Essays in Market Microstructure

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Dedication

To my family: my parents, Marilyn and Bill, my brother Matthew, and my sister Jocelyn.

Abstract

This thesis consists of three chapters on the topic of market microstructure, focusing on liquidity and information revelation in markets with asymmetrically informed traders. Market microstructure refers to the study of the mechanics of trading in financial markets, and how those mechanics affect the characteristics of financial markets.

One policy proposal that has received some recent attention in the realm of market microstructure is a financial transaction tax. An interesting question then is how such a tax might affect the functionality of financial markets, specifically, the liquidity and informational efficiency of markets, as these could have important welfare implications. In the first chapter, I analyze the effect of such a tax on liquidity and information revelation. I develop an insider trading model in the style of Kyle (1985) in which a single trader privately informed about the value of a financial asset chooses quantities to trade in a series of of auctions in an order-driven market. There are also uninformed traders who demand quantities for liquidity reasons and a risk-neutral market maker who sets the price equal to the expected value of the asset conditional on all public information. I implement a financial transaction tax on quadratic order flow in this market and prove the existence of a linear equilibrium in the model. I then analyze the effect of the transaction tax on liquidity

and information revelation in numerical parameterizations of the model. I find that when the tax does not affect the trading demands of uninformed traders, it improves liquidity in the market but reduces the speed of the private information revelation. However, if the tax reduces the amount of trading by uninformed traders, I show that it may be the case that both liquidity and information revelation are worsened by the tax.

In chapter 2, I conduct a study on the components of the bid-ask spread in markets with investors who trade based on private information. Recent research has suggested that informed traders may endogenously choose to trade when bid-ask spreads are low and liquidity is high. This would indicate that the bid-ask spread, a traditional measure of liquidity in a market, may then be a poor indicator because of information asymmetries. However, an interesting question is how the components of the spread, which is partly due to asymmetric information costs as well as costs associated with supplying liquidity, evolve in markets with informed trading, and whether a decomposition on the spread can lead to an improved measure of the existence of informed trading. To study such markets, I exploit an SEC disclosure rule to conduct an event study (as in Collin-Dufresne and Fos (2015)). Under the 1934 Securities Exchange Act, Schedule 13D forms ("beneficial ownership reports") must be filed with the SEC within 10 days of an investor acquiring 5% or more of a publicly traded company with an interest in influencing the management of the firm. SC13D filings frequently precede change in management events such as mergers and takeover. Since the run ups to SC13D filings frequently see abnormal returns above the value-weighted market index and higher share turnover than the periods before or after, and once the form is filed, the information becomes public, they are used as a proxy for private information revelation events. I use a structural model of price changes to decompose the bid-ask spreads of those firms on whom Schedule 13Ds were filed into an asymmetric information component and a component consisting of inventory and order processing costs. The price variance is also decomposed similarly. I study how the components of the spread and the price variance evolve over the period surrounding these filings. I find that the model predicted spreads increase substantially during the period preceding the filing date, with the asymmetric information component and the liquidity cost component increasing by 17% and 14%, respectively, from 20 days prior to the filing date to just prior.

In both my and Collin-Dufresne and Fos (2015)'s samples of SC13D filers, there appears to be a tendency for volume to be concentrated early on in the period when they possess valuable private information. In chapter 3, I develop another model in the style of Kyle (1985) to propose a possible explanation of this tendency, namely information leakage to the market. I then explore the effects of this possible explanation on liquidity and informational efficiency. In the model, an exogenous noisy signal correlated with the insider's information is gradually revealed to the market maker over the course of the trading period. I describe the necessary and sufficient conditions for the existence of a linear equilibrium in the model. In subsequent parameterizations, I find that this information leakage tends to improve market liquidity by substantially decreasing the price impact of order flow. It also improves the speed at which the insider's information is revealed to the market, thereby improving market efficiency. I find that under certain parameterizations, this can also explain the concentration of volume toward the beginning of trading. Lastly, I examine the effects on liquidity and information revelation in parameterizations where the informativeness of the noisy signal changes over time.

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Chapter 1

Financial Transaction Taxes and Dynamic Insider Trading

1.1 Introduction

How do financial transaction taxes (FTTs) affect financial markets with asymmetric information? In this article I investigate the effects on market liquidity, adverse selection costs, and information revelation of trade-size dependent transaction taxes in financial markets with informed and uninformed trading. In particular, I design a dynamic model in the style of Kyle (1985) with a monopolistically informed trader as well as uninformed traders who trade for liquidity reasons. The informed trader knows the liquidation value of the security that is publicly announced at a later date, and trades to maximize his profits based on this knowledge. The market maker observes the demand for the asset and sets the price with semi-strong efficiency, that is, equal to the conditional expected value of the asset based on all public information. However, traders must also pay a trade-size dependent tax on their transactions during each auction, which affects the decisions of the insider and hence the market characteristics.

Financial transaction taxes have attracted a renewed focus in recent years, having been implemented in a number of developed countries and proposed by various politicians in the United States. But the debate over the desirability of such a tax goes back many years, with Stiglitz (1989) and Summers and Summers (1989) supporting such a tax to curb excessive speculative trading, but Ross (1989) offers a refutation of their arguments. Most closely related to this paper, Subrahmanyam (1998) examines the effect of a financial transaction tax in a one-period Kyle model, with both an monopolistically informed trader as well as multiple competing informed traders. He finds that while in the monopolistic case, the tax improves market liquidity, it worsens liquidity when there are multiple informed traders. This is because the tax drives the competing informed traders to scale back their trading in equilibrium and the reduced competition among informed traders reduces information revelation and liquidity. He also develops a model of information acquisition under a transaction tax and finds that the tax can reduce the incentive to try to obtain private information earlier than others, and can lead to greater firm values. Dow and Rahi (2000) also examine the effect of a transaction tax in a one-period rational expectations model where uninformed traders have a hedging motive, and find the a tax can actually increase the informed speculators profits by causing prices to be less informative. Dávila (2020) finds the optimal level of transaction tax in a model with traders with heterogeneous beliefs about asset valuations and a social planner, and finds that the optimal tax depends on the relative beliefs of the traders.

A number of empirical studies have attempted to analyze the effect of transaction taxes in various countries that have implemented them. Liu (2007) studies the case of Japan's tax and Gomber et al. (2016) examine the case of France, with both finding that transaction taxes reduce liquidity and the efficiency price discovery. Cappelletti et al. (2017) study the effects of an FTT in Italy and also find some evidence that it decreases liquidity and increased volatility. Similarly, Habermeier and Kirilenko (2003) survey the literature as well as a number of cases of implemented taxes and find that transactions taxes can increase volatility as well as reduce liquidity. Conversely, Capelle-Blancard and Havrylchyk (2016) find that the French FTT had no significant effects on liquidity and volatility.

In this paper I focus on the effects over time of a financial transaction tax on liquidity and market efficiency, that is, the speed at which the price reflects the true value of the security. I find that in a dynamic market with a monopolistically informed trader, the tax both increases liquidity but reduces the speed of price discovery. I also examine a case in which the tax reduces the trading incentive of the liquidity traders, and find that in that case, it can both decrease liquidity and market efficiency.

The paper is organized as follows. The model and equilibrium are developed in section 1.2. Section 1.3 presents the results of parameterizations of the model. Section 1.4 concludes.

1.2 Model

1.2.1 Setup

The model follows the dynamic version of Kyle (1985) but with a financial transaction tax added to it. There are three market participants: the insider, the market maker, and liquidity traders. The insider and market maker are risk-neutral, and the liquidity traders are non-strategic. There are N auctions at which the market maker opens the floor for trading, $\{t_n\}_{n=1}^{N}$. The auctions are evenly spaced over time, and the interval of time $[t_n, t_{n+1})$ is called period n.

At each trading period n, the following sequence of events occurs. First, the insider observes private information about the fundamental value v of the asset. Next, the insider and liquidity traders simultaneously place market orders to buy or sell quantities of the asset x_n and y_n. The market maker is unable to distinguish between orders from the insider and orders from liquidity traders, and only observes the net volume of trade $z_n = x_n + y_n$. After observing the total volume of trade z_n, the market maker chooses the price p_n and trades to close all orders. This trading process continues until a fixed time N, when the fundamental value of the asset v is revealed. After the fundamental value of the asset is revealed, the insider loses his informational advantage, the price matches the value of the asset, and trading ends.

Liquidity traders do not trade strategically, and I assume that their orders $\{y_n\}_{n=1}^{N}$ are a sequence of independent and identically distributed (i.i.d.) normal random variables with mean 0 and variance Σ_y. The asset's liquidation value, v, is normally distributed with mean p_0 and variance Σ_0. The insider observes this value v prior to trading, but the market maker and the rest of the market participants know only the distribution of v. Using this information advantage, the insider trades strategically to maximize his expected net payoff over the N auctions.

At the beginning of each period n, the market maker commits to pricing rule that specifies the price p_n as a function of the volume of trade z_n and his information up to time n (which coincides with the public history of prices and total volumes of trade). His information in period n can then be denoted $\mathcal{F}_n^M = (z_0, p_0, \ldots, z_{n-1}, p_{n-1}, z_n)$. After the market maker's pricing rule is announced, the insider and liquidity traders place their orders, and all orders are executed at the end of the period. The insider's information in period n is represented by $\mathcal{F}_n^I = (v, x_0 z_0, p_0, \ldots, x_{n-1}, z_{n-1}, p_{n-1})$. That is, he knows the public history of trades and prices, his own past orders, and the fundamental value of the asset v.

Each period, the insider also pays a transaction tax τ on the quadratic of the total value of his order in that period, x_n^2. This assumption is made for two primary reasons. First and foremost, it leads to tractability in the model and the existence of a linear equilibrium. Second, the tax is trade size dependent (as are most real-world FTTs) and positive for both buys and sells. Costs are also frequently modeled as convex in the literature (such as in Subrahmanyam (1998)).

So, with the inclusion of the tax, for any trajectory $X = \{x_n\}_{n=1,\ldots,N}$ for the insider's trading and $P = \{p_n\}_{n=1,\ldots,N}$ for market prices, the insider's payoff starting in period n is

$$\Pi_n(P, X) = \sum_{k=n}^{N} [v - p_k] x_k - \tau x_k^2$$

Since the insider maximizes his expected payoff conditional on his information from each period n onward, his objective in period n will be to maximize

$$\mathbb{E}[\Pi_n(P, X)] = \mathbb{E}\left[\sum_{k=n}^{N} [v - p_k] x_k - \tau x_k^2 \middle| \mathcal{F}_k^I\right]$$

Next, we define strategies for the insider and market maker.

Definition 1 *A strategy for the market maker is a sequence of $\mathcal{F}_n^M$-measurable functions $P = \{p_n\}_{n=0}^N$ and a strategy for the insider is a sequence of $\mathcal{F}_n^I$-measurable functions $X = \{x_n\}_{n=0}^N$.*

1.2.2 Equilibrium

Definition 2 *The strategy profile (P, X) is an equilibrium if*

(i) Profit Maximization: Given P, X maximizes $\mathbb{E}[\Pi(P, X)]$, and

(ii) Market Efficiency: For any $n \geq 0$,

$$p_n = \mathbb{E}[v|X, \mathcal{F}_n^M]$$

Note that the market maker is not maximizing any particular objective. However, as is common in this literature, there is an implicit assumption that the market maker is a Bertrand competitor with other market makers, which drives the market maker to set the price equal to its expected value of the asset's liquidation value, given her information and the strategy of the insider. That is, the market maker sets the price equal to the conditional expectation of the asset's value given all public information, so that the market is semi-strong efficient.

As is common in the literature, I restrict attention to the linear equilibrium, where X and P are linear functions. A *linear equilibrium* is an equilibrium in which there exists sequences $\{\beta_n\}$ and $\{\lambda_n\}$ such that for every $n = 1, \ldots, N$,

$$x_n = \beta_n(v - p_{n-1}) \tag{1.1}$$

$$p_n = p_{n-1} + \lambda_n z_n \tag{1.2}$$

That is, the insider trades linearly in the price gap (the difference between the security's true value and the current price) and the market maker updates prices linearly in the total order flow.

Next, I prove the existence of such a linear equilibrium in the model ,where $\{\beta_n\}$ and $\{\lambda_n\}$ are deterministic sequences.

Proposition 1 *There exist sequences $\{\beta_n\}, \{\lambda_n\} \in \mathbb{R}_{++}$ such that the linear strategy profile defined by (1.1) and (1.2) is an equilibrium. In this equilibrium, $\{\Sigma_n\}$ is a deterministic sequence, and there exist deterministic sequences $\{\alpha_n\}, \{\gamma_n\}$ such that*

$$\mathbb{E}[\Pi_n(v, p_{n-1})] = \alpha_n(v - p_{n-1})^2 + \gamma_n \tag{1.3}$$

is the insider's expected payoff-to-go for all $n \geq 0$. Given Σ_0, the sequences $\{\beta_n, \lambda_n, \alpha_n, \gamma_n, \Sigma_n\}$ are the solution to the difference equation system

$$\alpha_n = \frac{1 + 4\alpha_{n+1}\tau}{4(\lambda_n + \tau - \alpha_{n+1}\lambda_n^2)} \tag{1.4}$$

$$\gamma_n = \alpha_{n+1}\lambda_n^2\Sigma_y + \gamma_{n+1} \tag{1.5}$$

$$\beta_n = \frac{1 - 2\alpha_{n+1}\lambda_n}{2(\lambda_n + \tau - \alpha_{n+1}\lambda_n^2)} \tag{1.6}$$

$$\lambda_n = \frac{\beta_n\Sigma_n}{\beta_n^2\Sigma_n + \Sigma_y} \tag{1.7}$$

$$\Sigma_{n+1} = \frac{\Sigma_n\Sigma_y}{\beta_n^2\Sigma_n + \Sigma_y} \tag{1.8}$$

subject to the terminal conditions $\alpha_{N+1} = \gamma_{N+1} = 0$, and the second order condition:

$$-2\lambda_n - 2\tau + 2\alpha_{n+1}\lambda_n^2 < 0$$

Proof: See Appendix A.1

It is not immediately clear from the difference equation system what effects the transaction tax will have on key market characteristics. I now turn to parameterizations of the model to investigate these effects.

1.3 Results

In this section I explore how different levels of transaction taxes affect market liquidity, the price impact of trading volume, the speed at which the informed trader's information is revealed to the market, and the aggressiveness with which the insider trades against the price gap. I first estimate a version of the model in which the liquidity traders are unaffected by the implementation of the tax, and then develop a rough approximation of how the results may change if the liquidity traders trade smaller amounts because of the tax.

1.3.1 Case: Liquidity Traders Unaffected by Tax

I parameterize the model by allowing for $N = 50$ auctions, with the ex ante mean and variance of the asset's liquidation value v equal to $p_0 = 0$ and $\Sigma_0 = 1$, respectively. The variance of the liquidity traders' demand is also given by $\Sigma_y = 1$. I examine 3 cases of the transaction tax, with τ set to $0, 0.25$, and 0.5. The results are presented in Figure 1.1.

As is evident from the figure, increasing the tax has the effect of decreasing the price impact of each trade, and increasing the market depth. This is because the transaction tax causes the insider to scale back his trading volume in each auction, as can be seen in the plot of β, his aggressiveness in trading in the price gap. Since the insider is trading less

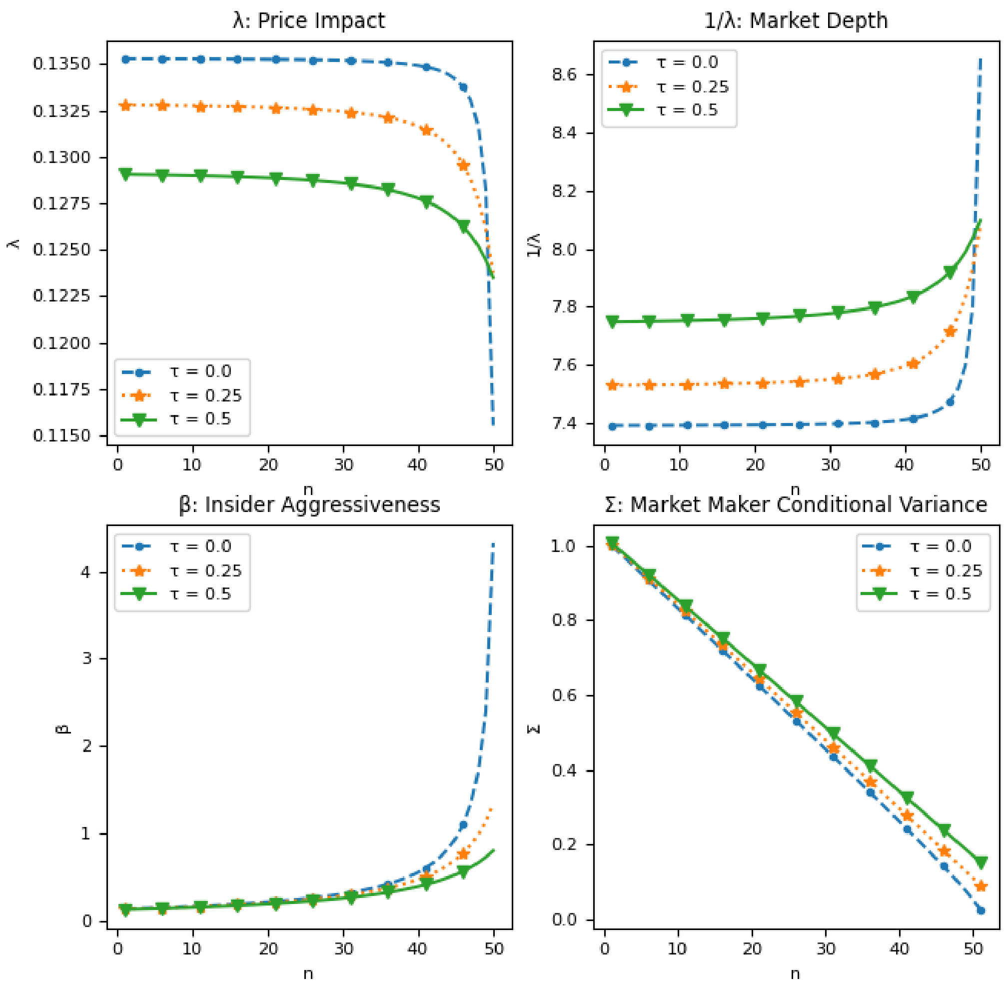

Figure 1.1: **Model Parameters for Different Levels of the Financial Transaction Tax.** All models are parameterized with $\Sigma_y = 1$, $\Sigma_0 = 1$, $p_0 = 0$, and $N = 50$.

aggressively, trades become relatively less informative for the market maker, and so order flow moves prices less. This also has the effect of increasing market depth, which is the primary measure of liquidity in this model, and is measured as the reciprocal of the price impact. Market depth measures how much volume the market can absorb without having large changes in prices. As in Kyle's model, a liquid market will tend to be deeper in this model, as this means that investors can trade larger amounts of the asset on average at near the current market price.

Despite the tax's positive effects on liquidity in the market, it does decrease the market efficiency, through decreasing the speed of price discovery. At any time over the course of trading, the price is less likely to reflect the fundamental value of the asset. This can be seen in the graph of the market maker's conditional variance, which is a measure of how much of the insider's information has been revealed to the public through his trades. Since the tax induces the insider to trade smaller quantities, the price is slower to converge to the true value of the asset. So in the case where the tax has no effect on the liquidity traders, it increases liquidity in the market but decreases the informational efficiency of prices.

1.3.2 Case: Variance of Liquidity Traders' Demand Decreasing in the Tax

I now turn to a rough approximation of what could happen if the transaction tax caused the uninformed traders to scale back their trading as well. While a more rigorous treatment of this case should include endogenizing the liquidity traders volume decisions and having them take the tax into account, such as in Dow and Rahi (2000) or Mendelson and Tunca (2004), for simplicity I assume that the variance of liquidity trader demand is simply decreasing in the size of the tax, so that $\Sigma_y = 1 - \tau$, with all other parameters identical to the previous case. The results are presented in Figure 1.2.

This example shows how the transaction tax could have negative effects on both liquidity and the market's informational efficiency if it reduces the volume of uninformed traders substantially. The insider continues to trade less aggressively in the presence of the tax, but because the tax diminishes the volume of the uninformed that the insider uses to disguise his own trades, the price impact is much larger in the presence of the tax. Since the price impact is larger, the depth of the market is significantly smaller, and hence the market would be considered less liquid.

While it could be the case that the smaller uninformed volume in the market would lead to an informationally more efficient market, this is not the case, as can be seen from the plot of the market maker's conditional variance of the asset's value. The smaller volume from liquidity traders does lead to more informative trades; however, this effect is not enough to offset the reduction in the insider's volume due to the costs imposed from the tax. The market with the is more efficient than in the case when the liquidity traders are unaffected by the tax, but remains less efficient than a market with no transaction tax.

Overall, these results suggest that transaction taxes reduce the speed with which prices in financial markets will reflect traders' private information, leading to less efficient financial markets. However, the effect on liquidity is ambiguous, and likely depends on the degree to which the tax discourages liquidity traders from trading. This matches the dual results of Subrahmanyam (1998), who models liquidity traders being unaffected by the tax, and Dow and Rahi (2000), who obtain a different result by endogenizing the liquidity traders' decisions, and extends these results to a dynamic setting with many sequential

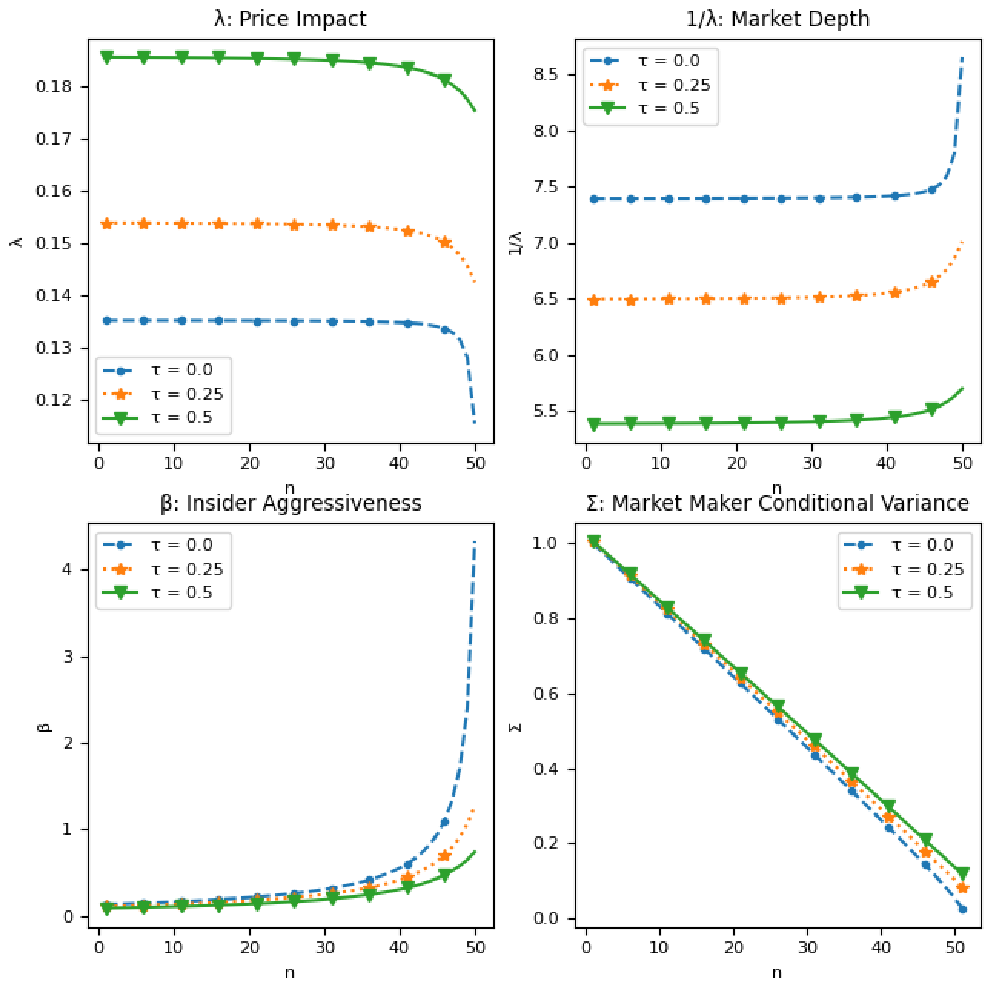

Figure 1.2: **Model Parameters for Different Levels of the Financial Transaction Tax with Decreasing Liquidity Trading.** All models are parameterized with $\Sigma_y = 1-\tau$, $\Sigma_0 = 1$, $p_0 = 0$, and $N = 50$.

auctions.

1.4 Conclusion

In this paper, I examine the effect of a financial transaction tax on liquidity and informational efficiency in a market with a monopolistically informed trader. I introduce a trade-size dependent tax in a series of sequential auctions in a semi-strong efficient market with asymmetric information, and derive an equilibrium of the model. I subsequently parameterize the model and investigate the effects of the tax on liquidity and the speed of price discovery. I find that the tax improves the liquidity in this type of market, but decreases the speed of price discovery, and hence the informational efficiency. However, in a second parameterization where the tax may affect the demands of the liquidity traders, I find that the tax can decrease both liquidity and informational efficiency.

Chapter 2

A Bid-Ask Spread Decomposition for SC13D Filers

2.1 Introduction

The bid-ask spread, the reasons for its existence, and the components of the spread have been an important subject of financial market microstructure research for decades. Generally speaking, past theoretical research has suggested that bid-ask spreads, as a measure of illiquidity, should be larger when there is a relatively high degree of information asymmetry in the market. That is, when there are more traders with valuable private information relative to uninformed traders, bid-ask spreads, price impact, and other measures of illiquidity should be larger. An extensive empirical literature attempts to decompose trading costs (as measured by the bid-ask spread) into different components, in particular, asymmetric information costs, order processing costs, and inventory costs. These decompositions rely on the fact that larger, more persistent price changes in response to trades should signal higher private information content in those trades, while smaller or more transitory price changes would signal less informative trades. Such spread decomposition models allow for the study of how the components of trading costs vary in dynamic settings, such as over the course of the trading day or in the days leading up to certain events, such as public announcements.

However, because events that definitively signal that extensive trading based on private information has occurred may be difficult to find, relatively less attention has been spent on spread decomposition in the run up to such occurrences. Since such information is valuable (and in some cases, illegal to trade on) traders would not willingly reveal when they were trading on private information nor what their private information may be. Due to a disclosure rule in the 1934 Securities Exchange Act, I am able to identify equities in which at least one relatively large investor is actively trading while relying on valuable private information during the period. After identifying these markets, I then use a structural spread decomposition model to estimate how the asymmetric information costs and liquidity supplying costs may evolve preceding and succeeding the disclosure event.

For this analysis to be valid, it must be the case that Schedule 13D filers are traders with valuable and long-lived information who actively trade on this information. Collin-Dufresne and Fos (2015) analyze a sample of such filers from 1998-2014, and by collecting the individual trades disclosed in the filings, find that Schedule 13D filers (1) trade frequently and trade substantial amounts of shares in the run up to the filing date, peaking around 10 days before the filing date (the "event date"), and (2) earn large returns on those shares. They also document that on days when they trade these filers trade large fractions of daily volume, and that cumulative buy-and-hold returns, volume, and share turnover are larger in the run up to the filing date. While I am unable to obtain data on the individual trades of the filers, I too document large increases in volume, share turnover, and abnormal buy-and-hold returns in the run up to filings in my sample of Schedule 13D filers.

This analysis is closely related to an extensive empirical literature that attempts to decompose bid-ask spreads, going back to Glosten (1987) and Glosten and Harris (1988). Most closely related is Madhavan et al. (1997) who develop a structural model to decompose the bid-ask spread into a component due to information asymmetry and a component due to inventory costs and order processing costs. They test their model on bid-ask spreads throughout the day, and find that while information asymmetry costs decrease through the trading day, transaction costs tend to increase. Huang and Stoll (1997) similarly develop a time-series model to perform a 3-way decomposition of the bid-ask spread into an information asymmetry component, an order processing component, and an inventory component. They also measure the model for differently sized trade groups and find a relationship between trade size and the spread components.

There is also another related literature investigating liquidity costs in the presence of trading based on valuable private information. Theoretical models of insider trading, such as Kyle (1985), Glosten and Milgrom (1985), Easley and O'hara (1987), have long theorized that illiquidity should be increasing in the degree of private information held by informed traders. Empirical studies such as Morse and Ushman (1983), Venkatesh and Chiang (1986), and Jennings (1994) investigate changes in bid-ask spreads around various kinds of financial announcements.

Interestingly, while cumulative abnormal returns continue to accumulate until after the filing date, Collin-Dufresne and Fos (2015) find that Schedule 13D filers trade less and less frequently and the asymmetric information component of the bid-ask spread decreases from about 5 days before the filing date until the filing date. This is consistent with the hypothesis that traders with valuable private information may trade more aggressively earlier relative to the date of the public announcement in order to offset possible information leakage that would be costly to their gains. Keown and Pinkerton (1981) and Kurov et al. (2019) find empirical evidence of such leakage by examining merger announcements and macroeconomic announcements. Furthermore, volume and share turnover also seem to decrease from a few days before the announcement date until the announcement date. This is in contrast to much theoretical research, where insiders either trade (in expectation) at a constant rate or increasing amounts up until the time of the public announcement. There are other possible hypotheses that would be consistent with this phenomenon, however, such as insider risk aversion (as in Kyle (1989)) or competition among multiple insiders (as in Holden and Subrahmanyam (1992)).

By decomposing the bid-ask spread into these components, I attempt to answer two important questions. First, to what degree do spread decomposition models that rely on persistent price impact actually capture the degree of information asymmetry in financial markets? In a recent empirical paper, Collin-Dufresne and Fos (2015) argue that standard measures of stock illiquidity do not reveal the presence of informed trading. In fact, they find negative relations between informed trading and measures of trading costs (such as price impact, bid-ask spreads, and other illiquidity measures). However, spread decomposition models may be more effective at isolating the asymmetric information costs from other liquidity supplying costs than sample adverse selection measures.

As evidence of this, I construct several adverse selection measures and document their levels in the period prior to Schedule 13D filings. I find no conclusive patterns in the levels of these illiquidity measures before, during, and after the Schedule 13D filings, suggesting that these measures are not effective at measuring adverse selection. In contrast, there are patterns in the levels of the asymmetric information component of the bid ask spread in the run up to these announcements.

Second, how do the costs of supplying liquidity relate to the presence of informed trading? Recent theoretical research (such as Collin-Dufresne and Fos (2016)) has argued that informed agents may endogenously choose to trade at times when markets are more liquid. Therefore, while we might see no changes in the spread (or even decreases) in the run up to private information revealing events, we may see a decrease in the component estimating the cost of supplying liquidity, while the asymmetric information component still increases. This would be consistent with insiders selecting the timing of their trades, but would also indicate that the model is effectively capturing the information asymmetry present in the market. However, in contrast to this, in my sample of Schedule 13D filers, I find that the liquidity cost component of the bid-ask spread is larger in the period immediately leading up to the filing date.

Price volatility in markets with higher amounts of informed trading has also long been of interest to researchers, traders, regulators, and other market participants. An advantage of using a spread decomposition model to study such markets is that it also allows for a partial decomposition of the price variance, so that asymmetric information, order processing costs and inventory cost components of price volatility can also be measured (as well as how new public information is affecting it). Analyzing the different components of volatility may have important implications for the efficiency in financial markets, since speed of price discovery may be closely related to the amount of volatility caused by information asymmetries. Importantly, theoretical research of markets with informed agents have frequently differed on the dynamics of price volatility, so that these empirical results could support or contradict existing theory.

Examining the components of price volatility due to asymmetric information and due to liquidity costs, we find that both tend to rise in the period prior to Schedule 13D filings (as well as the interaction term). Since average price variance is also high during the period preceding the announcement, but declines steeply just before the filing date, this has interesting implications for volatility patterns in insider trading models. Furthermore, since the spread decomposition model can decompose the variance of price changes as well, I can estimate to what extent these patterns are due to private information, public news shocks, and changes in liquidity costs.

The paper is organized as follows. Section 2.2 describes the data and the data sources used, as well as the algorithm used to determine trade direction in the data. Section 2.2 also includes some evidence of the existence of informed trading in the run-up to the filing date. Section 2.3 describes the structural spread decomposition model. Section 2.4 presents the results of the model. Section 2.5 concludes.

2.2 Data

2.2.1 Description of the Data

Under the 1934 Securities Exchange Act, disclosure Rule 13d-1(a) requires investors to file a Schedule 13D filing with the Securities and Exchange Commission (SEC) within

10 days of acquiring 5% or more of any class of securities of a publicly traded company if they have an interest in influencing the management of the company[1]. In other words, these are activist investors trading in securities of the company. In the Schedule 13D filings, Item 5(c) requires the activist investor(s) to report all trades in the target company in the preceding 60 days prior to the filing date. Using these filings and the reporting in Item 5(c), I identify securities markets in which certain investors are actively trading with substantial and valuable private information.

The data are for firms on whom a SC13D form was filed with the SEC in the year 2017 or 2018. The data on SC13D filings comes from the SEC's EDGAR database. Many of these filings include only privately negotiated security purchase agreements, merger agreements, or transactions involving bonds or derivatives of the security. To isolate the effect of private information in the market for the company's equity, I only include SC13D filers who under item 5(c) of the form reported transacting at least 1 share of the firm's common stock on the open market in the 60 days prior to the filings. SC13D/A filings, or amendments to previous SC13D filings, are omitted.

I retain only assets whose Center for Research in Security Prices (CRSP) share codes are 10 or 11, including equities and excluding certificates, ADRs, companies incorporated outside the United States, REITs, preferred stocks, closed-end funds, shares of beneficial interest, and units. I include those equities listed on the NYSE, NASDAQ, or AMEX (NYSE American) exchanges.

The final sample comprises all of the SC13D filings that satisfy the above criteria,

[1]Without the interest in influencing management, they may be able to file the less detailed Schedule 13G

consisting of 309 events in the years 2017 and 2018. For each event, information on the filing date and the underlying security are extracted.

For each filing, I match the firm to intraday trading data from the Trade and Quote (TAQ) consolidated trades and the National Best Bid and Offer (NBBO) data from Wharton Research Data Services (WRDS). I also obtain data on stock returns, volume, and prices from the Center for Research in Security Prices (CRSP) in order to describe basic market conditions during these periods. I include data from 60 days prior to each SC13D filing date to 40 days after the filing date. I follow Ellis et al. (2000) and determine trade direction in the following way[2]:

1. Trades occurring at the NBBO best ask and bid quotes are classified as buys and sells, respectively.

2. For trades occurring not at the NBBO best ask or bid quotes, then the tick test is used. The tick test classifies trades into four categories: upticks, downticks, zero-upticks, and zero-downticks:

    (a) A trade is an uptick if the price is higher than the price of the previous trade.

    (b) A trade is a downtick if the price is lower than the price of the previous trade.

    (c) A trade is a zero-uptick if the price of the trade is the same as the price of the previous trade, but the last price change was an uptick.

    (d) A trade is a zero-downtick if the price of the trade is the same as the price of the

[2] Ellis et al. (2000) compare several trade signing algorithms (the quote rule, tick test, Lee and Ready (1991)) on NASDAQ proprietary data that identifies trade direction, and propose a new classification algorithm that improves accuracy.

previous trade, but the last price change was a downtick.

The tick test classifies uptick and zero-uptick trades as buys and classifies downtick and zero-downtick trades as sells.

I limit my data to those trades taking place during the course of the trading day, since extended hours trading may exhibit different properties as liquidity is generally limited. Finally, I exclude observations in which less than 100 transaction took place during the trading day.

With this dataset, I estimate the model described in section (3). I denote by t the distance from the filing date (where $t = 0$ is the filing date) and estimate ρ (the autocorrelation of trades), α (the intercept), θ (the asymmetric information component of the half-spread), and ϕ (the order processing and inventory costs component of the half-spread) for each firm and each day t. Then, for each day relative to the filing date t, the average across firms is computed, so that my final data sample consists of the averages of each component across the 51 days surrounding the filing dates.

2.2.2 Volume, Share Turnover, and Cumulative Abnormal Returns

As noted in the introduction, volume, share turnover, and cumulative abnormal returns all see large increases in the periods leading up to the Schedule 13D filings. However, interestingly, volume and share turnover seem to have peaks at two points: around the event date (the date 10 days prior to the Schedule 13D filing), and just after the filing date.

Share turnover of the stocks with Schedule 13D filings is shown in Figure 2.1. Volume is in Figure 2.2, and cumulative abnormal buy-and-hold return is in Figure 2.3. As can be seen, share turnover and volume tend to be largest around the event date, decreases prior to the filing date, and then peaks again just after the announcement. The rolling average share turnover increases to over 20% in the runup to the filing before declining again until the public announcement. Assuming this increase in share turnover prior to the announcement were driven by trading by filers, this would further support that traders with inside information may choose to reveal their information through trading initially quickly, to offset the potential effects of information leakage or competing informed traders.

Volume tells a similar story. It gradually increases until a few days after the event date, then decreases until the public announcement, before increasing again. A possible explanation around increases in volume and share turnover after the announcement is that there is an adjustment period where investors are trying to discern the new true value of the asset after the information revelation. After the filing, investors may still be trying to discern the future value of the activist investors actions and are rebalancing their portfolios accordingly.

The cumulative buy-and hold returns in excess of the buy-and-hold returns of the value-weighted market index supply the strongest evidence that these filers possess valuable private and long lived information. As can be seen in Figure 2.3, cumulative abnormal returns are generally small or negative prior to the event date. From the event date until the filing date, however, these returns increase quickly, reaching about 3% on the day after the filing date. Interestingly, the cumulative abnormal returns increase most quickly in the period $(-1, 1)$, gaining from just above 0% to over 3%, even though most other measures (including in C-DF (2015), who have data on the individual trades of SC13D filers) imply that the filers trade less during this period than around the event date. This seems to indicate that there is still a substantial amount of valuable information that has not yet

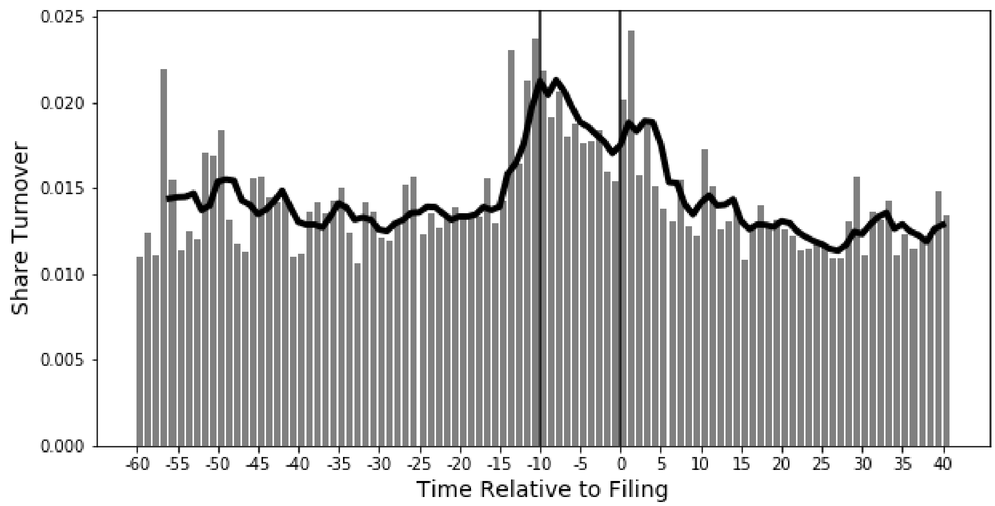

Figure 2.1: **Share Turnover around the Filing Date.** The bars represent share turnover and the line is a 5 day rolling window of average share turnover from 60 days prior to the filing date to 40 days afterwards.

been revealed through trading when the public announcement occurs.

Together, these measures seem to imply that these time periods in these markets contain significant trading costs due to adverse information. Therefore, I use them as a test of the ability of spread decomposition models to detect such adverse information costs, even while Schedule 13D filers may endogenously choose to trade when trading costs (including order processing costs, inventory costs, and possibly even asymmetric information costs from other informed traders with different information) are low.

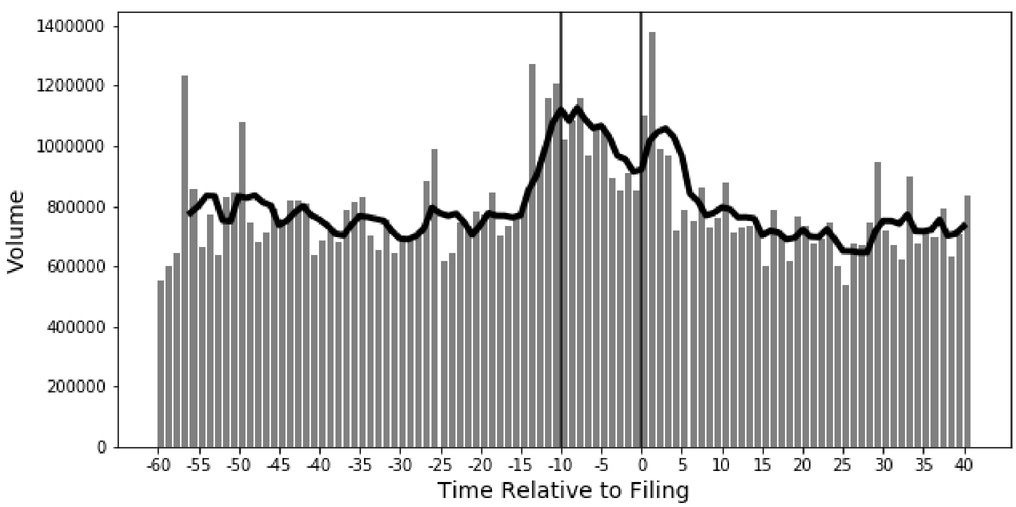

Figure 2.2: **Volume around the Filing Date.** The bars represent volume and the line is a 5 day rolling window of average volume from 30 days prior to the filing date to 20 days afterwards.

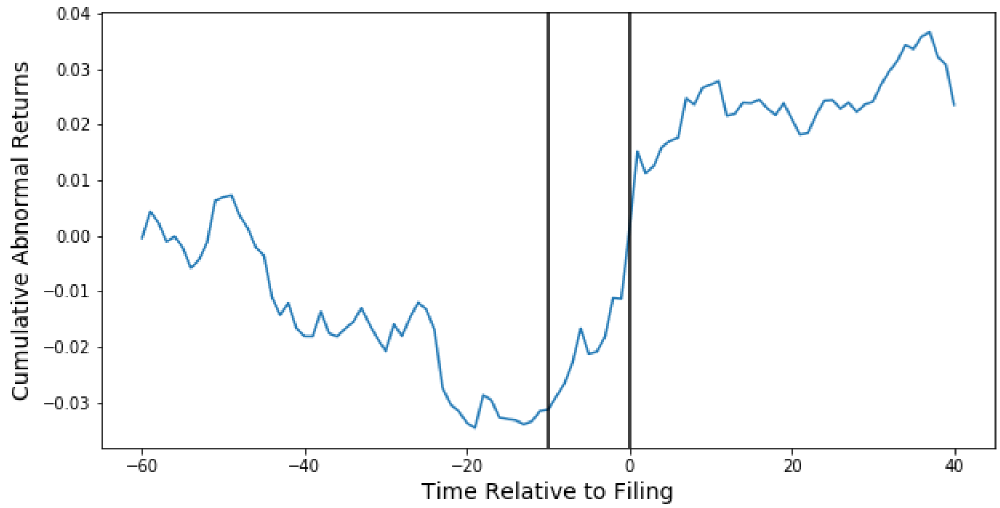

Figure 2.3: **Abnormal Cumulative Buy-and-hold Return around the Filing Date**

2.3 Structural Spread Decomposition Model

2.3.1 Structural Model

The structural model estimated is adapted from Madhavan et al. (1997). It describes a market for a risky security which is traded in an auction-exchange dealer mechanism, in which the liquidity providers quote bid and ask prices at which they are willing to trade (orders may also be executed within the quotes). Let p_t denote the transaction price at time t, and x_t be a trade initiation: $x_t = +1$ if the trade is buyer initiated, $x_t = -1$ if the trade is seller initiated.

It's assumed that buys and sells are unconditionally equally likely, that is, $\mathbb{E}[x_t] = 0$ and $var(x_t) = 1$.

Public beliefs may evolve for two reasons: (i) new public information announcements which are not associated with trading may cause revisions in beliefs, and (ii) order flow sends a noisy signal about future asset values. Let ϵ_t denote the innovation in beliefs between times $t-1$ and t due to new public information, and assume it is an iid random variable with mean $\mathbb{E}[\epsilon_t] = 0$ and variance $Var(\epsilon_t) = \sigma_\epsilon^2$. Secondly, if market makers believe some traders may possess valuable private information about the future value of the security, then buys are associated with an upward revision of beliefs about the asset value and sells with a downward revision. Then the change in beliefs based on the order flow can be written as

$$\theta(x_t - \mathbb{E}[x_t|x_{t-1}])$$

where $\theta \geq 0$ is the degree of information asymmetry or the so-called permanent impact of the order flow, and $(x_t - \mathbb{E}[x_t|x_{t-1}])$ is the "surprise" in order flow. Higher values of θ represent larger revisions in beliefs in response to order flow. In other words, the amount that each trade changes beliefs about the true value of the security is a measure of the private information contained in the trade.

Let μ_t denote the post-trade expected value of the stock conditional upon public information and the trade initiation variable x_t. The revision in beliefs is the sum of the change in beliefs due to new public information and order flow innovations, so that

$$\mu_t = \mu_{t-1} + \theta(x_t - \mathbb{E}[x_t|x_{t-1}]) + \epsilon_t \tag{2.1}$$

Further, let $\phi \geq 0$ represent the dealer's cost per share of providing liquidity (i.e., the dealer's compensation for transaction costs, inventory costs, risk-bearing, etc.). Then if we let p_t^a denote the pre-trade ask price at time t and p_t^b denote the pre-trade bid price at time t, we have that the ask price (i.e., the price conditional on a buy $x_t = 1$)

$$p_t^a = \mu_{t-1} + \theta(1 - \mathbb{E}[x_t|x_{t-1}]) + \phi + \epsilon_t$$

and the bid price (price conditional on a sell, $x_t = -1$) can be written as

$$p_t^b = \mu_{t-1} - \theta(1 + \mathbb{E}[x_t|x_{t-1}]) - \phi + \epsilon_t$$

and so ϕ here captures the temporary impact of trades on prices. Transactions that execute within the bid-ask spread are assumed to execute at the bid-ask midquote, $(p_t^a + p_t^b)/2$. Then the transaction price can be written as

$$p_t = \mu_t + \phi x_t + \xi_t \tag{2.2}$$

where ξ_t is a iid random variable capturing the effect of stochastic rounding errors produced by price discreteness. Combining equations (2.1) and (2.2) yields

$$p_t = \mu_{t-1} + \theta(x_t - \mathbb{E}[x_t|x_{t-1}]) + \phi x_t + \epsilon_t + \xi_t \tag{2.3}$$

To estimate the preceding equation, assumptions must be made about the temporal behavior of order flow. Assume order flow follows a general Markov process, where γ denotes the probability that a transaction at the ask (bid) follows a transaction as the ask (bid). That is, assume $\gamma = Pr[x_t = x_{t-1}]$. Since continuations are more likely than reversals (because large traders tend to break up their trades), $\gamma > \frac{1}{2}$. Let ρ denote the first order autocorrelation of trades,

$$\rho = \frac{\mathbb{E}[x_t x_{t-1}]}{Var(x_t)}$$

Then $\rho = 2\gamma - 1$. Now the conditional expectation $\mathbb{E}[x_t|x_{t-1}]$ can be computed. Observe that

$$\begin{cases} \text{if } x_{t-1} = 1, \implies \mathbb{E}[x_t|x_{t-1} = 1] = \rho \\ \\ \text{if } x_{t-1} = -1, \implies \mathbb{E}[x_t|x_{t-1} = -1] = -\rho \end{cases}$$

Therefore, $\mathbb{E}[x_t|x_{t-1}] = \rho x_{t-1}$. Using this and the fact that $\mu_{t-1} = p_{t-1} - \phi x_{t-1} - \xi_{t-1}$ (from equation (2.2)), we can write

$$p_t - p_{t-1} = (\phi + \theta)x_t - (\phi + \rho\theta)x_{t-1} + \epsilon_t + \xi_t - \xi_{t-1} \tag{2.4}$$

Equation (2.4) is our testable regression equation. This, along with the additional restriction for the autocorrelation, allows for the bid-ask spread components around SC13D filings to be estimated.

2.3.2 Model Estimation

The model involves 4 parameters to estimate: θ, the asymmetric information component of the bid-ask spread; ϕ, the inventory and order processing components of the spread; ρ, the autocorrelation in trade direction; and α, a constant drift we allow for in prices. Defining the error from the regression equation as

$$e_t = \Delta p_t - (\phi + \theta)x_t + (\phi + \rho\theta)x_{t-1} \tag{2.5}$$

We now have four population moments to match that exactly identify the four parameters to be estimated:

$$\mathbb{E}\begin{bmatrix} e_t - \alpha \\ (e_t - \alpha)x_t \\ (e_t - \alpha)x_{t-1} \\ x_t x_{t-1} - x_t^2 \rho \end{bmatrix} = 0 \tag{2.6}$$

These equations are the standard OLS equations along with an additional restriction for the autocorrelation that allows for the decomposition of the spread components (the last equation). The parameter vector, $\beta = (\phi, \theta, \rho, \alpha)$, is estimated by the Generalized Method of Moments (GMM) procedure to choose values for β to minimize a criterion function based on the moment conditions implied by the model. In particular, the sum of the squared deviations from the moment conditions (the L^2 norm) is minimized to find our estimate $\hat{\beta}$. Hansen (1982) proves that these estimates are consistent and asymptotically normally distributed. These parameter values are then chosen such that the sample moments, $h_T(\beta)$ from the model closely approximate the underlying population moments listed above. This method is chosen because it does not require strong additional distributional assumptions for the data and allows for adjustments for auto correlation and conditional heteroskedasticity.

In particular, the procedure developed by Newey and West (1986) is used to obtain a heteroskedasticity and autocorrelation consistent covariance matrix. The asymptotic variance-covariance matrix of $\hat{\beta}$ is

$$V_{\hat{\beta}} = [H_T' S_T^{-1} H_T]^{-1}$$

where H_T is the matrix of partial derivatives of the moment conditions with respect to the parameter vector, $\mathbb{E}[\frac{\partial h_T(\beta)}{\partial \beta}]/T$, and $S_T = \sum_{s=1}^{T} \mathbb{E}[h_t(\beta)h_s(\beta)']/T$. Using the Newey-West procedure, we estimate

$$\hat{S_T} = \hat{\Omega}_0 + \sum_{j=1}^{m} w(j,m)[\hat{\Omega}_j + \hat{\Omega}_j']$$

where

$$w(j,m) = 1 - [j/(m+1)]$$

and

$$\hat{\Omega}_j = \sum_{t=j+1}^{T} h_t(\hat{\beta})h_{t-j}(\hat{\beta})'/T$$

where m grows with the sample size appropriately. Newey-West showed that this estimate of the variance-covariance matrix is consistent in the presence of autocorrelation and conditional heteroskedasticity[3].

2.3.3 Price Variance Decomposition

As in Madhavan et al. (1997), the the price variance can also be decomposed into components from this method. The price variance can be written as

$$Var(\Delta p_t) = \sigma_\epsilon^2 + 2\sigma_\xi^2 + \left[(\phi+\theta)^2 + (\rho\theta+\phi)^2 - 2(\theta+\phi)(\theta\rho+\phi)\rho\right] \tag{2.7}$$

Equation (2.7) can be written in terms of a component of the volatility due to the asymmetric information component:

[3] Since the model is just identified, the choice of the weighting matrix in the GMM procedure does not matter, and we can simply use the identity matrix.

$$A = (1 - \rho^2)\theta^2$$

A component due to costs of providing liquidity:

$$B = 2(1 - \rho)\phi^2$$

An interaction term:

$$C = 2\phi\theta(1 - \rho^2)$$

And finally a term caused by the new public information and stochastic rounding errors:

$$D = \sigma_\epsilon^2 + 2\sigma_\xi^2$$

This decomposition of the components of price volatility is of particular interest in light of recent theoretical research examining effects of valuable private information on security prices. In particular, in dynamic models of insider trading, does price volatility increase, decrease, or remain constant as the date of information revelation nears? Since price volatility can be a measure of to what degree prices are incorporating valuable private information through trading, these results have important implications for the efficiency of financial markets. Through separating out the component of price variance due to asymmetric information and examining it before, during, and after trading on that information is taking place, we can attempt to reconcile theoretical models of price revelation with results from the data.

2.4 Results

2.4.1 Descriptive Statistics of Parameters

Table 2.1 presents the average values for several parameters of interest in 5 day windows around the filing date (time 0). Figures 2.4 and 2.5 also plot the average daily price volatility and average daily number of transactions for the stocks in the sample. From the figures and table, we see that price volatility roughly doubles from the time 20 days prior to the filing to the windows 10 days prior to the filing date (when the trader knows they will have to file), and then remains higher even after the filing. The bid-ask spread, interestingly, is actually lower prior to the filing date than 20 days prior, but spikes after the filing date by 1.5 cents. This is consistent with the observation by CDF (2015) that SC13D filers endogenously tend to trade more when spreads are lower. Unlike price volatility, however, bid-ask spreads decrease substantially again after an initial high period following the filing date, back to roughly the level 20 days prior to the filing.

Volume and the number of transactions also increase prior to the filing, peaking at the point just after the filing date. As with the bid-ask spread, they both first spike and then decrease sharply after the public filing occurs. The patterns in the bid-ask spread, volatility, and volume after the public filing could be consistent with increased uncertainty about the firm's value, as market makers and investors evaluate the purpose of the filing and its impact. The fact that these filings often precede mergers and takeovers, which typically involve protracted negotiations (and possibly litigation) and uncertain outcomes, offers a potential explanation for this observation.

Table 2.1: Descriptive Statistics for the Sample

Days to Filing	$[-20,-15)$	$[-15,-10)$	$[-10,-5)$	$[-5,0)$	$[0,5)$	$[5,10)$
Variance of ΔP	8.49	9.47	15.35	18.68	19.39	16.66
Spread (¢)	5.48	4.34	4.87	4.78	6.27	5.09
No. of Transactions	4806	5384	5120	5025	5570	4576
Volume (Thousands)	762	1078	1059	914	1030	778

Table 2.1 presents the average values for the variance of price changes (in ¢^{2}), bid-ask spreads (in cents), number of transactions per day, and volume per day of the sample of 309 stocks. The columns are 5 day windows relative to the day on which the SC13D form was filed.

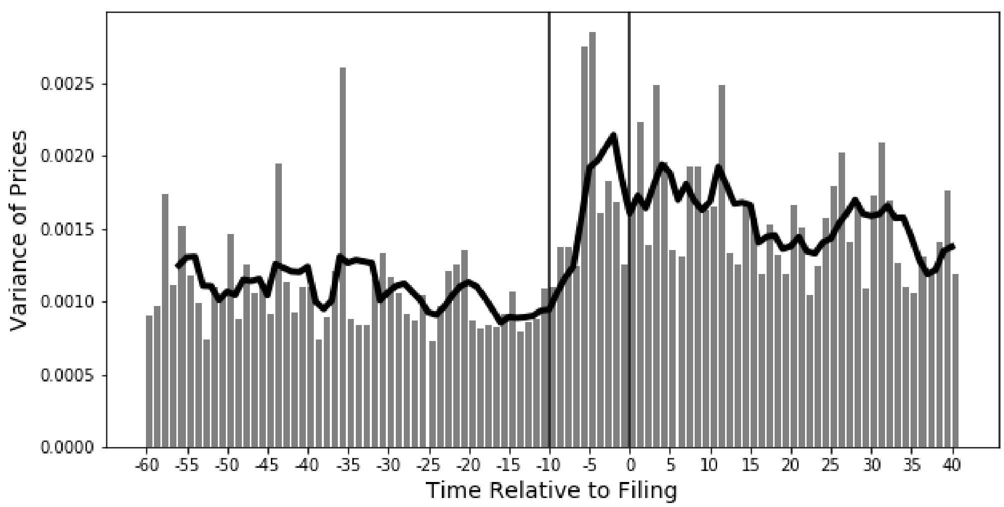

Figure 2.4: **Volatility of Price Changes around the Filing Date.** The bars are daily price change volatility, and the line is the rolling 5-day average.

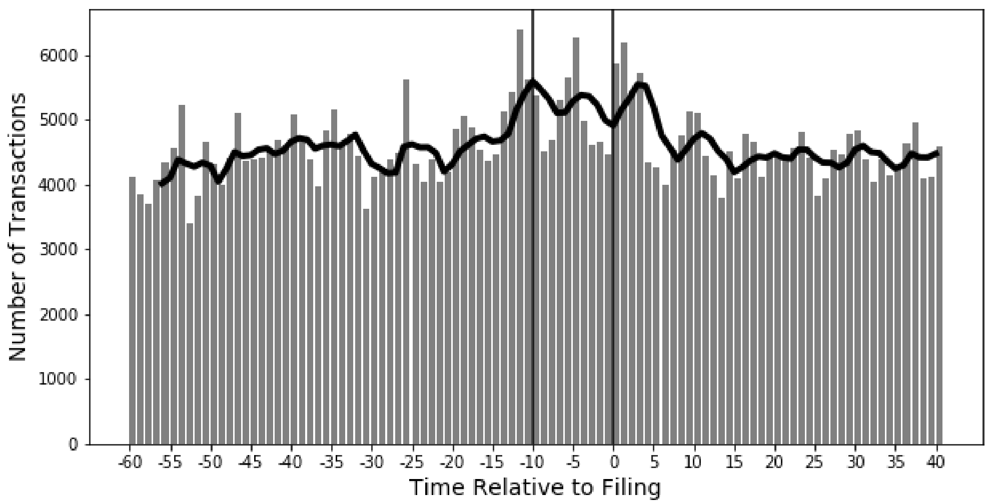

Figure 2.5: **Number of Transactions around the Filing Date.** The bars are the number of daily transactions. The line is the rolling 5-day average of the average number of daily transactions.

2.4.2 Parameter Estimates

Table 2.2 presents the primary results of the model. I report the means, standard errors, and standard deviations of the information asymmetry and liquidity cost components of the half spread as well as for the autocorrelation in trade directions. Figure 2.6 also plots the components half spread over time relative to the filing date. The information asymmetry component rises about 0.1 cents (17%) from twenty days prior to the filing date until just before the filing date. Even after the public filing, however, the information asymmetry component remains high. Interestingly, the liquidity cost component (representing transaction and inventory costs) also rises over this time period, about 0.1 cents (14%), and also remains high after the filing date. Accordingly, the estimated spread (which is twice the sum of the components of the half spread) follows a similar pattern, rising about 0.4 cents (15%) up to the filing date, and remaining high afterwards.

These results have several implications. First, it appears that the information asymmetry component is capturing the increase in informed trading prior to the filing date, while the bid-ask spread alone does not. Second, Figure 2.6 also appears consistent with the observation in Collin-Dufresne and Fos (2015) that informed traders may endogenously choose to trade when trading costs are low. The nadirs of both components, which are measures of trading costs, are just prior to the 10 day period before the filing, just as the trader would be accumulating shares and crossing the 5% ownership threshold.

Interestingly, the liquidity cost component also rises substantially during this period. Recall that ϕ measures the compensation to the dealer for providing liquidity, i.e., compensation for transaction costs, inventory costs, and risk-bearing. Assuming transaction costs are not systematically changing during these periods, the increase would be driven by inventory costs and additional risk-bearing taken on by the market makers during this period. This is also consistent with an increase in informed trading and/or decrease in liquidity traders during this period. As prices change in response to trades more frequently and by larger amounts, the risk the market maker incurs by holding a position in the security increases as well, and the liquidity cost component of the spread increases. Finally, both the information asymmetry and liquidity cost components remain at an elevated level even after the filing (as does the price variance). This result also points to an uncertain trading environment following the filing, as noted in the previous section.

The autocorrelation in trades, on the other hand, does not appear to show a pattern or much variation around these filings. In fact, they lie in a narrow band from 0.496 to 0.510. Consistent with previous research, the estimates are positive for every window.

One last feature to note is that the estimated spreads are substantially smaller than the actual spreads observed in these markets. The estimated spreads account for anywhere from one half to two thirds of the observed spreads in these markets. A possible explanation for this divergence is that as spreads widen, the execution of trades inside the quotes becomes more likely. Indeed, consistent with this explanation, the period in which the estimated spread is the smallest relative to the observed spread (the [0,5) window) is also the period with the largest observed spread.

2.4.3 Variance Decomposition

As noted in section 2.3.3, the model also allows for the decomposition of the variance of price changes into its components. The results are presented in table 2.3.

As is the case with the spread components,m the variance components tend to rise

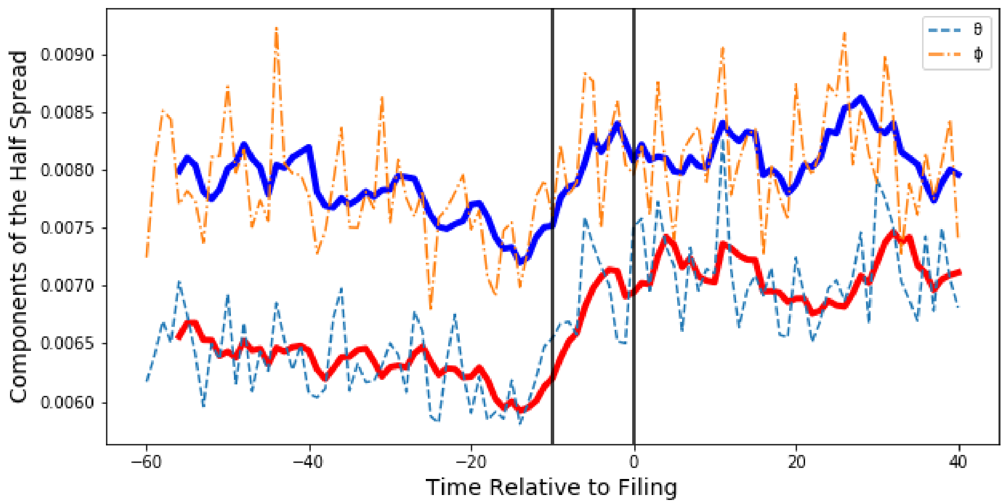

Figure 2.6: **Half-Spread Components around the Filing Date.** On top (dotted line) is the liquidity cost component (ϕ). On bottom (dashed line) is the asymmetric information component (θ). Solid lines overlaid are 5-day rolling averages.

sharply in the run up to the announcement, before peaking just after the announcement date and then steadily falling. Moreover, the largest part of the price variance from these estimates is due to the interaction term between the asymmetric information component and the liquidity cost component. However, clearly, from the size of price variance during these time periods relative to the amounts due to the spread components, the vast majority of price variance is caused by the new public information. So while changes in the spread components have substantial effects the size of the spread, their effect on price variance is fairly small.

2.5 Conclusion

In this article I use a structural model to decompose the bid-ask spread into components due to asymmetric information and liquidity costs in the lead-up to SC13D filings. I argue that these events represent the revelation of valuable private information, and that the filers' trading prior to the filing announcement can be considered informed trading. I then estimate the components of the bid-ask spread in the lead-up to the announcements, and find that the asymmetric information component increases substantially, in contrast to previous studies which have examined other measures of asymmetric information. I also find that liquidity costs rise substantially as well, as do model estimated bid-ask spreads (in contrast to the actual spread). Lastly, I decompose the variance similarly, and find that the amount of price variance attributable to the components of the spread increases somewhat during this period. However, these are dwarfed by the amount attributable to new public information.

Table 2.2: Parameter Estimates of the Model

Days to Filing	$[-20,-15)$	$[-15,-10)$	$[-10,-5)$	$[-5,0)$	$[0,5)$	$[5,10)$
θ (¢)						
Mean	0.594	0.609	0.682	0.691	0.745	0.706
(Avg. Std. Err.)	(0.117)	(0.113)	(0.135)	(0.131)	(0.140)	(0.13)
Std. Dev.	0.0196	0.0223	0.0446	0.0383	0.0276	0.0297
ϕ (¢)						
Mean	0.740	0.763	0.816	0.840	0.822	0.818
(Avg. Std. Err.)	(0.136)	(0.135)	0.156)	(0.157)	(0.159)	(0.151)
Std. Dev.	0.0311	0.0355	0.0513	0.0494	0.0397	0.0368
ρ						
Mean	0.509	0.510	0.494	0.497	0.504	0.498
(Avg. Std. Err.)	(0.0328)	(0.0327)	(0.0341)	(0.0333)	(0.0329)	(0.0338)
Std. Dev.	0.0103	0.0043	0.0094	0.0101	0.0104	0.0100
Est. Spread (¢)	2.68	2.77	3.01	3.09	3.15	3.07

Table 2.2 presents the means, average standard errors, and standard deviations for parameters estimated in the model on the sample of stocks. The parameters are the asymmetric information component of the half spread (θ), the liquidity cost component of the half spread (ϕ), and the autocorrelation in trades (ρ). The estimated spread is $2(\theta + \phi)$, twice the estimated half spread. The columns are 5 day windows relative to the day on which the SC13D form was filed.

Table 2.3: Amount of Price Variance due to Spread Components

Days to Filing	$[-20, -15)$	$[-15, -10)$	$[-10, -5)$	$[-5, 0)$	$[0, 5)$	$[5, 10)$
Asymmetric Information	0.263	0.280	0.353	0.362	0.413	0.375
Liquidity Cost	0.532	0.558	0.663	0.690	0.656	0.653
Interaction Term	0.649	0.686	0.836	0.861	0.902	0.854

Table 2.3 describes how the asymmetric information (A), liquidity cost (B), and interaction term (C), as derived in section 2.3.3, evolve around the announcement date. All units are ¢^{2}.

Chapter 3

Insider Trading with Information Leakage

3.1 Introduction

How does information leakage affect markets with insider trading? In this paper I develop and analyze a model of insider trading where the insider's private information is gradually and exogenously revealed to the market. In these types of insider trading models, the insider is privately informed of a financial asset's true value prior to trading, and chooses quantities to trade in a series of auctions in an order driven market. Uninformed traders also simultaneously place orders each auction, while a market maker sets the price after observing the orders that have been placed, and trades to close all orders. These models can be useful in analyzing the effects on liquidity and information revelation in order driven markets with different characteristics when some traders have private information.

In my model, a monopolistic informed trader receives an initial signal of the asset's value, while other traders and the market maker only receive subsequent noisy signals. The risk neutral insider trades on his private information, obscured by the uninformed noise traders, but part of that valuable private information is slowly revealed to the market. The market satisfies semi-strong efficiency in that the price each period equals the public's conditional expectation of the security's fundamental value. Trading ends at a fixed time and the asset's value is revealed to all market participants. I analyze how this information leakage may affect the insider's strategy, order flow, price revelation, and liquidity in the market.

There is a large literature on insider trading models, beginning with Kyle (1985) and Glosten and Milgrom (1985), who formalized the ideas of Bagehot (1971). Many subsequent papers have since extended the Kyle model to study informed trading in a variety of situations. Most closely related to this study are those papers that examine changes to the information structure of markets with informed trading. Admati and Pfleiderer (1988) develop a model in which information by multiple informed traders is short lived (one-period) and there exist both nondiscretionary liquidity traders, who must trade amounts, and discretionary liquidity traders, who demand liquidity but can be strategic in choosing when to exercise their trades. Similarly, Foster and Viswanathan (1990) develop a model in which the informed trader has different amounts of information on different days (or trading periods) and examine patters in interday volume, variance, and adverse selection costs.

Holden and Subrahmanyam (1992) considered a model with multiple competing and symmetrically informed insiders, which Foster and Viswanathan (1996) extended to the case of asymmetrically informed insiders. Both found that the increased competition among insiders leads to the rapid release of their private information. These models have a similar flavor to the model proposed in this paper, where the insider(s) trade(s) more aggressively because of the exogenous release of his information to the market. However, the lack of competition in this model leads to a much more gradual release of information. Spiegel and Subrahmanyam (1992) in a one-period model and Mendelson and Tunca (2004) in a dynamic model introduced endogenous liquidity traders that trade for hedging purposes.

Back and Pedersen (1998), Chau and Vayanos (2008), and Caldentey and Stacchetti (2010) introduced insider trading models in which the insider receives a flow of information each period. In fact, Chau and Vayanos (2008) also model a market in which the market maker continuously observes a noisy signal of the value of the asset. However, there a several differences between my model and theirs. They assume that trading has been taking place indefinitely, and so there is no initial information asymmetry. They also only examine the steady state of the model and not the dynamics of the model outside of the steady state. They assume that there is no public announcement and that trading takes place indefinitely. In contrast, my model only has initial information asymmetry, examines the dynamics of trading over time, and has a public announcement date.

Several papers have also found evidence of information leakage in empirical studies of markets. Keown and Pinkerton (1981) find evidence of excess returns earned by investors by examining trading in the periods prior to the first announcement of planned mergers. They find that leakage of inside information in the lead up to such announcements is a common occurrence. Kurov et al. (2019) examine prices around the announcement of US macroeconomic data. They find a frequent pre-announcement drift that they conclude could arise from information leakage and/or superior forecasting.

The paper is organized as follows. Section 3.2 presents the model, first in a one-period setting followed by a dynamic setting. Section 3.3 discusses results in a number of parameterizations of the dynamic model. Section 3.4 concludes.

3.2 Model

3.2.1 One Period Model

I first describe the model in a single period of trading before turning to the dynamic version. The market participants are the insider, the market maker, and noise/liquidity traders. Prior to trading, the liquidation value of the security v is realized and revealed only to the insider. There is a single auction in which the insider submits his order x and the noise traders simultaneously submit buy/sell orders y. After the traders submit their demands, the market maker receives a noisy signal of the asset's value $s = v + \epsilon$. The market maker observes the total volume of trade $z = x + y$ as well as the noisy signal, and sets the price p at which all trades are executed.

The distributions of the random variables are all normal and independent of one another. The asset's liquidation value is $v \sim N(p_0, \Sigma_0)$. The noise in the market maker's signal is $\epsilon \sim N(0, \Sigma_\epsilon)$ (so that $s \sim N(v, \Sigma_\epsilon)$) and the noise trader demand is $y \sim N(0, \Sigma_y)$. The market maker sets the price equal to the conditional expected value of the asset's liquidation value based on total trading volume and the noisy signal, $p = \mathbb{E}[v|z, s]$. The profits of the insider are given by $\Pi = (v - p)x$, and the insider is a risk-neutral agent that trades to maximize his expected profits. Since, as in Kyle's model, mixed strategies will not be optimal in what follows, we focus only on pure strategies. A strategy for the insider is then a measurable function $x = X(v)$, while the market maker's pricing rule can be similarly written as a function $p = P(z, s)$. As Π and p depend on X and P here, we write $\Pi(X, P)$ and $p(X, P)$ in what follows.

Definition 3 *An equilibrium is defined as a pair X, P such that the following two conditions hold:*

(i) Market Efficiency: Given X, P satisfies

$$p(X, P) = \mathbb{E}[v|z, s]$$

(ii) Profit Maximization: Given pricing rule P, for any alternate trading strategy X' and for any v,

$$\mathbb{E}[\Pi(X, P)|v] \geq \mathbb{E}[\Pi(X', P)|v]$$

Proposition 2 *There exists a unique equilibrium in which X and P are the linear functions:*

$$X = \beta(v - p_0), \quad P = (1 - \psi)(p_0 + \lambda z) + \psi s \tag{3.1}$$

Where constants β, λ, and ψ are defined by

$$\beta = \sqrt{\frac{\Sigma_y}{\Sigma_0}} \tag{3.2}$$

$$\lambda = \frac{1}{2}\sqrt{\frac{\Sigma_0}{\Sigma_y}} \tag{3.3}$$

$$\psi = \frac{\Sigma_0}{\Sigma_0 + 2\Sigma_\epsilon} \tag{3.4}$$

Proof: See Appendix A.2.

Before moving to the N-period model, I comment briefly on the form of the equilibrium in the single auction model, and compare it to the baseline of Kyle's model. The insider's optimal strategy in this model exactly reflects Kyle's model, and the insider trades exactly the same amount as he would without the existence of the noisy signal. Intuitively, this is because the insider selects his trading amount prior to the revelation of this signal, and as a risk neutral agent will only consider the expected value of this signal, which is just v. However, interestingly, this result will not hold in the dynamic model, as the insider will evaluate the effect of the noisy signal's variance on his continuation value.

While the value for λ is the same in Kyle's model as well, price impact (and hence market depth) and ex-post informativeness of prices are all affected by the noisy signal. The market maker sets the price as a convex combination of the sum of the equilibrium price in Kyle's model and the noisy signal. The weights in the convex combination, $(1 - \psi)$ and ψ, are driven by the relative size of the variance of the signal compared to the initial variance of the asset's fundamental value. As the signal's variance gets large (small), a larger (smaller) weight is placed on the Kyle equilibrium price, as the signal is less (more) informative relative to order flow and the ex ante expected asset value. Interestingly, in the one period model, ψ does not depend on the variance of the noise trader demand.

Price impact of order flow in this model is given by $(1 - \psi)\lambda$, which is smaller than in Kyle's model, as the market maker splits price movements between the two signals. Since price impact is smaller, market depth (the reciprocal of price impact in these models, which is a measure of market liquidity) is higher. The information leakage through the market signal reduces the adverse selection problem, and makes prices more informative. The posterior price variance is $\Sigma_1 = \frac{\Sigma_0 \Sigma_\epsilon}{\Sigma_0 + 2\Sigma_\epsilon}$ which is less than $\frac{1}{2}\Sigma_0$, what it is in baseline Kyle. Similarly, profits for the insider, given by $\Pi = \frac{(1-\psi)}{4\lambda}(v - p_0)^2$, are smaller than in the baseline Kyle model.

I now turn to the dynamic version of the model.

3.2.2 Dynamic Model

The dynamic model is a repeated version of the single auction model. Trading takes place at evenly spaced times $n = 1, \ldots, N$. The time between time intervals is denoted as Δ. During each period n, the sequence of events is the same as the single auction model. Noise traders submit buy/sell orders y_n, and simultaneously the insider chooses his order x_n. After the traders submit their demands, the market maker receives a noisy signal of the assets value $s_n = v + \epsilon_n$. The market maker then observes the total volume of trade $z_n = x_n + y_n$, and sets the price p_n at which all trades are executed. After all orders have been filled, the next trading period begins. This process continues until a fixed period N, at which point the true value of the security is revealed and trading ends.

The initial security value revealed to the insider is normally distributed, with $v \sim N(0, \Sigma_0)$. The noise traders are myopic agents and the sequence of noise trader demands $\{y_n\}_{n=1,\ldots,N}$ is a sequence of independent and identically distributed (i.i.d.) normally distributed random variables, with $y_n \sim N(0, \Sigma_y)$, where $\Sigma_y = \sigma_y^2 \Delta$. Similarly, the sequence of noise shocks to the market signal, $\{\epsilon_n\}_{n=1,\ldots,N}$, is a sequence of independent normally distributed random variables with $\epsilon_n \sim N(0, \Sigma_{\epsilon_n})$ and $\Sigma_{\epsilon_n} = \sigma_{\epsilon_n}^2 \Delta$. All market participants know the distributions of these random variables, but only the insider knows v. The market maker sets the price equal to the public's conditional expectation of the asset's value each period given the public information, so the market satisfies semi-strong efficiency.

Payoffs, Strategies, Information

The insider and the market maker are both risk neutral, and the insider trades to maximize his expected profit. That is, given a sequence $X = \{x_n\}_{n=1,...,N}$ of the insider's trading volume and $P = \{p_n\}_{n=1,...,N}$ for market prices, the payoff for the insider from date n onward is

$$\Pi_n(P, X) = \sum_{k=n}^{N} [v - p_k]x_k$$

The insider maximizes the expected value of his payoff at the end of trading:

$$\mathbb{E}[\Pi_n(P, X)] = \mathbb{E}\left[\sum_{k=n}^{N} (v - p_k)x_k\right]$$

Since the insider observes the asset's fundamental value, as well as market prices and demand each period, and the noisy signal (after he submits demand), his information set each period just prior to submitting his order can be denoted

$$\mathcal{F}_n^I = (v, p_0, x_1, z_1, p_1, \epsilon_1, \ldots, x_{n-1}z_{n-1}p_{n-1}, \epsilon_{n-1})$$

So that his expected profit-to-go from period n can be written as

$$\mathbb{E}[\Pi_n(P, X)] = \mathbb{E}\left[\sum_{k=n}^{N} (v - p_k)x_k \middle| \mathcal{F}_k^I\right]$$

Similarly, the market maker observes order flow each period, the noisy signal, and knows the history of prices, so her information set each period before setting the price can be denoted

$$\mathcal{F}_n^M = (p_0, z_1, p_1, s_1, \dots, z_{n-1}, p_{n-1}, s_{n-1}, z_n, s_n)$$

I now turn to defining strategies for each participant and an equilibrium.

Definition 4 *A strategy for the insider is a sequence of order flows $X = \{x_n\}_{n=1,\dots,N}$ adapted to his information set $\mathcal{F}_n^I$, and a strategy for the market maker is a sequence of prices $P = \{p_n\}_{n=1,\dots,N}$ adapted to her information set $\mathcal{F}_n^M$. A strategy profile (P, X) is an equilibrium if it satisfies the following two conditions:*

(i) Market Efficiency: The market maker sets the price equal to her conditional expectation of the asset's fundamental value each period:

$$p_n = \mathbb{E}\left[v \middle| \mathcal{F}_n^M\right] \quad \text{for } n = 1, \dots, N$$

(ii) Profit Maximization: Given P, for any alternate trading strategy X', any v, and all $n = 1, \dots, N$,

$$\mathbb{E}[\Pi_n(P, X)] \geq \mathbb{E}[\Pi_n(P, X')]$$

A (Markovian) *linear equilibrium* is one in which the market maker sets prices as a linear function of the previous price, order flow, and the market signal, and the insider trades linearly in the price gap. That is, an equilibrium in which there exist sequences $\{\beta_n, \delta_{1n}, \delta_{2n}, \delta_{3n}\}_{n=1,\ldots,N}$ such that:

$$p_n = \delta_{1n} p_{n-1} + \delta_{2n} z_n + \delta_{3n} s_n \tag{3.5}$$

$$x_n = \beta_n (v - p_{n-1}) \tag{3.6}$$

Necessary and Sufficient Conditions for Equilibrium

While I am unable at this point to prove the existence of a linear equilibrium in this formulation of the model, I can derive necessary and sufficient conditions for the existence of a linear equilibrium. I find that linear equilibria exist in at least some parameterizations of the model, and analyze these results numerically in section 3.

Proposition 3 *Given Σ_0, there exists a linear equilibrium if and only if there exist deterministic sequences $\{\beta_n, \lambda_n, \alpha_n, \gamma_n, \psi_n, \Sigma_n\}_{n=1,\ldots,N}$ that solve the following system of equations:*

$$\alpha_n = \frac{(1-\psi_n)}{4\lambda_n(1-\alpha_{n+1}\lambda_n(1-\psi_n))} \tag{3.7}$$

$$\gamma_n = \gamma_{n+1} + \alpha_{n+1}(1-\psi_n)^2\lambda_n^2\Sigma_y + \alpha_{n+1}\psi_n^2\Sigma_{\epsilon_n} \tag{3.8}$$

$$\beta_n = \frac{(1-2\alpha_{n+1}\lambda_n(1-\psi_n))}{2\lambda_n(1-\alpha_{n+1}\lambda_n(1-\psi_n))} \tag{3.9}$$

$$\lambda_n = \frac{\beta_n\Sigma_n}{\beta_n^2\Sigma_n + \Sigma_y} \tag{3.10}$$

$$\psi_n = \frac{\Sigma_n\Sigma_y}{\Sigma_n\Sigma_y + \beta_n^2\Sigma_n\Sigma_{\epsilon_n} + \Sigma_y\Sigma_{\epsilon_n}} \tag{3.11}$$

$$\Sigma_{n+1} = \frac{\Sigma_n\Sigma_y\Sigma_{\epsilon_n}}{\Sigma_n\Sigma_y + \beta_n^2\Sigma_n\Sigma_{\epsilon_n} + \Sigma_y\Sigma_{\epsilon_n}} = \psi_n\Sigma_{\epsilon_n} \tag{3.12}$$

subject to the boundary conditions $\alpha_{N+1} = \gamma_{N+1} = 0$ *and the second order condition.*

$$(1-\psi_n)\lambda_n(1-\alpha_{n+1}(1-\psi_n)\lambda_n) > 0 \tag{3.13}$$

If this linear equilibrium exists, it is defined by the above constants and the following equations:

$$x_n^* = \beta_n(v - p_{n-1}) \tag{3.14}$$

$$p_n = (1-\psi_n)(p_{n-1} + \lambda_n z_n) + \psi_n s_n \tag{3.15}$$

$$\Sigma_{n+1} = Var(v|\mathcal{F}_n^M) \tag{3.16}$$

$$\mathbb{E}[\Pi_n|\mathcal{F}_n^I] = \alpha_n(v - p_{n-1})^2 + \gamma_n \tag{3.17}$$

Proof: See Appendix A.3.

There are few interesting things to note about the model. The first is that in contrast to Kyle's model, the market maker's conditional variance of her estimate of the fundamental value decreases nonlinearly (and faster than linearly). Secondly, the rate at which this variance decreases is exactly ψ_n, the sequence of weights which the market maker places on the noisy signal when determining the new price. Since this rate is decreasing (as time goes on, the market maker will place less weight on the noisy signal, since she has a more accurate estimate of the value), this implies that information is incorporated into the market quickly at first, and then the rate slows. As in the one period model, the market maker sets the price exactly equal to a weighted combination of the previous price, the order flow, and the noisy signal, assigning weight $(1 - \psi_n)$ to the previous price, $\lambda_n(1 - \psi_n)$ (the price impact) to the order flow, and ψ_n to the noisy signal. In contrast to Kyle, the weight on the previous price is less than one, because of the additional weight the market maker puts on the noisy signal.

3.3 Results

In this section I analyze several sets of results for certain parameterizations of the model, focusing on the sequence of Σ_{ϵ_n}'s, the variance of the noisy part of the market signal, which is the primary parameter that distinguishes the model from Kyle's (as $\Sigma_{\epsilon_n} \to \infty$, the model collapses to baseline Kyle). First, I compare the model against the Kyle baseline model (with no noisy signal of the asset's value) against versions of the model with high, medium, and low variances of the noise in the information leakage. I then compare versions of the model in which this variance increases over time and decreases over time. Lastly, I compare versions of the model where the noise variance switches from high to low (low to high) at the midpoint of trading (the $N/2$th auction).

3.3.1 Comparing High, Medium, and Low Signal Variance with Baseline Kyle

I first compare cases for different levels of constant market signal variance. Figure 3.1 shows the parameters comparing three cases of the model's market signal variance, as well as the baseline Kyle model. As expected, the more informative the market signal, the more quickly the conditional price variance decreases, indicating more information is reflected in prices more quickly. We also see that in all cases, as the price gets closer to the fundamental value, both the pricing weight on the noisy signal and the price impact tend to decrease, indicating that later volume in later auctions moves prices less than earlier ones. The insider also trades more aggressively in the price gap in those situations when the noisy signal is more informative, indicating that a degree of information leakage could induce the insider to more quickly reveal his information to the market before it is revealed exogenously. Lastly, unlike in Kyle, where the price impact of order flow is fairly constant until the last few auctions, with information leakage it quickly decreases much earlier. Because the market maker now has two signals, and because the insider is trading more aggressively to gain maximize profits in the presence of the extra signal, the price quickly absorbs the information about the asset's value and both the noisy signal and the order flow have smaller effects on prices in later auctions.

This also has implications for market depth, which, as in Kyle's model, can be measured by the reciprocal of the price impact, and is one measure of liquidity in the market.

Specifically, market depth is a measure of how much quantity the market can absorb without having large changes in prices. With information leakage, market depth increases much more quickly and remains higher over the course of the trading period. This is a direct implication of the decrease in price impact noted above: with smaller price impact, it takes larger quantities of trading to move prices substantially. So information leakage leads to a "deeper" market.

Market resiliency, another measure of liquidity, is also improved by information leakage. Resiliency, roughly speaking, is a measure of how quickly prices tend to converge towards the asset's fundamental value, as well as the rate at which they bounce back from a random shock. Resiliency is also improved by the information leakage, as the prices converge much more quickly to the asset's value (as evidenced by the conditional variance graph) and will bounce back from noise trading shocks quickly (as evidenced by the insider's aggressiveness graph). Thus, information leakage actually improves two liquidity characteristics of the market.

I also examine the average informed traders volume, prices, and price volatility in 1000 simulations of these cases, as shown in Figure 3.2. In the most informative signal case, the low (L) signal variance, the insider trades very small amounts, the price converges in just a few auctions to very close to the asset's fundamental value, and price volatility, while high initially, quickly decreases. Interestingly, in both the medium (M) and high(H) signal variance cases, the insider trades larger amounts initially than even in the baseline Kyle model, and both volumes decrease initially before rising again in the last few auctions. In fact, in the high case, the informed trader's volume remains higher than the baseline for the duration of trading despite the leakage of information, and the fact that prices are

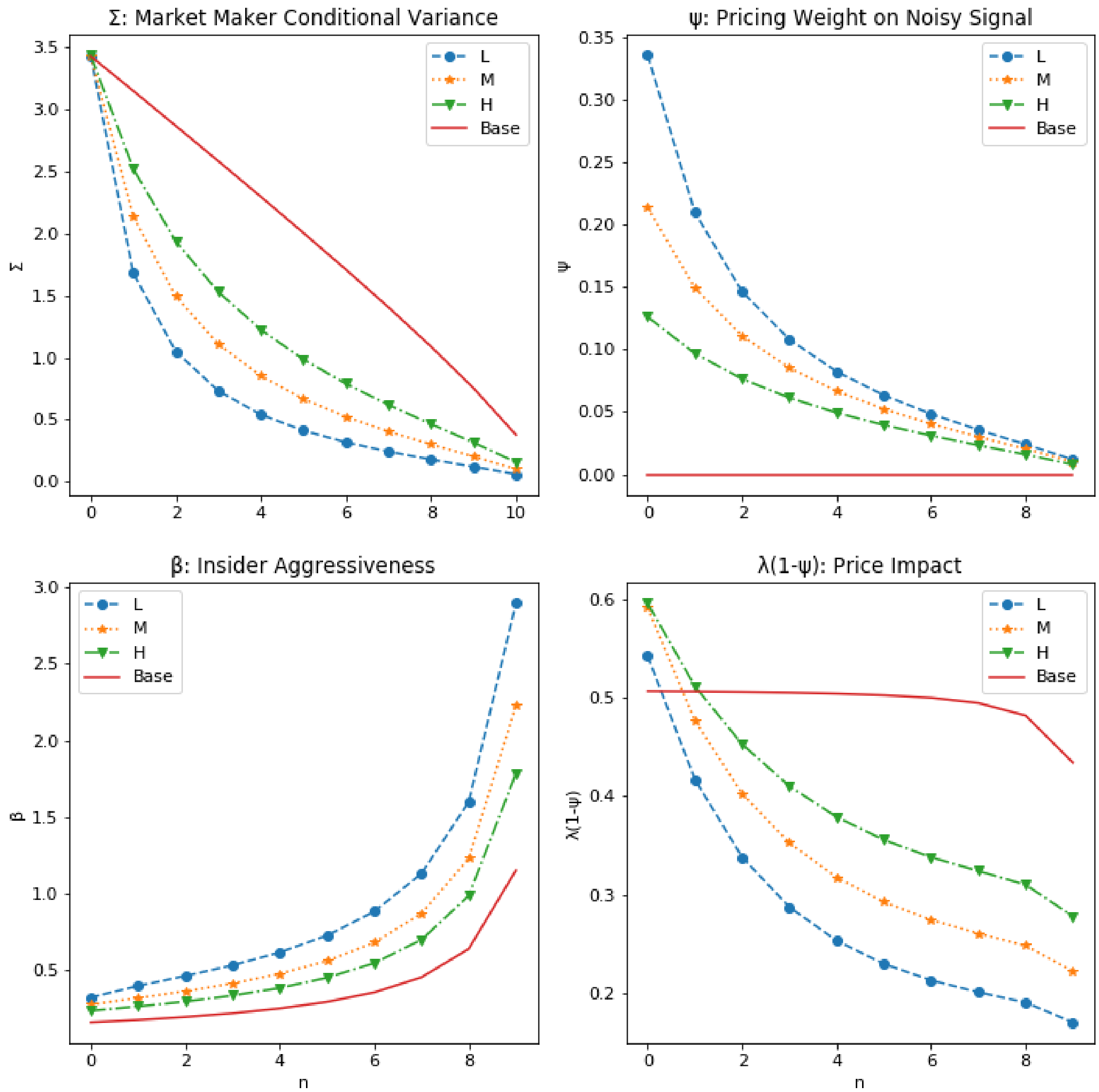

Figure 3.1: **Model Parameters for low (L), medium (M), high (H), and baseline (Base) levels of Signal Variance.** All models have $\Sigma_y = 1$, $\Sigma_0 = 3.43$ and $N = 10$. Low (L) sets $\Sigma_{\epsilon_n} = 5$. Medium (M) sets $\Sigma_{\epsilon_n} = 10$. High (H) sets $\Sigma_{\epsilon_n} = 20$. Base is baseline Kyle.

more informative (closer to the fundamental value) than in the baseline. The volume in the medium case falls below the baseline after a few auctions, but prices are even more informative. Price volatility also decreases over time in the medium case, whereas in the high case (as in the baseline) it rises initially before falling towards the end of the trading. It's possible that information leakage could explain some asset pricing phenomena, such as the tendency for insiders with long-lived information to trade large amounts early, and the positive volume-volatility relationship found by Gallant et al. (1992).

3.3.2 Comparing Increasing and Decreasing Signal Variance

An interesting comparison that this model allows for is when there are changes in the informativeness of the market signal over time. That is, considering what may happen when information leaked to the public is more or less informative earlier in the trading process. Figure 3.3 shows the parameters comparing versions of the model where the signal variance is increasing vs. decreasing (as well as constant and baseline Kyle). As can be seen in the figures, what matters most is how informative the signal is early on in trading. In the case when it's variance is increasing (that is, the signal is becoming less informative over time), prices are more informative, price impact is lower (and hence the market depth is higher), and the insider trades most aggressively. In the case where the variance is decreasing and the signal is becoming more informative over time, prices reflect information more slowly, and trades tend to have a large impact. This implies that markets where the private information is leaked more quickly initially will be more liquid and have lower adverse selection costs. However, interestingly, the conditional variance at the end of trading is very similar, regardless of the path of the variance of the signal. That is, the amount of

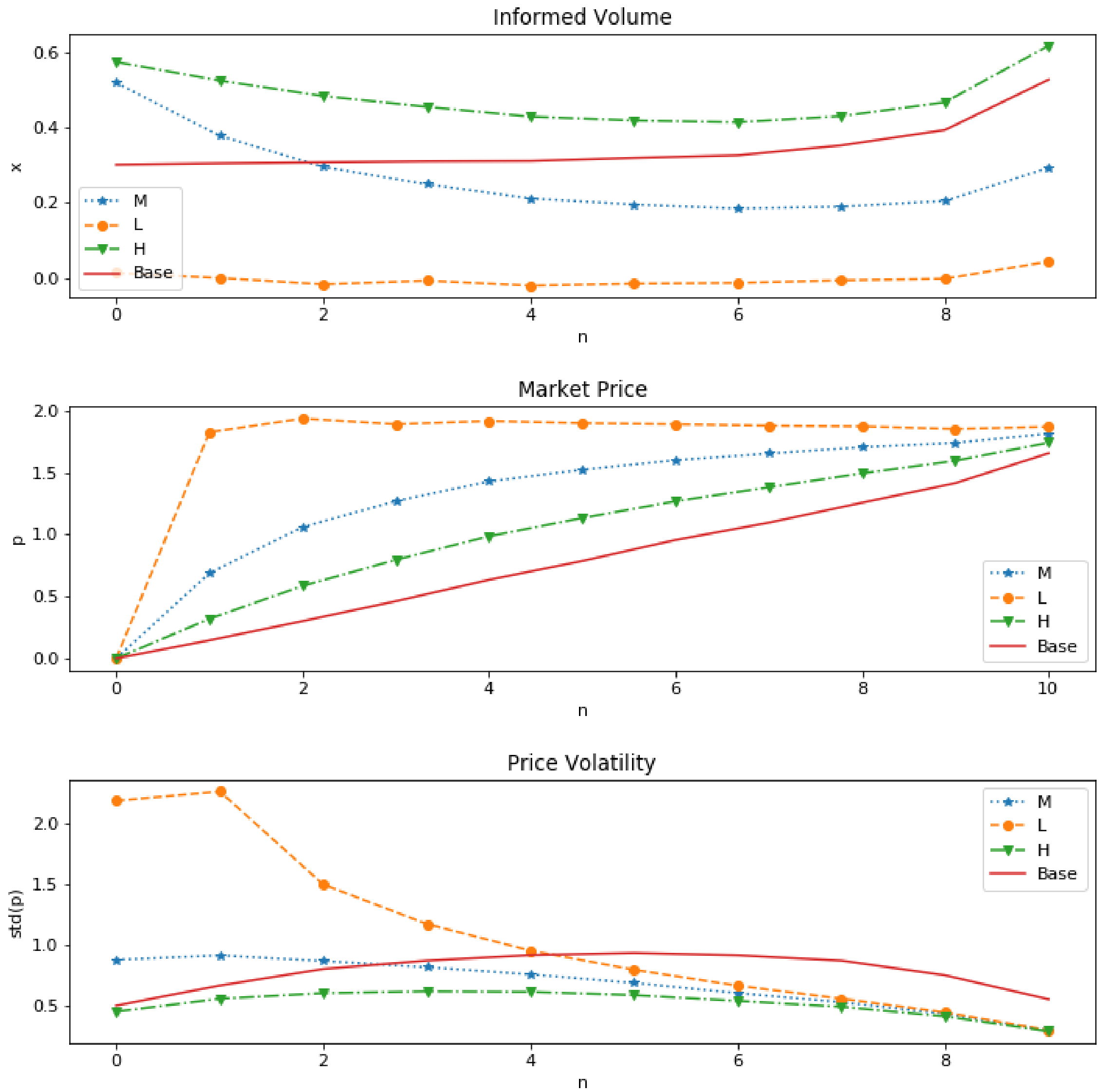

Figure 3.2: **Informed Volume, Market Prices, and Price Volatility for 1000 Simulations using low (L), medium (M), and high (H) levels of Signal Variance.** All models have $\Sigma_y = 1$, $\Sigma_0 = 3.43$, $v = \sqrt{3.5}$, and $N = 10$. Low (L) sets $\Sigma_{\epsilon_n} = 5$. Medium (M) sets $\Sigma_{\epsilon_n} = 10$. High (H) sets $\Sigma_{\epsilon_n} = 20$. Base is baseline Kyle.

information revealed by the end of last auction is almost exactly the same.

3.3.3 Comparing High-Low to Low-High

A second possible case when the informativeness of the market signal is dynamic is if there is a large change in it during the course of trading. Figure 3.4 shows the parameters comparing versions of the model where the signal variance switches from high-to-low at the $N/2$th auction vs. low-to-high. As in the case with increasing and decreasing signal variance, what matters most in this case is how informative the signal is towards the beginning of trading. The case where the signal variance begins at a low level and later jumps to a higher level has more informative prices, higher liquidity (lower price impact) and a more aggressively trading insider than the case where the variance begins high and jumps to a lower level. This is qualitatively similar to the increasing and decreasing signal variance analyzed in the previous section. One interesting feature to note, however, is that when the signal variance jumps from high to low (low to high), the slope of the price impact graph changes sharply, indicating that larger leaks of private information could quickly improve liquidity in markets and lessen the adverse selection costs.

3.4 Conclusion

In this paper, I analyze a model of insider trading in which the insider's private information is gradually and exogenously disseminated to the market over time. The insider is initially informed of the "true" or liquidation value of a security, and can trade along with noise traders to maximize his profits. However, this information is "leaked" to the market

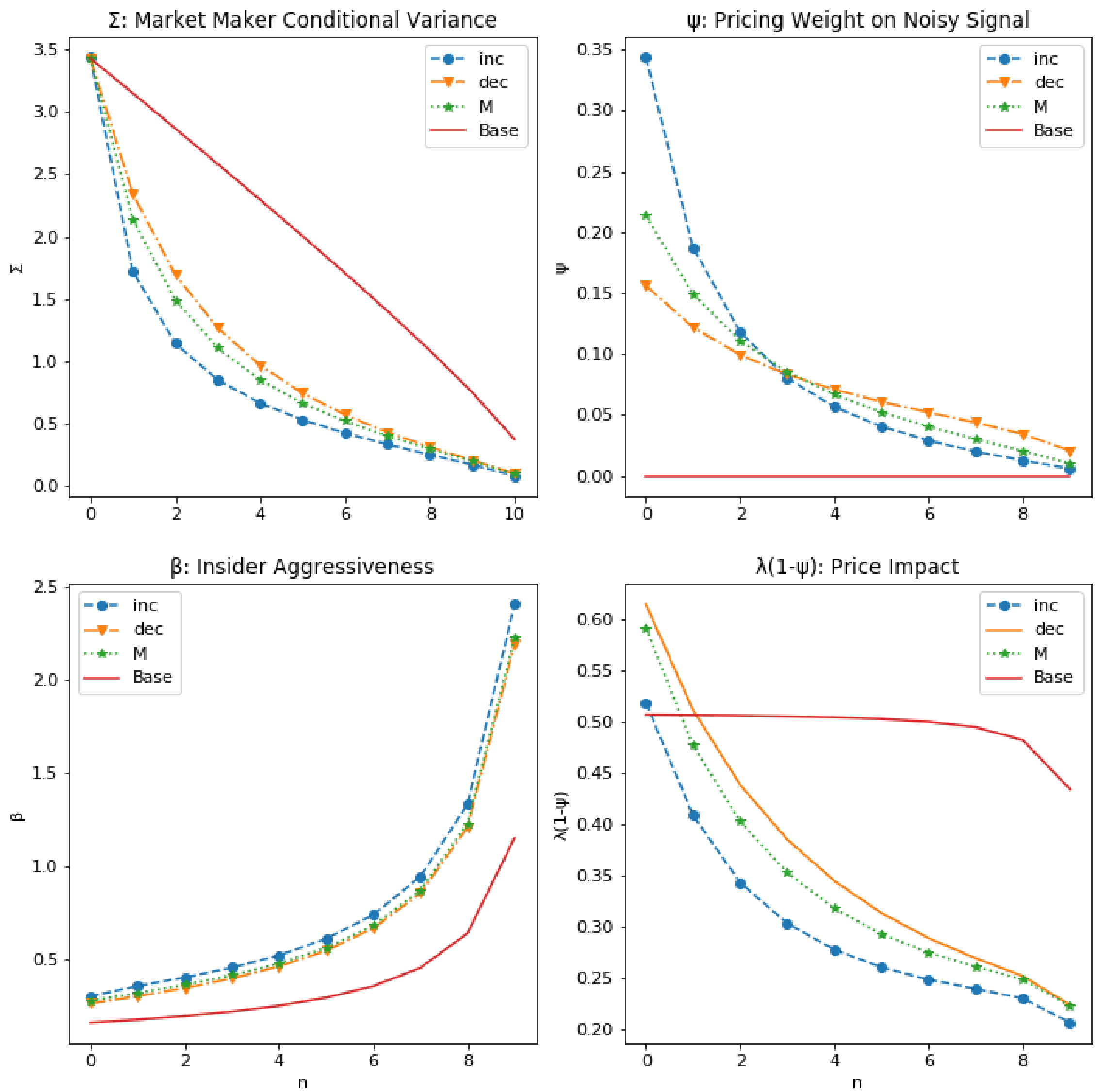

Figure 3.3: **Model Parameters for increasing (inc), decreasing (dec), constant (M), and baseline (Base) levels of Signal Variance.** All models have $\Sigma_y = 1$, $\Sigma_0 = 3.43$ and $N = 10$. Increasing (Inc) sets $\Sigma_{\epsilon_0} = 5$ and increases by 1 each period. Medium (M) sets $\Sigma_{\epsilon_n} = 10$. Decreasing (Dec) sets $\Sigma_{\epsilon_0} = 20$ and decreases by 1 each period. Base is baseline Kyle.

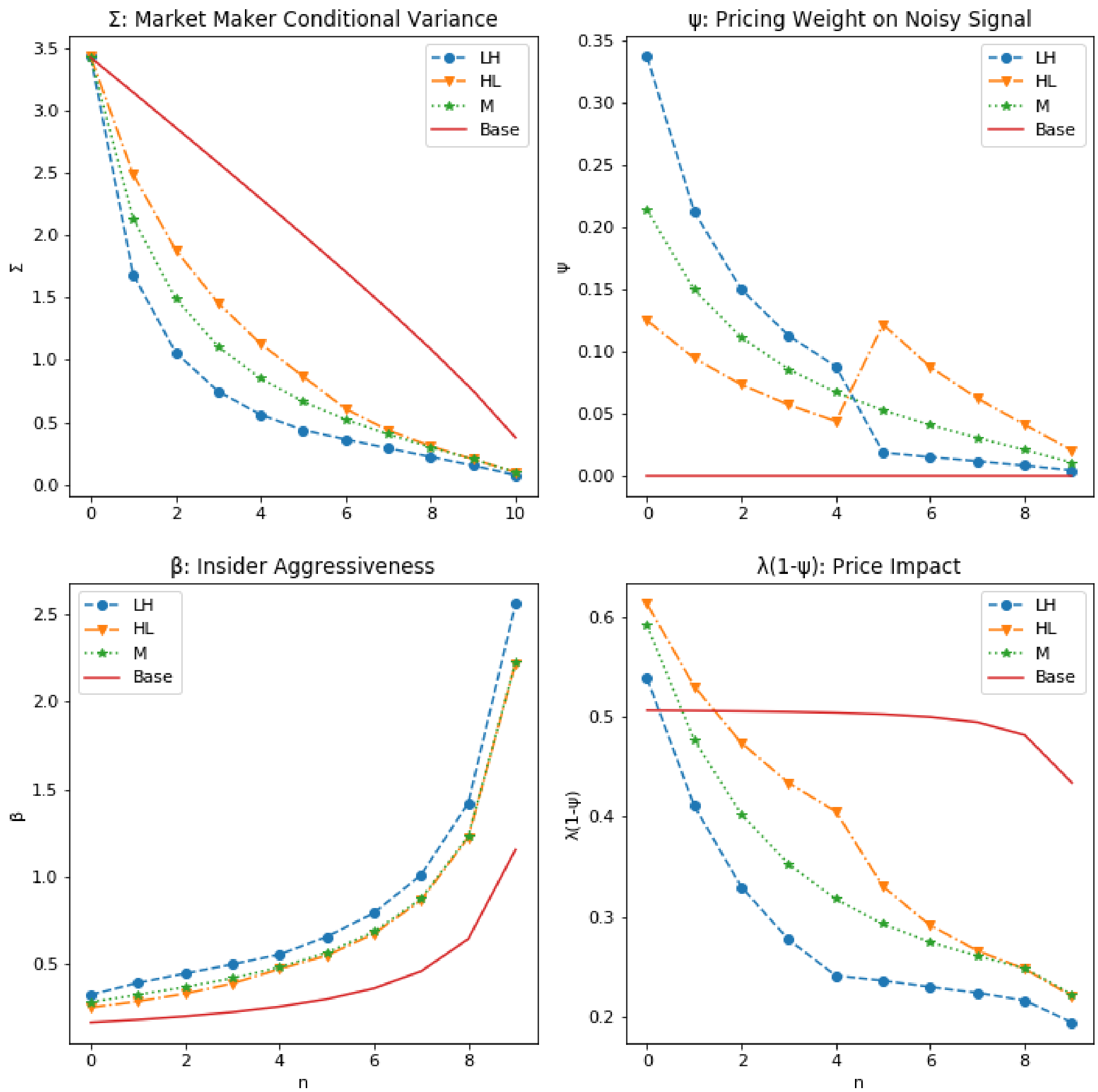

Figure 3.4: **Model Parameters for high-to-low (HL), low-to-high (LH), constant (M) and baseline (Base) levels of Signal Variance.** All models have $\Sigma_y = 1$, $\Sigma_0 = 3.43$ and $N = 10$. High-to-Low (HL) sets $\Sigma_{\epsilon_0} = 20$ and switches to 5 at the $N/2$th auction. Medium (M) sets $\Sigma_{\epsilon_n} = 10$. Low-to-High (LH) sets $\Sigma_{\epsilon_0} = 5$ and subsequently switches to 20 at the $N/2$th auction. Base is baseline Kyle.

maker in the form of a noisy signal with a mean equal to the liquidation value of the security. In both a one-period model and a dynamic model, I find that this information leakage tends to lead to higher liquidity, more informative prices, and lower adverse selection costs. I also examine cases where the informativeness of the signal varies over time, and find that the largest effects are generated by those cases where the signal is most informative (has the lowest variance) early on in the trading period.

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Appendix A

Proofs of Propositions

A.1 Proof of Proposition 1

First we'll consider the insider's problem in period n. Suppose the market maker plays a linear strategy, such that

$$p_n = p_{n-1} + \lambda_n z_n = p_{n-1} + \lambda_n (x_n + y_n)$$

Conjecture that the insider's continuation value, $\mathbb{E}[\Pi_{n+1}]$, is quadratic in the price gap:

$$\mathbb{E}[\Pi_{n+1}(X, P) | \mathcal{F}^I_{n+1}] = \alpha_{n+1}(v - p_n)^2 + \gamma_{n+1}$$

Then the insider chooses the amount to trade x_n to maximize his payoff-to-go:

$$\max_{x_n} \quad \mathbb{E}[(v - p_n)x_n - \tau x_n^2 + \alpha_{n+1}(v - p_n)^2 + \gamma_{n+1}]$$

Plugging in the market maker's strategy for p_n we have:

$$\max_{x_n} \quad \mathbb{E}[(v - p_{n-1} - \lambda_n(x_n + y_n))x_n - \tau x_n^2 + \alpha_{n+1}(v - p_{n-1} - \lambda_n(x_n + y_n))^2 + \gamma_{n+1}]$$

Expanding out and taking the expectation we have

$$\begin{aligned} \max_{x_n} \quad & vx_n - p_{n-1}x_n - \lambda_n x_n^2 - \tau x_n^2 \\ & + \alpha_{n+1}(v^2 + p_{n-1}^2 - 2vp_{n-1} + \lambda_n^2 x_n^2 + \lambda_n^2 \Sigma_y - 2v\lambda_n x_n + 2p_{n-1}\lambda_n x_n) + \gamma_{n+1} \end{aligned} \tag{A.1}$$

Taking the first order condition yields:

$$v - p_{n-1} - 2\lambda_n x_n - 2\tau x_n + 2\alpha_{n+1}\lambda_n^2 x_n - 2\alpha_{n+1}v\lambda_n + 2\alpha_{n+1}p_{n-1}\lambda_n = 0$$

Solving for x_n:

$$x_n = \frac{1 - 2\alpha_{n+1}\lambda_n}{2(\lambda_n + \tau - \alpha_{n+1}\lambda_n^2)}(v - p_{n-1})$$

So $\beta = \frac{1-2\alpha_{n+1}\lambda_n}{2(\lambda_n+\tau-\alpha_{n+1}\lambda_n^2)}$. Taking the second order condition gives:

$$-2\lambda_n - 2\tau + 2\alpha_{n+1}\lambda_n^2 < 0$$

We now will plug our expression for x_n back into equation (A.1) to verify that the value function is quadratic as conjectured. We had:

$$(v - p_{n-1})x_n - \lambda_n x_n^2 - \tau x_n^2 + \alpha_{n+1}((v - p_{n-1})^2 + \lambda_n^2 x_n^2 + \lambda_n^2 \Sigma_y - 2\lambda_n(v - p_{n-1})x_n) + \gamma_{n+1}$$

Plugging in for x_n gives:

$$\frac{(1+4\alpha_{n+1}\tau)}{4(\lambda_n+\tau-\alpha_{n+1}\lambda_n^2)}(v-p_{n-1})^2+\alpha_{n+1}\lambda_n^2\Sigma_y+\gamma_{n+1}$$

So the value function is indeed quadratic as conjectured, with

$$\alpha_n=\frac{1+4\alpha_{n+1}\tau}{4(\lambda_n+\tau-\alpha_{n+1}\lambda_n^2)},\ \text{and}\ \gamma_n=\alpha_{n+1}\lambda_n^2\Sigma_y+\gamma_{n+1}$$

Next, we'll consider the market efficiency condition. Suppose the insider is playing the linear strategy

$$x_n=\beta_n(v-p_{n-1})$$

for some constant β_n. The market maker wants to set the price, $p_n=\mathbb{E}[v|z_n,\mathcal{F}_{n-1}^M]$. Since the pair, (v,z_n) is a two-dimensional normally distributed random vector, we can use the projection theorem to write:

$$p_n=\mathbb{E}[v|z_n,\mathcal{F}_{n-1}^M]=\mathbb{E}[v|\mathcal{F}_{n-1}^M]+\frac{Cov(v,z_n|\mathcal{F}_{n-1}^M)}{Var(z_n|\mathcal{F}_{n-1}^M)}(z_n-\mathbb{E}[z_n|\mathcal{F}_{n-1}^M])$$

Then since $\mathbb{E}[v|\mathcal{F}_{n-1}^M]=p_{n-1}$, $z_n=x_n+y_n=\beta_n(v-p_{n-1})+y_n$, and $\mathbb{E}[z_n|\mathcal{F}_{n-1}^M]=0$, we have

$$\begin{aligned}p_n&=p_{n-1}+\frac{Cov(v,\beta_n(v-p_{n-1})+y_n|\mathcal{F}_{n-1}^M)}{Var(\beta_n(v-p_{n-1})+y_n|\mathcal{F}_{n-1}^M)}z_n\\&=p_{n-1}+\frac{\beta_n\Sigma_n}{\beta_n^2\Sigma_n+\Sigma_y}z_n\end{aligned}$$

This pricing rule satisfies condition (ii) of the equilibrium definition with λ_n as in equation (1.7). Furthermore,

$$\begin{aligned}
\Sigma_{n+1} &= Var(v|z_n, \mathcal{F}^M_{n-1}) \\
&= Var(v|\mathcal{F}^M_{n-1}) \left(1 - \frac{Cov(v, z_n|\mathcal{F}^M_{n-1})^2}{Var(v|\mathcal{F}^M_{n-1})Var(z_n|\mathcal{F}^M_{n-1})}\right) \\
&= \frac{\Sigma_n \Sigma_y}{\beta_n^2 \Sigma_n + \Sigma_y}
\end{aligned}$$

So $\{\Sigma_n\}$ is indeed a deterministic sequence as in equation (1.8).

Lastly, we need to show there exists a unique solution to the system of difference equations. We can write:

$$\lambda_n = \frac{\beta_n \Sigma_{n+1}}{\Sigma_y}$$

Then plugging in for β_n we have (as in Kyle) a cubic equation in λ_n:

$$\frac{\Sigma_y \lambda_n (\lambda_n + \tau - \alpha_{n+1}\lambda_n^2)}{\Sigma_{n+1}} + \alpha_{n+1}\lambda_n = \frac{1}{2}$$

with (given positive α_{n+1} and Σ_{n+1}) three real roots, only the middle of which satisfies the second order condition.

QED.

A.2 Proof of Proposition 2

Suppose first that the market maker's pricing rule is the linear function specified in equation (3.1). Then the insider's profit maximization problem is given by

$$\max_x \quad \mathbb{E}[(v-p)x] = \max_x \quad \mathbb{E}[(v - (1-\psi)(p_0 + \lambda z) - \psi s)x]$$

Taking the expectation gives:

$$\max_{x} \quad (1-\psi)(v-p_0)x - (1-\psi)\lambda x^2$$

The first order condition is:

$$(1-\psi)(v-p_0) - 2(1-\psi)\lambda x = 0$$

Solving gives the insider's optimal strategy

$$x = \frac{1}{2\lambda}(v-p_0)$$

So that $\beta = \frac{1}{2\lambda}$.

Now we turn to the market maker's problem. Suppose that the insider is playing a linear strategy of the form $\beta(v-p_0)$. The market maker is setting the price equal to

$$p = \mathbb{E}[v|z,s]$$

Before projecting v onto z and s, it is convenient to solve for the conditional variance $\Sigma_1 = Var(v|z,s)$. Projecting first v onto s (noting that $s = v+\epsilon$), we have

$$\begin{aligned}\Sigma_1 &= Var(v|z) - \frac{Cov^2(v, v+\epsilon|z)}{Var(v+\epsilon|z)} \\ &= Var(v|z) - \frac{Var^2(v|z)}{Var(v|z)+\Sigma_\epsilon}\end{aligned}$$

We can solve for $Var(v|z)$:

$$
\begin{aligned}
Var(v|z) &= Var(v) - \frac{Cov^2(v, \beta(v-p_0)+y)}{Var(\beta(v-p_0)+y)} \\
&= \Sigma_0 - \frac{\beta^2\Sigma_0^2}{\beta^2\Sigma_0+\Sigma_y} = \frac{\Sigma_0\Sigma_y}{\beta^2\Sigma_0+\Sigma_y}
\end{aligned}
$$

Plugging this back into the equation for Σ_1:

$$
\begin{aligned}
\Sigma_1 &= \frac{\Sigma_0\Sigma_y}{\beta^2\Sigma_0+\Sigma_y} - \frac{\left[\frac{\Sigma_0\Sigma_y}{\beta^2\Sigma_0+\Sigma_y}\right]^2}{\frac{\Sigma_0\Sigma_y}{\beta^2\Sigma_0+\Sigma_y}+\Sigma_\epsilon} \\
&= \frac{\Sigma_0\Sigma_y\Sigma_\epsilon}{\Sigma_0\Sigma_y+\beta^2\Sigma_0\Sigma_\epsilon+\Sigma_y\Sigma_\epsilon}
\end{aligned}
$$

Now, turning to $p = \mathbb{E}[v|z,s]$, we first project v onto s (again noting $s = v+\epsilon$):

$$
\begin{aligned}
p &= \mathbb{E}[v|z] + \frac{Cov(v, v+\epsilon|z)}{Var(v+\epsilon|z)}(v+\epsilon-\mathbb{E}[v+\epsilon|z]) \\
&= \mathbb{E}[v|z] + \frac{Var(v|z)}{Var(v|z)+\Sigma_\epsilon}(v+\epsilon-\mathbb{E}[v+\epsilon|z])
\end{aligned}
$$

Now project v and $v+\epsilon$ onto z:

$$
\begin{aligned}
p = \mathbb{E}[v] &+ \frac{Cov(v, \beta(v-p_0)+y)}{Var(\beta(v-p_0)+y)}(z-\mathbb{E}[z]) \\
&+ \frac{Var(v|z)}{Var(v|z)+\Sigma_\epsilon}(v+\epsilon-\mathbb{E}[v] - \frac{Cov(v+\epsilon, \beta(v-p_0)+y)}{Var(\beta(v-p_0)+y)}(z-\mathbb{E}[z])
\end{aligned}
$$

Noting that $\mathbb{E}[v] = p_0$ and that $\mathbb{E}[z] = 0$, we have

$$
p = p_0 + \frac{\beta\Sigma_0}{\beta^2\Sigma_0+\Sigma_y}z + \frac{Var(v|z)}{Var(v|z)+\Sigma_\epsilon}(v+\epsilon-p_0-\frac{\beta\Sigma_0}{\beta^2\Sigma_0+\Sigma_y}z)
$$

Note that

$$\frac{Var(v|z)}{Var(v|z)+\Sigma_\epsilon} = \frac{\frac{\Sigma_0\Sigma_y}{\beta^2\Sigma_0+\Sigma_y}}{\frac{\Sigma_0\Sigma_y}{\beta^2\Sigma_0+\Sigma_y}+\Sigma_\epsilon}$$
$$= \frac{\Sigma_0\Sigma_y}{\Sigma_0\Sigma_y+\beta^2\Sigma_0\Sigma_\epsilon+\Sigma_y\Sigma_\epsilon}$$
$$= \psi$$

Then, we can write the pricing function as

$$p = (1-\psi)(p_0+\lambda z)+\psi s$$

with ψ as above and with

$$\lambda = \frac{\beta\Sigma_0}{\beta^2\Sigma_0+\Sigma_y}$$

So we have the system of equations

$$\beta = \frac{1}{2\lambda} \tag{A.2}$$

$$\lambda = \frac{\beta\Sigma_0}{\beta^2\Sigma_0+\Sigma_y} \tag{A.3}$$

$$\psi = \frac{\Sigma_0\Sigma_y}{\Sigma_0\Sigma_y+\beta^2\Sigma_0\Sigma_\epsilon+\Sigma_y\Sigma_\epsilon} \tag{A.4}$$

Solving the system of equations gives the desired result, subject to the second order condition $(1-\psi)\lambda > 0$. Since ψ is by definition between 0 and 1, this really implies that λ (and thus β) is positive. QED.

A.3 Proof of Proposition 3

I first describe and solve the market maker's filtering problem, given that the insider plays a linear strategy, and then turn to the insider's maximization problem, conjecturing that the market maker plays a linear strategy.

A.3.1 Market Maker's Problem

Suppose the insider plays a linear strategy of the form

$$x_n = \beta_n(v - p_{n-1})$$

The Market maker observes $z_n = x_n + y_n$ and $s_n = v + \epsilon_n$. They set the price equal to

$$p_n = \mathbb{E}[v|z_n, s_n, \mathcal{F}^M_{n-1}]$$

The variance of the market maker's estimate of v after observing z_n and s_n is

$$\Sigma_{n+1} = Var(v|\mathcal{F}_{n-1}, s_n, z_n)$$

Variance Updating

First, we can evaluate the conditional variance. Projecting onto $s_n = v + \epsilon_n$ gives

$$\Sigma_{n+1} = Var(v|\mathcal{F}^M_{n-1}, z_n) - \frac{Cov^2(v, v + \epsilon_n|\mathcal{F}^M_{n-1}, z_n)}{Var(v + \epsilon_n|\mathcal{F}^M_{n-1}, z_n)} \tag{A.5}$$

$$= Var(v|\mathcal{F}^M_{n-1}, z_n) - \frac{Var^2(v|\mathcal{F}^M_{n-1}, z_n)}{Var(v|\mathcal{F}_{n-1}, z_n) + \Sigma_{\epsilon_n}} \tag{A.6}$$

Now note that we have (since p_{n-1} is known by the market maker and y_n is independent of v):

$$\begin{aligned} Var(v|\mathcal{F}_{n-1}^M, z_n) &= Var(v|\mathcal{F}_{n-1}^M) - \frac{Cov^2(v, \beta_n(v-p_{n-1}) + y_n|\mathcal{F}_{n-1}^M)}{Var(\beta_n(v-p_{n-1}) + y_n|\mathcal{F}_{n-1}^M)} \\ &= Var(v|\mathcal{F}_{n-1}^M) - \frac{\beta_n^2 Var^2(v|\mathcal{F}_{n-1}^M)}{\beta_n^2 Var(v|\mathcal{F}_{n-1}^M) + Var(y_n|\mathcal{F}_{n-1}^M)} \\ &= \Sigma_n - \frac{\beta_n^2\Sigma_n^2}{\beta_n^2\Sigma_n + \Sigma_y} = \frac{\Sigma_n\Sigma_y}{\beta_n^2\Sigma_n + \Sigma_y} \end{aligned}$$

Plugging this back into equation (A.6) gives

$$\Sigma_{n+1} = \frac{\Sigma_n\Sigma_y}{\beta_n^2\Sigma_n + \Sigma_y} - \frac{\left[\frac{\Sigma_n\Sigma_y}{\beta_n^2\Sigma_n + \Sigma_y}\right]^2}{\frac{\Sigma_n\Sigma_y}{\beta_n^2\Sigma_n + \Sigma_y} + \Sigma_{\epsilon_n}} = \frac{\frac{\Sigma_n\Sigma_y\Sigma_{\epsilon_n}}{\beta_n^2\Sigma_n + \Sigma_y}}{\frac{\Sigma_n\Sigma_y}{\beta_n^2\Sigma_n + \Sigma_y} + \Sigma_{\epsilon_n}}$$

Simplifying, we have our equation for variance updating:

$$\Sigma_{n+1} = \frac{\Sigma_n\Sigma_y\Sigma_{\epsilon_n}}{\Sigma_n\Sigma_y + \beta_n^2\Sigma_n\Sigma_{\epsilon_n} + \Sigma_y\Sigma_{\epsilon_n}} \tag{A.7}$$

Price Updating

Next, we can solve for the market maker's updating up prices. We know

$$p_n = \mathbb{E}[v|\mathcal{F}_{n-1}^M, s_n, z_n]$$

Projecting v onto $s_n = v + \epsilon_n$, we have

$$p_n = \mathbb{E}[v|\mathcal{F}_{n-1}^M, z_n] + \frac{Cov(v, v+\epsilon_n|\mathcal{F}_{n-1}^M, z_n)}{Var(v+\epsilon_n|\mathcal{F}_{n-1}^M, z_n)}(v + \epsilon_n - \mathbb{E}[v+\epsilon_n|\mathcal{F}_{n-1}^M, z_n])$$

Since v and ϵ_n are independent, we have

$$p_n = \mathbb{E}[v|\mathcal{F}^M_{n-1}, z_n] + \frac{Var(v|\mathcal{F}_{n-1}, z_n)}{Var(v|\mathcal{F}^M_{n-1}, z_n) + \Sigma_{\epsilon_n}}(v + \epsilon_n - \mathbb{E}[v + \epsilon_n|\mathcal{F}^M_{n-1}, z_n])$$

Projecting v and $v + \epsilon_n$ onto z_n we can get

$$\begin{aligned} p_n = \mathbb{E}[v|\mathcal{F}^M_{n-1}] + \frac{Cov(v, \beta_n(v - p_{n-1}) + y_n|\mathcal{F}^M_{n-1})}{Var(\beta_n(v - p_{n-1}) + y_n|\mathcal{F}^M_{n-1})}(z_n - \mathbb{E}[z_n|\mathcal{F}^M_{n-1}]) \\ + \frac{Var(v|\mathcal{F}_{n-1}, z_n)}{Var(v|\mathcal{F}^M_{n-1}, z_n) + \Sigma_{\epsilon_n}}(v + \epsilon_n - \mathbb{E}[v|\mathcal{F}^M_{n-1}] \\ - \frac{Cov(v + \epsilon_n, \beta_n(v - p_{n-1}) + y_n|\mathcal{F}^M_{n-1})}{Var(\beta_n(v - p_{n-1}) + y_n|\mathcal{F}^M_{n-1})}(z_n - \mathbb{E}[z_n|\mathcal{F}^M_{n-1}]) \end{aligned}$$

Now noting that $\mathbb{E}[v|\mathcal{F}^M_{n-1}] = p_{n-1}$, $\mathbb{E}[z_n|\mathcal{F}_{n-1}] = 0$, and that

$$\frac{Var(v|\mathcal{F}_{n-1}, z_n)}{Var(v|\mathcal{F}^M_{n-1}, z_n) + \Sigma_{\epsilon_n}} = \frac{\frac{\Sigma_n \Sigma_y}{\beta_n^2 \Sigma_n + \Sigma_y}}{\frac{\Sigma_n \Sigma_y}{\beta_n^2 \Sigma_n + \Sigma_y} + \Sigma_{\epsilon_n}} = \frac{\Sigma_n \Sigma_y}{\Sigma_n \Sigma_y + \beta_n^2 \Sigma_n \Sigma_{\epsilon_n} + \Sigma_y \Sigma_{\epsilon_n}} = \psi_n$$

Calling the above ψ_n.

Now, we can write

$$p_n = p_{n-1} + \frac{\beta_n \Sigma_n}{\beta_n^2 \Sigma_n + \Sigma_y} z_n + \psi_n(v + \epsilon_n - p_{n-1} - \frac{\beta_n \Sigma_n}{\beta_n^2 \Sigma_n + \Sigma_y} z_n)$$

Denoting:

$$\lambda_n = \frac{\beta_n \Sigma_n}{\beta_n^2 \Sigma_n + \Sigma_y}$$

we have the market maker's price updating equation:

$$p_n = (1 - \psi_n)p_{n-1} + (1 - \psi_n)\lambda_n z_n + \psi_n(v + \epsilon_n) \tag{A.8}$$

which we can write as

$$p_n = (1 - \psi_n)(p_{n-1} + \lambda_n z_n) + \psi_n s_n \tag{A.9}$$

with ψ_n and λ_n as defined above.

A.3.2 Insider's Problem

Now suppose the market maker uses the pricing rule

$$p_n = (1 - \psi_n)(p_{n-1} + \lambda_n z_n) + \psi_n s_n$$

Furthermore, suppose that there exist two constants α_{n+1} and γ_{n+1} such that

$$\Pi_{n+1}(P, X) = \alpha_{n+1}(v - p_n)^2 + \gamma_{n+1}$$

That is, conjecture that the insider's payoff-to-go is quadratic in the price gap. Then the insider's expected payoffs in period n are given by (assume all expectations are conditional on insider's information at the beginning of period n):

$$\mathbb{E}[\Pi_n] = \max_{x_n} \quad \mathbb{E}[(v - p_n)x_n] + \alpha_{n+1}\mathbb{E}[(v - p_n)^2] + \gamma_{n+1}$$

Insider's Optimal Strategy

Plugging into the conjectured value function the pricing rule for p_n, and evaluating the first expectation, we have

$$\max_{x_n} \quad (1-\psi_n)(v - p_{n-1} - \lambda_n x_n)x_n$$
$$+ \alpha_{n+1}\mathbb{E}[(v - (1-\psi_n)p_{n-1} - (1-\psi_n)\lambda_n(x_n + y_n) - \psi_n s_n)^2] + \gamma_{n+1}$$

We can write this as

$$\max_{x_n} \quad (1-\psi_n)(v - p_{n-1} + \lambda_n x_n)x_n$$
$$+ \alpha_{n+1}(1-\psi_n)^2\mathbb{E}\left[(v - p_{n-1} - \lambda_n(x_n + y_n) - \frac{\psi_n}{(1-\psi_n)}\epsilon_n)^2\right] + \gamma_{n+1}$$

Evaluating the expectation gives our value function (for the verify portion of the proof):

$$\max_{x_n} \quad (1-\psi_n)(v - p_{n-1} - \lambda_n x_n)x_n$$
$$+ \alpha_{n+1}(1-\psi_n)^2\Bigg[v^2 + p_{n-1}^2 + \lambda_n^2 x_n^2 + \lambda_n^2\Sigma_y + \left(\frac{\psi_n}{(1-\psi_n)}\right)^2\Sigma_{\epsilon_n}$$
$$- 2vp_{n-1} - 2\lambda_n v x_n + 2\lambda_n p_{n-1}x_n\Bigg] + \gamma_{n+1} \quad \text{(A.10)}$$

Differentiating with respect to x_n gives the first order condition:

$$\frac{\partial V}{\partial x_n} = (1-\psi_n)(v - p_{n-1} - 2\lambda_n x_n) + \alpha_{n+1}(1-\psi_n)^2(2\lambda_n^2 x_n - 2\lambda_n v + 2\lambda_n p_{n-1}) = 0$$

Solving for x_n^* yields:

$$x_n^* = \frac{(1 - 2\alpha_{n+1}\lambda_n(1-\psi_n))}{2\lambda_n(1 - \alpha_{n+1}\lambda_n(1-\psi_n))}(v - p_{n-1}) \quad \text{(A.11)}$$

This implies that

$$\beta_n = \frac{(1 - 2\alpha_{n+1}\lambda_n(1 - \psi_n))}{2\lambda_n(1 - \alpha_{n+1}\lambda_n(1 - \psi_n))}$$

We also obtain the second order condition:

$$(1 - \psi_n)\lambda_n(1 - \alpha_{n+1}(1 - \psi_n)\lambda_n) > 0 \tag{A.12}$$

Verify The Value Function is Quadratic

Next, we need to plug x_n^* back into our value function to verify that it is indeed quadratic in the price gap.

We have that

$$x_n^* = \frac{(1 - 2\alpha_{n+1}\lambda_n(1 - \psi_n))}{2\lambda_n(1 - \alpha_{n+1}\lambda_n(1 - \psi_n))}(v - p_{n-1}) = \beta_n(v - p_{n-1})$$

The value function is

$$\begin{aligned}\mathbb{E}[\Pi_n] = (1 - \psi_n)(v - p_{n-1} - \lambda_n x_n^*)x_n^* \\ + \alpha_{n+1}(1 - \psi_n)^2 \Big[v^2 + p_{n-1}^2 + \lambda_n^2 x_n^2 + \lambda_n^2 \Sigma_y \\ + \left(\frac{\psi_n}{(1 - \psi_n)}\right)^2 \Sigma_{\epsilon_n} - 2vp_{n-1} - 2\lambda_n v x_n^* + 2\lambda_n p_{n-1} x_n^*\Big]\end{aligned}$$

Subbing in for $x_n^* = \beta_n(v - p_{n-1})$, we can write

$$\mathbb{E}[\Pi_n] = (1-\psi_n)\left[\beta_n(v-p_{n-1})^2 - \lambda_n\beta_n^2(v-p_{n-1})^2\right]$$

$$+ \alpha_{n+1}(1-\psi_n)^2\left[(v-p_{n-1})^2 + \beta_n^2\lambda_n^2(v-p_{n-1})^2 - 2\lambda_n\beta_n(v-p_{n-1})^2\right]$$

$$+ \alpha_{n+1}(1-\psi_n)^2\lambda_n^2\Sigma_y + \alpha_{n+1}\psi_n^2\Sigma_{\epsilon_n}$$

So clearly, the value function is quadratic in the price gap and the induction hypothesis is satisfied. We have

$$\gamma_n = \alpha_{n+1}(1-\psi_n)^2\lambda_n^2\Sigma_y + \alpha_{n+1}\psi_n^2\Sigma_{\epsilon_n} + \gamma_{n+1}$$

We also have

$$\alpha_n = (1-\psi_n)\beta_n(1-\lambda_n\beta_n) + \alpha_{n+1}(1-\psi_n)^2\left[(1-\lambda_n\beta_n)^2\right]$$

Since $\beta_n = \frac{(1-2\alpha_{n+1}\lambda_n(1-\psi_n))}{2\lambda_n(1-\alpha_{n+1}\lambda_n(1-\psi_n))}$, we can simplify this to

$$\alpha_n = \frac{(1-\psi_n)}{4\lambda_n(1-\alpha_{n+1}\lambda_n(1-\psi_n))}$$

QED.