$\mathcal{N} = (0, 2)$ Supersymmetry and a Nonrenormalization Theorem

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Heterotic nonlinear sigma models

4D QCD	$\Rightarrow$	2D NLSM
$\mathcal{N}=2\,,\ U(N)$ theories	$\Rightarrow$	$\mathcal{N}=(2,2)\,,\ \mathrm{CP}(N-1)$
$\mathcal{N}=1$ deformation	$\Rightarrow$	$\mathcal{N}=(0,2)$ deformed model

Objective: to understand the perturbative aspects of the deformed $\mathrm{CP}(N-1)$ models.

Heterotic nonlinear sigma models

$$\mathcal{L}_{(2,2)} = G\left\{\partial^\mu\phi\partial_\mu\phi^\dagger + i\bar{\psi}\not{D}\psi + \text{four fermion interactions}\right\},$$

$$\begin{aligned}\mathcal{L}_{(0,2)} &= \mathcal{L}_{(2,2)} + \zeta_R^\dagger\, i\partial_L\,\zeta_R + \left[\gamma\,\zeta_R\,G\left(i\,\partial_L\phi^\dagger\right)\psi_R + \mathrm{H.c.}\right] \\ &\quad + \text{four fermion interactions}.\end{aligned}$$

- theory with two couplings.

$$\beta_{g^2} = -\frac{g^4}{2\pi} + \cdots, \quad \beta_\gamma = \frac{\gamma}{2\pi}(\gamma^2 - g^2) + \cdots.$$

- define $\rho = \frac{\gamma^2}{g^2}$, we have

$$\beta_\rho = \frac{g^2}{2\pi}\rho(2\rho - 1) + \cdots.$$

- The factorization sustains to all loop level. [CS, 10]

$\mathcal{N} = (0,2)$ supersymmetry

- $\mathcal{N} = (0,2)$ superspace:

$$\begin{matrix} x_L & & x_R & & & x_L & & x_R & \\ \theta_L & \theta_L^\dagger & \theta_R & \theta_R^\dagger & \Rightarrow & & & \theta_R & \theta_R^\dagger \end{matrix}$$

- super-derivatives

$$D_R\,,\bar{D}_R \Rightarrow \{D_R, \bar{D}_R\} = 2i\partial_L$$

∂_R commutes with everything!

- chiral superfields

bosonic	ϕ	ψ_L		
fermionic		ψ_R	F	

A simplified model with flat target space

In a bid to understand the deformation and to get some higher loop correction, we consider the following model, which corresponds to the limiting case $\frac{\gamma^2}{g^2} \to \infty$ of the previous model.

$$\mathcal{L}_{\rm linear} = \frac{1}{2} \int d^2\theta_R \, iA^\dagger \overleftrightarrow{\partial_R} A + B^\dagger B + \mathcal{B}^\dagger \mathcal{B} - \left(\gamma \mathcal{B} B A^\dagger + {\rm H.c.}\right) \,.$$

- background symmetry

$$A \to A + \alpha + \alpha^\dagger A^2 \;\Rightarrow\; A \to A + \alpha \,.$$

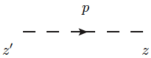
- fermion flavor symmetry:

$$\begin{pmatrix} \mathcal{B} \\ \frac{B}{1 + A^\dagger A} \end{pmatrix} \Rightarrow \begin{pmatrix} \mathcal{B} \\ B \end{pmatrix} \,.$$

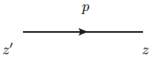
Supergraph as a probe

Feynman rules to remember:

- Propagators:

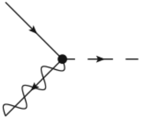
$T[A_z, A^\dagger_{z'}] =$ 

$= -\frac{i}{p^2}\delta^1(\theta_R - \theta'_R)$

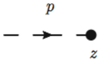
$T[B_z, B^\dagger_{z'}] =$ 

$= -\frac{i}{p_L}\delta^1(\theta_R - \theta'_R)$

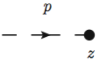
- vertices:


$= \frac{i\gamma}{2}$

- (anti)chiral projectors:

A 

$= \bar{D}_R(p, \theta_R, \theta^\dagger_R) A_z$

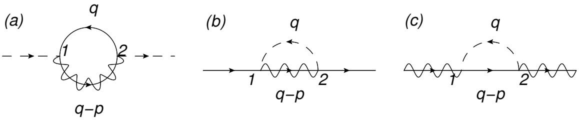
$A^\dagger$ 

$= A^\dagger_z \overleftarrow{D}_R(p, \theta_R, \theta^\dagger_R)$

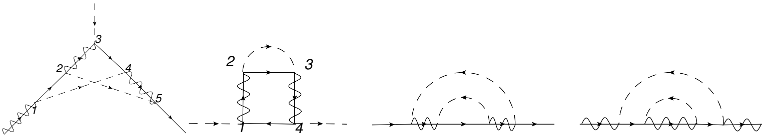
Supergaph as a probe

Feynman diagrams to calculate:

- one-loop diagrams:



- two-loop diagrams:



Finally, two loop contribution to $\beta(\gamma)$ vanishes!

Background field method as a proof

Key idea: shift symmetry of $\theta_R^\dagger$ preserved classically and quantum mechanically $\Rightarrow \int d\theta_R d\theta_R^\dagger$ constant = vanishing loop correction!
Need: nontrivial background!
Upshot:

- something $\bar{Q}_R$-closed, ie,

$$[\bar{Q}_R, X] = 0\,.$$

- F term:

$$\int d^2x\, [Q_R, X]|_{\theta_R=\theta_R^\dagger=0} \neq 0\,,$$

  we end up at the usual nonrenormalization theorem.
- D term:

$$\int d^2x\, \{Q_R, [\bar{Q}_R, X]\}|_{\theta_R=\theta_R^\dagger=0} = 0\,!$$

Background field method as a proof

The way out: target space symmetry helps!

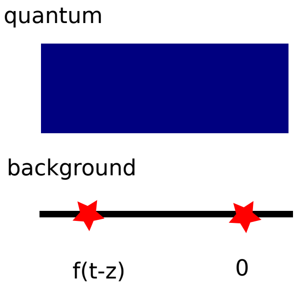
- consider an equivariant version of the previous story,

$$[\bar{Q}_R, \bar{X}] = \bar{0}\,, \quad \int d^2x\, \{Q_R, [\bar{Q}_R, X]\}|_{\theta_R=\theta_R^\dagger=0} \neq 0\,.$$

- Target space symmetry is given by

$$A_{bk} \to A_{bk} + f(x_R)\,, \quad A_{qu} \to A_{qu}\,.$$

Quantum correction is the same for all possible backgrounds in the equivalent class, but the background could be nontrivial!



Nonperturbative regime

Following Seiberg's argument [Seiberg, 93] Good news:

- can promote γ to a superfield $\Rightarrow$ chirality is preserved!
- can assign R-charges!

Bad news:

- no F term, no holomorphic function!

So you will get,

$$f\left(\gamma\mathcal{B}BA^\dagger, \frac{\gamma\mathcal{B}B}{A}, |\mathcal{B}|^2, |B|^2, |\gamma|^2\right) .$$

Now apply target space symmetry, the holomorphic piece will save our day. Similarly one could analyze Z_A.

Conclusion

Why it is so good?

- Flat target space, so quantum correction is uniformal, in the sense of respecting target space symmetry.
- $\mathcal{N} = (0, 2)$ makes sure that the translation is powerful.

$\Rightarrow$ generalization to other linear models will be straightforward.
What about nonlinear models?

- For aforementioned heterotic $\mathbb{CP}^1$ model, Z_A and Z_γ are free from $|\gamma|^{2n}$-correction at n loop order.
- We shall explore the mixing contribution by g and γ at higher loop order.

Thank you!