GLOBAL EXISTENCE FOR A CLASS OF NON-FICKIAN POLYMER-PENETRANT SYSTEMS

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IMA Preprint Series # 1184

November 1993

Global Existence For A Class Of Non-Fickian Polymer-Penetrant Systems*

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Abstract

This paper deals with a class of strongly coupled and highly degenerate nonlinear parabolic systems, which arises from a model describing non-Fickian diffusion of penetrant into glassy polymers. By means of a fixed point argument and a priori estimates, we establish global existence and uniqueness for the systems.

1. Introduction

Diffusion in swollen media is most commonly described by Fick's law

$$J(x,t) = -D(c)\frac{\partial c}{\partial x}, \tag{1.1}$$

where x is the spatial variable, c is the concentration of the penetrant, J is the fluid mass intake, or *flux*, and $D(c)$ is the diffusion coefficient. Fick's law is supplemented by the conservation of mass

$$\frac{\partial c(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x}. \tag{1.2}$$

Swollen rubbery polymers obey the (concentration-dependent) Fick's law. However, it cannot adequately describe diffusion in glassy polymers. In fact, diffusion in glassy polymers causes micro-mechanical changes in the polymer structure and this results in a non-Fickian diffusion.

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Suppose that one face of a slab of dry polymer is in contact with fluid (penetrant); then the fluid will penetrate into the slab through this face. Cohen and White [3] proposed the following model to describe the diffusion of the penetrant in the glassy polymer. They take

$$J = -D(c)\frac{\partial c}{\partial x} - E(c)\frac{\partial \sigma}{\partial x} + Mc, \tag{1.3}$$

where σ is the stress; $E(c)\frac{\partial \sigma}{\partial x}$ is the component of the flux representing the viscoelastic effect with the appropriate proportionality factor E, and Mc represents convective effects. Using some viscoelastic models which relate σ to the strain ε and imposing some relationship between ε and c, they derive the relation (for more details see [3] and [4])

$$\frac{\partial \sigma}{\partial t} + \beta(c)\sigma = \rho(c)c + \gamma(c)\frac{\partial c}{\partial t}, \tag{1.4}$$

where $\beta(c), \rho(c), \gamma(c)$ are smooth functions of c (see also [4, Chapter 4] for more details).

The system (1.2), (1.3) and (1.4) needs to be supplemented with initial and boundary conditions. For one dimensional case, the complete problem is

$$c_t = (Dc_x + E(c)\sigma_x)_x - (Mc)_x \quad \text{for } 0 < x < 1, t > 0, \tag{1.5}$$

$$\sigma_t + \beta(c)\sigma = \rho(c)c + \gamma(c)c_t \quad \text{for } 0 < x < 1, t > 0, \tag{1.6}$$

$$c(0,t) = \theta > 0, \qquad c(1,t) = 1 > \theta \quad \text{for } t > 0, \tag{1.7}$$

$$c(x,0) = c_0(x) \geq 0 \quad \text{for } 0 < x < 1, \tag{1.8}$$

$$\sigma(x,0) = \sigma_0(x) \quad \text{for } 0 < x < 1, \tag{1.9}$$

where θ is a constant.

DEFINITION: *We refer to the system* (1.5) – (1.9) *as Problem* 1.

From the mathematical point of view Problem 1 is a strongly coupled (second derivatives are coupled) and highly degenerate nonlinear parabolic system, whose "diffusion matrix" has rank one only. In the case where $D = E(c) = d$, a positive constant, and $\rho(c) = 1, \gamma(c) = 0$, Problem 1 is just a special case of the more general strongly degenerate semilinear parabolic systems considered in H. Amann [1]. Well posedness and global existence are established there by using semigroup theory.

In view of particular choices of functions $E(c), \beta(c), \rho(c), \gamma(c)$ based on the physical considerations (see, for example, [3] [4]), we will assume, throughout this paper, that they are smooth enough so that they have the required differentiability, and

$$E(0) = 0; \quad E(c) > 0; \quad 0 \leq E'(c) \leq E_1; \quad |E''(c)| \leq E_1 \quad \text{for } c \geq 0, \tag{1.10}$$

$$\beta_0 \le \beta(c) \le \beta_1; \quad 0 \le \beta'(c) \le \beta_2 \quad \text{for } c \ge 0, \tag{1.11}$$

$$0 \le \rho(c) \le \rho_1; \quad |\rho'(c)| \le \rho_2 \quad \text{for } c \ge 0, \tag{1.12}$$

$$\rho(c) = 0 \qquad \text{for } c \ge c^*, \tag{1.13}$$

$$0 \le \gamma(c) \le \gamma_1; \quad |\gamma'(c)| \le \gamma_2 \quad \text{for } c \ge 0, \tag{1.14}$$

where $E_1, \beta_0, \beta_1, \beta_2, \rho_1, \rho_2, c^*, \gamma_1, \gamma_2$ are positive constants.

In addition to the assumption (1.10), we further assume that

$$\int_0^\eta \frac{ds}{E(s)} < +\infty \qquad \text{for } 0 < \eta < +\infty, \tag{1.15}$$

which holds for a large class of physically acceptable functions $E(c)$ (see [2]).

For simplicity, we also assume that $D(c) = 1$, $\sigma_0(x) = 0$ and $M = 0$. Our method extends to the more general $D(c)$, $\sigma_0(x)$ and M with necessary modifications. The initial condition $c_0(x)$ is approximately equal to 0. It is assumed that $c_0(x) \in C^{2+\alpha}[0,1]$ for some $0 < \alpha < 1$ and that $0 \le c_0(x) \le 1$. Moreover, the first order compatibility conditions at $(0,0)$ and at $(1,0)$ will also be assumed.

This paper is organized as follows. In section 2, by introducing a nonlinear transformation, we decoupled the second derivatives and reduce (1.5)–(1.9) to an equivalent problem, which consists of a nonlinear parabolic equation and a nonlinear integral equation with their first x derivatives coupled. Local existence and uniqueness are established in section 3. Finally, in section 4, we derive a priori estimates for the solution and prove global existence.

2. Reduction of Problem 1

Set $w(x,t) = \varphi(c(x,t))$, where

$$\varphi(c) = \int_0^c \frac{1}{E(s)} ds \qquad \text{for } c \ge 0.$$

Since $\varphi'(c) = 1/E(c) > 0$ for $c > 0$, there exists an inverse function of $w = \varphi(c)$, and we denote it by $c = \psi(w)$. By (1.10), $\int_0^\infty \frac{dw}{E(w)} = +\infty$, so ψ is well defined for all $0 \le w < \infty$. Introduce a new function $u = w + \sigma$. We can now deduce from (1.5) and (1.6) that

$$\psi'(w) w_t = (\psi'(w) u_x)_x \qquad \text{for } 0 < x < 1, t > 0, \tag{2.1}$$

$$(u - w)_t + B(w)(u - w) = P(w) + (R(w))_t \qquad \text{for } 0 < x < 1, t > 0, \tag{2.2}$$

where $B(w) = \beta(\psi(w)), P(w) = \rho(\psi(w)) \cdot \psi(w)$ and $R(w) = \int_0^{\psi(w)} \gamma(s)ds$.

By (2.2),

$$(1 + R'(w))w_t = u_t + B(w)(u - w) - P(w). \tag{2.3}$$

Substituting (2.3) into (2.1), we get

$$u_t = (1 + R'(w))u_{xx} + \frac{\psi''(w)}{\psi'(w)}(1 + R'(w))w_x u_x - B(w)(u - w) + P(w). \tag{2.4}$$

On the other hand, multiplying (2.2) with $e^{\int_0^t B(w(x,s))ds}$ and then integrating by parts, we obtain

$$\begin{aligned} w + R(w) = u \quad &+ \quad \int_0^{c_0(x)} \gamma(s)ds \cdot e^{-\int_0^t B(w)ds} \\ &+ \quad \int_0^t R(w)B(w)e^{-\int_\tau^t B(w)ds} d\tau \\ &- \quad \int_0^t P(w)e^{-\int_\tau^t B(w)ds} d\tau. \end{aligned} \tag{2.5}$$

Here we used that $\psi(w(x, 0)) = c(x, 0) = c_0(x)$, $R(w) = \int_0^{\psi(w)} \gamma(s)ds$ and $u(x, 0) - w(x, 0) = \sigma(x, 0) = 0$.

To complete our new problem we need corresponding initial and boundary conditions for equation (2.4). Recalling that $u = w + \sigma$ and $c(0, t) = \theta$, and solving $\sigma(0, t)$ from (1.6), we find that

$$\begin{aligned} u(0, t) &= w(0, t) + \sigma(0, t) \\ &= \int_0^\theta \frac{ds}{E(s)} + \frac{\rho(\theta)\theta}{\beta(\theta)}(1 - e^{-\beta(\theta)t}) \qquad \text{for } t > 0. \end{aligned} \tag{2.6}$$

Similarly,

$$u(1, t) = \int_0^1 \frac{ds}{E(s)} + \frac{\rho(1)}{\beta(1)}(1 - e^{-\beta(1)t}) \qquad \text{for } t > 0. \tag{2.7}$$

Finally, since $\sigma(x, 0) = 0$ for $0 < x < 1$, the initial condition for (2.4) is

$$u(x, 0) = w(x, 0) = \int_0^{c_0(x)} \frac{ds}{E(s)}. \tag{2.8}$$

DEFINITION: *We refer to the system* (2.4) – (2.8) *as Problem* 2.

We shall need some basic parabolic estimate in our proofs. We state it in the following lemma.

Let $Q_T = \Omega \times (0,T)$ and let $u(x,t)$ be a solution of the following equation

$$\begin{aligned} u_t &= \sum_{i,j=1}^{n} a_{ij}(x,t)u_{x_i x_j} + \sum_{i=1}^{n} b_i(x,t)u_{x_i} + c(x,t)u + f(x,t) \qquad \text{in } Q_T, \\ u &= U_0 \qquad \text{on } \partial_p Q_T, \end{aligned}$$

where $0 < T \leq T^*$.

LEMMA 2.1. *Suppose that $\partial\Omega \in C^2$ and*

$$\begin{aligned} &\lambda I \leq (a_{ij}(x,t)) \leq \Lambda I, \quad |b_i(x,t)|,\ |c(x,t)| \leq \Lambda, \quad \textit{for } (x,t) \in Q_T, \\ &a_{ij} \textit{ is continuous on } \overline{Q_T}. \end{aligned}$$

Suppose that U_0 can be extended to a function on $\overline{Q_{T^*}}$ such that

$$\|U_0\|_{W_p^{2,1}(Q_{T^*})} < \infty.$$

Then for $p > n+2$, we have

$$\|u\|_{C^{1+\beta,(1+\beta)/2}(\overline{Q_T})} \leq C\Big(\|U_0\|_{W_p^{2,1}(Q_{T^*})} + \|f\|_{L^p(Q_T)}\Big), \tag{2.9}$$

where $\beta = 1-(n+2)/p$, and C depends on $\lambda, \Lambda, \Omega, T^*, p$ and the modulus of continuity of a_{ij}. It is independent of T.

Proof. The estimate (2.9) is essentially the L^p estimates and the Sobolev's embedding theorem (see [8]). The Sobolev's embedding theorem, however, depends on the domain, and hence the lower bound of T. It is clear that the embedding theorem is invalid as $T \to 0+$. In order to make sure that the constant C is independent of T, we let

$$\tilde{a}_{ij}(x,t) = \begin{cases} a_{ij}(x,t) & \text{for } t \leq T \\ a_{ij}(x,T) & \text{for } t > T \end{cases}, \qquad \tilde{b}_i(x,t) = \begin{cases} b_i(x,t) & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases},$$

and

$$\tilde{f}(x,t) = \begin{cases} f(x,t) & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases}.$$

Let $\tilde{u}$ be the corresponding solution in Q_{T^*}. Then the L^p estimates and the Sobolev's embedding theorem immediately give us the estimates (2.9) for $\tilde{u}$ on Q_{T^*}. Since $u \equiv \tilde{u}$ for $t \leq T$ and $\tilde{f} = 0$ for $t > T$, (2.9) follows immediately. $\square$

3. Local Existence and Uniqueness

We begin with a local existence theorem for Problem 2. We shall show at the end of this section, any solutions $c(x,t)$ and $w(x,t)$ of Problem 2 are non-negative. However, technically we shall allow both $c(x,t)$ and $w(x,t)$ to be negative in the following contraction mapping arguments. For this reason, we need to extend all functions of c involved so that these functions are defined for negative numbers. We extend $\beta(c), \rho(c), \gamma(c)$ such that they become even functions; hence they are non-negative and Lipshitz continuous for all c. We also extend $E(c)$, $\varphi(c)$ and $\psi(w)$ such that they become odd functions; thus they are continuously differentiable and their first derivatives are non-negative; their second derivatives are locally bounded.

Define

$$U_0(x,t) = \int_0^{c_0(x)} \frac{ds}{E(s)} + \frac{\rho(c_0(x))c_0(x)}{\beta(c_0(x))}(1 - e^{-\beta(c_0(x))t}).$$

Clearly, $U_0(x,t) \in C^{2,1}([0,1] \times [0,\infty))$ and $u = U$ for (x,t) on the parabolic boundary of $Q_T = (0,1) \times (0,T)$. Set

$$\begin{aligned} X \ = \ &\Big\{u(x,t) \in C^0(\overline{Q_T}); \quad u|_{\partial_p Q_T} = U_0|_{\partial_p Q_T}, \|u\|_{L^\infty(Q_T)} \le M_0, \\ &\|u_x\|_{L^\infty(Q_T)} \le M_1, \|u\|_{C^{\delta,\delta/2}(\overline{Q_T})} \le M_\delta\Big\}, \end{aligned}$$

where T, δ $(0 < \delta < 1)$ and M_δ are constants to be determined; $M_0 = 2\|U_0(\cdot,0)\|_{L^\infty[0,1]}$ and $M_1 = 2\|U_{0x}(\cdot,0)\|_{L^\infty[0,1]}$.

For simplicity, we denote by C the generic constants depending only on the known data and the upper bound of T, unless otherwise indicated. It may, however, vary from a line to another.

Lemma 3.1. *For each $u(x,t) \in X$, there exists a unique solution $w(x,t)$ to the integral equation* (2.5) *for any $(x,t) \in Q_T$. Moreover, there hold:*

$$|w(x,t)| \le C \quad \text{for } (x,t) \in Q_T, \tag{3.1}$$

$$|w_x(x,t)| \le C \quad \text{for } (x,t) \in Q_T, \tag{3.2}$$

$$\|w\|_{C^{\delta,\delta/2}(\overline{Q_T})} \le C(\delta, M_\delta), \tag{3.3}$$

the constants in (3.1) *and* (3.2) *are independent of δ and M_δ.*

Proof. Let $W = f(w)$, where $f(w) = w + R(w)$. Then $f'(w) = 1 + R'(w)$. Since $R'(w) = \gamma(\psi(w))\psi'(w) \ge 0$, $W = f(w)$ has an inverse function, we denote it by $w = g(W)$. Moreover, clearly $f'(w) \ge 1$ for all w, which implies that

$$0 < g'(W) \le 1 \qquad \forall \quad W. \tag{3.4}$$

Now we write (2.5) in terms of W to get an integral equation as follows

$$W = u + \int_0^{c_0(x)} \gamma(y)dy \cdot e^{-\int_0^t \widehat{B}(W)ds} + \int_0^t G(t,\tau,W)d\tau, \tag{3.5}$$

where

$$G(t,\tau,W) = [(W - g(W))\widehat{B}(W) - \widehat{P}(W)]e^{-\int_\tau^t \widehat{B}(W)ds},$$

and

$$\widehat{B}(W) = B(g(W)), \qquad \widehat{P}(W) = P(g(W)).$$

Notice that $\beta_0 \leq \widehat{B}(W) \leq \beta_1$, and $\widehat{B}'(W) = \beta'(\psi(g(W)))\psi'(g(W))g'(W)$, which is locally bounded since $\psi'(g(W)) = E(\psi(g(W)))$ is locally bounded and $g'(W)$ is bounded by (3.4); also $|\widehat{P}(W)| \leq C_0$, where $C_0 = \max_{0\leq c\leq c^*} \rho(c)c$, and similarly $\widehat{P}'(W)$ is also locally bounded. Therefore, it is clear that there exists a unique solution $W(x,t)$ to (3.5) for any $x \in [0,1]$ and $0 \leq t < T_0$ where T_0 is some small number (see, for instance, [6]).

To prove global existence, we need to establish a priori bound for $W(x,t)$ for $(x,t) \in Q_T$. Clearly

$$\begin{aligned}
|G(t,\tau,W)| &\leq \beta_1|W - g(W)| + C_0 \\
&= \beta_1|W - g(W) + g(0)| + C_0 \\
&\leq \beta_1|W| + C_0,
\end{aligned}$$

where we used the fact $g(0) = 0$ and inequality (3.4). Therefore by (3.5),

$$|W(x,t)| \leq |u(x,t)| + \gamma_1 + \int_0^t (\beta_1|W(x,\tau)| + C_0)d\tau,$$

so that

$$|W(x,t)| \leq (M_0 + \gamma_1 + C)e^{\beta_1 t} \tag{3.6}$$

for $(x,t) \in Q_T$ by Gronwall's inequality. As a consequence of (3.6), we can extend the local solution $W(x,t)$ of (3.5) globally, that is, (3.5) has a unique solution $W(x,t)$ for $0 \leq t \leq T$, as long as $u(x,t)$ is well defined for $0 \leq t \leq T$. Once the solution $W(x,t)$ is found, (2.5) has a unique global solution $w(x,t) = g(W(x,t))$.

Next, it follows from the standard theory of integral and differential equations that W_x exists as long as u_x exists (see [6]). Differentiating (3.5) with respect to x and using (3.6), we can easily find that

$$|W_x| \leq |u_x| + \gamma_1|c_{0x}| + C\int_0^t |W_x|d\tau. \tag{3.7}$$

By Gronwall's inequality, (3.7) implies that

$$|W_x| \leq C(M_1 + \gamma_1|c_{0x}|)e^{Ct}. \tag{3.8}$$

Since $w = g(W), g(0) = 0$ and $0 < g(W) \leq 1$, the assertions (3.1) and (3.2) follow immediately. The prove (3.3), we first notice that the Hölder continuity in x direction is already contained in (3.2). Therefore one needs only to establish the Hölder continuity in t direction, which is a direct consequence of applying Gronwall's inequality to (3.5). The proof is complete. $\square$

Now we define a map $\mathcal{M}$ on X in the following way. For each $u(x,t) \in X$, there exists a unique solution $w(x,t)$ to (2.5); substituting this $w(x,t)$ into (2.4) and using (3.1) and (3.2), we obtain a unique solution $\overline{u}(x,t)$ to the linear problem (2.4), (2.6)–(2.8). Define $\mathcal{M}u(x,t) = \overline{u}(x,t)$ for each $u(x,t) \in X$. Then $\mathcal{M}$ is clearly well defined on X. We wish to prove that it is a contraction.

LEMMA 3.2. *The map $\mathcal{M}$ maps X into X and is a contraction if T is sufficiently small.*

Proof. We first show that $\mathcal{M}$ maps X into X provided T is small. For each $u(x,t) \in X$, let $w(x,t)$ be the unique solution of the integral equation (2.5). Set $F = \max_{(x,t)\in Q_T} |B(w)w + P(w)|$. Then $F \leq C$ by (3.1). Clearly, $U(x,t) = M_0/2 + Ft$ and $V(x,t) = -M_0/2 - Ft$ are supersolution and subsolution of problem (2.4), (2.6)–(2.8) respectively. So by the comparison principle, $V(x,t) \leq \overline{u}(x,t) \leq U(x,t)$. Hence

$$|\overline{u}(x,t)| \leq M_0/2 + CT \leq M_0 \qquad \text{for } (x,t) \in Q_T, \tag{3.9}$$

provided T is sufficiently small.

Notice that $\psi''(w)/\psi'(w) = E'(\psi(w))$ is bounded due to (3.1). A direct application of the standard Hölder estimate to (2.4) (see [7]) yields

$$\|\overline{u}\|_{C^{\tilde{\delta},\tilde{\delta}/2}(\overline{Q_T})} \leq C^*, \tag{3.10}$$

for some positive constant $\tilde{\delta}(0 < \tilde{\delta} < 1)$, and C^* depends only on the known data and upper bound of T. We now take $\delta = \tilde{\delta}$, $M_\delta = C^*$ in the definition of X. By Lemma 2.1,

$$\|\overline{u}_x\|_{C^{\alpha,\alpha/2}(\overline{Q_T})} \leq C(\|U_0\|_{W_p^{2,1}(Q_{T^*})} + \|B(w)w + P(w)\|_{L^p(Q_T)}) \leq C, \tag{3.11}$$

for some $\alpha \in (0,1)$. Thus

$$\|\overline{u}_x\|_{L^\infty(\overline{Q_T})} \leq \frac{M_1}{2} + CT^{\alpha/2} \leq M_1, \tag{3.12}$$

provided T is small enough. Therefore $\overline{u}(x,t) \in X$. It follows that $\mathcal{M}$ maps X into X if T is sufficiently small.

Now, let $u_i(x,t) \in X (i=1,2)$ and let $w_i(x,t)(i=1,2)$ be the corresponding solutions of (2.5); and let $\overline{u}_i(x,t)$ $(i=1,2)$ be the solutions of (2.4) corresponding to $w_i(x,t)(i=1,2)$ respectively. Consider $U = u_1 - u_2, W = w_1 - w_2, \overline{U} = \overline{u}_1 - \overline{u}_2$. By using (3.1), we can easily deduce from (2.5) that

$$|W(x,t)| \le C|U(x,t)| + C\int_0^t |W(x,\tau)|d\tau \qquad \text{for } (x,t) \in Q_T. \tag{3.13}$$

Similarly,

$$|W_x(x,t)| \le C|U_x(x,t)| + C|W(x,t)| + C\int_0^t (|W(x,\tau)| + |W_x(x,\tau)|)d\tau, \tag{3.14}$$

for $(x,t) \in Q_T$. This implies, recalling (3.13), that

$$|W_x(x,t)| + |W(x,t)| \le C(|U_x(x,t)| + |U(x,t)|) + C\int_0^t (|W_x(x,\tau)| + |W(x,\tau)|)d\tau, \tag{3.15}$$

so that

$$\|W\|_{W^{1,0}_\infty(Q_T)} \le C\|U\|_{W^{1,0}_\infty(Q_T)} \tag{3.16}$$

by Gronwall's inequality. The function $\overline{U}(x,t)$ satisfies

$$\overline{U}_t = (1+R'(w_1))\overline{U}_{xx} + E'(\psi(w_1)(1+R'(w_1))w_{1x}\overline{U}_x - B(w_1)\overline{U} + F(x,t) \tag{3.17}$$

for $(x,t) \in Q_T$, where

$$\begin{aligned} F(x,t) \quad = \quad & \Big[R'(w_1) - R'(w_2)\Big]\overline{u}_{2xx} + \Big[E'(\psi(w_1)(1+R'(w_1))w_{1x} \\ & -E'(\psi(w_2)(1+R'(w_2))w_{2x}\Big]\overline{u}_{2x} + \Big[B(w_1) - B(w_2)\Big]\overline{u}_2 \\ & +B(w_1)w_1 + P(w_1) - B(w_2)w_2 - P(w_2) \end{aligned} \tag{3.18}$$

It follows from (3.16) that

$$|F(x,t)| \le C\|U\|_{W^{1,0}_\infty(Q_T)}(|\overline{u}_{2xx}| + |\overline{u}_{2x}| + |\overline{u}_2| + 1). \tag{3.19}$$

By Lemma 2.1,

$$\|\overline{U}\|_{C^{1+\beta,(1+\beta)/2}(\overline{Q_T})} \le C\|U\|_{W^{1,0}_\infty(Q_T)}(\|\overline{u}_2\|_{W^{2,1}_p(Q_T)} + T^{1/p}) \le C\|U\|_{W^{1,0}_\infty(Q_T)}, \tag{3.20}$$

where we used the fact that $\overline{U}|_{\partial_p Q_T} = 0$ and that $\|\overline{u}_2\|_{W^{2,1}_p(Q_T)}$ is bounded. Recalling that $\overline{U} = \overline{U}_x = 0$ for $t = 0$, (3.20) implies that

$$\|\overline{U}\|_{W^{1,0}_\infty(Q_T)} \le CT^{\beta/2}\|U\|_{W^{1,0}_\infty(Q_T)}, \tag{3.21}$$

Hence the operator $\mathcal{M}$ is a contraction provided T is small enough. $\square$

We proved

THEOREM 3.4. *There exists a unique solution for Problem* 2 *provided* T *is small.*

As an immediate corollary, we have

COROLLARY 3.5. *There exists a unique classical solution for Problem* 1 *for short time* T.

Proof. It is easy to see that $c = \psi(w)$ and $\sigma = u - w$ solve Problem 1 except that c might be negative. We now prove that $c > 0$ for $(x,t) \in Q_T$, as a result, Problem 1 has a unique solution. We may derive directly from (3.5), (3.11) that $\|w_x\|_{C^{\alpha,\alpha/2}(\overline{Q_T})} \leq C$. Hence by Schauder's estimate on the equation (2.4) we obtain that $u_{xx}(x,t)$ is Hölder continuous for $(x,t) \in \overline{Q_T}$.

Our evenly extended functions in (3.5) can not be differentiated twice. For this reason, we first assume that $c_0(x) > 0$ for $0 \leq x \leq 1$. We claim that $c(x,t) > 0$ as long as the solution exists. If this is not true, then we let $\widetilde{T}$ $(\leq T)$ be the smallest time such that $c(\tilde{x}, \widetilde{T}) = 0$ for some $\tilde{x}$ and $c(x,t) > 0$ for $t < \widetilde{T}$. Since $c(x,t) \geq 0$, $w(x,t) = \varphi(x,t) \geq 0$. Therefore the functions involved in (3.5) are twice differentiable. Differentiating (3.5) twice with respect to x and using Gronwall's inequality we can find that this implies $w_{xx}(x,t)$ is bounded for $(x,t) \in Q_T$; and thus $\sigma_{xx}(x,t)$ is bounded where $\sigma = u - w$. Since c satisfies

$$c_t = c_{xx} + E'(c)c_x\sigma_x + E(c)\sigma_{xx}$$

for $(x,t) \in Q_T$, and $E(0) = 0$, it follows from the maximum principle that $c(x,t) > 0$ for $t = \widetilde{T}$, which is a contradiction.

The more general case where $c_0(x) \geq 0$ can always be uniformly approximated by functions $c_0^\varepsilon(x) > 0$. $\square$

4. Global Existence for Problem 1

It is clear that we can always extend the local solution obtained in the last section as long as $\|u\|_{W_\infty^{1,0}(Q_T)} < \infty$. Thus if $(0, T^*)$ is the maximal existence interval for the solution and $T^* < \infty$, then we must have $\|u\|_{W_\infty^{1,0}(Q_{T^*})} = \infty$. In this section we shall derive L^∞ a priori estimates on the solution u and u_x, then the global existence will follow. Once again we use C_T to denote generic constants depending only on the known data and the upper bound of T.

LEMMA 4.1. *Let u, w be the local solution of Problem 2 for $0 \le t \le T$. Then*

$$0 \le u(x,t) \le C_T, \quad 0 \le w(x,t) \le C_T \quad \textit{for } (x,t) \in Q_T. \tag{4.1}$$

Proof. Since $c(x,t) \ge 0$ as proved in Corollary 3.5, $w(x,t) = \varphi(c(x,t)) \ge 0$. We write (2.4) in the following form

$$u_t - (1 + R'(w))u_{xx} - E'(c)(1 + R'(w))w_x u_x + B(w)u = B(w)w + P(w) \tag{4.2}$$

Clearly, the right-hand side of (4.2) is nonnegative and $u(x,t) \ge 0$ on the parabolic boundary of Q_T. Therefore by the maximum principle, $u(x,t) \ge 0$ for $(x,t) \in Q_T$. Since $W = w + R(w)$, $W(x,t) \ge 0$. Thus by (3.4) and (3.5),

$$W(x,t) \le u(x,t) + \gamma_1 + \beta_1 \int_0^t W(x,\tau)d\tau, \tag{4.3}$$

for $(x,t) \in Q_T$. By Gronwall's inequality,

$$\int_0^t W d\tau \le C_T + C_T \int_0^t u d\tau. \tag{4.4}$$

Now we rewrite (4.2) as follows

$$u_t - (1 + R'(w))u_{xx} - E'(c)(1 + R'(w))w_x u_x = \xi(x,t), \tag{4.5}$$

where $\xi(x,t) = B(w)(w-u) + P(w)$. Notice that $\beta_0 \le B(w) \le \beta_1, 0 \le P(w) \le C_0$, and $w - u \le w + R(w) - u = W - u$. We deduce from (4.3) that

$$\xi(x,t) \le \beta_1(\gamma_1 + \beta_1 \int_0^t W d\tau) + C_0. \tag{4.6}$$

Substituting (4.4) into (4.6), we get

$$\xi(x,t) \le C_T + C_T \int_0^t u(x,t)d\tau. \tag{4.7}$$

Therefore it follows from comparison principle (cf. the proof of Lemma 3.2) that

$$\|u\|_{L^\infty(Q_T)} \le C_T + C_T T \int_0^T \|u\|_{L^\infty(Q_\tau)} d\tau,$$

where we used the fact that $u \ge 0$. So by Gronwall's inequality, we conclude

$$\|u\|_{L^\infty(Q_T)} \le C_T.$$

The assertion (4.1) is proved. $\square$

LEMMA 4.2.
$$\int_0^T\int_0^1 u_x^2 dxdt \leq C_T. \tag{4.8}$$

Proof. Rewrite (2.4) as

$$u_t = ((1+R'(w))u_x)_x + a(x,t)w_x u_x + B(w)(w-u) + P(w), \tag{4.9}$$

where $a(x,t) = \psi''(w)(1+R'(w))/\psi'(w) - R''(w)\psi'(w)$.

Note that

$$\begin{aligned}
\psi'(w) &= E(c) = E(\psi(w)),\\
R'(w) &= \gamma(\psi(w))\psi'(w),\\
\psi''(w) &= E'(\psi(w))\psi'(w),\\
R''(w) &= \gamma'(\psi(w))E^2(\psi(w)) + \gamma(\psi(w))\psi''(w).
\end{aligned}$$

By (4.1), it is clear that

$$\begin{aligned}
|a(x,t)| \;\leq\; & E'(\psi(w))(1+\gamma_1 E(\psi(w)))\\
&+\Big(\gamma_2 E^2(\psi(w)) + \gamma_1 E'(\psi(w))E(\psi(w)\Big)E(\psi(w)) \;\leq\; C_T.
\end{aligned}$$

Now we multiply (4.9) with $[e^{\lambda(u-U_0)^+}-1]$ and $[e^{\lambda(u-U_0)^-}-1]$, where λ is a positive constant to be determined. After integrating by parts, using Cauchy's inequality, we obtain

$$\begin{aligned}
\frac{\lambda}{2}\int_0^T\int_0^1 u_x^2 e^{\lambda(u-U_0)^+}\chi_{\{u\geq U_0\}}dxdt \;&\leq\; C_T\int_0^T\int_0^1 |w_x u_x| e^{\lambda(u-U_0)^+}\chi_{\{u\geq U_0\}}dxdt + C_{\lambda,T},\\
\frac{\lambda}{2}\int_0^T\int_0^1 u_x^2 e^{\lambda(u-U_0)^-}\chi_{\{u\leq U_0\}}dxdt \;&\leq\; C_T\int_0^T\int_0^1 |w_x u_x| e^{\lambda(u-U_0)^-}\chi_{\{u\leq U_0\}}dxdt + C_{\lambda,T},
\end{aligned}$$

where $C_T, C_{\lambda,T}$ are constants depending only on λ, T and the known data, and χ is the characteristic function. Combining these two inequalities, we get

$$\frac{\lambda}{2}\int_0^T\int_0^1 u_x^2 e^{\lambda|u-U_0|}dxdt \leq C_T\int_0^T\int_0^1 |w_x u_x| e^{\lambda|u-U_0|}dxdt + C_{\lambda,T} \tag{4.10}$$

On the other hand, recalling the following inequality established earlier (cf. (3.7))

$$|w_x| \leq C_T(1+|u_x| + \int_0^t |w_x|d\tau).$$

By Gronwall's inequality, we easily derive

$$|w_x| \leq C_T(1+|u_x| + \int_0^t |u_x|d\tau). \tag{4.11}$$

It follows that

$$\int_0^T \int_0^1 |w_x u_x| e^{\lambda|u-U_0|} dxdt \tag{4.12}$$
$$\leq \int_0^T \int_0^1 \left[C_T(1+u_x^2) + (C_T+1)\int_0^t u_x^2 d\tau \right] e^{\lambda|u-U_0|} dxdt.$$

Substituting (4.12) into (4.10) with $\lambda = 2C_T + 2$, we obtain

$$\int_0^T \int_0^1 u_x^2 e^{\lambda|u-U_0|} dxdt \leq C_{\lambda,T} + C_T \int_0^T \int_0^t \int_0^1 u_x^2 dx e^{\lambda|u-U_0|} d\tau dt. \tag{4.13}$$

Since $u - U_0$ is bounded, (4.13) yields

$$\int_0^T \int_0^1 u_x^2 dxdt \leq C_T(1 + \int_0^T \int_0^t \int_0^1 u_x^2 dx d\tau dt).$$

The assertion (4.8) now follows immediately from Gronwall's inequality. □

Next we shall derive a Hölder estimate for u and w.

LEMMA 4.3.

$$\|u\|_{C^{\delta,\delta/2}(\overline{Q_T})} \leq C_T \tag{4.14}$$

for some constants $\delta (0 < \delta < 1)$ and C_T.

Proof. The equation (4.9) is of the form

$$u_t = ((1 + R'(w))u_x)_x + a(x,t,u,w,u_x,w_x). \tag{4.15}$$

Using (4.11) and Cauchy's inequality, we derive

$$a(x,t,u,w,u_x,w_x) \leq C_T(u_x^2 + 1 + (\int_0^t |u_x| d\tau)^2).$$

Let $\tilde{\varphi} = (\int_0^t |u_x| d\tau)^2$, then

$$\begin{aligned} \|\tilde{\varphi}\|_{L^{1,4}(Q_T)} &= (\int_0^T \|\tilde{\varphi}\|^4_{L^1[0,1]} dt)^{1/4} \\ &= (\int_0^T (\int_0^1 (\int_0^t |u_x| d\tau)^2 dx)^4 dt)^{1/4} \\ &\leq (T^2(\|u_x\|^2_{L^2(Q_T)})^4)^{1/4} \\ &\leq C_T \qquad \text{(by (4.8))}. \end{aligned}$$

We can now apply the Theorem 1.1 in [8] (page 419) with $q = 1, r = 4, k_1 = 1/4$ to conclude the inequality (4.14). □

As a consequence of (4.14), we have

$$\|w\|_{C^{\delta,\delta/2}(\overline{Q_T})} \leq C_T \tag{4.16}$$

for some constants $\delta (0 < \delta < 1)$ and C_T. Once these Hölder estimates are established, we can obtain the estimates for all of the derivatives.

LEMMA 4.4.

$$\|u\|_{C^{2+\delta,1+\delta/2}(\overline{Q_T})} \leq C_T. \tag{4.17}$$

Proof. Since all the coefficients are Hölder continuous (owing to to (4.14) and (4.16)), we may apply Schauder's estimates on the equation (see [4]) (4.2) to get

$$\|u\|_{C^{2+\delta,1+\delta/2}(\overline{Q_T})} \leq C_T(\|w_x u_x\|_{C^{\delta,\delta/2}(\overline{Q_T})} + 1). \tag{4.18}$$

Clearly,

$$\begin{aligned}\|w_x u_x\|_{C^{\delta,\delta/2}(\overline{Q_T})} &\leq \|w_x\|_{L^\infty(Q_T)}\|u_x\|_{C^{\delta,\delta/2}(\overline{Q_T})} + \|w_x\|_{C^{\delta,\delta/2}(\overline{Q_T})}\|u_x\|_{L^\infty(Q_T)} \\ &\leq C_T\|u_x\|_{L^\infty(Q_T)}\|u_x\|_{C^{\delta,\delta/2}(\overline{Q_T})}.\end{aligned}$$

By interpolation inequalities,

$$\begin{aligned}\|u_x\|_{C^{\delta,\delta/2}(\overline{Q_T})} &\leq C_T\|u\|^{1/2}_{C^{\delta,\delta/2}(\overline{Q_T})}\|u\|^{1/2}_{C^{2+\delta,1+\delta/2}(\overline{Q_T})} \\ &\leq C_T\|u\|^{1/2}_{C^{2+\delta,1+\delta/2}(\overline{Q_T})},\end{aligned}$$

$$\begin{aligned}\|u_x\|_{L^\infty(Q_T)} &\leq C_T\|u\|^{(1-\delta)/2}_{C^{2+\delta,1+\delta/2}(\overline{Q_T})}\|u\|^{(1+\delta)/2}_{C^{\delta,\delta/2}(\overline{Q_T})} \\ &\leq C_T\|u\|^{(1-\delta)/2}_{C^{2+\delta,1+\delta/2}(\overline{Q_T})}.\end{aligned}$$

Therefore

$$\|w_x u_x\|_{C^{\delta,\delta/2}(\overline{Q_T})} \leq C_T\|u\|^{(1-\delta/2)}_{C^{2+\delta,1+\delta/2}(\overline{Q_T})}. \tag{4.19}$$

Substituting (4.19) into (4.18), we get

$$\|u\|_{C^{2+\delta,1+\delta/2}(\overline{Q_T})} \leq C_T(\|u\|^{(1-\delta/2)}_{C^{2+\delta,1+\delta/2}(\overline{Q_T})} + 1), \tag{4.20}$$

from which the assertion (4.17) follows immediately. □

The estimates (4.1) and (4.17) immediately implies that the local solution can be extended for all $t > 0$. So we proved

THEOREM 4.5. *There exists a unique solution for Problem* 1 *for all* $t > 0$.

REMARK 4.1. As indicated in the introduction, our methods can be extended to solve a more general problem in which M and σ_0 may not be zero; for example,

σ_0 can be any smooth function, M can be a smooth function of (x, t), and $D(c)$ can be any smooth function of c such that $D(c) > 0$ for $c \geq 0$. They also work for a problem with more general boundary conditions as well as problems with other boundary conditions, say, the Neumann boundary condition.

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