Resilience of Lake Eutrophication Models to Changes in Phosphorus Inputs

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1 Introduction

Eutrophication is a biological process where the phytoplankton population increases significantly in an aquatic environment due to the presence of excess nutrients, especially phosphorus [1]. It is extremely detrimental to the environment and has several negative consequences, such as increased frequency of fish deaths due to anoxic events and excessive plant and algae growth [2]. It happens due to excess nutrients being added to the water, especially phosphorus, coming from agricultural runoff, sewage, and industrial discharges, and the restoration of an eutrophic lake can have high associated costs, depending on the current state of the lake [3]. The reversal of eutrophication varies depending on their responses to the reduction of the phosphorus input. Certain lakes, called "reversible", are able to leave the eutrophic state solely by decreasing the phosphorus input. Others require a combination of decreasing it and some outside intervention, such as chemical treatment, and are called "hysteretic" [3].

There have been multiple models developed to study this phenomenon. A number of these focus on Lake Mendota, WI, which is considered by some "the most studied lake in the world" [4]. This paper will focus on the 1999 Carpenter, Ludwig, and Brock model and the 2005 Carpenter model, both of which use data from Lake Mendota [2, 3].

2 Resilience

Before understanding the work that was done, first, the concept of resilience must be defined. Resilience is the characteristic that systems can absorb an imposed change and keep working, even if in a different basin of attraction from a different equilibrium state, and two different methods are further discussed [5, 6]. The resilience model used in this work is the method introduced by Cessi, where the chosen way to impose a change is to start from one of the stable equilibrium solutions and change the parameters of the system for enough time to force the original solution to switch to the basin of attraction of the other equilibrium solution [7].

3 Single Equation Model

Essentially, lake eutrophication depends on the Phosphorus input, the rate of loss, and the recycling of Phosphorus in the lake water [3]. This can be modeled by the equation below.

$$\frac{dP}{dt} = \ell - sP + \frac{rP^q}{m^q + P^q}$$

where ℓ is the P input, s is the rate of loss of P per unit time, r is the maximum rate of recycling, and m is the P value where the recycling reaches half of its maximum rate [3]. To determine the resilience of this model, the parameter that was varied was the Phosphorus input ℓ. Starting with the equation at $\ell = 2.5$, the coordinate for the non-eutrophic equilibrium was used as the initial condition. Then, the system was switched to the new parameter,the new value of ℓ, for a certain period of time. The coordinate of where the new system was after this time was then returned to the original equation where $\ell = 2.5$ and was let freely flow to see is the system would return to the non-eutrophic equilibrium. This process was repeated to determine the maximum time length the system can remain with the new parameter values before it does not return to the non-eutrophic state once the system is switched back to $\ell = 2.5$, instead tending towards the eutrophic equilibrium.

This procedure was repeated for several values of ℓ, where $\ell = 3$ is the minimum value where a switch between equilibrium solutions happens. The results are plotted in the figure below.

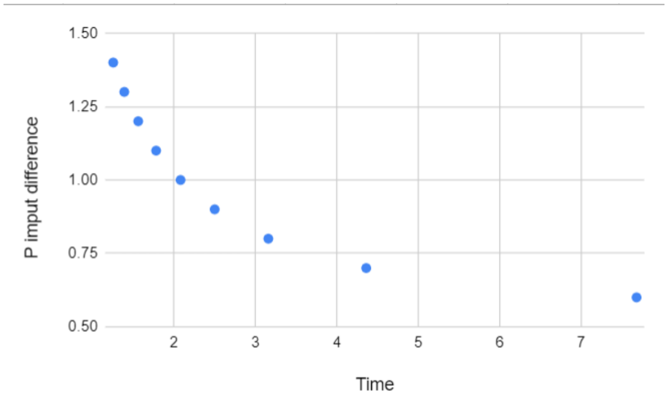

Figure 1: The minimum time length of a perturbation required to switch equilibrium solutions based on the difference from $\ell = 2.5$.

The results found a match with the expected results from Cessi, which also showed the resilience of the system to be in the form of a power function [7]. This is interpreted by there being an inverse relation between Phosphorus input and time, where the larger the input, the less time there is before the lake becomes eutrophic. The reverse, however, is not always true, since depending on the initial input, the lake remains eutrophic even if there is no more phosphorus input coming into the lake.

4 Triple Equation Model

A more complex model was developed to include the effect of Phosphorus in the soil and sediments in the process of eutrophication [2]. The model is composed of the three differential equations below.

$$\frac{dU}{dt} = W + F - H - cU$$

$$\frac{dP}{dt} = cU - (s + h)P + rMf(P)$$

$$\frac{dM}{dt} = sP - bM - rMf(P)$$

$$f(P) = \frac{P^q}{m^q + P^q}$$

where U is the phosphorus density in the soil, P is this density in lake water, and M is the phosphorus density in surface sediments [2]. Also F and W are the agricultural and non-agricultural inputs of phosphorus respectively, H is the annual export of phosphorus from the lake, and $c, s, h, r, b.m$ are constant parameters.

Using the values determined by Carpenter for c, s, h, r, b, q, m , we can see that for values of $W + F - H$ between around $0.38\frac{g}{y*m^2}$ and $0.85\frac{g}{y*m^2}$ there are multiple equilibria [2]. The two stable equilibria represent the eutrophic and the non-eutrophic states of the lake.

To determine the resilience of the three-equation model, the value of $W + F - H$ is varied while the others remain constant. Starting with $W + F - H = 0.6\frac{g}{y*m^2}$, the initial coordinates for the system were chosen to be $P = 0.1\frac{g}{m^2}$ and $M = 100\frac{g}{m^2}$, and then were let freely flow for 100 years. Then, the system was switched to $W + F - H = 1\frac{g}{y*m^2}$ for another 100 years. Lastly, the system was switched to the new parameter value of $W + F - H$ for a certain period of time. The coordinate of where the new system was after this time was then returned to the original equation where $W + F - H = 0.6\frac{g}{y*m^2}$ and was let freely flow to see is the system would return to the non-eutrophic equilibrium. This process was repeated to determine the maximum time length the system can remain with the new parameter values before it does not return to the non-eutrophic state once the system is switched back to $W + F - H = 0.6\frac{g}{y*m^2}$, instead tending towards the eutrophic equilibrium. This process was repeated for several values of $W + F - H$, and the results are plotted in Figure 2.

Once again, the results match with what was expected from Cessi since the resilience is in the form of a power function [7]. This result shows that there is very little time to reverse the effects of eutrophication. Some values of $W+F-H$

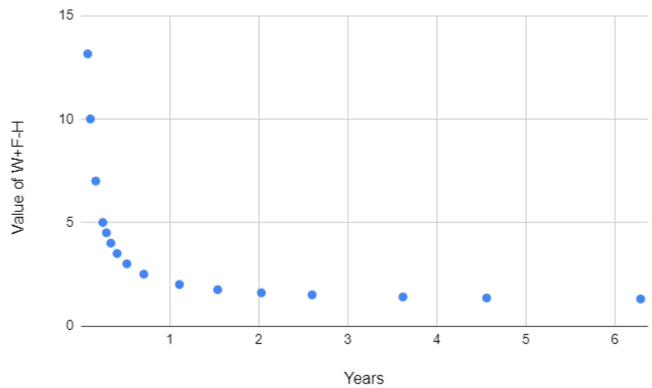

Figure 2: The minimum time length of a perturbation required to switch the system to the eutrophic equilibrium

have only months before reducing the phosphorus input is not enough to prevent eutrophication.

5 Discussion and Conclusion

For the three-equation model, the simulation was run in this fashion to simulate pre-agricultural, agricultural, and agricultural with other human interference levels. The increased use of fertilizers for agricultural practices has increased the leakage of nutrients, such as Phosphorus and Nitrogen, into almost all bodies of water, especially in freshwater [8]. The effects of leakage are seen very quickly since algae grows extremely fast when compared with other aquatic species. Generally, when it is possible to reverse eutrophication, it still takes hundreds of years for the lake to return to its original state [2]. This shows that once a body of water is eutrophied, it will still take a very long time for any significant difference to be made. Thus, the main strategy to prevent eutrophication is to control the input of phosphorus, especially due to agriculture and other human sources.

In this paper, we have explored the response of different models of lake eutrophication as the parameter for phosphorus input changes. We observed that a sufficiently large change in input induces the models to switch to new steady states. We saw that the systems remain in their new states, even if the input is returned to its original values. We quantified the relationship between the magnitude of the change in input and the amount of time that the change must be maintained to reached the tipping point. These models, especially the three-equation Carpenter's model, illustrate the urgency of mitigating the amount of phosphorus added to bodies of water to prevent eutrophication, for which it is difficult to return.

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