MONEY IN THE PRODUCTION FUNCTION: AN INTERPRETATION OF EMPIRICAL RESULTS

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Uri Ben-Zion and Vernon W. Ruttan

Discussion Paper No. 74-44, July 1974

Center for Economic Research
Department of Economics
University of Minnesota
Minneapolis, Minnesota 55455

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In a recent article in this journal, Professors Sinai and Stokes [8] have presented a very interesting test of the hypothesis that money enters the production function, the idea had been previously suggested by several writers. Their empirical findings seem to be very strongly consistent with their hypothesis, and they were led to some "far reaching conclusions" with regard to a large body of literature on production functions and technological change. For example, they suggest that "real balances could be a missing variable that has contributed to the unexplained 'residual' being attributed to technological changes".

The theory of induced innovation, as presented by Fellner [4] and Schmookler [9, 10] suggests that market condition may effect the demand for innovation and the realized technological changes. This is clearly stated by Barzel [1] "innovations are induced since they are more profitable with expansion of output".[2] Since money may be regarded as a proxy for short-run fluctuations in the aggregate demand, this theory suggests that money effects output and technological changes as a demand factor rather than as a factor of production.

In this note we suggest the appropriate test to distinguish between the two alternative hypotheses, and present some empirical results.

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*We have benefited from the helpful comments from Professors Hans P. Binswanger, T. Paul Schultz and other members of the Trade and Development and Macro Workshops at the University of Minnesota. We also acknowledge the financial support of the U. S. Agency of International Development through the Center of Economic Development of the University of Minnesota. Needless to say, the authors are solely responsible for any remaining errors.

The Two Alternative Model

Sinai and Stokes hypothesis was written as

$$y_t = f\ (K_t,\ L_t,\ M_t,\ t) + u_t \tag{1}$$

where K and L are capital and labor services, M is the stock of real balances, t is time trend to represent long-run trend in factor productivity, and u is a random disturbance.

The induced innovation approach can be written as

$$y_t = g\ (K_t, L_t, t) + V_t + W_t \tag{2}$$

where V_t is a measure of the short-run market fluctuation (e.g., expansion or decline) and W_t is a random disturbance. Using the monetary approach to income determination we can write the market fluctuation term V_t as

$$V_t = h\left(\left(\frac{\Delta M}{M}\right)_t,\ \left(\frac{\Delta M}{M}\right)_{t-1} \cdots \left(\frac{\Delta M}{M}\right)_{t-n},\ M_{t-n}\right) \tag{3}$$

where $\left(\frac{\Delta M}{M}\right)_t$ is the rate of change in the real money supply in period t, and similarly $\left(\frac{\Delta M}{M}\right)_{t-1} \cdots \left(\frac{\Delta M}{M}\right)_{t-n}$ are the previous rates of change in the real money supply in n previous periods. M_{t-n} is the level of the real balances at the initial point t - n.[3] n is the average lag between money and technological changes. As was emphasized by Friedman [5] the view of money as a determinant of market demand attaches more importance to the rate of changes in money rather than levels in explaining short-term fluctuation. Combining (2) and (3) we can write the induced hypothesis approach as

$$y_t = g\ (K_t,L_t,t) + h\left(\left(\frac{\Delta M}{M}\right)_{t-1}, \left(\frac{\Delta M}{M}\right)_{t-2} \cdots \left(\frac{\Delta M}{M}\right)_{t-n}, M_{t-n}\right) + W_t \qquad (4)$$

Comparing (1) and (4) we see that the two alternative approach suggests that output will be positively related to the real balances variables. There are, however, two main differences between the approaches.

(a) The induced innovation approach suggests that rates of changes in the real money balances may be more important than the level of the real balances. The production factor approach does not distinguish between "old money" and "new money:, and the effect of changes in real money supply is the same as the effect of the initial level M_{t-n} .

(b) According to the production factor approach, money held by firms plays a more important role in determining output than money held by individuals. According to the demand-induced innovation approach, since firms may adjust their cash balances faster than individuals, money held by individuals may play a more important role in influencing the aggregate demand.

Following Sinai and Stokes we assume that the production function (1) is of Cobb-Douglas form. We use the following identity

$$\ell g\ M_t = \ell g\ M_{t-n} + \sum_{j=1}^{n} \left(\frac{\Delta M}{M}\right)_{t-j} \qquad (5)$$

to combine (1) and (4) as follows:

$$\ell g\ y_t = \alpha_o + \alpha_1\ \ell g\ L_t + \alpha_2\ \ell g\ K_t + \alpha_3\ t \qquad (6)$$

$$+ \beta_o\ \ell g\ M_{t-n} + \beta_1 \left(\frac{\Delta M}{M}\right)_{t-1} \cdots\ \beta_n \left(\frac{\Delta M}{M}\right)_{t-n} + W_t$$

(6) will serve as the basic equation to distinguish between the alternative hypothesis with regard to (a) above. The production function approach suggests that we get the same coefficients for different "fractions" of the real money balances $\beta_o = \beta_1 = \ldots = \beta_n$ while the induced innovation approach suggests that the level coefficient β_o will be less important.

Similarly we can extend (6) to test the different predictions (b) as follows:

$$\ell g\ y_t = \alpha_o + \alpha_1\ \ell g\ L_t + \alpha_2\ \ell g\ K_t + \alpha_3 \cdot t \qquad (7)$$

$$+ \gamma_o\ \ell g\ MF_{t-n} + \gamma_1 \left(\frac{\Delta MF}{MF}\right)_{t-1} + \gamma_2 \left(\frac{\Delta MF}{MF}\right)_{t-2} + \ldots + \gamma_n \left(\frac{\Delta MF}{MF}\right)_{t-n}$$

$$+ \delta_o\ \ell g\ MC_{t-n} + \delta_1 \left(\frac{\Delta MC}{MC}\right)_{t-1} + \delta_2 \left(\frac{\Delta MC}{MC}\right)_{t-2} + \ldots + \delta_n \left(\frac{\Delta MC}{MC}\right)_{t-n} + W_{2_t}$$

MF_{t-n} and MC_{t-n} are the levels of the real balances held by firms and individuals respectively at period $t - n$. Similarly, the rate of changes in real balances held by firms and consumers is denoted by $\left(\frac{\Delta MF}{MF}\right)_{t-i}$ and $\left(\frac{\Delta MC}{MC}\right)_{t-i}$ respectively. $i = 1 \ldots n$.

In this formulation the production function approach suggests that since money held by the firm is more related to "productive money input" we will expect that the set of its coefficients will be statistically significant with $\gamma_o = \gamma_1 = \ldots = \gamma_n$ while the coefficients of money held by individuals are expected to be much weaker and probably insignificant. The induced innovation approach suggests that money held by consumer has more important effect on induced innovation than money held by firms and lead basically to an opposite prediction.

The empirical test to distinguish between the two hypotheses that were suggested above is performed in the following section.

The data and the empirical tests

The production data of output labor services and capital services were based on data calculated by Christensen and Jorgenson [3] for the period 1929-1967 which are the same data used by Sinai and Stokes [8]. For the data on nominal money balances we have used the average annual data given in Friedman and Schwartz [6]. In order to test the two components of money separately we have used data on money held by corporation, based on Goldsmith [7] for the period 1925-1939 and on the "Handbook of Basic Economic Statistics" [11] for the period 1939-1967. The two series of money held by corporations and money held by other sectors (calculated as a residual) were deflated by price indexes of producers and consumers goods respectively. These indexes are given in Christensen and Jorgenson [3].[4] The sum of the two deflated series was defined as total real money balances. In the estimation of (6) and (7) we have used OLS Regression technique, and have corrected the data for first order serial correlation. The results of the estimation are given in Tables 1 and 2 respectively.

Table 1 clearly indicates that rates of changes in the real money supply seem to have stronger and more significant effects than the level of real balances. This is clearly consistent with the induced innovation approach but not with the production function approach.

Table 2 indicates that the level and the rates of changes in the level of real money balances held by firms seem to have no significant effect on output. The level and rates of changes in money held by consumers retain their significance. These results are again consistent with the induced innovation approach but not with the production factor approach.

Summary and Conclusion

The main conclusion of this paper is that money as factor of demand seems to play an important role in explaining "induced technological changes". The results, however, do not support the initial strong conclusion of Sinai and Stokes that money is an important "omitted input" in the aggregate production function.

FOOTNOTES

[1]The research was completed in May 1973, while the two authors were affiliated with the University of Minnesota, postdoctoral fellow, Department of Economics [Ben-Zion], and Director of the Economic Development Center [Ruttan]. A shorter version of the work (dated June 1974) is forthcoming in the Review of Economics and Statistics.

[2]See Barzel [1], page 354. For more extensive analysis, see Ben-Zion and Ruttan [2].

[3]We use the level of money at period $t - n$ rather than at period t, since M_t is already included in the previous rates of change in the money supply which appear as separate variables.

[4]For the period 1925-1928 we have used the consumer price index to deflate the two series.

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Table 1

REGRESSION RESULTS OF (6) THE RELATION OUTPUT

TO PRODUCTION FACTORS, TIME AND REAL BALANCES

(t-values of the coefficients are given in parentheses)

Variable	1	2	3	4	5
Constant	- .2828 (2.7)	- .1715 (4.5)	-1.020 (4.0)	- .8629 (4.6)	- .6825 (4.2)
L	1.136 (9.0)	.7985 (6.4)	.9917 (7.8)	.9671 (9.2)	.9434 (9.6)
K	.2581 (1.9)	.5311 (4.1)	.3346 (3.7)	.3484 (4.5)	.3263 (4.3)
T	.0105 (2.1)	.0019 (0.4)	.0086 (2.7)	.0082 (3.1)	.0090 (3.2)
M_t		.2523 (4.6)			
M_{t-1}			.1109 (2.5)		
$\left(\frac{\Delta M}{M}\right)_{t-1}$			.4307 (5.9)	.3477 (5.6)	.3612 (6.3)
$\left(\frac{\Delta M}{M}\right)_{t-2}$				.3412 (5.0)	.3243 (5.2)
M_{t-2}				.0991 (2.7)	
$\left(\frac{\Delta M}{M}\right)_{t-3}$					.2158 (3.8)
M_{t-3}					.0947 (2.7)
R^2	.9818	.9789	.9966	.9975	.9975
F	648.78	420.21	2120.18	2404.21	2094.07
DW	1.13	1.18	1.44	1.52	1.52

Table 2

REGRESSION RESULTS OF (7) THE RELATION BETWEEN OUTPUT, PRODUCTION FACTORS, TIME, AND THE TWO COMPONENTS OF THE REAL BALANCES

(t-ratios of the coefficients are given in parentheses)

Variable	1	2	3	4
Constant	- .1988 (2.9)	- .4174 (2.5)	- .7082 (3.0)	- .5130 (2.1)
L_t	.6922 (4.8)	.8564 (6.7)	.8646 (7.5)	.8238 (8.6)
K_t	.5896 (4.2)	.2903 (2.5)	.3756 (4.0)	.3618 (4.6)
T	.0018 (0.4)	.0120 (3.2)	.0096 (3.2)	.0115 (4.5)
(MF_t)	- .0223 (0.2)			
(MC_t)	.2106 (3.7)			
$\left(\frac{\Delta MF}{MF}\right)_{t-1}$		- .0629 (0.6)	- .0239 (0.3)	.0289 (0.4)
$\left(\frac{\Delta MC}{MC}\right)_{t-1}$		.3469 (5.0)	.3022 (4.6)	.2947 (5.3)
$(MF)_{t-1}$		.1178 (1.0)		
$(MC)_{t-1}$		.0833 (1.7)		
$\left(\frac{\Delta MF}{MF}\right)_{t-1}$			.1044 (1.0)	.1266 (1.4)

Table 2 (Continued)

Variable	1	2	3	4
$\left(\frac{\Delta MC}{MC}\right)_{t-1}$			.2341 (3.5)	.2428 (4.2)
$(MF)_{t-2}$			.0052 (0.1)	
$(MC)_{t-2}$			.0981 (2.3)	
$\left(\frac{\Delta MF}{MF}\right)_{t-3}$				.0067 (0.1)
$\left(\frac{\Delta MC}{MC}\right)_{t-3}$				.1987 (3.3)
$(MF)_{t-3}$				- .1073 (1.1)
$(MC)_{t-3}$				.1271 (3.5)
R^2	.986	.996	.998	.998
F	509.48	1190.46	1633.19	2130.53
DW	1.03	1.10	1.22	1.53

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