UNCERTAINTY AND THE BIDDING FOR INCENTIVE CONTRACTS

by

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In many procurement contracts, the monetary reward for the winning bidder is a function of the bid and of the *ex post* production cost. Therefore, contract incentives can affect the strategic relationship between a firm's bid and its estimates of production expenses. In the existing literature on incentive contract bidding, the effects of contract incentives on a firm's bidding strategy are usually analyzed with the assumption that expectations about rival bids are exogenous. That is, these expectations are summarized by a probability distribution that is assumed to be unchanged as incentives and other structural parameters are varied. But any factor that would alter one bidder's strategy would presumably alter rivals' bidding strategies as well, thereby changing the observed distribution of rival bids.

An alternative approach is to require that bidders' expectations satisfy an equilibrium or rationality condition. A tractable model with endogenously determined expectations is possible when all bidders have similar preferences and opportunities. This model is outlined in section I, and the optimal bidding decision of a single firm is discussed in section II. In section III, the unique Nash equilibrium bidding strategies of all firms are characterized in detail. The focus in sections IV and V is on the equilibrium effects of changes in incentives and other structural parameters on bidding behavior and expected procurement costs.

I. The Model

The awarding of contracts on the basis of bidding competition is a frequently used resource allocation mechanism in both the public and private sectors. Consider a model of this competition in which the buyer announces a contract for the procurement of a specified commodity. It is assumed that bids are solicited and the contract is awarded to the lowest bidder. This model is only a first approximation of many procurement processes. For example, John G. Cross observed that sometimes the U.S. Department of Defense will propose a "rough contract", invite bids, and negotiate the final details with the winner. Often, a government agency or other buyer will engage in competitive negotiations with several bidders simultaneously.[1] Negotiations may be focused on price, but other considerations such as design and contract incentives may be involved. These non-price dimensions will be ignored in this paper, and therefore, the propositions derived will be more relevant to situations in which price competition is important.

It is assumed that there are N bidders for a particular contract, and that the firm submitting the lowest bid is awarded the contract. In general, the contract specifies this firm's total receipts from the buyer as a function of the bid p and of other variables such as the *ex post* cost of production c. For the case of a linear incentive contract, the firm's total contract revenue is determined:

(1) $\alpha p + \beta(p - c) + c$

where $0 \leqq \alpha \leqq 1$ and $0 \leqq \beta \leqq 1$. If $\beta = 0$, the firm receives the production cost plus a "fixed fee" αp. A "fixed price" contract is one for which $\alpha = 0$ and $\beta = 1$; the winning firm's profit equals the full difference between the bid price and the production cost. Note that if the cost exceeds the bid price, this "overrun" is penalized by a factor of β. Underruns are rewarded symmetrically. The parameters α and β are sometimes referred to as the "guaranteed profit rate" and the "sharing rate" respectively.

Many contracts that are awarded on the basis of competitive bidding are for tailor-made jobs that cannot be readily purchased in the existing markets. This is one of the reasons why there may be considerable uncertainty about the cost of completing the contract job. Note that a reduction in the sharing rate β means that the government or other procuring agent agrees to accept both a greater share of the risk of a cost overrun and a greater share of the potential savings resulting from an underrun. A common justification for the use of incentive contracts is that if bidders are risk averse, then the procuring agent may be able to reduce its expected procurement cost by accepting a greater share of the risk. The assumption is that firms bidding for contracts cannot insure against the risk of cost overruns because of moral hazard problems: overruns are sensitive to the firm's own decisions. If the owners of a firm are risk averse, the management may be reluctant to undertake a relatively large contract with significant cost uncertainties that cannot be insured. In addition, the managers themselves may be risk averse.[2]

First, consider the bidding decision of a specific firm. It is assumed that this firm's perferences toward profit risks can be represented by a VonNeumann-Morgenstern utility function $U(\cdot)$.[3] It is also assumed that the firm does not know its production cost with certainty at the time of bidding. Let the expected value of the production cost be denoted $\bar{c}$. The actual ex post production cost c is determined:

$$(2) \quad c = \bar{c} + \gamma$$

where γ is the realization of a continuous random variable with mean 0 . The probability density function (p.d.f.) and distribution function (d.f.) of γ will be denoted by $h(\cdot)$ and $H(\cdot)$ respectively. It is assumed that $h(\cdot)$ is strictly positive on a finite interval, zero elsewhere.

Suppose that the firm under consideration is the ith firm and that its bid is p_i . It is apparent from (1) that this firm's contract profit as a function of both the bid p_i and the realized cost c is $\alpha p_i + \beta(p_i - c)$. Then it follows from (2) that its expected profit, denoted by $\bar{z}$, is $(\alpha + \beta)\, p_i - \beta\bar{c}$. The firm's expected utility of the contract profit, denoted by $E_c(p_i)$ is:

$$(3) \quad E_c(p_i) \equiv \int_{-\infty}^{\infty} U(\bar{z} - \beta\gamma)\, h(\gamma)\, d\gamma$$

Following John McCall, firms that lose the contract are assumed to use their resources in the private sector. The ith firm's profit from the alternative private sector operation will be denoted r_i . The alternative profit r_i is assumed to be known to firm i at the time the bid is submitted. Therefore, the firm's utility is $U(r_i)$ in the event that the contract is lost. If θ is the probability of winning the contract, then the firm's expected utility can be written:

$$(4) \quad \theta E_c(p_i) + [1 - \theta] U(r_i)$$

In general, θ will be a function of the bid p_i ; high bids will be less likely to win. The implicit assumption in David Baron's (1972) analysis of incentive contract bidding is that the functional relationship between θ and p_i is independent of the structural parameters of the model. But the relation between p_i and θ surely depends on the bidding strategies of rival firms. For example, consider a change in contract incentives that would cause p_i to change for a given θ function. It seems reasonable to expect this type of change to alter rival bids as well, and therefore the ith firm's probability of winning with a bid p_i should be altered.

In order to determine the equilibrium effects of structural changes on bidding behavior, it is necessary to analyze the bidding decisions of all bidders simultaneously. The model is manageable if all bidders are assumed to have similar preferences and opportunities. Specifically, it is assumed that all N firms have the same utility function for profit which is twice differentiable, strictly increasing, and exhibits risk aversion: $U''(\cdot) < 0$. In addition, all firms are assumed to be equally efficient in the sense that the probability distribution of the cost c is the same for each firm. Thus the ith firm's expected utility as a function of p_i and r_i is given in (4) for firms $i = 1, \ldots, N$.

Although a firm's bid is often referred to as a "target cost", these bids will be analyzed as strategic moves, not as forecasts or goals.[4] A bidding strategy specifies a relationship between the bid price and the firm-specific parameter r_i. This alternative profit opportunity may differ from firm to firm at any particular time because of differences in planned investment projects. However, firms have similar opportunities in the sense that the profit alternatives are assumed to be independent realizations of a continuous random variable with a p.d.f. and d.f. denoted by $g(\cdot)$ and $G(\cdot)$ respectively. It is assumed that $g(\cdot)$ is strictly positive on a finite interval (a,b).[5] This density is known by all so that firms have symmetric information about the profit opportunities of rivals.[6]

The bidding competition for contracts will be analyzed as a noncooperative game. Because of the symmetry of information and preferences, only symmetric Nash equilibria will be considered. In a symmetric equilibrium the common Nash strategy will have the property that it maximizes the expected utility of each bidder when all rivals are known to be using the same strategy. Recall that a bidding strategy is a function of the alternative profit opportunity. A Nash equilibrium strategy, together with the distribution of rival profit alternatives, determines the distribution of rival bids. Equivalently, the equilibrium strategy maximizes the firm's expected utility with respect to this distribution of rival bids. If all firms use the Nash strategy, each firm's subjective probability distribution over rival bids corresponds to the distribution of rival bids prior to the drawing of profit alternatives. A unique Nash point in strategies generates a unique set of "rational expectations" in the sense of R.E. Lucas and E. C. Prescott. Any other set of firm's expectations will be unstable; at least one firm would have an incentive to alter its bidding strategy, causing the ex ante probability distributions of bids to differ from the distributions which summarize firms' expectations. In equilibrium, expectations are determined by assuming that each firm uses the Nash bidding strategy.

II. A Firm's Bidding Decision

The focus of this section is on the optimal bidding strategy for a typical firm i when the $N-1$ rivals are known to use a strategy $p_j = p(r_j)$, $j \neq i$. The next step is to determine the $p(\cdot)$ function that is the firm's optimal strategy when all rivals are known to be using the same strategy function; we are looking for a fixed point in strategy functions. The strategy function that satisfies this requirement will be a Nash equilibrium strategy for the symmetric game outlined in the previous section.

First, suppose that the ith firm's $N-1$ rivals are using a strategy $p_j = p(r_j)$ which is differentiable and strictly increasing on $r \in (a,b)$. Then there is an inverse strategy

$\pi(\cdot)$ such that $\pi(p(r_j)) = r_j$ and $\pi'(\cdot) > 0$. If $y = \min_{j \neq i}\{r_j\}$, then it follows from the assumed monotonicity of $p(\cdot)$ that the lowest rival bid is $p(y)$. Thus the ith firm will win the contract if $p_i < p(y)$, or equivalently, if $\pi(p_i) < y$.

It is now straightforward to determine the probability that firm i will win the contract with a bid of p_i. Let the p.d.f. and d.f. of y be denoted by $f(\cdot)$ and $F(\cdot)$ respectively. Recall that y is the lowest order statistic in a sample of size N-1 from a distribution with d.f. $G(\cdot)$. Therefore, $F(y)$ and $f(y)$ are determined:

$$(5) \quad F(y) = 1 - [1-G(y)]^{N-1}$$

and $f(y) = dF(y)/dy$. With this notation, the probability of winning the contract with a bid of p_i is $1 - F(\pi(p_i))$.

Given the rivals' bidding behavior determined by the $p(\cdot)$ function and its inverse $\pi(\cdot)$, the ith firm's expected utility in (4) can be expressed:

$$(6) \quad E_c(p_i)\,[1-F(\pi(p_i))] + U(r_i)\,F(\pi(p_i))$$

The necessary condition for determining the optimal bid p_i is

$$(7) \quad E_c'(p_i)\,[1-F(\pi(p_i))] - [E_c(p_i) - U(r_i)]\,f(\pi(p_i))\,\pi'(p_i) = 0$$

where $E_c'(p_i) \equiv dE_c(p_i)/dp_i$. For a given rival strategy $p(\cdot)$ corresponding to $\pi(\cdot)$, equation (7) specifies a bid p_i for any $r_i \in (a,b)$.

III. The Equilibrium Bidding Strategy

The next step is to determine a particular bidding strategy which is optimal when all rival bidders are known to be using the <u>same</u> strategy. Such a strategy will constitute a Nash equilibrium because no firm could increase its expected utility by deviating from the common equilibrium strategy.

In a symmetric Nash equilibrium, all firms will use the same strategy function: $p_i = p(r_i)$ and $r_i = \pi(p_i)$ for $i = 1, \ldots N$. If one replaces r_i in (7) with $\pi(p_i)$, the result is a differentiatial equation in the equilibrium $\pi(\cdot)$ function. Equivalently, the equilibrium bidding strategy function $p(\cdot)$ must satisfy the differential equation:

$$(8)\quad [1-F(r_i)]\, E_c'(p(r_i))\, p'(r_i) \;-\; [E_c(p(r_i)) - U(r_i)]\, f(r_i) \;=\; 0$$

because $1/\pi'(p_i) = p'(\pi(p_i)) = p'(r_i)$. Equation (8) must be satisfied for all r_i on the interval (a,b). Therefore, (8) can be integrated from any arbitrary value of $r \in (a,b)$ to b, and the integrated equation is

$$(9)\quad \int_r^b [1-F(s)] \frac{dE_c(p(s))}{ds}\, ds \;=\; \int_r^b [E_c(p(s)) \;-\; U(s)]\, f(s)\, ds$$

because $E_c'(p(r_i))\, p'(r_i) = dE_c(p(r_i))/dr_i$. One can use integration by parts and the fact that $F(b) = 1$ to express (9):

$$(10)\quad [1-F(r)]\, E_c(p(r)) \;=\; \int_r^b U(s)\, f(s)\, ds$$

This equation implicitly defines the equilibrium bidding strategy $p(r)$.

Without an assumption of risk neutrality or constant absolute risk aversion, it is impossible to obtain a closed form expression for the equilibrium bidding strategy from equation (10). However, it can be shown that the equilibrium strategy is unique. More important, the qualitative effects of changes in contract parameters on bids and contract profits can be determined regardless of the assumed degree of risk aversion.

First, it is convenient to introduce some new notation. Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the respective p.d.f. and d.f. of the random variable $\varepsilon = \beta\gamma$, so that

$$(11) \quad \Phi(\varepsilon) = H(\varepsilon/\beta) , \quad \text{and} \quad \phi(\varepsilon) = \frac{1}{\beta} h(\varepsilon/\beta)$$

Then it follows from (3) and the definition of ε that

$$(12) \quad E_c(p) = \int_{-\infty}^{\infty} U(\bar{z} - \varepsilon) \; \phi(\varepsilon) d\varepsilon = U(\bar{z} - \Delta_\varepsilon)$$

where Δ_ε is a positive risk premium.[7] With this notation, equation (10) which determines the equilibrium bid can be written:

$$(13) \quad U(\bar{z} - \Delta_\varepsilon) = U^*(r)$$

where

$$(14) \quad U^*(r) \equiv \frac{\int_r^b U(s) \; f(s) \; ds}{1-F(r)}$$

It follows from (12) and (13) that for each value of r , $U^*(r)$ is equal to the expected utility in the event of winning the contract with the equilibrium bid $p(r)$. Note that $U^*(r)$ in (14) can be interpreted as being the expected utility of the minimum rival profit alternative y conditional on the event that $y > r$. Then (13) states that this conditional expected utility equals the expected utility of the contract profit evaluated at the equilibrium bid.

For each value of $r \in (a,b)$, it can be shown that equation (13) uniquely determines the expected contract profit $\bar{z}$ that corresponds to the equilibrium bid $p(r)$. First note that $U(\bar{z} - \Delta_\varepsilon)$ defined in (12) is a continuous, strictly increasing function of $\bar{z}$ as shown in figure 1. The unique equilibrium value of $\bar{z}$ that solves (13) is determined by the intersection of the $U(\bar{z} - \Delta_\varepsilon)$ line with the horizontal $U^*(r)$ line. Also, it is apparent from (14) that $U(r) < U^*(r) < U(b)$. If $\bar{z} = r$, then $U(\bar{z} - \Delta_\varepsilon) < U(r) < U^*(r)$ and it follows that the equilibrium solution for $\bar{z}$ is greater than r. For each value of r, the equilibrium level of $\bar{z}$ will be denoted by $\bar{z}(r)$.

The Nash bid for each value of r is determined from the equilibrium level of $\bar{z}(r)$:

$$(15)\quad p(r) = \frac{\beta\bar{c} + \bar{z}(r)}{\alpha + \beta}$$

If the utility function exhibits constant absolute risk aversion, then the risk premium Δ_ε defined in (12) will be a constant, independent of $\bar{z}$.[8] In this special case, it follows from (13) that $\bar{z} = \Delta_\varepsilon + U^{-1}(U^*(r))$, where $U^{-1}(\cdot)$ denotes the inverse of $U(\cdot)$. The equilibrium bidding strategy can be determined explicitly from (15) in this case.

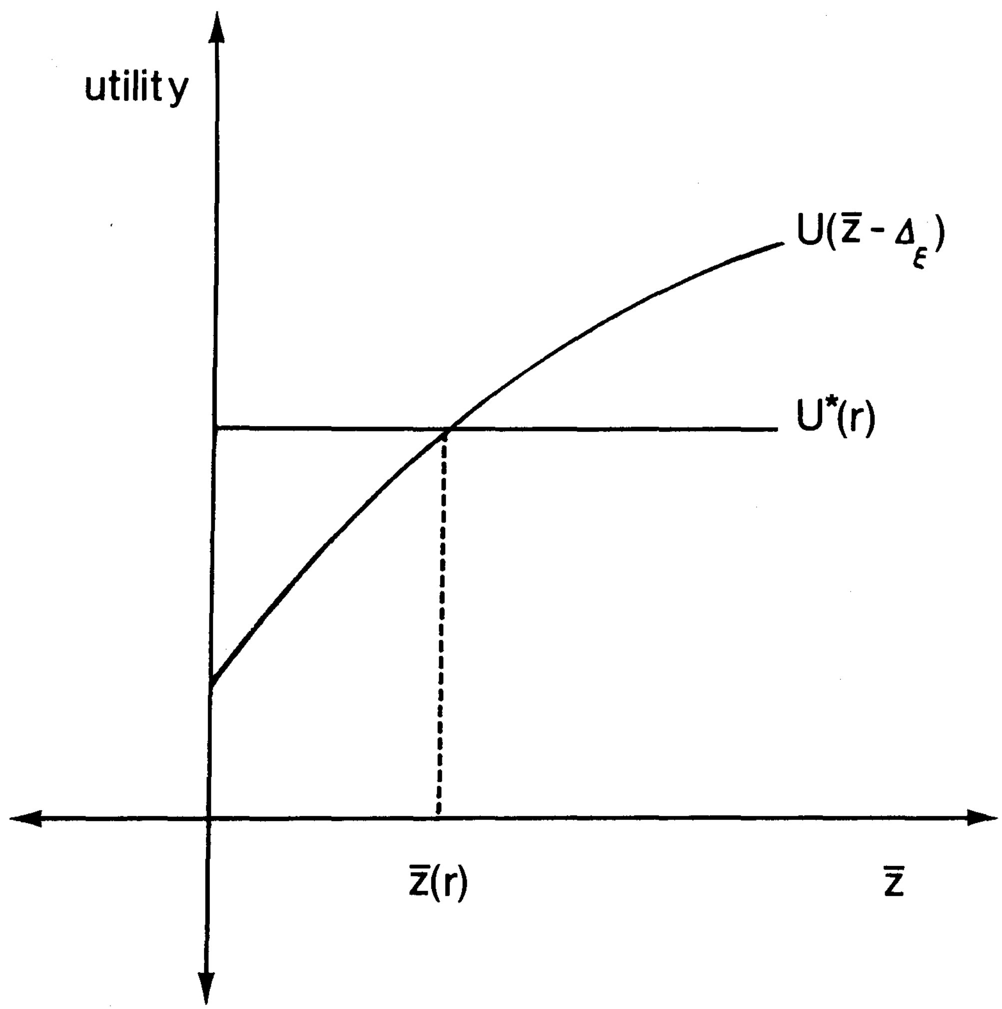

FIGURE 1

To summarize, there is a unique bidding strategy determined by (13) for $r \in (a,b)$. It follows from (13),(14), and the differentiability of $U(\cdot)$ that this bidding strategy is a strictly increasing, differentiable function of r . Furthermore, this strategy is unique in the class of strictly increasing, differentiable strategy functions on (a,b): First consider the terminal condition. It follows from L'Hospital's Rule that the terminal condition implicit in (10) is: $\lim_{r \to b} E_c(p(r)) = U(b)$. It can be shown 1) that any symmetric equilibrium strategy must satisfy this condition, and 2) that the uniqueness of the equilibrium strategy follows from the uniqueness of the terminal condition.[9] Finally, it can be shown that the bids determined by (13) are indeed Nash equilibrium bids; no firm can increase its expected utility by submitting a bid $p \neq p(r)$ when all $N-1$ rivals are known to use the equilibrium strategy function $p(r)$ implicit in (13) and (15).[10]

IV. The Equilibrium Effects of Structural Parameters

A structural change is an event that alters the decision environment of all potential bidders. The bidding behavior of all competitors is generally affected by a structural change, and therefore, each bidder's expectations concerning rival bids should be affected.

First, consider the contract parameters α and β . Note that $U^*(r)$ defined in (14) is obviously independent of α and

β . Therefore, it follows from (13) that the expected utility of the contract profit (evaluated at the equilibrium bid) is independent of the contract parameters.

However, an increase in β will increase equilibrium bids. To see this, consider the effect of a change in β on the distribution of ε . It follows from (11) that an increase from β_1 to β_2 will change the d.f. of ε from $\Phi_1(\varepsilon) \equiv H(\varepsilon/\beta_1)$ to $\Phi_2(\varepsilon) \equiv H(\varepsilon/\beta_2)$. The mean of the random variable ε is 0 for each value of β , but $\Phi_2(\varepsilon) > \Phi_1(\varepsilon)$ if $\varepsilon < 0$, $\Phi_2(\varepsilon) = \Phi_1(\varepsilon)$ if $\varepsilon = 0$, and $\Phi_2(\varepsilon) < \Phi_1(\varepsilon)$ if $\varepsilon > 0$. Next, it can be shown that $\Phi_1(\varepsilon)$ is strictly greater than $\Phi_2(\varepsilon)$ in the sense of second degree stochastic dominance, i.e. that

$$\int_{-\infty}^{\varepsilon} [\Phi_2(x) - \Phi_1(x)]\, dx \geqq 0$$

for all ε and > for at least one value of ε . This second degree stochastic dominance inequality obviously holds for $\varepsilon \leqq 0$. If $\varepsilon > 0$,

$$\int_{-\infty}^{\varepsilon} [\Phi_2(x) - \Phi_1(x)]\, dx =$$
$$\int_{-\infty}^{\infty} [\Phi_2(x) - \Phi_1(x)]\, dx - \int_{\varepsilon}^{\infty} [\Phi_2(x) - \Phi_1(x)]\, dx$$

The first integral on the right side of this equation is zero because the means of each d.f. are equal. Recall that $\Phi_2(\varepsilon) < \Phi_1(\varepsilon)$ for $\varepsilon > 0$, and therefore, the second degree stochastic dominance inequality holds for $\varepsilon > 0$ as well.

Given the assumption that $U''(\cdot) < 0$, it follows from a strict version of the second degree stochastic dominance theorem that the expected value of $U(\bar{z} - \beta_1\gamma)$ is strictly greater than the expected value of $U(\bar{z} - \beta_2\gamma)$ for each fixed value of $\bar{z}$.[11] Equivalently, an increase in β, with $\bar{z}$ fixed, will reduce $U(\bar{z} - \Delta_\varepsilon)$. This downward shift in the $U(\bar{z} - \Delta_\varepsilon)$ curve in figure 1 will increase the equilibrium level of $\bar{z}(r)$, and therefore, the equilibrium bids determined by (15) will increase. With all firms bidding for greater contract profits, the expected procurement cost will increase.

The conclusion is that, in this model, the government or other procuring agent could reduce its expected procurement cost by accepting all risk $(\beta = 0)$. However, this conclusion is subject to two qualifications: First, a risk averse procuring agent may not be willing to accept all risk. Second, an increase in β increases the penalty for cost overruns, and therefore, high values of β might induce firms to keep costs down once the contract has been awarded.[12]

The effects of changes in β can be summarized:

<u>Proposition 1</u>: If 1) firms are risk averse, 2) production costs are uncertain, and 3) there are no cost reduction opportunities, then an increase in the sharing rate β will increase equilibrium bids. In addition, the expected value of the contract profit evaluated at each firm's equilibrium bid will increase. The expected procurement cost is, therefore, increasing in β.

Next consider a shift in the α parameter. Recall that $\varepsilon = \beta\gamma$, and therefore, the distribution of ε will not be altered.

Thus for a fixed level of $\bar{z}$, both $U(\bar{z} - \Delta_\varepsilon)$ in (12) and $U^*(r)$ in (14) are unaffected by the shift in α . It now follows from (13) that there is no change in $\bar{z}(r)$, the expected value of the contract profit evaluated at each firm's equilibrium bid. Consequently, the expected procurement cost is independent of α . But an increase in α will decrease the equilibrium bid for each value of r because $\bar{z}(r)$ is unchanged and $\bar{z}(r) = (\alpha + \beta)\, p(r) - \beta\bar{c}$. Note that an increase in α that reduces equilibrium bids will generally increase the probability that there will be a cost overrun. To summarize:

Proposition 2: An increase in the guaranteed profit rate α will tend to decrease equilibrium bids and increase the probability of cost overruns. However, the resulting overruns do not indicate excessive procurement expenditures because expected procurement costs are independent of α .[13]

Finally, consider the effect of an increase in the number of bidders N . Recall that $U^*(r)$ in (14) is the expected utility of the minimum rival profit opportunity y , conditional on the event that $y \geqq r$. As the number of rival bidders increases, the minimum rival profit opportunity is likely to be reduced. Therefore, one would expect that $U^*(r)$ is a decreasing function of N for each value of r . A proof of this conjecture is given in Holt, pp. 62-66. Thus, the increase in N shifts the $U^*(r)$ line downward in figure 1, and therefore $\bar{z}(r)$ decreases. It follows that an increase in the number of bidders reduces equilibrium bids and expected procurement costs.

As the number of bidders approaches infinity, one would expect that if y is greater than r, it is not likely to be very much greater than r. Recalling the previous interpretation of $U^*(r)$ as a conditional expected utility, one would expect that $\lim_{N \to \infty} U^*(r) = U(r)$. This limit is verified in Holt, pp. 69-71. In the limit, (13) becomes $U(\bar{z} - \Delta_\varepsilon) = U(r)$.

In order to interpret this limiting strategy, note that $U^*(r)$ in (14) exceeds $U(r)$. For finite N, the equilibrium strategy implicit in (13) has the property that $U(\bar{z}(r) - \Delta_\varepsilon) > U(r)$. In other words, it is optimal to submit a bid for which the expected utility of the contract profit exceeds the utility of the alternative profit in the event of a loss. If this were not the case, there would be no reason to bid! However, as the number of bidders approaches infinity, the competition induces each bidder to bid for a contract profit that has the same expected utility as the alternative profit. This limiting strategy is the type of strategy used by McCall in an important analysis of incentive contract bidding when firms have different production costs.

V. Uncertainty and Risk Aversion

It is interesting to consider the interrelationships between firms' aversions to risk and the riskiness of profits. There are several plausible measures of the riskiness of a random variable, but the comparisons in this section are between distributions with identical means. A mean preserving increase in risk for a random variable $\tilde{x}$ is defined to be the creation of a new random variable $\tilde{x} + \tilde{\eta}$, where $\tilde{\eta}$ is independent of $\tilde{x}$ and $E\{\tilde{\eta}\} = 0$. M. Rothschild and J. E. Stiglitz show that a mean preserving increase in risk will reduce the expected utility of a risk averter. Equivalently, the risk premium is increased.

Baron (1972) analyzes the effect of cost uncertainty on a particular firm's bid when the firm's probability distribution of the minimum rival bid is fixed and independent of the firm's production cost uncertainty. Baron shows that the optimal bid for a deterministic production cost $\bar{c}$ is less than the optimal bid if costs are uncertain with an expected value of $\bar{c}$.[14] One interesting interpretation of Baron's proposition is that if there are a large number of firms, any two firms might have very similar distributions for the minimum rival bid. If the two firms are identical in other respects, the firm that is uncertain about the production cost will submit a greater bid. This is important because it suggests that bid differences may not reflect cost differences between firms.

On the other hand, a game theoretic approach makes it possible to evaluate conditions which affect the cost uncertainty for all potential bidders. For example, suppose that the production cost uncertainty for all firms is increased as a result of volatile factor prices or imprecise project specifications. If $U''(\cdot) < 0$, a mean preserving increase in the risk associated with the production cost will increase the risk premium Δ_ε determined by (12). As a result, the $U(\bar{z} - \Delta_\varepsilon)$ line in figure 1 shifts downward for each value of $\bar{z}$, and the equilibrium level of $\bar{z}(r)$ is increased. In a Nash equilibrium, bids are increased because bidders demand a greater expected contract profit as compensation for the increased risk.

Next, consider the effect of an increase in the risk aversion for <u>each</u> firm. For example, if $U(\cdot)$ exhibits decreasing absolute risk aversion, then the imposition of a lump sum tax of τ will increase risk aversion:

$$\frac{-U''(w - \tau)}{U'(w - \tau)} > \frac{-U''(w)}{U'(w)}$$

for each level of w . An increase in risk aversion will increase the risk premium Δ_ε in (12), and this shifts the $U(\bar{z} - \Delta_\varepsilon)$ line downward in figure 1. However, recall that $U^*(r)$ in (14) is an expected utility expression, and therefore, an increase in risk aversion reduces the expected utility $U^*(r)$. As a result, the $U^*(r)$ line in figure 1 shifts downward, and the equilibrium effect of this increase in risk aversion is indeterminate.[15]

The results of this section can be summarized:

<u>Proposition 3</u>: If bidders are risk averse, then a mean preserving increase in risk associated with <u>each</u> firm's production cost will increase both the equilibrium bid $p(r)$ and the equilibrium contract profit $\bar{z}(r)$ for each value of r . However, the effects of an increase in each firm's absolute risk aversion are indeterminate.

VI. Conclusion

In equilibrium, the structural factors that form the decision environment of all bidders will have an impact on the probability distribution of bids relevant for each firm's decision. Consequently, it is not meaningful to consider the effect of a structural change on a firm's bidding behavior when expectations about rival bids are assumed to be unchanged. Expectations about rival bids were assumed to be exogenous in Baron's (1972) analysis, and the effects of contract incentives on bidding behavior were generally indeterminate. This is because the probability distribution representing a firm's expectations was essentially arbitrary; no equilibrium condition was imposed on these expectations.

For the symmetric model analyzed in this paper, firms' expectations about rival bids are endogenous and rational when firms use their Nash equilibrium bidding strategies. The equilibrium effects of contract incentives and other structural parameters on both bidding strategies and expected procurement costs are determined. Briefly, the effect of an increase in the "guaranteed" profit rate is to lower bids to the extent that each firm's expected contract profit for the bid tendered is unchanged. Thus a change in the profit guarantee will not affect expected procurement costs, but it is shown than the frequency of cost overruns may be affected. Consequently, procurement efficiency is not necessarily associated with the absence of observed cost overruns. On the other hand, an increase in the sharing rate will induce risk averse firms to bid for greater expected contract profits as compensation for accepting a larger share of the risk associated with uncertain production costs. The main results are stated more precisely in propositions 1 - 3.

REFERENCES

D. P. Baron, "Incentive Contracts and Competitive Bidding," Amer. Econ. Rev., June 1972, 62, 384-94.

__________, "Incentive Contracts and Competitive Bidding: Reply," Amer. Econ. Rev., Dec. 1974, 64, 1072-73.

V. S. Bawa, "Optimal Rules for Ordering Uncertain Prospects," J. Fin. Econ., Mar. 1975, 2, 95-121.

C. C. Blaydon and P. W. Marshall, "Incentive Contracts and Competitive Bidding: Comment," Amer. Econ. Rev., Dec. 1974, 64, 1070-71.

J. G. Cross, "A Reappraisal of Cost Incentives in Defense Contracts," Western Econ. J., June 1968, 6, 205-25.

J. M. Cummins, "Incentive Contracting for National Defense: A Problem of Optimal Risk Sharing," Bell J. Econ., Spring 1977, 8, pp. 168-85.

C. A. Holt, "Bidding for Contracts," unpublished doctoral dissertation, Carnegie-Mellon Univ., Mar. 1977.

R. E. Lucas and E. C. Prescott, "Investment Under Uncertainty," Econometrica, Sept. 1971, 39, 659-81.

J. J. McCall, "The Simple Economics of Incentive Contracting," Amer. Econ. Rev., Dec. 1970, 60, 837-46.

A. Ortega-Reichert, "Models for Competitive Bidding Under Uncertainty," Tech. Report No. 103, Dept. of Operations Research, Stanford University, Jan. 1968.

J. W. Pratt, "Risk Aversion in the Small and in the Large," Econometrica, Jan./Apr. 1964, 32, 122-36.

M. Rothschild and J. E. Stiglitz, "Increasing Risk: I. A Definition," J. Econ. Theor. , Sept. 1970, 2, 225-43.

F. M. Scherer, "The Theory of Contractural Incentives for Cost Reduction," Quart. J. Econ., May 1964, 78, 257-80.

W. Vickrey, "Counterspeculation, Auctions, and Competitive Sealed Tenders," J. Finance, Mar. 1961, 16, 8-37.

R. B. Wilson, "Competitive Bidding with Disparate Options," Manage. Sci., Mar. 1969, 15, 446-48.

FOOTNOTES

1. It is not uncommon for buyers to negotiate contract terms with a single firm. However, government agencies are often required by law to seek competitive offers unless only one firm is capable of providing the desired commodity.

2. On the basis of case studies, F. M. Scherer, p. 275, reported a prevalent concern that short run contract losses might be interpreted as managerial failure requiring reorganization.

3. The justification of a utility function for the firm should be based on the preferences of managers and owners, and on the implicit contracts made by these agents under conditions of uncertainty. In my opinion, this is an important, unsolved problem.

4. The game theoretic approach taken in this paper is based on W. Vickrey's initial analysis of bidding games, and on the subsequent work of A. Ortega-Reichert and R. B. Wilson.

5. It is straightforward to generalize the analysis to cover the cases in which a and b are not finite. See C. A. Holt, p. 85.

6. Note that the only relevant difference between firms is the r_i parameter, because the $U(\cdot)$ function and the density functions $h(\cdot)$ and $g(\cdot)$ are the same for each firm. If any of these symmetry assumptions were relaxed, the bidding strategies would depend on all firm-specific parameters.

7. A risk premium is defined by J. W. Pratt to be the quantity Δ that satisfies: $E\{U(x)\} = U(E\{x\} - \Delta)$, where the expectation is taken with respect to the distribution of the random variable x.

8. See Pratt for a discussion of constant absolute risk aversion and risk premiums.

9. A proof of uniqueness can be found in Holt, p. 90.

10. A proof can be found in Holt, p. 90.

11. See V. S. Bawa, Theorem 2, pp. 101-102.

12. Both Scherer and J. M. Cummins discuss this cost reduction effect in the context of a negotiated contract. A competitive bidding model with cost reduction opportunities can be found in Holt, pp. 102-112.

13. Cummins, p. 179, reaches the same conclusion in his analysis of negotiated contracts, i.e. that cost overruns do not necessarily indicate excessive procurement costs.

14. See Baron (1972), Proposition 4. The proof of this proposition is based on the assumption that the Arrow-Pratt measure of absolute risk aversion, $-U''(w)/U'(w)$, is a nonincreasing function of w .

15. See C. C. Blaydon and P. W. Marshall and Baron (1974) for a clarification of the effect of an increase in a single firm's risk aversion on its optimal bid. Briefly, this effect is indeterminate if the production cost is uncertain, as is case for the game theoretic analysis of a symmetric increase in the risk aversion of each firm.