Warm inflation model building and CMB predictions

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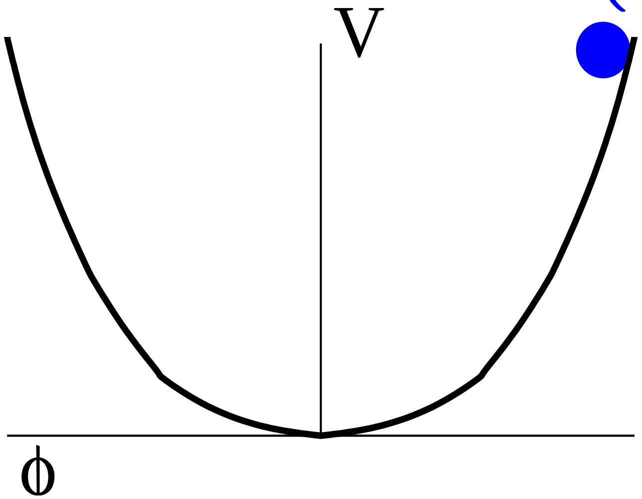
Overview

- Review warm inflation
- Computing dissipative coefficients
- Models of warm inflation
- Comparing observational predictions from dissipative processes during inflation to WMAP and expected Planck data

Scalar field ("inflaton") dynamics



$$\rho = \frac{\dot{\phi}}{2} + \mathrm{V}(\phi) + \frac{(\nabla\phi)^2}{2R^2}$$

$$p = \frac{\dot{\phi}}{2} - \mathrm{V}(\phi) - \frac{(\nabla\phi)^2}{6R^2}$$

- Cold inflation: Just Choose $\mathrm{V}(\phi)$

$$\ddot{\phi} + 3\mathrm{H}\dot{\phi} + \mathrm{V}'(\phi) = 0$$

Potential energy dominated $3\mathrm{H}\dot{\phi} \gg \ddot{\phi}$, "slow-roll"

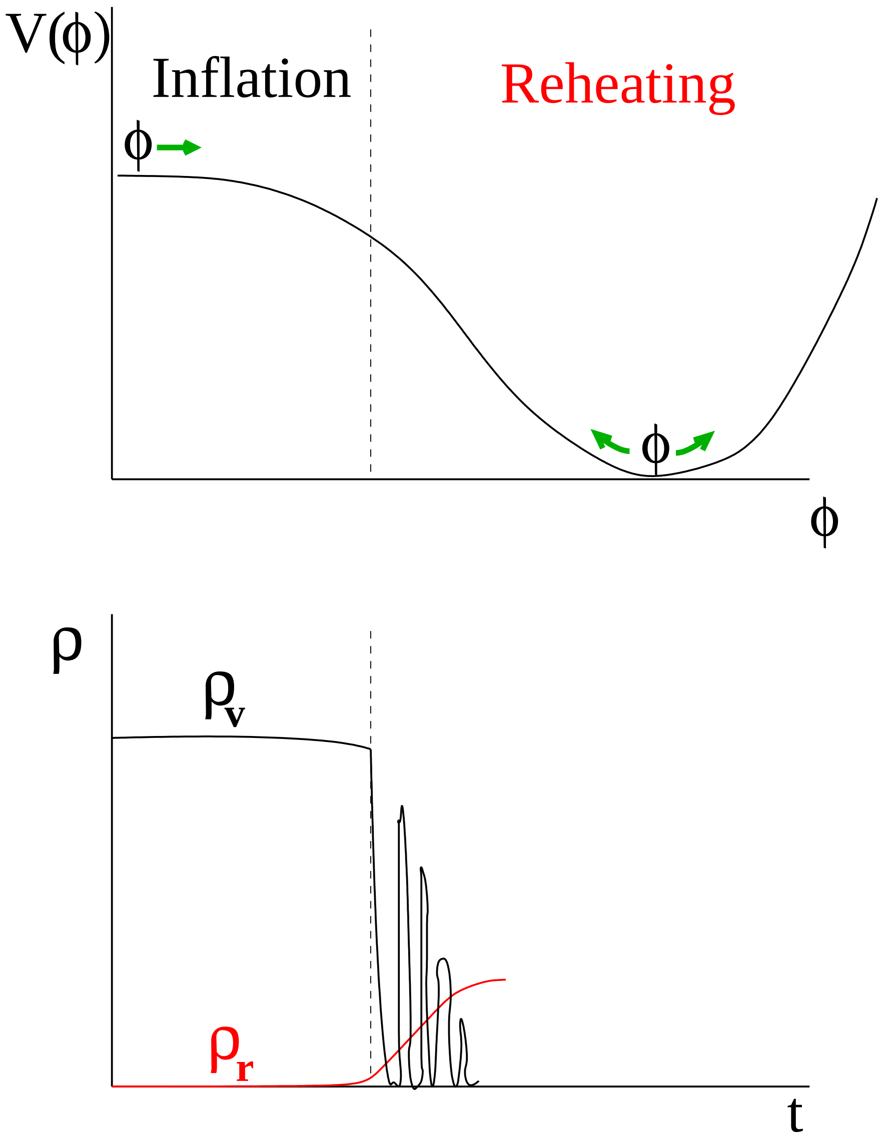
- Warm Inflation: Υ dominates

$$\ddot{\phi} + 3\mathrm{H}\dot{\phi} + \Upsilon\dot{\phi} + \mathrm{V}'(\phi) = 0$$

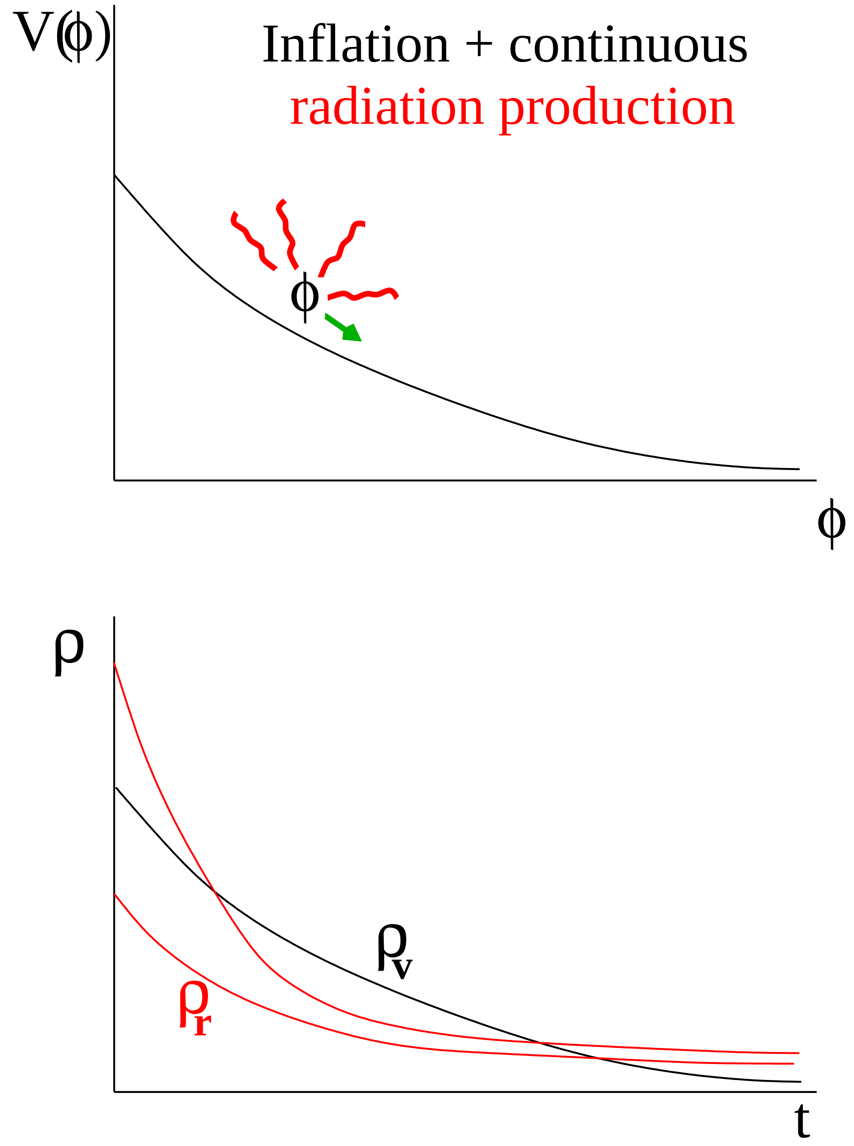
Slow-roll now means $\Upsilon\dot{\phi} \gg 3\mathrm{H}\dot{\phi}, \ddot{\phi}$, overdamped

Two basic inflation pictures

Cold Inflation



Warm Inflation

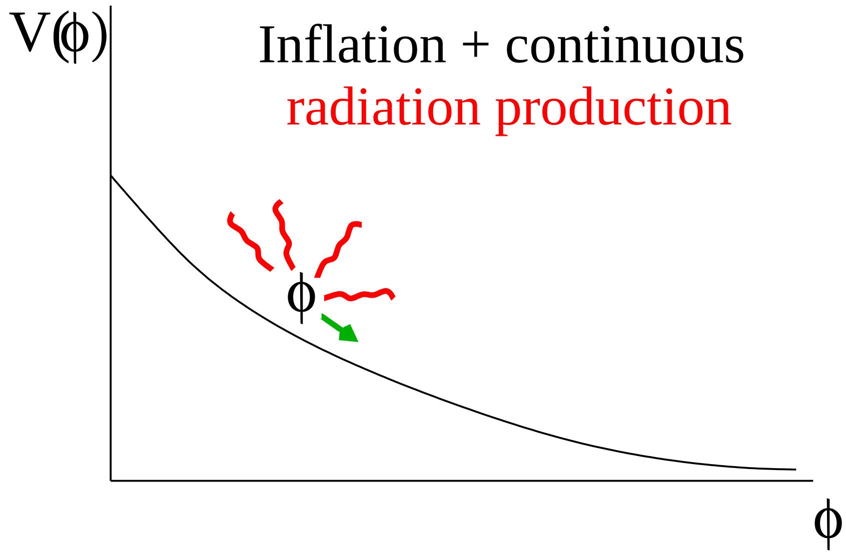


Warm inflation

(AB, PRL 75, 1995)

Stochastic Evolution equation includes includes dissipation and noise (AB and Fang, PRL74, 1912 (1995)):

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3\mathrm{H}\dot{\phi} + \Upsilon\dot{\phi} + \mathrm{V}'(\phi) = \xi$$

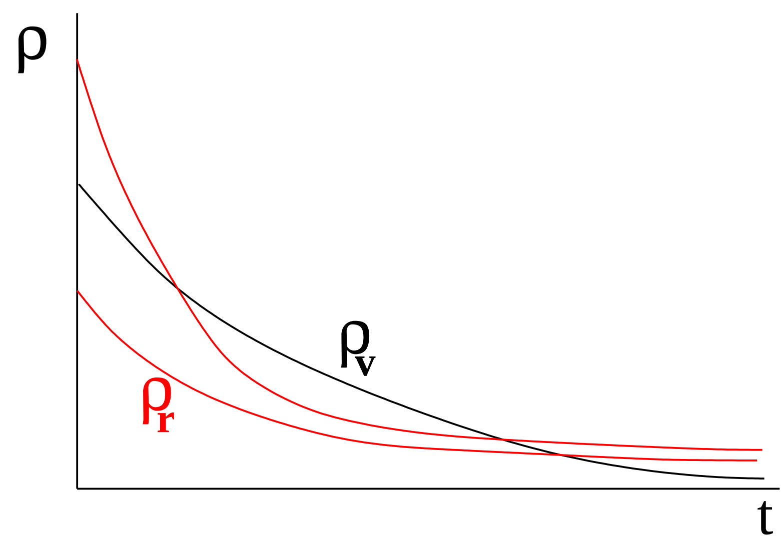


Dissipation term leads to radiation production during inflation,

$$\dot{\rho}_r = -4H\rho_r + \Upsilon\dot{\phi}^2$$

Density perturbations:

$$R(k) = \frac{H}{\dot{\phi}}\delta\phi, \qquad \delta\phi = \frac{k_F^{1/2}T^{1/2}}{2\pi}$$



Strong dissipative regime:

$$\Upsilon > 3H,\ T > H,\ k_F = \sqrt{\Upsilon H}$$

Weak dissipative regime:

$$\Upsilon < 3H,\ T > H,\ k_F = H$$

Energetics

- Consider GUT scale inflation: $M \sim 10^{15}\text{GeV}$

$$\Rightarrow \qquad \rho_v \sim M^4 \sim 10^{60}\text{GeV}^4 \quad \Rightarrow \quad \text{H} \sim 10^{10}\text{GeV}$$

  $\text{T} > \text{H}$ requires > 1 part in $\sim 10^{20}$ of $\rho_v \to \rho_r$
  (influences structure formation)
  $\text{T} > 1\text{GeV}$ requires > 1 part in $\sim 10^{60}$ of $\rho_v \to \rho_r$
  (makes reheating unnecessary)

- Inflation pictures
  1. Cold inflation: basic assumption is no dissipation
  2. Warm inflation: radiation production inherent (AB, PRL **75**, 3218 (1995))

- Theoretical consideration
  Equipartition Hypothesis of Statistical Mechanics: *Scalar field should distribute its energy evenly amongst all degrees of freedom.*

  Dynamical Question: *Will relevant time scales during inflation prohibit the minute' radiation production given above?*

Scalar Field (Φ) Dynamics

$$\mathcal{L}_\Phi = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{m_\phi^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4$$

$$\mathcal{L}_I = -\Phi \sum_{j=1}^{N_\psi} h_j \bar{\psi}_j \psi_j, \; -\frac{1}{2}\sum_{j=1}^{N_\chi} g_j^2 \Phi^2 \chi_j^2 + \cdots$$

- Decompose into background φ and fluctuations ϕ: $\Phi = \varphi + \phi$
- Effective equation of motion (EOM) for $\varphi(t)$:

$$\ddot{\varphi}(t) + 3H\dot{\varphi}(t) + \xi R \varphi(t) + m_\phi^2 \varphi(t) + \frac{\lambda}{6}\varphi^3(t) + \varphi(t)\sum_{j=1}^{N_\chi} g_j^2 \langle \chi_j^2 \rangle + \cdots = 0$$

with $\langle \phi^2 \rangle$, $\langle \phi^3 \rangle$ and $\langle \chi_j^2 \rangle$ etc... evaluated perturbatively

Two stage dissipative mechanism

(AB and R. Ramos, PRD **63**, 103509 (2001); I. G. Moss and C. Xiong, hep-ph/0603266; Bastero-Gil, AB, Ramos, 1008.1929 [hep-ph])

Basic Lagrangian - inflaton field coupled to heavy field ($> T$) which in turn coupled to light fields ($< T$)

Examples:

$\phi \to \chi \to \psi$ with $\quad m_\chi > 2m_\psi > m_\phi$

$$\mathcal{L}_I = -g_1^2\phi^\dagger\phi\chi^\dagger\chi - h_2[\chi^\dagger\bar{\psi}_\sigma P_R\psi_\sigma + \chi\bar{\psi}_\sigma P_L\psi_\sigma], \qquad \Upsilon = 0.11 g_1^4 h_2^4 \varphi^2 \frac{T^7}{m_{\psi_\chi}^2}$$

$\phi \to \chi \to y$ with $\quad m_\chi > 2m_\sigma > m_\phi$

$$\mathcal{L}_I = -g_1^2\phi^\dagger\phi\chi^\dagger\chi - h_1 M[\chi^\dagger\sigma^2 + \chi(\sigma^\dagger)^2], \qquad \Upsilon = 0.026 g_1^4 h_1^4 \varphi^2 \frac{T^3 M^4}{m_\chi^8}$$

$\phi \to \psi_\chi \to \psi_d, \sigma$ with $\quad m_\chi > m_{\psi_\sigma} + m_\sigma > m_\phi$

$$\mathcal{L}_I = -\frac{1}{\sqrt{2}} g_2 \varphi \bar{\psi}_\chi \psi_\chi - h_3[\sigma^\dagger\bar{\psi}_\chi P_R\psi_\sigma + \sigma\bar{\psi}_\chi P_L\psi_\sigma], \qquad \Upsilon = 0.0072 g_2^2 h_3^4 \frac{T^5}{m_{\psi_\chi}^4}$$

Calculating the dissipation coefficient

(AB+Ramos, PRD63, 103509 (2001))

$$\begin{aligned}\Upsilon &= 4g^4\varphi(t)^2\int_{-\infty}^{t}\int\frac{d^3\mathbf{p}}{(2\pi)^3}(t'-t)\mathrm{Im}[G_{\chi 11}(\mathbf{p},t,t')]^2_{t>t'}\\ &= 4ig^4\varphi(t)^2\int_{-\infty}^{t}\int\frac{d^3k}{(2pi)^3}\int\frac{d^3p}{(2\pi)^3}(t'-t)F(\mathbf{k},t,t')\rho(\mathbf{p}-\mathbf{k},t,t')\end{aligned}$$

Evolution equations:

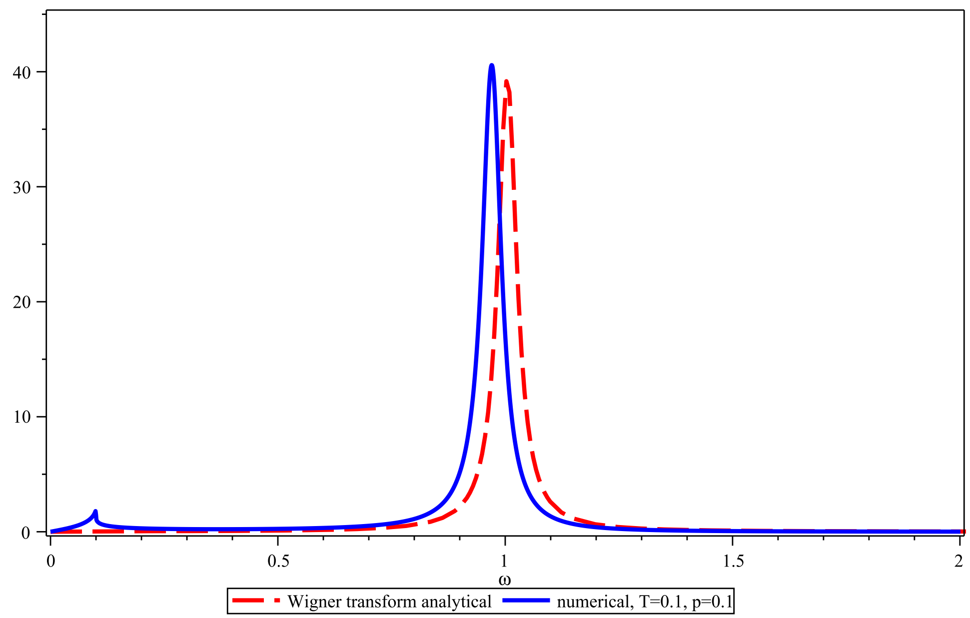
$$\begin{aligned}\left[\frac{\partial^2}{\partial t^2}+\mathbf{k}^2+M^2\right]\rho(\mathbf{k},t,t') &= i\int_{t'}^{t}dt''\Sigma_\rho(\mathbf{k},t,t'')\rho(\mathbf{k},t'',t')\\ \left[\frac{\partial^2}{\partial t^2}+\mathbf{k}^2+M^2\right]F(\mathbf{k},t,t') &= i\int_{t_0}^{t}dt''\Sigma_\rho(\mathbf{k},t,t'')F(\mathbf{k},t'',t')\\ &- i\int_{t_0}^{t'}dt''\Sigma_F(\mathbf{k},t,t'')\rho(\mathbf{k},t'',t')\end{aligned}$$

Spectral function

Equilibrium approximation:

$$\rho_j(p_0, \mathbf{p}) = \frac{4\omega_j(\mathbf{p})\Gamma_j(p_0, \mathbf{p})}{\left[-p_0^2 + \omega_j(\mathbf{p})^2\right]^2 + \left[2\omega_j(\mathbf{p})\Gamma_j(p_0, \mathbf{p})\right]^2},$$

$$F(\mathbf{p}, \omega) = \frac{1}{2}[1 + 2n(\omega)]\rho(\mathbf{p}, \omega)$$



Analytical: Moss+Xiong; Bastero-Gil,AB,Ramos

Numerical: AB, Moss, Ramos, in preparation

Physical picture of two stage dissip. mech.

(Morikawa+Sasaki '84; Berera+Ramos '05; Moss+Graham '08)

Number density: $\langle \hat{a}^\dagger(\mathbf{p_1},t)\hat{a}(\mathbf{p_2},t)\rangle = 2\omega_{\mathbf{p_1}}(2\pi)^3\delta(\mathbf{p_1}-\mathbf{p_2})n(\mathbf{p_1})$

$\hat{a}(\mathbf{p},t) = \omega_{\mathbf{p}}\hat{\sigma}(\mathbf{p},t) + i\hat{\pi}(-\mathbf{p},t) \qquad \hat{a}^\dagger(\mathbf{p},t) = \omega_{\mathbf{p}}\hat{\sigma}(-\mathbf{p},t) - i\hat{\pi}(\mathbf{p},t)$

In terms of Green's function:

$\langle \hat{\sigma}(\mathbf{p_1},t_1)\hat{\sigma}(\mathbf{p_2},t_2)\rangle = (2\pi)^3\delta(\mathbf{p_1}-\mathbf{p_2})G_{21}(\mathbf{p_1},t_1,t_2)$

$$\Rightarrow n(\mathbf{p},t) = \frac{1}{\omega_{\mathbf{p}}}(\omega_{\mathbf{p}} - i\partial_{t_1})(\omega_{\mathbf{p}} + i\partial_{t_2})G_{21}(\mathbf{p},t_1,t_2)$$

Particle production rate of radiation bath particles:

$$\dot{n}(\mathbf{p},t) = \frac{-i}{2\omega_{\mathbf{p}}}\left[(\partial_{t_1}^2+\omega_{\mathbf{p}}^2)(\omega_{\mathbf{p}}+i\partial_{t_2}) - (\omega_{\mathbf{p}}-i\partial_{t_1})(\partial_{t_2}^2+\omega_{\mathbf{p}}^2)\right]G_{21}(\mathbf{p},t_1,t_2)$$

Noting $\rho_r = \int \frac{d^3p}{(2\pi)^3}\omega_{\mathbf{p}} n(\mathbf{p},t)$ implies:

$$\Upsilon = \frac{\dot{\rho}_r}{\dot{\varphi}^2} = \frac{1}{\dot{\varphi}^2}\int \frac{d^3p}{(2\pi)^3}\left[\omega_{\mathbf{p}}\dot{n}(\mathbf{p},t)\right]$$

Interaction generic in inflation models

- $g^2\phi^2\chi^2$ generic to inflation models (for reheating)
- for $g \gtrsim 10^{-3}$ require SUSY for flat potential
- Minimal SUSY model with this interaction
  $W = \sqrt{\lambda}\Phi^3 + \frac{g}{\sqrt{2}}\Phi X^2 + \frac{h}{\sqrt{2}} X Y^2$

  ($\Phi = \phi + \theta\psi + \theta^2 F$, $X = \chi + \theta\psi_\chi + \theta^2 F_\chi$, $Y = y + \theta\psi_y + \theta^2 F_y$)

  $$\Longrightarrow \mathcal{L}_{int} \sim \frac{1}{4}g^2|\phi|^2|\chi|^2 + \frac{1}{4}g\phi\psi_\chi\psi + \frac{g}{2}\phi\bar{\psi}_\chi\psi_\chi + \frac{1}{\sqrt{2}}h\chi\bar{\psi}_y\psi_y + \cdots$$

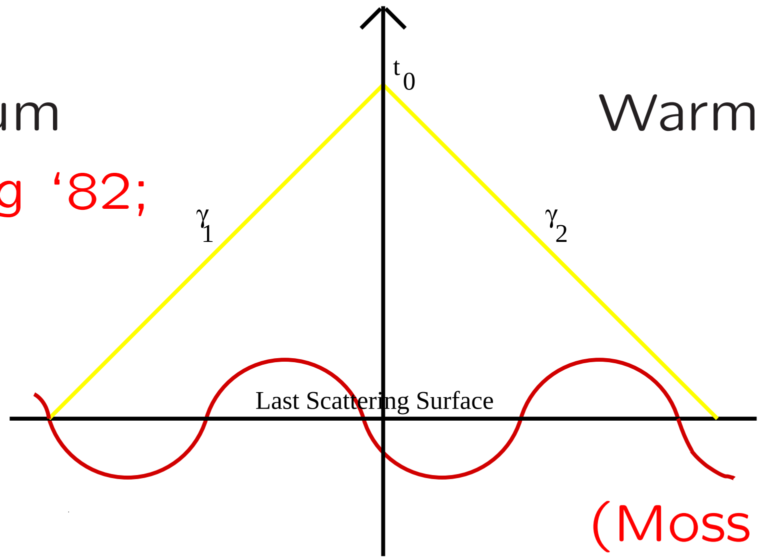
- For $\langle\phi\rangle \equiv \varphi \neq 0$ SUSY is broken with $m_{\chi 1} \gg m_{\psi_\chi} \gg m_{\chi 2}$
  ($V_{1-loop} \sim g\lambda\varphi^4 < V_{tree} \sim \lambda\varphi^4$, so flatness preserved)
- $\Longrightarrow$ dissipative mechanism through $\phi \rightarrow \chi \rightarrow \bar{\psi}_y + \psi_y, 2y, \ldots,$

  just like our toy model, so all results follow

Nature of the fluctuations

Thermal/quantum fluctuations leave "ripples" in early universe

- imprinted on CMB at Last Scattering
- amplified by gravity to make structure



Cold inflation: Quantum

(Guth, Pi '82; Hawking '82;

Starobinsky '82;

Bardeen *et. al.* '83)

$$\delta\phi = \frac{H}{2\pi}$$

Warm Inflation: Thermal

$$\delta\phi = \frac{k_F^{1/2} T^{1/2}}{2\pi}$$

$\Upsilon < 3H : k_F = H$

(Moss '85, AB,Fang '95)

$\Upsilon > 3H : k_F = \sqrt{\Upsilon H}$

(AB '99)

$$\frac{\delta\rho}{\rho} = \frac{H\delta\phi}{\dot{\phi}}\Big|_{\text{HORIZON EXIT}}$$

Linear order evolution of curvature perturbation

(Bastero-Gil,AB, Ramos, JCAP 1107, 030 (2011))

$$\ddot{y}_k + 3H(1+Q)\dot{y}_k + H^2\left[z^2 + 3\eta(1+Q) - 3mQ\frac{\dot{\phi}}{H\phi}\right]y_k = \left(\frac{k}{a}\right)^{3/2}\xi_k - 3QcH^2 w_k$$

$$\dot{w}_k + H(4-c)w_k = \frac{H}{3Q}z^2 u_k + 2\dot{y}_k$$

$$\dot{u}_k + 3H(1+z^2\bar{\zeta}_s)u_k = -3QH\left(\frac{w_k}{3} + y_k\right)$$

$z = k/(aH)$ y_k - inflaton fluctuation w_k - radiation fluctuation

u_k - radiation momentum fluctuation

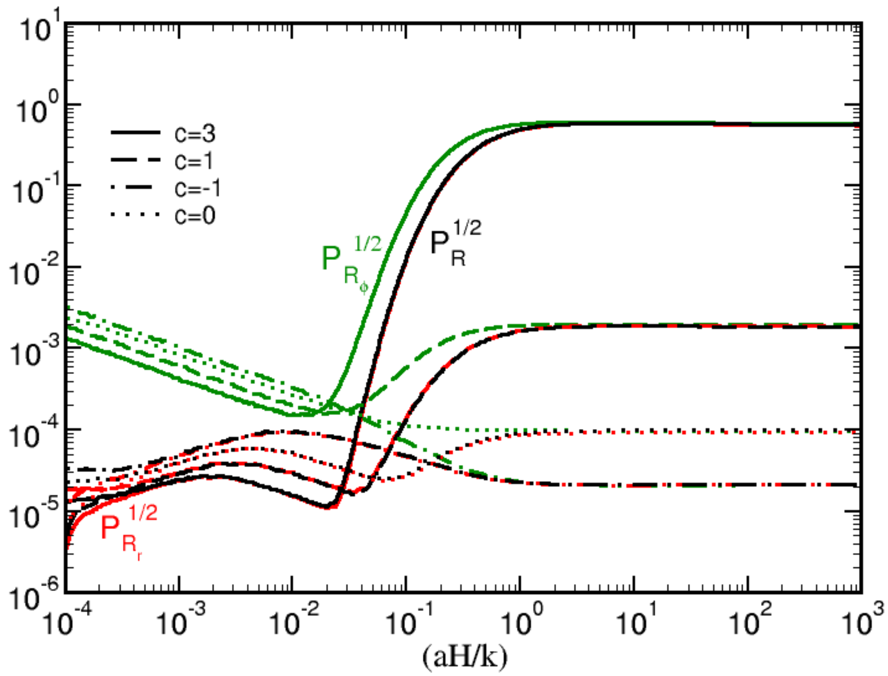
Dissipative coefficient : $Q = \frac{\Upsilon}{3H}$ $\Upsilon = C_\phi \frac{T^c}{\phi^{c-1}}$

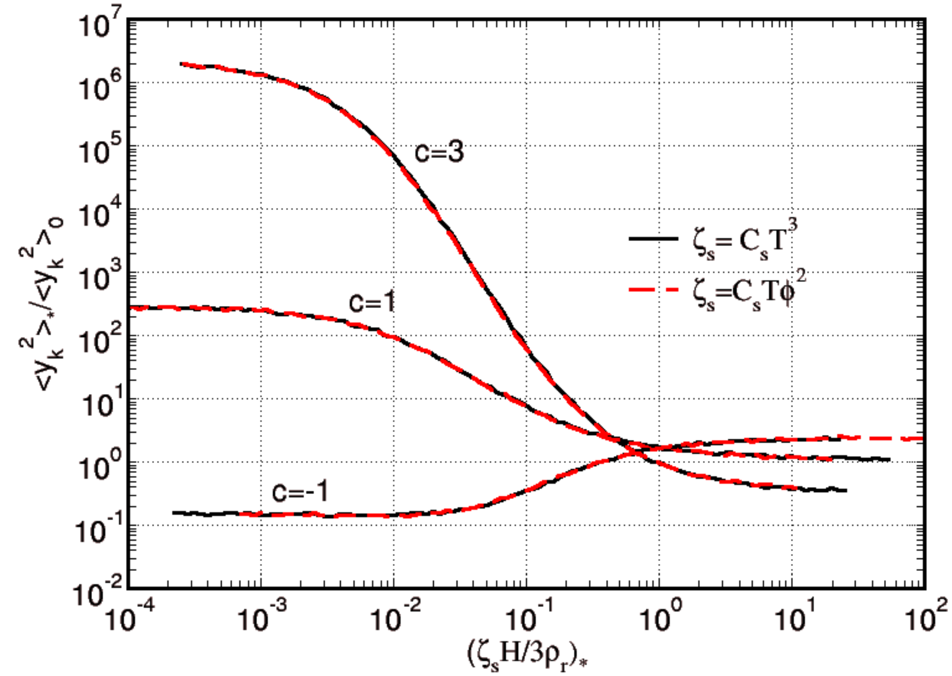
Shear viscosity : $\bar{\zeta}_s = \frac{4}{9}\frac{\zeta_s H}{\rho_r + p_r}$

Stochastic noise : $\langle \xi(t,x)\xi(t',x')\rangle = \delta(t-t')\delta^{(3)}(x-x')$

Effect of shear viscosity

(Bastero-Gil,AB, Ramos, JCAP 1107, 030 (2011))





Here $\quad P_{\mathcal{R}} \simeq \left(\frac{H}{\dot\phi}\right)^2 \frac{(H+\Upsilon)T}{\pi^2}\langle y_k^2\rangle_*$

$$\Upsilon \simeq 0.1 h^4 \mathcal{N}_\chi \mathcal{N}_\sigma^2 \frac{T^3}{\phi^2} \qquad \zeta_s \simeq 127 N_\sigma (1+0.03h)\frac{T^3}{h^4}$$

Condition for no growing mode: $h^4 \lesssim 128 \frac{\mathcal{N}_\sigma}{g_*}\frac{H}{T} \simeq 69\frac{H}{T}$

Bulk viscosity tends to be small

Worked example - no η-problem

Model: $V = \frac{m^2}{2}\varphi^2$

$$\Longrightarrow N_e \approx 2\sqrt{2}\frac{\varphi_0}{m}\frac{\Upsilon}{m_P},\ T \approx \frac{m^{3/4} m_P^{1/4} \varphi_0^{1/4}}{\Upsilon^{1/4}},\ \frac{\delta\rho}{\rho} \approx \left(\frac{\varphi_0}{m}\right)^{3/8}\left(\frac{\Upsilon}{m_P}\right)^{9/8}$$

For $N_e = 60$, $\frac{\delta\rho}{\rho} = 10^{-5} \Longrightarrow \frac{\varphi_0}{m} \approx 6 \times 10^8$, $\frac{\Upsilon}{m_P} \approx 4 \times 10^{-8}$

e.g., $m = 10^9\,\mathrm{GeV} \Longrightarrow \frac{H}{m} \approx 0.17$, $\frac{\varphi_0}{m_P} \approx 6 \times 10^{-2}$, $T \approx 10^4 m$

- No η-problem: $m > H$
- No φ amplitude problem: $\varphi < m_P$
- No graceful exit problem: inflation $\rightarrow$ RD automatic
- No quantum-to-classical trans. problem: $\delta\varphi$ classical

Non-gaussianity

Power spectrum: $\langle R(\mathbf{k_1})R(\mathbf{k_2})\rangle = (2\pi)^3\delta(\mathbf{k_1}+\mathbf{k_2})P_R(k_1)$

Bispectrum:

$\langle R(\mathbf{k_1})R(\mathbf{k_2})R(\mathbf{k_3})\rangle = (2\pi)^3\delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_R(k_1,k_2,k_3)$

e.g. local model: $B_R(k_1,k_2,k_3) = -\frac{6}{5}f_{NL}\left(P_R(k_1)P_R(k_2) + perm.\right)$

f_{NL} - Nonlinearity parameter

WMAP bounds:

$-10 < f_{NL}^{local} < 74 \ \ (k_3 \ll k_1 \sim k_2); \ \ -214 < f_{NL}^{equil} < 266 \ \ (k_3 \sim k_1 \sim k_2)$

Planck $(f_{NL} \sim O(10))$

standard inflation models $f_{NL} \sim 0(\epsilon,\eta)$ -

- Single field inflation
- canonical kinetic terms
- slow-roll
- initial vacuum state

(Maldacena '03, Acquaviva, *et al.*, '03)

Non-gaussianity in warm inflation

(Gupta, *et al.*, PRD66, 043510 (2002); Moss and Xiong, JCAP 0704, 007 (2007))

$$\ddot{\Phi}(x) + (3H + \Upsilon)\dot{\Phi}(x) + \Upsilon a^{-2} v_\alpha \partial_\alpha \Phi(x) - a^{-2}\partial^2\Phi(x) + V_\Phi = \xi(x)$$

2nd order term

V - scalar velocity perturbation

Bispectrum

$$B_r^v(k_1, k_2, k_3) \approx 18L(Q) \sum_{cyclic} P_R(k_1)P_R(k_2)\left(\frac{1}{k_1^2} + \frac{1}{k_2^2}\right) k_1 k_2$$

$$\Rightarrow \quad L(Q) \equiv \ln(1 + Q/14)$$

Relation for f_{NL}:

$$-15L(Q) < f_{NL}^v < \frac{33}{2} L(Q)$$

$$Q \sim 100 \Rightarrow |f_{NL}| \sim O(30)$$

Summary of WMAP data

WMAP 7 year data and angular power spectra:

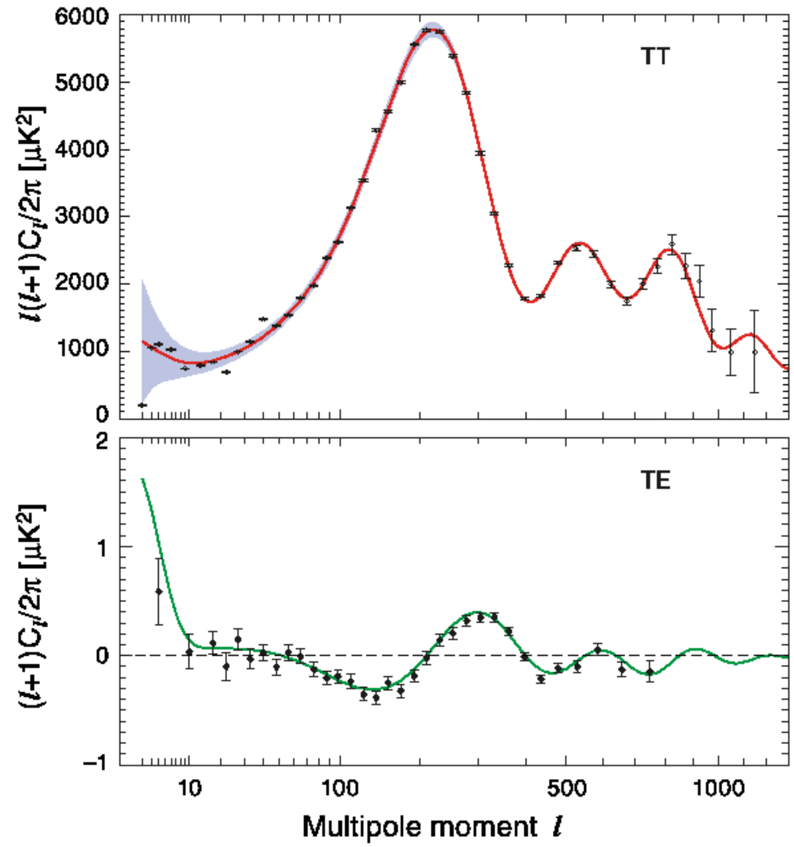
Scalar amplitude: $A_S = (2.43 \pm 0.11)^{-9}$

Scalar spectral index: $n_S = 0.963 \pm 0.014$

Running of spectral index:
$dn_S/d\ln k = -0.034 \pm 0.026$

Tensor to scalar ratio: $r < 0.36(95\%CL)$

Nongaussianty: $f_{NL}^{loc} = 32 \pm 21$ (68% CL)
Consistent with Gaussian within 95% CL

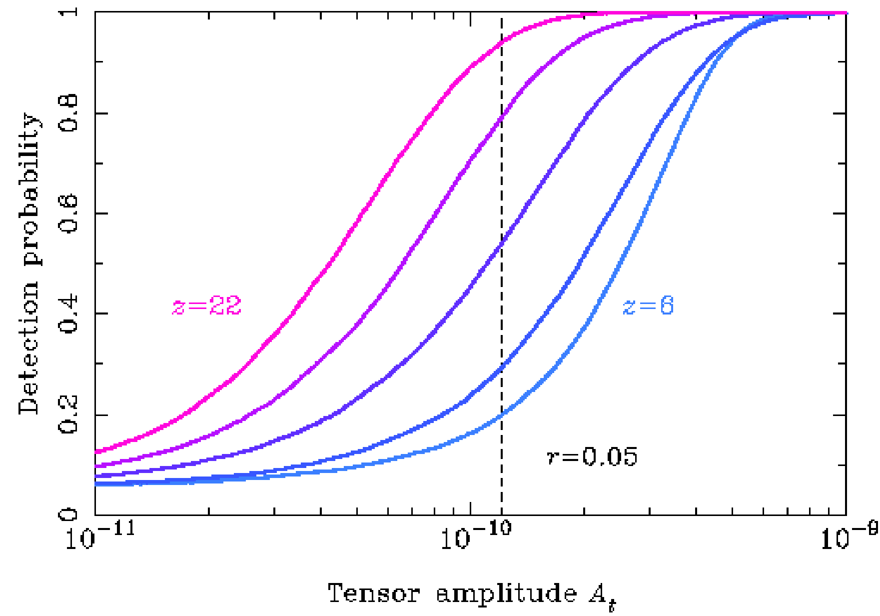


Yadav and Wandelt, 2008 found in WMAP 3 year data
$27 < f_{NL}^{loc} < 147$ $(95\%\ CL)$ with rejection of $f_{NL}^{loc} = 0$ at 2.8σ

Parameter forecast for Planck

PARAMETER FORECASTS FOR WMAP AND PLANCK

Parameter	Input Value	June'03	June'03 +2dF	*WMAP*$_4$	*Planck*	*WMAP*$_4$ ACT/SPT
Flat+weak priors						
ω_b	0.2240	0.00095	0.00090	0.00047	0.00017	0.00025
ω_c	0.1180	0.011	0.007	0.0039	0.0016	0.0035
n_S	0.9570	0.026	0.024	0.0125	0.0045	0.0080
τ	0.108	0.059	0.056	0.020	0.005	0.021
+running						
ω_b	0.2240	0.00162	0.00090	0.00047	0.00017	0.00025
ω_c	0.1180	0.0158	0.007	0.0039	0.0016	0.0035
$n_S(k_n)$	0.9570	0.055	0.024	0.0125	0.0045	0.0080
n_{run}	0.0	0.033	0.029	0.025	0.005	0.0092
τ	0.108	0.112	0.074	0.019	0.006	0.0266



ESA Planck Bluebook, 2005

Experiments	f_{NL} (Bispectrum)	f_{NL} (Skewness)
COBE	600	800
WMAP	20	80
Planck	5	70
Ideal	3	60

Bartolo, *et al.*, 2005

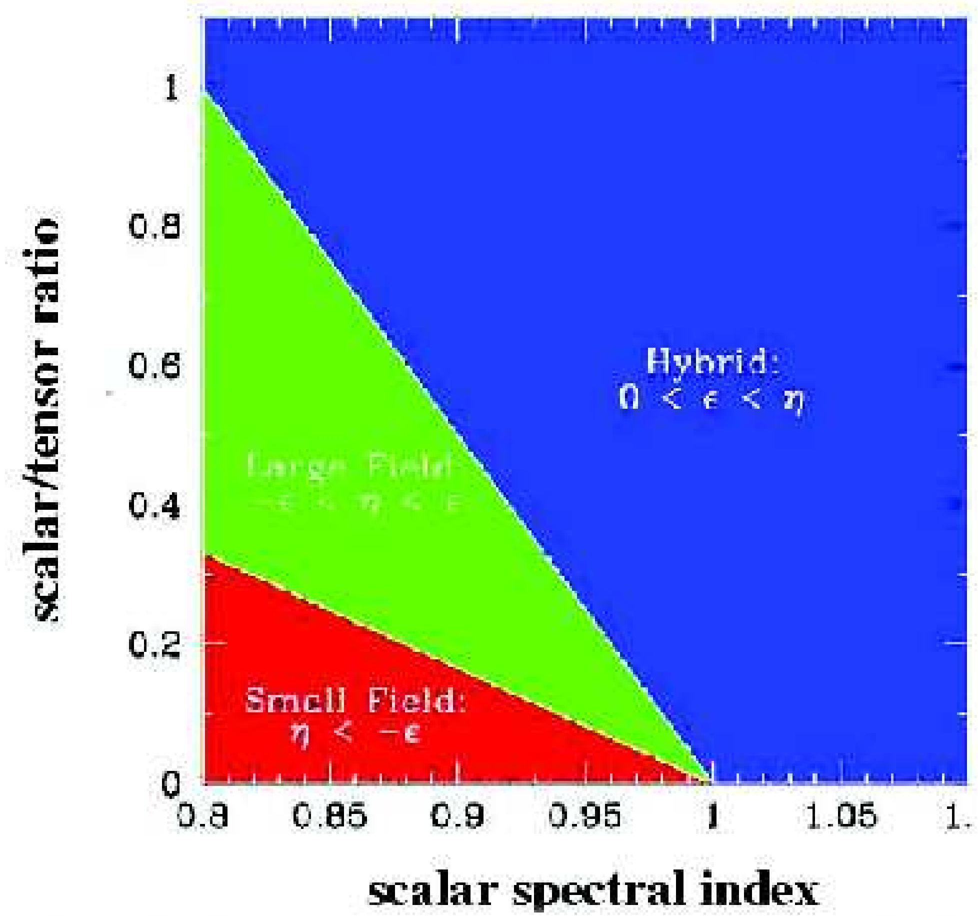
Error on scalar index reduced to half a percent

Tensor-scalar ratio detectable down to ~ 0.05 (optimistic)

If no detection of r in Planck $\Rightarrow$ low energy scale $V^{1/4}$ of inflation,

$V^{1/4} = 3.3 \times 10^{16} r^{1/4}$GeV

Standard inflation models - predictions



Kinney, *et al.*, 2001

Slow roll parameters:

$$\epsilon \equiv \frac{m_{pl}^2}{16\pi}\left(\frac{V'}{V}\right) \ll 1$$

$$\eta \equiv \frac{m_{pl}^2}{8\pi}\left(\frac{V''}{V}\right) \ll 1$$

$f_{NL} \lesssim 1$ in all models

$p = 4$ ruled out by WMAP data

If $r < 0.1$ is found by Planck, monomial cold inflation models ruled out

If $f_{NL} \gtrsim 5$ is found by Planck all these simplest of inflation models would be ruled out

Warm inflation models

(AB and Ramos, PLB 607, 1 (2005))

Superpotential:

$$W = W(\Phi) + g\Phi X^2 + hXY^2$$

Dissipative coefficient:

$$\Upsilon \approx C_\phi \frac{T^3}{\phi^2}$$

$$C_\phi \equiv 0.64 h^4 N_\chi N_{decay}^2$$

Slow-roll:

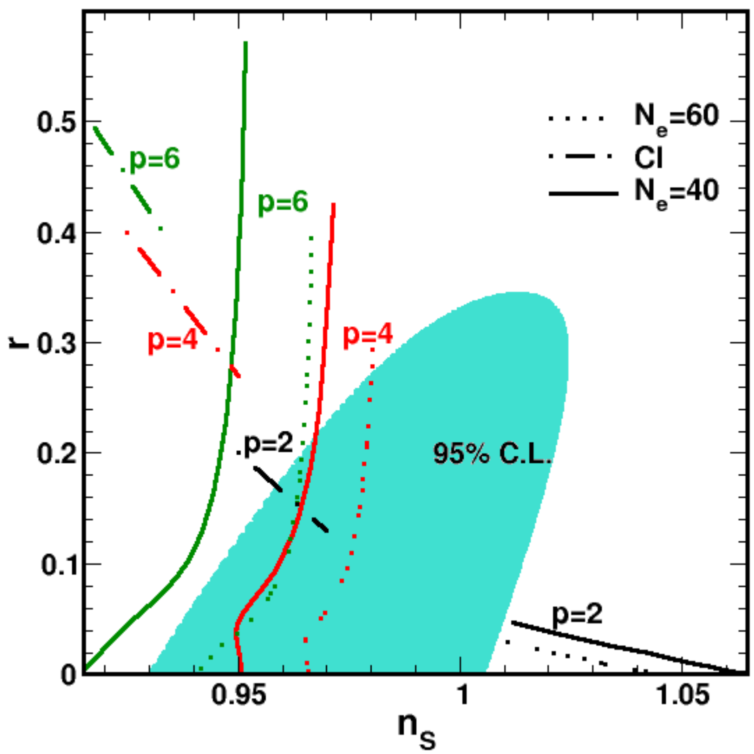
$$\dot{\phi} \approx -\frac{V_\phi}{[3H(1+Q)]} \qquad Q \equiv \Upsilon/(3H)$$

$$4\rho_R \approx 3Q\dot{\phi}^2 \qquad \rho_R = \pi^2 g_* T^4/30$$

Warm inflation models - monomial potential

(Bastero-Gil and AB, Int. J. Mod. Phys A24, 2207 (2009))

$$V = V_0 \left[\frac{\phi}{m_p}\right]^p$$



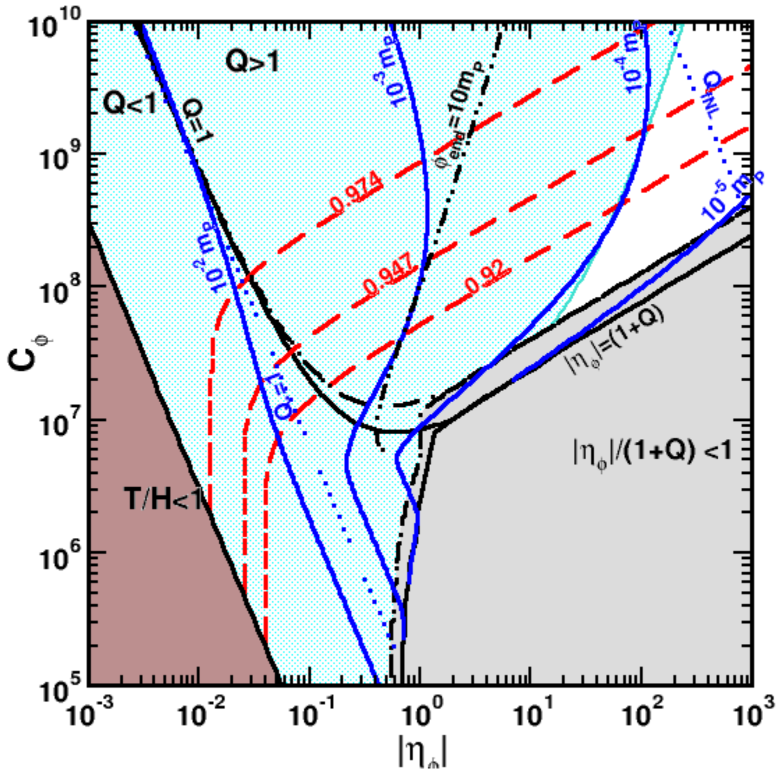
- $\frac{d(T/H)}{dN_e} > 0$
- Weak DR $\longrightarrow$ Strong DR
- Solves "eta" - problem, $m_\phi > H$
- Solves large ϕ amplitude problem - $\phi < m_P$
- $C_\phi \equiv 0.16 N_\chi N_{decay}^2$ $\sim 10^6 - 10^8$

$T/H > 1$ $\qquad$ $f_{NL} < 110$

Warm inflation models - hilltop

(Sanchez, *et al.*, PRD77, 123527 (2008))

$$V = V_0 \left[1 - \gamma \left(\frac{\phi}{M_p} \right)^2 \right] + \cdots$$



- $\frac{d(T/H)}{dN_e} > 0$
- Strong DR $\longrightarrow$ Weak DR
- n_S red-tilted
- solid blue lines - max value of V
  $r < 0.22$ for $V < 10^{-2} m_P$
- WI $\longrightarrow$ kination $\longrightarrow$ RD

Stringy warm inflation

(AB, Kephart, PRL 83, 1084 (1999))

- Recurrent problem in embedding inflation in string models is the "eta" - problem, i.e. quantum corrections and SUGRA contributions to inflaton potential ruin required flatness Warm inflation solution is large dissipation $\Upsilon \gg H \Rightarrow V'' \gg H^2$, i.e. much bigger than scale for SUGRA corrections.

- Necessary ingredient for warm inflation is large number of fields - naturally available in string theory, i.e. moduli fields, branes, Kaluza-Klein modes come in the hundreds of thousands.

- Example: trapped warm inflation (Bastero-Gil, *et al.*, 0904.2195) - scalar inflaton field trapped in decaying oscillation about ESP, coupling to other fields leads to warm inflation.

- Example: brane-antibrane warm inflation (Bastero-Gil, *et al.*, 1103.5623) - D$\bar{\text{D}}$-branes with intersecting branes introducing light matter.

Refining the theory

- Finite temperature effective potential in SUSY models
- Origin of dissipation from more general non-equilibrium derivation
- Higher loops, resummations etc....
- Apply to more models

Summary of warm inflation

- Treats dynamical effects of inflaton interacting with other fields during inflation

- Model Building
  - Can have inflation models with $m_\phi > H$
  - Particle physics during inflation phase, eg. magnetic fields baryogenesis...

- Observational implications
  - Running spectral index in simple models
  - Blue and red spectra are possible
  - Non-gaussianity at strong dissipation $f_{NL} \sim O(10)$
  - low tensor-scalar ratio $r \ll 0.1$

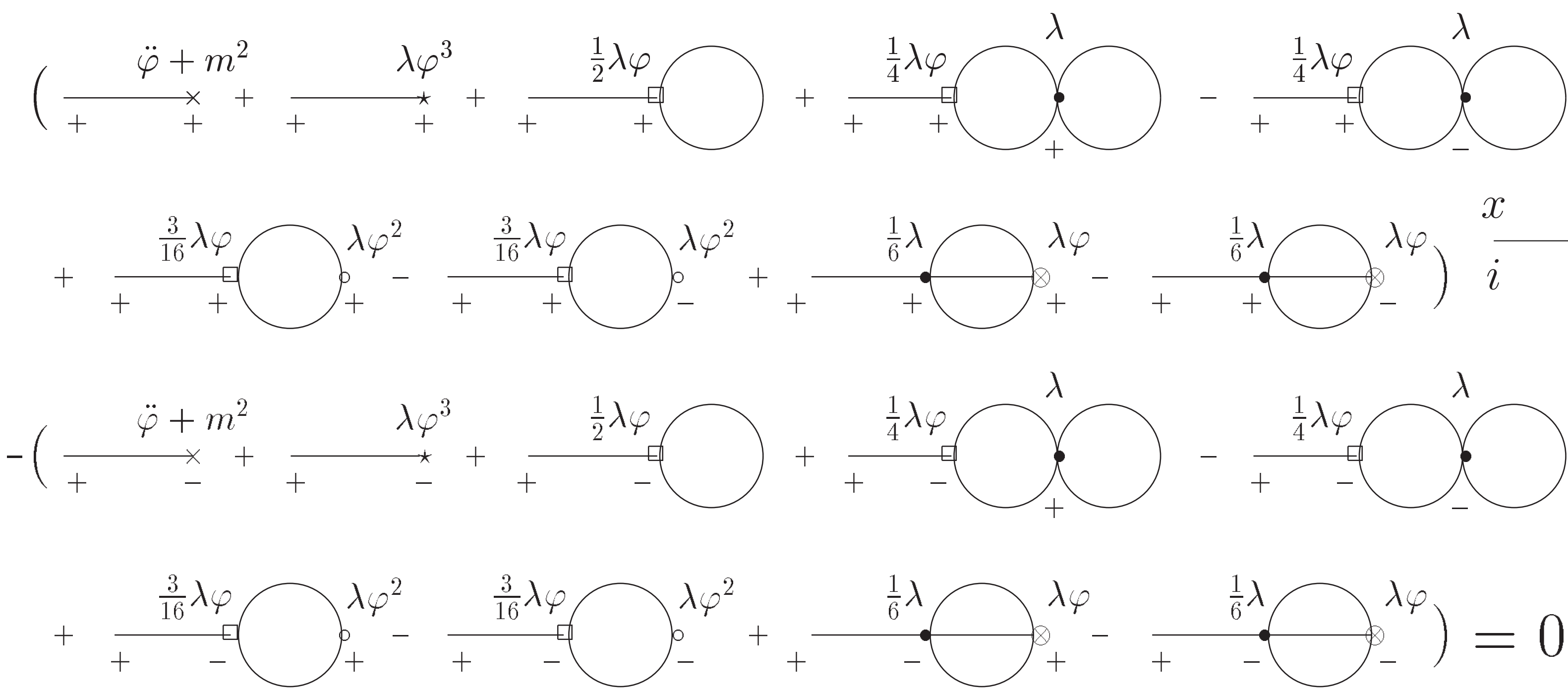
How to obtain the φ-effective EOM

Tadpole Method: demand $\langle\phi\rangle = 0$ and compute

Example: $S = \int d^4x \left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{m^2}{2}\Phi^2 - \frac{\lambda}{4!}\Phi^4\right]$, $\quad \Phi = \varphi + \phi \Longrightarrow$

$$S = \int d^4x[-\varphi\frac{1}{2}[\Box + m^2]\varphi - \phi\frac{1}{2}[\Box + m^2]\phi - \phi[\Box + m^2]\varphi - \frac{\lambda}{4!}(\varphi^4 + 4\varphi^3\phi + 6\varphi^2\phi^2 + 4\varphi\phi^3 +$$

$\langle\phi\rangle = 0$ at lowest nontrivial order $\Longrightarrow$



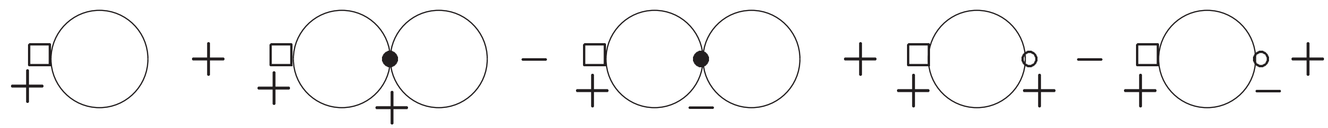
$x \quad x'$, $i \quad j$: $\equiv G^{0ij}_\phi(x, x')$

$i, j = \pm$

Tadpole Method (cont.) - Explicit EOM

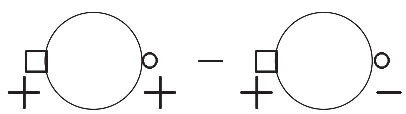
( ) =0 $\Longrightarrow$

$0 = \int d^4x' G_\phi^{++}(x,x')$ [$\times$ + $\star$ +



$\Longrightarrow$ [ ] = 0, now convert to analytic expression

e.g.



$$= \tfrac{3}{16}\lambda^2\varphi(t')\int d^4y[G_\phi^{++}(x',y)G_\phi^{++}(x',y) - G_\phi^{+-}(x',y)G_\phi^{+-}(x',y)]\varphi^2(t_y)$$

$$= \tfrac{3}{8}\lambda^2\varphi(t')\int d^4y Im[G_\phi^{++}(x',y)G_\phi^{++}(x',y)]\varphi^2(t_y), \quad \text{etc} \ldots$$

Final Expression:

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi + \frac{\lambda}{6}\varphi^3 + \overset{local\ \ 1-loop\ \ V_{eff}}{terms} + \frac{3}{8}\lambda^2\varphi(t')\int d^4y Im[G_\phi^{++}(x',y)G_\phi^{++}(x',y)]\varphi^2(t_y)$$

$$+ \frac{1}{3}\lambda^2\int d^4y Im[G_\phi^{++}(x',y)G_\phi^{++}(x',y)G_\phi^{++}(x',y)]\varphi(t_y) = 0$$

Closed Time Path approach - Goal

Compute Observable

$$\langle \hat{O}(t) \rangle \equiv \frac{\text{Tr}(\hat{\rho}(t)\hat{O})}{\text{Tr}(\hat{\rho}(t))}$$

Thermal initial state at $T^<$:

$$\rho(T^<) = \exp(-\beta H) = U(T^< - i\beta, T^<)$$

($U(t,t') \equiv \exp[-iH(t-t')]$, time evolution operator)

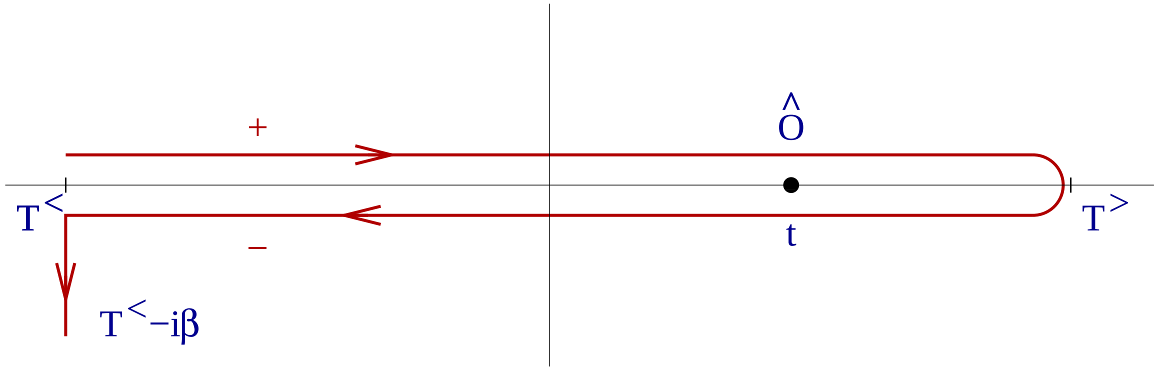
Thus

$$\langle \hat{O}(t) \rangle = \frac{\text{Tr}[U(T^< - i\beta, T^<)\, U(T^<, t)\, \hat{O}\, U(t, T^<)]}{\text{Tr}[U(T^< - i\beta, T^<)]}$$

Also add large positive time $T^>$

$$\langle \hat{O}(t) \rangle = \frac{\text{Tr}[U(T^< - i\beta, T^<)\, U(T^<, T^>)\, U(T^>, t)\, \hat{O}\, U(t, T^<)]}{\text{Tr}[U(T^< - i\beta, T^<)]}$$

Closed Time Path approach - Method



Can express as a path integral

recall $\quad U(t,t') \equiv \int \mathcal{D}\Phi \, \exp\left(i\int_{t'}^{t} d^4x \mathcal{L}[\Phi]\right)$

$$Z[J^+,J^-,J^\beta] \;=\; \mathrm{Tr}[U(T^< - i\beta, T^<; J^\beta)U(T^<, T^>; J^-)U(T^>, T^<; J^+)]$$

$$= \int \mathcal{D}\Phi^+\mathcal{D}\Phi^-\mathcal{D}\Phi^\beta \exp\left(i\int_{T^<}^{T^>} d^4x[\mathcal{L}^{J^+}[\Phi^+] - \mathcal{L}^{J^-}[\Phi^-]] + i\int_{T^<}^{T^<-i\beta} d^4x\mathcal{L}^{J^\beta}[\Phi^\beta]\right)$$

e.g. scalar field theory:

$$\mathcal{L}^J[\Phi] = \frac{1}{2}[\partial_\mu\Phi\partial^\mu\Phi - m^2\Phi^2] - \frac{\lambda}{4!}\Phi^4 + J\Phi$$

Green's function $G_\phi^{ij}(x,x')$ - Basic Properties

$$G_\phi(x,x') = \begin{pmatrix} G_\phi^{++}(x,x') & G_\phi^{+-}(x,x') \\ G_\phi^{-+}(x,x') & G_\phi^{--}(x,x') \end{pmatrix} = \begin{pmatrix} i\langle T_+\phi(x)\phi(x')\rangle & i\langle \phi(x')\phi(x)\rangle \\ i\langle \phi(x)\phi(x')\rangle & i\langle T_-\phi(x)\phi(x')\rangle \end{pmatrix}$$

Fourier space: $G_\phi(x,x') = i\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}.(\mathbf{x}-\mathbf{x}')}\tilde{G}_\phi(\mathbf{q},t-t')$

$$\begin{aligned}
\tilde{G}_\phi^{++}(\mathbf{q},t-t') &= \tilde{G}_\phi^{>}(\mathbf{q},t-t')\theta(t-t') + G_\phi^{<}(\mathbf{q},t-t')\theta(t'-t), \\
\tilde{G}_\phi^{--}(\mathbf{q},t-t') &= \tilde{G}_\phi^{>}(\mathbf{q},t-t')\theta(t'-t) + G_\phi^{<}(\mathbf{q},t-t')\theta(t-t'), \\
\tilde{G}_\phi^{+-}(\mathbf{q},t-t') &= \tilde{G}_\phi^{<}(\mathbf{q},t-t'), \\
\tilde{G}_\phi^{-+}(\mathbf{q},t-t') &= \tilde{G}_\phi^{>}(\mathbf{q},t-t')\,.
\end{aligned}$$

Hermiticity: $\tilde{G}_\phi^{>*}(\mathbf{q},t-t') = \tilde{G}_\phi^{>}(\mathbf{q},t'-t)$

Continuity: $\frac{d}{dt}\left[\tilde{G}^{>}(\mathbf{q},t-t') - \tilde{G}_\phi^{>}(\mathbf{q},t'-t)\right]|_{t=t'} = i\delta(t-t')$

Green's Function - equilibrium approximation

Gives lower bound estimate of dissipative effects

(Moss and Xiong, hep-ph/0603266)

Low temperature regime ($m_\chi < T$):

$$G_{\text{equil}}(\mathbf{k},t) = \frac{i}{2(\omega_{\mathbf{k}} - i\Gamma_\chi)} \exp[-i(\omega_{\mathbf{k}} - i\Gamma_\chi)t] + f(\omega_{\mathbf{k}} - i\Gamma_\chi, t) - f(\omega_{\mathbf{k}} + i\Gamma_\chi, t)$$

where $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m_\chi^2}$,

$$f(\omega, |\mathbf{k}|, t) = \frac{\exp(i\omega t)}{4\pi\omega} E_1(i(|\mathbf{k}| + \omega)t) - \frac{\exp(-i\omega t)}{4\pi\omega} E_1(i(|\mathbf{k}| - \omega)t)$$

At small time, i.e. $t \sim \tau_\chi = \Gamma_\chi^{-1} \log \frac{m_\chi^2}{\Gamma_\chi^2}$, the behavior same as the exponential decay approximation.

At larger time, power-law decay behavior.

Leads to dissipative coefficient:

$$\Upsilon_{\text{equil}}(\varphi, T) = 4 \times 10^{-2} g^2 h^4 \left(\frac{g\phi}{m_\chi}\right)^4 \frac{T^3}{m_\chi^2}$$

Green's Function - equil. approx. (cont)

(Hosoya and Sakagami, PRD **29**, 2228 (1984);
Berera, Ramos, Gleiser, PRD **58**, 123508 (1998))

High Temperature limit:

$$\Upsilon \approx \frac{132}{\pi T}\varphi^2 N_\chi \ln\left(\frac{2T}{2\mu(T)}\right)$$

where thermal mass

$$\mu(T) = \frac{gT}{\sqrt{12}}$$

Green's functions relations

(AB, Moss and Ramos, in preparation 2007)

One-loop effective equation of motion:

$$\left[\Box + M_\phi^2\right] \varphi_c(x) + \frac{\lambda}{3!}\varphi_c^3(x) - \frac{\varphi_c(x)}{2}\int d^3x' \varphi_c^2(\mathbf{x}',t)\mathcal{D}_1(\mathbf{x}-\mathbf{x}',0) - \int d^3x' \varphi_c(\mathbf{x}',t)\mathcal{D}_2(\mathbf{x}-\mathbf{x}',0$$
$$+\varphi_c(x)\int d^3x' \int_{-\infty}^{t} dt' \varphi_c(\mathbf{x}',t')\dot{\varphi}_c(\mathbf{x}',t')\mathcal{D}_1(\mathbf{x}-\mathbf{x}',t-t') + \int d^3x' \int_{-\infty}^{t} dt' \dot{\varphi}_c(\mathbf{x}',t')\mathcal{D}_2(\mathbf{x}-\mathbf{x}',t$$
$$= \varphi_c(x)\xi_1(x) + \xi_2(x)$$

where defining $\mathcal{C}_i(\mathbf{x}-\mathbf{x}',t-t') = -\frac{\partial}{\partial t'}\mathcal{D}_i(\mathbf{x}-\mathbf{x}',t-t')$

$$\mathcal{C}_1(\mathbf{x}-\mathbf{x}',t-t') = \lambda^2 \text{Im}\left[G_\phi^{++}(x,x')\right]^2 \text{sgn}(t-t') + 4g^4 \text{Im}\left[G_{\chi_j}^{++}(x,x')\right]^2 \text{sgn}(t-t')$$

$$\mathcal{C}_2(\mathbf{x}-\mathbf{x}',t-t') = \frac{\lambda^2}{3}\text{Im}\left[G_\phi^{++}(x,x')\right]^3 \text{sgn}(t-t') + 4g^4 \text{Im}\left[G_\chi^{++}(x,x')G_\phi^{++}(x,x')G_\chi^{++}(x,x')\right] \text{sg}$$

$$\langle\xi_1(x)\xi_1(x')\rangle \equiv \mathcal{N}_1(\mathbf{x}-\mathbf{x}',t-t') = \frac{\lambda^2}{2}\text{Re}\left[G_\phi^{++}(x,x')\right]^2 + 2g^4 \text{Re}\left[G_\chi^{++}(x,x')\right]^2$$

$$\langle\xi_2(x)\xi_2(x')\rangle \equiv \mathcal{N}_2(\mathbf{x}-\mathbf{x}',t-t') = \frac{\lambda^2}{6}\text{Re}\left[G_\phi^{++}(x,x')\right]^3 + 2g^4 \text{Re}\left[G_\chi^{++}(x,x')^2 G_\phi^{++}(x,x')\right]$$

Fluctuation dissipation theorem - relation

Relation betweem real and imaginary parts of Green's function leads to generalized fluctuation-dissipation theorem (in Fourier space):

$$\tilde{\mathcal{N}}_1(\mathbf{p},\omega) = 2\omega\left[n(\omega)+\frac{1}{2}\right]\tilde{\Gamma}_1(\mathbf{p},\omega)$$

where $\tilde{\mathcal{D}}_i(\mathbf{p},\omega) = 2\tilde{\Gamma}_i(\mathbf{p},\omega)$

Relates dissipation and noise

In local form, i.e. when $\mathcal{N}(t-t') \equiv \mathcal{N}_0\delta(t-t')$, get familiar expression:

$$N_0 = \tilde{\mathcal{N}}(0) = 2T\tilde{\Gamma}(0)$$

Local limit of noise and dissipation

(AB, Moss and Ramos, in preparation 2007)

$f(t)$ *slowly varying on timescale* τ if Fourier transform $\tilde{f}(\omega)$ satisfies

$$\tilde{f}(\omega) = 0 \quad \text{for} \quad \omega > 2\pi/\tau$$

kernel function $\mathcal{K}(t)$ described as *localized on a timescale* τ *with accuracy* $1-\epsilon$ if Fourier transform $\tilde{\mathcal{K}}(\omega)$ satisfies

$$\left|\frac{\tilde{\mathcal{K}}(\omega) - \tilde{\mathcal{K}}(0)}{\tilde{\mathcal{K}}(0)}\right| < \epsilon \quad \text{for} \quad \omega < 2\pi/\tau$$

$\Rightarrow$ $f(t)$ slowly varying on a timescale τ and $\mathcal{K}(t)$ localized on a timescale τ,

$$\int_{-\infty}^{\infty} K(t-t')f(t')dt' = \tilde{\mathcal{K}}(0)f(t) + \mathcal{O}(\epsilon)$$

$\Rightarrow \mathcal{K}(t) \approx \tilde{\mathcal{K}}(0)\delta(t-t')$

(Note: if $\tilde{\mathcal{K}}(\omega)$ analytic in $\omega \Rightarrow$ kernel can be localised and kernel admits a local derivative expansion. In general, derivative expansion might not exist, even when the kernel is localized in above sense.)

Local limit of one-loop kernels

Homogeneous field $\varphi_c(x) = \varphi_c(t)$

$$\tilde{\mathcal{N}}_1(\omega) \simeq g^4 \left(e^{\beta\omega} + 1\right) \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} n(\omega') n(\omega - \omega') \tilde{\rho}_\chi(\mathbf{k}, \omega') \tilde{\rho}_\chi(\mathbf{k}, \omega - \omega')$$

$$\equiv g^4 h^4 \frac{\mathcal{M}^4 T^4}{m_\chi^8} F_1(\beta\omega)$$

$x \equiv \omega/T$

Observational tests of inflation

- Spectra of energy density fluctuations (scalar spectra)
- Spectra of gravitational waves (tensor spectra)
- Non-gaussian deviations
- Isocurvature fluctuations
- Present day cosmological constant
- Particle Spectra

EXPERIMENTS

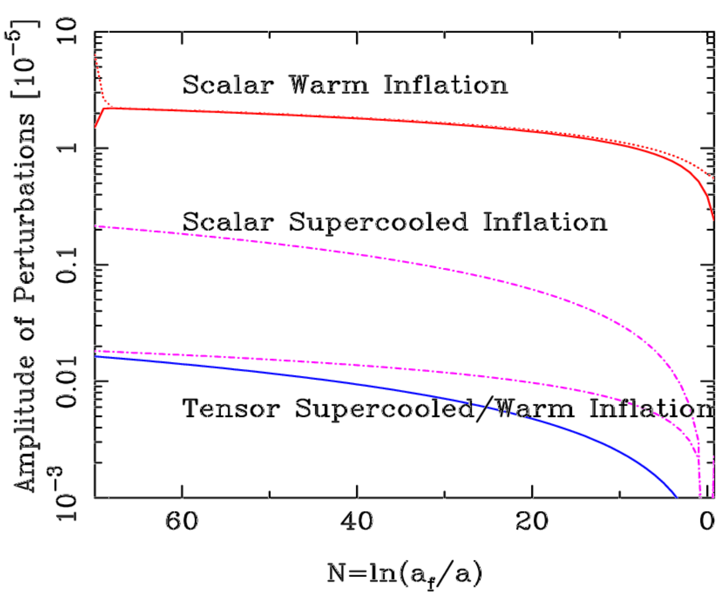
- CMB - COBE, MAP, Planck, Boomerang, Maxima
- Redshift surveys - Sloan, 2df ...
- Big Bang Nucleosynthesis
- Supernovae 1A
- Hubble Space Telescope

Signatures for inflation dynamics

(Taylor and AB, PRD **62**, 083517 (2000))

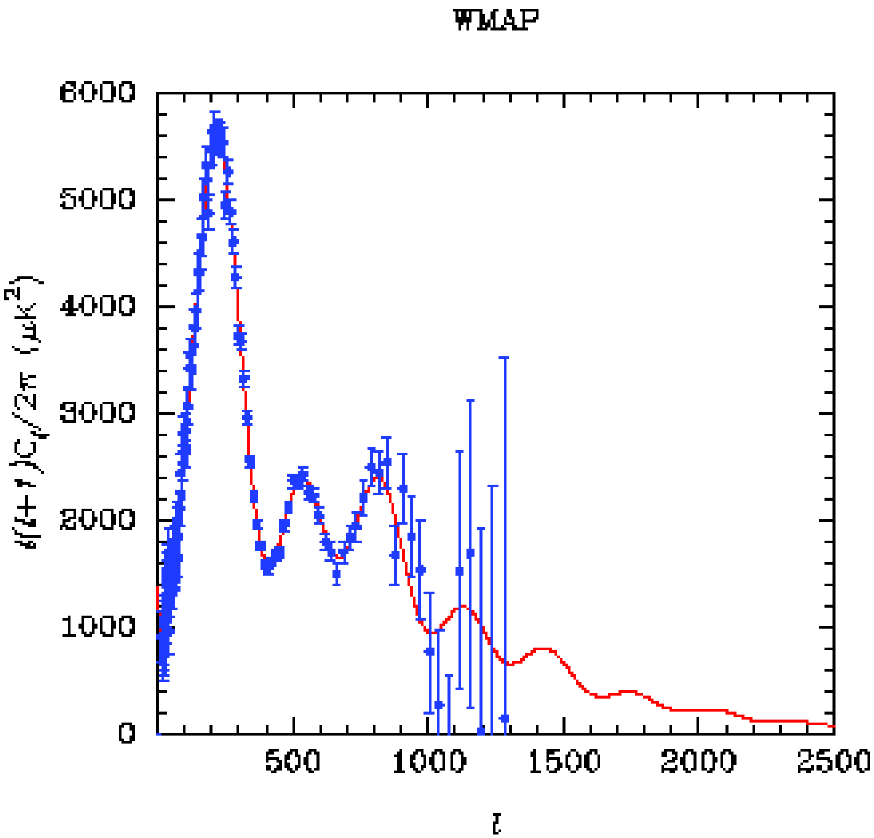
- Standard inflation has no interactions so universe is supercooled
- Warm inflation includes interactions

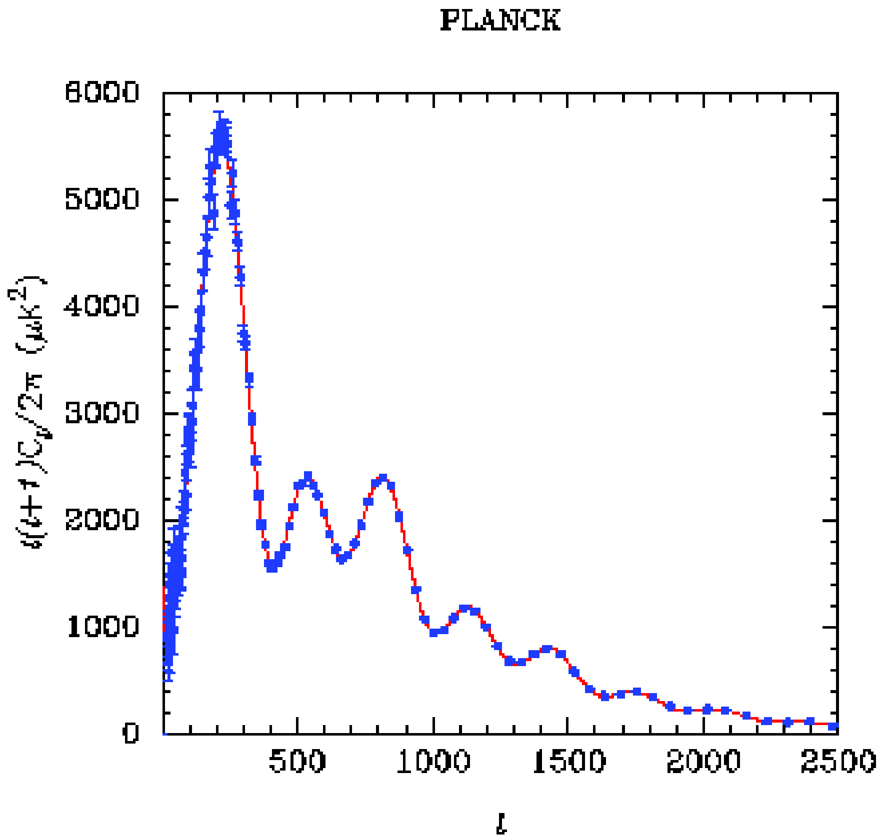
Amplitude of perturbations ($V = \frac{1}{2}m^2\phi^2$):



- Warm inflation predicts different tensor-to-scalar ratio than cold inflation.
- No consistency relation in warm inflation.
- Mechanism for isocurvature modes - which may be detectable by Planck.

What Planck can achieve

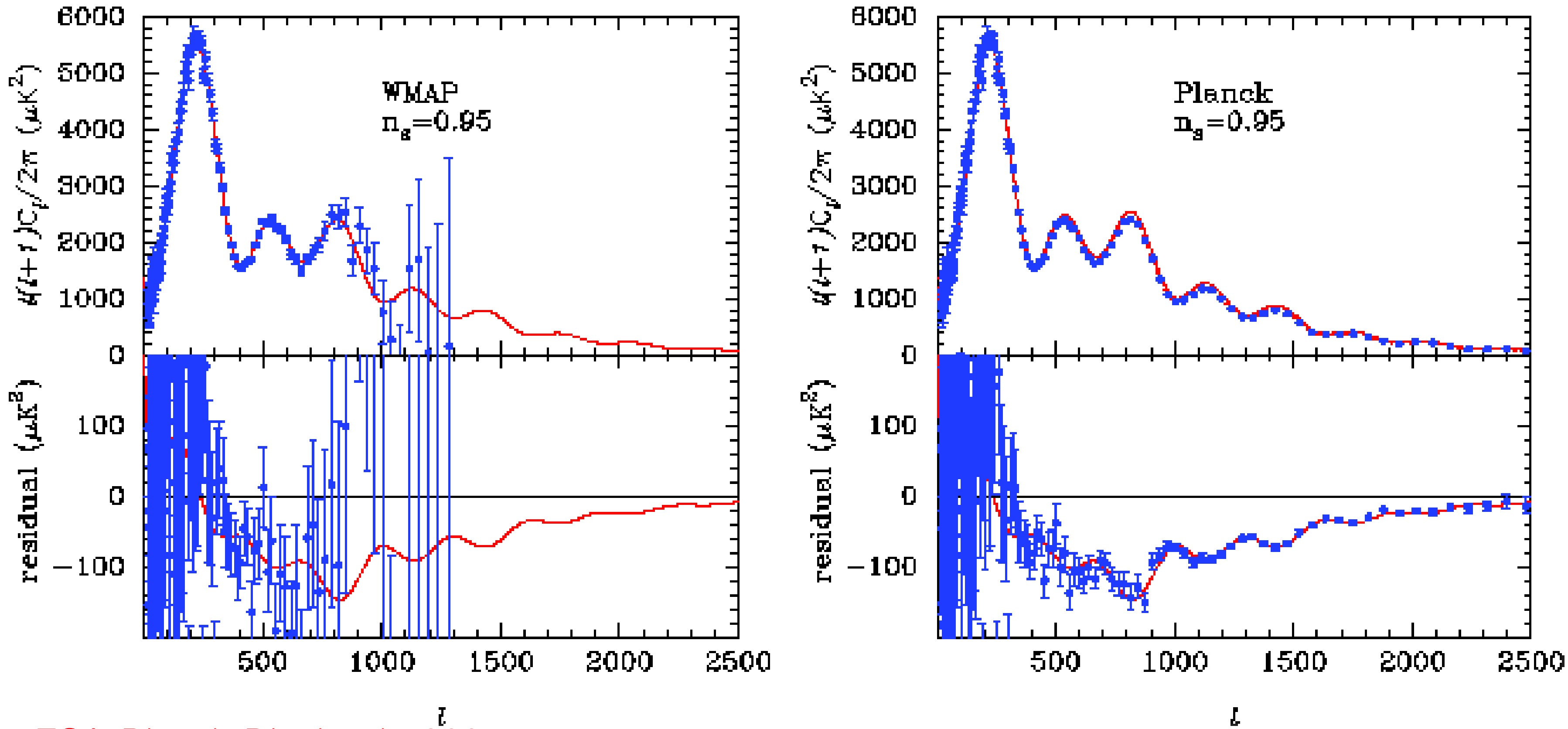




ESA Planck Bluebook, 2005

CMB power spectrum for concordance ΛCMB model (red line) compare WMAP to (projected) Planck data

Distinguishing models



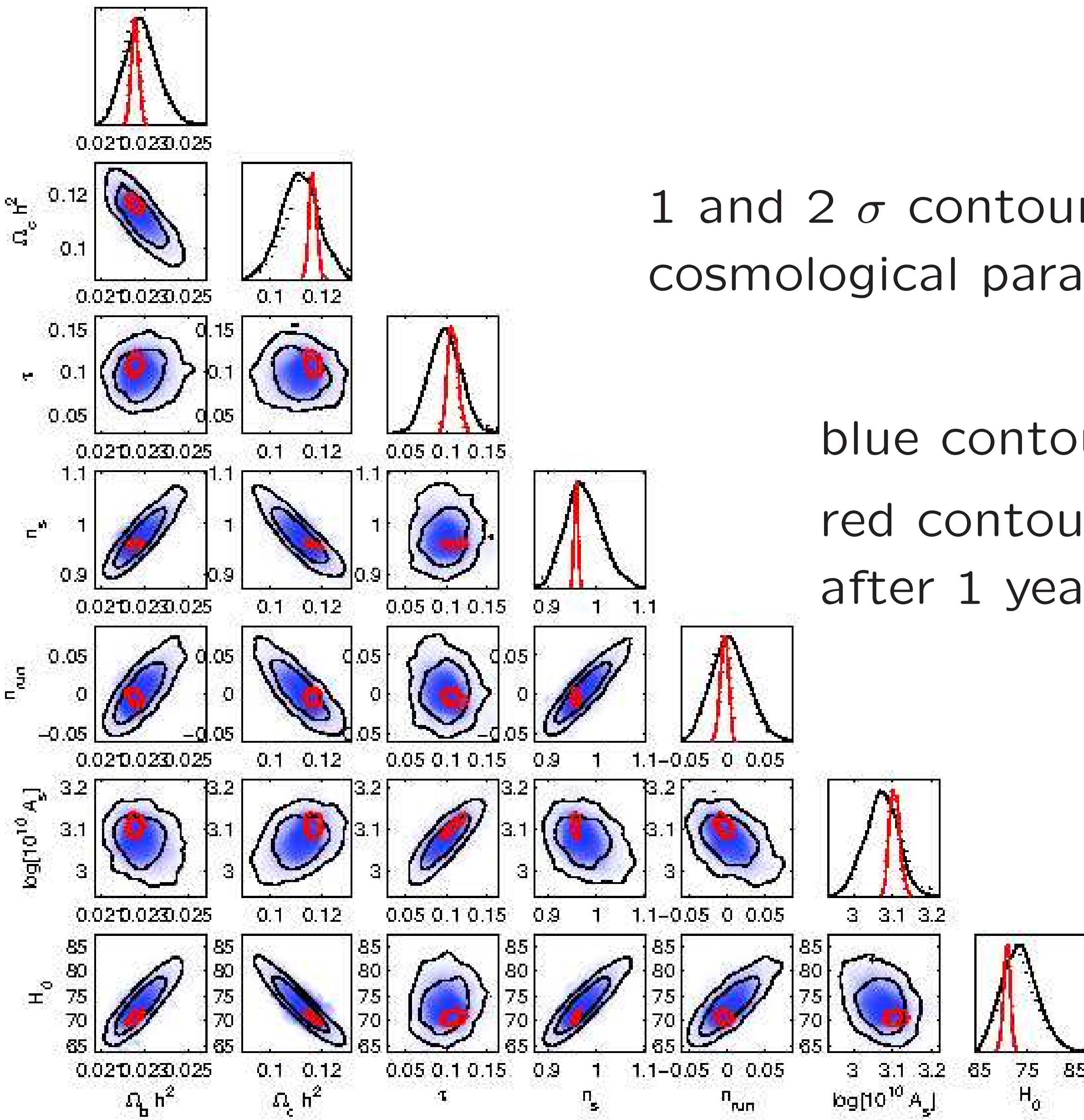
ESA Planck Bluebook, 2005

Solid red lines are concordance ΛCDM model with spectral index $n_S = 0.95$ and 1

WMAP has difficulty distinguishing between the models vs. Planck can distinguish very well

Improving parameter estimation

1 and 2 σ contour regions for cosmological parameters

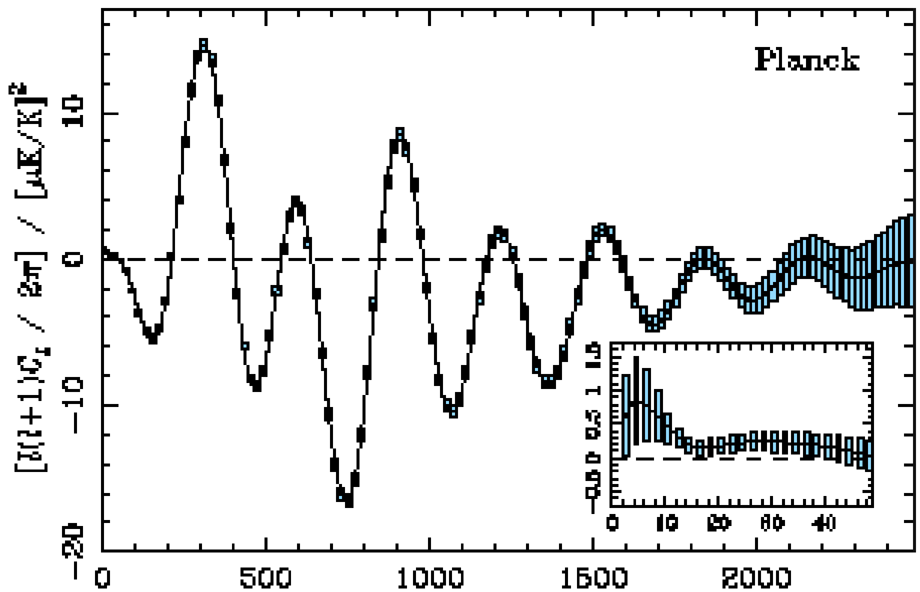


blue contours WMAP 4

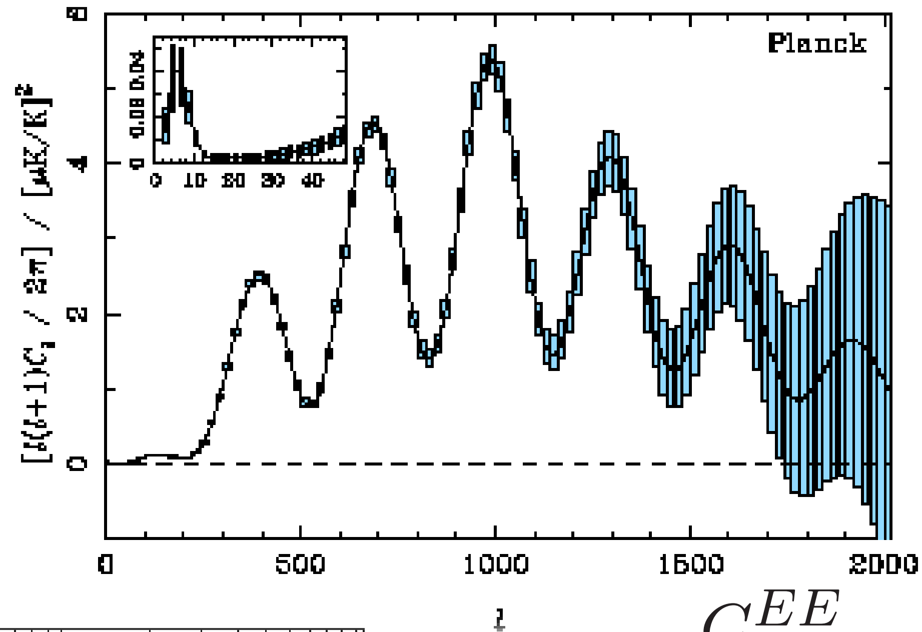
red contours Planck projected after 1 year of observation

ESA Planck Bluebook, 2005

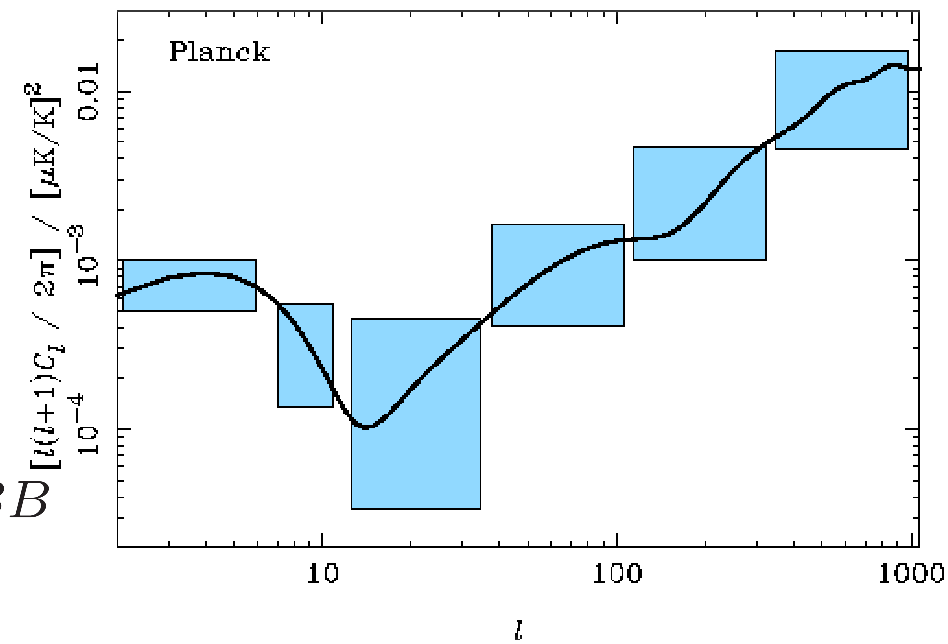
Projected Planck polarization spectra



C_ℓ^{TE}



C_ℓ^{EE}



C_ℓ^{BB}

Warm inflation on a computer

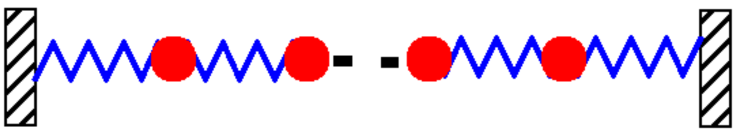
[AB, G. Lacagnina (lattice gauge), C. Verdozzi (condensed matter)]

Purpose:

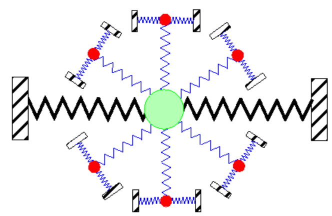
- Study of overdamped motion and its universal features
- study of how equipartition is achieved

Feasible goals:

- Numerical simulations of classical/quantum models from condensed matter:



Fermi-Paste-Ulam



Caldeira-Leggett

- Simulations of lattice quantum field theory models: Caldeira-Leggett, ϕ^4 ...

Example of overdamping in the classical regime