

Essays on Dynamic Macroeconomics

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

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July, 2020

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Acknowledgements

I am deeply indebted to my advisor V.V. Chari for all his help, encouragement, and support. His insights have always improved my understanding of economics. I thank Chari for sharing with me his passion for research.

Special thanks to Larry Jones for being a great teacher. My gratitude is to Todd Schoellman for the three great years at the Federal Reserve Bank of Minneapolis. I thank Todd for treating me as a colleague rather than a research assistant at the Fed.

I am grateful to Mariacristina De Nardi for many hours, no matter how busy she was, she spent helping me with my work. I thank Erzo Luttmer, Anmol Bhandari, Tim Kehoe and Ellen McGrattan for many insightful conversations I had with them over the years.

Many of my friends have contributed to this work through various discussions and helpful comments. In particular, I would like to thank Ivan Pylnev, Carlos Esquivel, Konstantin Platonov and Salomón García. My special thanks go to Egor Malkov for being such a great friend for so many years.

Diana, you have been with me throughout this ride and that was the best part of it all. I am glad and grateful we made it such good years.

Most importantly, I want to thank my parents and my sister for their love and support. I dedicate this work to them.

Dedication

Моим родителям.

Abstract

This dissertation consists of three chapters which contribute to theoretical and quantitative understanding of business cycles.

The first essay studies firm dynamics over the business cycle. I present evidence from the United Kingdom that more rapidly growing firms are born in expansions than in recessions. Using administrative records from Census data, I find that this observation also holds for the last four recessions in the United States. I also present suggestive evidence that financial frictions play an important role in determining the types of firms that are born at different stages of the business cycle. I then develop a general equilibrium model in which firms choose their managers' span of control at birth. Firms that choose larger spans of control grow faster and eventually get to be larger, and in this sense have a larger target size. Financial frictions in the form of collateral constraints slow the rate at which firms reach their target size. It takes firms longer to get up to scale when collateral constraints tighten; therefore, businesses with the largest target size are affected disproportionately more. Thus, fewer entrepreneurs find it profitable to choose larger projects when financial conditions deteriorate. Using Bayesian methods, I estimate the model using micro and aggregate data from the United Kingdom. I find that financial shocks account for over 80% of fluctuations in the formation of businesses with a large target size, and TFP and labor wedge shocks account for the remaining 20%. An independently estimated version of the model with no choice over the span of control needs larger aggregate shocks in order to account for the same data series, suggesting that the intensive margin of business formation is important at business cycle frequencies. The model with the choice over the span of control generates an empirically relevant and non-targeted collapse in the right tail of the cumulative growth distribution among firms started in recessions, while the model without such a choice does not. The paper also discusses implications for micro-targeted government stimulus policies.

The second essay is based on joint work with Makoto Nakajima. We investigate cyclicity of variance and skewness of household labor income risk in a unified estimation framework using PSID data. We make five findings. First, we find that head's labor income exhibits

countercyclical variance and procyclical skewness. Second, cyclical volatility of hourly wage is muted, suggesting that head's labor income risk is mainly coming from volatility of hours. Third, the second earner lowers the cyclical volatility and skewness of labor income risk. Fourth, government taxes and transfers reduce fluctuations of income risk even further, making it nearly invariant with respect to business cycle. Finally, among heads with strong labor market attachment, cyclical volatility of labor income becomes weaker, while cyclical skewness remains.

The third essay explores the aggregate implications of countercyclical heteroscedasticity of TFP shocks. First, by drawing on data of publicly traded U.S. firms, I find that both the investment rate and the probability of investment spikes decrease as macroeconomic volatility rises, with this effect weakening as firms age. Second, to rationalize empirical patterns, I build an uncertainty regime-switching general equilibrium model of investment with capital adjustment costs and firms' entry and exit. In the model, the option value of waiting for younger businesses is larger than for mature firms, since the former are further away from their target size than the latter. As a result, young businesses respond stronger to heightened uncertainty as compared to older ones. Finally, in contrast to existing literature, I show that the entry/ exit margin can be quantitatively important, especially during high-uncertainty episodes such as the Great Recession.

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Chapter 1

Compositional Nature of Firm Growth and Aggregate Fluctuations

1.1 Introduction

In this paper, I study the business formation and growth of young enterprises over the business cycle. It is well known that the entry rate falls in recessions. This paper puts forward an idea that recessions particularly discourage the formation of rapidly growing enterprises, which results in relatively few rapidly growing and relatively more slowly growing new firms (*compositional effect*). I provide evidence for the compositional effect in the data by showing that fewer rapidly growing firms are started during economic downturns. I also provide suggestive evidence that rapidly growing firms are more financially constrained than slower growing enterprises. Therefore, access to financing can play an important role in determining which firm types are started at different stages of the business cycle. I then develop a general equilibrium model with financial frictions in which entrepreneurs choose the optimal size of their projects upon entry. I use the model to quantitatively assess the compositional effect and study the implications for the government stimulus policy.

Studying firm dynamics and business formation is important, since the behavior of young firms can have sizable aggregate implications. New enterprises constitute a small fraction of employment and fixed assets, but they contribute disproportionately to aggregate job creation and investment ([Decker, Haltiwanger, Jarmin and Miranda, 2014](#); [Gonzalez-Uribe](#)

and Paravisini, 2019).¹ In fact, young enterprises in the United States accounted for one-half of the overall decline in employment during the Great Recession (Sedláček and Sterk, 2017).² Therefore, a reduction in the number of rapidly growing firms during recessions can have a large impact on aggregate investment and employment growth. Moreover, provided that the compositional shift from rapidly growing to more slowly growing enterprises in a recession determines the subsequent growth of firms, the *intensive* margin of business formation can have a long-lasting impact on the economy and delay the recovery in the aftermath of economic crises.

I start by documenting a substantial heterogeneity in the subsequent growth rates of firms after they are born. I use firm-level data from the United Kingdom to show that the mean cumulative growth rate of firms that are born in expansions (*expansionary firms*) is larger than that of firms started in recessions (*recessionary firms*). I further show that the observed difference in mean cumulative growth rates is primarily driven by the collapse in the right tail of the cumulative growth rate distribution, whereas I find the left tail to be stable.

I also apply a clustering algorithm proposed by Bonhomme and Manresa (2015), which optimally assigns firms to groups so that businesses within each group share similar life cycle profiles. The advantage of this method is that it implements the optimal grouping, taking the effect of observables into account. Since the life cycles of firms can be affected by aggregate conditions, I implement the clustering algorithm controlling for sequences of aggregate shocks each firm went through over its tenure. I measure aggregate shocks as a cyclical component of GDP series from HP-filtering. I find that the mass of recessionary firms assigned to the group with the most rapidly growing businesses is significantly smaller compared with that of expansionary firms. I repeat this exercise controlling for both aggregate shocks and a measure of balance sheet strength—financial leverage—and find similar results.

My results for the United Kingdom are based on a firm-level accounting dataset, which covers only one recessionary episode (the 2008-2009 crisis). I additionally use administrative records from the U.S. Census Bureau’s Longitudinal Business Database (LBD) to show that each of the last four recessions in the United States saw a collapse in the right tail of firms’

¹As for the U.S. manufacturing sector, Eslava, Haltiwanger and Pinzon (2019) find that in the absence of entry, incumbents’ share of employment would have shrunk by 40% over the course of a decade.

²In the data, the growth rate of young firms fell stronger than that of mature enterprises during the last 3 aggregate contractions in the U.S. (Figure A.7). Thus, it is likely that the importance of young businesses for the aggregate job creation is not limited to the Great Recession.

growth distribution.³ The LBD is the most comprehensive source of information on U.S. businesses and spans the time period 1976-2016.⁴ The extensive coverage of the LBD allows me to document this finding across several recessions.

I then provide suggestive evidence that rapidly growing firms are more financially constrained relative to more slowly growing enterprises. I exploit a unique feature of the U.K. data—one can observe a measure of the residential property of firm owners⁵—to study the response of firm-level investment to idiosyncratic fluctuations in the residential real estate of their owners. I find that firms with high subsequent growth rates are more responsive to idiosyncratic fluctuations in housing wealth than more slowly growing enterprises. Following a large empirical literature on the collateral effects of firm financing (*e.g.*, [Chaney, Sraer and Thesmar, 2012](#); [Bahaj, Foulis and Pinter, 2018](#)), I interpret this finding as strong, suggestive evidence that fast growing firms are more financially constrained (as identified through the collateral channel) than slow growing businesses.⁶ This finding also implies that financial frictions may affect the type of firms that are born at different stages of the business cycle. I then build a general equilibrium model of firm dynamics over the business cycle, in which the behavior of incumbents is formalized similarly to that in [Khan and Thomas \(2013\)](#): heterogeneous firms need physical capital to produce, and they accumulate it subject to a collateralized borrowing constraint. I extend their model along several dimensions. First, I assume that firms differ in their managers’ spans of control. Firms that have larger spans of control grow faster and eventually get to be larger, and in this sense have a larger target size.⁷ The model’s assumption that the life cycle of projects is to a certain extent determined by an ex ante fixed component captures the idea that adjusting organizational capabilities is costly (see, for example, [Hannan and Freeman, 1984](#) and [Henderson and Clark, 1990](#)).⁸

³The list of recessions includes 1980-1982, 1990-1991, 2001-2002 and 2008-2009.

⁴The LBD covers all sectors of the U.S. economy and encompasses all businesses with at least one paid employee ([Jarmin and Miranda, 2002](#)).

⁵The data provide information on the residential property of firm *directors*—a group of people (or a single person) responsible for running and promoting the firm. However, over 70% of directors are shareholders in corresponding firms ([Bahaj, Foulis and Pinter, 2018](#)).

⁶[Davis and Haltiwanger \(2019\)](#) draw on the LBD and document that movements in local housing prices affect firm creation. This paper makes a related point using another measure of financial shocks: the value of the residential real estate of the firm’s directors.

⁷The idea that firms differ in their ex ante type is particularly attractive in light of a recent study by [Sedláček, Pugsley and Sterk \(2018\)](#) in which they decompose firms’ growth profiles into a fixed “business quality” component and a sequence of ex post shocks. This approach reveals that ex ante heterogeneity is an important determinant of firms’ life cycles. Firms grow large not only because of pure luck (a sequence of high demand, low cost shocks, and so on), but also, and more importantly, because the very idea behind the business plan was good.

⁸In the model, firms are subject to idiosyncratic productivity shocks. Therefore, firms’ life cycles are

Second, potential entrants can *target* their start-up attempts toward projects of different optimal size. This assumption allows my model to capture both margins of business formation. On the *extensive* margin, economic crises can reduce the number of entering firms (Clementi, Khan, Palazzo and Thomas, 2015; Clementi and Palazzo, 2016). On the *intensive* margin, relatively few firms with a large span of control can get started during recessions. The focus of this paper is on the latter channel (the compositional shift), but the model is designed to capture both channels. Thus, my model differs from classic industry dynamics frameworks (Jovanovic, 1982; Hopenhayn, 1992) in that entrepreneurs can *a priori* decide how large their businesses can be.⁹ The ability of entrepreneurs to control the target size of their businesses also finds support in survey data: a significant fraction of business owners have no intention to grow their business to a large size, and some of them start firms for non-pecuniary reasons (Hurst and Pugsley, 2011).

Motivated by the empirical evidence, I assume that firms are subject to financial frictions. In particular, firms can finance their investment expenditures using three sources: retained earnings, debt financing and equity issuance.¹⁰ Debt financing is subject to a collateral constraint, and issuing equity is costly. As I discuss in Section 1.3, both frictions are necessary to make firms financially constrained. Finally, my model has three aggregate stochastic processes: TFP, financial shocks and labor wedge shocks, which I estimate using Bayesian methods on 40 years of quarterly data from the United Kingdom.

I make several findings in this paper. First, I decompose fluctuations in the entry intensity of firm types over time into contributions of different aggregate shocks and find that the financial shock accounts for 82% of fluctuations in the entry intensity of fast growing businesses. Aggregate TFP and labor wedge shocks account for the remaining 20%. As for more slowly growing businesses, financial shock accounts for only 5% of fluctuations in the entry intensity of such firms. This finding suggests that rapidly growing businesses are

allowed to be different *within* types.

⁹Such a theoretical construct also allows me to break the tight link between size and age in classic models, where young firms are less productive and small at the beginning, but over time their productivity mean-reverts and makes them grow (Gavazza, Mongey and Violante, 2018). In the data, roughly half of 10-year-old firms hire less than 10 workers—a size close to 6 employees hired by a typical entrant (Karahan, Pugsley and Şahin, 2019).

¹⁰The typical assumption in firm dynamics models with financial frictions is that firms have to deliver a non-negative dividend stream (see, *e.g.*, Khan and Thomas, 2013; Ottonello and Winberry, 2018). In this paper, however, I relax this assumption because equity financing is non-negligible among young firms (Robb and Robinson, 2014). Moreover, small and old firms behave differently with respect to their capital structure over the business cycle (Begenau and Salomao, 2018): large and old firms substitute between debt and equity, whereas small firms (and young ones) have procyclical debt and equity financing policies.

particularly sensitive to aggregate financial conditions.

Second, my model can account for two-thirds of the collapse in the right tail of the cumulative growth distribution during the 2008-2009 recession, even though it was not directly targeted in the estimation process. To demonstrate that this feature of the data arises because of the intensive margin of firm entry, I develop and independently estimate a version of the model in which the composition of entrants is business cycle invariant. I show that such a model cannot account for the collapse in the right tail of the cumulative growth distribution.

Third, I find that the compositional effect is quantitatively large at business cycle frequencies. To this end, I confront the predictions of my baseline model and the model with no compositional effect during the Great Recession episode. I find that the model with no intensive margin of business formation explains 25% less of the drop in investment and hours observed during the crisis, suggesting that the model needs larger shocks to account for the same dynamics of aggregate economic series. The key reason is that adverse aggregate shocks affect rapidly growing firms disproportionately more than more slowly growing firms, precisely because it takes the former more time and resources to get up to scale than the latter. In the model, bad aggregate conditions discourage potential entrants from pursuing financially demanding projects, and they switch to less ambitious ideas. The relative entry of projects with a large target size declines and leads to a compositional shift, for which I find support in the data. Given that businesses with a high span of managerial control account for a significant share of aggregate investment and employment, a smaller fraction of such firms makes the recession deeper.¹¹ The model without an endogenous choice of project types does not capture this amplification effect, which forces the model to have larger shocks in order to account for the same aggregate dynamics. This finding, coupled with the result that formation of rapidly growing businesses is particularly sensitive to financial conditions, suggests a potentially important role for government stimulus policies.

Fourth, I argue that the amplification mechanism is relevant. To do so, I compare the predictions of the two models with respect to a key non-targeted series: the extensive margin of business formation (the entry rate). I show that the model with an intensive margin that is business cycle invariant predicts a counterfactually larger decline in the entry rate during the financial crisis, whereas the prediction of the baseline model is much closer to the data.

¹¹On a related matter, [Eslava, Haltiwanger and Pinzon \(2019\)](#) compare firm dynamism in the U.S. and Colombia, and find that in Colombia employment growth of an average firm is slower due to a less enthusiastic contribution of fast-growing firms.

Intuitively, this occurs because the benchmark model can deliver an empirically realistic fall in macroeconomic aggregates during downturns by reducing the entry intensity of rapidly growing firms, which is associated with a small overall decline in the entry rate (there are relatively few fast growing projects as compared with slow growing ones). The model without compositional effects induces an equal decline in entry of all project types, which translates into a larger fall in the entry rate overall.

Finally, I study the welfare consequences of government policies. The first case I consider is based on the assumption of full information on behalf of the government; it knows the assignment of firms across the types and targets the policy toward firms with the highest growth potential. The second case assumes that the government is agnostic about the firm-type assignment; therefore, it applies the policy to all firm types. The particular policy I consider takes the form of a reduction in the cost of entry at the expense of lower tax rebate to the household. I find that welfare benefits *(i)* are large in the first case (about 0.3% of lifetime consumption) and *(ii)* are negligible in the second case. This highlights the importance of the micro-targeted government policies as they help achieve welfare gains due to cost-efficiency.

Related literature This paper contributes to understanding the effect of aggregate economic conditions upon firms' inception on their subsequent growth. On a conceptual level, this paper is most closely related to the studies by [Moreira \(2016\)](#) and [Sedláček and Sterk \(2017\)](#), although they abstract from financial constraints. [Ouyang \(2009\)](#) studies the scarring and cleansing effects of recessions. In her model, entering businesses are uncertain about their productivity, and learn it over time. Economic downturns reduce the amount of time firms can stay in the market learning their productivity, which forces some high-quality businesses to exit as they cannot bear to learn as long as during good times. My paper extends the aforementioned studies by introducing financial frictions and studying how aggregate economic conditions affect the subsequent growth of firms. [Albert and Caggese \(2019\)](#) study a multi-country entrepreneurial survey and find that adverse financial shocks discourage the entry of fast growing businesses. My paper complements that study by quantifying the importance of the intensive margin of firm entry for the business cycle in a model with heterogeneous firms.

My paper contributes to the literature that attempts to measure the impact of access to

financing on entrepreneurship. I use a structural general equilibrium model to interpret the empirical evidence from the firm-level data, whereas the existing literature typically draws on the household-side data. Such literature indirectly infers how much potential and existing entrepreneurs would like to borrow and at which price. While some studies find evidence of borrowing constraints (see, *e.g.*, [Gentry and Hubbard, 2004](#) for an empirical investigation, and [Evans and Jovanovic, 1989](#); [Buera, 2006](#), and [Cagetti and De Nardi, 2006](#) for estimated structural models), others argue that liquidity constraints might not be such a strong force in discouraging business formation, at least in the United States (see [Hurst and Lusardi, 2004](#) for a notable example).

Finally, this paper contributes to the literature on financial shocks at business cycle frequencies. In particular, I develop a model in which firms are financially constrained and have *permanent* heterogeneity in the optimal size. To the best of my knowledge, my paper is the first to study the transmission of financial shocks in such a framework. In an influential study, [Jermann and Quadrini \(2012\)](#) find that standard real business cycle models cannot account for aggregate fluctuations in financial flows; thus, they highlight the importance of financial shocks. [Chari \(2014\)](#) argues that financial frictions in representative firm models—when brought to the data to match aggregate series of financial flows—will have no effect. The reason is that, in the aggregate, available funds for firms exceed aggregate investment expenditures. He suggests that any quantitatively successful model with financial frictions has to feature firm heterogeneity. This idea was further developed in a quantitative study by [Zetlin-Jones and Shourideh \(2017\)](#), which shows that financial shocks transmit through their impact on private—rather than public—firms. My paper, in turn, argues that firms with larger spans of control are more sensitive to aggregate financial conditions, and, therefore, financial shocks transmit through the response of the rapidly growing enterprises.¹²

The rest of the paper is organized as follows. Section 1.2 presents the empirical results. In Section 1.3, I lay out a heterogeneous agent model of firm dynamics with endogenous entry and financial frictions, which I bring to the data in Section 1.4. Section 1.5 provides the main results. Section 1.6 explores policy implications, and Section 1.7 concludes.

¹²I also find that a TFP shock affects all firm types proportionately, and therefore permanent heterogeneity in growth profiles does not play an important role for the propagation of aggregate productivity shocks. This is related to findings of [Clementi, Khan, Palazzo and Thomas \(2015\)](#) and [Smirnyagin \(2018\)](#) in that the effects of TFP shocks in rich heterogeneous firms and simplified models are similar.

1.2 Empirical Results

This section describes my empirical results. I first show that recessionary cohorts of firms grow slower than expansionary ones because of fewer fast growing enterprises. Second, I use fluctuations in firm directors’ residential wealth as an identified financial shock to argue that rapidly growing businesses are more financially constrained than slow growing ones.

1.2.1 Data

Here I provide a brief description of the data. Subsection 1.2.1 introduces the U.K. data, and subsection 1.2.1 - the U.S. data. Please refer to Appendix A.1 for a more detailed discussion of the data sources.

U.K. data

Firm-Level Data The key empirical results presented in this paper are based on a large panel dataset of firms’ financial accounts called Financial Analysis Made Easy (FAME), provided by the Bureau van Dijk (BvD).¹³ The data covers the corporate universe of U.K. firms for the period 1995-2017, and encompasses approximately 1.5mln private and public firms per year. The data includes both the firms’ balance sheet (assets and liabilities, debt structure, issued capital) and income statements (operating profit, turnover, cost of sales, etc.). Information on firms’ directors—a group of people (or a single person) responsible for running and promoting a firm—is also reported.

Real Estate Price Data Residential housing data comes from the Land Registry’s Price Paid dataset (covering England and Wales) and the Registers of Scotland. These datasets cover the universe of residential property transactions since 1995. BvD provides residential addresses of firms’ directors¹⁴, enabling one to measure the residential wealth of directors whose property was bought or sold at some point after 1995.¹⁵

¹³This dataset is different from commonly used Orbis and Amadeus in that it covers only U.K. registered firms. FAME is a live panel meaning that its information is accurate only at the moment of filing and not for historical reference. In order to improve the coverage, identify entry and exit of firms, multiple vintages of FAME have been combined - see [Bahaj, Foulis and Pinter \(2018\)](#) for a detailed explanation of this process.

¹⁴By UK law, directors must report several pieces of information (such as their name, date of birth and residential address among other things) when they register a firm. BvD contains this information.

¹⁵Strictly speaking, not all directors are complete owners of their houses. [Bahaj, Foulis and Pinter \(2018\)](#) made use of the Product Sale Database (PSD)—an administrative data on all residential mortgages since 2005 at origination and on the stock of outstanding mortgages in 2015. The PSD is provided by the UK Financial Conduct Authority. The FCA Product Sales Data include regulated mortgage contracts only, and

Sample Selection Standard cleaning procedures were applied to the raw data. Financial and real estate sectors (FIRE), as well as firms which do not comply with Companies' Act, were excluded. Outliers and observations for which the balance sheet identity did not hold were dropped. See Appendix A.1 for details as well as for summary statistics).

U.S. Data

My data source for the U.S. is the LBD, which is an administrative panel dataset that covers the universe of non-farm establishments in the U.S. private sector with at least one paid employee (Jarmin and Miranda, 2002). The unit of observation in the LBD is an establishment, which is defined as a single physical location where business is conducted.¹⁶ The LBD is a set of annual snapshots of the U.S. private sector (with establishment-level longitudinal identifiers); currently, the LBD includes annual observations from 1976 to 2016. I perform my analysis at the establishment-level in order to avoid difficulties associated with the firm-level data (*e.g.* firms operating several establishments in different locations and industries). However, given that my focus is on young businesses, few entering firms operate more than 1 establishment. Besides, establishment managers have a substantial independence in making hiring and investment decisions in the U.S. (Bloom, Sadun and Van Reenen, 2012). I nevertheless check that my results still hold at the firm level.

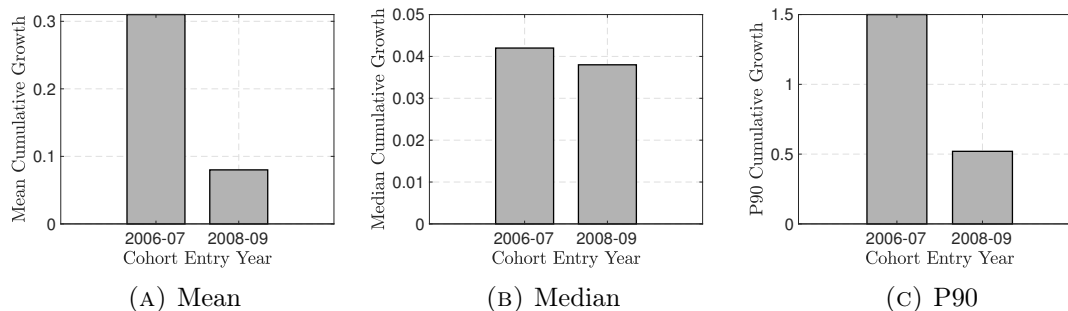
1.2.2 Compositional Effect of Recessions

In this subsection, I document that recessionary cohorts are growing slower than expansionary ones, and this is driven by the feature that there are fewer rapidly growing firms started in recessions.

therefore exclude other regulated home finance products such as home purchase plans and home reversions, and unregulated products such as second charge lending and buy-to-let mortgages. PSD contains information on the borrower's date of birth and the mortgaged property's full postcode. Therefore, by linking directors' home values with mortgage data, it is possible to construct a measure of *home equity*. They find that results remain largely unaffected when residential *equity* is used instead of residential real estate. This is not surprising given that approximately 90% of directors are homeowners.

¹⁶It is possible to aggregate the establishment-level information to the firm-level, by using appropriate firm identifiers. The aggregation process is associated with several well-known issues (a new firm identifier emerges in the LBD if, for example, a firm merges with another firm). See Appendix A.1 for a discussion on how to aggregate data to firm-level in a way robust to ownership and control changes.

FIGURE 1.1: MEAN, MEDIAN AND TOP DECILE OF CUMULATIVE GROWTH OVER THE FIRST 5 YEARS OF TENURE BY YEAR OF FIRM BIRTH



Notes: Figure 1.1 contains 3 panels. Panel A plots the average cumulative growth, Panel B—the median cumulative growth, and Panel C—the top decile of cumulative growth. Cumulative growth is in terms of employment, and is defined as $\log\left(\frac{y_{i5}}{y_{i1}}\right)$, where y_{i5} and y_{i1} are numbers of employees at firm i at ages 1 and 5, respectively. Source: BvD.

Evidence from the U.K.

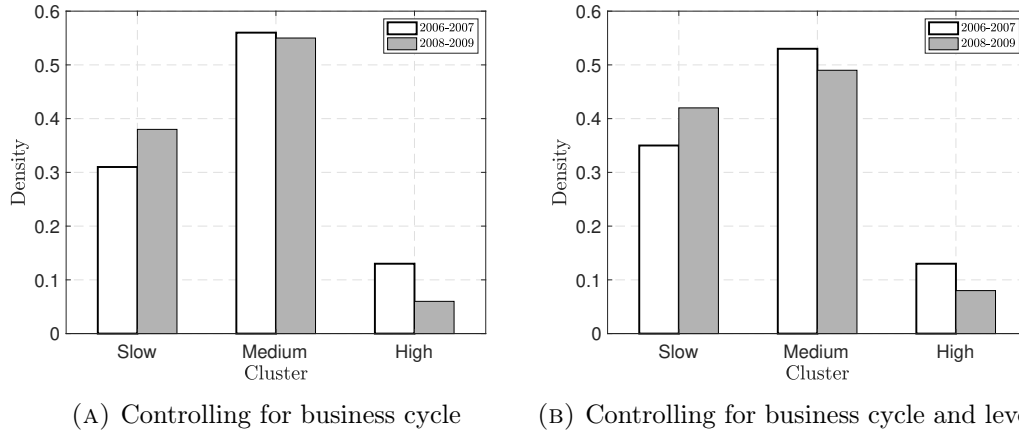
The U.K. economy went through the Great Recession in 2008-2009, and was expanding in 2006-2007. Figure 1.1 plots the mean, the median and the top decile of the cumulative growth distribution for expansionary (born in 2006-2007) and recessionary (born in 2008-2009) cohorts. According to Panel A, expansionary firms grew on average by 35% between ages 1 and 5, and recessionary—by only 8%.

The observed difference in the average growth between cohorts is mainly driven by the right tail of the cumulative growth distribution. In particular, Panel B shows that the median firm in recessionary and expansionary cohorts grew at a similar rate of 4%. However, the 90th percentile appears to be strikingly different (Panel C): the top decile of firms born in the expansion grew 3 times faster over the course of 5 years, as compared to the top 10% of businesses entered in the recession.

Disentangling Aggregate Effects and Firms’ Growth Profiles Overall, the data shows that recessionary cohorts grow slower than expansionary ones because of fewer rapidly growing firms. One potential concern with Figure 1.1 is that it ignores aggregate effects: the fast growth of expansionary firms can be fueled by favorable aggregate conditions.

In order to assuage this concern, I apply a *grouped fixed effects* clustering algorithm developed by [Bonhomme and Manresa \(2015\)](#). The benefit of this approach is that it optimally assigns firms into the pre-specified number of groups based on how similar firms’ time profiles

FIGURE 1.2: BONHOMME AND MANRESA (2015) GROUPED FIXED EFFECTS ESTIMATION



Notes: Figure 1.2 plots the distribution of firms across clusters based on the grouped fixed effects estimator by Bonhomme and Manresa (2015). White bars correspond to the firms started in 2006-2007, and gray ones—to the firms which entered in 2008-2009. The following linear model is considered:

$$y_{ia} = \alpha_{g_i a} + x'_{ia} \theta + \varepsilon_{ia}, \quad i = 1, \dots, N, \quad a = \overline{1, 8},$$

where y_{ia} is the real book value of assets of firm i of age a , and x_{ia} is a vector of control variables. Panel A plots the resulting distribution of firms across clusters when x_{ia} contains the cyclical component of GDP from the HP filter with smoothing parameter 100 (annual frequency). The vector of controls in Panel B also includes the liability-based leverage of firms (ratio of total liabilities to total assets). Clustering was performed on a 20% random subsample of manufacturing firms, which I tracked for up to 8 years. The choice of age 8 as an upper bound was dictated by the panel's duration: this is the maximal observable age of firms born in 2009. The exercise was repeated numerous times in order to ensure that the results are robust to different draws. Source: BvD.

of observables look like after the effect of controls has been taken out. By applying this estimator to the firm-level data, I can assign firms into growth types (fast, medium and slow) *controlling* for the sequence of aggregate shocks each firm went through.

I experiment with 2 different sets of controls. The first one includes only the cyclical (from HP filtering) component of GDP in order to control for the business cycle (Panel A in Figure 1.2). The second one adds a leverage ratio¹⁷ as a proxy for firms' financial conditions (Panel B). In both cases, I obtain broadly similar results: the mass of rapidly growing firms is roughly 40% smaller in recessionary cohorts, which is reflected in a relatively large mass of slow types.¹⁸

¹⁷I use a liability-based leverage: a ratio of total liabilities to total assets.

¹⁸In Appendix A.1.4, I report the average growth within each firm type. For example, "high" type firms grew 5 times faster between ages 1 and 5 than "slow" type businesses.

1.2.3 Evidence from the U.S.

The results sourced from the U.K. data are based on one recessionary episode—the financial crisis of 2008-2009, which was particularly deep and prolonged. In this subsection, I show that this feature of the data—the right tail of the cumulative growth distribution is lower among firms entered during economic downturns—has occurred during the last 4 recessions in the U.S.

I rely on the administrative data from the LBD—a comprehensive dataset on the U.S. private sector businesses housed by the U.S. Census Bureau. According to Figure 1.3, the left tail of the cumulative growth distribution in the U.S.—defined as the difference between the median and the bottom 10th percentile—is very stable across the entire sample. At the same time, the right tail (difference between the 90th and 50th percentiles) fluctuates over time, and falls during the NBER recessions. Consistent with the U.K. evidence, the Great Recession caused a large collapse in the right tail, potentially reflecting a financial nature of that economic crisis.¹⁹

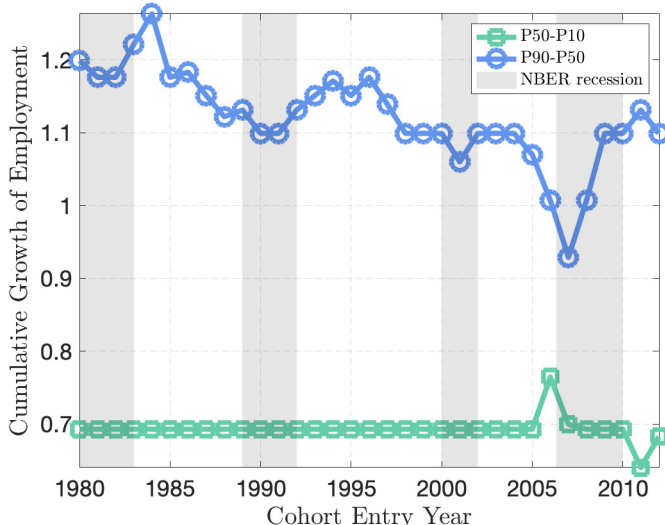
In order to study the impact of initial aggregate conditions on the subsequent growth of firms, I fit several linear probability models in Appendix A.3. In particular, I estimate the effect of being born in a recession on the probability for the establishment to exhibit a 5-year cumulative growth rate above the 10th (50th and 90th) percentile of the corresponding distribution. I find that the fact that an establishment was started during the NBER recession has no effect on the odds of having a cumulative growth rate above the 10th percentile (Table A.4). At the same time, the adverse aggregate conditions upon inception reduce the probability of the cumulative growth rate to exceed the 50- and 90th percentiles by 2pp and 5pp, respectively (Tables A.5 and A.6). The results are robust to the inclusion of a rich set of controls, such as industry, region and type of operation fixed effects.

1.2.4 Heterogeneous Response to Collateral Shocks

This section studies the investment response of fast and slow growing firms to fluctuations in the residential real estate of their directors. Following a large empirical literature ([Chaney](#),

¹⁹These patterns are unlikely to be driven by an more intensive exit of firms started in recessions. I calculate the survival rates at 3-, 5- and 7-year horizons and find them to fluctuate very little over time (Figure A.6).

FIGURE 1.3: TOP AND BOTTOM TAILS OF CUMULATIVE GROWTH DISTRIBUTION IN THE U.S.



Notes: Figure 1.3 plots 2 lines. The blue line with circle markers is the right tail of the cumulative growth distribution. The green line with square markers is the left tail of the cumulative growth distribution. The right tail is the difference between the 90th and 50th percentiles of the cumulative growth distribution among establishments born in a specific year. Analogously, the left tail is the difference between the 50th and 10th percentiles. Cumulative growth of each establishment is measured as a log change in employment between ages 1 and 5. The vertical gray bars represent NBER recession dates. Data source: LBD.

Sraer and Thesmar, 2012; Bahaj, Foulis and Pinter, 2018 among others), I interpret my findings as strong, suggestive evidence of an extent to which different firm types are financially constrained. I first describe the classification of firms into growth types, and subsequently lay out my empirical strategy.

Assignment of Firms to Growth Groups I follow a parsimonious approach and group firms based on how their growth rate of total assets—a uniformly reported measure of size—relates to the growth rates of their peers (same industry firms born in the same year). I assign a firm to the “fast” type if it grew faster than its median peer in at least half of the years of its tenure.²⁰ I assign a firm to the “slow” type if it grew slower than its median peer in at least half of the years of its tenure. In Appendix A.1, I run several robustness checks with respect to this classification: in particular, I redefine the thresholds (40- and

²⁰Technically, I compute median growth rates of total assets in 3 dimensional cells over age, 2-digit SIC code and year. Subsequently, for each firm, I calculate the number of years it grew faster than its peers, and divide it by the total number of years this firm was observed in the panel. I classify a firm into “fast” type if the resulting fraction exceeds 0.5. Similarly, I assign a firm to the “slow” type if it grew slower than its median peer in at least 50% of the periods in which it was observed in the sample.

60th percentiles instead of the median), increase the number of years a firm has to be above (below) its median peer in order to be classified as a “fast” (“slow”) type, and drop firms which did not survive through age 5.

Empirical Strategy Once I obtain the assignment of firms into groups, I study the response of investment of different firm types to residential collateral shocks. Following Bahaj, Foulis and Pinter (2018), instead of measuring residential property of firm directors in every year independently, the real estate of each firm’s director is fixed at its 2002 level, and subsequently rolled forward based on the local housing price index. This approach is advantageous as it isolates fluctuations in residential wealth from potentially endogenous decisions of directors to move into bigger/smaller houses depending on the performance of their firms. Therefore, throughout this section, directors’ residential real estate for firm i at time t is measured as:

$$Residential\ RE_{it} = |D_i| \frac{\sum_{d \in \tilde{D}_i} L_{i,2002}^d L_{h_d,t}^P}{|\tilde{D}_i|}, \quad (1.1)$$

where $L_{i,2002}^d$ is the estimated value of a house where the director d working at firm i lived in 2002, and $L_{h_d,t}^P$ is the house price index of the region h_d where that director lived in 2002 (with a normalization $L_{h_d,2002}^P = 1$). According to Equation (1.1), the residential real estate for firm i is the average value of property across matched directors \tilde{D}_i , multiplied by the total number of directors D_i .²¹ The benchmark specification takes the following form:

$$Investment_{it} = \alpha_i + \delta_{kt} + \mu_{lt} + \sum_{j \in \mathbf{J}} \eta_j \times \mathbf{1}_{\{i \in j\}} \times Residential\ RE_{it} + \gamma \times controls_{it} + \varepsilon_{it}, \quad (1.2)$$

where $Investment$ is the change in fixed assets plus depreciation, and $\mathbf{J} = \{\text{slow}, \text{fast}\}$. α_i , δ_{kt} and μ_{lt} are firm, region-time and industry-time fixed effects. Indicator function $\mathbf{1}_{\{i \in j\}}$ takes a value of 1 if a firm i was assigned to group j , and 0 otherwise. Standard errors are clustered at the level of the firm’s region. All monetary variables are scaled by lagged fixed assets²², which provides a pound-to-pound interpretation of the coefficients. Thus, coefficients of interest $\{\eta_j\}_{j \in \mathbf{J}}$ show by how many pounds investment of type j firms will change if their directors’ residential wealth appreciates by £1.

²¹As described in Bahaj, Foulis and Pinter (2018), the residential property of roughly 60% of directors was successfully valued.

²²Chaney, Sraer and Thesmar (2012) also use lagged fixed assets as a scaling variable.

Identification Before getting to the results, it is worth discussing some potential endogeneity concerns. The firm fixed effect α_i absorbs any time-invariant omitted factors which affect firm’s behavior. The list of such factors includes the initial values of directors’ homes $L_{i,2002}^d$, as well as the number and composition of directors in 2002. It is also the case that $L_{h_d,t}^P$ —the house price index for each director’s region—is typically correlated with the firm’s real estate price index $L_{j,t}^P$. In turn, $L_{j,t}^P$ could affect the firm’s investment opportunities; for example, by way of fueling local consumption (Mian and Sufi, 2014). I include region-time fixed effects δ_{kt} in order to address this. Following Chaney, Sraer and Thesmar (2012), the vector of controls includes firm-level characteristics: a measure of the balance sheet strength (financial leverage), and a measure of cash flow (operating profits). As Bahaj, Foulis and Pinter (2018) point out, residential real estate does not naturally scale up with the firm’s size as, for instance, corporate real estate would; therefore, the vector of controls further includes the inverse of lagged fixed assets in order to eliminate any spurious size effects.

Results Table A.7 reports my baseline results. It shows that the investment of rapidly growing firms is more responsive to idiosyncratic fluctuations in directors’ real estate than that of more slowly growing businesses. In particular, according to the tightest specification considered in Column (8), a £1 appreciation of directors’ residential housing is associated with a 1.3p (pence) increase in a fast type firms’ investment, and with only 0.6p for the slow type enterprises. This suggests that fast growing firms are more financially constrained than slow growing firms since the collateral shock leads to a twice bigger investment response among the former group.

Table A.8 splits the data into young (under the age of 5) and old subsamples (Columns 2 and 3), and shows that young businesses respond stronger to collateral shocks as compared to more mature enterprises (point estimates are now 1.4p and 1p for fast and slow types, respectively). This is consistent with an idea that young businesses are usually below their target size, grow fast, and exhibit high investment demand. Furthermore, Columns (4) and (5) in Table A.8 show that larger firms (≥ 50 employees) respond stronger to fluctuations in residential wealth, potentially reflecting the higher investment expenditures such businesses need to undertake.²³

Arguably, one needs to observe a firm long enough in order to properly classify it. For

²³The subsamples with respect to size are constructed based on the time-average of employment within each firm. Therefore, a firm might be in the “large” subsample but be small (< 50 employees) at some point.

that reason, Table A.9 reports the estimates of the baseline specification for firms which survive through age 5. I find that results are barely affected. Finally, in Table A.10, I check how robust estimates are to alternative groupings of firms. I find that the estimates are largely unaffected when the requirement to spend half of tenure above (below) the median peer is increased to 75% (Column 2), or when the threshold is shifted from the median to 40th percentile for the slow type and the the 60th percentile for the fast type (Column 3). Finally, I re-estimate the model on the largest possible sample with no controls and total asset growth as a dependent variable (Column 4), and qualitatively confirm my baseline results.

1.2.5 Taking Stock

The first part of the critical empirical evidence presented in this section established that recessionary cohorts of firms grow slower than expansionary ones. In particular, I documented a collapse of the right tail of cumulative growth distribution among firms born during recessions, which implies that there are fewer rapidly growing firms started during economic contractions.

The second part of this section studied the investment response of fast and slow growing businesses to idiosyncratic fluctuations in the residential wealth of their owners. I found that rapidly growing enterprises respond stronger to collateral shocks than slower growing businesses, which suggests that the former are more financially constrained than the latter. Next, I build a structural model of firm dynamics which is useful to interpret these empirical findings.

1.3 Model

In this section, I develop a heterogeneous firms real business cycle model with financial frictions. The model builds on [Khan and Thomas \(2013\)](#) and extends it along several meaningful dimensions. First, since the focus of this paper is on business formation, it is important to have a realistic description of young firms' financing decisions. In particular, in the model I allow firms to issue equity—an important source of funding for young firms ([Robb and Robinson, 2014](#)). Second, I introduce heterogeneity in firms' growth profiles. This feature of the model will allow me to study how and why the composition of startups changes

over the cycle, and to quantify the aggregate implications of such compositional shifts. Finally, potential entrants will get to *choose* the project type upon entry, and, therefore, determine the target long-run size of their businesses. This modeling choice differentiates this paper from classic papers on industry dynamics (Jovanovic, 1982; Hopenhayn, 1992). The entry mechanism in this paper adapts tools from the search literature, and is reminiscent of that in Sedláček and Sterk (2017).

1.3.1 Environment

Time in the model is discrete and runs forever: $t = 0, 1, \dots$. The economy is populated by 4 types of agents: potential entrepreneurs, incumbent firms, a government, and a representative household. Next, I describe the physical environment.

There is a finite number of projects types $J \in \mathbb{N} \setminus \{0, 1\}$, each of them indexed by a parameter μ_j .²⁴ This parameter characterizes the optimal size of the firm at the steady-state, which is formally incorporated into the firms' production function as a Lucas (1978) span of control:

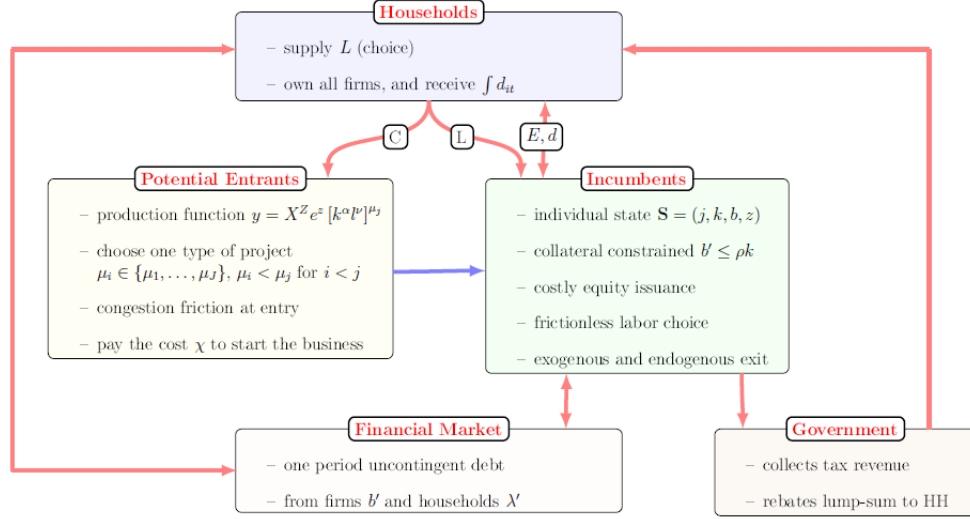
$$y_{ijt} = Z_t e^{z_{it}} [k_{ijt}^\alpha n_{ijt}^\nu]^{\mu_j}, \quad (1.3)$$

where k_{ijt} and n_{ijt} denote capital and labor inputs, and y_{ijt} stays for the output of type- j firm i at time t . The permanent firm-level heterogeneity captured by μ_j will allow me to study compositional shifts in business formation over the cycle. The assumption that the life cycle of projects is determined by an ex ante fixed component captures the idea that organizational capabilities are costly to adjust (Hannan and Freeman, 1984; Henderson and Clark, 1990). Parameters $\{\mu_j\}$ will be estimated in Section 1.4 to generate a realistic firm-size distribution.

Parameters α and ν are assumed to be strictly positive, with their sum being less than 1, $\alpha + \nu < 1$. The decreasing returns technology implies that every firm has a finite target size. The production function (1.3) is scaled by 2 factors: a time-varying idiosyncratic productivity z_{it} and an aggregate TFP Z_t . I postpone the description of aggregate shocks until Subsection 1.4.4, and at this point only specify the process for idiosyncratic productivity, which is

²⁴Hereafter, \mathbf{J} will denote a set $\{1, \dots, J\}$.

FIGURE 1.4: STRUCTURE OF THE MODEL



assumed to be an AR(1):

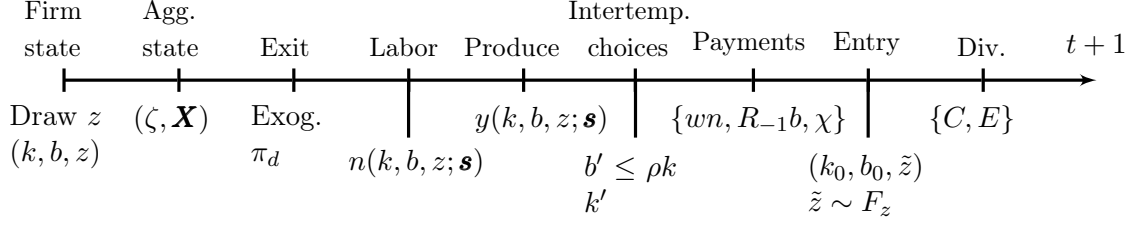
$$z_{it} = \rho_z z_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N}(0, \sigma_z^2), \quad (1.4)$$

where ε_{it} is i.i.d. across time and space.

All firms are owned by a representative household, and their objective is to maximize the discounted stream of dividends. In order to finance investment expenses, firms can use internal and external funds, each of them subject to a friction. On one hand, firms can finance investment by way of reducing current payments to shareholders, or may even raise equity if necessary (in this case, firms bear additional costs). On the other hand, firms can also borrow external funds from a competitive financial intermediary; this channel is also subject to a friction. Following a wide body of literature (Khan and Thomas, 2013; Zetlin-Jones and Shourideh, 2017 among others), I assume that the amount of external borrowing is limited by a fraction of the firms' installed capital—their collateral. I provide a formal description of firms' problem in Subsection 1.3.2. Both frictions are necessary in equilibrium: without the collateral constraint, firms can obtain any level of capital by issuing debt. With zero equity issuance cost, firms can instantaneously get to the optimal size by way of financing their investment through equity issuance—even if the collateral constraint binds.

Every period the economy is confronted with a large mass of ex-ante identical potential

FIGURE 1.5: TIMELINE OF THE MODEL



entrants. In order to make an entry attempt, they have to pay a cost χ denominated in units of the final good. They subsequently endogenously choose a project type μ_j . The entry process is subject to a coordination friction, meaning that some entry attempts will be unsuccessful. Successful entrants are endowed with an exogenous amount of capital k_0 and debt b_0 . They draw their initial idiosyncratic productivity z from a distribution $F(z)$. Subsection 1.3.3 describes the entry process in full detail.

The tax authority (government) levies a tax on firms' operating profits and rebates the proceedings back to households in a lump-sum manner. Figure 1.4 graphically shows the environment of the model.

The timing of the events within a period is as follows:

1. the aggregate state is realized;
2. each incumbent firm observes the realization of an exit shock, which is a Bernoulli random variable with parameter π_d . Firms that received the shock have to exit the economy at the end of the period, after the production stage takes place. Other firms may continue into the next period;
3. the production stage takes place: firms choose the optimal labor input and produce. At this point, firms which received an exit shock leave the economy. The rest of firms make intertemporal decisions k' and b' ;
4. the influx of new firms (successful potential entrants) enters the economy;
5. representative household consumes.

The verbal description of events is summarized in Figure 1.5. Next, I provide the recursive formulations of the individual optimization problems.

1.3.2 Incumbent Firms

The state vector of the incumbent firm contains four elements: physical capital k , financial position b , idiosyncratic productivity z , and its type j . Since the model features aggregate uncertainty, the state vector \mathbf{s} includes both the aggregate shocks \mathbf{X} and the distribution of firms over the state space ζ . Therefore, the aggregate state vector is $\mathbf{s} = (\zeta, \mathbf{X})$. In order to streamline the exposition, I postpone the discussion of aggregate shocks until Subsection 1.4.4.

At the start of the period, each firm first learns the realization of an exit shock, which is i.i.d. across time and space. The value of the firm at the beginning of the period and prior to the realization of an exit shock is:

$$v_j^0(k, b, z; \mathbf{s}) = \pi_d v_j^1(k, b, z; \mathbf{s}) + (1 - \pi_d) v_j^2(k, b, z; \mathbf{s}), \quad (1.5)$$

where $v_j^1(\cdot)$ and $v_j^2(\cdot)$ are the values the firm attains, conditional on the realization of an exit shock.

Firms can use several sources to finance their investment expenditures. First, businesses can use retained earnings Π —funds left after they sell output, pay the wagebill and the interest on their outstanding debt. Second, they can use debt financing and borrow b' (throughout the paper it is understood that $b' > 0$ means borrowing, and $b' < 0$ corresponds to savings). Finally, firms can raise funds by issuing equity E . Negative values of E correspond to the case when the firm pays out dividends, while positive values are interpreted as equity issuance. The firm's budget constraint takes the following form:

$$i_j(k, b, z; \mathbf{s}) = \Pi_j(k, b, z; \mathbf{s}) + E_j(k, b, z; \mathbf{s}) + b'_j(k, b, z; \mathbf{s}) - b, \quad (1.6)$$

where subscript j denotes the firm's "type", and the left-hand side variable $i_j(\cdot)$ is the firm's investment choice. Operating profit $\Pi_j(\cdot)$ is formally defined as:

$$\Pi_j(k, b, z; \mathbf{s}) = \max_{n \in \mathbb{R}_+} (1 - \tau) [y(k, n, z; \mathbf{s}) - wn - (R(\mathbf{s}_{-1}) - 1) b] + \tau \delta k, \quad (1.7)$$

where $R(\mathbf{s}_{-1})$ is a gross interest rate from the preceding period and τ is the tax rate. Physical

capital accumulation process takes the form:

$$k'_j(k, b, z; \mathbf{s}) = (1 - \delta)k + i_j(k, b, z; \mathbf{s}) - \Phi(k'_j, k, b, z; \mathbf{s}). \quad (1.8)$$

Function $\Phi(\cdot)$ in Equation (1.8) denotes the capital adjustment costs. I introduce these costs in order to match important moments of the investment rate distribution. I assume that function $\Phi(\cdot)$ takes a standard quadratic form:

$$\Phi(k'_j, k, b, z; \mathbf{s}) = \frac{\varphi^K}{2} \left(\frac{k'_j(k, b, z; \mathbf{s})}{k} - 1 \right)^2 k, \quad (1.9)$$

where parameter φ^K governs the extent to which adjustment costs prevail in the model.

It is instructive to continue the exposition of the model with the problem of a firm which did not receive an exit shock. Such firm is allowed to continue into the next period after choosing a new level of capital k' and issuing a new level of debt b' . The recursive formulation of a continuing firm's problem is:

$$v_j^2(k, b, z; \mathbf{s}) = \max_{k', b'} -E_j(k, b, z; \mathbf{s}) - C(E_j(k, b, z; \mathbf{s})) + \mathbb{E} [d(\mathbf{s}'|\mathbf{s})v_j^0(k', b', z'|\mathbf{s}')] \quad (1.10)$$

$$i_j(k, b, z; \mathbf{s}) = \Pi_j(k, b, z; \mathbf{s}) + E_j(k, b, z; \mathbf{s}) + b'_j(k, b, z; \mathbf{s}) - b,$$

$$b'_j(k, b, z; \mathbf{s}) \leq \rho k, \quad (1.11)$$

$$\mathbf{s}' \sim \Gamma(\mathbf{s}),$$

where the first constraint is the budget constraint, the second is the collateral constraint, and the third is the law of motion for the aggregate state.

Function $C(\cdot)$ in Equation (1.10) captures the equity issuance cost. Parameter ρ in the collateral constraint governs the tightness of financial frictions in the economy: high values of ρ allow firms to pledge a larger fraction of currently installed physical capital, and, therefore, borrow more. Conversely, with smaller values of ρ , firms need larger amounts of capital in order to be able to borrow the same.

Since all firms in this economy belong to the household, the future stream of dividends in Equation (1.10) is priced according to the stochastic discount factor $d(\mathbf{s}'|\mathbf{s})$, which converts the value of future resources in terms of current ones.

Now I turn to the problem of a firm which is forced to exit at the end of the period. Such

a firm produces in the current period, sells its undepreciated capital, pays the outstanding debt, and leaves the economy. Exiting firms do not make any intertemporal choices, their value is then simply given by:

$$v_j^1(k, b, z; \mathbf{s}) = \Pi_j(k, b, z; \mathbf{s}) + (1 - \delta)k - b, \quad (1.12)$$

where $\Pi_j(\cdot)$ is defined as in Equation (1.7).

1.3.3 Entry

This paper departs from a wide body of literature on firm dynamics in that potential entrants can *choose* a type of business opportunity upon entry. To operationalize this, I introduce a coordination friction which is reminiscent of the literature on directed search, and is formulated similarly to [Sedláček and Sterk \(2017\)](#).

Let $\{\psi^j\}_{j \in \mathbf{J}}$ be a distribution of available to potential entrants business opportunities of types \mathbf{J} in period t . In order to enter, an aspiring potential entrant has to pay a cost χ denominated in units of the final good. This entrance cost is designed to capture expenditures associated with market research, developing a business plan and alike. Upon paying the cost, a potential entrant gets to choose one of projects $j \in \mathbf{J}$, and, subsequently, has a chance to seize one of the available ideas ψ^j . Therefore, parameters $\{\psi^j\}$ can be thought of governing the relative probability of success of starting different project types.

Given a coordination friction, not all business opportunities are seized, while others are seized by several aspiring entrepreneurs. This friction is modeled by a matching function which returns the mass of successful entrants of type j , m_t^j :

$$m_t^j = \left(e_t^j\right)^\phi \left(\psi^j\right)^{1-\phi}, \quad (1.13)$$

where e_t^j is the mass of potential entrants who decided to pursue a project of type j , and $\phi \in (0, 1)$ is an elasticity of successful entrants with respect to startup attempts.

While the equilibrium of the model will be laid out later, it is convenient to formulate one equilibrium condition here. Potential entrants are indifferent with respect to which project to start in equilibrium; this consideration gives rise to a set of free-entry conditions (one for each type). The free-entry condition states that the cost of starting a business has to match

the associated expected benefit:

$$\underbrace{\chi}_{\text{cost of entry}} = \underbrace{\frac{m_t^j}{c_t^j}}_{\text{success probability}} \underbrace{\int_z v_j^0(k_0, b_0, z; \mathbf{s}) dF(z)}_{\text{exp. value of type-}j \text{ project}} \quad \forall j \in \mathbf{J} \quad z \sim F(z), \quad (1.14)$$

where the ratio on the right-hand side is a success probability of starting a project of type j .

Business cycle fluctuations (shifts in elements of \mathbf{s}) trigger a change in the expected benefit of starting and running projects; as discussed above, different project types can be affected differentially. But the free-entry condition (1.14) ensures that the success probability of projects with a relatively high value adjusts correspondingly (downwards), so that the right-hand side of (1.14) remains unchanged.

By way of combining the matching friction (1.13) with free-entry conditions (1.14), and defining $\tilde{v}_j^0(k_0, b_0; \mathbf{s}) := \int_z v_j^0(k_0, b_0, z; \mathbf{s}) dF(z)$, one can derive the following equation for the mass of successful entrants of each type:

$$m_t^j = \chi^{\frac{\phi}{\phi-1}} [\tilde{v}_j^0(k_0, b_0; \mathbf{s})]^{\frac{\phi}{1-\phi}} \psi^j \quad \forall j \in \mathbf{J}. \quad (1.15)$$

The right-hand side of Equation (1.15) contains a product of 3 terms. Given that $\phi < 1$, larger entry costs χ reduce the mass of entrants m_t^j . Furthermore, an increase in the value $\tilde{v}_j^0(\cdot)$ and/or project availability ψ^j stimulates the entry of type- j projects. Critically, Equation (1.15) shows how time-varying aggregate conditions affect entry decisions of potential entrepreneurs; it is also clear that if the value \tilde{v}_j^0 responds differently across the firm types to the same aggregate shock, one should expect a differential change in the intensity of entry of those firm types.²⁵

²⁵My model does not capture “waiting” decisions of potential entrepreneurs; in other words, they do not postpone entry decisions for later when aggregate conditions deteriorate. In order to introduce such a mechanism, one needs to study business formation through the lens of occupation choice models, and this is beyond the scope of this paper. However, based on the evidence from the Business Formation Statistics (BFS)—a new dataset developed by the U.S. Census—the waiting time to form businesses declined during the Great Recession in the U.S. (see Figure A.4 in Appendix A.2). This implies that, if anything, the financial crisis did not generate a pronounced delayed entry effect.

1.3.4 Households

The economy is populated by a unit mass of identical households. Each household consumes, supplies labor, and saves into a risk-free bond λ' and firms' shares $\omega(k, b, z, j)$. The price of current shares is $\rho_0(\cdot)$, and the purchase price of new shares is $\rho_1(\cdot)$. The net risk-free interest rate is $q_0(\mathbf{s})^{-1} - 1$. The household's dynamic programming problem is:

$$W(\omega, \lambda; \mathbf{s}) = \max_{c, n, \lambda', \omega'} [U(c, n) + \beta \mathbb{E}W(\omega', \lambda'; \mathbf{s}')] \quad (1.16)$$

subject to the budget constraint and the law of motion for the aggregate state:

$$c + q_0(\mathbf{s})\lambda' + \int_{K \times B \times Z \times \mathbf{J}} \rho_1(k', b', z', j; \mathbf{s}) d\omega' \leq w(\mathbf{s})n + \lambda + \int_{K \times B \times Z \times \mathbf{J}} \rho_0(k, b, z, j; \mathbf{s}) d\omega + T,$$

$$\mathbf{s}' \sim \Gamma(\mathbf{s}).$$

The right-hand side of the budget constraint represents resources available to the household: it consists of firms' shares coming from the previous period, tax rebate from the government T , as well as labor income. The left-hand side shows that a part of these resources is consumed, and the rest is reinvested into new firm shares as well as into a risk-free bond. Let $C(\omega, \lambda; \mathbf{s})$ be the household's consumption policy function, and $N(\omega, \lambda; \mathbf{s})$ be a labor supply policy function. Also, let $\Xi(k', b', z', \omega, \lambda, j; \mathbf{s})$ be a number of shares purchased in firms of type j which start tomorrow with capital k' , debt b' , idiosyncratic productivity z' . Finally, $\Lambda(\omega, \lambda)$ is the policy function with respect to a risk-free bond.

1.3.5 Equilibrium

Let Σ_K , Σ_B and Σ_Z be Borel sigma algebras over K , B and Z . The state space is $S = K \times B \times Z$ with (k, b, z) being an element of that space. Let Σ_S be the sigma algebra on the state space with typical set $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, and (S, Σ_S) be the corresponding measurable space. Let $\zeta_j : \Sigma_S \rightarrow [0, 1]$ be a distribution of type- j firms across the state space at the beginning of the period after idiosyncratic uncertainty has been revealed.

A *recursive competitive equilibrium* for this economy is a collection of functions:

$$\left\{ v_j^0, v_j^1, v_j^2, k_j', b_j', n_j, w, \rho_0, \rho_1, W, C, N, \Lambda, \Xi, d, m^j, e^j, \Gamma \right\}_{j \in \{1, \dots, J\}},$$

such that:

1. W solves the household's problem (1.16), and (C, N, Ξ, Λ) are the associated policy functions,
2. $\{v_j^0, v_j^1, v_j^2\}$ solve the firms' problem (1.5)-(1.12), and $\{k'_j, b'_j, n_j\}$ are the corresponding policy functions,
3. $\{m^j\}$ satisfy the free-entry conditions (1.14),
4. consistency condition satisfies $\forall(k, b, z, j) \in K \times B \times Z \times \mathbf{J}$

$$\Xi(k', b', z', \{\zeta\}, \lambda, j; \mathbf{s}) = \zeta'_j(k', b', z'),$$

5. labor market clears

$$N(\{\zeta\}, \lambda; \mathbf{s}) = \sum_{j=1}^J \int_S n_j(k, b, z; \mathbf{s}) d\zeta_j,$$

6. stochastic discount factor satisfies

$$d(\mathbf{s}'|\mathbf{s}) = \beta \frac{U'_C(C(\{\zeta'\}, \lambda', \mathbf{s}'), N(\{\zeta'\}, \lambda', \mathbf{s}'))}{U'_C(C(\{\zeta\}, \lambda, \mathbf{s}), N(\{\zeta\}, \lambda, \mathbf{s}))},$$

7. goods market clears

$$\begin{aligned} C(\{\zeta\}, \lambda; \mathbf{s}) = & \sum_{j=1}^J \int_S [e^z [k_j^\alpha n_j^{1-\alpha}]^{\mu_j} - (1 - \pi_d)(k'_j - (1 - \delta)k_j) + \\ & + \pi_d((1 - \delta)k_j) - m^j k_0] d\zeta_j - \sum_{j=1}^J \chi e^j, \end{aligned}$$

8. government runs a balanced budget (transfers = tax revenue)

$$T = \tau \sum_{j=1}^J \int_S [y_j - wn_j - (R(\mathbf{s}_{-1} - 1)b_j) - \delta k_j] d\zeta_j$$

9. bonds market clears (by Walras law),
10. the law of motion for the aggregate state vector is consistent with firms' policy functions.

Following [Khan and Thomas \(2008\)](#), in Appendix A.4.1 I discuss how to combine household’s and firms’ programming problems for the efficient computation of the equilibrium.

1.3.6 Solution Method

In general, it is difficult to solve for the recursive competitive equilibrium in models of the type laid out in Section 1.3, since the firms’ policy functions depend on the aggregate state vector \mathbf{s} , which includes an infinitely dimensional object—distribution of firms across the state space. The standard method in the literature to solve such models is [Krusell and Smith \(1998\)](#). In particular, for the model laid out in Section 1.3, the method of [Krusell and Smith \(1998\)](#) requires stipulating the law of motion for the marginal utility and future mean capital stock as functions of a finite number of moments of the cross-sectional distribution. The resulting forecasting rule would include many level and cross-sectional terms, rendering the approach both impractical and slow. Since the model has to be solved many times in order to estimate aggregate processes, the solution procedure must be fast.

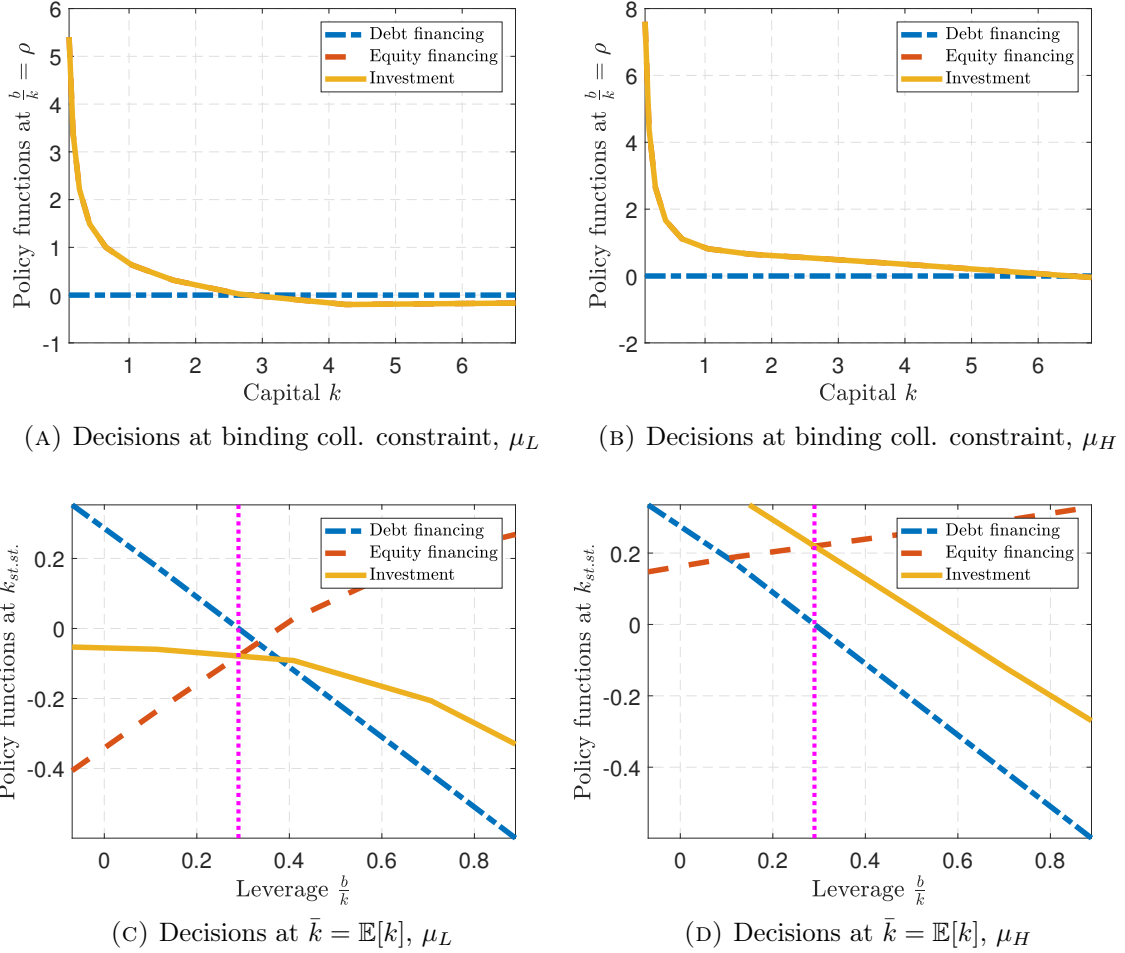
This reasoning motivates me to follow an alternative approach - perturbation method of [Reiter \(2009\)](#): it involves solving the firms’ decisions rules globally at the deterministic steady-state, and then perturbing the solution with respect to aggregate shocks. This approach, therefore, preserves the full non-linearity of the firms’ policy rules with respect to idiosyncratic states, and perturbs these policies linearly with respect to aggregate shocks ([Mongey and Williams, 2016](#)). Further details on the solution method are relegated to Appendix A.4.

1.3.7 Characterization of Steady-State

In order to demonstrate the workings of the model, in this section I plot the firms’ decision rules at the calibrated steady-state (aggregate shocks are turned off). Given the dimensionality of the problem, I follow [Guo \(2019\)](#) and first show how different firms finance their investment expenditures when they can no longer issue debt (collateral constraint (1.11) binds), and subsequently plot firms’ decisions by tracing out leverage ratios at the cross-sectional mean level of capital $\bar{k} = \mathbb{E}[k]$.

To facilitate the visual inspection, here I focus on the lowest (μ_L) and highest (μ_H) types only. Idiosyncratic productivity is fixed at the unconditional mean ($z = 0$). The policy functions for investment demand, debt financing and equity financing are depicted in Figure

FIGURE 1.6: DECISION RULES AT STEADY-STATE



Notes: Figure 1.6 consists of 4 panels. Panels (A) and (B) plot decision rules when collateral constraint (1.11) binds, and Panels (C) and (D) plot decision rules when firms' capital is fixed at the cross-sectional mean. Panels (A) and (C) correspond to $\mu = \mu_L$, while Panels (B) and (D)—to $\mu = \mu_H$. Vertical pink dotted lines in Panels (C) and (D) correspond to the calibrated tightness of the collateral constraint (parameter ρ). All decision rules are computed before profits ($\Pi = 0$), and are normalized by the idiosyncratic level of capital.

1.6. The top row plots the decisions for when the collateral constraint binds, and the bottom row—at the cross-sectional mean level of capital \bar{k} .

Along the size dimension (top row), there is a significant variation in investment and equity financing policies, but little variation in debt financing policies. Given the decreasing returns to scale technology (1.3) and mean-reverting productivity process (1.4), firms have a finite optimal target size. Therefore, small firms, which are further away from the optimal size, exhibit higher demand for investment. Conversely, large firms are at or around the optimal size, and their investment demand is small or even slightly negative. This is the main reason

why the investment policy is decreasing with size. Given that the collateral constraint binds, firms cannot issue debt to finance their investment; therefore, businesses finance their investment expenditures primarily through equity issuance (corresponding lines lie on top of each other). It is also straightforward to see that firms with a larger span of control have uniformly higher investment rates, and their target size (determined by the point where investment demand crosses 0) is larger.

Along the leverage ratio dimension (bottom row in Figure 1.6), there is a significant variation in financing policies. The cross-sectional mean level of capital is above the target size of low-type firms, but below the target size of high-type businesses. This observation is reflected in the way decision rules look like for these firm types: investment expenditures are negative for μ_L and positive for μ_H firms. With an increase in the leverage ratio, the residual capacity of debt financing decreases, so the debt financing declines and the equity financing increases correspondingly.²⁶ Due to the equity issuance cost, firms do not raise equity one-to-one with investment demand, and investment decreases with the leverage ratio. Businesses with a larger span of control reduce their investment demand more than low type firms as their leverage rises. Besides, while μ_H firms issue equity (red dashed line is in positive region across all leverage ratios), μ_L firms pay out dividends when leverage ratio is relatively low, and start issuing equity only when they become highly leveraged.

In the next section, I bring the model to the data.

1.4 Parametrization and Estimation

In my quantitative exercise, I set number of types equal $J = 3$, which is both computationally feasible and aligns with empirical investigation from Section 1.2.

Strategy I split all parameters into 3 groups. Parameters in the first group are standard in macro literature, and I assign values to them without solving the model (Subsection 1.4.1). The second group contains parameters which govern the behavior of the model economy at the steady-state; I assign values to these parameters so that the model-generated moments match a set of empirical targets (Subsection 1.4.2). In Subsection 1.4.3, I show that the

²⁶In the data, young firms issue significant amounts of debt (see Figure A.2 in Appendix A.2). In line with the model's predictions, equity issuance is most pronounced in the first several years of firm tenure (Figure A.3).

TABLE 1.1: FIXED PARAMETERS

Parameter	Description	Value
β	Discount factor	0.98
α	Capital share	0.33
ν	Labor share	0.67
δ	Depreciation rate	0.025
b_0	Initial debt position	0.00
τ	Tax rate	0.24
π_d	Exit probability	0.03

model performs well along the dimensions that were not directly targeted at the estimation step.

The last group consists of persistence and variance parameters for aggregate stochastic processes, which I jointly estimate using Bayesian methods in Subsection 1.4.4. Finally, before presenting the main results of the paper in Section 1.5, Subsection 1.4.5 validates the model by way of comparing the model-implied business cycle statistics against the U.K. data.

1.4.1 Fixed Parameters

The period in the model is one quarter, which is a suitable frequency to study business cycles. Therefore, I set the discount factor $\beta = 0.98$. I set labor share $\nu = 0.67$ and capital share $\alpha = 0.33$, and then estimate $\{\mu_j\}_{j=1}^J$ in Subsection 1.4.2. I will restrict the values of $\{\mu_j\}$ to be less than 1 in order to guarantee decreasing returns to scale (and a finite target size).

I set depreciation rate $\delta = 0.025$ so that the aggregate investment is 10% per annum. Tax rate is set equal to 0.24, which is the average corporate tax rate in the U.K. over the last decade. Entering firms start with zero initial debt $b_0 = 0$. Quarterly exit probability is set to 0.03 in order to get an annual 12% exit rate.

I let the instantaneous utility function of the household be separable between consumption and labor:

$$U(C(\mathbf{s}), N(\mathbf{s})) = \frac{C(\mathbf{s})^{1-\eta}}{1-\eta} - A \frac{N(\mathbf{s})^\zeta}{\zeta}, \quad (1.17)$$

where Frisch elasticity of substitution is assumed to be infinite ($\zeta = 1$). Parameter A is the disutility from labor, and is estimated in Section 1.4 to make the household devote a third

of its time endowment to market work. Following [Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry \(2018\)](#), I also assume a log consumption utility function ($\eta = 1$). With this, the utility function (1.17) implies that the following equilibrium condition must hold:

$$w(\mathbf{s}) = AN(\mathbf{s})^{\zeta-1}C(\mathbf{s})^\eta = AC(\mathbf{s}). \quad (1.18)$$

Equation (1.18) represents a link between equilibrium aggregate consumption and the wage rate, and allows for the efficient computation of equilibrium. Table 1.1 summarizes the pre-set parameters.

1.4.2 Internally Estimated Parameters

In this section, I estimate parameters which govern the behavior of the model at the steady-state. In what follows, I provide a heuristic identification argument that justifies the choice of the target moments. Even though every targeted moment is simultaneously affected by all the parameters, in this section I discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power. For the most part I follow literature in picking the moments to target.

Since the firm-level data comes at annual frequency, and the model period is set to a quarter, I have to make the model- and data-based moments comparable. To do so, at each step of the estimation procedure, I draw a panel from the model and subsequently aggregate the data to annual frequency in order to compute the corresponding moments.

The elasticity of the matching function ϕ is informative about how volatile firm entry is over the business cycle (see Equation (1.14)); I, therefore, target the relative volatility of the entry rate with respect to GDP.²⁷ Parameter χ is picked so that the total amount of resources devoted to entry equals 7.3% of GDP at the steady-state ([Sedláček and Sterk, 2017](#)). Collateral constraint parameter ρ directly affects the degree to which firms are financially constrained in the model; I, therefore, require the model to deliver the mean leverage ratio of 0.27 as in the data.

Idiosyncratic productivity process parameters ρ_z and σ_z along with a capital adjustment cost parameter φ_K affect investment decisions of firms the most. I, therefore, include the first two moments of the investment rate distribution as well as the frequency of investment

²⁷I obtain quarterly data from OECD: it reports the index of firm entry between 2006Q1 and 2018Q2, with normalization of 1 for year 2007.

TABLE 1.2: ESTIMATED PARAMETERS

Parameter	Description	Value	Target	Data	Model
A	Preference for leisure	2.04	N	—	0.33
ϕ	Elast. of match. fun.	0.72	$\frac{\sigma(\text{entry})}{\sigma(Y)}$	2.78	2.29
χ	Entry cost, %	0.32	$\frac{\sum_{j=1}^J \chi^{e_j}}{Y}$	0.07	0.12
ρ	Tightness of coll. const.	0.31	$\mathbb{E} \left[\frac{\rho}{k} \right]$	0.27	0.27
σ_z	Std of idios. inn.	0.03	$\mathbb{E} \left[\frac{z}{k} \right]$	0.29	0.30
ρ_z	Pers. of idios. inn.	0.98	$\sigma \left[\frac{z}{k} \right]$	0.68	0.71
φ^E	Equity iss. cost	0.06	$\mathbb{E} \left[\frac{E}{k} \mid \text{age} < 5 \right]$	0.06	0.05
φ^K	Adjustment cost	0.13	$\mathbb{E} \left[\left \frac{z}{k} \right > 0.2 \right]$	0.36	0.42
k_0	Capital endowment	0.20	$\frac{\mathbb{E}[n \mid \text{age}=0]}{\mathbb{E}[n]}$	0.32	0.32
μ_2	Medium DRS	0.80	Emp. share $n < 50$	0.37	0.37
$\Delta\mu$	DRS spread	0.05	Emp. share $n \geq 250$	0.48	0.25
ψ_1	Availability of low type	0.40	Firm share $n < 50$	0.95	0.95
ψ_2	Availability of med. type	0.33	Firm share $n \geq 250$	0.005	0.005

spikes²⁸ in the set of empirical targets.

I assume that the cost of issuing equity is cubic in Equation (1.10):

$$C(E_j) = \varphi^E [\max\{0, E_j\}]^3.$$

Such formulation implies that firms find it increasingly hard to raise equity. From the technical perspective, I found that a cubic function smooths out the kink, which results in a more stable solution. Equity issuance cost φ^E is chosen so that the model generates an average (among firms up to age 5) issued equity-to-capital ratio of 0.06.

Capital endowment of successful entrants k_0 is set to make entrants' size 32% of the average incumbent's size (in terms of employment). New firms are assumed to draw initial idiosyncratic productivity from a time-invariant normal distribution $F(z)$ with a mean -0.1 and variance σ_z .²⁹

Span of control parameters $\{\mu_j\}_{j=1}^J$ are chosen to generate a skewed firm-size distribution (see Figure A.1 in Appendix A.1.3). In particular, I target the share of firms with less than 50 and more than 250 employees. Finally, availability parameters $\{\psi_j\}_{j=1}^J$ are picked to match the share of aggregate employment in firms with less than 50 employees, and share of employment in firms with at least 250 workers. Table 1.2 reports the estimates for the

²⁸Investment spike is a situation when the firm's investment rate exceeds 20% in absolute value.

²⁹Entering businesses are estimated to be 10% less productive than incumbents (see Table A.3). Lee and Mukoyama (2015) obtain similar findings using Census data.

structural parameters.

1.4.3 Model Validation

In this section, I first explore the firms' average life cycle profiles. Subsequently, I check how well the model performs with respect to the moments not directly targeted in estimation: specifically, I will be looking at the investment and financial heterogeneity, as well as at the distribution of employment growth rates. Finally, I show that, in the model, rapidly growing firms are more responsive to collateral shocks than slowly growing businesses, mirroring findings from Section 1.2.

Average life cycle Figure 1.7 plots the central to this paper dimensions of firms' life cycle profiles. Panel A shows how different target sizes are across firm types; while the average low type firm ends up being around 3 times bigger than a newborn business (in terms of labor), the average high type firm takes off pretty fast and exceed the size of entrants by a factor of 13 about 5 years into their tenure.³⁰ It is also visible that it takes around 5 quarters for a low-type firm to reach its target size, while the high type business is still growing even 5 years out.

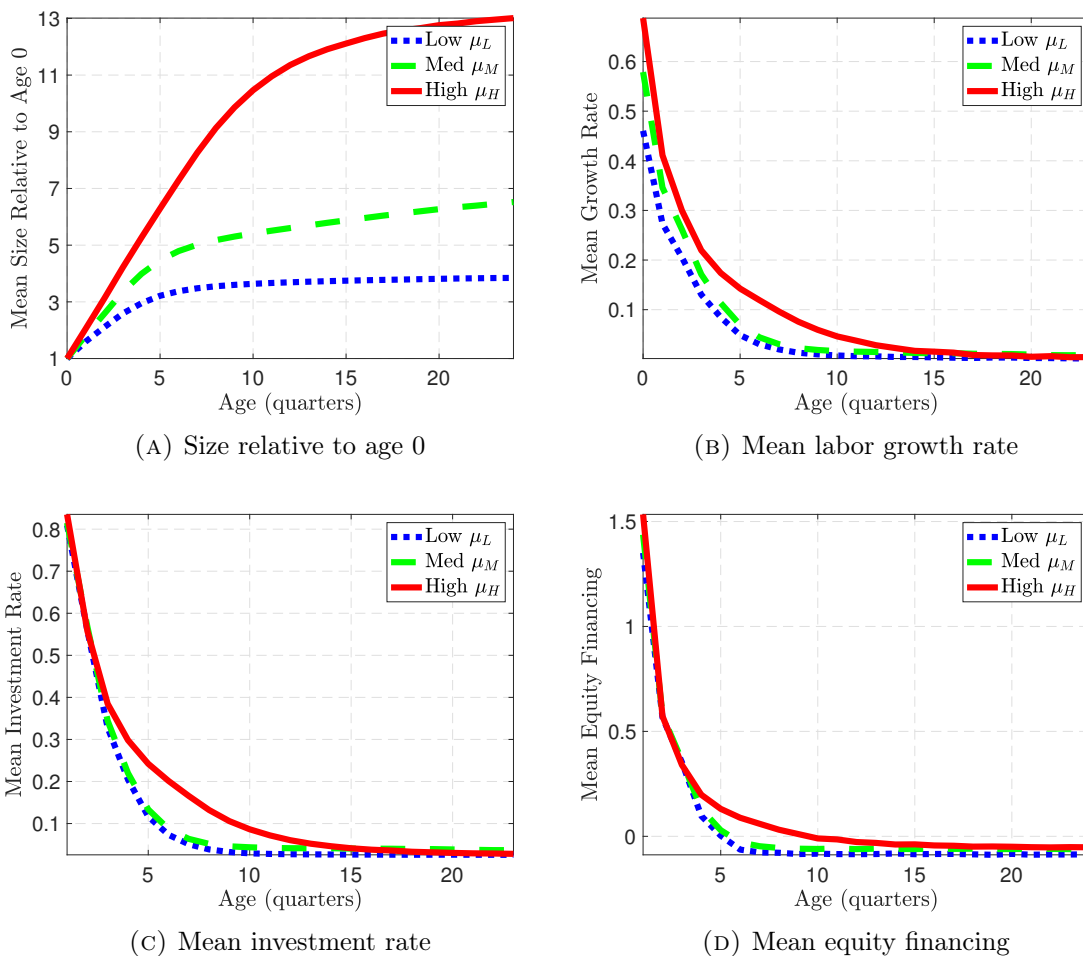
The patterns observed in Panel A translate into growth rate terms in Panel B, where the mean growth rate of labor is depicted. It illustrates how the high type businesses exhibit high (relative to low and medium type firms) growth rates throughout their life cycles.

Growth in terms of labor is accompanied by capital accumulation through investment (Panel C). Entering businesses are endowed with low physical capital holdings (around 10% of the level of an average incumbent), and, therefore, exhibit high investment rates early in their tenure. High investment expenditures are financed through 3 sources (retained earnings, debt and equity issuance). Panel D shows that young firms rely a lot of equity financing, and start paying out dividends only several years after entry (it takes longer for high type firms as their target size is larger).

Figure 1.7 is key for understanding the compositional effects I study in this paper. Provided that it takes high type firms substantially more time (and resources—see Panel D) to get up to scale as compared to low type businesses, tightening of the collateral constraint affects the former stronger than the latter. At the extreme, consider a firm for which its target

³⁰Note that the relative size of firms is calculated with respect to employment in the first *quarter* of their tenure.

FIGURE 1.7: AVERAGE LIFE CYCLE PROFILES



Notes: Figure 1.7 plots the average (across idiosyncratic productivity) life cycle profiles of firms in the first 25 quarters after their birth. Panels A, B, C and D plot the mean size (in terms of labor) relative to entrants, the mean growth rate of labor, the mean investment rate, and the mean equity financing, respectively. The growth rate of labor is calculated as $\Delta_t = (n_t - n_{t-1}) / \frac{1}{2}(n_t + n_{t-1})$. The equity financing is normalized by the firm's current size (capital).

size coincides with its initial size—such firm will not be affected by collateral shocks at all. Therefore, financial shock is expected to affect μ_H firms stronger than other firm types.

TFP shocks change target sizes of all firm types *proportionately*, therefore, equally affecting the values of low, medium and high target size businesses. Thus, the impact of the aggregate productivity shock is expected to affect similarly the entry intensity of different firm types.

Investment and leverage heterogeneity In order to characterize the investment and leverage heterogeneity in the model—moments not directly targeted in the calibration—I follow [Ottonello and Winberry \(2018\)](#) and compare (auto)correlations of investment rates

TABLE 1.3: INVESTMENT AND LEVERAGE HETEROGENEITY

Moment	Description	Data	Model
Investment heterogeneity			
$\mathbb{E} \left[\frac{i}{k} \right]$	Mean investment rate (targeted)	0.29	0.30
$\sigma \left[\frac{i}{k} \right]$	Std investment rate (targeted)	0.68	0.71
$\rho \left(\frac{i}{k}, \frac{i}{k-1} \right)$	Autocorrelation investment rate	0.12	0.58
Joint heterogeneity			
$\rho \left(\frac{b}{k}, \frac{b}{k-1} \right)$	Autocorrelation leverage	0.76	0.84
$\rho \left(\frac{i}{k}, \frac{b}{k} \right)$	Correlation of investment and leverage	-0.15	-0.51

Notes: Table 1.3 reports the statistics regarding the cross-sectional distribution of investment rates and leverage ratios in the data (BvD) and in the model (steady-state). Model-generated data has been aggregated to annual frequency. A balanced 10-year long panel is considered. The first two moments reported (mean and std. of investment rates) have been targeted during the estimation step in Section 1.4.

and leverage in the model-simulated panel against the data.

I compute statistics in a balanced panel conditional on observing firms for 10 years.³¹ Overall, Table 1.3 shows that the model is broadly consistent with the investment and financial heterogeneity observed in the data. Even though I targeted only the first moment of the leverage distribution and the first two moments of the investment distribution, the model does a reasonable job in picking up the joint heterogeneity.

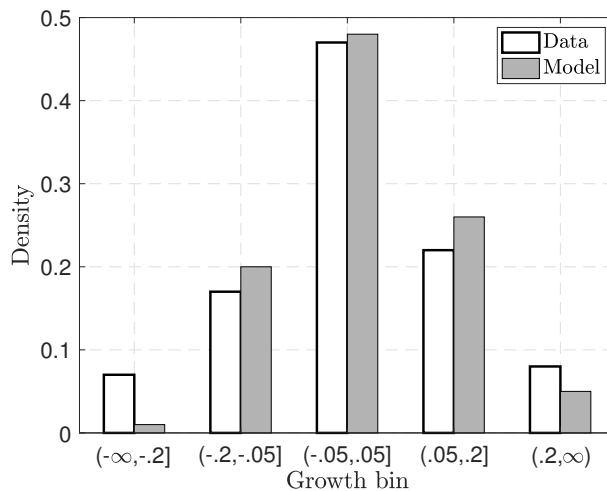
Growth Rates Distribution Figure 1.8 displays a distribution of employment growth rates in the model and in the data. In order to comply with the empirical counterpart, the model-generated data was aggregated to annual frequency.

Figure 1.8 shows that the model-implied distribution of employment growth rates looks reasonably similar to its empirical analog, even though it was not directly targeted at the parametrization step. The model generates a spike at the bin corresponding to growth rates between -5% and 5%, as in the data. The size of the spike is very close to the one observed in the data.

Heterogeneity in Responsiveness to Collateral Shocks In Section 1.2, I argued that fast growing businesses are more financially constrained than slow growing firms by projecting firms' investment on the orthogonal idiosyncratic fluctuations in the residential wealth of firms' directors. Clearly, it is not feasible to run the exactly same regression on the model-simulated data; however, it is possible to study the *relative* responsiveness of firms'

³¹Empirical moments are computed on a balanced panel for years 2001-2010.

FIGURE 1.8: DISTRIBUTION OF EMPLOYMENT GROWTH RATES



Notes: Figure 1.8 plots the empirical and the model-generated employment growth distribution. Simulated data was aggregated to annual frequency. Underlying data are 10-year long balanced panels. Employment growth of firm i at time t was calculated as $2(n_{it} - n_{it-1})/(n_{it} + n_{it-1})$. Data source: BvD.

investment to fluctuations in their collateral.

Given the formulation of the financial constraint in Equation (1.11), I mimic fluctuations in the collateral by generating independent (across time and space) shocks to firms' capital. In particular, I simulate a panel of firms without collateral shocks, but at the same time I also record in each period what the investment of each firm would have been had they experienced a shock to capital. Let k_{it} be the capital stock of firm i at time t . I assume that collateral shocks are drawn from the uniform distribution with the support $[-k_{it}, k_{it}]$, so that the after-shock capital holding is non-negative. I classify firms into fast and slow growing types exactly as it was described in Subsection 1.2.4. I then regress investment after a collateral shock on an interaction of the shock's size and firm's type (including the same controls as in the benchmark empirical specification (1.2)).

Table 1.4 shows that, in the model, fast growing firms are also more responsive to collateral shocks, providing validation for the theoretical framework developed in Section 1.3. In particular, fast growing firms are 1.8 times more responsive to collateral shocks than slow growing businesses; this is close to the estimate of 2.2 observed in the data. Next, I proceed with the estimation of aggregate shocks.

TABLE 1.4: HETEROGENEOUS RESPONSE TO COLLATERAL SHOCKS

	Data	Model	
		With controls	Without controls
Slow	0.006	0.55	0.56
Fast	0.013	1.00	1.01
Ratio	2.2	1.8	1.8
Firm FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes

Notes: Table 1.4 reports OLS estimates from projecting firms' investment on the interaction of the collateral shock and firm growth group. Classification of firms into slow and fast types was performed as in Subsection 1.2.4. The vector of controls includes firms' profits, the inverse scale and the leverage. Variables were lagged and normalized by the idiosyncratic level of capital. Firm and time fixed effects were included.

1.4.4 Estimation of Aggregate Shocks

I introduce 3 exogenous aggregate stochastic processes: the first governs the tightness of the collateral constraint ρ , the second affects the disutility of labor A , and the third is an aggregate productivity shock Z . I assume that shocks follow AR(1) processes in logs:

$$\begin{aligned} \log \tilde{X}_{t+1} &= \rho_X \log \tilde{X}_t + \varepsilon_{X,t+1}, \\ \varepsilon_{X,t+1} &\sim \mathcal{N}(0, \sigma_X^2) \\ X &\in \{\rho, A, Z\}, \end{aligned} \tag{1.19}$$

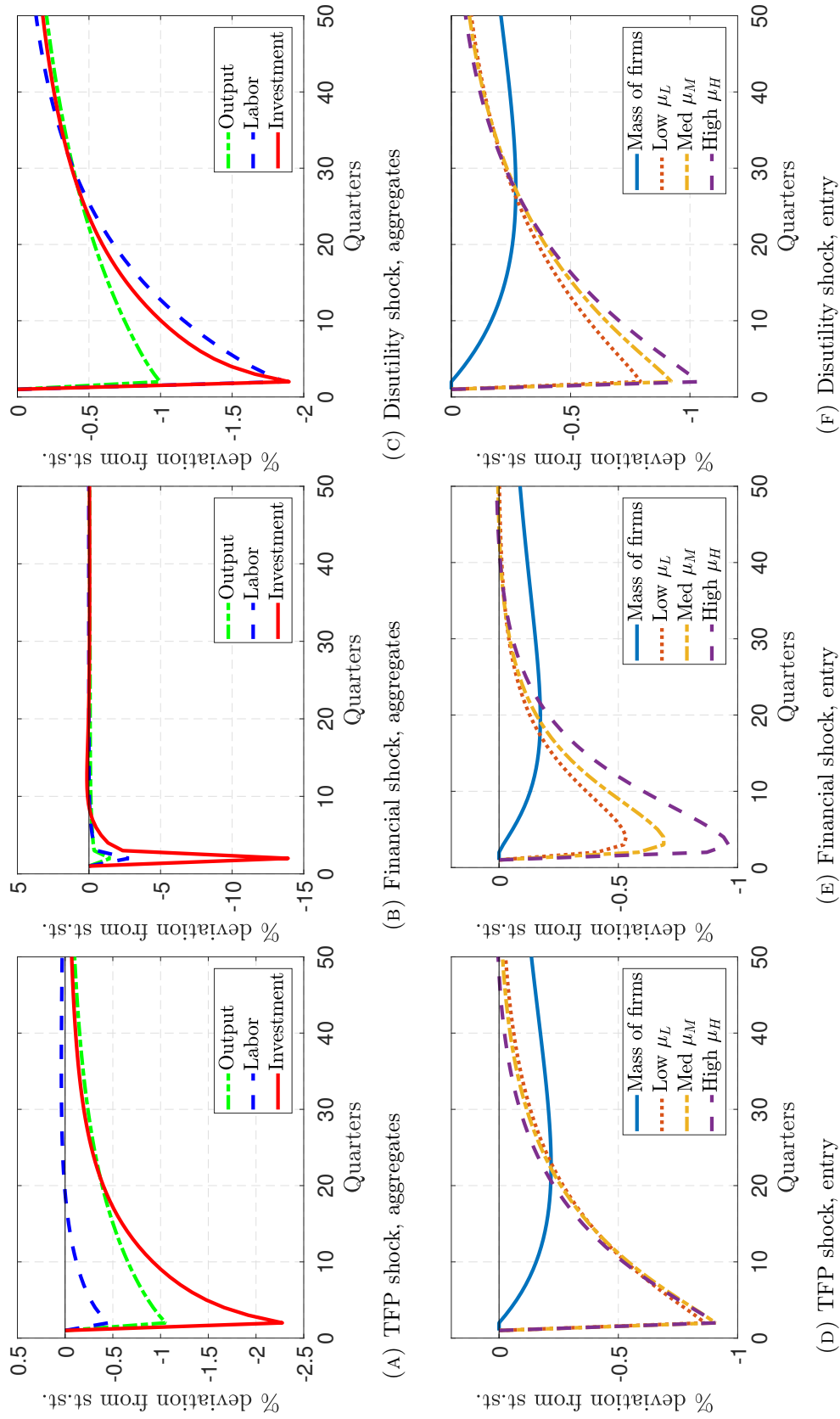
where $X_t = X_{ss} \times \tilde{X}_t$. The collateral shock directly affects the ease with which firms in the model can issue debt and finance their investment. The model developed in Section 1.3 features frictionless labor markets; therefore, in order to account for the distressed labor markets during recessions, I introduce shocks to the disutility of labor A —it is a parsimonious way to model disruptions originating in the labor market. Finally, aggregate TFP shock is a reduced-form way to account for alternative, deeper mechanisms, not captured by the benchmark model (Kehoe, Midrigan and Pastorino, 2018).³²

I use employment, GDP and investment expenditures aggregate series for the U.K. economy in order to estimate the persistence and volatility of 3 stochastic processes (see Figure A.8 in the Appendix). I make a standard in the literature choice³³ and use the Beta and Inverse

³²Chari, Kehoe and McGrattan (2007) develop a business cycle accounting procedure and argue that an empirically successful business cycle model should feature mechanisms which would manifest themselves as labor and efficiency wedges in a *prototype* model. I introduce these wedges directly into my model by including labor disutility and TFP shocks.

³³See, for example, An and Schorfheide (2007).

FIGURE 1.9: IMPULSE-RESPONSE FUNCTIONS TO AGGREGATE SHOCKS



Notes: Figure 1.9 plots impulse-response functions to innovations in TFP (panels A and B), financial (panels C and D), and labor disutility (panels E and F) stochastic processes. Size of shocks was calibrated to generate a 1% decline in output upon impact. Persistence and volatility of exogenous stochastic processes are as in Table 1.5.

TABLE 1.5: ESTIMATION OF AGGREGATE SHOCKS

Param.	Type	Prior		Posterior (flexible)			Posterior (fixed)		
		Mean	Std	Mode	5%	95%	Mode	5%	95%
Persistence									
ρ_z	Beta	0.5	0.05	0.964	0.964	0.995	0.996	0.981	0.996
ρ_r	Beta	0.5	0.05	0.858	0.851	0.913	0.889	0.885	0.968
ρ_n	Beta	0.5	0.05	0.989	0.954	0.995	0.983	0.925	0.997
Standard deviation									
σ_z	Inv. Gamma	0.25	0.06	0.009	0.008	0.011	0.008	0.008	0.009
σ_r	Inv. Gamma	0.25	0.06	0.101	0.089	0.113	0.096	0.088	0.117
σ_n	Inv. Gamma	0.25	0.06	0.007	0.007	0.008	0.007	0.007	0.008

Notes: Table 1.5 reports the results of the Bayesian estimation of aggregate exogenous stochastic processes. Each process is characterized by its persistence and the standard deviation of innovations—see Equation (1.19). Table reports the priors for each parameter as well as the mode and 95% confidence bounds based on 10000 draws from the posterior distribution. See text and Appendix A.4.4 for more details on the estimation procedure. “Flexible” refers to a benchmark model, and “fixed” - to a model with a fixed composition of firm types.

gamma distributions as priors for persistence and volatility parameters, respectively. Table 1.5 reports the estimation results. Anticipating a quantitative assessment of the intensive margin of business formation in Section 1.5, I independently estimate 2 models: a benchmark model along with a fixed composition version of the model (to be described in Subsection 1.5.1).

Identification How does the aggregate data identify the parameters of the exogenous stochastic processes? Figure 1.9 plots the impulse-response functions for 3 aggregate stochastic processes considered.³⁴ For each shock, it separately plots the response of key macro aggregates - output, hours and investment (top row), and the intensity of entry for each type (bottom row). For comparability reasons, the size of each shock was set to generate a 1% decline in output upon impact.

Each shock has different quantitative implications for the behavior of macroeconomic aggregates. It is clear that the financial shock affects investment stronger than output and hours. At the same time, the labor disutility shock leads to an equally strong decline in hours and investment. The aggregate productivity shock affects output stronger than hours, which is the opposite to the effect of the labor shock. Therefore, the identification of shocks comes from the way aggregate series move relative to each other over the cycle.

³⁴Aggregate processes have the estimated persistence and volatility parameters from Table 1.5.

TABLE 1.6: UNCONDITIONAL BUSINESS CYCLE STATISTICS

Statistic	Volatility		Statistic	Cyclicalilty	
	Data	Model		Data	Model
$\sigma(Y_t)$	1.38%	1.67%			
$\sigma(C_t)/\sigma(Y_t)$	0.91	0.64	$\rho(C_t, Y_t)$	0.81	0.86
$\sigma(I_t)/\sigma(Y_t)$	3.20	2.72	$\rho(I_t, Y_t)$	0.79	0.81
$\sigma(N_t)/\sigma(Y_t)$	1.20	0.98	$\rho(N_t, Y_t)$	0.74	0.75

Notes: Table 1.6 reports the empirical and model-generated unconditional business cycle statistics. The aggregate data has quarterly frequency and spans the period 1975Q1-2016Q4. Consumption is “Real Consumption Expenditures in the U.K.,” Investment is “Real Investment Expenditures in the U.K.,” Output corresponds to “Real GDP for U.K.,” and Hours worked is “Total actual hours worked”. The data comes from the ONS and FRED databases. “Model” refers to an estimated model from Section 1.3. Prior to computing the statistics, the data was logged and HP-filtered with the smoothing parameter of 1600.

Aggregate Shocks and Business Formation Aggregate shocks have important implications for the formation of different firm types. Panel D in Figure 1.9 shows that a negative TFP shock leads to a nearly identical drop in the entry of low, medium and high type firms. This occurs precisely because the TFP shock affects the expected benefit *proportionately* across the types according to Equation (1.14), resulting in a similar response in the mass of successful entrants. Financial and labor disutility shocks affect high-type firms *disproportionately* more—given that such businesses require more resources to get up to scale—and depress their entry intensity more as compared to other firms with a lower span of control.

1.4.5 Business Cycle Statistics

As an additional validation check, I compute standard unconditional business cycle statistics in the model and data (volatility and cyclicalilty), and report them in Table 1.6. I find that my model does a good job in picking up the key business cycle moments of the U.K. economy. Investment is almost 3 times more volatile than GDP, while consumption is less volatile. The model-implied volatility of hours is lower than in the data, which is a common issue that many business cycle models share.

On the cyclicalilty side, consumption, investment and hours are found to be strongly positively correlated with output, in line with the data. I also find that the model-based cyclicalilty moments are quantitatively very close to empirical ones.

1.5 Compositional Effects and Business Cycle

In this section, I study the quantitative implications of compositional shifts for the U.K. business cycle. I start by introducing in Subsection 1.5.1 a version of the baseline model which features no intensive margin of firm entry over the business cycle. I then show in Subsection 1.5.2 that the benchmark model with the intensive margin of business entry can generate a collapse of the right tail of the cumulative growth distribution in recession, while the model without this mechanism cannot. Subsequently, I assess the quantitative relevance of the intensive margin mechanism by way of comparing the performance of the two estimated versions of the model during the Great Recession episode in Subsection 1.5.3. I then argue that the mechanism is empirically relevant in Subsection 1.5.4. Finally, in Subsection 1.5.5, I quantify the contribution of each of exogenous aggregate forces in driving the intensive margin of business formation.

1.5.1 Fixed Composition Model

A natural version of the model laid out in Section 1.3 but without compositional shifts can be obtained by modifying the timing of the entry problem described in Subsection 1.3.3. In particular, I now assume that potential entrants still know the distribution of *type availability* (parameters $\{\psi^j\}_{j \in \mathbf{J}}$), but can no longer target their entry efforts to any particular type. Instead, each potential entrant is assigned the type randomly—according to the *induced* availability probability mass function³⁵—upon successful entry. Under this formulation, the distribution of entrants across types will mechanically be business cycle invariant.

The new free-entry condition takes the following form:

$$\underbrace{\chi}_{\text{cost of entry}} = \underbrace{\frac{m_t}{e_t}}_{\text{success probability}} \underbrace{\int_z \sum_{j \in \mathbf{J}} \psi^j v_j^0(k_0, b_0, z; \mathbf{s}) dF(z)}_{\text{exp. value of entry}}. \quad (1.20)$$

The difference between (1.14) and (1.20) is in the integrand on the right-hand side: now potential entrants cannot know which project they will end up operating, and so in equilibrium they have to balance the entry cost χ with an expected (over types) value of running a firm. Therefore, it is the extensive (overall mass of entrants)—not the intensive (distribution of

³⁵In particular, this means that in a fixed composition model, the share of new firms of type $j \in \mathbf{J}$ among entrants is $\frac{\psi_j}{\sum_{j=1}^{\mathbf{J}} \psi_j}$. These shares are business-cycle invariant, since the parameters $\{\psi_j\}$ are scalars.

firms over types)—margin of firm entry which is allowed to fluctuate over time. Therefore, the difference in predictions between the two models is fully accounted by the compositional effect.

1.5.2 Collapse of the Right Tail of Cumulative Growth Distribution

I now illustrate that the benchmark model is able to generate the collapse in the right tail of the cumulative growth distribution, while the model with no compositional effect cannot. To do so, I simulate both models during the Great Recession episode, and calculate the moments of the cumulative growth distribution based on the model-generated panels.

TABLE 1.7: MOMENTS OF THE CUMULATIVE GROWTH: MODEL VERSUS DATA

	Data	Full model	Fixed comp. model
$\Delta P90$	-66%	-45%	-8%
$\Delta P50$	-13%	-5%	-7%

Notes: Table 1.7 reports the by how many percent the median and the top 10th percentile of the cumulative growth distribution change in recession (2008-2009) relative to the expansion (2006-2007). “Data” corresponds to the BvD evidence from Section 1.2. See Subsection 1.5.1 for the description of the fixed composition model.

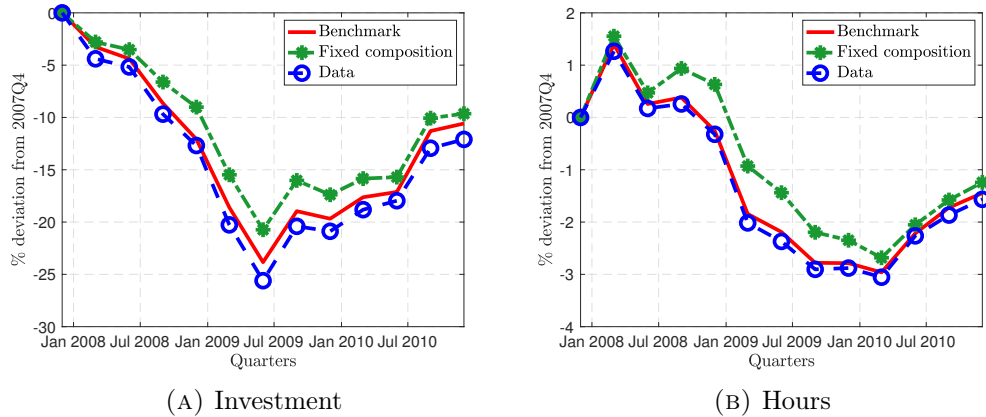
Table 1.7 shows that the full model generates a collapse in the right tail of the cumulative growth distribution, which is about two thirds of the size observed in the data. Note that this change was not targeted at the estimation step.

The fixed composition model does not generate any sizable change in the right tail because the median and the top 10th percentile declined similarly in a recession. Intuitively, this occurs because in that version of the model, there is no protraction mechanism which can make the effect of a recession persistent. Even though adverse aggregate shocks also affect high type firms stronger in the fixed composition model, these firms get up to full potential when aggregate conditions improve. As a result, I do not detect any sizable change in the right tail of the 5 year-long growth in this version of the model.

1.5.3 Great Recession and the Intensive Margin of Firm Entry

In order to assess the quantitative importance of compositional changes, I now compare the predictions of two versions of the model during the Great Recession episode. Using the Kalman smoother, I obtain sequences of exogenous aggregate processes from the benchmark model. Subsequently, using the finite representation of the model (see Appendix A.4.4 for

FIGURE 1.10: GREAT RECESSION: DATA VERSUS MODEL (2007Q4 = 0)



Notes: Figure 1.10 contains 2 panels. Panel A corresponds to aggregate investment, and panel B - to aggregate hours. Each panel has 3 lines. The blue dashed line is data, the red solid line corresponds to the benchmark model with flexible composition, and the green dash-dotted line—to a version of the model with fixed composition. Each series represents the percentage deviation from the corresponding level in 2007Q4.

further details), I simulate two sets of endogenous states: one for the benchmark and one for the fixed composition model. Effectively, I will be comparing the predictions of the two models by feeding in the identical series of aggregate shocks.

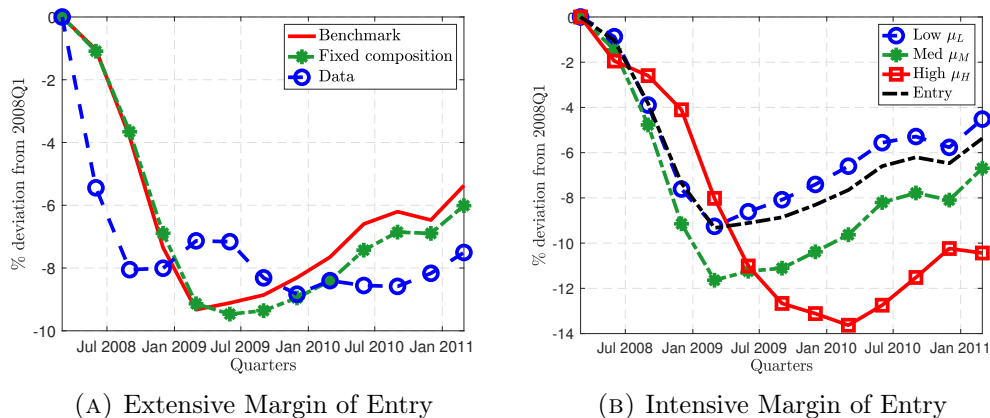
Figure 1.10 plots the recovered series of aggregate investment and hours for the two versions of the model along with the data during the Great Recession episode. By construction, the benchmark model almost fully accounts for the data, since I fed in the shocks filtered through that model. In order to facilitate the visual inspection, I plot the deviation of aggregates relative to the last quarter of 2007. I find that the fixed composition model requires larger shocks in order to explain the same dynamics of macro aggregates. Around mid 2009—when the economy reached the trough of the contraction—benchmark model falls roughly 25% deeper than the model with no compositional shifts. After the trough of the recession, the series corresponding to the two models get closer to each other, potentially reflecting the restored intensity of the high type business formation. I investigate this mechanism in full detail in the next subsection.

1.5.4 Discussion of the Amplification Mechanism

An important question is whether we actually need an amplification mechanism; in other words, maybe shocks were indeed bigger than what the benchmark model implies?

To this end, I look at the key non-targeted dimension of model performance—the extensive

FIGURE 1.11: GREAT RECESSION: EXTENSIVE AND INTENSIVE MARGINS OF BUSINESS FORMATION



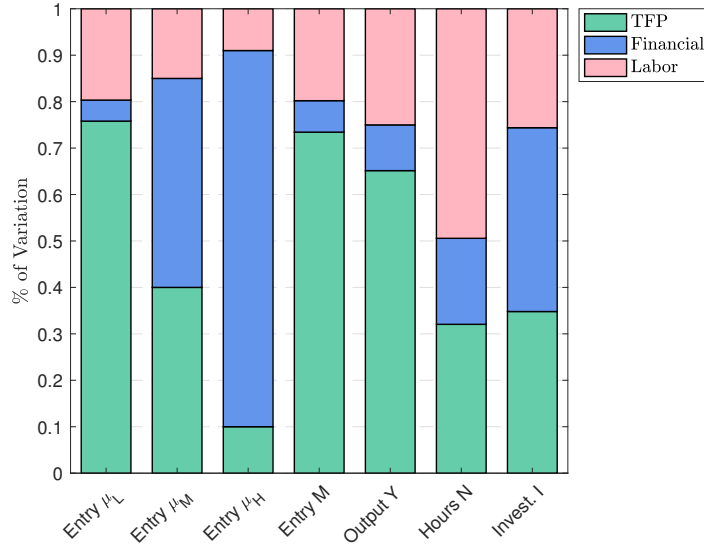
Notes: Figure 1.11 contains 2 panels. Panel A plots the extensive margin of business formation for the benchmark model (red solid), the fixed composition model (green dot dashed) and in the data (blue dashed). Panel B unpacks the extensive margin (black dot-dashed) into the entry intensity of the low type (blue with circle markers), middle type (green with star markers) and high type (red with square markers). Data source: OECD.

margin of business entry—and confront it with the data. Panel A in Figure 1.11 plots 3 lines over the 12 quarters period starting from 2008Q1. The solid red line is the benchmark model, the green line with star markers is the fixed composition model, and the dashed blue line with circle markers is data.

According to Panel A of Figure 1.11, both models exhibit a delayed—relative to the data—drop in the entry rate. However, it is also clear that the benchmark model delivers a more realistic dynamics of business formation rate during the Great Recession as compared to the fixed composition model. This implies that the fixed composition model needs to compress the extensive margin of firm entry counterfactually more in order to explain the same series of investment, output and hours, suggesting that the amplification effect of the intensive margin of business formation is empirically relevant.

The key reason behind the observed discrepancy between the models is that, intuitively, the full model has two degrees of freedom (intensive and extensive margins of firm entry) to account for the dynamics of aggregate series, while the fixed composition model has only one. Panel B in Figure 1.11 shows that the entry intensity of high type firms declines stronger during the Great Recessions as compared to low and medium type businesses. Therefore, the full model can account for the fall in macroeconomic aggregates during the recession by reducing the entry intensity of rapidly growing firms, which is associated with a small

FIGURE 1.12: VARIANCE DECOMPOSITION



Notes: Figure 1.12 reports the decomposition of aggregate fluctuations of several variables across the 3 aggregate shocks considered. Each bar is split into green (TFP), blue (financial) and pink (labor wedge) parts; the height of each part equals the share of the overall variance accounted by each shock. The following variables are considered: entry of low types (“entry μ_L ”), entry of medium types (“entry μ_M ”), entry of high types (“entry μ_H ”), overall entry (“entry M ”), output (“ Y ”), hours (“ N ”) and investment (“ I ”).

overall decline in the entry rate (there are relatively few fast growing projects as compared to slow growing ones). The model without compositional effects induces an equal decline in entry of all project types, which translates into an overall larger fall in the entry rate.

1.5.5 Variance Decomposition of Aggregate Shocks

Which aggregate shocks are driving the intensive margin of business formation over the cycle? This question is important for two reasons. First, as was shown in Subsection 1.5.3, aggregate implications of compositional effects are quantitatively sound. Second, one needs to evaluate the role financial shocks play relative to other aggregate forces in order to assess the room for government intervention.

Figure 1.12 reports the results of the variance decomposition of business formation of low, medium and high types along with several key macro aggregates with respect to aggregate forces. The financial shock accounts for about 82% of variation in the formation of the high types. TFP and labor wedge shocks account for the remaining 18%. As for the low type firms, TFP shock explains almost 80% in the entry intensity of these firms. Remarkably, the relative contribution of the financial shock declines substantially from 82% down to 5%.

This suggests that the formation of rapidly growing firms is very sensitive to conditions on financial markets. This finding coupled with the quantitative soundness of compositional shifts documented in Subsection 1.5.3 hints at potentially large benefits from government stimulus policies. Next, I turn to the quantitative assessment of such policies.

1.6 Policy Implications

In this section, I study the policy implications of my model. First, I illustrate that the policy which stimulates the formation of all firm types by reducing the cost of entry during aggregate contractions does not yield any welfare benefits; however, the government can generate a sizable welfare improvement if it targets the policy toward businesses of the highest growth potential. I argue that it occurs due to cost-inefficiency of the non-targeted policy.

In reality, however, the firm type is not observable. Thus, I explore the welfare benefits associated with the policy targeted at firms with a high leverage ratio—the object which is observable by the policymaker. The goal of this exercise is to illustrate the magnitude of welfare benefits associated with this feasible micro-targeted government policy, rather than to strongly advocate for a particular implementation.

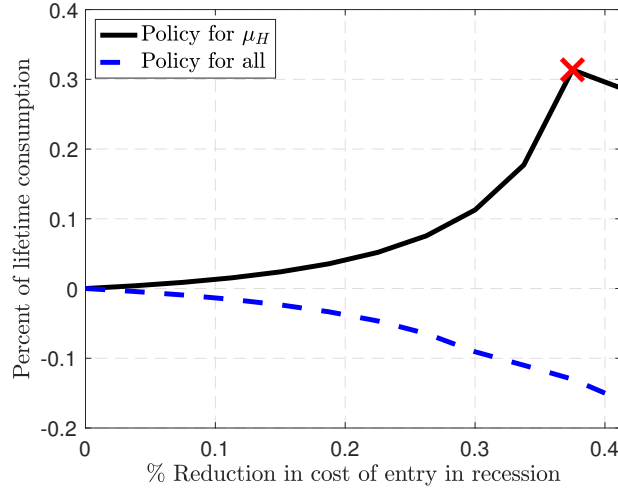
Subsection 1.6.1 explains the welfare criterion I use to evaluate different policies. I highlight the importance of the micro-targeted policies in Subsection 1.6.2. Finally, Subsection 1.6.3 describes the countercyclical policy targeted at high leveraged firms.

1.6.1 Welfare Calculation

I use [Lucas \(1987\)](#) measure to evaluate welfare changes associated with policy interventions. This criterion calculates the percent of additional lifetime consumption that must be endowed at all future dates and states to a representative household under no policy so that the expected welfare is the same as in the economy where policy is implemented. Technically, the welfare criterion takes the following form:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(\widehat{C}(1+x), \widehat{N}) \right] = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(\widetilde{C}, \widetilde{N}) \right], \quad (1.21)$$

FIGURE 1.13: WELFARE ASSESSMENT OF ENTRY COST REDUCTION POLICY



Notes: Figure 1.13 summarizes the welfare benefits associated with the entry cost reduction government policy. The solid black line corresponds to the policy targeted toward μ_H firms. The blue dashed line corresponds to the policy applied to all firm types. Refer to Subsection 1.6.2 for details.

where hats denote a status-quo allocation (no policy), and wiggles—an outcome under the policy considered. Parameter x governs the percent of lifetime consumption which makes the household in the status-quo economy indifferent between the two economies. In other words, I measure welfare benefits in consumption equivalent units.

1.6.2 Subsidizing the Entry Cost

The policy I consider takes the form of a reduction in the cost of entry in recessions. In particular, the cost of starting a business becomes time-varying, and takes the following form:

$$\tilde{\chi}_t = \chi \times (1 - d \times \mathbf{1}_{\{Y_t < Y_{ss}\}}), \quad d \in [0, 1], \quad (1.22)$$

where d is the share of the cost subsidized by the government. Note that this policy will reduce the tax rebate to the household; therefore, such a stimulus policy will yield welfare gains as long as the benefits associated with a more enthusiastic business formation exceed the lower tax rebate from the government.

The quantitative results of this policy are summarized in Figure 1.13. I find that if the government is able to target this policy toward μ_H firms only, the welfare gains can reach 0.3% of lifetime consumption. Intuitively, this occurs because there are very few high type firms, but which collectively account for a significant share of investment and employment

TABLE 1.8: WELFARE ASSESSMENT OF GOVERNMENT COUNTERCYCLICAL POLICY, % OF LIFETIME CONSUMPTION

	$\iota = 0.2$	$\iota = 0.25$
$\xi = \mathbb{E}[n] \times w_{ss}$	0.12	0.17
$\xi = 1.5 \times \mathbb{E}[n] \times w_{ss}$	0.13	0.21

Notes: Table 1.8 reports the results of welfare assessment of the policy described in Subsection 1.6.3. The reported numbers are in percent of lifetime consumption (see Subsection 1.6.1 for details). w_{ss} is the equilibrium wage rate in the steady state.

growth. Therefore, the government can generate a welfare improvement by devoting a small fraction of resources to subsidizing the formation of such businesses.

In stark contrast with the targeted policy case, my simulations show no welfare benefits associated with the policy which subsidizes the formation of all firm types. This occurs because such a policy is not cost-efficient: the welfare benefits associated with stimulating the formation of low and medium type businesses do not outweigh the loss in the tax revenue.

1.6.3 Subsidizing Leveraged Firms

Next, I consider the policy which takes the form of a subsidy to highly leveraged firms in recessions. Formally, the new budget constraint of a firm becomes:

$$i_j(k, b, z; \mathbf{s}) = \Pi_j(k, b, z; \mathbf{s}) + E_j(k, b, z; \mathbf{s}) + \Delta b_j(k, b, z; \mathbf{s}) + \xi \mathbf{1}_{\{Y_t < Y_{ss}\}} \mathbf{1}_{\{\frac{b}{k} \geq \iota\}}, \quad (1.23)$$

According to Equation (1.23), in a recession—whenever output falls below the steady-state level $Y_t < Y_{ss}$ —the highly levered firms (those with leverage ratio exceeding ι) get additional resources of ξ . By varying parameters ι and ξ , I explore the generosity of the proposed policy. Table 1.8 reports the results. I find the largest welfare gains when the government targets the policy towards the most constrained firms ($\iota = 0.25$). I also find that welfare gains fall more when the size of the policy is large ($\xi = 1.5\mathbb{E}[n]w_{ss}$), and the government applies it to less constrained firms. This mirrors the cost-efficiency logic from the previous subsection.

1.7 Conclusion

Motivated by the empirical evidence that there are less fast growing firms started in recessions than in expansions, and that rapidly growing firms are more financially constrained, I build and estimate a general equilibrium model of firm dynamics where these patterns

arise naturally. The two key model ingredients are the ability of potential entrepreneurs to target their startup efforts towards project of different target size, and that firms' growth is hampered by financial frictions. The estimated model shows that the intensive margin of firm entry is quantitatively pronounced at business cycle frequencies, and that financial shocks account for a large share of fluctuations in the formation of rapidly growing firms. I also discuss the implications for the government stimulus policy.

I see several fruitful avenues for future research. While this paper studied the intensive margin from the perspective of the industry dynamics literature, new insights can be obtained by looking at this phenomenon through the lens of models with occupational choice. For instance, such frameworks can speak to the labor market implications of the compositional effect, and, thus, lead to new policy prescriptions.

Second, more empirical work is needed to better understand the decisions of entrepreneurs to start firms with different growth potential. Individual labor market experiences as well as educational, family and demographic factors can all play an important role in this process; new empirical evidence can help inform structural models.

Finally, my model can have important financial market implications. For example, by way of incorporating firms' default decisions into my model, one can study firm-level spreads and, therefore, the risk dimension of business formation. Such a model can shed light on, for example, the behavior of stock prices for innovative (high growth) firms ([Pástor and Veronesi, 2009](#)). I leave these issues to future research.

Chapter 2

Cyclical Labor Income Risk

2.1 Introduction

How does individual labor income risk change between economic expansions and contractions? What drives the cyclical nature of labor income risk—wages or hours? How effective are private (through the second earner) and public (through government taxes and transfers) insurance channels in stabilizing labor income risk? We address these questions by estimating time-varying second and third moments of earnings shocks for several income types by drawing on the Panel Study of Income Dynamics (PSID).

At the conceptual level, we systematically analyze cyclicity of risk for several income definitions within an unified estimation framework. We extend the econometric technique proposed by [Storesletten, Telmer and Yaron \(2004\)](#), which is designed to estimate the parameters of income shocks with a time-varying second moment (variance), by extending it to the case with a time-varying third moment (skewness). The list of income definitions we consider includes individual (head's) hourly wages and labor income, joint (spousal) labor income, and post-government (taxes and transfers) joint labor income. We also consider head's labor income for a subsample of heads with strong labor market attachment ([Abowd and Card, 1989](#); [Meghir and Pistaferri, 2004](#); [Guvenen, Ozkan and Song, 2014](#)), identified by labor income exceeding a certain income threshold. For the sake of convenience, hereafter we refer to the head's labor income defined this way as a *narrowly defined* head's labor income.

We make several findings. First, head's labor income exhibits countercyclical income risk, in the sense that both variance (countercyclical variance) and right skewness (procyclical

skewness) increase in contractions. Second, head's hourly wage is less cyclical than head's labor income in both variance and skewness. This implies that changes in hours, possibly due to unemployment, are behind the cyclicity of head's labor income risk.

Third, we investigate the role of private (through labor income of the second earner) and public (through government taxes and transfers) insurance in mitigating the level and cyclicity of labor income risk. We find that existence of the second earner lowers the overall *level* of skewness of income risk, as well as its *cyclicity*. Effects on variance of labor income risk are limited. On the other hand, government taxes and transfers are found to lower the cyclicity of both variance and skewness. These results are found to have important implications for the probability of disasters: both private and public insurance channels mitigate by a factor of 2 the fall in labor income following a large (3 standard deviations) negative income shock in recessions.

Finally, we reconcile seemingly conflicting evidence between [Storesletten, Telmer and Yaron \(2004\)](#), who find countercyclical variance of labor income risk, and [Güvenen, Ozkan and Song \(2014\)](#) and [Busch, Domeij, Güvenen and Madera \(2018\)](#), who find acyclical variance and procyclical skewness of labor income risk. We find that using narrowly-defined individual labor income can partially explain the differences between the two. When we look at narrowly-defined individual labor income, cyclicity of variance is found to be sizably weaker than that of head's labor income. Besides, government taxes and transfers also ameliorate cyclicity of variance. On the other hand, narrowly-defined head's labor income exhibits cyclicity of skewness almost as strongly as head's labor income.

We contribute to the literature by studying cyclicity of both variance and skewness of labor income risk for various definitions of labor income. [Storesletten, Telmer and Yaron \(2004\)](#) serves as the classic benchmark in the literature. They propose a novel estimation methodology of the countercyclical income risk, and find that standard deviation of labor income shocks is 80% higher in recessions than in expansions. We extend their study in two ways. First, we allow for time-varying skewness in addition to time-varying volatility. Second, we investigate 5 definitions of labor income, which allows us to study roles of various factors affecting the cyclicity of labor income risk, such as second earners and government taxes and transfers.

Deepening our understanding of labor income risk is important as labor income risk, together with market incompleteness, is found to be crucial for many important questions in

macroeconomics. The literature that studies the role of labor income risk in macroeconomics goes back at least to [Deaton \(1997\)](#). [Storesletten, Telmer and Yaron \(2001\)](#) argue that cost of business cycles changes significantly if cyclical movement of labor income risk is taken into account. [Kaplan and Violante \(2010\)](#) find that the degree of consumption smoothing depends on the nature of labor income risk—whether it is persistent or transitory. [Heathcote, Storesletten and Violante \(2010b\)](#) explore the welfare implications of the increasing wage volatility in the U.S., and find that it benefits recent generations of workers as higher educational premium improves college attainment and redistributes labor within the household. [Güvenen, Karahan, Ozkan and Song \(2019\)](#) document a related empirical regularity: they show that in the data, income growth rate is very small for most individuals, while there is a considerable mass of people with very large growth rates. Therefore, high-order moments (kurtosis in this case) are important features of income growth distribution. In macro-finance literature, [Constantinides and Duffie \(1996\)](#) and [Storesletten, Telmer and Yaron \(2007\)](#) find that considering labor income risk could partially solve the risk premium puzzle.

The rest of paper is organized as follows. Sections 2.2 and 2.3 describe the data and lay out the estimation methodology. In Section 2.4, we estimate and analyze individual labor income risk with time-varying variance for various definitions of labor income. We extend the methodology to allow for time-varying skewness in Section 2.5. We provide economic interpretation of our results in Section 2.6. Section 2.7 concludes.

2.2 Data

Background We draw on the Panel Study of Income Dynamics (PSID) data, which is the longest publicly-available panel data on the U.S. population. PSID started in 1968 with more than 2,000 U.S. families being interviewed on a broad set of topics. The “split-off” families (when family members move out and establish their own households) are also interviewed. PSID spans the time period 1968-2014.

Different Labor Income Types The advantage of PSID for our purposes is the possibility to simultaneously observe several types of labor income. In particular, we consider 5 different definitions of labor income:

1. head’s hourly wage¹,
2. head’s labor income,
3. head’s labor income (narrow definition),
4. joint labor income (head and spouse combined),
5. post-government (taxes and transfers) joint labor income.²

As it was mentioned in Section 2.1, *narrowly defined* head’s labor income refers to those observations for which the labor income exceeds some minimum threshold. This is intended to capture individuals with a strong labor-market attachment. In particular, the income threshold is defined as half of an hourly minimum wage multiplied by 520 hours (13 weeks at 40 hours per week). We use the minimum wage data from the Federal Reserve Economic Data.³ The post-government joint labor income is equal to the joint labor income (head and spouse’s income combined) plus government transfers (unemployment compensation, disability insurance and alike) minus taxes (federal and state). PSID provides imputed values of taxes for a subset of years (1978-1990), but we opt to use as many years of data as possible and, therefore, use TAXSIM to obtain our own estimates of state and federal government tax liabilities for the sample of households. TAXSIM, however, is capable of computing the taxes starting from 1978—no state-level tax regulations are available prior to that year—which forces us to restrict the sample to years 1978-2014. Detailed explanations of how the variables have been constructed are relegated to Appendix B.1.1.

PSID is a survey data which suffers from well-known issues, such as top-coding of labor income, potential misreporting of income and small sample size. We, however, argue that PSID is an appropriate source of data for our exercise for several reasons. First, we are interested in several types of labor income, which necessitates the knowledge of labor income separately for each spouse, transfers received, wages and hours. This information is typically not simultaneously available in other datasets. Second, while the top-coding problem is

¹Throughout the paper, we stick to PSID terminology and call a male earner (husband) a household’s head, unless it is a family with a female being the only earner (in this case, wife is the head). A natural alternative is to mark the top-earner within the family as its “head”; this, however, will make our exercise not directly comparable to previous studies based on PSID data, and we, therefore, opt to use a conventional definition instead.

²In order to make income definition comparable, we normalize both joint and post-government labor incomes by way of dividing them by 2 (equal split between spouses).

³Around 5% of family heads per year earn below the minimum threshold (see Table B.3 in the Appendix).

TABLE 2.1: SUMMARY STATISTICS

	Wage	Head's LI	Head's LI (narrow)	Joint LI	Post-Govt LI
Min	0.0	0.0	1554.3	0.0	2352.3
Max	1080.5	588716.5	588716.5	294925.4	197278.9
Median	23.0	52532.5	54199.9	37623.5	36822.2
Std	23.1	45419.2	44490.5	27252.4	19428.5
Bottom 5%	0.0	3684.4	14954.3	10035.3	13074.1
Top 5%	61.7	578327.4	578327.4	294358.3	195537.5

Notes: Table 2.1 reports the summary statistics for the PSID-based sample spanning the years 1978-2014 (see Appendix B.1.2 for details on data construction). The description of income variables (wages, head's labor income, head's labor income (narrow), joint labor income, post-government joint labor income) is in Appendix B.1.1. All nominal variables were deflated by CPI with 2010 being the base year. The statistics were calculated using PSID sample weights.

particularly acute for studying income inequality (especially at the right tail of distribution), in this paper we are primarily interested in general swings of income risk over the business cycle. We, therefore, do not expect this issue to affect our results considerably.

Finally, there is a technical issue with changing frequency: years 1969-1995 are covered with annual frequency, and 1996-2014 with biannual frequency. As it will become clear in Section 2.3, it is straightforward to handle gaps in the data using our methodology.

The rest of this section—Subsections 2.2.1 and 2.2.2—discusses sample selection and identification of business cycles.

2.2.1 Sample Selection

We broadly follow the sample selection strategy of [Storesletten, Telmer and Yaron \(2004\)](#). In PSID, the object of analysis is a family unit (FU). We track heads of FUs as follows: if FU contains a married couple, then the husband is arbitrarily assigned to be the head. A woman can be the head only if husband is missing. In our analysis, we treat split-off families as new independent families: that is, when the head of the household changes, we record it as a new family unit.

Next, we apply a series of selection criteria to construct our dataset. First, a FU is in our sample as long as the age of the head is between 23 and 60 years old. By doing so, we pick only those households where the head is likely to have finished the educational phase of his or her life and entered the labor market.

Second, we drop all families with zero or negative total labor income in any year. We

also drop families with extreme labor income growth rates⁴ (Meghir and Pistaferri, 2004). Observations with top-coded values are also dropped.⁵

Finally, we drop families which are part of the Survey of Economic Opportunity or Latino subsample.⁶ This leaves us with approximately 55,000 observations. Table 2.1 provides the summary statistics for the final dataset.

Appendix B.1.2 contains more details on the process of sample selection. Table B.1 reports the number of observations retained at each step of data preparation.

2.2.2 Identifying Business Cycles

There is no unique way to classify years into “expansions” and “contractions”. Even though PSID is the longest available panel data, its time span covers few recessionary periods as defined by NBER.⁷ It has become a working standard in the literature to classify years into stages of business cycle based on whether the growth rate of some macro aggregate was above or below the long-run mean in that particular year: for example, Storesletten, Telmer and Yaron (2004) used GNP per capita and Lee and Mukoyama (2015) used real GDP. We opt to use the real GNP per capita growth rate as a determinant of economic expansions and contractions in our estimation exercise, leaving the analysis of alternatives to Appendix B.2. One of the reasons we prefer GNP per capita growth rate is that it keeps our exercise close to Storesletten, Telmer and Yaron (2004) which is important for comparison purposes. Appendix B.1.7 shows that classifications based on GDP and GNP per capita yield comparable results.

2.3 Estimation Methodology

We follow the estimation methodology proposed by Storesletten, Telmer and Yaron (2004) for three reasons. First, this is a parsimonious way to estimate the time-varying variance,

⁴That is, we keep household i as long as

$$\Delta \ln(y_{it}) \in \left(\frac{1}{20}, 20 \right) \quad \forall t.$$

⁵In order to ensure that our estimates are not affected by extremely large and very small labor incomes, we drop the top and bottom percentiles of labor income distribution.

⁶Such families have 1968 interview ID in the interval [5000, 7000] (Survey of Economic Opportunity), or above 7000 (Latino subsample).

⁷Recessionary years according to NBER are 1970, 1974-75, 1981-82, 1991, 2001, 2008-09.

and the estimates reported by the aforementioned study serve as a natural reference point. Second, the parametric assumptions this methodology relies on help mitigate small-sample size issues which are typical for easy-to-access datasets. Third, this methodology can be extended to allow for a time-varying skewness (see Section 2.5.3). Finally, the methodology can easily accommodate the change in the frequency of PSID from annual to biennial in the middle of the sample period (from 1996 onwards). In what follows, we give a brief summary of the estimation methodology in Subsection 2.3.1, and then provide an identification argument for this method in Subsection 2.3.2.

2.3.1 Overview of the Method

Let y_{it}^h be the natural logarithm of labor income of household i of age h in year t . We first project log labor income on a set of observables:

$$y_{it}^h = g(x_{it}^h, Y_t) + u_{it}^h, \quad (2.1)$$

where x_{it}^h is the deterministic component of household-specific income attributable to age, education and family size. Y_t is a measure of aggregate conditions at time t , which picks up the business cycle component of individual labor income.

The residual u_{it}^h is a random component which under standard assumptions satisfies the orthogonality condition

$$\mathbb{E}(u_{it}^h | Y_t, x_{it}^h) = 0 \quad \forall t.$$

Intuitively, the residual captures variation in labor income which cannot be attributed to personal characteristics (such as differences in education), and is not explained by aggregate conditions (information contained in Y_t).

Next, a certain structure is imposed on the $\{u_{it}^h\}$ process. In particular, it is assumed that the idiosyncratic earnings component u_{it}^h follows the process:

$$\begin{aligned} u_{it} &= \alpha_i + z_{it} + \varepsilon_{it} \\ z_{it} &= \rho z_{i,t-1} + \eta_{it}. \end{aligned} \quad (2.2)$$

Here α_i is a time-invariant fixed effect which household i draws at the beginning of its labor market life. Next, ε_{it} is a purely transitory component, while z_{it} is a persistent earnings

component which follows an AR(1) process. Random variables α_i , ε_{it} and η_{it} are independent across time and space, and are drawn from some distributions characterized by the mean, variance and skewness:

$$\begin{aligned}\alpha_i &\sim F^\alpha(\mu_1^\alpha, [\mu_2^\alpha]^2, \mu_3^\alpha), \\ \varepsilon_{it} &\sim F^\varepsilon(\mu_1^\varepsilon, [\mu_2^\varepsilon]^2, \mu_3^\varepsilon), \\ \eta_{it} &\sim F^\eta(\mu_1^\eta, [\mu_{2,t}^\eta]^2, \mu_{3,t}^\eta).\end{aligned}$$

The means of the corresponding distributions are set equal to zero: $\mu_1^\alpha = \mu_1^\varepsilon = \mu_1^\eta = 0$.

The model is capable of picking up the time-varying labor income risk, since it allows both the variance and skewness of innovations to the persistent component η_{it} to be a function of the aggregate state:

$$\mu_{2,t}^\eta = \begin{cases} \mu_2^E & \text{if expansion at } t \\ \mu_2^C & \text{if contraction at } t, \end{cases} \quad (2.3)$$

and

$$\mu_{3,t}^\eta = \begin{cases} \mu_3^E & \text{if expansion at } t \\ \mu_3^C & \text{if contraction at } t. \end{cases} \quad (2.4)$$

Therefore, there are 7 parameters to estimate in total:

$$\Theta = \{\rho, \mu_2^\alpha, \mu_2^\varepsilon, \mu_2^E, \mu_2^C, \mu_3^E, \mu_3^C\}.$$

We estimate Θ by the Generalized Method of Moments (GMM), using the moment conditions that relate the cross-sectional variance and skewness of estimated residuals \hat{u}_{it}^h with the history of shocks households experienced throughout their labor market life.⁸ For the rest of this section, we restrict ourselves to a discussion of a time-varying volatility, and will get back to the case of cyclical skewness in Section 2.5.

Using an independence assumption, we can express the variance of a labor income shock of

⁸We assume that individuals enter labor market at the age of 23.

family i with the head aged h in year t as:

$$\begin{aligned}
\text{Var}[u_{it}^h] &= \text{Var}[\alpha_i + z_{it} + \varepsilon_{it}] \\
&= [\mu_2^\alpha]^2 + [\mu_2^\varepsilon]^2 + \text{Var}[\rho z_{it-1} + \eta_{it}] \\
&= [\mu_2^\alpha]^2 + [\mu_2^\varepsilon]^2 + \sum_{j=0}^{h-1} \rho^{2j} [I_{t-j}[\mu_2^E]^2 + (1 - I_{t-j})[\mu_2^C]^2]. \tag{2.5}
\end{aligned}$$

In Equation (2.5), I_t is an indicator of an aggregate expansion in year t . The sample analog of the population moment (2.5) takes the form:

$$\frac{1}{N_{ht}} \sum_{i=1}^{N_{ht}} \left\{ [u_{it}^h]^2 - ([\mu_2^\alpha]^2 + [\mu_2^\varepsilon]^2) - \sum_{j=0}^{h-1} \rho^{2j} [I_{t-j}[\mu_2^E]^2 + (1 - I_{t-j})[\mu_2^C]^2] \right\} = 0. \tag{2.6}$$

Here, N_{ht} is the number of families at time t with a head aged h . Note that μ_2^α and μ_2^ε are not identified separately.⁹ In this paper, we are not interested in either of those parameters separately and thus we estimate their sum $\mu_2^\alpha + \mu_2^\varepsilon$.¹⁰

There are $H \times T$ moments of type (2.6) in total, with H denoting the number of different ages in the data, and T —the number of years.¹¹ Furthermore, we aggregate moment conditions so that the number of observations in any $\{H, T\}$ cell does not fall below 100. To accomplish this, we break down all feasible ages 23-60 into 4 age groups indexed by $h \in \{25, 35, 45, 55\}$, and make each group contain ages ± 5 from the mean age within the group.¹² These adjustments help us balance the two opposing forces: on the one hand, the more moments conditions we use, the more information we extract from the data. On the other hand, more moment conditions lead to some (age, year) cells being too small. Following [Altonji and Segal \(1996\)](#), we weight all moment conditions equally as it was shown that the identity weighting matrix dominates an asymptotically optimal weighting matrix in small samples.

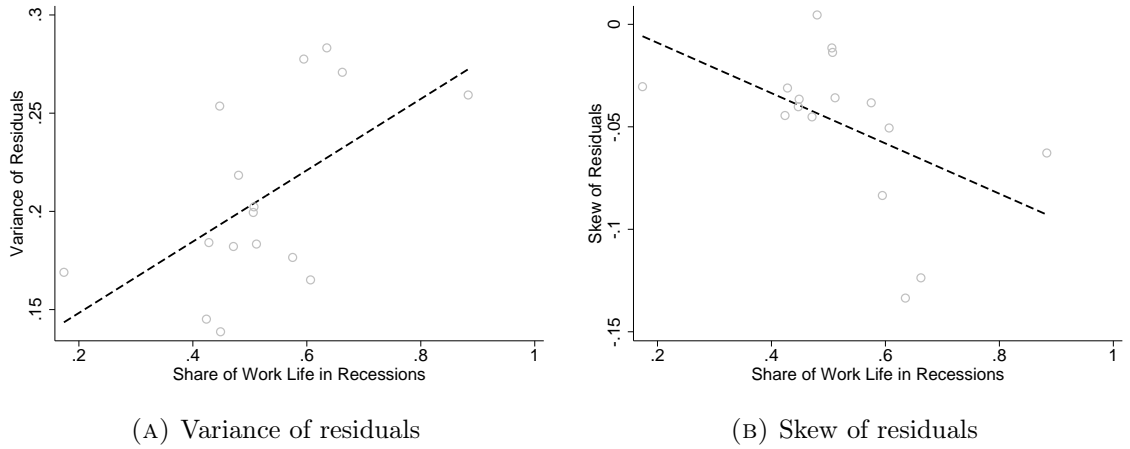
⁹[Storesletten, Telmer and Yaron \(2004\)](#) use additional moment conditions for the autocovariances of u_{it}^h in order to disentangle these two parameters. Specifically, they obtain $\widehat{\mu_2^\alpha + \mu_2^\varepsilon} = 0.316$ when they do not disentangle the two, and $\widehat{\mu_2^\alpha} = 0.201$ and $\widehat{\mu_2^\varepsilon} = 0.123$ when the two variances are separately identified.

¹⁰As we find below, our estimates of $\mu_2^\alpha + \mu_2^\varepsilon$ are close to what [Storesletten, Telmer and Yaron \(2004\)](#) report.

¹¹This amounts to $37 \times 38 = 1,406$ moment conditions.

¹²Precise distribution of ages across 4 groups is as follows: group $h = 25$ contains ages 23-30, group $h = 35$ contains ages 31-40, group $h = 45$ encompasses ages 41 – 50, and group $h = 55$ aggregates the remaining ages 51-60.

FIGURE 2.1: ILLUSTRATION OF THE ACCUMULATION ARGUMENT



Notes: Figure 2.1 is based on PSID data over the period 1978-2014. Panel A plots the variance of residuals \hat{u} against the share of working life spent in recessions. Panel B plots the skew (measured by the third central moment) of residuals. The statistics were aggregated within 4 age groups (23-30, 31-40, 41-50 and 51-60); the graphs depict the data corresponding to years 1980, 1990, 2000 and 2010.

2.3.2 Identification

The way estimation is set up in Section 2.3.1 highlights its benefits (see Equation 2.5): even though there are few families in the dataset whose working life we observe entirely—from the year when its head enters the labor market till the year when he or she retires—we can still incorporate the entire history of business cycle fluctuations that every household experienced over its lifetime into the estimation. In other words, the use of cross-sectional moments for identification allows us to exploit macroeconomic information that predates the micro panel, and thus include more business cycles in the analysis than covered by the sample.

The basic idea behind the entire approach is to exploit how the distribution of persistent idiosyncratic shocks accumulates over time: if the income process is persistent (values of ρ are close to 1 in Equation (2.2)), then as a cohort ages, the cross-sectional income distribution at any age becomes a function of the sequence of shocks experienced by the cohort's members. If the variance of income shocks is higher in recessionary years than in expansionary ones, then a cohort that lived through more contractions will have a higher income variance at a given age than a cohort of the same age that lived through fewer contractions. Panel A in Figure 2.1 illustrates this intuition: it shows that the cross-sectional variance of \hat{u} tends to increase as the share of labor market life spent in recessions rises. Each circle corresponds to the variance of \hat{u} computed across households of a certain age in a specific year; the

location of markers along the horizontal axis is determined by the share of working life a particular cohort spent in recessionary years. Figure B.4 in Appendix B.1.5 confirms that the aforementioned upward sloping relationship is present in all 4 age groups considered. The upward sloping pattern is more pronounced among younger cohorts, but the overall level of variance is higher for older households, reflecting their longer labor market history. Our extension of that approach in Section 2.5.3 is based on the insight that a similar “accumulation” argument holds for skewness. If the probability of a large positive income shock is lower during an aggregate contraction, then the skewness of the shock in a recessionary period will be smaller (more negative) than in an expansion. Therefore, by way of comparing two cohorts of the same age, the distribution of residual income for the cohort that lived through more recessions will exhibit a smaller (more negative) cross-sectional skewness. Panel B in Figure 2.1 illustrates this logic: the skewness of income shocks decreases as the share of labor market life spent in recessions rises. Figure B.5 in Appendix B.1.6 additionally shows that this negative pattern is present in all 4 age groups.

2.4 Volatility of Idiosyncratic Labor Income Risk

In this section, we study how variance of labor income risk fluctuates across economic expansions and contractions. Conceptually, our exercise is reminiscent of [Storesletten, Telmer and Yaron \(2004\)](#) in that we estimate the same parameters using the same moment conditions, but we diverge from them in that we explore the nature of fluctuations in riskiness of several income definitions ([Storesletten, Telmer and Yaron \(2004\)](#) use joint labor income *after* transfers but *before* tax).

By studying different types of labor income, we are able to shed more light on the origins of income risk fluctuations. For example, by moving from hourly wage to head’s labor income, we can speak to the quantitative importance of hours (including both intensive and extensive, i.e., employment and unemployment, adjustments) in shaping labor income risk. The intra-family insurance channel can be evaluated through the juxtaposition of risk between head’s and joint (head and spouse) labor incomes. Finally, in order to quantify the role of government policy—including both taxes and transfers—in alleviating the cyclicity of labor income risk, we assess to what extent (pre-government) joint labor income is more volatile than post-government income.

We first conduct a graphical analysis in Subsection 2.4.1, before providing the estimates in Subsection 2.4.2.

2.4.1 Graphical Analysis

In order to shed light on the (counter)cyclical nature of idiosyncratic income shock volatility, we need to obtain the residuals u_{it}^h . We estimate Equation (2.1) by running a pooled regression.¹³ The estimation results are provided in Table B.2 in Appendix B.1.3.

We consider the following specification of function $g(\cdot)$:

$$g(x_{it}^h, Y_t) = \theta_0 + \theta_1' \mathbf{D}(Y_t) + \theta_2' x_{it}^h,$$

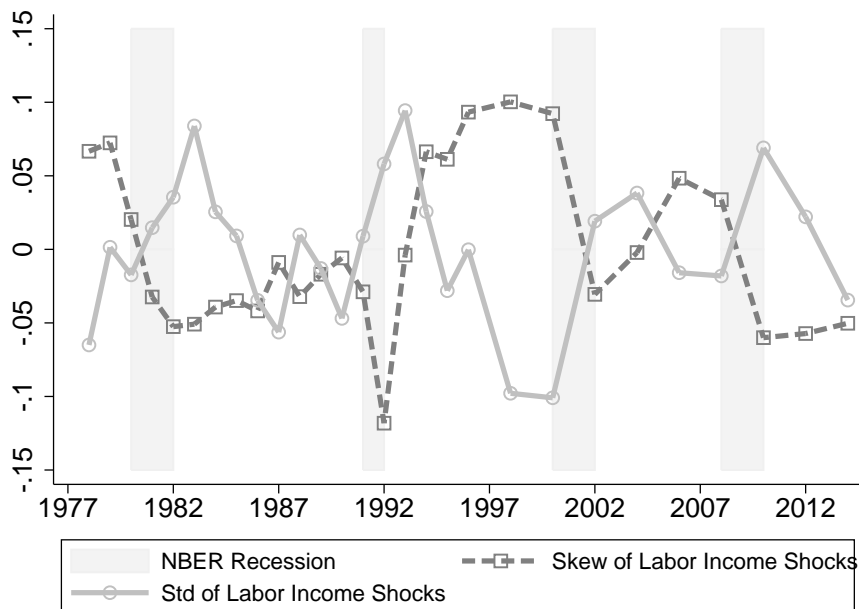
where x_{it}^h includes the following list of observables: cubic polynomial in age, education of head, and the size of the family. Aggregate effects are absorbed by a full set of year fixed effects $\mathbf{D}(Y_t)$. Results in Table B.2 are broadly consistent with a wide body of literature: the earnings age profile is concave and increasing in education, large family sizes are associated with high labor incomes. All estimates are statistically significant and have the expected sign.

We subsequently retrieve \widehat{u}_{it}^h as residuals from the estimated Equation (2.1). Figure 2.2 plots the time series of the second and third moments of \widehat{u}_{it}^h for our sample years. The figure displays two strong patterns: the variance of labor income shocks is countercyclical, decreasing in expansions and increasing in recessions, while the skewness is strongly procyclical.

While Figure 2.2 indicates that the countercyclical income risk is a robust feature of the data, it is silent about the nature of this cyclicity. We, therefore, categorize every sample year into one of 3 bins, depending on the growth rate of real GNP per capita in that year: if GNP per capita grew by a lot (in top tercile of growth rate distribution), we place that year in bin 3. Conversely, if the growth was in a bottom tercile of the growth rate distribution, that year falls in bin 1. Table 2.2 reports the mean and median GNP growth rate for each tercile. Subsequently, we take the average (across years which are sorted in a particular bin) deviation of a corresponding statistic from its long-run mean, and do it for all 5 different income definitions (Figure 2.3).

¹³We also experimented with estimating a panel regression, but the results did not change significantly.

FIGURE 2.2: SECOND AND THIRD MOMENTS OF \hat{u}_{it}^h OVER SAMPLE PERIOD



Notes: Figure 2.2 is based on PSID data over the period 1978-2014. The standard deviation of labor income shocks in year t is a cross-sectional standard deviation of \hat{u}_{it}^h —residuals from the estimated Equation (2.1). The skew of labor income shocks in year t is a cross-sectional Kelley measure. We also subtract linear trends from the resulting series, which chiefly eliminates the long-run mean (the slope coefficient is nearly zero). Grey bars represent NBER recessions.

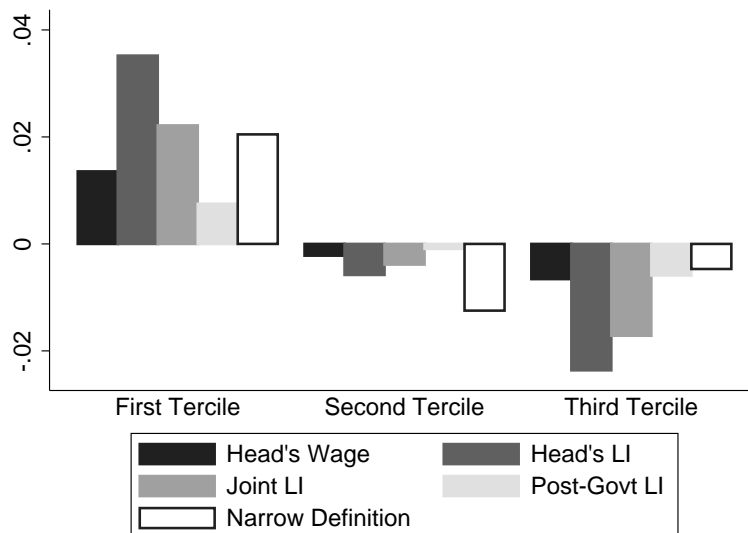
TABLE 2.2: MEAN AND MEDIAN GNP PER CAPITA GROWTH RATE IN EACH BIN

	Tercile 1	Tercile 2	Tercile 3
Mean growth rate, %	-2.8	1.9	6.5
Median growth rate, %	-1.6	1.8	5.3

Notes: Table 2.2 reports the mean and median for each tercile of the distribution of GNP per capita growth rates.

Figure 2.3 anticipates several findings. At the very least, it shows that the volatility of income risk exhibits countercyclicity, and this pattern is robust across different labor income definitions. However, we can say more than that. First, consistently with findings of [Guvenen, Ozkan and Song \(2014\)](#), the narrowly-defined head’s labor income exhibits relatively modest fluctuations in risk over the cycle. Second, head’s wages are less volatile and less cyclical than head’s labor income, pointing at the importance of hours in driving individual labor income risk fluctuations. Third, joint labor income, if anything, exhibits fluctuations in income risk which are smaller than those of the head’s labor income. And, finally, cyclical patterns of post-government and narrowly-defined labor incomes are moderate.

FIGURE 2.3: VOLATILITY OF IDIOSYNCRATIC INCOME RISK BY GNP PER CAPITA GROWTH TERCILE



Notes: Figure 2.3 is based on PSID data over the period 1978-2014. Each year from the period 1978-2014 is classified in one of 3 bins, depending on which tercile the growth rate of GNP per capita in that year falls into. Tercile 1 contains years with the lowest growth rate of GNP per capita, while tercile 3 contains years with the highest growth rates. The standard deviations shown are averages over years in each bin. Each tercile contains standard deviations for 5 measures of labor income: head's wage, head's labor income, joint labor income, post-government joint labor income, and head's labor income (narrow definition).

These cyclical properties we observe here are robust to alternative definitions of the business cycle. In Appendix B.2, we provide analogous figures where we categorize years based on the NBER definition of recessions (B.2.1) and by real GDP growth rate (Section B.2.2).

2.4.2 Estimation Results

In Subsection 2.4.1, we provided suggestive evidence on the countercyclical nature of income shocks volatility. We now take a step forward and estimate the vector of structural parameters which govern the income process (2.2). As it has been discussed above, there are in total $H \times T$ moment conditions of type (2.6). In Section 2.3 we argued that we cannot use all of them, as the sample size of certain age-year cells becomes too small to obtain precise estimates. Instead, we focus on a subset of moment conditions which correspond to ages 25, 35, 45 and 55. We check that there are at least 100 observations in each cell.

Table 2.3 provides GMM estimates for all 5 income definitions. Our results reconcile the findings of previous studies with seemingly conflicting results. On one hand, individual labor income exhibits a sizable countercyclical risk. Specifically, the estimated standard deviation

TABLE 2.3: GMM ESTIMATION RESULTS: TIME-VARYING VOLATILITY

	Head's wage	Head's LI	Joint LI	Post-govt LI	Narrow defn.
μ_2^E	0.12*** (0.01)	0.12*** (0.01)	0.06*** (0.01)	0.05** (0.02)	0.12*** (0.01)
μ_2^C	0.11*** (0.01)	0.18*** (0.01)	0.11*** (0.01)	0.08*** (0.03)	0.12*** (0.01)
ρ	0.96*** (0.01)	0.96*** (0.01)	0.99*** (0.00)	0.90*** (0.06)	0.96*** (0.00)
$\sqrt{\mu_\alpha^2 + \mu_\varepsilon^2}$	0.52*** (0.01)	0.61*** (0.01)	0.48*** (0.01)	0.44*** (0.01)	0.54*** (0.01)
$\mu_2^C - \mu_2^E$	-0.01	0.06	0.05	0.03	0.00

Notes: Table 2.3 reports the estimation results for Θ by GMM based on the moment conditions (2.6). Standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

is 0.12 in expansions and 0.18 in recessions. The ratio of our estimates (0.18/0.12=1.5) is somewhat lower than what [Storesletten, Telmer and Yaron \(2004\)](#) report (0.16/0.09=1.8), but still within the range of estimates they provide. This small difference might arise because of transfers: our definition of individual labor income corresponds to labor earnings before government transfers *and* taxes.¹⁴

On the other hand, the narrowly-defined head's labor income exhibits no countercyclicality of shock volatility. The ratio of the standard deviation in recessions and that in expansions is 1. This finding mirrors the results of [Güvenen, Ozkan and Song \(2014\)](#) who find that the second-moment of income risk is flat with respect to the business cycle. Remarkably, we obtain that result despite several differences in methodology, including the way we identified the income shock (residual from OLS regression, rather than income growth), the estimation approach (parametric, rather than non-parametric), and different data used (PSID vs. Social Security Administration records).

Wage rate also exhibits no countercyclicality, hinting towards an important quantitative role of hours (most likely, employment and unemployment). This finding mirrors the observation from Figure 2.3. Moving from head's labor income to joint labor income, we see that intra-family insurance channel through the added worker effect reduces the level of income risk both in expansions and recessions considerably. Finally, government taxes and transfers further mitigate both the level and cyclicity of income risk.

¹⁴Consistent with this logic, [Heathcote, Perri and Violante \(2010a\)](#) study the distributional effects of taxes and transfers and find that they compress the earnings inequality, especially at the bottom of the distribution.

2.5 Skewness of Idiosyncratic Labor Income Risk

In this section, we extend our analysis to allow for a time-varying skewness of income shocks. While countercyclical variance can tell us that tail events (large positive and negative shock realizations) become more likely during economic downturns, there is a growing body of literature highlighting the importance of the third moment (*inter alia*, [Salgado, Guvenen and Bloom, 2016](#); [Guvenen, Ozkan and Song, 2014](#); [Busch, Domeij, Guvenen and Madera, 2018](#)). Non-zero skewness implies that some extreme shock realizations are likely to be either positive or negative—depending on the sign of the coefficient of skewness. This also implies that constant skewness—something our analysis has implicitly assumed so far—can mask a rich heterogeneity between left- and right-tail events. We proceed as follows. First, in Subsection 2.5.1 we graphically visualize the presence of procyclical skewness for all 5 different definitions of labor income that we consider. In Subsection 2.5.2, we employ an alternative way of measuring income shocks, used by [Guvenen, Ozkan and Song \(2014\)](#), to further facilitate the comparison between our findings and theirs. Finally, in Subsection 2.5.3 we estimate the skewness of income shocks using GMM.

2.5.1 Graphical Analysis

Throughout our analysis of skewness, we consider the following two conventional measures:

1. third central moment:

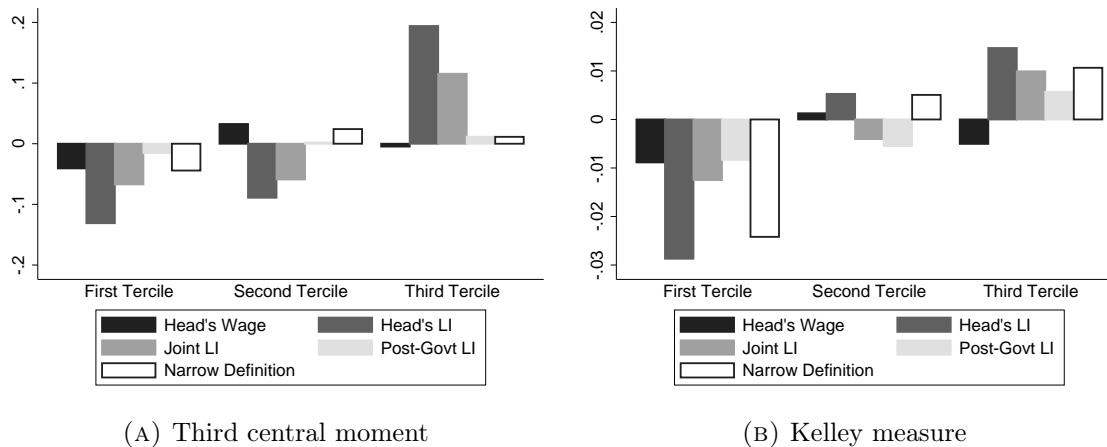
$$\mu_{th}^3 = \frac{\sum_i (\hat{u}_{it}^h - \bar{\hat{u}}_{it}^h)^3}{n_t},$$

2. Kelley measure:

$$\text{Kelley}_t = \frac{(P90_t - P50_t) - (P50_t - P10_t)}{P90_t - P10_t}.$$

The first statistic is a sample analog of the third central moment, while the second one—Kelley measure—is a function of several percentiles of \hat{u}_{it}^h -distribution, which makes it robust to “extreme” observations (note that it is independent from the first and last deciles of the underlying distribution). Intuitively, Kelley measure computes the difference in inequality between the right ($P90 - P50$) and left ($P50 - P10$) tails, and relates it to the overall variation in the sample ($P90 - P10$). If the right tail is heavier than the left one (underlying distribution of \hat{u}_{it}^h has a positive skew), then the Kelley measure is positive. And the other way around, a heavier left tail makes the Kelley measure negative. Procyclical skewness

FIGURE 2.4: SKEW OF IDIOSYNCRATIC LABOR INCOME RISK, BY GNP PER CAPITA GROWTH TERCILE



Notes: Figure 2.4 is based on PSID data over the period 1978-2014. Panel A plots the third central moment, Panel B plots the Kelley measure. Each year from the period 1978-2014 is classified in one of 3 bins, depending on which tercile the growth rate of GNP per capita in that year falls into. Tercile 1 contains years with the lowest growth rate of GNP per capita, while tercile 3 contains years with the highest growth rates. The measures of skewness shown are averages over years in each bin. Each tercile contains skewness measures for 5 different types of labor income: head's wage, head's labor income, joint labor income, post-government joint labor income and head's labor income (narrow definition).

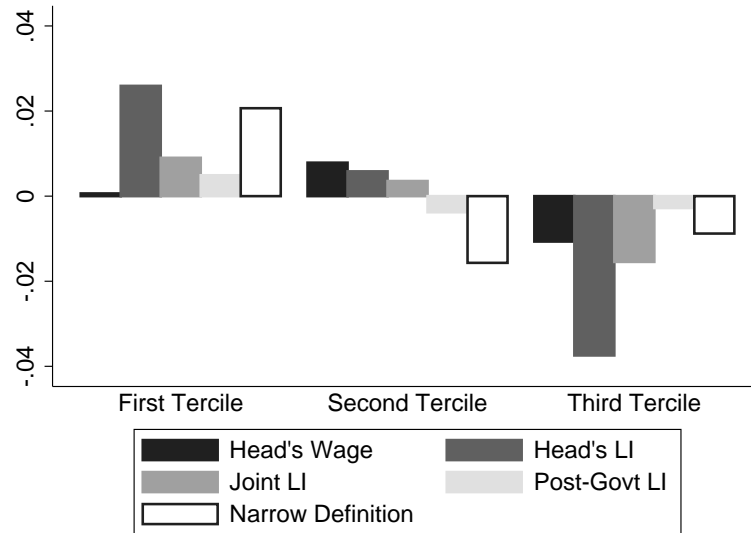
implies that during economic upturns (downturns), the right (left) tail of income shocks thickens, leading to a disproportionate bigger fraction of large positive (negative) shocks. At the same time, the odds of receiving a large negative (positive) shock go down.

In order to shed light on how skewness of income shocks moves over the business cycle, in Figure 2.4 we plot the average skewness of income shocks for expansions and contractions defined in accordance with GNP per capita growth terciles. Panel A plots the third central moment, and Panel B plots the Kelley measure. The figure confirms that the skewness is procyclical, with Kelley measure exhibiting a somewhat stronger cyclical pattern. That means that during economic downturns, a large negative income shock is more likely than an equally large positive one.

A closer inspection of Figure 2.4 reveals that the skewness of post-government joint labor income barely changes over the cycle. It implies that the odds of getting a very negative shock for that income definition co-move with the odds of getting a very positive shock. Both measures of skewness rank post-government income as the one with the most stable skewness.

When we look at household's income without transfers and taxes (joint labor income), we find that the cyclicity of skewness becomes stronger: the probability of getting a large

FIGURE 2.5: LABOR INCOME RISK AS IN [GUVENEN ET AL. \(2014\)](#): VOLATILITY OF LABOR INCOME RISK BY GNP PER CAPITA GROWTH TERCILE



Notes: Figure 2.5 is based on PSID data over the period 1978-2014. Each year from the period 1978-2014 is classified in one of 3 bins, depending on which tercile the growth rate of GNP per capita in that year falls into. Tercile 1 contains years with the lowest growth rate of GNP per capita, while tercile 3 contains years with the highest growth rates. The standard deviations shown are averages over years in each bin. Each tercile contains standard deviations for 5 measures of labor income: head's wage, head's labor income, joint labor income, post-government joint labor income, and head's labor income (narrow definition).

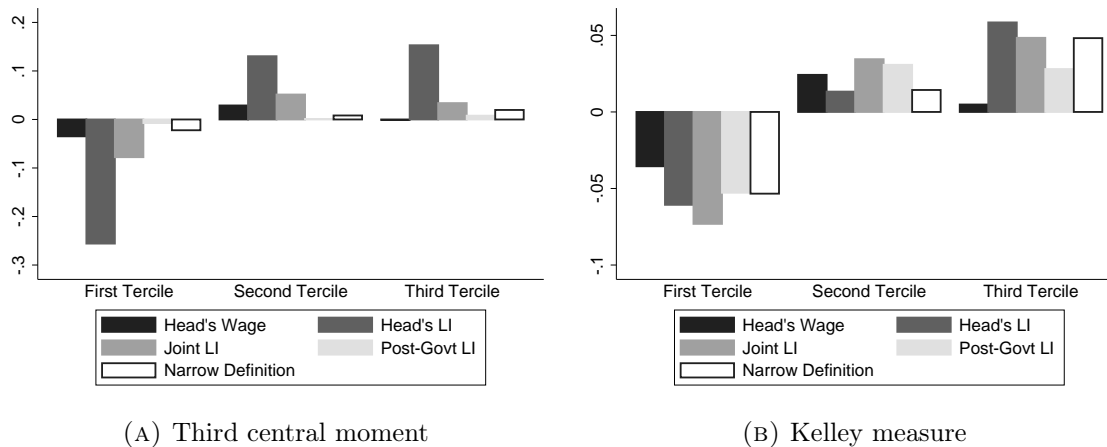
negative income shock increases by more than the odds of getting a large positive shock during economic contractions. Head's labor income exhibits the strongest fluctuations in skewness. This observation might reflect intra-family insurance through an added worker effect: during economic downturns the probability of getting laid off increases, and spouse can step in and compensate for the head's job loss (by working more hours, getting an extra job, etc.). The head's wage exhibits relatively moderate fluctuations in skewness. The narrowly defined labor income shows sizable shifts in skewness.

Figures B.7 and B.9 in Appendix B.2 display similar patterns and confirm that the above observations are broadly robust to alternative ways of business cycle identification.

2.5.2 Labor Income Risk As in [Guvnen, Ozkan and Song \(2014\)](#)

This section explores the cyclical nature of shocks when those are identified as the growth rate of income ([Guvnen, Ozkan and Song, 2014](#); [Busch, Domeij, Guvnen and Madera, 2018](#)). Their approach is non-parametric, and allows to study fluctuations in risk with few

FIGURE 2.6: LABOR INCOME RISK AS IN [GUVENEN ET AL. \(2014\)](#): SKEW OF LABOR INCOME RISK, BY GNP PER CAPITA GROWTH TERCILE



Notes: Figure 2.6 is based on PSID data over the period 1978-2014. Panel A plots the third central moment, Panel B plots the Kelley measure. Each year from the period 1978-2014 is classified in one of 3 bins, depending on which tertile the growth rate of GNP per capita in that year falls into. Tertile 1 contains years with the lowest growth rate of GNP per capita, while tertile 3 contains years with the highest growth rates. The measures of skewness shown are averages over years in each bin. Each tertile contains skewness measures for 5 different types of labor income: head's wage, head's labor income, joint labor income, post-government joint labor income and head's labor income (narrow definition).

identifying assumptions.¹⁵ While in this paper we opt to follow [Storesletten, Telmer and Yaron \(2004\)](#) and use the parametric approach given the size and changing frequency of the dataset, it is important to establish the connection between these two approaches. Taking into account the fact that PSID became biannual starting from 1996, we take the 2-year growth rate as our alternative measure of income shocks.¹⁶

First, regarding the cyclical nature of volatility of idiosyncratic labor income shocks, Figure 2.5 shows that the countercyclicality of the volatility carries over to this alternative way of shock identification. By way of comparing 5 definitions of labor income, one can observe that the head's labor income exhibits the strongest cyclical movement of volatility, while the narrowly-defined head's labor income fluctuates less. This is consistent with our benchmark result using the parametric approach. Moreover, post-government joint labor income exhibits little cyclical volatility, especially when compared with sizable cyclical volatility of individual labor income. This indicates the role public insurance plays in lowering income volatility.

Second, Figure 2.6 confirms that procyclicality of skewness, measured as the third central

¹⁵[Güvenen, Ozkan and Song \(2014\)](#) differentiate between the transitory and persistent components of income. *Transitory* component is measured as a 1-year growth rate ($\Delta \log y_{it}$). *Persistent* component is a 5-year growth rate ($\Delta_5 \log y_{it}$).

¹⁶[Busch, Domeij, Güvenen and Madera \(2018\)](#), who also use the PSID, use 1-year growth rate before 1996 and 2-year growth rate starting from 1996 to measure income shocks.

moment (Panel A) and Kelley measure (Panel B), is also preserved when shocks are measured as growth rates of income. Both measures of skewness exhibit strong procyclicality, declining sharply during contractions. Overall, our key qualitative results are robust to the non-parametric way of income risk measurement; for various definitions of labor income, variance of income risk is countercyclical, while skewness is procyclical, but the cyclicity of variance is dampened when labor income is narrowly defined.

2.5.3 Joint Estimation of Cyclical Volatility and Skewness

Motivated by evidence from Subsection 2.5.1 that different income categories exhibit systematic procyclical skewness, in this section we extend the methodology laid out in Section 2.3 to handle the time-varying skewness of income innovations.

Conceptually, our insight is based on the accumulation argument we graphically shown in Figure 2.1 Panel B: the distribution of residual income for cohorts that lived through more recessions exhibits a smaller (more negative) cross-sectional skewness than cohorts which lived through fewer recessionary episodes. Therefore, we identify the time-varying skewness of income shocks by relating the skewness of labor incomes of cohorts with different aggregate histories.

Theoretically, assuming non-skewed distributions for α and ε , we can express the skewness of the residual labor income of cohort aged h in year t as

$$\mu_{3,t}^h = \sum_{j=0}^{h-1} \rho^{3j} [I_{t-j} \mu_3^E + (1 - I_{t-j}) \mu_3^C]. \quad (2.7)$$

where μ_3^E and μ_3^C are coefficients of skewness for the persistent innovation η in expansions and recessions, respectively. The sample analog of (2.7) is given by

$$\frac{1}{N_{ht}} \sum_{i=1}^{N_{ht}} \left\{ \mu_{3,t}^h - \sum_{j=0}^{h-1} \rho^{3j} [I_{t-j} \mu_2^E + (1 - I_{t-j}) \mu_2^C] \right\} = 0. \quad (2.8)$$

Table 2.4 reports the results of GMM estimation. There are several observations. First, the skewness appears to be procyclical, confirming graphical analysis in Subsection 2.5.1. Second, the skewness of wages is similar across expansions and recessions, while there is a large procyclical skewness in case of individual labor income; this points at the importance of hours worked—rather than wages—in accounting for a large increase in probability of labor

TABLE 2.4: GMM ESTIMATION RESULTS: TIME-VARYING SKEWNESS

	Head's wage	Head's LI	Joint LI	Post-govt LI	Narrow defn.
μ_3^E	-0.01*** (0.00)	-0.09*** (0.00)	-0.01*** (0.00)	-0.00*** (0.00)	-0.07*** (0.00)
μ_3^C	-0.01*** (0.00)	-0.13*** (0.00)	-0.03*** (0.00)	-0.01*** (0.00)	-0.10*** (0.00)
$\mu_3^C - \mu_3^E$	0.00	0.04	0.02	0.01	0.03

Notes: Table 2.4 reports the estimation results for Θ by GMM based on the moment conditions (2.8). Standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

income drops during economic downturns.

Furthermore, moving from individual to joint labor income, we find that the latter exhibits twice smaller movements in skewness over the cycle (0.04 and 0.02, respectively). Remarkably, the level of skewness is also substantially smaller in case of joint labor income. This observation signals about potentially large private (within family) insurance channels during adverse aggregate conditions.

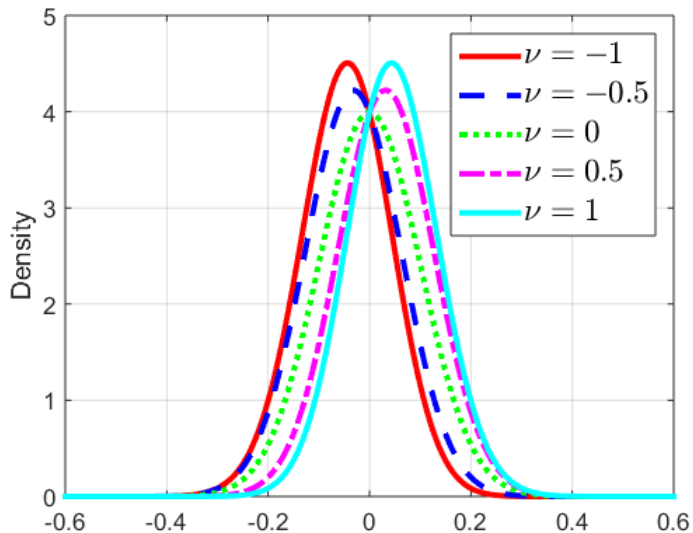
Labor income after taxes and transfers shows very modest shifts in skewness (-0.01 in recessions and 0.00 in expansions), reflecting a quantitatively important role government policy plays in reducing the downside income risk during contractions. Finally, the narrowly defined labor income shows sizable movements in skewness (-0.10 in recessions and -0.07 in expansions); thus, it is not surprising that [Güvenen, Ozkan and Song \(2014\)](#) found procyclical skewness along with acyclical variance.

While the estimates of skewness and variance are suggestive, they are not directly informative about the income risks households face at different stages of the business cycle. The key reason is that both the volatility *and* skewness of income shocks simultaneously determine the probability of tail events. In the next section, we explore the economic implications of the reported estimates.

2.6 Economic Interpretation

In this section, we provide economic interpretations for our estimates. First, we argue that the skew normal distribution is a reasonable representation of income shocks (Subsection 2.6.1), and subsequently we graphically show how shock distributions change depending on the aggregate economic state (Subsection 2.6.2). Third, in Section 2.6.3, we quantify the

FIGURE 2.7: SKEW NORMAL DENSITY FOR DIFFERENT VALUES OF ν



Notes: Figure 2.7 plots the skew normal density for 5 values of $\nu \in \{-1, -0.5, 0, 0.5, 1\}$. The shape parameter ω is 0.1, and location parameter ζ is set to 0.

changes in probabilities of positive and negative income shocks over the business cycle. Our results imply a substantial heterogeneity in probabilities of tail events across income types. Finally, in Section 2.6.4, we discuss how to interpret the magnitude of cyclical changes in labor income risk.

2.6.1 Skew Normal Distribution

We assume that innovations to a persistent component η_{it} are drawn from a *skew normal distribution*, which is a generalization of a normal distribution to the case with a non-zero skewness:

$$\eta_{it} \sim \mathcal{SN}(\zeta, \omega_t, \nu_t). \quad (2.9)$$

The skew normal distribution is a family of probability distributions governed by 3 parameters: location ($\zeta \in \mathbb{R}$), scale ($\omega \in \mathbb{R}_{++}$), and shape ($\nu \in \mathbb{R}$).¹⁷ We assume that the location parameter is business cycle invariant, and we normalize it to 0.

Figure 2.7 shows how the shape parameter ν governs the skewness: the corresponding density

¹⁷The p.d.f. of the skew normal distribution is $f(x) = 2\phi(x)\Phi(\nu x)$, where $\phi(x)$ and $\Phi(x)$ are p.d.f. and c.d.f. of the standard normal distribution, respectively. In case of symmetric distribution ($\nu = 0$), the formula collapses to a standard normal p.d.f.

tends to be skewed towards more positive values (positive skew) if $\nu > 0$, and towards negative values (negative skew) if $\nu < 0$.

We make the variance and skewness be state-dependent by allowing the shock structure to change between expansions and contractions. In particular, the support of both ω_t and ν_t consists of two points:

$$\omega_t = \begin{cases} \omega_E & \text{if expansion at } t \\ \omega_C & \text{if contraction at } t \end{cases}$$

and

$$\nu_t = \begin{cases} \nu_E & \text{if expansion at } t \\ \nu_C & \text{if contraction at } t. \end{cases}$$

Given our estimates of variance and skewness (Tables 2.3 and 2.4), we recover the vector of scale and shape parameters $\{\omega_E, \omega_C, \nu_E, \nu_C\}$ for each income category.¹⁸

Note that while the skew normal distribution has bounded skewness¹⁹, at no point in our estimation step we imposed that restriction. Rather, we verify ex post that the skewness never goes outside that interval, and thus the skew normal distribution is a reasonable choice for the representation of persistent income shocks.

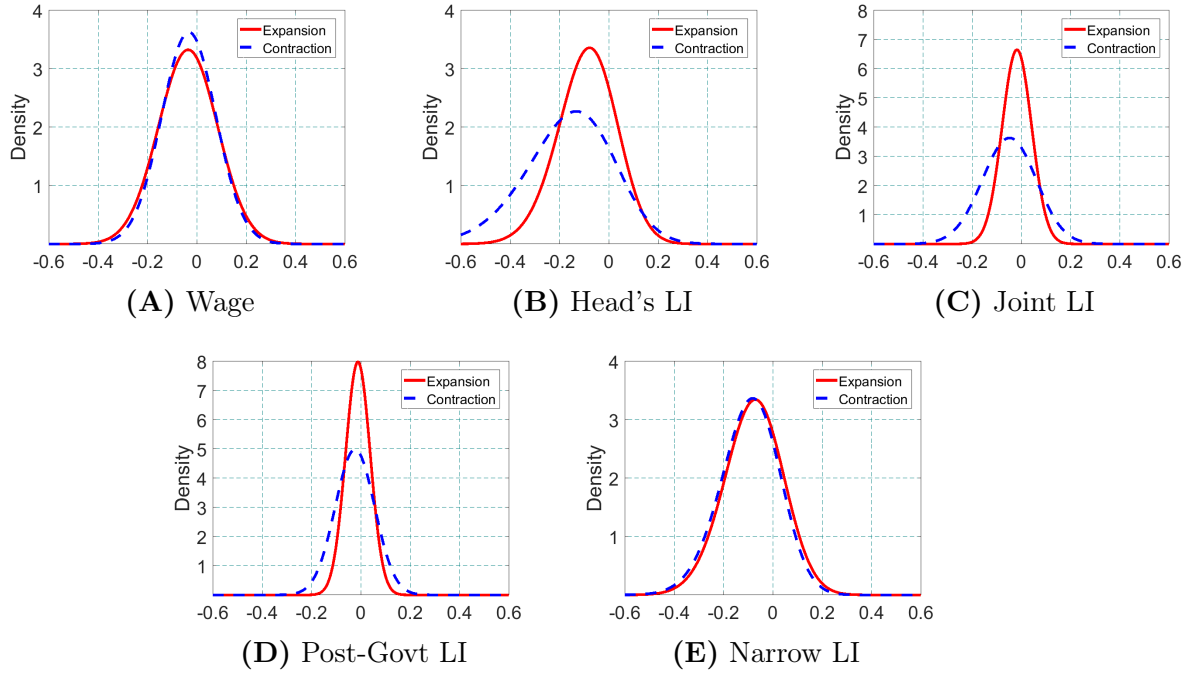
2.6.2 Implied Distributions of Income Shocks

In this subsection, we plot the implied distributions of persistent income shocks η_{it} using the estimates from Tables 2.3 and 2.4. Figure 2.8 shows distributions of shocks η_{it} in expansions (solid red lines) and in recessions (dashed blue lines), for 5 labor income definitions. The most striking difference between expansion and contraction distributions of income shocks is observed in case of the head's labor income (Panel B): while the solid red line (expansion) is fairly symmetric around 0 with a small variance, the blue dashed line (contraction) is substantially more dispersed with a heavy left tail. This observation implies that individuals are more likely to be hit by a large negative (rather than positive) shock during economic contractions. However, when the economy is expanding, tail events, both positive and

¹⁸The variance of a skew normal random variable with parameters ζ, ω and ν is $\omega^2 \left[1 - \frac{2\delta^2}{\pi}\right]$, where $\delta = \frac{\nu}{1+\nu^2}$. The skewness is given by $\frac{4-\pi}{2} \frac{(\delta\sqrt{2/\pi})^3}{(1-2\delta^2/\pi)^{3/2}}$. Therefore, we recover parameters ω and ν by equating the estimated variance and skewness to these expressions, and simultaneously solving the resulting system of two equations.

¹⁹The skewness of a skew normal random variable lies in the interval $[-1, 1]$.

FIGURE 2.8: IMPLIED DISTRIBUTIONS OF PERSISTENT INCOME SHOCKS: EXPANSIONS AND CONTRACTIONS



Notes: Figure 2.8 plots the estimated distributions of persistent income shocks η_{it} for economic expansions (solid red) and contractions (dashed blue). The parameters of those distributions are taken from Tables 2.3 and 2.4.

negative, become substantially less likely.

Panel A of Figure 2.8 confirms again that it is the number of hours worked rather than hourly wages that drives a significant portion of countercyclical head’s labor income risk. In particular, the recessionary and expansionary distributions of wage shocks nearly coincide thus attributing all the fluctuations in labor income risk to hours.

The role of intra-family insurance channels is visible in Panel C of Figure 2.8—as compared to head’s labor income, variance of shocks is substantially lower both in expansions and recessions, and probability of large negative shocks during downturns is reduced. Government transfers and taxes (Panel D) smooth out countercyclical risk and procyclical skewness significantly, even after the spousal channel has been taken into account. Finally, labor income risk for people with a strong market attachment (narrow definition) exhibits more moderate swings in volatility and skewness over the cycle as compared with individual labor income (Panels E vs. B).

TABLE 2.5: CHANGE IN PROBABILITY OF POSITIVE AND NEGATIVE INCOME SHOCKS (RECESSIONS VS. EXPANSIONS), %

Income Category	$\Delta\%P[\eta > 0]$	$\Delta\%P[\eta < 0]$
Head's Wage	0.0	0.0
Head's LI	-21.9	4.8
Joint LI	-14.4	8.8
Post-Govt Joint LI	-5.8	3.9
Head LI (narrow)	-15.5	5.4

Notes: Table 2.5 reports the percentage change in probabilities of positive and negative income shocks in recessions as compared to expansions. The table is based on the estimates reported in Tables 2.3 and 2.4.

2.6.3 Positive and Negative Income Shocks

In this subsection, we quantify the importance of income risk cyclicalities by evaluating how the probability of positive and negative income shocks changes between recessions and expansions for all 5 different income definitions.

Table 2.5 reports the percentage change in probabilities of positive and negative income shocks in recessions as compared to expansions using the implied skew normal distributions discussed in Subsection 2.6.1.

Several observations are in order. First, shocks to wages are drawn from similar across expansions and recessions distributions (Panels A in Figure 2.8): this results in insignificant changes in probabilities of both positive and negative shocks between booms and busts.

Other income types exhibit sizable procyclical skewness, which implies that probability of negative shocks increases in recessions and probability of positive shocks declines. Individual labor income is 22% less likely to increase during aggregate contractions, and at the same time is 5% more likely to go down. Joint and post-government labor incomes also exhibit asymmetry over the business cycle which is somewhat less pronounced as compared to head's labor income. Narrowly-defined labor income has a time-invariant but relatively high variance, which exacerbates changes in probabilities of positive and negative shocks over the cycle.

2.6.4 Evaluating Large Negative Events

The discussion so far was set in terms of *residual income*, but it is important to understand how large income shocks are relative to *labor income*. This subsection attempts to shed light on the quantitative relevance of income shocks.

TABLE 2.6: EVALUATING MAGNITUDE OF INCOME SHOCKS

	Wage	Head's LI	Joint LI	Post-Govt LI	Head's LI (narrow)
Recession	-26%	-37%	-30%	-23%	-32%
Expansion	-26%	-30%	-21%	-19%	-28%

Notes: Table 2.6 reports the percentage change in labor income following an extreme (3 standard deviations) negative labor income shock during aggregate expansion and recession. See text for more details.

In particular, we will be looking at the percentage change in labor income when household is hit by a large negative income shock. In order to avoid complicated simulations and to focus on the quantitative magnitude of income shocks, we assume that both persistent and transitory components of residual income (z_{it} and ε_{it}) are zero, and only innovations to a persistent component η_{it} are allowed to change. Furthermore, to mimic a large negative income shock, we set $\eta_{it} = -3\mu$, where μ is a standard deviation of the corresponding shock distribution (see Table 2.3).

Table 2.6 reports the results. For each income category, we calculate the percentage change in income following a large negative income shock which occurred in recession (first line) or in expansion (second line). In order to facilitate the comparison of different income categories, we focus on a prime age male (40 years old) with a spouse, who is a college graduate and has no children. We obtain labor income corresponding to such a household by substituting estimates from Table B.2 into Equation (2.1).

Our results imply that the cyclical nature of income shocks is quantitatively pronounced: a reduction in labor income following an extreme negative shock during aggregate expansion is sizably weaker as compared to when the shock hits in recession. In line with earlier discussion, the starkest differences are found in case of head's and joint labor incomes, while cyclical differences are weaker for wage and post-government labor income.

2.7 Conclusion

In this paper, we analyze volatility and skewness of labor income risk over the business cycle. We systematically apply our analysis to 5 definitions of labor income, which allows us to disentangle the role of hourly wage, hours, second earners, government taxes and transfers, and labor market attachment. We make several findings. First, for head's labor income, both variance and right skewness increase in contractions. Second, head's hourly wage is less cyclical than head's labor income in both variance and skewness, implying that

changes in hours, possibly due to unemployment, are behind the cyclical risk of head's labor income risk. Third, we find that existence of the second earner (private insurance) lowers the cyclical risk of skewness, and also compresses volatility. On the other hand, government taxes and transfers (public insurance) are found to lower both the level and the cyclical risk of labor income volatility. Both channels help mitigating increasing risk of negative tail events in recessions. Finally, among heads with strong labor market attachment, cyclical risk of labor income volatility becomes much weaker, while cyclical risk of skewness remains. This implies that whether one focuses on heads with strong labor market attachment or not helps reconcile seemingly conflicting evidence about cyclical risk of labor income risk in the recent literature.

We see several fruitful avenues for future research. The first one is to quantify macroeconomic impact of countercyclical labor income risk using a quantitative macro model. Another potential line of research can explore the opposite direction of causality and identify macro shocks that generate the observed fluctuations in income risk. Finally, more research—both empirical and theoretical—is needed on the intra-family insurance channel (through the second earner), given that our results suggest its quantitative soundness.

Chapter 3

Uncertainty Driven Entry and Exit over the Business Cycle

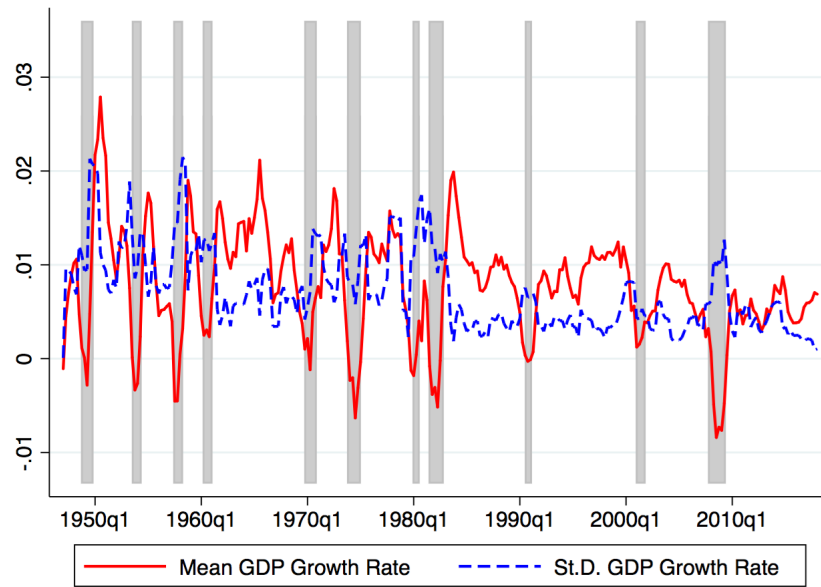
3.1 Introduction

The main contribution of this paper is to show that time-varying heteroscedasticity of aggregate TFP shocks (which throughout the paper will be measured as realized macro volatility) is a quantitatively important determinant of the investment decisions of firms — more so of young firms — and, through this channel, of the short- and medium-run behavior of the economy. I base my argument on a quantitative general equilibrium model in which firms face non-convex capital adjustment costs. The irreversibility of investment implies that firms have an inaction region where they prefer not to adjust their capital stock as the option value of waiting for them is large ([Abel, Dixit, Eberly and Pindyck, 1996](#)). When macro volatility rises, the inaction region expands and aggregate investment falls.

Figure 3.1 shows that macro volatility rises during almost all NBER recessions in the postwar period, and, if anything, appears to be countercyclical. The recession of 2007-2009 received a lot of attention in the literature, and several studies document spikes in uncertainty during that episode.¹ Quantifying the effect of volatility shocks on macro aggregates is important in order to understand the recovery in the aftermath of economic contractions.

¹[Bloom \(2009\)](#) uses the implied stock market volatility as measured by VXO index and shows that it displayed a large burst of uncertainty. [Bloom \(2009\)](#) find a significant increase in establishment-level dispersion of TFP shocks during the Great Recession in the U.S. manufacturing sector. [Jurado, Ludvigson and Ng \(2015\)](#) measure uncertainty as a conditional volatility of purely unforecastable component of future series, and reveal the 2007-2009 recession as an episode of heightened uncertainty.

FIGURE 3.1: MEAN AND STANDARD DEVIATION OF REAL GDP GROWTH RATE IN THE U.S. SINCE 1947



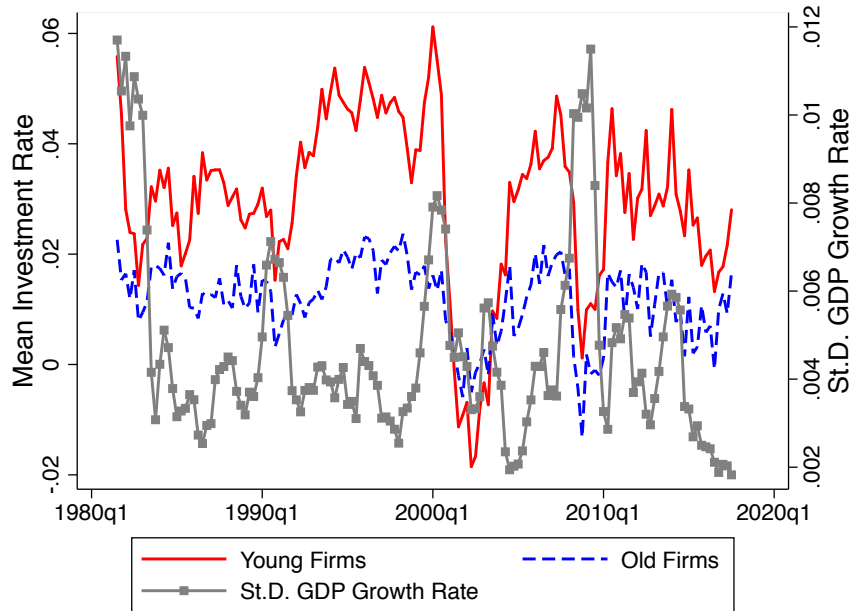
Notes: Figure 3.1 plots the evolution of the mean and standard deviation of real GDP growth rate in the U.S. since 1947. The mean and the standard deviation are computed over the 5-quarter rolling windows as in Equation (3.2). The data at quarterly frequency comes from the FRED website. Grey bars indicate NBER recessions dates.

The focus of this paper is to quantify impact of the heteroscedasticity of TFP shocks on the behavior of firms through the investment channel. Figure 3.2 plots the mean investment rate of Compustat firms against the measure of macroeconomic volatility over the last 4 decades. The red solid line represents the mean investment rate for young firms (5 years or younger), and the blue dashed line corresponds to older enterprises. Visually, investment decisions of older firms are more stable throughout the period than those of younger businesses; we also can see that at times of heightened measured volatility, young firms on average cut their investments stronger.

The paper consists of three parts. On the empirical side, it first shows that, in the sample of listed U.S. firms, macroeconomic heteroscedasticity is a statistically and economically significant determinant of firm-level investments. I consider 2 different measurable dimensions of firm investment decisions: investment rates and investment spikes.² Quantitatively, an increase in macro volatility of 1 standard deviation reduces the firm’s investment rate by 0.2 pp. This is a non-negligible number given that the mean quarterly investment rate in

²A case when the investment rate exceeds 20% in absolute value is called an investment spike.

FIGURE 3.2: MEAN INVESTMENT RATE BY AGE GROUP, COMPUSTAT



Notes: Figure 3.2 plots the average investment rate for young and old firms over the period 1982-2018. The data comes from the Compustat Quarterly. Young firms are those which are aged 5 or below. Old firms are the remaining businesses. Investment rate is computed as discussed in Subsection 3.3.2.

the sample is 2%. However, the drop is less pronounced for older firms, losing 0.01pp with every additional quarter of firm's tenure. Similarly, the probability of an investment spike goes down by 0.4% (with a sample mean of 3%), and the size of the drop decreases by 0.01% for each quarter of age. This relation is robust to the inclusion of various controls, such as real GDP growth rate, size and leverage of firms.

I then introduce time varying macroeconomic uncertainty along with firm entry and exit in a general equilibrium model of investment, and show that a calibrated version of the model can account for the patterns observed in the data. In particular, the model can reproduce the age profile of a cohort's aggregate employment: in the first couple of years, the amount of labor hired by the cohort shrinks reaching the trough at the age of 3. After that, the cohort employment grows. For that to happen, the model must correctly balance out 2 forces: the intensity of firm exit and growth of surviving businesses. I also show that the model generates a realistic distribution of investment rates.

In the model validation exercise, I simulate the model and extract a panel of firms of age 10 and above: this way, I mirror the selection of firms into the Compustat dataset. I then estimate the same regression models on the simulated data, and find that the estimates are

close to the ones obtained with Compustat data.

Finally, I use the calibrated model to study the behavior of the economy during a recession. I model a recession as either an unanticipated decrease in aggregate TFP, or as a decrease in TFP accompanied by a switch to a high-uncertainty regime. Under a one-shock recession (only negative TFP shock), the response of the two models is virtually the same, which confirms the findings of macro literature that entry/exit margin contributes little to aggregate fluctuations. However, under a two-shock recession, I show that in the model with firm entry aggregate output contracts 25% stronger, and the recovery is delayed. The intuition is as follows: young firms are on average less productive than incumbents, and their productivity grows over time (conditional on their survival). It is, therefore, crucial for them to know what their productivity will be tomorrow, since they have not have enough time to accumulate a capital stock which complies with their long-run productivity. Hence, those firms are more likely to freeze their investments in a high-uncertainty regime. Conversely, mature firms are likely to operate at the efficient scale dictated by their long-run productivity, and they would not have invested much even if uncertainty had remained low. Hence, this mechanism generates excess sensitivity of young firms.

A stronger response to a volatility shock in the economy with entry and exit can, however, be generated by a smaller number of entrants in high-uncertainty times, reducing the quantitative importance of the key mechanism of this paper. To check whether it is the case, I simulate a version of the model where potential entrants can observe a common TFP component, but do not see a switch in uncertainty regime. I find that in case of high volatility and low TFP, the number of entering firms is higher than in case of low TFP and low uncertainty (which I argue is due to a mean-reverting nature of the productivity process), and that the change in the extensive margin is quantitatively not important.

The rest of the paper is organized as follows. In Section 3.2, I provide a literature review. Section 3.3 quantifies the effect of macro uncertainty on investment decisions of firms by drawing on Compustat data. In Section 3.4, I build a general equilibrium model of investment. Sections 3.5 and 3.6 define the recursive competitive equilibrium and discuss the calibration strategy. I validate the model in Section 3.7 and inspect the mechanism in Section 3.8. Then, I proceed with the quantitative exploration of the model in Section 3.9. Section 3.10 concludes.

3.2 Literature Review

This paper is related to two broad strands of the literature, and serves as a link between them. First, this paper focuses on the aggregate implications of uncertainty shocks, which recently has been a very active area of research. In his seminal paper, [Bloom \(2009\)](#) argues that second-moment shocks are able to generate a sizable drop in aggregate output by propagating through the investment channel.³ [Bloom et al. \(2018\)](#) extend the analysis to the general equilibrium and show that the model calibrated to U.S. data generates a 2.5% drop in aggregate output in response to uncertainty shock. However, [Bachmann and Bayer \(2013\)](#) do not find a large role for uncertainty shocks in a model calibrated to German data. [Mongey and Williams \(2016\)](#) estimate a model featuring a set of standard aggregate macro shocks along with a second-moment shock. Their variance decomposition analysis reveals that the uncertainty shock explains a substantial portion of the cross-sectional dispersion of sales, but does not contribute much to the variation in the aggregates. Conversely, standard macroeconomic shocks explain almost all variation in aggregates, but do not appear to be important determinants of the cross-sectional dispersion. [Schaal \(2017\)](#) introduces time-varying volatility into a search model of the labor market. In his model, uncertainty has an ambiguous effect on employment decisions of the firm: on one hand, hiring is paused at times of heightened uncertainty as a relatively high productivity today may quickly revert tomorrow. On the other hand, separation is paused as well, since it will be costly to search for new workers in case of a rise in future productivity. Calibrated to U.S. data, his model implies that uncertainty is important for several business cycle moments, but overall plays a minor role in accounting for the magnitude and persistence of unemployment during the period 2007-2009.

Papers mentioned above have a real-option value of waiting (so-called “wait-and-see” effect) at the heart of their propagation mechanisms: when uncertainty rises and it is costly to revert previous decisions, economic agents rationally decide to wait and this translates into lower aggregate output. Nevertheless, the “wait-and-see” effect is not the only propagation mechanism recent research has looked into. For example, [Arellano, Bai and Kehoe \(2019\)](#) argue that at times of higher uncertainty the risk premium increases and the cost of external financing rises. As a consequence, firms have a limited potential to expand. [Zeke](#)

³There is work exploring the opposite direction of causality, that is, how uncertainty endogenously evolves over the business cycle. [Bachmann and Moscarini \(2013\)](#) is an example of such studies.

(2016) studies an environment where employment decisions of firms are subject to operating leverage: he shows that volatility shocks have a large impact on the magnitude of labor losses.

Fernández-Villaverde et al. (2015) consider an environment with risk-averse agents where spikes in uncertainty discourage investors from undertaking risky but high-return investments due to a precautionary motive. This again has an adverse effect on output, slows down growth and protracts the recovery. Gopinath et al. (2017) find that the dispersion of productivity shocks in Spain was declining during 1999-2007, which through the alleviation of precautionary motive led to a lower dispersion of marginal revenue product of capital (MRPK) and higher TFP.

This paper incorporates entry and exit of firms, and is therefore related to the literature on firm dynamics. Even though young establishments are small relative to incumbent establishments, the former contribute disproportionately to the net job creation.⁴ Decker et al. (2014) find that startups and fast-growing businesses in the U.S. (most of them are young) contribute 70% to the gross job creation. Young establishments have a high exit rate, losing 50% of original employment to exit in the first 5 years. However, there is a small share of young establishments that grow very fast, and nearly compensate the drop in employment due to exit within their cohort. Decker et al. (2014) conclude that those startups have a long-lasting effect on the net job creation process in the U.S.

Hopenhayn (1992) has become a workhorse model of firm dynamics. It, however, lacks aggregate shocks, which would be at the heart of my quantitative exercise. Clementi and Palazzo (2016) build on Hopenhayn (1992) and incorporate an aggregate TFP shock into the model. They find that the entry/exit margin is quantitatively important for the propagation of the aggregate shock: it protracts the transition of the economy following the aggregate shock due to the *missing generation* effect. In particular, when a positive TFP shock hits the economy, the number of exiting firms is lower than the number of entrants. Since entering firms are on average less productive than incumbent firms, and because the productivity process is mean-reverting, some young firms (conditional on survival) start growing fast, protracting the effect of a TFP shock. The missing generation effect takes place after a negative productivity shock, when the number of entering firms fails to replenish the pool of exiting firms. In this case, the economy lacks those fast-growing startups, and this delays

⁴Startups create 3mln jobs per annum, with 1.5mln net job creation in total in the U.S. private sector.

the recovery.

Clementi, Khan, Palazzo and Thomas (2015) extend Clementi and Palazzo (2016) framework to the general equilibrium setup. They find that the general equilibrium feedback mitigates the effect of the entry-exit margin: the response of the economy with endogenous entry and exit and the one without it exhibit similar dynamics following an aggregate productivity shock. The intuition here is that when the economy is hit by a first-moment (TFP) shock, it effectively “scales” the productivities of all the firms in the economy. Since incumbent firms are larger, the marginal effect of young firms (which are small) is negligible. However, Clementi et al. (2015) show that when a shock affects young firms *disproportionately*, the role of young firms in the business cycle dynamics might increase. Specifically, they show that an increase in the operating costs, which is more likely to affect the young firms, exacerbates the missing generation effects and raises the importance of the entry/exit margin. This paper argues that uncertainty shocks have a stronger impact on less productive firms with low stocks of capital — the characteristics pertaining to startups — and, therefore, increase the quantitative role of entry and exit in aggregate dynamics.

3.3 Suggestive Evidence

In this section, I provide suggestive evidence for the main mechanism I focus on in this paper. In particular, I draw on a rich firm-level panel data (Compustat) on a subset of U.S. businesses — the publicly traded firms — and explore the quantitative impact of aggregate volatility on investment decisions of firms. In Subsection 3.3.1, I argue why the dataset is suitable for the purposes of this paper. Subsection 3.3.3 provides the evidence supporting the view that younger firms exhibit excess sensitivity of investment to aggregate volatility.

3.3.1 Data

In order to shed light on how firms of different age respond to aggregate volatility, the dataset has to satisfy certain criteria. First, to quantify the propagation of uncertainty shocks through the investment channel, the data on capital has to be available at business cycle frequencies (quarterly). Second, the researcher has to observe the age of the firm. Third, the firms need to be observed for sufficiently long time periods, so that the firm-level fixed effects are estimated precisely. And, finally, the dataset needs to be long enough to

cover periods of economic expansions and contractions.

I draw on a panel of publicly traded firms from the (quarterly) Compustat dataset. The focus of my analysis is an (unbalanced) panel of firms spanning the period 1970-2018 at a quarterly frequency. I apply cleaning criteria which are standard in the literature (Clementi and Palazzo (2019), Ottonello and Winberry (2018)), and exclude the regulatory (SIC codes in range 4900-4999) and financial (SIC codes between 6000-6999) firms to reduce the impact of finance-related factors on the estimates. Appendix C.1.1 discusses the process of data preparation in more detail.

The Compustat dataset is subject to certain limitations. First, it covers only listed firms, which, of course, do not represent the universe of U.S. businesses. Second, Compustat does not provide a direct measure of firm’s age. Given these two drawbacks, the main objective of this section is to provide *suggestive* evidence of the main mechanism I explore in this paper, as well as to help validate the model I lay out in Section 3.4. Following Salgado, Guvenen and Bloom (2016) and Ottonello and Winberry (2018), I will proxy the firm’s age as the number of periods since IPO. My strategy is as follows: First, I will calibrate the model to hit the moments of the U.S. business dynamics reported elsewhere (see Section 3.6 for a detailed discussion). Subsequently, in order to validate the quantitative predictions of the model, I will re-estimate the empirical models from Subsection 3.3.3 on the model-generated data. To mirror the selection of firms into the Compustat sample, I will focus on the subset of firms aged 10 and above. This exercise will allow me to overcome the shortcomings of the Compustat, and instead will enable me to exploit its useful features: long time dimension, detailed capital/investment information at the firm level, and quarterly frequency.

3.3.2 Construction of Variables and Summary Statistics

The key variables are capital, investment, and aggregate volatility. I measure the investment rate as:

$$\text{i.rate}_{it} = \frac{K_{jt+1} - K_{jt}}{\frac{1}{2}(K_{jt+1} + K_{jt})}, \quad (3.1)$$

where K_{jt} is the amount of capital of firm i in the end of period t . I also consider investment spikes: I call an observation a “spike” if the investment rate exceeds 20% in absolute value. The detailed discussion of variables based on Compustat is relegated to Appendix C.1.2.

TABLE 3.1: FIRM-LEVEL VARIABLES: SUMMARY STATISTICS

Statistic	$\log(K)$	$\text{i.rate} = \frac{K_{jt+1} - K_{jt}}{\frac{1}{2}(K_{jt+1} + K_{jt})}$	$\mathbf{1}_{\{\text{i.rate} > 0.2\}}$
<i>Quarterly</i>			
Mean	4.00	0.02	0.03
Std	2.56	0.09	0.18
Median	3.85	0.00	0.00
Top 5%	8.37	0.15	0.00
Bottom 5%	0.03	-0.07	0.00
<i>Annual</i>			
Mean	4.05	0.07	0.18
Std	2.54	0.24	0.38
Median	3.93	0.03	0.00
Top 5%	8.38	0.50	1.00
Bottom 5%	0.11	-0.25	0.00

Notes: Table 3.1 reports the summary statistics of the firm-level data from Compustat. The top part of the table corresponds to quarterly data, while the bottom part — to the data aggregated to annual frequency. The data has been processed as discussed in Appendix C.1.1. The panel covers years 1970-2018.

Table 3.1 provides summary statistics for the data at both quarterly and annual frequencies. It is worth noting that even though the data covers the subset of very large firms in the U.S., the investment-related moments align well with the numbers reported in [Cooper and Haltiwanger \(2006\)](#) who worked with establishment-level data. For example, the mean annual investment rate in Compustat is 0.07, and it is 0.11 when estimated for the manufacturing establishments. Furthermore, the mean annual frequency of investment spikes almost coincides in two datasets (0.18 and 0.19).

In this paper, I follow [Fogli and Perri \(2015\)](#) and measure aggregate uncertainty as a standard deviation of real GDP growth rate over a 5-quarter long window around a quarter of interest. In particular, the volatility in year t is given by:

$$\sigma_t = \sqrt{\frac{1}{5} \sum_{j=-2}^{+2} g_{t+j}^2 - \left(\frac{1}{5} \sum_{j=-2}^{+2} g_{t+j} \right)^2}, \quad (3.2)$$

where g_t is a growth rate of real GDP from quarter $t - 1$ to t . In Subsection 3.3.3, I quantify how aggregate volatility affects the investment decisions of firms at *quarterly* frequency. In Appendix C.1.4, I aggregate the firm-level data to a yearly frequency, and study the impact of uncertainty on annual firm investment profiles.

3.3.3 Heterogeneous Response by Age

I consider two separate measures of investment: the investment rate and the investment spikes. It makes sense to consider these 2 measures, given that in the data investment behavior of firms is lumpy (see, for example, [Cooper and Haltiwanger \(2006\)](#)).

Investment Spikes

I ask: are young firms less likely to make large (in absolute value) investments as macroeconomic uncertainty rises?

The baseline specification takes the following form:

$$\mathbf{1}_{\{i.\text{rate}_{it}>0.2\}} = \beta_1 \text{Volatility}_t + \beta_2 \text{Volatility}_t \times \text{Age}_{it} + \mathbf{\Gamma}' Z_{it} + u_{it}. \quad (3.3)$$

The dependent variable is binary, taking a value of 1 if the investment rate exceeds 20% in absolute value, and 0 otherwise. The vector of controls, Z_{it} , includes mean GDP growth rate (computed over the same window as volatility), mean GDP growth rates interacted with firm's age, firm's size (measured by total assets), firm's leverage (ratio of total debt and total assets) and firm-level fixed effects. I control for leverage to make sure the estimates are not affected by the financial conditions of the firm; this is important, given that the model I develop in Section 3.4 does not speak to financial decisions of enterprises. The mean GDP growth rate controls for the business cycle, leaving the *volatility* variable the role of capturing the “wait-an-see” effect: the first and second moments move in opposite directions (as can be seen in Figure 3.1), and without controlling for the first moment, the estimates will merely reflect the fact that investment is declining during economics downturns.

I make an extra transformation of the data to make the economic interpretation of coefficients transparent: I standardize the mean and standard deviation of GDP growth rates, so that the units of these two variables are the number of standard deviations from the sample mean. The coefficients of interest are β_1 and β_2 , which under standard exogeneity assumptions capture (i) how the aggregate volatility affects the probability of investment spikes, and (ii) how the effect changes as the firm ages.

Economic Interpretation Table 3.2 reports the estimation results of Equation (3.3). Since the dependent variable is a dummy variable, the estimated equation is a linear probability model. I run 5 different specifications of Equation (3.3), including the regressors one by

TABLE 3.2: AGGREGATE UNCERTAINTY AND PROBABILITY OF INVESTMENT SPIKES

Variable	(1)	(2)	(3)	(4)	(5)
Volatility	-0.006*** (0.0005)	-0.003*** (0.0006)	-0.003*** (0.0006)	-0.004*** (0.0006)	-0.004*** (0.0006)
Volatility×Age	0.0001*** (0.00007)	0.0001*** (0.00001)	0.0001*** (0.00001)	0.0001*** (0.00001)	0.0001*** (0.00001)
Mean Growth		0.006*** (0.0003)	0.006*** (0.0005)	0.006*** (0.0005)	0.006*** (0.0005)
Mean Growth×Age			-0.0000 (0.0000)	0.000*** (0.0000)	0.000*** (0.0000)
Size				-0.004*** (0.0002)	-0.004*** (0.0002)
Leverage					-0.010*** (0.002)
FE	✓	✓	✓	✓	✓
N	627,960	627,960	627,960	627,960	627,960
R^2	0.02	0.02	0.01	0.01	0.01

Notes: Table 3.2 reports the estimation results of the Equation (3.3). The firm-level data comes from Compustat; see Appendix C.1 for details. **Volatility** and **Mean GDP growth rate** variables have been standardized, and their units reflect the number of standard deviations from the sample mean. **Age** is proxied by the number of quarters since IPO, **size** is measured by the logarithm of total assets, and **leverage** is a share of total debt in total assets. *** denotes 1%, ** denotes 5%, and * denotes 10% significance, respectively.

one, and show that the coefficient estimates ($\hat{\beta}_1$ and $\hat{\beta}_2$) do not change much across them. Overall, the results indicate that the probability of investment spikes across age 0 firms goes down by 0.5 p.p. when the aggregate volatility increases by 1 s.d. Besides, with every extra quarter of tenure, the probability of an investment spike increases by 0.01 p.p.. These numbers are economically significant given that the average probability of investment spikes is 3% in the sample. The estimates also suggest that 12-year old (and above) firms barely react to volatility shocks.

The results of Table 3.2 support the view that younger firms exhibit excess sensitivity to macro volatility as compared to older businesses. The effect is both statistically (at 1% level) and economically significant. I have also shown that the result is not driven by the aggregate business cycle, and that it is unlikely to be driven by firms' size and/or their financial conditions.

In Appendix C.1.4, I show that the impact of volatility on *annualized* probability of investment spikes is qualitatively consistent with quarterly results, but quantitatively larger. A 1 st.d. increase in macro volatility decreases the probability of an investment spike by 5 p.p.,

with that effect disappearing at the rate of 0.5 p.p. per extra year of firm's tenure.

Investment Rates

In the previous subsection, I explored to which extent aggregate volatility affects the decisions of young firms to make a large investment. However, investment spikes occur only in 3% of cases, and the mean investment rate is 2% (see Table 3.1). This suggests that in a majority of cases firms adjust their capital stocks marginally. Therefore, in this section I explore to which extent the intensive margin of firms' investment responds to aggregate uncertainty, and how this effect varies by age.

The baseline specification takes the following form:

$$\text{in.rate}_{it} = \beta_1 \text{Volatility}_t + \beta_2 \text{Volatility}_t \times \text{Age}_{it} + \mathbf{\Gamma}' Z_{it} + u_{it}. \quad (3.4)$$

The discussion of this formulation is identical to the one provided in Subsection 3.3.3. The only difference is that here the left-hand side variable is the investment rate of firm i at date t , defined as in Equation (3.1).

Economic Interpretation Results of Table 3.3 indicate that a 1 standard deviation increase in aggregate volatility reduces the investment rate of age 0 firms by 0.004. Every extra quarter of tenure reduces the investment rate response to aggregate uncertainty by 0.0001. This effect is statistically significant at 1% level across all specifications which I ran. Again, firm's size and financial conditions do not appear to affect the estimates both quantitatively and qualitatively. The estimated effects are economically significant, given that the sample mean of investment rate is 0.02. To put these estimates into context, note that a 40-quarter old firm (10 years old) will barely change its investment decision following an increase in aggregate volatility. Therefore, this gives extra support towards the view that young firms' investments respond stronger to increased aggregate volatility, and that the effect weakens with age.

In Appendix C.1.4, I repeat the exercise for annual investment rates, and find that 1 st.d. increase in volatility decreases the investment rate by 0.03 on average, and that the effect decays at the rate of 0.002 per year of tenure.

In this section, I shed light on the excess sensitivity of young firms to macro-volatility by exploiting the Compustat firm-level data. I presented the empirical support towards the

TABLE 3.3: AGGREGATE UNCERTAINTY AND FIRMS' INVESTMENT RATES

Variable	(1)	(2)	(3)	(4)	(5)
Volatility	-0.004*** (0.0003)	-0.004*** (0.0003)	-0.001** (0.0003)	-0.002*** (0.0003)	-0.002*** (0.0003)
Volatility×Age	0.0001*** (0.00004)	0.0001*** (0.00003)	0.0001*** (0.00003)	0.0001*** (0.00001)	0.0001*** (0.00004)
Mean Growth		0.006*** (0.0003)	0.005*** (0.0002)	0.005*** (0.0005)	0.005*** (0.0005)
Mean Growth×Age			-0.0000 (0.0000)	0.000*** (0.0000)	0.000*** (0.0000)
Size				0.003*** (0.0002)	0.003*** (0.0002)
Leverage					-0.033*** (0.001)
FE	✓	✓	✓	✓	✓
N	627,960	627,960	627,960	627,960	627,960
R^2	0.01	0.02	0.01	0.01	0.01

Notes: Table 3.3 reports the estimation results of the Equation (3.4). The firm-level data comes from Compustat; see Appendix C.1 for details. **Volatility** and **Mean GDP growth rate** variables have been standardized, and their units reflect the number of standard deviations from the sample mean. **Age** is proxied by the number of quarters since IPO, **size** is measured by the logarithm of total assets, and **leverage** is a share of total debt in total assets. *** denotes 1%, ** denotes 5%, and * denotes 10% significance, respectively.

main mechanism I explore in this paper along two margins of investment choice: both the probability of investment spikes and the investment rate decline stronger among the younger businesses as compared to mature firms. In the next section, I develop a general equilibrium model of firm dynamics where these patterns arise naturally. In particular, my model has three key ingredients. First, firms will be heterogeneous with respect to idiosyncratic productivity and capital, which will allow me to capture the salient patterns of the U.S. firm dynamics. Second, firms' investment will be costly: in order to increase or reduce its capital stock, the firm will have to pay an adjustment cost; this is paramount for the “real option of waiting” effect I focus on in this paper. Finally, my model features entry and exit of firms. This extends the model along the firm age dimension, and enables me to study the evolution of industry composition over time.

3.4 Model

I build a model of firm dynamics in the spirit of [Clementi et al. \(2015\)](#). Time in the model is discrete and runs forever. Three different types of agents populate the economy: households, active firms, and potential entrants. There is a single final good produced by firms, which can be either consumed or invested. Households own shares in firms, supply labor, and consume the final good. First, I describe the physical environment, and then proceed with the recursive formulation of the agents' problems.

3.4.1 Preferences and Technology

Every active firm i has access to a Cobb-Douglas production technology with decreasing returns to scale (DRS):

$$y_{it} = z_t s_{it} F(k_{it}, l_{it}) = z_t s_{it} k_{it}^\alpha l_{it}^\nu$$

with $\alpha, \nu > 0$ and $\alpha + \nu < 1$. I impose the DRS property to pin down the size of the firm.⁵ Every firm produces a homogeneous output y_{it} by combining capital, k_{it} , and labor, l_{it} , with weights α and ν , respectively. The production function is scaled by two productivity components: a common component, z_t , and an idiosyncratic component, s_{it} . Common and idiosyncratic productivities evolve according to the following AR(1) processes in logs:

$$\log(z_{t+1}) = \rho_z \log(z_t) + \underbrace{\sigma_{z,t}}_{\text{time-varying risk}} \varepsilon_{t+1} \quad (3.5)$$

$$\log(s_{it+1}) = \rho_s \log(s_{it}) + \sigma_s \kappa_{it+1}, \quad (3.6)$$

where $\varepsilon_t, \kappa_t \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, 1)$. The persistence parameters satisfy $\rho_s, \rho_z \in (0, 1)$. Note that $\sigma_{z,t}$ has a time subscript, which captures the notion of risk in my model. When $\sigma_{z,t}$ is high, the current compound productivity of the firm (that is, the product of idiosyncratic and aggregate components) becomes less informative about its future productivity, and, therefore, it is less likely to make an investment choice which will align with the realized future productivity. I allow for 2 different values of σ_z , so that the model transitions between them in a Markovian fashion. Let θ_i with $i \in \{L, H\}$ denote the probability that the uncertainty regime remains the same in the next period.

⁵The alternative to assuming the DRS property of the production function is to make the firms monopolistic competitors. This, however, makes the model computationally more intensive without extra insight.

Over time, firms go through different histories of idiosyncratic shock realizations, thus they end up holding different amounts of capital. The distribution of firms across capital holdings and idiosyncratic productivities is an important object, since it determines the total amount of labor demanded and, thus, the equilibrium wages. Let the aggregate state vector of the economy at time t be \mathbf{S}_t , such that:

$$\mathbf{S}_t = (z_t, \mu_t),$$

where z_t is the value of the common productivity component, and μ_t is the distribution of active firms across k and s . Anticipating a recursive formulation ahead, in what follows I drop the time subscript and use primes to denote values of variables in the next period. This model is a regime switching model, where the economy fluctuates between high and low uncertainty states. I do not include the current regime in the state vector \mathbf{S} ; rather, I solve for two sets of value functions (described below), with superscript $i \in \{L, H\}$ standing for the uncertainty regime.

An important ingredient in my model is non-convex capital adjustment costs. This feature of the model makes the firm's investment a risky endeavor: the firm has to pay an extra cost in case of over- or under-accumulation of its capital stock. The adjustment cost function has two components. The first one is a fixed cost of investment, which is intended to capture the frequency of non-zero investments. The second component is a standard quadratic cost function which intuitively controls how large firms' investment choices are. [Cooper and Haltiwanger \(2006\)](#) argued that a mix of convex and non-convex capital adjustment costs functions provides the best fit for the observed distribution of investment rates. Formally, I assume the following functional form:

$$AC(k, s) = Ck\mathbf{1}_{I \neq 0} + \frac{\gamma}{2} \left(\frac{I(k, s)}{k} \right)^2 k. \quad (3.7)$$

The first component in Equation (3.7) is made proportional to the stock of capital in the firm, in order to avoid any size effects. As is well known, the “kinked” adjustment cost function generates an inaction region⁶ in the investment choice of the firm, which will be the driving force for the wait-and-see effect I explore in this paper.⁷

⁶The firm adjusts its capital only if its current stock is sufficiently far away from the target level of capital which the firm's productivity implies.

⁷The irreversibility of investment is an empirically relevant model's feature which generates a negative

On the households' side, I assume that the instantaneous utility function has the following form:

$$U(C, N) = \frac{C^{1-\eta}}{1-\eta} - \theta \frac{N^\chi}{\chi}. \quad (3.8)$$

Households value consumption, C , and get disutility from labor, N . Households can smooth their consumption over time by investing in firms' shares.

Every period there is a constant mass of potential entrants, $M > 0$. In my model, M is independent from the aggregate state of the economy, which is different from [Clementi et al. \(2015\)](#) who assume a fixed mass of production locations in the economy. Every potential entrant receives a signal $q \sim Q(q)$ about her idiosyncratic productivity in the first period of activity if she decides to enter. The signal is imperfectly informative: the probability of getting a better draw of productivity in case of entry increases in the level of a signal. Entering the industry bears a cost c_e , which is denominated in terms of the final good. For technical reasons (to avoid jumps in entry over the cycle), I assume that c_e is independently, identically distributed across time and space, and is drawn from the uniform distribution $c_e \sim U[0, \bar{c}_e]$.

Now that the environment is described, I proceed with a detailed description of the individual optimization problems.

3.4.2 Incumbent Firm

The incumbent firm enters the period and observes the aggregate state $\mathbf{S} = (z, \mu)$ along with the realization of idiosyncratic productivity, s_{it} . Individual firms perceive the law of motion for the aggregate state of the form $\mathbf{S}' = \Gamma(\mathbf{S})$. I assume a completely frictionless labor market, where firms demand labor and the representative household supplies it. The firm makes its labor choice in the beginning of the period by solving the following static profit maximization problem:

$$\pi(k, s; \mathbf{S}) = \max_l [zsk^\alpha l^\nu - w(\mathbf{S})l].$$

I opt to make the firm's exit exogenous: each period, every business is subject to a death

relationship between investment and volatility [Leahy and Whited \(1996\)](#).

shock.⁸ The probability of survival is a function of the firm's size:

$$\mathbf{P}(\text{continue}|k) := c(k) = 1 - \kappa e^{-\iota k}, \quad (3.9)$$

where k is the amount of idiosyncratic capital, and $(\kappa, \iota) \in \mathbb{R}_+^2$. A parsimonious formulation (3.9) along with parameters κ and ι allows to capture the empirical regularity that smaller firms are more likely to exit as compared to large ones. While ι governs the relative probability of exit with respect to the firm's size, κ shifts the probability schedule uniformly for all businesses.

Next, I describe the firm's investment decision. Let superscript i denote the current uncertainty regime. Given that investment is subject to adjustment costs (3.7), it is convenient to write the firm's value prior to the investment decision but after the production has taken place in the following form:

$$\tilde{v}^i(k, s; \mathbf{S}) = \max\{v_a^i(k, s; \mathbf{S}), v_n^i(k, s; \mathbf{S})\}. \quad (3.10)$$

According to (3.10), the value of staying in business, \tilde{v} , is the maximum between 2 options: the firm can either adjust its capital stock and get value v_a , or it can avoid paying adjustment costs by allowing its capital stock to passively depreciate till the next period, and get value v_n .

Let $d(\mathbf{S}'|\mathbf{S})$ denote the stochastic discount factor. In what follows, it will be convenient to introduce an additional piece of notation, and denote the expected (with respect to the future regime) continuation value as $\bar{v} := \theta_i v^i + (1 - \theta_i) v^{-i}$, where θ_i is a probability that the regime will not change the next period. Then the value of adjusting the capital stock is:

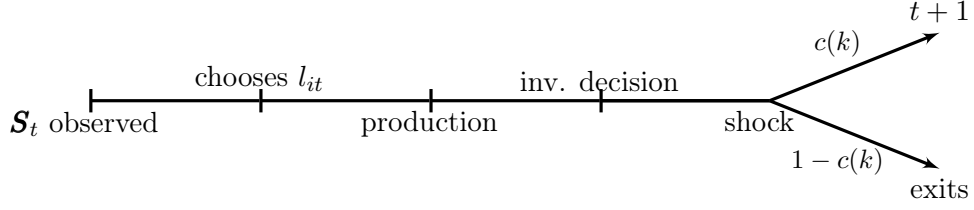
$$v_a^i(k, s; \mathbf{S}) = \max_{k' \in \mathbb{R}^+} \{-(k' - (1 - \delta)k) - AC(k, s; \mathbf{S}) + c(k) \mathbb{E} [d(\mathbf{S}'|\mathbf{S}) \bar{v}(k', s'; \mathbf{S}') | s, k; \mathbf{S}]\}. \quad (3.11)$$

Here $v(\cdot)$ corresponds to the value of the firm as of the beginning of the period (to be defined below), and $c(k)$ is an exogenous probability of survival. The value of a non-adjusting firm is:

$$v_n^i(k, s; \mathbf{S}) = c(k) \mathbb{E} [d(\mathbf{S}'|\mathbf{S}) \bar{v}(k(1 - \delta), s'; \mathbf{S}') | s, k; \mathbf{S}]. \quad (3.12)$$

⁸There are several ways in the literature of making firms exit the economy. For example, one can assume that firms are subject to some operation cost shock as in [Hopenhayn \(1992\)](#). Alternatively, the firm exits if the value of some outside option exceeds the continuation value of staying, as in [Jovanovic \(1982\)](#).

FIGURE 3.3: INCUMBENT IN PERIOD t



The firm's value at the beginning of the period can therefore be written as follows:

$$v^i(k, s; \mathbf{S}) = \pi(k, s; \mathbf{S}) + \tilde{v}^i(k, s; \mathbf{S}). \quad (3.13)$$

The beginning of the period value $v(\cdot)$ is the sum of the firm's profit (note that all active firms produce and receive profits, the exit shock hits later) and the continuation value. The continuation value is the maximum between 2 options: the firm might either pay the cost and update its capital stock, or it can simply let its stock depreciate.

Let $l(k, s; \mathbf{S})$ and $k'(k, s; \mathbf{S})$ denote the labor and capital policy functions, respectively. The verbal description of a timeline for an incumbent firm in period t is summarized in Figure 3.3.

3.4.3 Potential Entrant

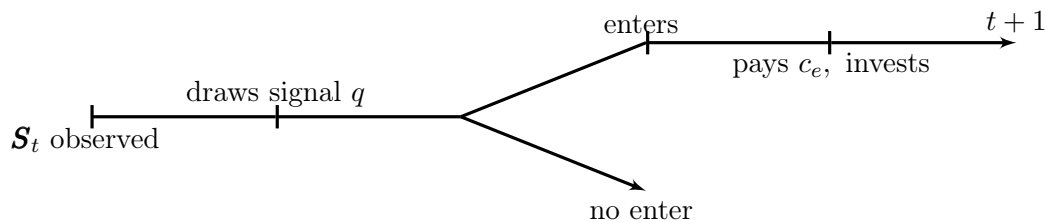
Every period the model economy confronts a constant mass of potential entrants, $M > 0$. First, every potential entrant observes the current aggregate state, \mathbf{S} , and receives a signal about its idiosyncratic productivity, $q \sim Q(q)$. The signal is drawn from a distribution with support identical to the one for idiosyncratic productivities: $\{q_1, \dots, q_{N_q}\} = \{s_1, \dots, s_{N_s}\}$. Let the probability of drawing a signal q_b be π_b^e . If a potential entrant decides to enter, his idiosyncratic productivity in the first period of operation will be drawn according to the same conditional distribution as for incumbents: $\mathbf{P}(s = s_i | q = q_b) = \mathbf{P}(s = s_i | s = s_b) \forall (i, b)$.

The entrant's problem (conditional on receiving a signal q) is:

$$v_e^i(q; \mathbf{S}) = \max_{k' \in \mathbb{R}^+} \left\{ -k' + \mathbb{E} [d(\mathbf{S}' | \mathbf{S}) \tilde{v}(k', s'; \mathbf{S}') | q; \mathbf{S}] \right\}. \quad (3.14)$$

Note how in (3.14) the continuation value is not scaled by the survival probability; it is

FIGURE 3.4: POTENTIAL ENTRANT IN PERIOD t



assumed that if in period t the entrant decides to produce, she makes it to the production stage in period $t + 1$ with probability 1.

Let $k'_e(q; \mathbf{S})$ denote the optimal choice of capital upon entry. The entrance decision is associated with paying a sunk cost c_e , which is denominated in units of a consumption good. A potential entrant decides to enter the economy whenever $v_e(q; \mathbf{S}) \geq c_e$. For technical reasons⁹, I assume that $c_e \sim U[0, \bar{c}_e]$, so that the probability of entry for those with signal q is:

$$\mathbf{P}(\text{enter}|q) = \begin{cases} 1, & \text{if } v_e^i(q; \mathbf{S}) \geq \bar{c}_e \\ \frac{v_e^i(q; \mathbf{S})}{\bar{c}_e}, & \text{if } 0 < v_e^i(q; \mathbf{S}) < \bar{c}_e \\ 0, & \text{if } v_e^i(q; \mathbf{S}) < 0. \end{cases} \quad (3.15)$$

The timing of events for a potential entrant is illustrated in Figure 3.4.

The next subsection formulates the household's problem.

3.4.4 Households

The economy is populated by a unit mass of identical households. Each household consumes, supplies labor, and saves into firms' shares. Investment in firms is a device households use in order to smooth consumption over time. Note, however, that I could have augmented the model with a set of risk-free one period ahead securities, but it is innocuous not to consider them, as those securities must be in zero net supply in equilibrium. The wealth is held in one-period shares in firms, $\xi(k, s)$. The price of current shares is ρ_0 , and the purchase price of new shares is ρ_1 . The household's dynamic programming problem is:

$$W^i(\xi; \mathbf{S}) = \max_{c, n, \xi'} [U(c, 1 - n) + \beta \mathbb{E}_{z'} \bar{W}(\xi'; \mathbf{S}')] \quad (3.16)$$

⁹In my numerical implementation, the signal space consists of a finite number of points. Therefore, without "convexifying" the entry decision with the help of the entry cost random variable, the entry margin will move discretely.

subject to

$$c + \int_{\mathcal{K} \times \mathcal{S}} \rho_1(k', s'; \mathbf{S}) d\xi' \leq w(\mathbf{S})n + \int_{\mathcal{K} \times \mathcal{S}} \rho_0(k, s; \mathbf{S}) d\xi. \quad (3.17)$$

The right-hand side of (3.17) represents the resources available to the consumer: it consists of firm shares coming from the previous period, as well as labor income. Part of these resources is consumed, and the rest is reinvested into firm shares.

Let $C(\xi; \mathbf{S})$ be the household's consumption policy function, and $N(\xi; \mathbf{S})$ be a labor supply policy function. Also, let $\Xi(k', s'; \mathbf{S})$ be a number of shares purchased in firms which start tomorrow with capital k' and idiosyncratic productivity s' .

I show in Appendix C.2 how to reduce the computational burden by way of scaling Bellman equations with the marginal utility of the household. After making the necessary transformations (and denoting the adjusted value functions with capital letters), I next define the *recursive competitive equilibrium* for my environment.

3.5 Equilibrium

The Recursive competitive equilibrium for this economy consists of the following functions:

$$\left\{ V^i, V_x^i, V_a^i, V_n^i, k', \tilde{V}^i, \chi^i, V_e^i, l, k'_e, w, \rho_0, \rho_1, W^i, C, N, \Xi, d \right\}_{i \in \{L, H\}},$$

such that:

1. W^i solves the household's problem (3.16), and (C, N, Ξ) are the associated policy functions,
2. $V_x^i, \tilde{V}^i, V^i, V_a^i, V_n^i$ solve the incumbent's problem (C.4)-(C.7), and (k^a, k^n, l, χ) are the corresponding policy functions,
3. V_e^i solves the potential entrant's problem (C.8), and k'_e is the resulting policy function,
4. consistency condition satisfies $\forall (k, s) \in \mathcal{K} \times \mathcal{S}$

$$\Xi(k', s'; \mathbf{S}) = \mu'(k', s'),$$

5. labor market clears

$$N(\mu, \mathbf{S}) = \int_{\mathcal{K} \times \mathcal{S}} l(k, s; \mathbf{S}) d\mu,$$

6. stochastic discount factor satisfies

$$d(\mathbf{S}'|\mathbf{S}) = \beta \frac{U'_C(C(\mu', \mathbf{S}'), N(\mu', \mathbf{S}'))}{U'_C(C(\mu, \mathbf{S}), N(\mu, \mathbf{S}))},$$

7. goods market clears

$$\begin{aligned} C(\mu, \mathbf{S}) = & \int_{\mathcal{K} \times \mathcal{S}} \{z s F(k, l(k, s; \mathbf{S})) - [\mathbf{1}_{\text{adj}}(k'(k, s; \mathbf{S}) - (1 - \delta)k)]\} d\mu - \\ & - \int_q \int_0^{v_e^i(q; \mathbf{S})} x \mathbf{P}(\text{enter}|q, \mathbf{S}) dx dQ - \int_{\mathcal{K} \times \mathcal{S}} \mathbf{1}_{\text{adj}} AC(k, s) d\mu + \\ & + \int_{\mathcal{K} \times \mathcal{S}} c(k) k d\mu - M \int \mathbf{P}(\text{enter}|q, \mathbf{S}) k'_e(q; \mathbf{S}) dQ, \end{aligned}$$

where $\mathbf{1}_{\text{adj}} = \mathbf{1}_{\{V_a^i(k, s; \mathbf{S}) > V_n^i(k, s; \mathbf{S})\}}$.

8. law of motion for the aggregate state vector, $\Gamma(\cdot)$, is consistent with firms' policy functions.

In Section 3.6, I discuss the calibration of the baseline model.

3.6 Calibration

In this section, I describe my calibration strategy. First, in Subsection 3.6.1, I assign values to the parameters which are relevant to the behavior of the model at the steady state. Next, calibration of parameters relevant for aggregate fluctuations is provided in Subsection 3.6.2. Subsection 3.6.3 explores the model's success in capturing non-targeted moments and firm dynamics.

3.6.1 Steady State Parameters

The period in the model is one quarter, which is a suitable frequency to study business cycle fluctuations. I split all parameters into two groups: externally and internally calibrated. The former group includes standard macroeconomic parameters; the remaining parameters from the latter are calibrated by the simulated method of moments (SMM).

In total, my model has 14 parameters. The first 7 of them, $(\beta, \alpha, \nu, \delta, \rho_s, \rho_z, \sigma_z)$, are “external”. The discount factor β implies an annual interest rate of 4%. Capital and labor input shares are 0.25 and 0.64, respectively. Parameter δ is assigned a value of 2.5%, which implies a 10% depreciation rate per annum. I assume that the quarterly persistence of the idiosyncratic productivity is 0.95, consistent with [Khan and Thomas \(2008\)](#). I follow the literature

and assume that z is very persistent with $\rho_z = 0.95$. I borrow the value of $\sigma_z = 0.0067$ from [Bloom et al. \(2018\)](#). Panel A of Table 3.4 summarizes the externally calibrated parameters. Parameters from the second group are calibrated internally. Technically, I solve the model for the stationary equilibrium by turning off the aggregate shocks, that is I set $z_{t-1} = z_t = 1$ and $\sigma_{z,t-1} = \sigma_{z,t} = \bar{\sigma}_z$. Then I simulate a large number of firms for many periods from the stationary distribution, and compute the moments from the simulated data. The parameter values used in subsequent analysis minimize the distance between the model generated moments and their empirical counterparts.

In what follows, I provide a heuristic identification argument that justifies the choice of the moments used in the calibration. Even though every targeted moment is simultaneously affected by all parameters, in this section I discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power. For the most part I follow the literature in picking the moments to target.

Following [Clementi et al. \(2015\)](#), I use the mean investment rate, the standard deviation of the investment rate and the fraction of firm-year observations with investment rate exceeding 20% (so-called “spikes”) to inform the standard deviation of the idiosyncratic productivity process, σ_s , as well as parameters of the capital adjustment cost function, (γ, C) . These moments come from [Cooper and Haltiwanger \(2006\)](#), who obtained these statistics from the balanced panel of establishments based on the Longitudinal Research Database (LRD). In order to be consistent with their calculations, I simulate a large number of establishments from the stationary distribution for many years, and keep only those establishments which survive throughout. Then I compute the moments from my simulated data. Following the [Cooper and Haltiwanger \(2006\)](#) methodology, the investment rate for the firm i at time t is computed by taking the ratio of investment and the current capital stock:

$$\text{ir}(k, s) = \frac{k'(k, s) - (1 - \delta)k}{k},$$

where $k'(k, s)$ is the policy function regarding k' for the firm with current capital stock k and idiosyncratic productivity s . The aggregate state \mathbf{S} is suppressed since the model is calibrated to the steady-state. The frequency of investment spikes is:

$$\text{Frequency of spikes} = \frac{\sum_{i,t} \mathbf{1}_{\{\text{ir}_{i,t} > 0.2\}}}{NT},$$

TABLE 3.4: CALIBRATED PARAMETERS

Parameter	Description	Value	Target	Data	Model
<i>Panel A: Externally calibrated parameters</i>					
β	Discount factor	0.989			
α	Capital share	0.25			
ν	Labor share	0.64			
δ	Depreciation rate	0.025			
ρ_s	Persist. of id.prod.	0.95			
ρ_z	Persist. of ag.prod.	0.95			
σ_z	Std of innovations to z	0.0067			
<i>Panel B: Internally calibrated parameters</i>					
σ_s	Std of id. innov.	0.022	$\mathbb{E}\left[\frac{z}{k}\right]$	0.12	0.09
\bar{c}_e	Up. bnd. of entry cost	0.2	$P(\mathbf{1}_{\{i/k>0.2\}})$	0.19	0.11
κ	Param. of surv. fun.	0.05	$\sigma\left[\frac{z}{k}\right]$	0.34	0.36
ι	Param. of surv. fun.	0.4	$P[\text{exit}]$	0.02	0.02
γ	Param. of AC fun.	0.7	$\frac{\mathbb{E}[n]}{\mathbb{E}[n \text{age}=0]}$	0.6	0.55
C	Param. of AC fun.	0.002	$\frac{\mathbb{E}[s]}{\mathbb{E}[s \text{age}=0]}$	0.75	0.79
M	Mass of p. entrants	0.0507	$\int \mu_{ss} = 1$	—	1

Notes: Table 3.4 consists of 2 parts. Panel A reports the assigned values to the externally calibrated parameters. Panel B reports the values for the internally calibrated parameters. Each parameter in this group is associated with an empirical moment for which it (intuitively) has the highest identification power (see text).

where N and T denote the number of firms and number of periods, respectively. Here $ir_{i,t}$ is an investment rate of firm i in period t .

In the model, potential entrants draw a signal from the exponential distribution with parameter λ , and decide to enter whenever $V_e(q; \mathbf{S}) \geq c_e$. The distribution of entry costs, parametrized by c_e , as well as parameters of the survival function (3.9), (κ, ι) , jointly determine the relative productivity and size of entrants along with the entry rate.¹⁰ These empirical moments are taken from Lee and Mukoyama (2015). They draw on the Annual Survey of Manufacturers (ASM) portion of LRD constructed by the U.S. Census Bureau for the period 1972-1997. The advantage of the ASM is that this dataset has a yearly frequency, which makes it suitable to study business cycle fluctuations.¹¹ Lee and Mukoyama (2015)

¹⁰I follow Dunne, Roberts and Samuelson (1988) to compute statistics in the model generated data. Specifically, let $NE(t)$ be a number of establishments entering between $t-1$ and t , $NT(t)$ is a total number of establishments at time t (including $NE(t)$). Next, let $NX(t-1)$ be a number of exiting establishments between $t-1$ and t . $QE(t)$ and $QT(t)$ denote the size of entering and all establishments at time t , respectively. Lastly, $QX(t-1)$ is a size of exiting establishments. Then the entry rate is defined as $ER(t) := \frac{NE(t)}{NT(t-1)}$. Exit rate is $XR(t) := \frac{NX(t-1)}{NT(t-1)}$. The average size of entering establishments relative to incumbents is $ERS(t) := \frac{QE(t)/NE(t)}{(QT(t)-QE(t))/(NT(t)-NE(t))}$. Finally, the average size of exiting firms relative to continuing ones is defined as $XRS(t-1) := \frac{QX(t-1)/NX(t-1)}{(QT(t-1)-QX(t-1))/(NT(t-1)-NX(t-1))}$.

¹¹The Census of Manufacturers (CM) dataset, which is also a part of the LRD, is conducted every 5

report corresponding statistics for both expansion and contraction periods, but I am using the averages over time as targets.

In the model simulated data, the aforementioned moments are calculated as follows. First, the entry rate at time t is¹²:

$$\text{er}_t = \frac{M \int \mathbf{P}(\text{enter}|q) dQ}{\int_{\mathcal{K} \times \mathcal{S}} \mathbf{1}_{\{k,s\}} d\mu^{\text{ss}}},$$

where M is the mass of potential entrants, and μ^{ss} is the stationary distribution of firms. Productivity is measured simply as a realization of the random variable s . Therefore, the relative productivity of entrants is:

$$\text{rp}_t = \frac{\text{average productivity of entrants}_t}{\text{average productivity of incumbents}_t} = \frac{\int \mathbf{P}(\text{enter}|q) [\sum_s \Pi(s|q)s] dQ / \int \mathbf{P}(\text{enter}|q) dQ}{\int_{\mathcal{K} \times \mathcal{S}} s d\mu^{\text{ss}} / \int_{\mathcal{K} \times \mathcal{S}} \mathbf{1}_{\{k,s\}} d\mu^{\text{ss}}}.$$

Finally, the relative size of entrants is:

$$\text{rs}_t = \frac{\text{average size of entrants}_t}{\text{average size of incumbents}_t} = \frac{\int \mathbf{P}(\text{enter}|q) k'(q) dQ / \int \mathbf{P}(\text{enter}|q) dQ}{\int_{\mathcal{K} \times \mathcal{S}} k d\mu^{\text{ss}} / \int_{\mathcal{K} \times \mathcal{S}} \mathbf{1}_{\{k,s\}} d\mu^{\text{ss}}}.$$

Since the model period is one quarter, and the statistics reported in the literature come from data with annual frequency, I aggregate the model generated data appropriately before computing the moments. For example, when I compute model-implied investment moments, I extract a balanced panel of firms from the simulated data, and then compute the annual rates.

Finally, the mass of potential entrants, M , simply scales the distribution of active firms. I adjust M so that the stationary distribution of firms μ integrates up to 1 (i.e., becomes a probability distribution). The results of the calibration are summarized in Table 3.4.

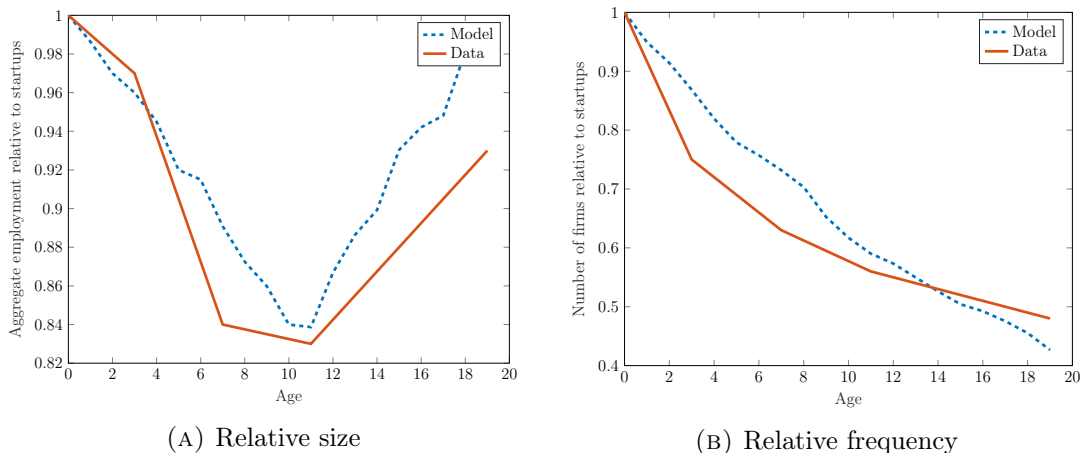
3.6.2 Aggregate Fluctuations Parameters

The model switches between high- and low-uncertainty regimes stochastically. In the model, low regime is associated with a low dispersion of innovations in Equation (3.5), and high uncertainty regime corresponds to a high value of dispersion. That is, $\sigma_z \in \{\sigma_L, \sigma_H\}$ with $\sigma_L < \sigma_H$.

years, and thus the CM-based moments are not quite suitable for business-cycle frequency models.

¹²As it will be discussed below, the stationary distribution in my model integrates up to 1, and so the denominator in the expression for the entry rate is 1.

FIGURE 3.5: EVOLUTION OF AN ENTERING COHORT, DATA AND MODEL



Notes: Figure 3.5 depicts the fit of the model with respect to the (A) relative size of a cohort, (B) relative number of firms. The measures plotted in the Figure are constructed relative to age 0 firms (startups). The data comes from BDS, the corresponding lines reflect author’s calculations.

The transition between low and high values of σ is governed by the first-order Markov chain with a corresponding transition matrix Π^σ . This matrix has the following structure:

$$\Pi^\sigma = \begin{bmatrix} 1 - \pi_{L,H}^\sigma & \pi_{L,H}^\sigma \\ 1 - \pi_{H,H}^\sigma & \pi_{H,H}^\sigma \end{bmatrix}.$$

I use the quarterly estimates of $\pi_{L,H}^\sigma$ and $\pi_{H,H}^\sigma$ provided by [Bloom et al. \(2018\)](#). They estimate a GARCH(1,1) using data on quarterly TFP growth from 1972Q1 to 2010Q4, and find that the quarterly probability of switching into a high uncertainty state is $\pi_{L,H}^\sigma = 2.6\%$. The persistence of a high uncertainty state is estimated to be $\pi_{H,H}^\sigma = 94\%$.

3.6.3 Model Fit

While the targeted moments are matched closely, it is crucial to check how the model does with respect to non-targeted moments. First, I check how well the model captures the so-called “up-or-out” dynamics of young firms. It has been empirically documented ([Decker et al. \(2014\)](#)) that young firms are losing roughly 50% of employment to exit within the first 5 years, but the remaining firms grow so fast that they nearly compensate for that loss.

Figure 3.5 confronts the model’s predictions about the evolution of the entering cohort of firms with the data. I used the BDS data to construct the evolution of the relative (in terms of employment) size of the cohort, and the relative number of firms. Unfortunately, U.S.

Census reports the required data only up to the age of 6, and then aggregates ages in wide bins. I, therefore, restrict myself to the first few years of the firm’s life-cycle.

The left panel of Figure 3.5 shows how the entering cohort of firms loses employment to exit in the first couple of years, but then, after the “cleansing” effect has taken place and only sufficiently productive firms have survived, the employment rebounds. Rather neatly, the model is capable of picking up this rich dynamics observed in the data, and captures remarkably well both the timing of the “trough” and its magnitude: 3 year old firms hire the least number of employees across all ages, and this is 15% lower than what the youngest firms employ.

Panel B depicts how the typical entering cohort shrinks with time as firms exit. The model tracks the data pretty closely; roughly 50% of new firms exit by the age of 5.

The propagation mechanism I explore in this paper works through the investment channel: when the conditions become more volatile, firms decide to postpone their investment decisions for later to avoid paying the associated adjustment cost. Even though I targeted several investment-related moments in Subsection 3.6.1, it is crucial to see whether the distribution of investment rates delivered by the model is realistic. I use the data from [Cooper and Haltiwanger \(2006\)](#), who report the distribution of investment rates for a balanced panel of manufacturing establishments. Correspondingly, I extract a balanced panel of firms from the model simulated data.

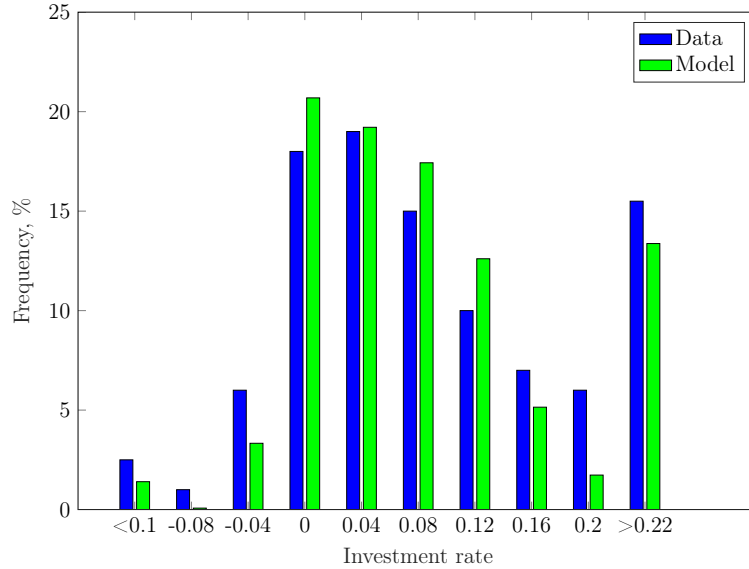
Figure 3.6 confirms that the model does a reasonable job in generating a realistic distribution of investment rates. As in the data, the distribution is skewed to the right, with a large mass concentrated around 0.

Overall, I conclude that the model is a good approximation of reality since it matches the key to the main point of this paper dimensions of the data — the life-cycle of firms and the distribution of investment rates.

3.7 Model Validation

Does the model generate excess sensitivity of young firms with respect to aggregate volatility — the empirical regularity discussed in Section 3.3? In this section, I use the model generated data to argue that the calibrated model captures the response of firms’ investment to measured macro volatility remarkably well.

FIGURE 3.6: EMPIRICAL AND MODEL-IMPLIED DISTRIBUTIONS OF INVESTMENT RATES



Notes: Figure 3.6 plots the empirical and the model-implied distribution of investment rates. The data comes from Cooper and Haltiwanger (2006). Each number along the horizontal axis corresponds to a 4% interval around that value (i.e. 0 must be interpreted as $[-2\%, 2\%]$, and so on).

I proceed in two steps. First, I simulate a large panel of firms from the stochastic equilibrium of the model, and report the moments of both investment rates and investment spikes for a subset of firms mirroring Compustat (those which survived for at least 10 years¹³). I show that the generated moments align well with the data moments reported in Table 3.1. I then run the same panel regressions from Section 3.3 on model-generated data, and confirm that the model captures the excess sensitivity of young firms both qualitatively and quantitatively. Furthermore, I also fit the empirical models from Section 3.3 on a full sample of firms (not necessarily mirroring Compustat), and find that the excess sensitivity of younger enterprises rises.

Table 3.5 suggests that the model is capable of doing a reasonable job in matching the empirical investment moments found in Compustat data. The calibrated model slightly underpredicts the mean investment rate and its standard deviation, but, overall, aligns well with the data. Note that this is not a mechanical result as it has not been directly targeted at the calibration stage in Section 3.6: target moments there characterized the entire distribution of firms' investment rates (of all ages and sizes), while Table 3.5 reports the statistics essentially for the right tail of firm-size distribution.

¹³Wilmer et al. (2017) find that the median time to IPO ranges between 6 to 8 years.

TABLE 3.5: DATA VS. MODEL: SUMMARY STATISTICS

Statistic	Data		Model	
	i.rate	$\mathbf{1}_{\{i.rate>0.2\}}$	i.rate	$\mathbf{1}_{\{i.rate>0.2\}}$
Mean	0.02	0.03	0.01	0.02
Std	0.09	0.18	0.05	0.11
Median	0.00	0.00	0.00	0.00
Top 5%	0.15	0.00	0.10	0.00
Bottom 5%	-0.07	0.00	-0.04	0.00

Notes: Table 3.5 reports the summary statistics for both the data and the model. The data portion corresponds to Compustat. The data has been processed as described in Appendix C.1.1, and has quarterly frequency.

TABLE 3.6: DATA VS. MODEL: REGRESSIONS

	Investment rate		P(Investment spikes)	
	Model	Data	Model	Data
Volatility	-0.004 (0.001)	-0.002 (0.0003)	-0.004 (0.001)	-0.004 (0.0006)
Volatility×Age	0.00004 (0.00001)	0.0001 (0.00004)	0.00003 (0.00001)	0.0001 (0.00001)
R^2	0.01	0.01	0.01	0.01

Notes: Table 3.6 reports the estimation results for models (3.3) and (3.4) both for the Compustat and model-simulated data. The dependent variables in the table are: investment rate in the left part of the table, and an indicator of investment spikes in the right part of the table. The “data” estimates repeat those from Tables 3.2 and 3.3.

Next, I fit the empirical models (3.3) and (3.4) on the model-simulated data (Table 3.6). The model is successful in capturing the excess sensitivity of young firms both qualitatively (the estimates have the expected signs) and quantitatively. The discrepancies in the estimates between the model and the data might arise for several reasons (mirroring Compustat by conditioning on age is not perfect, omitted variable biases, etc.), but, overall, the model’s predictions are very close to the data.

As it has been discussed in Section 3.3, firm-level data with age and investment information is very scarce; that led me to use data on a subset of large firms. In the model, however, I can compute any object of interest, including the investment rates of very small and young businesses. I have already established the model’s potential of picking up the empirical patterns which characterize the right tail of the firm-size distribution. Therefore, it is natural to investigate what the model implies regarding the response of investment rates to macro volatility *without* conditioning on very large firms. Technically, I re-estimate equations (3.3) and (3.4) on the full model-generated panel.

TABLE 3.7: MODEL-GENERATED DATA: FULL SAMPLE

	Model (Full Sample)	
	Investment rate	\mathbf{P} (Investment spike)
Volatility	-0.007 (0.001)	-0.005 (0.0002)
Volatility \times Age	0.0002 (0.00001)	0.0001 (0.00002)
R^2	0.01	0.01

Notes: Table 3.7 reports the estimation results for models (3.3) and (3.4) on the full model-generated panel. The dependent variables in the table are: investment rate in Column 2, and an indicator of investment spikes (Column 3). Data has quarterly frequency.

Table 3.7 reports the results. Perhaps, it is not surprising that qualitatively the response of investment-related variables to macro volatility has increased: now we include both the old and young firms into the estimation. Consistently with the logic of the model developed in Section 3.4, young firms are growing fast (conditional on survival) and, therefore, they represent the majority of capital-adjusting firms. When uncertainty rises, the response of investment is stronger when I consider a full sample, because it incorporates a more pronounced response of younger enterprises.

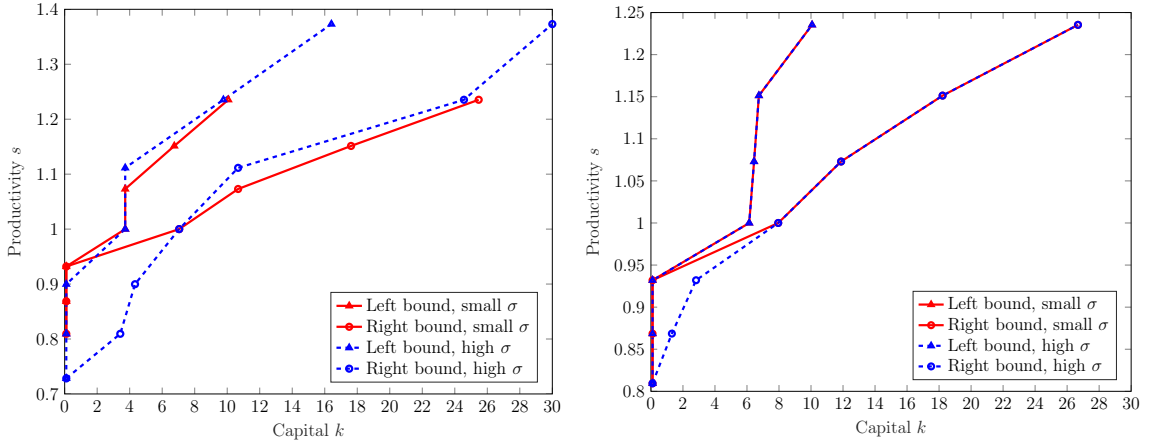
In this section, I established a tight connection between the data and the model which is based on quantitatively similar predictions regarding the impact of macro volatility on investment behavior of businesses in different age categories. Next section describes the workings of the model in more detail.

3.8 Inspecting the Mechanism

Before I turn to the aggregate implications of macro-volatility, it is instructive to pause an exposition and discuss how the model built in Section 3.4 is capable of generating the wait-and-see effect during volatile aggregate conditions, which is more pronounced for younger firms.

The decision of an individual firm about whether to postpone its investment or not is fully governed by the value functions for cases of “adjustment” (Equation 3.11) and “no adjustment” (Equation 3.12). These functional equations are affected by the time-varying general equilibrium prices which affect the decision-making firm through the stochastic discount factor.

FIGURE 3.7: INACTION REGION IN (k, s) -SPACE



Notes: Figure 3.7 consists of 2 panels: panel A plots the inaction regions in (k, s) -space for the model with no aggregate fluctuations, while panel B plots them for the full model. Each panel contains 2 lines: the red solid lines pertain to the case of low volatility, and blue solid lines — to the case of high volatility. See text for details.

In order to assess to which extent the investment sensitivity of young firms works through the general equilibrium forces in the full model, I first consider the model at the steady-state. In particular, I compare 2 equilibria — one pertaining to a low value of volatility (σ_s), and the other to a high one — and plot the *inaction regions* on the same plot (panel A of Figure 3.7). The difference in volatilities across the two models quantitatively matches σ_H/σ_L ratio in the full model. Inaction region is a set of points in (k, s) -space where an individual firm finds it optimal to wait:

$$\text{Inaction region}_\sigma = \{(\tilde{k}, \tilde{s}) : v_n(\tilde{k}, \tilde{s}|\sigma) \geq v_a(\tilde{k}, \tilde{s}|\sigma)\}.$$

Red solid lines depict the bounds for the case of low σ_s , while blue dashed ones — for the case of high σ_s . The inaction region is an area between the corresponding lines (which I call bounds).

First, the bounds are upward-sloping, merely reflecting the fact that high productivity necessitates high levels of capital.¹⁴ Second, the regions tend to fan out as productivity increases. This occurs because once the firm has accumulated a large stock of capital, the incentives to reduce it decrease: if anything, the firm can simply let it depreciate (and avoid paying the adjustment cost by doing so), which in some time brings the stock to the desirable level.

¹⁴When volatility increases in the steady-state model, the values of s change along: this is a mechanical consequence of the method I used to discretize process (3.6). This does not occur in the full model because volatility is governed by the separate process (3.5).

When volatility increases, the bounds move. I find that at high productivity levels (above $s = 1$) the bounds are very close to those pertaining to the low-volatility case, while the most pronounced divergence occurs at low productivity levels. When volatility is low, the least productive firms always prefer to adjust their stock (red solid left- and right-bounds coincide). When volatility increases, the bounds no longer coincide: the inaction region for the least productive firms increases *disproportionately*. It is precisely the key reason of why the heightened macro-volatility affects young firms stronger: this group of businesses is typically located in the south-west corner of the (k, s) -space.

I next analyze the inaction areas for the full model (panel B of Figure 3.7). In this case, the value functions incorporate the time-varying equilibrium prices which are necessarily absorbed by the inaction regions. Overall, the way inaction regions move across the uncertainty regimes resemble that of the steady-state case. I find that the bounds visually coincide at high productivity levels, and that they diverge for low values of s . This suggests that the excess sensitivity of young firms' investment is not an artifact of general equilibrium forces of the stochastic equilibrium, and that it is to a large extent driven by the real option of waiting effect.

3.9 Aggregate Implications

The main objective of this section is to assess quantitatively the role that the excess sensitivity of young firms plays in the aggregate. I first simulate the model unconditionally, and compare the model-implied business cycle properties against the U.S. data in Subsection 3.9.1. I then study the behavior of the economy in the aftermath of the crisis. Subsection 3.9.2 considers a recession which is driven by an unexpected drop in a common productivity component (TFP). Subsection 3.9.3 analyzes a recession driven by a negative TFP shock accompanied by an increase in macro volatility.

3.9.1 Business Cycle Properties

In this subsection, I compute the standard business cycle statistics for both models — with and without firm entry and exit — and compare it against the U.S. data.

Table 3.8 shows that, in the data, investment is 5 times more volatile than output, while consumption is less volatile than output, and labor is slightly more volatile. All series are

TABLE 3.8: BUSINESS CYCLE STATISTICS

	Data			Model w/o EE			Model w/ EE		
	$\sigma(X)$	$\frac{\sigma(X)}{\sigma(Y)}$	$\rho(X, Y)$	$\sigma(X)$	$\frac{\sigma(X)}{\sigma(Y)}$	$\rho(X, Y)$	$\sigma(X)$	$\frac{\sigma(X)}{\sigma(Y)}$	$\rho(X, Y)$
<i>Y</i>	1.6	1.0	1.0	1.5	1.0	1.0	1.5	1.0	1.0
<i>I</i>	7.4	4.6	0.8	8	5.3	0.6	7.8	5.0	0.8
<i>C</i>	1.2	0.8	0.8	1.7	1.1	0.4	1.6	1.1	0.4
<i>L</i>	1.9	1.2	0.9	0.7	0.5	0.8	1.0	0.7	0.9

Notes: Table 3.8 reports the business cycle statistics computed from the model, as well as their empirical counterparts from the U.S. data. The U.S. data covers the period 1947Q1-2018Q1 and is imported from the FRED website. The series used are: real gross GDP, real gross private domestic investment, real personal consumption expenditures, and total non-farm business sector hours. First, all series have been expressed in logarithms, and subsequently HP-filtered with a smoothing parameter 1600. Y,I,C,L denote output, investment, consumption and labor, respectively.

highly correlated with aggregate output, with the coefficient of correlation exceeding 0.8.

Both models are to a great extent capable of picking up the investment- and consumption-related moments observed in the data. The model with no entry/exit margin has hard times generating enough volatility in hours (0.7 against 1.9 in the data) and is somewhat far from capturing high the correlation between consumption and output (0.4 against 0.8 in the data).

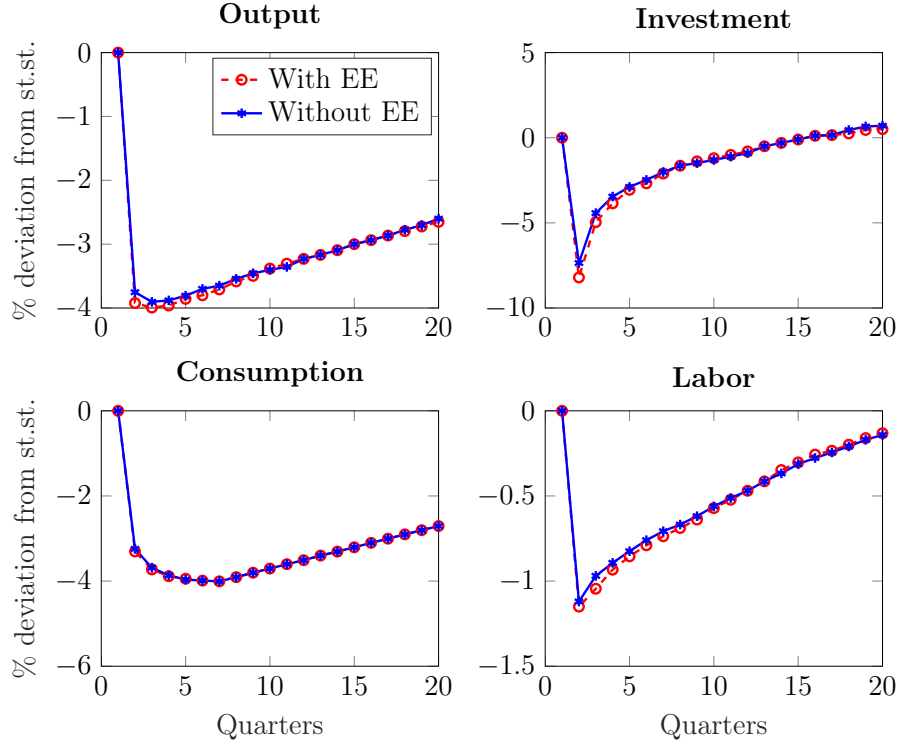
Overall, the model with entry and exit comes closer in hitting the data moments, as compared to the benchmark model. The relative success of the full model is visible the most in terms of investment-related moments: both the volatility of investment and its correlation with output get closer to the empirical counterparts. The overall volatility of labor increases (1.0 against 0.7 in the model with no entry), which represents another improvement over the model with no firm dynamics.

3.9.2 Recession: Drop in TFP Only

In this and the following subsections, I undertake the main quantitative exercise of the paper: I explore the behavior of the economy in the aftermath of a recession.

I start off with a case of a recession initiated by a 4% drop in the aggregate productivity

FIGURE 3.8: IMPULSE RESPONSE FUNCTIONS: NEGATIVE TFP SHOCK



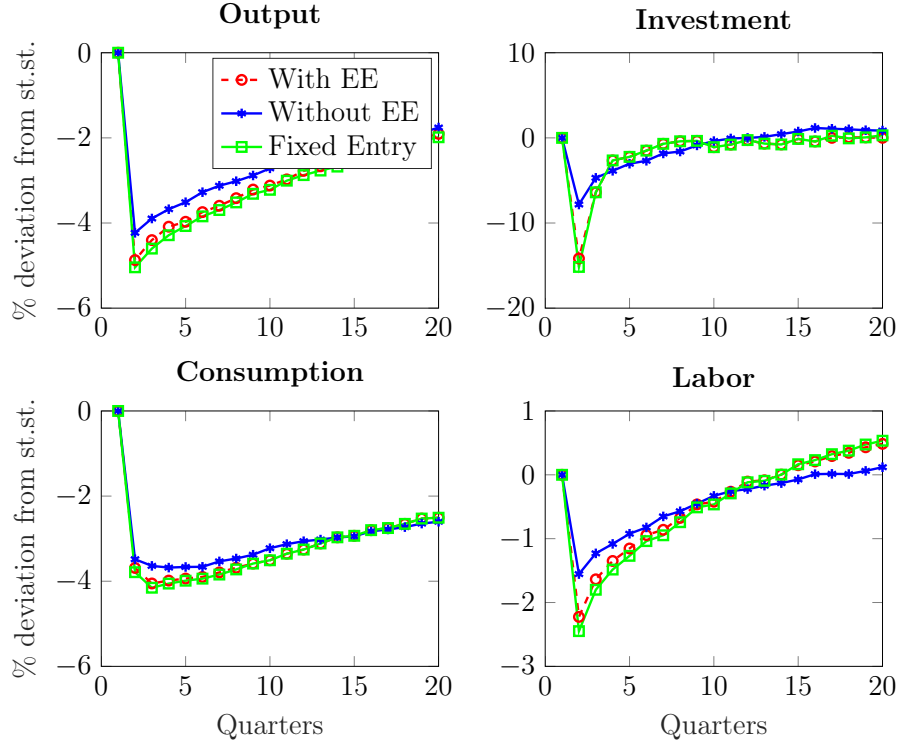
Notes: Figure 3.8 plots the impulse response functions for the case of a 4% drop in TFP only. Numbers along the vertical axis denote the deviation (in percent) from the steady-state. Time is measured in quarters and is marked along the horizontal axis. Blue solid lines with asterisk markers stay for the response of the model with no entry and exit, while the red dashed lines with hollow circle markers indicate the response of the full economy with entry/exit.

component z .¹⁵

Figure 3.8 displays the response of key macro aggregates — output, investment, consumption, and hours — to a negative TFP shock for two models: with no firm entry (blue solid lines with asterisk markers), and a full model with firm entry/exit (red hollow markers, dashed lines). The impulse-response functions are very close across the two models, which reiterates the well-known finding of macro literature that entry/exit margin has a quantitatively small role in the aggregate when considering aggregate TFP shocks only. The reason is that a TFP shock affects all firms *uniformly*: both small and large firms contract by the same

¹⁵The computational procedure for the general case of 2 shocks is as follows (“TFP shock only” case is handled by a straightforward simplification). I obtain the impulse-response functions by simulating a large number of independent economies for T_{IRF} periods. All economies are evolving normally until time T_{shock} , when I mechanically impose a high uncertainty state in all of them ($\sigma_{z, T_{\text{shock}}} = \sigma_H$); at the same time, I also reduce the aggregate TFP z by 4%. Then, for periods $T_{\text{shock}} + 1, \dots, T_{\text{IRF}}$ everything evolves normally again. The percentage deviation of variable X from the steady-state t periods after the shock is computed as $X_t^{\text{IRF}} = 100 \log \left(\frac{\bar{X}_{T_{\text{shock}}+t}}{\bar{X}_{T_{\text{shock}}-1}} \right)$, where \bar{X}_t denotes the cross-sectional mean of variable X at time t .

FIGURE 3.9: IMPULSE RESPONSE FUNCTIONS: NEGATIVE TFP SHOCK AND INCREASE IN VOLATILITY



Notes: Figure 3.9 plots the impulse response functions for the case of a 4% drop in TFP accompanied by a shift to a high-uncertainty regime. Numbers along the vertical axis denote the deviation (in percent) from the steady-state. Time is measured in quarters and is marked along the horizontal axis. Blue solid lines with asterisk markers stay for the response of the model with no entry and exit, while the red dashed lines with hollow circle markers indicate the response of the full economy with entry/exit. Green lines with square markers denote the response of the model with entry and exit, but where the mass of entrants is not affected by uncertainty regime.

proportion. Given that young firms are typically small, their contribution to aggregates is also small.

3.9.3 Recession: Drop in TFP and Increase in Volatility

Motivated by Figure 3.1, I next consider the impulse-response functions to a 2-shock recession: a negative TFP shock accompanied by an increase in macro-volatility. The response of an economy with firm entry and exit to a 2-shock recession is quantitatively different from the response of an economy with a fixed mass of firms (see Figure 3.9).

Overall, the model with entry and exit exhibits a more pronounced drop than the model without firm entry. For example, output decreases 25% stronger on impact (5% drop in a full model vs. 4% drop in a benchmark model). Discrepancy in response of investment

across the two models is striking: investment in the model with firm entry drops by 100% stronger (7% against 15% drop). This observation is consistent with the excess sensitivity of young firms to volatility shocks: even though young firms are small, they contribute disproportionately to aggregate investment. In the model it occurs because idiosyncratic productivity of mature firms is at (or near) the stationary state of s , which implies that those firms are unlikely to undertake large investment decisions. In turn, young firms are typically less productive than incumbents, and their idiosyncratic productivity (conditional on survival) grows over time. These firms, therefore, find it optimal to accumulate capital which their increasing productivity dictates, and push their investment expenditures up. It is precisely due to the young firms' excess response why the aggregate investment drops so much following an increase in volatility.

By comparing the speed of recovery between Figures 3.8 and 3.9, one can notice that even though the economy contracts by more after a 2-shock recession, it seems to recover faster as well: 20 quarters after a 2-shock recession, investment and labor are completely recovered, and output is 2% below the steady-state level (as opposed to almost 3% in case of a TFP shock). I argue that this happens due to the mean-reversion property of productivity process (3.5): low aggregate productivity z along with high uncertainty implies better prospects about the evolution of future aggregate productivity as compared to low productivity and low uncertainty. Intuitively, this happens because in the case of low uncertainty, economic agents expect the “bad” aggregate conditions to persist for longer, while under higher volatility the probability of z to go up increases by more than its probability to go further down. I provide a formal proof in Appendix C.3 which supports this intuition.

This property of a mean-reverting process implies, in particular, that the continuation value in the recursive problem of a potential entrant (Equation (3.14)) is higher when the economy's current state is characterized as low z with high volatility: a potential entrant with signal \tilde{q} is more likely to enter under low z and high σ , than under low z and low σ . One, therefore, might be concerned with whether the discrepancies in IRFs found in Figure 3.9 are driven by the excess sensitivity of existing firms, or they simply reflect a more intense inflow of new firms. In order to disentangle these two forces, I simulate the economy where — at the moment of a shock — potential entrants see a change in TFP, but do not observe heightened volatility. I, therefore, obtain an economy where existing firms respond to both shocks, but the extensive margin of firms dynamics remains intact: the difference in IRFs

between the models can then be fully attributed to the excess sensitivity of young enterprises to heightened macro volatility.

Green lines with square markers in Figure 3.9 correspond to the economy with fixed entry. The red lines with asterisks and green lines with squares virtually coincide: this suggests that the extensive margin of the firm entry explains a small portion of observed differences in response of macro aggregates.

3.10 Conclusion

In this paper, I explored the aggregate implications of volatility shocks through the investment channel. I first provided suggestive evidence that young businesses exhibit excess sensitivity to fluctuations in macro volatility as compared to mature enterprises. To rationalize the empirical findings, I built a general equilibrium model of investment with firm entry and exit. The mechanism works through the combination of non-convex capital adjustment costs and gradually rising productivities of firms over the average life-cycle. By studying the impulse response functions, I also showed that this mechanism has important implications in the aggregate: the response of output, investment, consumption, and hours is more pronounced in the model with firm entry and exit, and it cannot be fully attributed to an extensive margin of firms' entry. This suggests that — as opposed to previous macro literature — firm entry and exit can be quantitatively important for business cycle fluctuations.

There are several potential directions for future research. While in this paper firms born at different stages of the business cycle differed only in terms their initial productivity level, more empirical work is needed to understand the composition of startups across booms and busts. Access to finance — the dimension this paper did not speak to — might be another important determinant of firm creation across different stages of the cycle.

Recent research by [Salgado, Guvenen and Bloom \(2016\)](#) reveals that skewness of several firm-level variables (such as the growth rate of profit and sales) is strongly procyclical. The economic interpretation of this observation is that during recessions a disproportionate mass of firms experiences very large negative shocks. The distributional and aggregate consequences of third-moment fluctuations might be another interesting avenue to pursue.

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Appendix A

Appendix to Chapter 1

A.1 Empirical Appendix

This appendix provides further details for the empirical part of the paper, including data background, sample selection and additional empirical results, referenced in the main text.

A.1.1 Bureau van Dijk Data (BvD) - Background

The objective of this section is to provide a brief institutional background for the firm-level accounting data in the U.K. Please see [Bahaj, Foulis and Pinter \(2018\)](#) for full details on data construction.

Companies House is the Registrar of firms in the U.K. At the end of a fiscal year, every firm must prepare a set of statutory annual accounts which they file with Companies House. This information includes the balance sheet, as well as profit and loss accounts. All limited firms are required to file, but reporting standards differ across firms of different sizes (see Companies House guide for details). Firms must file accounts every 18 months. Firms which do not report every 12 months are excluded from the analysis.

Companies House is the original source of the data, but the direct source is BvD, which provides a workable interface to access it. For the U.K., BvD provides microdata through the product called FAME (Financial Analysis Made Easy). This is a separate product from commonly used Amadeus and Orbis - also provided by BvD - which cover firms at the European and Global levels, respectively. The data comes on a monthly basis in DVD (and, subsequently, Blu-Ray) discs. A disc contains a snapshot of the FAME database for the U.K. firms in a particular month. The database is updated on the monthly basis, and incorporate new information as firms file new accounts, or when firms conduct report-driven filings (for example, if director's information changes).

The discs were sampled at 6 month frequency for two reasons. First, as has been mentioned above, firms might have irregular filing periods or may conduct event-driven irregular filings, therefore, biannual sampling mitigates this issue. Second, this frequency has been chosen to balance the capacity required to store the information and the amount of new information which is gained by adding additional discs. Increasing the frequency beyond biannual was found to bring very little extra information.

Each firm in the UK is assigned a unique Companies House Registration Number (CRN) upon formation which stays with the firm throughout its lifetime. The CRN may change if

Companies House chooses to adopt a new numbering format, but this did not happen over the sample period. This firm-level identifier was used to construct a panel from multiple vintages of the FAME dataset.

A.1.2 Selection Criteria and Summary Statistics

The sample was constructed to satisfy the following selection criteria:

1. only observations with “Live” status were retained;
2. observations with extreme growth rates of real total assets were dropped (i.e., those in 1st or 99th percentile of the corresponding distribution of growth rates);
3. similarly, observations with extreme investment rates were excluded. Investment rate for firm i in year t is calculated as follows:

$$irate_{it} = \frac{\Delta \text{fixed assets}_{it} + \text{depreciation}_{it}}{\text{fixed assets}_{it-1}},$$

where fixed assets_{it} is the firm’s i amount of fixed assets at the end of period t ;

4. drop firms which do not comply with Companies’ Act: “Economic European Interest Grouping”, “Guarantees”, “Industrial Provident”, “Limited Liability Partnership”, “Not Companies Act”, “Royal Charter”, “Unlimited”, “Public Investment Trust”, “Limited Partnership” and “Charity”;
5. exclude firms in services, financial and real estate sectors ($\text{sic} > 5$).
6. drop observations with negative leverage, or with leverage exceeding 1. I consider 3 different financial leverages:
 - (a) short-term leverage, which is a ratio of short-term debt and overdrafts to total assets;
 - (b) long-term leverage, defined as a ratio of long-term debt to total assets;
 - (c) full liability-based leverage:

$$leverage_{it} = \frac{(\text{total assets}_{it} - \text{shareholders' funds}_{it})}{\text{total assets}_{it}}.$$

7. observations with negative `turnover` and/or `total assets` were also dropped.

TABLE A.1: SUMMARY STATISTICS

	Empl.	Collat.	ST lev.	LT lev.	Total lev.	Age
Private firms						
Mean	103	0.30	0.19	0.17	0.29	12
Bottom 25%	10	0.07	0.04	0.04	0.13	3
Median	44	0.21	0.12	0.11	0.26	7
Top 25%	97	0.47	0.27	0.24	0.42	16
Public firms						
Mean	737	0.39	0.12	0.13	0.24	26
Bottom 25%	32	0.17	0.02	0.03	0.11	9
Median	113	0.36	0.07	0.09	0.21	19
Top 25%	628	0.58	0.16	0.19	0.34	37

Notes: Table A.1 reports the descriptive statistics. All nominal variables have been deflated with the CPI (2014 being base year). Table reports summary statistics for private and public firms separately. Private firms include “Private Limited” category. Public firms include the following categories: “Public, Quoted”, “Public, Not Quoted”, “Public, Quoted OFEX”, “Public AIM”. `Collateral` is the ratio of fixed assets to total assets. `ST leverage` is short-term debt and overdrafts divided by total assets. `LT leverage` is long-term debt to total assets. `Total leverage` is a sum of `short-term debt`, `overdrafts` and `long-term debt` divided by total assets. All ratios are winsorized at 1/99 percentiles.

In the end, I am left with ≈ 7 mln (firm, year) observations, which corresponds to approximately 1.5mln distinct firms. Table A.1 reports summary statistics for the final dataset.

A.1.3 Firm-Size Distribution

The dataset covers the corporate universe in the U.K., but as it was mentioned earlier in Appendix A.1.1, reporting requirements vary dramatically by the size of the firm. In general, larger firms have to report more information. The `total assets` variable — the book value of firm’s assets — is reported uniformly in the data, but `employment` is available for only about 10% of observations.

In order to circumvent reporting issues and gauge how the sample firm-size distribution looks like, I follow [Haltiwanger, Jarmin, Kulick and Miranda \(2017\)](#) and [Dinlersoz, Kalemli-Ozcan, Hyatt and Penciakova \(2018\)](#) and construct weights by way of fitting probit regressions. In particular, I split all (firm, year) couples into one of 3 groups: entering, continuing and exiting businesses. Subsequently, within each group I fit a probit model with left-hand side variable R_{it} being an indicator which takes a value of 1 if firm i reports employment in year t , and 0 otherwise:

– continuing firms

$$R_{it} = \alpha + \gamma_1 \log(\text{TotalAssets}_{it}) + \gamma_2 \text{Age}_{it} + \gamma_3 D10_{it} + \gamma_4 AG_{it} + ind + \varepsilon_{it}$$

– births

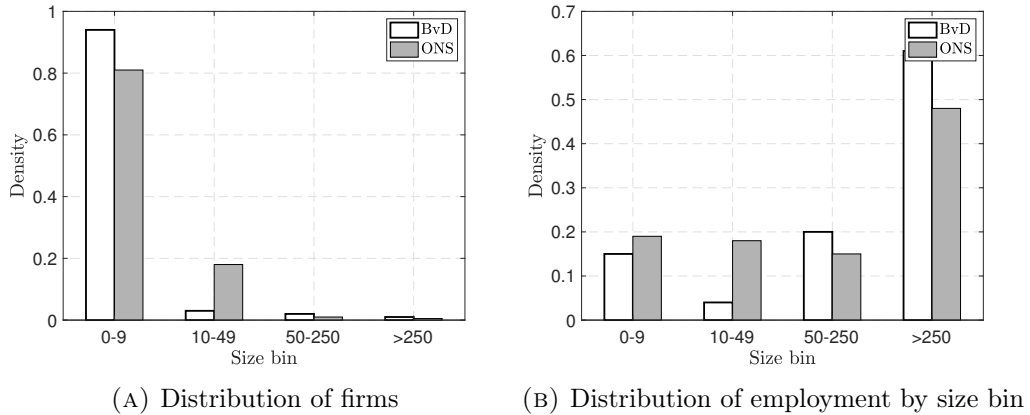
$$R_{it} = \alpha + \gamma_1 \log(\text{TotalAssets}_{it}) + ind + \varepsilon_{it}$$

– deaths

$$R_{it} = \alpha + \gamma_1 \log(\text{TotalAssets}_{it}) + \gamma_2 \text{Age}_{it} + \gamma_3 D10_{it} + ind + \varepsilon_{it},$$

where $D10$ is an indicator whether the firm is older than 10 years, AG is asset growth rate (7 groups), and ind is 2-digit SIC industry dummy.

FIGURE A.1: FIRM-SIZE DISTRIBUTION IN 2014: BvD AND ONS



Notes: Figure A.1 contains 2 panels. Panel A plots the distribution of firms across size bins: ONS data is gray, BvD is white. Size bins are in terms of employment. Panel B reports distribution of employment across size groups. See Appendix A.1.3 for details on how firm-size distribution was constructed in BvD data.

Figure A.1 is constructed using weights which are the inverse of predicted probabilities from probit regressions. The data gets reasonably close to the official firm-size distribution: it captures correctly the share of small firms in population, as well as their share of the overall employment.

A.1.4 Bonhomme and Manresa (2015) Grouped Fixed Effects Estimator

This appendix describes the application of Bonhomme and Manresa (2015) grouped fixed effect estimator (GFE) on BvD data. This clustering algorithm optimally assigns firms into the pre-specified number of homogeneous groups after controlling for observables. Specifically, let y_{it} be a some observable of firm i at time t . Let g_i denote the group firm i belongs to (the total number of groups is pre-set by researcher). Bonhomme and Manresa (2015) consider the following linear model:

$$y_{it} = \alpha_{g_{it}} + x'_{it}\theta + \varepsilon_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T. \quad (\text{A.1})$$

In Equation (A.1), x_{it} is a vector of covariates whose effect one wishes to partial out. As a result, units whose time profiles of observable - net of impact of covariates - are most similar, are grouped together in estimation.

There were several features of the data and estimator itself that determined the nature of this exercise for the purposes of this project. First of all, GFE is a computationally intensive procedure which is suitable for relatively small panels. This feature of the algorithm rendered its application on the entire BvD data impossible. To circumvent this issue, I followed “trial and error” approach and found the largest sample of firms I could work with. In particular, GFE was applied on a 20% random subsample of manufacturing firms born between years 2006 and 2009. GFE was applied numerous times in order to ensure that results are consistently similar across different draws.

Since small firms face different from big firms reporting requirements in the U.K., few entrants report employment consistently throughout their life cycle. Therefore, uniformly reported total assets were used as a measure of size. I also restricted analysis to firms reaching the age of 8.¹ This was done for two reasons. First, the model developed in Section 3.4 views long-term differences in firm growth as a consequence of permanent heterogeneity. In order to pick up this heterogeneity in the data, one needs to observe firms for sufficiently long period of time. Second, longer time dimension improves the performance of GFE.

Finally, clustering was performed based on the life cycle of firms’ sizes. In practice, I found that clustering based on growth rates is very unstable. Moreover, Table A.2 shows that even

¹This is the longest life cycle I can observe for a firm born in 2009, given that BvD spans years through 2017. No noticeable differences were found when a different sample was used (firms surviving through age of 5).

though firms were clustered based on their size, the groups preserve ranking with respect to cumulative growth. This is not surprising given that for young firms growth and size are highly correlated.

Figure 1.2 in main text reports the results. In panel A, vector of controls includes a cyclical component of GDP. Panel B adds liability-based leverage into vector of controls. I consistently find a shift toward less fast-growing firms started during recessionary years 2008-2009.

TABLE A.2: AVERAGE 5-YEAR CUMULATIVE GROWTH BY CLUSTER

	$\mathbb{E}[\log(y_{age5}) - \log(y_{age1})]$
Slow	0.12
Medium	0.45
High	0.58

Notes: Table A.2 reports the average growth of total assets between ages 1 and 5 for firms clustered into “slow”, “medium” and “high” groups using [Bonhomme and Manresa \(2015\)](#) grouped fixed effects estimator. See Appendix A.1.4 for further details on application of that algorithm.

A.1.5 Longitudinal Business Database (LBD)

LBD is the most comprehensive panel dataset covering the universe of U.S. businesses and spanning the years 1976-2016.² The unit of observation in LBD is an establishment, which is defined as a single physical location where business operations take place. A firm is then defined as a set of establishments that are under common ownership or control.

LBD is based on several sources, such as the Business Register (also known as the Standard Statistical Establishment Listing - SSEL), Economic Censuses, and surveys. The LBD offers the most reliable and complete data on births, deaths, and age of establishments operating in the US. There are several data issues potentially leading to measurement errors in identification of business formation (for example, non-administratively registered establishments may not be correctly identified, or gaps in the records of establishments). See [Jarmin and Miranda, 2002](#)) for more details on which efforts have been undertaken to mitigate this issues in the process of construction of the LBD.

Identification of Birth and Death

The unit of observation in LBD is an establishment, and variable `lbdnum` - which is robust to mergers and acquisitions - is used to track establishments over time.

²Years 2017 and 2018 will soon become available.

Establishments The age of an establishment is measured as the number of years elapsed since the first year that establishment appeared in the data.³ Since LBD contains establishments with at least one paid employee, it might be the case that some businesses are older than what can be measured in the data (if that establishment had no paid employees in the first several years of operation and then evolved to an employer business).

Firms Identification of firm birth and death is associated with the construction of firm linkages over time. I follow a standard approach in the literature which is robust to ownership changes and acquisitions (Haltiwanger, Jarmin and Miranda, 2013). A new firm identifier emerges in the LBD either because a new firm is born or because an existing businesses undergoes a change of ownership and control (e.g. merger and acquisition, divestitures). A new firm is registered when all of its establishments are of age 0. Accordingly, when a new firm identifier arises through a merger of two preexisting firms, it is not treated as a firm birth and is assigned the age of the oldest continuing establishment of the newly combined business. The firms are then allowed to age naturally regardless of mergers and acquisitions as long as the ownership and control does not change. A firm death is determined when a firm identifier disappears and all associated establishments cease operations and exit.

Employment

Establishments Employment (LBD variable `emp`) is defined as the number of full- and part-time employees⁴ as of March 12th of each calendar year.

Firms Firms can own a single establishment or many establishments, which may span multiple geographic areas and industries. I compute firm-level characteristics based on the characteristics of the set of establishments of the firm. Naturally, firm-level employment is calculated as the sum of employment across the establishments belonging to that firm.

³Age cannot be measured for establishments born prior to 1976 - the first year covered by the LBD. For that reason, in my empirical exercise I exclude the cohort of 1976. Besides, (Moscarini and Postel-Vinay, 2012) argue that there are concerns with the first couple of LBD cohorts; I, therefore, use cohorts starting from 1978.

⁴Including employees who are on paid sick leave, holidays, and vacations. The reported number also includes salaried officers and executives of corporations, but it excludes sole proprietors and partners of unincorporated businesses.

Industry

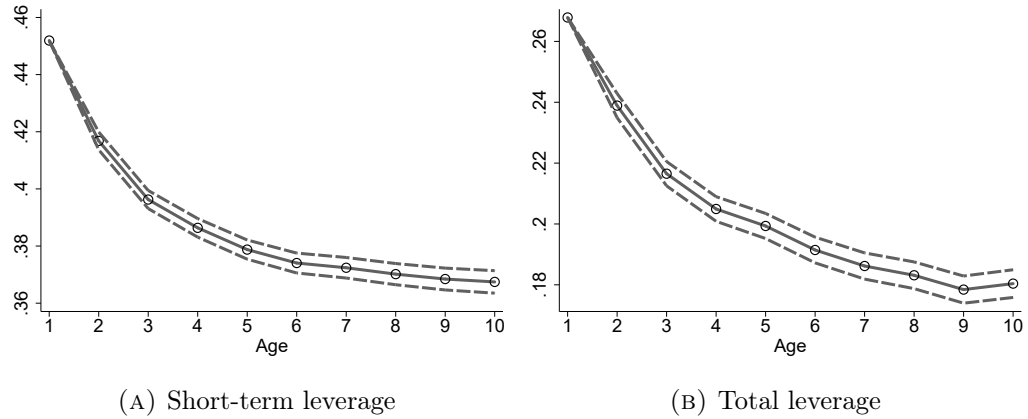
The LBD also includes detailed information on industry classification. The period of analysis coincides with the transition from SIC to NAICS industry classifications, which leads to classification issues. I, therefore, use a consistent NAICS 2002 industry classification variable constructed by [Fort and Klimek \(2016\)](#) (variable `fk_naics12`).

Firms frequently encompass establishments from several industries. For that reason, I assign to the firm an industry of its largest (in terms of employment) establishment.

A.2 Figures

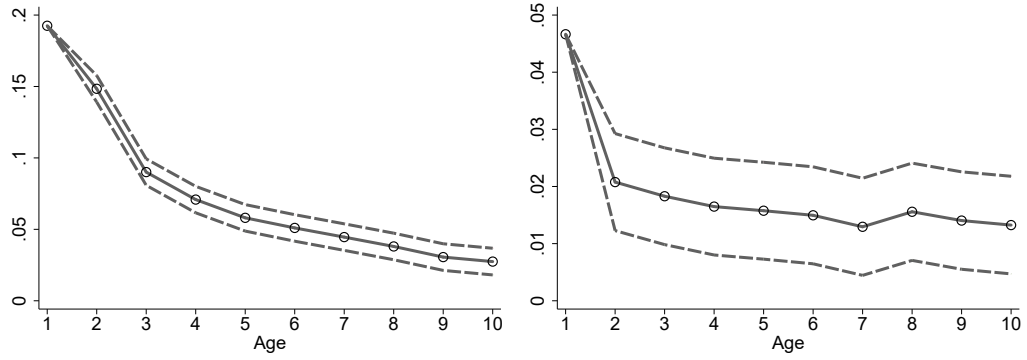
This section contains additional figures referenced in the main text.

FIGURE A.2: SHORT- AND LONG-TERM LEVERAGE BY AGE



Notes: Figure A.2 plots the estimated age fixed effects (plus constant) from projecting short-term leverage (defined as a ratio of short-term debt and overdrafts to total assets) and long-term leverage (long-term debt to total assets) on firms' log assets, collateral, turnover, age dummies and a full set of industry-year fixed effects. All ratios were winsorized at 2/98 percentiles. Source: BvD.

FIGURE A.3: DEBT AND EQUITY FINANCING BY AGE

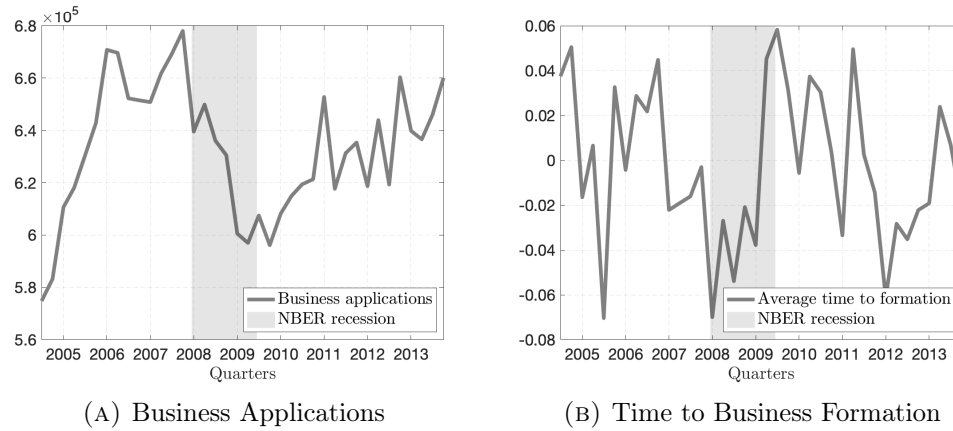


(A) Debt financing

(B) Equity financing

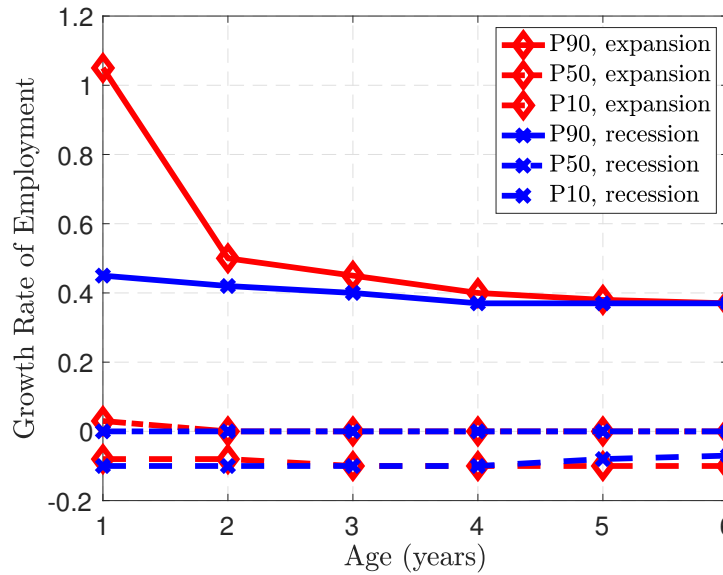
Notes: Figure A.3 plots the estimated age fixed effects (plus constant) from projecting a change in total liabilities (Panel A) and equity financing (Panel B) on age dummies and a full set of industry-year fixed effects. Equity financing is defined as a change in issued capital and share premium account. All ratios were winsorized at 2/98 percentiles. Source: BvD.

FIGURE A.4: BUSINESS APPLICATIONS AND TIME TO BUSINESS FORMATION IN THE U.S.



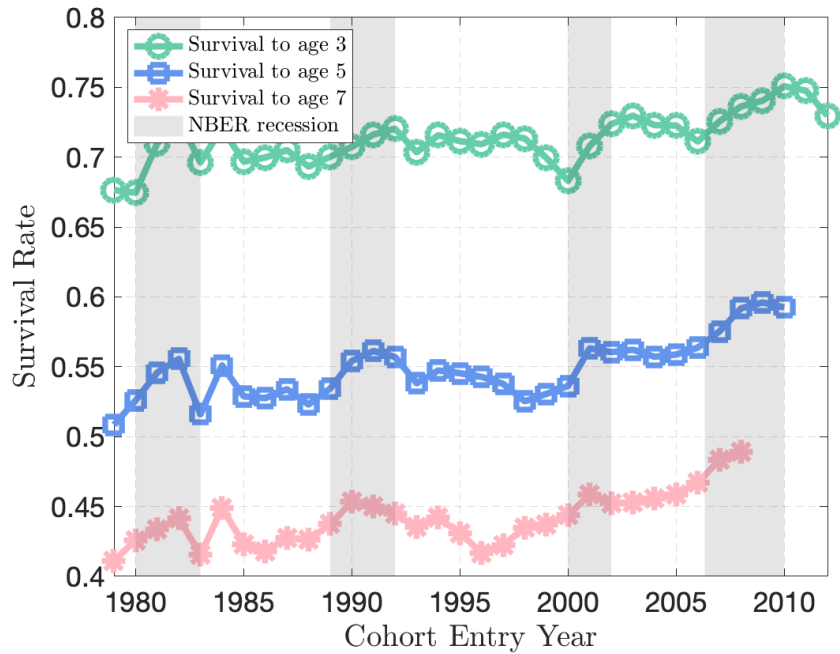
Notes: Figure A.4 consists of 2 panels. Panel A plots the number of business application in the U.S., while Panel B plots the average time between application and business formation in the U.S. The data is from the Business Formation Statistics and spans the period 2004Q3 to 2013Q4 at quarterly frequency. Business application is identified as a filing of the IRS Form SS-4. Time to business formation is an average (in quarters) time between the filing of the SS-4 form and the first quarter when positive payroll is recorded. Panel B plots the detrended series (linear time trend is subtracted).

FIGURE A.5: DISTRIBUTION OF EMPLOYMENT GROWTH OF CONTINUERS BY AGE



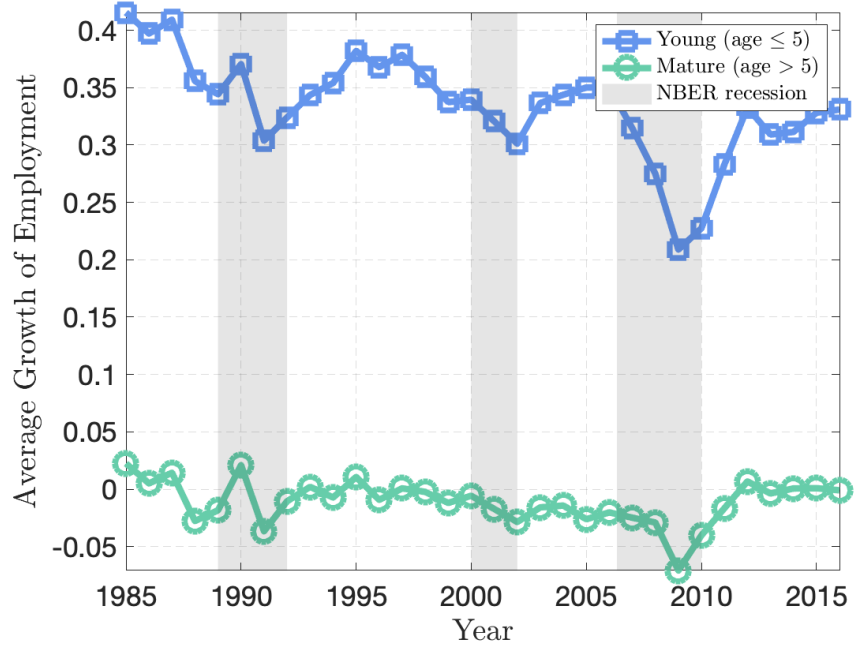
Notes: Figure A.5 plots the distribution of employment growth rates for continuing firms born in the recession (2008-2009) and in the expansion (2006-2007). Dashed lines correspond to the 10th percentile of the employment growth distribution (traced by age), the dot-dashed lines mark the median growth, and solid lines - the 90th percentile. Blue lines with circle markers denote recessionary cohorts, and red lines with triangle markers - expansionary ones. Growth rate of firm i at time t is computed as $2(E_{it} - E_{it-1}) / (E_{it} + E_{it-1})$. Source: BvD.

FIGURE A.6: SURVIVAL RATES IN THE U.S.



Notes: Figure A.6 plots the survival rates in the U.S. private sector. The line with circle markers corresponds to survival rates to the age of 3, the line with square markers—to the age of 5, and the line with star markers—to the age of 7. The survival rate is calculated as a number of establishments which make it to a specific age divided by the total number of establishments started in a specific year. The gray vertical bars represent NBER recessions. Data source: LBD.

FIGURE A.7: MEAN GROWTH RATE OF EMPLOYMENT IN THE U.S.: YOUNG VERSUS OLD



Notes: Figure A.7 plots the average growth of employment for young (age under 5 years) and old (age above 5 years) establishments. The average growth rate is employment-weighted. The growth rate is defined as a one-year log change in employment. The vertical gray bars represent NBER recession dates. Data source: LBD.

A.3 Tables

TABLE A.3: RELATIVE PRODUCTIVITY OF ENTRANTS

	Labor productivity		TFP	
	Manufacturing	All	Manufacturing	All
2005-2006	0.92	0.88	0.91	0.90
2007-2009	0.93	0.90	0.94	0.92
2010-2012	0.95	0.90	0.96	0.92

Notes: Table A.3 reports the relative productivity of entrants with respect to incumbents for all industries considered (see sample selection criteria in Appendix A.1.2) and for manufacturing sector separately. Two productivity measures are considered: labor productivity and TFP (defined as a residual from production function). Entrants are firms of ages 1 and 2, incumbents are firms older than 2. Labor productivity is calculated as log turnover minus log of number of employees. Production function was estimated for each 1-digit industry using [Olley and Pakes \(1996\)](#) methodology. Fixed assets were used as a measure of capital, and investment expenditures were used as a proxy for unobserved productivity.

TABLE A.4: INITIAL AGGREGATE CONDITIONS AND PROBABILITY OF CUMULATIVE GROWTH TO BE ABOVE THE 10TH PERCENTILE: U.S. DATA

	(1)	(2)	(3)	(4)	(5)
$\mathbf{1}_{\{\text{Recession}\}}$	0.0008*** (0.000)	0.0009*** (0.000)	0.0023*** (0.000)	0.0026*** (0.000)	-0.0012*** (0.000)
Const.	0.9180*** (0.000)	0.9181*** (0.000)	0.9180*** (0.000)	0.9180*** (0.000)	0.9208*** (0.000)
N (in 1000's)	3752	3744	3576	3569	2358
Adj. R^2	0.01	0.01	0.01	0.01	0.01
Industry FE	Yes	Yes	Yes	Yes	Yes
Region FE	No	Yes	Yes	Yes	Yes
Legal Form of Organization FE	No	No	Yes	Yes	Yes
Multi-Unit Status FE	No	No	No	Yes	Yes
Type of Operation FE	No	No	No	No	Yes

Notes: Table A.4 reports the OLS estimates of the following linear probability model:

$$\mathbf{1}_{\{\text{c.growth}_{it} \geq P10\}} = \beta \mathbf{1}_{\{\text{entered in recession}\}} + \text{controls} + \varepsilon_{it},$$

where the dependent variable equals 1 if the 5-year cumulative growth of establishment i is above the 10th percentile, and 0 otherwise. The key independent variable is a binary variable, which takes a value of 1 if establishment i entered in a recession, and 0 otherwise. Recessionary years are defined according to the NBER classification. The vector of controls includes the following categorical variables: 4-digit NAICS (imputed from a 6-digit NAICS variable `fk_naics12` constructed by [Fort and Klimek, 2016](#)), county FIPS, legal form of organization (LBD variable `lfo`), multi-unit status (LBD variable `mu`), and type of operation (LBD variable `toc`). The cumulative growth for each establishment i is calculated as a log change in employment between ages 1 and 5. Subsequently, the 10th percentile of the cumulative growth distribution is constructed based on the pooled sample of cumulative growth rates across all establishment for which the cumulative growth is defined. Data source: LBD.

TABLE A.5: INITIAL AGGREGATE CONDITIONS AND PROBABILITY OF CUMULATIVE GROWTH TO BE ABOVE THE 50TH PERCENTILE: U.S. DATA

	(1)	(2)	(3)	(4)	(5)
$\mathbf{1}_{\{\text{Recession}\}}$	-0.0041*** (0.001)	-0.0040*** (0.001)	-0.0043*** (0.001)	-0.0038*** (0.001)	-0.0087*** (0.001)
Const.	0.4794*** (0.000)	0.4794*** (0.000)	0.4794*** (0.000)	0.4792*** (0.000)	0.4719*** (0.000)
N (in 1000's)	3752	3744	3576	3569	2358
Adj. R^2	0.01	0.01	0.01	0.01	0.02
Industry FE	Yes	Yes	Yes	Yes	Yes
Region FE	No	Yes	Yes	Yes	Yes
Legal Form of Organization FE	No	No	Yes	Yes	Yes
Multi-Unit Status FE	No	No	No	Yes	Yes
Type of Operation FE	No	No	No	No	Yes

Notes: Table A.5 reports the OLS estimates of the following linear probability model:

$$\mathbf{1}_{\{\text{c.growth}_{it} \geq P50\}} = \beta \mathbf{1}_{\{\text{entered in recession}\}} + \text{controls} + \varepsilon_{it},$$

where the dependent variable equals 1 if the 5-year cumulative growth of establishment i is above the 50th percentile, and 0 otherwise. The key independent variable is a binary variable, which takes a value of 1 if establishment i entered in a recession, and 0 otherwise. Recessionary years are defined according to the NBER classification. The vector of controls includes the following categorical variables: 4-digit NAICS (imputed from a 6-digit NAICS variable `fk_naics12` constructed by [Fort and Klimek, 2016](#)), county FIPS, legal form of organization (LBD variable `lfo`), multi-unit status (LBD variable `mu`), and type of operation (LBD variable `toc`). The cumulative growth for each establishment i is calculated as a log change in employment between ages 1 and 5. Subsequently, the 50th percentile of the cumulative growth distribution is constructed based on the pooled sample of cumulative growth rates across all establishment for which the cumulative growth is defined. Data source: LBD.

TABLE A.6: INITIAL AGGREGATE CONDITIONS AND PROBABILITY OF CUMULATIVE GROWTH TO BE ABOVE THE 90TH PERCENTILE: U.S. DATA

	(1)	(2)	(3)	(4)	(5)
$\mathbf{1}_{\{\text{Recession}\}}$	-0.0039*** (0.000)	-0.0039*** (0.000)	-0.0046*** (0.000)	-0.0043*** (0.000)	-0.0053*** (0.001)
Const.	0.0947*** (0.000)	0.0947*** (0.000)	0.0948*** (0.000)	0.0946*** (0.000)	0.0957*** (0.000)
N (in 1000's)	3752	3744	3576	3569	2358
Adj. R^2	0.01	0.01	0.01	0.02	0.02
Industry FE	Yes	Yes	Yes	Yes	Yes
Region FE	No	Yes	Yes	Yes	Yes
Legal Form of Organization FE	No	No	Yes	Yes	Yes
Multi-Unit Status FE	No	No	No	Yes	Yes
Type of Operation FE	No	No	No	No	Yes

Notes: Table A.6 reports the OLS estimates of the following linear probability model:

$$\mathbf{1}_{\{\text{c.growth}_{it} \geq P90\}} = \beta \mathbf{1}_{\{\text{entered in recession}\}} + \text{controls} + \varepsilon_{it},$$

where the dependent variable equals 1 if the 5-year cumulative growth of establishment i is above the 90th percentile, and 0 otherwise. The key independent variable is a binary variable, which takes a value of 1 if establishment i entered in a recession, and 0 otherwise. Recessionary years are defined according to the NBER classification. The vector of controls includes the following categorical variables: 4-digit NAICS (imputed from a 6-digit NAICS variable `fk_naics12` constructed by [Fort and Klimek, 2016](#)), county FIPS, legal form of organization (LBD variable `lfo`), multi-unit status (LBD variable `mu`), and type of operation (LBD variable `toc`). The cumulative growth for each establishment i is calculated as a log change in employment between ages 1 and 5. Subsequently, the 90th percentile of the cumulative growth distribution is constructed based on the pooled sample of cumulative growth rates across all establishment for which the cumulative growth is defined. Data source: LBD.

TABLE A.7: INVESTMENT AND RESIDENTIAL WEALTH OF DIRECTORS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Slow	0.0004 (0.000)	0.0160*** (0.001)	0.0189*** (0.001)	0.0191*** (0.001)	0.0189*** (0.001)	0.0059*** (0.001)	0.0062*** (0.001)	0.0062*** (0.001)
Fast	0.0082*** (0.000)	0.0261*** (0.001)	0.0269*** (0.001)	0.0271*** (0.001)	0.0270*** (0.001)	0.0148*** (0.001)	0.0126*** (0.001)	0.0130*** (0.001)
Leverage					-0.1847*** (0.016)	-0.1790*** (0.016)	-0.1492*** (0.015)	-0.1500*** (0.015)
Inv. scale						20.2029*** (1.091)	18.4466*** (1.031)	18.3335*** (1.029)
Profit							0.0766*** (0.003)	0.0774*** (0.003)
HPI								-0.0001 (0.000)
Const.		0.1371*** (0.002)						
<i>N</i>	108013	105109	105072	105004	104998	104998	104998	104072
Adj. <i>R</i> ²	0.03	0.18	0.20	0.20	0.20	0.21	0.23	0.23
Firm FE	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Region-time FE	No	No	No	Yes	Yes	Yes	Yes	Yes

Notes: Table A.7 reports OLS estimates of Equation (1.2). The sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1.1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm 'Fixed Assets'. All ratios are winsorized at the median \pm 5 times interquartile range. Standard errors, clustered by firm region, are in parentheses. Column (1) reports the effect of residential real estate on firms' investment without other controls. Column(2) adds firm fixed effects. Column (3) further adds industry-time (2-digit SIC classification) fixed effects. Column (4) adds region-year fixed effects. Column (5) adds firms' leverage. Column (6) additionally controls for potentially spurious size effects and includes the inverse of (lagged) fixed assets. Columns (7) and (8) further include firms' profits and a firm-region house price index. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

TABLE A.8: COLLATERAL CHANNEL OF RESIDENTIAL PROPERTY: SPLIT BY AGE AND SIZE

	Baseline		Age group		Size group	
	(1)	(2)	(3)	(4)	(5)	(5)
Slow	0.0062*** (0.001)	0.0108* (0.006)	0.0065*** (0.001)	0.0067*** (0.002)	0.0173*** (0.003)	0.0173*** (0.003)
Fast	0.0130*** (0.001)	0.0140** (0.006)	0.0135*** (0.001)	0.0105*** (0.001)	0.0209*** (0.003)	0.0209*** (0.003)
Leverage	-0.1500*** (0.015)	-0.2224*** (0.085)	-0.1588*** (0.017)	-0.1463*** (0.032)	-0.1767*** (0.021)	-0.1767*** (0.021)
Inv. scale	18.3335*** (1.029)	32.8427*** (4.274)	18.4368*** (1.109)	21.3020*** (1.555)	14.3574*** (5.373)	14.3574*** (5.373)
Profit	0.0774*** (0.003)	0.0873*** (0.018)	0.0762*** (0.003)	0.0545*** (0.005)	0.1065*** (0.005)	0.1065*** (0.005)
HPI	-0.0001 (0.000)	-0.0004 (0.001)	-0.0001 (0.000)	0.0002 (0.000)	-0.0002 (0.000)	-0.0002 (0.000)
<i>N</i>	104072	5019	99844	30382	52329	52329
Adj. <i>R</i> ²	0.23	0.38	0.22	0.23	0.20	0.20
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Industry-time FE	Yes	Yes	Yes	Yes	Yes	Yes
Region-time FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Table A.8 reports OLS estimates of Equation (1.2 for different sample splits. The full sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1.1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables are scaled by the lag of firm "Fixed Assets". All ratios are winsorized at the median \pm 5 times interquartile range. Column (1) provides baseline estimates. Columns (2) and (3) report estimates of Equation (1.2) for young (ages 1-5) and old (ages 5+) subsamples. Columns (4) and (5) report estimates for small (less than 50 employees) and large (more than 50 employees) subsamples. Size is determined as a time-series average of firm-level employment. Standard errors, clustered by firm region, are in parentheses. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

TABLE A.9: INVESTMENT AND RESIDENTIAL WEALTH OF DIRECTORS, CONDITIONAL ON REACHING AGE 5

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Slow	0.0005* (0.000)	0.0162*** (0.001)	0.0191*** (0.001)	0.0193*** (0.001)	0.0191*** (0.001)	0.0060*** (0.001)	0.0063*** (0.001)	0.0064*** (0.001)
Fast	0.0082*** (0.000)	0.0260*** (0.001)	0.0269*** (0.001)	0.0271*** (0.001)	0.0270*** (0.001)	0.0148*** (0.001)	0.0126*** (0.001)	0.0130*** (0.001)
Leverage					-0.1852*** (0.016)	-0.1794*** (0.016)	-0.1496*** (0.015)	-0.1503*** (0.015)
Inv. scale						20.2223*** (1.123)	18.4575*** (1.064)	18.3352*** (1.061)
Profit							0.0769*** (0.003)	0.0776*** (0.003)
HPI								-0.0001 (0.000)
Const.	0.1370*** (0.002)							
<i>N</i>	107767	104949	104912	104844	104837	104837	104837	103916
Adj. <i>R</i> ²	0.03	0.18	0.20	0.20	0.20	0.21	0.23	0.23
Firm FE	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Region-time FE	No	No	No	Yes	Yes	Yes	Yes	Yes

Notes: Table A.9 reports OLS estimates of Equation (1.2). The sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1.1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm 'Fixed Assets'. All ratios are winsorized at the median \pm 5 times interquartile range. Standard errors, clustered by firm region, are in parentheses. Column (1) reports the effect of residential real estate on firms' investment excluding other controls. Column(2) adds firm fixed effects. Column (3) further adds industry-time (2-digit SIC classification) fixed effects. Column (4) adds region-year fixed effects. Column (5) adds firms' leverage. Column (6) additionally controls for potentially spurious size effects and includes the inverse of (lagged) fixed assets. Columns (7) and (8) further include firms' profits and firm-region house price index. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

TABLE A.10: INVESTMENT AND RESIDENTIAL WEALTH OF DIRECTORS: ROBUSTNESS OF FIRMS' ASSIGNMENT

	(1)	(2)	(3)	(4)
Slow	0.0064*** (0.001)	0.0052 (0.004)	0.0043** (0.002)	0.0045*** (0.001)
Fast	0.0130*** (0.001)	0.0184*** (0.003)	0.0139*** (0.001)	0.0370*** (0.001)
Leverage	-0.1503*** (0.015)	-0.2308*** (0.035)	-0.1716*** (0.021)	
Inv. scale	18.3352*** (1.061)	14.9576*** (3.906)	17.4575*** (1.667)	
Profit	0.0776*** (0.003)	0.0984*** (0.009)	0.0874*** (0.006)	
HPI	-0.0001 (0.000)	-0.0004 (0.001)	-0.0003 (0.000)	
<i>N</i>	103916	16353	46983	439113
Adj. <i>R</i> ²	0.23	0.29	0.24	0.12
Firm FE	Yes	Yes	Yes	Yes
Industry-time FE	Yes	Yes	Yes	Yes
Region-time FE	Yes	Yes	Yes	Yes

Notes: Table A.10 reports OLS estimates of Equation (1.2). The sample covers reporting UK firms over the period 2002-2014. The dependent variable in Columns (1) - (3) is investment (change in fixed assets plus depreciation) and change in Total Assets in Column (4). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1.1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm "Fixed Assets". All ratios are winsorized at the median \pm 5 times interquartile range. Standard errors, clustered by firm region, are in parentheses. Column (1) is the baseline. Column(2) reports estimates for when the firm has to spend more than 75% of its tenure above the median growth rate in order to be classified as "high" type. Column (3) reports estimates when the threshold is moved from 50th percentile to 30th and 60th for "slow" and "fast" type, respectively. Column (4) corresponds to the case with no controls and growth rate of total assets as a dependent variable. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

A.4 Model Appendix

The computation of the model can be broadly divided into 3 parts: (1) simplification of programming problems by way of combining household's and firms' optimization problems, (2) computation of the model at the steady-state, and (3) solving for the model with aggregate fluctuations using perturbation techniques. In what follows, I lay out the key details of the numerical algorithm. To facilitate the exposition, I assume there is only one type of firms $\mathbf{J} = 1$.

A.4.1 Analysis of the Model

The model outlined in Section 3.4 incorporates optimization problems for three distinct types of agents: representative household, incumbent firms and potential entrants. This implies that, first, I need to solve 3 programming problems, and then make sure that the agents' decisions are consistent with each other, and markets clear. Fortunately, it is possible to combine the optimality conditions for the households and the firms' Bellman equations, and thus reduce the computational complexity of the problem at hand. Using $C(\mathbf{s})$ and $N(\mathbf{s})$ to denote the market clearing values of household consumption and hours worked it is straightforward to show that market-clearing requires:

1. the real wage w be equal to the household marginal rate of substitution between leisure and consumption:

$$w(\mathbf{s}) = \frac{U'_2(C(\mathbf{s}), 1 - N(\mathbf{s}))}{U'_1(C(\mathbf{s}), 1 - N(\mathbf{s}))};$$

2. the risk-free bond price q_0^{-1} be equal to the expected gross real interest rate:

$$q_0(\mathbf{s}) = \beta \mathbb{E} \left[\frac{U'_1(C(\mathbf{s}'), 1 - N(\mathbf{s}'))}{U'_1(C(\mathbf{s}), 1 - N(\mathbf{s}))} \right];$$

3. firms' state-contingent discount factors be consistent with the household marginal rate of substitution between consumption across states:

$$d(\mathbf{s}'|\mathbf{s}) = \beta \frac{U'_1(C(\mathbf{s}'), 1 - N(\mathbf{s}'))}{U'_1(C(\mathbf{s}), 1 - N(\mathbf{s}))}.$$

Following [Khan, Senga and Thomas \(2014\)](#), I compute for the recursive competitive equilibrium effectively substituting the equilibrium implications of household optimization into the

recursive problems faced by the firms. Let $p(\mathbf{s})$ be the marginal utility of the household with respect to equilibrium consumption $C(\mathbf{s})$. Then equations (1.5) - (1.12) can be rewritten by way of expressing each firm's value in terms of the marginal utility of the household.

$$V_j^0(k, b, z; \mathbf{s}) = \pi_d V_j^1(k, b, z; \mathbf{s}) + (1 - \pi_d) V_j^2(k, b, z; \mathbf{s}). \quad (\text{A.2})$$

$$V_j^2(k, b, z; \mathbf{s}) = \max_{k' \geq 0, b' \in \mathcal{B}} p(\mathbf{s}) [-E_j(k, b, z; \mathbf{s}) - \varphi^E \max\{E_j, 0\}^3] + \beta \mathbb{E} [V_j^0(k', b', z' | \mathbf{s}')]. \quad (\text{A.3})$$

$$V_j^1(k, b, z; \mathbf{s}) = p(\mathbf{s}) [\Pi_j(k, b, z; \mathbf{s}) + (1 - \delta)k - b]. \quad (\text{A.4})$$

Next, I lay out the algorithm which I used to solve for the equilibrium.

A.4.2 Steady-State

I use collocation methods to solve the firm's functional equations (A.2)-(A.4). In practice, I use Chebyshev polynomials to approximate the value functions.

I set up a grid of collocation nodes $\mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, with N_i nodes in each dimension, $i \in \{\mathcal{K}, \mathcal{B}, \mathcal{Z}\}$. Throughout the algorithm, I compute expectations with respect to idiosyncratic productivity shocks using Gauss-Hermite quadrature. The computation of the stationary state of the model proceeds in the following 4 steps:

1. guess the equilibrium wage rate, w ;
2. solve for individual decision rules (k', b') ;
3. given the decision rules, compute stationary histogram (distribution of firms over the state space);
4. compute the excess demand on the labor market. If it exceeds some prespecified tolerance, adjust the wage guess correspondingly and go back to Step 2. Otherwise, terminate.

Approximation of Value Functions

I approximate 2 (normalized by the household's marginal utility) value functions: $V^0(\cdot)$ and $V^2(\cdot)$, which are defined in (A.2) and (A.3), respectively. In particular, I represent these value functions as weighted sums of orthogonal polynomials:

$$\begin{cases} V^0(k, b, z) &= \sum_{i,j,k=1,1,1}^{N_{\mathcal{K}}, N_{\mathcal{B}}, N_{\mathcal{Z}}} \theta_0^{ijk} T^i(k) T^j(b) T^k(z) \\ V^1(k, b, z) &= \sum_{i,j,k=1,1,1}^{N_{\mathcal{K}}, N_{\mathcal{B}}, N_{\mathcal{Z}}} \theta_1^{ijk} T^i(k) T^j(b) T^k(z), \end{cases}$$

where $\{\theta_0^{i,j,k}, \theta_1^{i,j,k}\}$ are approximation coefficients, and $T^i(\cdot)$ is the Chebyshev polynomial of order i .

I use collocation method to simultaneously solve for $\{\theta_0^{i,j,k}, \theta_1^{i,j,k}\}$. Collocation method requires setting the residual equation to hold exactly at $N = N_{\mathcal{K}} \times N_{\mathcal{B}} \times N_{\mathcal{Z}}$ points ; therefore, I essentially solve for $2 \times N$ unknown coefficients. I compute the basis matrices for Chebyshev polynomials using [Miranda and Fackler \(2002\)](#) Compecon toolbox. Subsequently, I solve for a vector of unknown coefficients using Newton's method. A much slower alternative is to iterate on the value function. Given the current guess of coefficients, I solve for the optimal policies $k'(k, b, z)$ and $b'(k, b, z)$ using nested vectorized golden search. After I solve for the policy function, I recompute decision rules on a finer grid, and, subsequently, compute the stationary distribution.

Stationary Distribution

When I solve for a stationary distribution, I iterate on a mapping using firms' decisions rules:

$$L' = \mathbf{Q}'L + L^e,$$

where L is a current distribution of incumbents across the state space, and L^e is a distribution of successful entrants. Matrix \mathbf{Q} is a transition matrix, which determines how mass of firms shifts in the (k, b, z) -space. It is a direct product of three transition matrices \mathbf{Q}_k , \mathbf{Q}_b and \mathbf{Q}_z :

$$\mathbf{Q} = \mathbf{Q}_k \odot \mathbf{Q}_b \odot \mathbf{Q}_z,$$

which govern the shift of mass along k - , b - and z -dimensions, respectively. While \mathbf{Q}_z is completely determined by the exogenous stochastic process (1.4), matrices \mathbf{Q}_k and \mathbf{Q}_b are

constructed so that the model generates an unbiased distribution in term of aggregates.⁵ More precisely, element (i, j) of the transition matrix \mathbf{Q}_x with $x \in \{k, b\}$ informs which fraction of firms with the current idiosyncratic state (k, b, z) will end up having x_j tomorrow. Therefore, this entry of the matrix is computed as:

$$\mathbf{Q}_x(i, j) = \left[\mathbf{1}_{x' \in [x_{j-1}, x_j]} \frac{x' - x_j}{x_j - x_{j-1}} + \mathbf{1}_{x' \in [x_j, x_{j+1}]} \frac{x_{j+1} - x'}{x_{j+1} - x_j} \right].$$

Tensor product of matrices \mathbf{Q}_k , \mathbf{Q}_b and \mathbf{Q}_z is computed using the `dprod` function from [Miranda and Fackler \(2002\)](#) toolkit.

A.4.3 Model with Aggregate Shocks

Once a finite representation of the system at the steady-state is obtained, I can write down equilibrium conditions as a system of difference equations, where some equations are backward-looking (e.g., the evolution of the distribution), and forward-looking (Bellman equations). Following a standard approach of using the expectation errors in forward-looking equations (denoted η_{t+1}), the equilibrium with aggregate uncertainty can be written as the finite non-linear system

$$\Gamma(\Theta_t, \Theta_{t+1}, \eta_{t+1}, \varepsilon_{t+1}) = \mathbf{0}, \tag{A.5}$$

where the vector Θ_t contains state and jump variables (such as histogram and collocation parameters for value functions approximation), and ε_{t+1} is a vector of Gaussian disturbances to exogenous aggregate stochastic processes. The vector Θ_t also contains an $M \times 1$ vector \mathbf{g}_t , which collects observables used in the estimation step. This vector is general can be a non-linear function of other elements of Θ_t .

With this representation at hand, the solution of the steady-state boils down to solving for the value of $\bar{\Theta}$ when aggregate shocks are turned-off; that is, it has to satisfy

$$\Gamma(\bar{\Theta}, \bar{\Theta}, \mathbf{0}, \mathbf{0}) = \mathbf{0}.$$

Subsequently, one can express (A.5) in terms of log deviations from the steady state, $\hat{\Theta}_t = \log(\Theta_t) - \log(\bar{\Theta})$, and take a first-order Taylor expansion. This delivers a linear system of

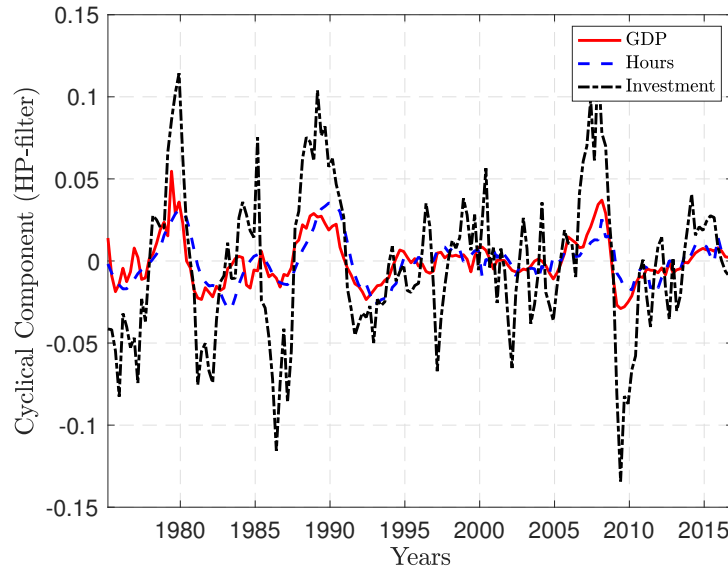
⁵See [Young \(2010\)](#) for more details.

equations, which provides a SVAR representation of the model:

$$\Gamma_0 \hat{\Theta}_{t+1} = \Gamma_1 \hat{\Theta}_t + \Psi \varepsilon_{t+1}. \quad (\text{A.6})$$

The matrices Γ_0 and Γ_1 contain first-order partial derivatives of equilibrium conditions with respect to elements of Θ_t , which are computed numerically using automatic differentiation.⁶ I solve the resulting system of equations (A.6) using the rational expectations solver **Gensys** provided by Sims (2002).

FIGURE A.8: U.K. QUARTERLY AGGREGATE DATA, 1975Q1-2016Q4



Notes: Figure A.8 plots 3 aggregate series used for Bayesian estimation: real GDP, total annual hours worked, and investment expenditures. Series have been logged and HP-filtered with smoothing parameter 1600. Nominal values have been converted to real using CPI index (2015 being base year). Source: Office for National Statistics.

A.4.4 Estimation

In this section, I describe the estimation procedure of $\Omega = \{\rho_x, \sigma_x\}_{x \in \{z, r, n\}}$ - parameters of aggregate stochastic processes introduced in Subsection 1.4.4.

The solution to Equation (A.6) along with measurement equation form the following system

⁶I use myAD toolkit written by SeHyoun Ahn, which is available at <https://github.com/sehyoun/MATLABAutoDiff>

of equations:

$$\begin{aligned}\hat{\Theta}_{t+1} &= A(\Omega)\hat{\Theta}_t + B(\Omega)\varepsilon_{t+1} \\ Y_t &= C\hat{\Theta}_t + D\zeta_t,\end{aligned}$$

where $A(\Omega)$ describes the evolution of the model's state and $B(\Omega)$ is an impact matrix. The second equation is a measurement equation: it relates the observable series $\{Y_t\}_{t=1}^T$ to a latent state $\{\hat{\Theta}_t\}_{t=1}^T$. With the representation above, one can compute the likelihood of any sequence of $\{Y_t\}_{t=1}^T$ using Kalman filter (see [An and Schorfheide \(2007\)](#) and [Mongey and Williams \(2016\)](#) for the description of that procedure).

Given a current draw of parameters Ω , let $P(\{Y_t\}_{t=1}^T|\Omega)$ denote the likelihood of the observed data. The posterior can be computed by combining the likelihood with the prior:

$$P(\Omega|\{Y_t\}_{t=1}^T) \propto P(\{Y_t\}_{t=1}^T|\Omega) P(\Omega).$$

In order to quantify the uncertainty about parameter estimates, I characterize the posterior by drawing from it using Markov Chain Monte Carlo; I use Metropolis-Hastings algorithm to accomplish this step.

I estimate parameters of 3 exogenous stochastic processes (6 parameters in total) using aggregate series for GDP, hours worked and investment expenditures. The data is quarterly and spans the time period 1975Q1:2016Q4 (168 data points in total) – see Figure A.8. Table 1.5 in main text reports the prior distributions used in the estimation step, and characterizes the posteriors: their modes and 5%, 95% bounds.

Appendix B

Appendix to Chapter 2

B.1 Data

In this section, we first describe the way we construct variables from PSID data (Subsection B.1.1), and then we discuss the process of sample selection (Subsection B.1.2).

B.1.1 Definition of Variables

We break down all the variables into 2 categories: demographic (Section B.1.1) and income-related (Section B.1.1).

Demographic and Socioeconomic Variables

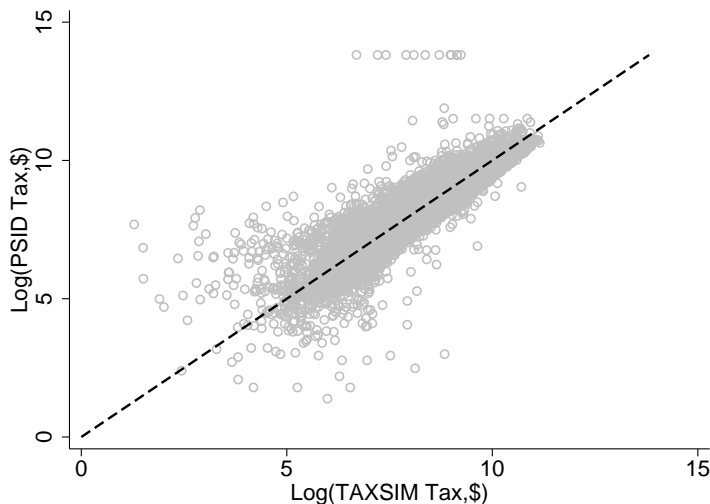
- **Head** We identify current heads as those individuals within the family unit with Sequence Number equal to 1. In the PSID, the male is labeled as the household’s head and the female as his spouse conditional on this family being full (married couple). A female is assigned to be the household’s head only if she is unmarried. We select a new head in case of split-off families.
- **Age** Prior to 1996, PSID interviews were conducted annually (and biannually since then). However, the interview dates were not exactly a year apart, and, therefore, it could be the case that individuals report either the same age or numbers 2 years apart in consecutive waves. We create a consistent age variable by taking the age reported in the first year a particular individual appears in the survey, and add 1 to this variable in each subsequent year (2 for when the survey became biannual).
- **Education** This paper is focused on people in the labor market stage of their lives (age 23-60); hence, it is natural to assume that individuals are typically done with their education by the time they are first interviewed. Our measure of education is equal to the number of complete years of education. This variable is, however, not reported consistently over years (sometimes, only bracketed information is available). We, therefore, reconstruct this variable by taking the maximum number reported for each individuals over the years he/she was in the sample.
- **Family size** We found that this variable is consistently reported throughout the waves.

Income Variables

- **Head’s labor income** This is among the most consistently reported income-related variables in PSID, available throughout all waves. However, this variable is bracketed—and, therefore, useless for our analysis—in waves 1968 and 1969. The PSID reporting standards are wave-specific: if, for example, head’s labor income in waves 1970-1982 is bounded by \$99,999, in subsequent 10 waves (1983-1992) this variable is capped at \$999,999. We make sure to drop all “capped” observations. Besides, another issue associated with this variable is that starting from 1994 wave, PSID stopped including labor portions of farm and business incomes into head’s labor income. We correct for this by adding those income sources (labor part of Farm income and labor part of Business income) for waves 1994 onwards.
- **Wife’s labor income** We follow similar (to the head’s labor income) steps to construct this variable. The only difference is that PSID stopped including labor portions of farm and business incomes for spouses starting from wave 1993 (not 1994). We correct for that inconsistency.
- **Joint labor income** This variable is a sum of the head’s and wife’s labor incomes.
- **Post-government labor income** We add family-wide transfers and subtract taxes from the spousal labor income to get the labor-related portion of the family-wide disposable income. Transfers and taxes are reported poorly in PSID; we discuss them next.
- **Transfers** Transfer data is reported inconsistently across the years. In general, we consider the old-age, survivor, and disability insurance (routinely abbreviated as OASDI), unemployment insurance (coming from the household’s head, spouse and from other family members (OFUMs)), food stamps, as well as some other minor categories (bonuses, miscellaneous transfers, transfers received by OFUMS).
- **Taxes** Taxes are imputed by PSID analysts for waves 1978-1990. We want to use as many waves as possible, and so we need to construct a measure of household-wide taxes which will be consistent throughout the years. We use TAXSIM system to impute the federal- and state-level liabilities for individuals in our sample.¹ In particular, we feed

¹TAXSIM is able to compute state taxes for years 1978 onwards; therefore, we drop years prior to 1978

FIGURE B.1: PSID TAX AGAINST THE TAXSIM DATA, 1978-1990



Notes: Figure B.1 depicts a scatter plot of PSID tax data and the tax liabilities imputed through the TAXSIM. Every circle represents a particular family-year pair, with years covered being 1978-1990.

in the following information: primary earner's (head) labor income, secondary earner's (spouse) labor income, state (needed to compute state-level tax liabilities) and number of dependents (family size minus 1 or 2 depending on the family composition). In order to check how consistent our measure of tax liabilities is with the tax data reported in PSID, we plot a scatter plot for these 2 measures pooled across years when both of them are available (1978-1990).

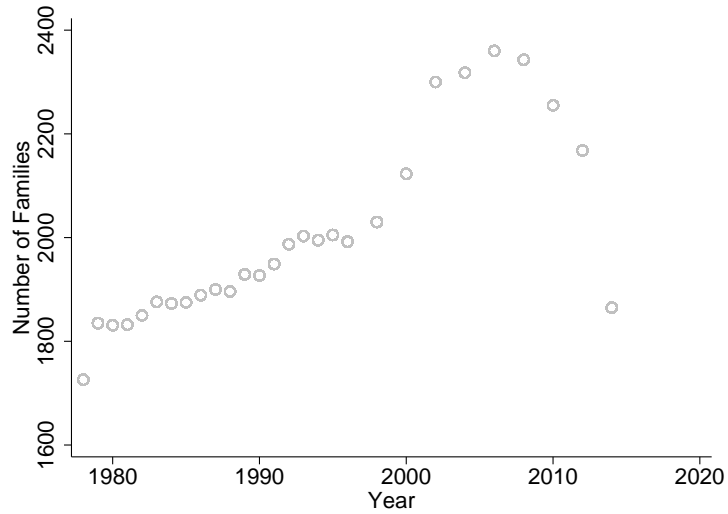
Figure B.1 shows that the data imputed through the TAXSIM does a reasonable job, as the majority of observations lie on or close to the 45-degree line.

- **Head's labor income (narrow)** is obtained from the head's labor income data, when we drop observations which are below the minimum threshold. The threshold is defined as half of the current minimum hourly wage multiplied by 520 hours (13 weeks, 40 hours in each). The data on the minimum hourly wage comes from Federal Reserve Economic Data.²

in our final dataset.

²<https://fred.stlouisfed.org>

FIGURE B.2: NUMBER OF FAMILIES IN THE FINAL SAMPLE



Notes: Figure B.2 is based on PSID data over the period 1978-2014. Each dot indicates the number of families in a given year, after selection criteria outlined in Appendix B.1.2 have been applied.

B.1.2 Sample Selection

Our baseline sample is based on PSID Family files and spans the time period 1969-2014. We use Individual Files to track individuals over time, and subsequently use information from the Family Files to obtain family-wide variables. We track only heads of households; therefore, any “split-off” family we treat as a new family. In what follows, we provide a step-by-step algorithm of data preparation. Table B.1 shows the number of observations retained at each step of the preparation process.

Figure B.2 illustrates the number of families in the final sample across years.

1. We start off with downloading the 1968-2014 PSID files;
2. keep only complete couples;
3. families which are part of SEO along with a Latino subsample are dropped;
4. only households’ heads are tracked;
5. observations with missing or non-positive head’s labor incomes are dropped;
6. observations with negative spousal labor incomes are dropped;
7. years prior to 1978 are dropped;

TABLE B.1: NUMBER OF OBSERVATIONS KEPT AT EACH STEP

Step	Observations Retained
Start	3,011,697
Only heads	278,119
Only complete couples	158,229
No CEO, Latino subsamples	98,478
Working age	78,148
Missing/negative head's income	78,144
Negative spouse income	78,143
Years ≥ 1978	
No outliers	
State-code is 1-51	
Income growth within $[1/20, 20]$	
Years of education are 1-17	

Notes: Table B.1 reports the total number of remaining observations after each step of data preparation (see Appendix B.1.2).

8. heads aged between 23 to 60 are considered;
9. trim the top and bottom 1% of the household labor income by year;
10. drop households with income growth anomalies (annual log growth rate is between $\frac{1}{20}$ and 20);
11. keep observations with properly reported state code (1-51);
12. keep observations with head's years of education 1-17.

B.1.3 Estimation of Equation 2.1

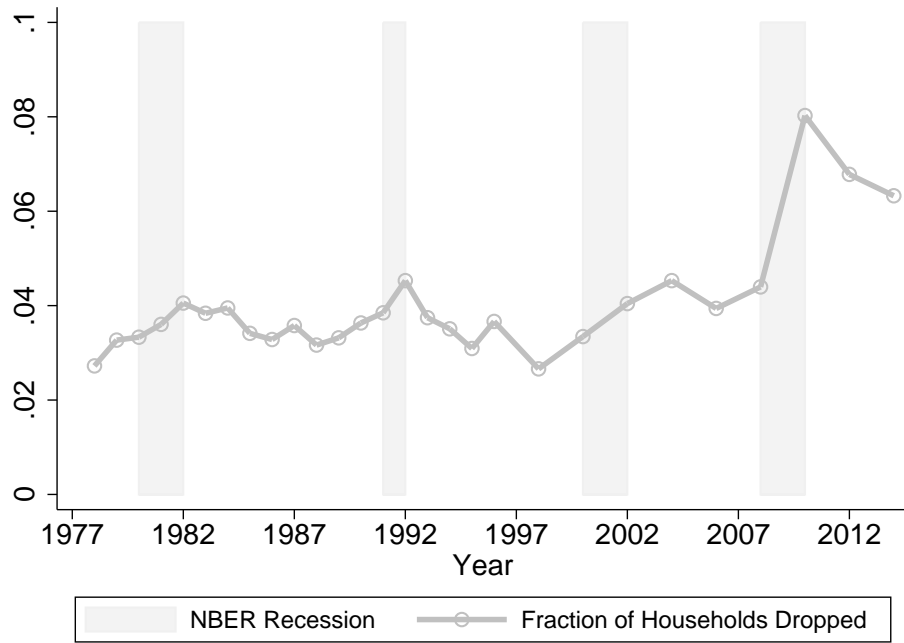
TABLE B.2: ESTIMATION OF EQUATION 2.1

	DEPENDENT VARIABLE $\log(y_{it})$				
	Post-govt LI	Joint LI	Head's LI	Wage	Head's LI (narrow)
Age	0.1269*** (0.012)	0.1105*** (0.016)	0.1318*** (0.019)	0.1414*** (0.016)	0.1479*** (0.017)
Age ²	-0.0020*** (0.000)	-0.0012*** (0.000)	-0.0019*** (0.000)	-0.0024*** (0.000)	-0.0023*** (0.000)
Age ³	0.0000*** (0.000)	-0.0000 (0.000)	0.0000* (0.000)	0.0000*** (0.000)	0.0000*** (0.000)
log(Educ.)	1.0392*** (0.013)	1.3021*** (0.018)	1.2959*** (0.020)	1.1498*** (0.017)	1.2667*** (0.018)
log(F. size)	-0.0326*** (0.004)	-0.0786*** (0.006)	0.0430*** (0.007)	0.0286*** (0.006)	0.0497*** (0.006)
<i>N</i>	55675	55139	53705	51453	53432
Adj. <i>R</i> ²	0.24	0.20	0.15	0.16	0.17

Notes: Table B.2 reports the results of OLS estimation, and is based on PSID data over the period 1978-2014. **Age** is the age of a household's head, **Education** is a number of completed (by the head) years of education. **Fam.size** is a number family members in a family unit. Standard errors are in parentheses. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

B.1.4 Share of Heads with Weak Labor Market Attachment

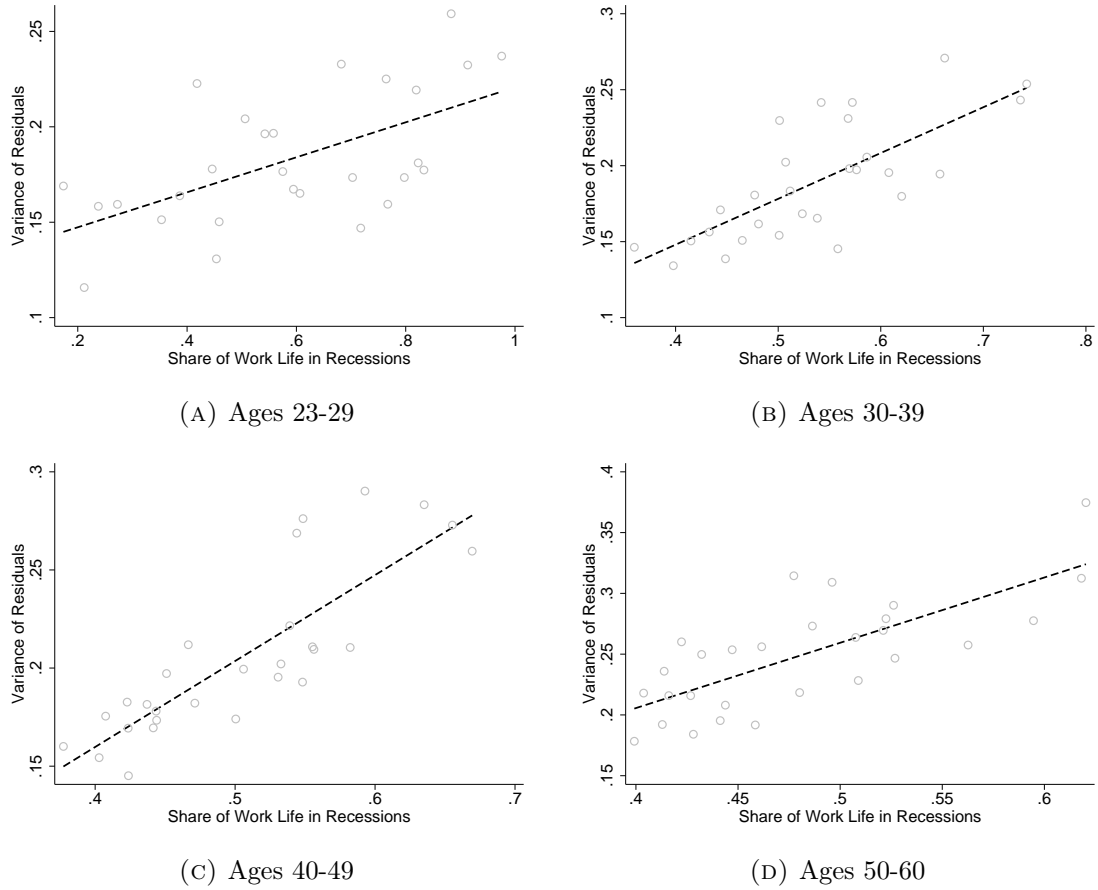
FIGURE B.3: SHARE OF HEADS WITH WEAK LABOR MARKET ATTACHMENT



Notes: Figure B.3 is based on PSID data over the period 1978-2014. The line plots the share of heads with weak labor market attachment (see Section 2.2 for details). Grey bars represent NBER recession dates.

B.1.5 Variance of Residuals as Function of Work History

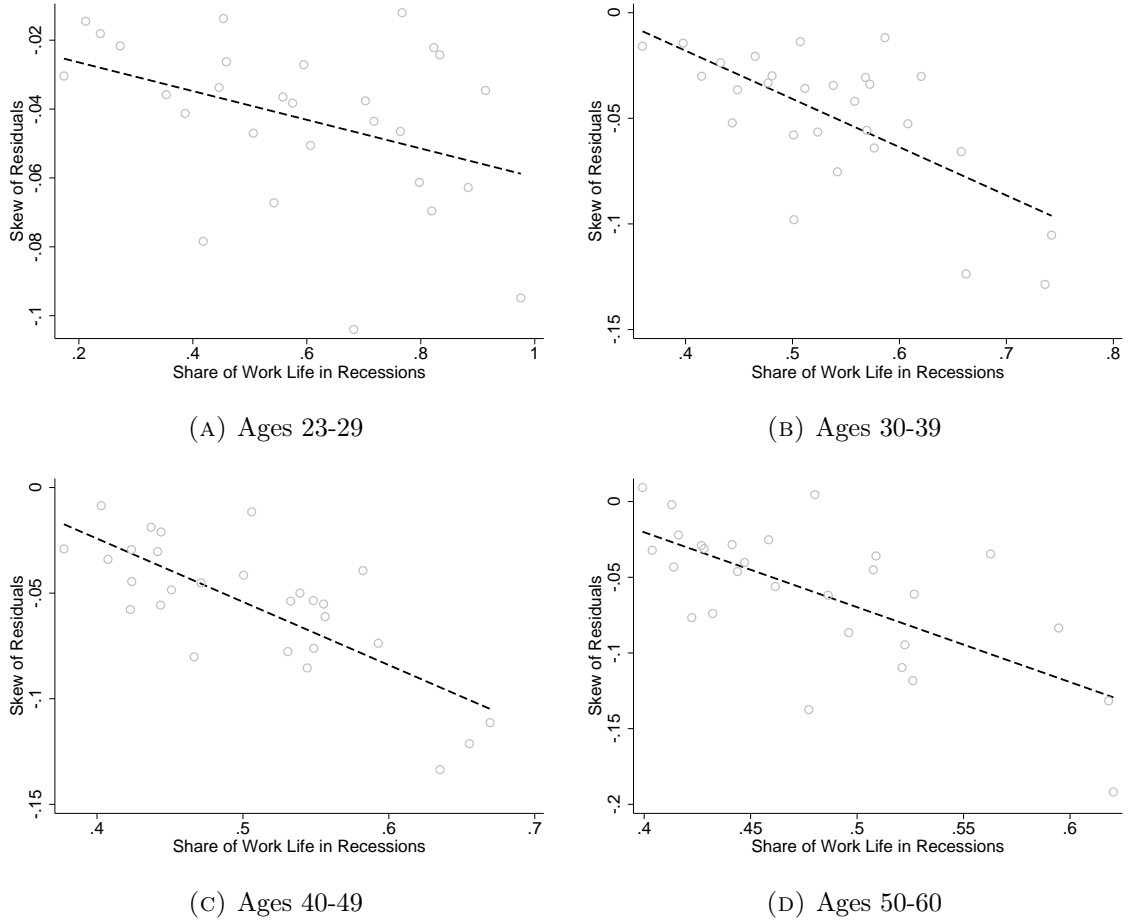
FIGURE B.4: VARIANCE OF RESIDUALS AS FUNCTION OF WORK HISTORY



Notes: Figure B.4 plots the variance of residuals \hat{u} against the share of working life spent in recessions for 4 age groups: 23-29, 30-39, 40-49, and 50-60. The graphs depict the data corresponding to years 1980, 1990, 2000 and 2010. Share of work history spent in recessions is a continuous variable bounded between 0 and 1; it represents a share of (working) life each cohort spent in recessions.

B.1.6 Skew of Residuals as a Function of Work History

FIGURE B.5: SKEW OF RESIDUALS AS FUNCTION OF WORK HISTORY



Notes: Figure B.5 plots the skewness (third central moment) of residuals \hat{u} against the share of working life spent in recessions for 4 age groups: 23-29, 30-39, 40-49, and 50-60. The graphs depict the data corresponding to years 1980, 1990, 2000 and 2010. Share of work history spent in recessions is a continuous variable bounded between 0 and 1; it represents a share of (working) life each cohort spent in recessions.

B.1.7 Recessionary and Expansionary Years: A Comparison

TABLE B.3: CLASSIFICATION OF YEARS INTO RECESSIONS AND EXPANSIONS

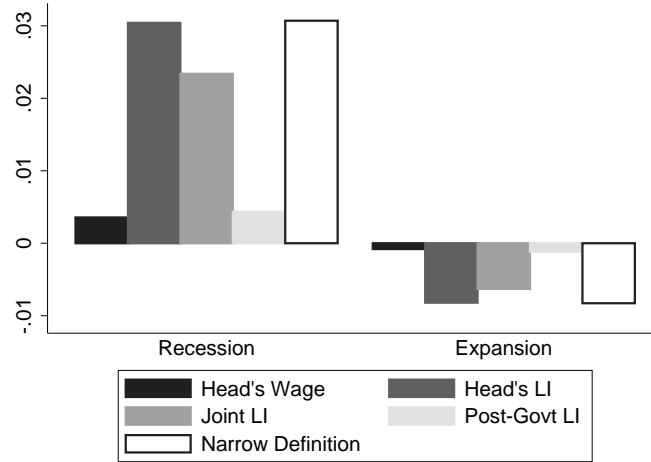
Year	NBER	GDP	GNP per capita
1978			
1979		✓	✓
1980		✓	✓
1981	✓	✓	✓
1982	✓	✓	✓
1983			
1984			
1985			
1986			
1987			✓
1988			
1989		✓	✓
1990		✓	✓
1991	✓	✓	✓
1992		✓	✓
1993			✓
1994			
1995		✓	✓
1996		✓	✓
1997			
1998			
1999			
2000			
2001	✓	✓	✓
2002		✓	✓
2003		✓	✓
2004			
2005			
2006			
2007		✓	✓
2008	✓	✓	✓
2009	✓	✓	✓
2010		✓	✓
2011			✓
2012			✓
2013			✓
2014			

Notes: The table reports the classification of sample years (1978-2014) into recessions and expansions. Recessionary years are marked with a checkmark. The table provides 3 classification: based on real GDP, real GNP per capita, and NBER recessionary years.

B.2 Alternative Measures of Business Cycle

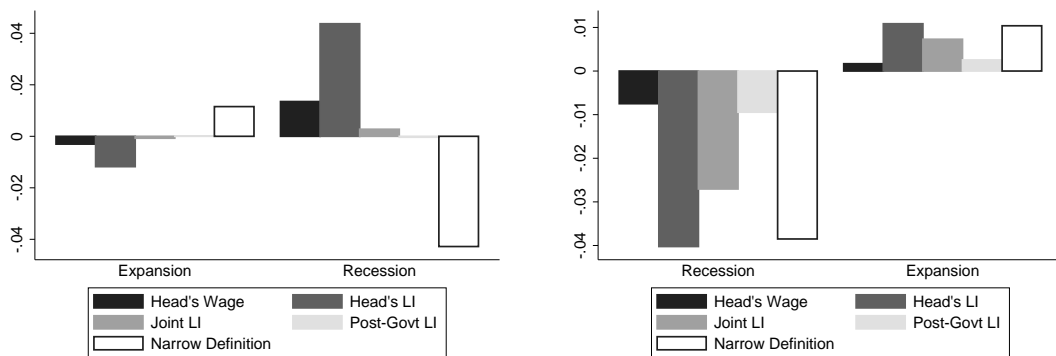
B.2.1 NBER Recession Dates

FIGURE B.6: VOLATILITY OF IDIOSYNCRATIC LABOR INCOME RISK BY NBER RECESSION DATES



Notes: Figure B.6 is based on PSID data over the period 1978-2014. Each year from the period 1978-2014 is classified as a NBER recession or expansion. The standard deviations shown are averages over years in each bin. Each bin contains standard deviations for 5 measures of labor income: head's wage, head's labor income, joint labor income, post-government joint labor labor income, and head's labor income (narrow definition).

FIGURE B.7: SKEW OF IDIOSYNCRATIC LABOR INCOME RISK, BY NBER RECESSION DATES



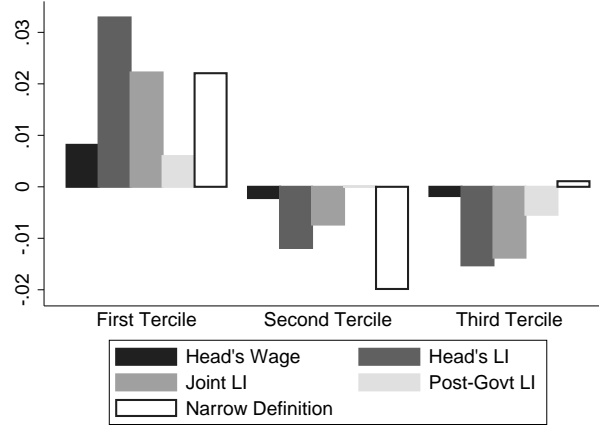
(A) Third central moment

(B) Kelley measure

Notes: Figure B.7 is based on PSID data over the period 1978-2014. Panel A plots the third central moment, Panel B plots the Kelley measure. Each year from the period 1978-2014 is classified as a NBER recession or expansion. The measures of skewness shown are averages over years in each bin. Each bin contains skewness measures for 5 different types of labor income: head's wage, head's labor income, joint labor income, post-government joint labor labor income and head's labor income (narrow definition).

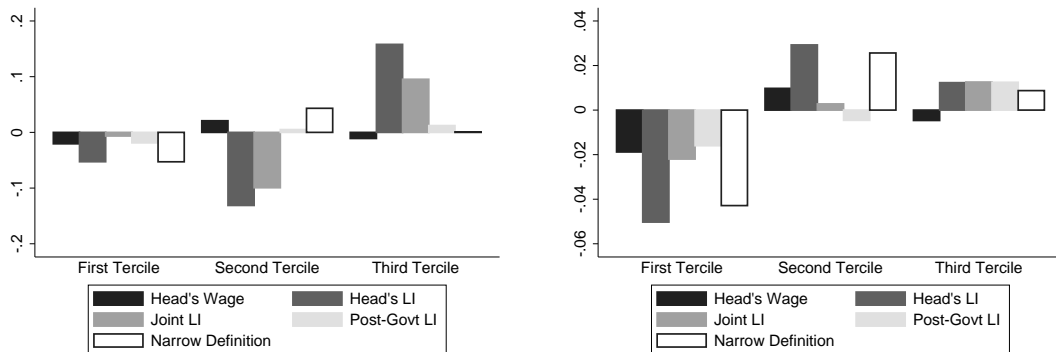
B.2.2 Real GDP Growth

FIGURE B.8: VOLATILITY OF IDIOSYNCRATIC LABOR INCOME RISK BY REAL GDP GROWTH TERCILE



Notes: Figure B.8 is based on PSID data over the period 1978-2014. Each year from the period 1978-2014 is classified in one of 3 bins, depending on which tertile the growth rate of real GDP in that year falls into. Tertile 1 contains years with the lowest growth rate of GDP, while tertile 3 contains years with the highest growth rates. The standard deviations shown are averages over years in each bin. Each tertile contains standard deviations for 5 measures of labor income: head's wage, head's labor income, joint labor income, post-government joint labor income, and head's labor income (narrow definition).

FIGURE B.9: SKEW OF IDIOSYNCRATIC LABOR INCOME RISK, BY REAL GDP GROWTH TERCILE



(A) Third central moment

(B) Kelley measure

Notes: Figure B.9 is based on PSID data over the period 1978-2014. Panel A plots the third central moment, Panel B plots the Kelley measure. Each year from the period 1978-2014 is classified in one of 3 bins, depending on which tertile the growth rate of real GDP in that year falls into. Tertile 1 contains years with the lowest growth rate of GDP, while tertile 3 contains years with the highest growth rates. The measures of skewness shown are averages over years in each bin. Each tertile contains skewness measures for 5 different types of labor income: head's wage, head's labor income, joint labor income, post-government joint labor income and head's labor income (narrow definition).

Appendix C

Appendix to Chapter 3

C.1 Empirical Work

This appendix describes the firm-level variables used in the empirical analysis of the paper, based on quarterly Compustat. First, in Appendix C.1.1 I discuss the data preparation steps I made to clean the data. Variables of interest are defined in Appendix C.1.2. Appendix C.1.3 describes the process of dataset construction.

C.1.1 Sample Selection

The empirical analysis excludes:

1. Firms in finance, insurance, and real estate sectors (2-digit SIC codes in range [60,67]) and public administration (SIC codes in range [91, 97]),
2. observations in the top and bottom 0.5% of the investment rate distribution,
3. observations with leverage exceeding 10,
4. firms with “extreme” real sales growth (a firm is dropped from the sample if in any period the growth rate of real sales exceeds 5 or is below -5).

C.1.2 Variable Construction

Capital is constructed using the perpetual inventory method. For every firm, I find the first period when the variable `ppentq` (gross plant, property, and equipment) is available. From that period onwards, I reconstruct a series of capital using the interperiod changes in `ppentq` (net plant, property, and equipment):

$$k_{jt+1} = k_{jt} + \text{ppentq}_{jt+1} - \text{ppentq}_{jt}.$$

`ppentq` has a substantially better coverage than `ppentq`, but I linearly interpolate in periods when this variable is not reported.

Investment rate is measured as

$$\text{ir}_{jt} = \frac{k_{jt+1} - k_{jt}}{\frac{1}{2}(k_{jt+1} + k_{jt})}.$$

As it has been discuss in the main text, this measure represents the investment along the intensive margin. I also consider the extensive margin of investment decisions, and call an

investment decision a “spike”, if the absolute value of investment rate exceeds 20%:

$$\mathbf{1}_{\{|ir_{jt}|>0.2\}}.$$

Leverage is defined as the ratio of total debt (sum of `dlcq` and `dlttq`) to total assets (`atq`).

Size is approximated by the logarithm of total assets.

Sales are simply equal to `salesq`, deflated by CPI. I take a natural logarithm of real sales when I compute the sales’ growth rate.

Sectors covered include (with 2-digit SIC codes in parentheses):

- agriculture, forestry, and fishing (<10),
- mining ([10, 14]),
- construction ([15, 17]),
- manufacturing ([20, 39]),
- transportation, communications, electric, gas, and sanitary services ([40, 49]),
- wholesale trade ([50, 51]),
- retail trade ([52, 59]),
- services ([70, 89]).

C.1.3 Details on Dataset Construction

I begin by retrieving quarterly firm-level data from WRDS website¹. The raw data spans years 1961-2018, and contains ≈ 1.1 mln firm-quarter observations. As it has been mentioned in Appendix C.1.1, the chosen method of construction of a capital measure necessitates the knowledge of `ppegtq`. Therefore, for each firm, I find the first non-missing value of this variable, and drop all the preceding observations. This results in dropping ≈ 273 K firm-quarter observations. Next, I drop finance and real estate firms, which amounts to another 55K dropped observations. Firms with anomaly growth rates of sales (outside interval $[-5, 5]$ on the quarterly basis) are eliminated, which reduces the sample size by 20K. I drop firms

¹<https://wrds-web.wharton.upenn.edu/wrds/index.cfm>

TABLE C.1: AGGREGATE UNCERTAINTY AND PROBABILITY OF INVESTMENT SPIKES

Variable	(1)	(2)	(3)	(4)	(5)
Volatility	-0.050*** (0.003)	-0.037*** (0.003)	-0.039*** (0.003)	-0.053*** (0.003)	-0.053*** (0.003)
Volatility×Age	0.003*** (0.0001)	0.003*** (0.0001)	0.003*** (0.0001)	0.005*** (0.0001)	0.005*** (0.0001)
Mean Growth		0.014*** (0.001)	0.005*** (0.002)	0.002 (0.002)	0.001 (0.002)
Mean Growth×Age			0.001 (0.0001)	0.001*** (0.0001)	0.001*** (0.0001)
Size				0.031*** (0.001)	0.031*** (0.001)
Leverage					-0.029*** (0.006)
FE	✓	✓	✓	✓	✓
N	144,094	144,094	144,094	144,094	144,094
R ²	0.02	0.02	0.01	0.01	0.01

Notes: Table C.1 reports the estimation results of the Equation (3.3). The firm-level data comes from Compustat; see Appendix C.1 for details. **Volatility** and **Mean GDP growth rate** variables have been standardized, and their units reflect the number of standard deviations from the sample mean. **Age** is proxied by the number of years since IPO, **size** is measured by the logarithm of total assets, and **leverage** is a share of total debt in total assets. *** denotes 1%, ** denotes 5%, and * denotes 10% significance, respectively.

which ever reported negative or zero capital (20K observations). Extra 61K are dropped because the measured leverage exceeds the values of 10. After this procedure, ≈ 650 K firm-quarter observations remain.

C.1.4 Robustness Checks

This paper studies business cycle fluctuations, and, therefore, quarterly frequency is preferred (a period in the model in Section 3.4 is one quarter). However, the abundance of data in the literature comes with yearly frequency, thus, in order to make my results comparable, I assess the impact of aggregate volatility on investment decisions of firms at annual frequency. Technically, I keep observations which correspond to the first quarter of each year — in total I have approximately 145K observations. I next compute the annual investment rates, and trim the sample by dropping cases where investment rate is below 0.5% or above 99.5% percentile of the corresponding distribution. Tables C.1 and C.2 report the results.

TABLE C.2: AGGREGATE UNCERTAINTY AND FIRMS' INVESTMENT RATES

Variable	(1)	(2)	(3)	(4)	(5)
Volatility	-0.029*** (0.002)	-0.021*** (0.002)	-0.022*** (0.002)	-0.037*** (0.002)	-0.037*** (0.002)
Volatility×Age	0.002*** (0.0001)	0.002*** (0.0001)	0.002*** (0.0001)	0.004*** (0.0001)	0.004*** (0.0001)
Mean Growth		0.012*** (0.001)	0.006*** (0.001)	0.003* (0.001)	0.002 (0.001)
Mean Growth×Age			0.0004 (0.0001)	0.001*** (0.0001)	0.001*** (0.0001)
Size				0.033*** (0.001)	0.032*** (0.001)
Leverage					-0.077*** (0.004)
FE	✓	✓	✓	✓	✓
N	144,094	144,094	144,094	144,094	144,094
R ²	0.01	0.01	0.01	0.01	0.02

Notes: Table C.2 reports the estimation results of the Equation (3.4). The firm-level data comes from Compustat; see Appendix C.1 for details. **Volatility** and **Mean GDP growth rate** variables have been standardized, and their units reflect the number of standard deviations from the sample mean. **Age** is proxied by the number of years since IPO, **size** is measured by the logarithm of total assets, and **leverage** is a share of total debt in total assets. *** denotes 1%, ** denotes 5%, and * denotes 10% significance, respectively.

C.2 Analysis of the Model

The model outlined in Section 3.4 incorporates optimization problems for three distinct types of agents: representative household, incumbent firms and potential entrants. This implies that, first, I need to solve 3 programming problems, and then make sure that the agents' decisions are consistent with each other, and markets clear. Obviously, this is a hard computational task. Fortunately, it is possible to combine the optimality conditions for the households and the firms' Bellman equations, and thus reduce the computational complexity of the problem at hand.

Maximizing (3.16) subject to (3.17) yields a expression for the wage as being equal to the ratio of marginal utilities with respect to labor and consumption:

$$w(\mathbf{S}) = \frac{U'_N(C(\mathbf{S}), N(\mathbf{S}))}{U'_C(C(\mathbf{S}), N(\mathbf{S}))}. \quad (\text{C.1})$$

The household's problem also implies that the firms' stochastic discount factor agrees with

the household's marginal rate of substitution for consumption across aggregate states tomorrow and today:

$$d(\mathbf{S}'|\mathbf{S}) = \beta \frac{U'_C(C(\mathbf{S}'), N(\mathbf{S}'))}{U'_C(C(\mathbf{S}), N(\mathbf{S}))}, \quad (\text{C.2})$$

where C and N are the market clearing consumption and labor, respectively.

The trick is to think of $p(\mathbf{S}) := U'_C(C(\mathbf{S}), N(\mathbf{S}))$ as a price with which firms value current output, and substitute expression (C.2) into the Bellman equations for firms (3.10)-(3.14).

By doing so, I effectively scale all the Bellman equations with the household's marginal utility, $p(\mathbf{S})$, and the original problem is now equivalent to solving the set of redefined functional equations. For brevity, I will show how one of the Bellman equations will look like, and delegate the remaining equations and details to Appendix C.2.

Consider the Bellman equation for the firm which decided to adjust its capital stock, Equation (3.11). Invoking (C.2), and using the upper case to denote the adjusted value functions (e.g., $V_a(\cdot) := v_a(\cdot)p$), one can rewrite (3.11) as:

$$V_a^i(k, s; \mathbf{S}) = \max_{k' \in \mathbb{R}^+} \left\{ -(k' - (1 - \delta)k)p^i(\mathbf{S}) - AC(k, s; \mathbf{S})p^i(\mathbf{S}) + c(k)\beta \mathbb{E} [\bar{V}(k', s'; \mathbf{S}')|s, k; \mathbf{S}] \right\}. \quad (\text{C.3})$$

The reformulated programming problem is obviously simpler, since now (i) I can express all firm functional equations in terms of V , and so there is no need to solve explicitly for all of them, and (ii) the household's functional equation is taken care of as we incorporated the resulting optimality conditions into (C.1) and (C.2).

The remaining (transformed) Bellman equations are described below. The value for an active firm at the start of the period is given by

$$V^i(k, s; \mathbf{S}) = p^i(\mathbf{S})\pi(k, s; \mathbf{S}) + \tilde{V}^i(k, s; \mathbf{S}). \quad (\text{C.4})$$

The continuation value of staying in the industry is

$$\tilde{V}^i(k, s; \mathbf{S}) = \max\{V_a^i(k, s; \mathbf{S}), V_n^i(k, s; \mathbf{S})\}. \quad (\text{C.5})$$

The value of an “adjusting” firm satisfies

$$V_a^i(k, s; \mathbf{S}) = \max_{k' \in \mathbb{R}^+} \{ -p^i(\mathbf{S})(k' - (1 - \delta)k) + \beta c(k) \mathbb{E} [\bar{V}(k', s'; \mathbf{S}') | s, k; \mathbf{S}] \}. \quad (\text{C.6})$$

Similarly, the value of a non-adjusting firm becomes:

$$V_n^i(k, s; \mathbf{S}) = \beta c(k) \mathbb{E} [\bar{V}(k', s'; \mathbf{S}') | s, k; \mathbf{S}]. \quad (\text{C.7})$$

Finally, the value the potential entrant gets if it decides to enter is provided by the following expression

$$V_e^i(q; \mathbf{S}) = \max_{k' \in \mathbb{R}^+} \{ -p^i(\mathbf{S})k' + \beta \mathbb{E} [\bar{V}(k', s'; \mathbf{S}') | q; \mathbf{S}] \}. \quad (\text{C.8})$$

In Appendix C.4, I lay out the algorithm which I used to solve for the equilibrium.

C.3 Proof

Consider the following process:

$$d_{t+1} = \rho d_t + \sigma \nu_{t+1},$$

where $\nu_{t+1} \sim \mathcal{N}(0, 1)$ and $|\rho| < 1$. Standard deviation σ is strictly positive. Suppose that $d_t < 0$ at time t . Consider $\varepsilon > 0$ small enough that both $d_t - \varepsilon$ and $d_t + \varepsilon$ are less than 0 (the steady-state value of d). Next, I show that the probability of event $(d_{t+1} > z + \varepsilon)$ grows by more than probability of $(d_{t+1} < z - \varepsilon)$ when σ increases.

Let the probability of these events be P^H and P^L , respectively. These probabilities can be expressed as follows:

$$P^L = \mathbf{P}(\rho d_t + \sigma \nu_{t+1} < d_t - \varepsilon) = \Phi\left(\frac{d_t - \varepsilon - \rho d_t}{\sigma}\right)$$

and

$$P^H = \mathbf{P}(\rho d_t + \sigma \nu_{t+1} > d_t + \varepsilon) = 1 - \Phi\left(\frac{d_t + \varepsilon - \rho d_t}{\sigma}\right).$$

The derivatives of P^L and P^H with respect to σ :

$$P_\sigma^{L'} = \phi\left(\frac{d_t - \varepsilon - \rho d_t}{\sigma}\right) \frac{\rho d_t - d_t + \varepsilon}{\sigma^2}$$

$$P_\sigma^{H'} = \phi\left(\frac{d_t + \varepsilon - \rho d_t}{\sigma}\right) \frac{-\rho d_t + d_t + \varepsilon}{\sigma^2}$$

First, it is straightforward to notice that both derivatives are strictly positive, which is intuitive: when variance rises, tail events become more likely. Second, the term-by-term comparison of the derivatives implies that $P^{H'} > P^{L'}$:

$$d_t - \varepsilon - \rho d_t < d_t + \varepsilon - \rho d_t < 0 \implies \phi\left(\frac{d_t - \varepsilon - \rho d_t}{\sigma}\right) < \phi\left(\frac{d_t + \varepsilon - \rho d_t}{\sigma}\right)$$

and

$$2\rho d_t < 2d_t \implies \rho d_t - d_t + \varepsilon < d_t + \varepsilon - \rho d_t \implies \frac{\rho d_t - d_t + \varepsilon}{\sigma^2} < \frac{-\rho d_t + d_t + \varepsilon}{\sigma^2}.$$

This implies that when the variance increases, the probability for the process to go up increases by more than to go down. *Q.E.D.*

C.4 Computational Details

I use collocation methods to solve the firm's functional equations (C.4)-(C.8). I rely on approximate aggregation techniques (Krusell and Smith, 1998) when I solve for the equilibrium with aggregate fluctuations, and approximate the distribution of firms by its first moment, the aggregate capital, K . Therefore, the state vector in equations (C.4)-(C.8) is $(k, s; z, K)$. I set up a grid of collocation nodes $R = K \times S \times Z \times AK$, with N_i nodes in each dimension ($i \in \{K, S, Z, AK\}$). In practice, I set 10 points in K -dimension, 7 points for both types of productivities, and 5 points along the aggregate capital dimension. The grid for the aggregate capital is centered around its value at the steady-state, $K_{s.s.}$, and spans the interval $[0.75K_{s.s.}, 1.25K_{s.s.}]$. I fit a cubic spline for individual capital, and linear splines for the remaining 3 dimensions. When I solve for a distribution or aggregate quantities, I obtain firms' policy functions on a dense grid of 100 points along the k -dimension.

Instead of solving for the value functions directly, it is possible to solve only for 2 functions (expected continuation values under low- and high-uncertainty regimes), and subsequently recover the value functions of interest. Let $i \in \{L, H\}$, and θ_i be a probability that the economy will be subject to the same uncertainty regime in the next period. The expected value under regime i can be written as:

$$\begin{aligned} V_E^i(k, s; z, K) &= \\ &= (\mathbf{I}_{NAK} \otimes \Pi_{z'|z} \otimes \Pi_{s'|s} \otimes \mathbf{I}_{NK}) [\theta_i \max\{V_a^i(k, s; z, K), V_n^i(k, s; z, K)\} + \\ &\quad + (1 - \theta_i) \max\{V_a^{-i}(k, s; z, K), V_n^{-i}(k, s; z, K)\}], \quad (\text{C.9}) \end{aligned}$$

where V_a^i, V_n^i are given by:

$$V_a^i(k, s; z, K) = p^i (\pi(k, s; z, K) - k'(k, s; z, K) + (1 - \delta)k) + \beta c(k) V_E^i(k', s; z, K'), \quad (\text{C.10})$$

$$V_n^i(k, s; z, K) = p^i \pi(k, s; z, K) + \beta c(k) V_E^i(k(1 - \delta), s; z, K'). \quad (\text{C.11})$$

The superscript $-i$ in (C.9) stays for the case of a complementary uncertainty regime. I approximate the expected continuation functions (C.9) with splines with $N = N_K \times N_S \times N_Z \times N_{AK}$ unknown coefficients. I compute the basis matrices using Miranda and Fackler (2002) Compecon toolbox. Subsequently, I solve for a vector of unknown coefficients using

the Newton's method. An (much slower) alternative is to iterate on the value function. The prices, $\{p^i\}$, and the future aggregate capital are unknown, and I, therefore, use 4 log-linear forecast rules for them which I feed into the firms' programming problems (there are 2 forecast equations for each uncertainty state, one for the contemporaneous equilibrium marginal utility, and one for the future mean capital). I iterate on the coefficients of forecast rules in the outer loop. In the inner loop, I solve individual problems by iterating on coefficients for approximands. Given the current guess of coefficients, I solve for the optimal policy $k'(k, s; z, K)$ using the vectorized golden search. Once the coefficients have converged, I simulate the economy forward from a steady-state distribution for a large number of periods using the following law of motion:

$$L' = \mathbf{Q}'(c(k)\mathbf{1}_{\{\text{no exit}\}}L) + \mathbf{Q}_e'(MP(\text{enter}|q)L_e),$$

where L_e is a distribution of potential entrants over the signal space. Matrices \mathbf{Q} and \mathbf{Q}_e are two transition matrices, which determine how mass of firms shifts in the (k, s) -space. Each of those matrices is a tensor product of two transition matrices \mathbf{Q}_k and \mathbf{Q}_s , which govern the shift of mass along k - and s -dimensions. While \mathbf{Q}_s is completely determined by the Markov process, which approximates (3.6), matrix \mathbf{Q}_k is constructed so that the model generates an unbiased distribution in term of aggregates. More precisely, element (i, j) of the transition matrix \mathbf{Q}_k informs which fraction of firms with the current capital stock k_i will have capital k_j tomorrow. Therefore, this entry is computed as:

$$\mathbf{Q}_k(i, j) = \left[\mathbf{1}_{k' \in [K_{j-1}, k_j]} \frac{k' - k_j}{k_j - k_{j-1}} + \mathbf{1}_{k' \in [k_j, k_{j+1}]} \frac{k_{j+1} - k'}{k_{j+1} - k_j} \right].$$

While I use the forecasts rule to solve for the firms' policy functions, I do not use the price rule during the simulation step. Instead, each period I close the market by iterating over the household's marginal utility, p . In practice, I break an interval $[p_{min}, p_{max}]$ centered around the steady-state value of marginal utility into $P - 1$ subintervals, where P is the number of available CPUs². Then, at each node I compute the implied consumption using (7), and the excess quantity $1/p_{quess} - C(p_{quess})$. Subsequently, I find the two adjacent nodes p_{i-1} and p_i , where the excess function changes its sign, and use that interval in the next step.

After simulation, I update the forecast rules by running an OLS regression on the simulated

²This method requires at least 3 CPUs.

data, and start over again. The model is solved when the coefficients converge (the guessed coefficients are close to the implied ones).