}rW > (c\frac{r}{\lambda}+c\phi)(W-K)$. Meanwhile, $\pi^{\dagger} = \frac{c\frac{r}{\lambda}+c\phi+\lambda\phi(N-1)\frac{c}{\lambda}}{rW+\lambda\phi(N-1)(W-K)}$ and $\pi^{\ddagger} = \frac{c\frac{r}{\lambda}+c\phi+\lambda(N-1)\frac{c}{\lambda}}{rW+\lambda(N-1)(W-K)}$.

$$
\begin{aligned}
&\pi^{\dagger} < \pi^{\ddagger} \\
\Longleftrightarrow & \frac{c\frac{r}{\lambda} + c\phi + \lambda\phi(N-1)\frac{c}{\lambda}}{rW + \lambda\phi(N-1)(W-K)} < \frac{c\frac{r}{\lambda} + c\phi + \lambda(N-1)\frac{c}{\lambda}}{rW + \lambda(N-1)(W-K)} \\
\Longleftrightarrow & [c\frac{r}{\lambda} + c\phi + \lambda\phi(N-1)\frac{c}{\lambda}](W-K) < \\
& \frac{c}{\lambda}[rW + \lambda\phi(N-1)(W-K)] \\
\Longleftrightarrow & (c\frac{r}{\lambda} + c\phi)(W-K) < \frac{c}{\lambda}rW
\end{aligned}
$$

□

B.6 Proof of Proposition 3.3

Proof. The argument for $\pi < \pi^{\ddagger}$ is logically the same as Proposition 3.1. Since $\overline{V'}^{Pub}(\pi) = \frac{\lambda W + (N-1)\lambda K}{r + N\lambda} < W = V_I'(\pi)$, $\overline{V}^{Pub}(\pi) < V_I(\pi)$ for all $\pi > \pi^{\ddagger}$. To see the second part of the statement, suppose there exists a π such that $V_H(\pi) = \overline{V}^{Pub}(\pi)$, i.e.

$$
\begin{aligned}
& -\frac{c\,(1-\phi)}{(\lambda+r)} + \frac{\lambda(W + (N-1)(K + \frac{c(1-\phi)}{\lambda+r}))}{N\lambda + r}\pi + C_1\,(1-\pi)\left(\frac{1-\pi}{\pi}\right)^{\frac{r+\lambda}{(N-1)\lambda}} = \frac{\lambda\pi W + (N-1)\lambda\pi K - c(1-\phi)}{r + N\lambda} \\
\Longleftrightarrow & C_1\,(1-\pi)\left(\frac{1-\pi}{\pi}\right)^{\frac{r+\lambda}{(N-1)\lambda}} = \frac{c\,(1-\phi)}{(\lambda+r)} - \frac{c(1-\phi)}{r+N\lambda}(\frac{\pi\lambda}{\lambda+r} + 1)
\end{aligned}
$$

Notice the RHS is positive as

$$
\begin{aligned}
& \frac{c\,(1-\phi)}{(\lambda+r)} - \frac{c(1-\phi)}{r+N\lambda}(\frac{\pi\lambda}{\lambda+r} + 1) > 0 \\
\Longleftrightarrow & \frac{1}{\lambda+r} > \frac{\frac{\pi\lambda}{\lambda+r} + 1}{r+N\lambda} \\
\Longleftrightarrow & r + N\lambda > \pi\lambda + \lambda + r
\end{aligned}
$$

The last line holds as $N \geq 2$ and $\pi \leq 1$. However the LHS is negative as $C_1 < 0$, which is contradictory. Thus $V_H(\pi) < \overline{V}^{Pub}(\pi)$. □

B.7 Proof of Proposition 3.4

Proof. The first half of the statement is the same as Proposition 3.2. For the second half, notice, in this case, $\pi^{\ddagger} < \pi^{\dagger}$. Thus $\overline{V}^{Pub}(\pi^{\dagger}) < V_I(\pi^{\dagger}) = V_H(\pi^{\dagger})$. The first inequality comes from $\overline{V'}^{Pub}(\pi) < V_I'(\pi)$ and the second equality comes from value matching condition at $\pi^{\dagger}$.

$$\begin{aligned}&\overline{V}^{Pub}(\pi) - V_H(\pi)\\ =&\frac{c\,(1-\phi)}{(\lambda+r)} - \frac{c(1-\phi)}{r+N\lambda}(\frac{\pi\lambda}{\lambda+r}+1) - C_1\,(1-\pi)\left(\frac{1-\pi}{\pi}\right)^{\frac{r+\lambda}{(N-1)\lambda}}\end{aligned}$$

which implies $\lim\limits_{\pi\to 1} \overline{V}^{Pub}(\pi) - V_H(\pi) = \frac{c(1-\phi)}{(\lambda+r)} - \frac{c(1-\phi)}{r+N\lambda}(\frac{\lambda}{\lambda+r}+1) = \frac{c(1-\phi)}{(\lambda+r)}(1-\frac{2\lambda+r}{N\lambda+r}) > 0$. As both $\overline{V}^{Pub}(\pi)$ and $V_H(\pi)$ are continuous functions. By intermediate value theorem, there exists a $\pi^\star$ such that $\overline{V}^{Pub}(\pi) > V_H(\pi)$ if $\pi > \pi^\star$.

$\pi > \pi^\star$ is also a sufficient condition for $\overline{V}^{Pub}(\pi) > V_H(\pi)$. This is because $V_H(\pi)$ is a convex function in the high preemption scenario. Thus $V_H(\pi)$ has at most two intersections with the linear function $\overline{V}^{Pub}(\pi)$. However if there were two intersections, $V_H(\pi)$ will be strictly greater than $\overline{V}^{Pub}(\pi)$ for all π larger than the second intersection. This is contradictory to the fact that $\overline{V}^{Pub}(\pi) - V_H(\pi)$ at a neighbor of $\pi = 1$. Therefore $\overline{V}^{Pub}(\pi)$ intersects with $V_H(\pi)$ only once. □

B.8 Proof of Lemma 3.4

Proof. Suppose $a_{St} = \phi$, then

$$\bar{V_S} = \frac{\lambda\phi\pi_0(W+K)}{r+2\lambda\phi}$$

$\bar{V_S} > \pi_0 W - \frac{c}{\lambda}$ if and only if

$$c > \lambda\frac{r\pi_0 W + \lambda\phi\pi_0(W-K)}{r+2\lambda\phi}$$

which is contradictory to equation (3.13). To verify $a_{St} = 1$ is indeed optimal, if so

$$\bar{V_S} = \frac{\lambda\pi_0 W + \lambda\phi\pi_0 K - c(1-\phi)}{r+\lambda+\lambda\phi}$$

$\bar{V}_S < \pi_0 W - \frac{c}{\lambda}$ if and only if

$$c < \lambda \frac{r\pi_0 W + \lambda\phi\pi_0(W-K)}{r+2\lambda\phi}$$

Thus $a_{St} = 1$, $a_{Pt} = \phi$ and $\bar{V}_S = \frac{\lambda\pi_0 W + \lambda\phi\pi_0 K - c(1-\phi)}{r+\lambda+\lambda\phi}$ if $\pi_2 < \breve{\pi}$. □

B.9 Proof of Proposition 3.5

Proof. We first show $\breve{\pi} > \hat{\pi}$. Thus P's strategy follows a simple switching rule. Suppose not, then

$$\begin{aligned}
&\breve{\pi} < \hat{\pi} \\
\Longleftrightarrow &\frac{c(r+\lambda+\lambda\phi)}{\lambda(rW+\lambda(W-K))} < \frac{c\left(\phi+\frac{r}{\lambda}\right)}{rW+((W-K)\lambda-c)} \\
\Longleftrightarrow &rW+\lambda(W-K) < c(r+\lambda+\lambda\phi) \\
\Longleftrightarrow &\breve{\pi} > 1
\end{aligned}$$

which is contradictory to the fact that $\breve{\pi} \leq 1$ as a probability.

S's Optimal Action. Suppose $a_P = 1$ if $\pi > \breve{\pi}$ (we will validate this is indeed the case later). We start with defining the indifferent curve for S as

$$V_{SI}\left(\pi_{Pt}\right) = \pi_0 W - \frac{c}{\lambda} - \pi_{Pt}\left(1-\pi_{Pt}\right)V'_{SI}\left(\pi_{Pt}\right)$$

which follows a general solution

$$V_{SI}\left(\pi_{Pt}\right) = \pi_0 W - \frac{c}{\lambda} + C_{SI}\frac{1-\pi_{Pt}}{\pi_{Pt}}$$

C_{SI} is a constant pinned down by the boundary condition

$$V_{SI}(\breve{\pi}) = \bar{V}_S$$

Suppose locally to the right of $\breve{\pi}$, S exerts full effort $a_{St} = 1$, then

$$r\overline{V}_{H,S}\left(\pi_{Pt}\right) = -c\left(1-\phi\right) + \lambda\left(\pi_0 W - \overline{V}_{H,S}\left(\pi_{Pt}\right)\right) + \lambda\left(\pi_0 K - \overline{V}_{H,S}\left(\pi_{Pt}\right)\right)$$
$$- \lambda\pi_{Pt}\left(1-\pi_{Pt}\right)\overline{V}'_{H,S}\left(\pi_{Pt}\right) \quad \text{(B.2)}$$

ODE (B.2) has a general form of solution as $\overline{V}_{H,S}\left(\pi_P\right) = \frac{-c(1-\phi)+\lambda\pi_0 W+\lambda\pi_0 K}{(r+2\lambda)} + \overline{C}_{HS}\frac{\lambda}{r+2\lambda}(\frac{1-\pi_P}{\pi_P})^{\frac{r+2\lambda}{\lambda}}$, where $\overline{C}_{HS}$ is pinned down by the same boundary condition $\overline{V}_{H,S}(\breve{\pi}) = \bar{V}_S$. Similarly, if $a_{St} = \phi$ to the right of $\breve{\pi}$, then

$$r\underline{V}_{H,S}\left(\pi_{Pt}\right) = \lambda\phi\left(\pi_0 W - \underline{V}_{H,S}\left(\pi_{Pt}\right)\right) + \lambda\left(\pi_0 K - \underline{V}_{H,S}\left(\pi_{Pt}\right)\right) - \lambda\phi\pi_{Pt}\left(1-\pi_{Pt}\right)\underline{V}'_{H,S}\left(\pi_{Pt}\right) \quad \text{(B.3)}$$

ODE (B.2) has a general form of solution as $\underline{V}_{H,S}\left(\pi_{Pt}\right) = \frac{\lambda\phi\pi_0 W+\lambda\pi_0 K}{(r+\lambda+\lambda\phi)} + \underline{C}_{HS}\frac{\lambda\phi}{r+\lambda+\lambda\phi}(\frac{1-\pi_P}{\pi_P})^{\frac{r+\lambda+\lambda\phi}{\lambda\phi}}$, where $\underline{C}_{HS}$ is pinned down by the same boundary condition $\overline{V}_{H,S}(\breve{\pi}) = \bar{V}_S$.

First consider S's indifferent curve with the boundary condition $V_{SI}(\breve{\pi}) = \bar{V}_S = \frac{\lambda\pi_0 W+\lambda\phi\pi_0 K-c(1-\phi)}{r+\lambda+\lambda\phi}$. This generates

$$\begin{aligned} C_{SI}\frac{1-\breve{\pi}}{\breve{\pi}} &= (\frac{\lambda\pi_0 W+\lambda\phi\pi_0 K-c(1-\phi)}{r+\lambda+\lambda\phi} - \pi_0 W + \frac{c}{\lambda}) \\ &= \bar{V}_S - (\pi_0 W - \frac{c}{\lambda}) \\ &< 0 \end{aligned}$$

where the second line comes from Lemma **??**. Thus $C_{SI} < 0$ and $V'_{SI} = -C_{SI}\frac{1}{\pi_P^2} > 0$.

Suppose $a_S = 1$ if $\pi > \breve{\pi}$. Then $\overline{V}_{H,S}\left(\pi_P\right) = \frac{-c(1-\phi)+\lambda\pi_0 W+\lambda\pi_0 K}{(r+2\lambda)} + \overline{C}_{HS}\frac{\lambda}{r+2\lambda}(\frac{1-\pi_P}{\pi_P})^{\frac{r+2\lambda}{\lambda}}$. By value matching condition ($\overline{V}_{H,S}(\breve{\pi}) = \bar{V}_S$),

$$\overline{C}_{HS}\frac{\lambda}{r+2\lambda}(\frac{1-\breve{\pi}}{\breve{\pi}})^{\frac{r+2\lambda}{\lambda}} = \frac{\lambda\pi_0 W+\lambda\phi\pi_0 K-c(1-\phi)}{r+\lambda+\lambda\phi} - \frac{-c(1-\phi)+\lambda\pi_0 W+\lambda\pi_0 K}{(r+2\lambda)}$$

Notice if $K < \frac{\lambda\pi_0 W-c(1-\phi)}{\pi_0(r+\lambda)}$, the RHS is positive and thus $\overline{C}_{HS} > 0$. This implies $\overline{V}'_{H,S} = -\overline{C}_{HS}(\frac{1}{\pi_P} - 1)^{\frac{r+2\lambda}{\lambda}}\frac{1}{\pi_P^2} < 0$. By $V_{SI}(\breve{\pi}) = \overline{V}_{H,S}(\breve{\pi})$ and the monotonicity of them two, $\overline{V}_{H,S} < V_{SI}$ if $\pi > \breve{\pi}$ and $a_S = 1$ is optimal. Notice $K < \frac{\lambda\pi_0 W-c(1-\phi)}{\pi_0(r+\lambda)}$ is a sufficient but unnecessary condition. Denote the solution of equation (B.2) with respect to K as $\overline{V}_{H,S}(\pi; K)$. $\overline{V}_{H,S}(\pi; K)$ is continuous function of K and $\frac{\partial\overline{V}_{H,S}(\pi;K)}{\partial K} > 0$. This implies there exists $\underline{K}$ such that $\overline{V}_{H,S} < V_{SI}$ for all $\pi > \breve{\pi}$ if $K < \underline{K}$.

Suppose $a_S = \phi$ if $\pi > \breve{\pi}$. Then $\underline{V}_{H,S}(\pi_{Pt}) = \frac{\lambda\phi\pi_0 W + \lambda\pi_0 K}{(r+\lambda+\lambda\phi)} + \underline{C}_{HS}\frac{\lambda\phi}{r+\lambda+\lambda\phi}(\frac{1-\pi_P}{\pi_P})^{\frac{r+\lambda+\lambda\phi}{\lambda\phi}}$. By value matching condition ($\underline{V}_{H,S}(\breve{\pi}) = \bar{V}_S$),

$$\begin{aligned}\underline{C}_{HS}\frac{\lambda\phi}{r+\lambda+\lambda\phi}(\frac{1-\breve{\pi}}{\breve{\pi}})^{\frac{r+\lambda+\lambda\phi}{\lambda\phi}} &= \frac{\lambda\pi_0 W + \lambda\phi\pi_0 K - c(1-\phi)}{r+\lambda+\lambda\phi} - \frac{\lambda\phi\pi_0 W + \lambda\pi_0 K}{(r+\lambda+\lambda\phi)} \\ &= \frac{\lambda(1-\phi)(\pi_0 W - \pi_0 K - \frac{c}{\lambda})}{r+\lambda+\lambda\phi}\end{aligned}$$

Notice $\underline{C}_{HS} < 0$ if $K > W - \frac{c}{\lambda\pi_0}$. Meanwhile,

$$\begin{aligned}&\underline{V}_{H,S}(\pi_P) - V_{SI}(\pi_P) \\ =&\int_{\breve{\pi}}^{\pi_P} \underline{V'}_{H,S} d\pi - \int_{\breve{\pi}}^{\pi_P} V'_{SI} d\pi \\ =&\underline{C}_{HS}\frac{\lambda\phi}{r+\lambda+\lambda\phi}(\frac{1-\pi_P}{\pi_P})^{\frac{r+\lambda+\lambda\phi}{\lambda\phi}} - \underline{C}_{HS}\frac{\lambda\phi}{r+\lambda+\lambda\phi}(\frac{1-\breve{\pi}}{\breve{\pi}})^{\frac{r+\lambda+\lambda\phi}{\lambda\phi}} \\ &-(C_{SI}\frac{1-\pi_{Pt}}{\pi_{Pt}} - C_{SI}\frac{1-\breve{\pi}}{\breve{\pi}}) \\ =&\underbrace{\underline{C}_{HS}\frac{\lambda\phi}{r+\lambda+\lambda\phi}(\frac{1-\pi_P}{\pi_P})^{\frac{r+\lambda+\lambda\phi}{\lambda\phi}} - C_{SI}\frac{1-\pi_{Pt}}{\pi_{Pt}}}_{f(\pi_P)} \\ &+\frac{\lambda\phi\pi_0 W + \lambda\pi_0 K}{(r+\lambda+\lambda\phi)} - \pi_0 W + \frac{c}{\lambda}\end{aligned}$$

Take derivatives of $f(\pi_P)$. We have $f'(\pi_P) = (C_{SI} - \underline{C}_{HS}(\frac{1-\pi_P}{\pi_P})^{\frac{r+\lambda}{\lambda\phi}})\frac{1}{\pi_P^2}$ and $f'(\pi_P) > 0$ if and only if $C_{SI} - \underline{C}_{HS}(\frac{1-\pi_P}{\pi_P})^{\frac{r+\lambda}{\lambda\phi}} > 0$. If $\underline{C}_{HS} < 0$, the function $C_{SI} - \underline{C}_{HS}(\frac{1-\pi_P}{\pi_P})^{\frac{r+\lambda}{\lambda\phi}}$ is decreasing in π_P and $\lim_{\pi_p \to 1} f'(\pi_P) = C_{SI} < 0$. Thus it's sufficient to check $f(1) + \frac{\lambda\phi\pi_0 W + \lambda\pi_0 K}{(r+\lambda+\lambda\phi)} - \pi_0 W + \frac{c}{\lambda} > 0$, which is equivalent to

$$\begin{aligned}&\frac{\lambda\phi\pi_0 W + \lambda\pi_0 K}{(r+\lambda+\lambda\phi)} - \pi_0 W + \frac{c}{\lambda} > 0 \\ \Rightarrow &K > \frac{(r+\lambda)\pi_0 W - \frac{c}{\lambda}(r+\lambda+\lambda\phi)}{\lambda\pi_0} \\ &= W - \frac{c}{\lambda\pi_0} + \frac{1}{\lambda\pi_0}(r\pi_0 W - c\frac{r}{\lambda} - c\phi)\end{aligned}$$

Let $\bar{K} = W - \frac{c}{\lambda\pi_0} + \frac{1}{\lambda\pi_0}(r\pi_0 W - c\frac{r}{\lambda} - c\phi) > W - \frac{c}{\lambda\pi_0}$. The statement is proved.

P's Optimal Action. The part of proof when $a_S = 1$ is the same as Proposition 3.2. Consider the case when $a_S = \phi$ and $a_P = 1$ for all $\pi > \breve{\pi}$. Then P's value function follows

$$rV_P(\pi_{Pt}) = -c(1-\phi) + \lambda(\pi_{Pt}W - V_P(\pi_{Pt})) + \lambda\phi\pi_{Pt}(K - V_P(\pi_{Pt})) - \lambda\phi\pi_{Pt}(1-\pi_{Pt})V_P'(\pi_{Pt}) \tag{B.4}$$

Use the value matching condition at $\breve{\pi}$,

$$r(\breve{\pi}W - \frac{c}{\lambda}) = \lambda\phi\breve{\pi}\left(K - W + \frac{c}{\lambda} + (1-\breve{\pi})W\right) - \lambda\phi\breve{\pi}(1-\breve{\pi})V_P' + c\phi$$
$$\Longleftrightarrow \breve{\pi}(rW + \lambda\phi(W - K - \frac{c}{\lambda})) - c(\frac{r}{\lambda} + \phi) = \lambda\phi\breve{\pi}(1-\breve{\pi})(W - V_P')$$

Thus $V_P' < W$ if and only if $\breve{\pi} > \frac{c(\frac{r}{\lambda}+\phi)}{rW+\lambda\phi(W-K-\frac{c}{\lambda})}$. When $K > W - \frac{c}{\lambda}$, $\frac{c(\frac{r}{\lambda}+\phi)}{rW+\lambda\phi(W-K-\frac{c}{\lambda})} < \frac{c(\frac{r}{\lambda}+\phi)}{rW+\lambda(W-K-\frac{c}{\lambda})} = \hat{\pi}$ as $\phi < 1$. We've already shown that $\breve{\pi} > \hat{\pi}$. Thus $V_P' < W$. Suppose V_P crosses V_I from the bottom at some $\pi^\star > \breve{\pi}$. Then it must be the case that $V_P(\pi^\star) = \pi^\star W - \frac{c}{\lambda}$ and $V_P'(\pi^\star) > W$. By equation (B.4),

$$\begin{aligned} rV_P(\pi^\star) &< \lambda(\pi^\star W - \pi^\star W - \frac{c}{\lambda}) + \lambda\phi\pi^\star(K - \pi^\star W - \frac{c}{\lambda}) - \lambda\phi\pi^\star(1-\pi^\star)W - c(1-\phi) \\ &= \lambda\phi\pi^\star\left(K - W + \frac{c}{\lambda} + (1-\pi^\star)W\right) - \lambda\phi\pi^\star(1-\pi^\star)W + c\phi \\ &= \lambda\phi\pi^\star\left(K - W + \frac{c}{\lambda}\right) + c\phi \end{aligned}$$

However, the last line is larger than $r(\pi W - \frac{c}{\lambda})$ if and only if $\pi^\star < \frac{c(\frac{r}{\lambda}+\phi)}{rW+\lambda\phi(W-K-\frac{c}{\lambda})} < \breve{\pi}$, which is contradictory. □

Proof of Proposition 3.6

Proof. 1) Following the proof of Proposition 3.5, $\overline{V}_{H,S}(\pi_P) = V_{pub}(\pi_0) + \overline{C}_{HS}\frac{\lambda}{r+2\lambda}(\frac{1-\pi_P}{\pi_P})^{\frac{r+2\lambda}{\lambda}}$. Value matching condition ($\overline{V}_{H,S}(\breve{\pi}) = \bar{V}_S$) implies $\overline{C}_{HS} > 0$ if $\bar{V}_S > V_{pub}(\pi_0)$. Then it follows $\overline{V}_{H,S}(\pi_0) > V_{pub}(\pi_0)$. This implies player is better off with no disclosure if the other is disclosing given that $\bar{V}_S > V_{pub}(\pi_0)$.

2) If $K < \frac{\lambda\phi}{r+\lambda\phi}W$ and both players are not disclosing, then they follow a simple switching strategy with threshold $\underline{\pi} = \frac{c\left(\frac{r}{\lambda}+N\phi\right)}{\lambda\phi(N-1)(W-K)+rW}$. Since $\frac{\lambda\phi}{r+\lambda\phi}W < \frac{\lambda\pi_0 W - c(1-\phi)}{\pi_0(r+\lambda)}$, by the proof of Proposition 3.5, both players use $a_i = 1$ if $\pi > \breve{\pi}$. Notice $\breve{\pi} < \underline{\pi}$ by $K < \frac{\lambda\phi}{r+\lambda\phi}W$. Now consider P's decision. No matter he switches to no-disclosure or not, his value function follows equation (3.8). The only difference is that the boundary is shifted to the right if he deviates.

Denote $V_P(\pi;\tilde{\pi})$ as the solution to (3.8) when the boundary condition is $V_P(\tilde{\pi}) = V_I(\tilde{\pi})$. One could easily show that $\frac{\partial V_P(\pi;\tilde{\pi})}{\partial \tilde{\pi}} > 0$. Thus deviation is optimal for P. $\square$