Insights from Harvest Scheduling Applications in Minnesota[1]

by

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Introduction

From a basic mathematical perspective, linear programming (LP) is a relatively new tool. A solution strategy for LP problems was not developed until the 1940s. Today LP is applied to many important and widely different real-world problems. Applications of LP were instrumental for strategic planning by the military during World War II. Development and improvement of computer technologies have revolutionized its use in terms of the size of problems that can easily be solved mathematically using a desktop computer. LP has been used to support forest planning for approximately 50 years. It can help support decision-making for the forest as a whole by simultaneously addressing and integrating multiple objectives over multiple forest cover types and multiple stand-level management intensities. It can address traditional forest regulation concepts like area control and volume control, as well as economic and ecological objectives. LP has been used extensively by forest industry, especially in the southern and western US and Canada. The USDA Forest Service has invested heavily in LP, spending millions of dollars with separate applications for each national forest in the country. Many recent applications have expanded on basic formulations to help address environmental conditions and the spatial arrangement of the forest. The recent applications for the two national forests in Minnesota are good examples (USDA Forest Service 2004). With demands on forests likely only to increase in the future, it would not be surprising to see LP and other analytical tools from operations research as key tools for developing good forest plans. Although LP analysis is science based, applying it well is often considered as much of an art as it is a science.

The Minnesota Department of Natural Resources (MnDNR) is the largest single forest landowner in Minnesota. The MnDNR has a long history of forest planning. Yet the MnDNR began using LP only within the last 10 years. The MnDNR is clearly interested in utilizing this computer-based technology to help explore the almost unlimited number of possible ways of combining stand-level management options to address environmental conditions and resource flows for the forest as a whole. Most MnDNR foresters have minimal knowledge about LP. Minnesota Forest Industries (MFI) has applied LP-based harvest scheduling models to help identify and suggest effective management strategies for MnDNR lands. Results of MFI and MnDNR analyses have had substantial differences in associated forest management schedules, thus leading to questions about what is best for supporting MnDNR planning and management. Recently, the MnDNR and MFI have been working together to better understand their LP modeling differences. Both parties welcomed involvement by the Interagency Information Cooperative (IIC) coordinated by the Department of Forest Resources at the University of Minnesota. This paper lists and briefly summarizes aspects of LP and the forest management situation in Minnesota that are important to consider when applying LP to help support forest management planning. Intent is to keep the paper simple, general, and short, with references provided to more detailed information. Most references are not recent documents, reflecting a fairly basic and longstanding nature of LP applications in forestry. Yet integrating many facets of forest management situations within an LP framework is challenging, with it important for models to be complex enough to adequately address important detail yet still simple enough to be useful.

Harvest scheduling models are economic models

In practice, economics is often confused with financial analyses. Financial analyses only consider market resources while economics analyses address resources in addition to those traded in the market. If economics were simplified to consider only financial resources and financial returns, it is understandable why “economics” would be viewed quite negatively by stakeholders concerned most about environmental conditions of the forest. Economic textbooks emphasize that economics is the science of allocating scarce resources, addressing more than financial returns. The Timber Resource Allocation Model (RAM) (Navon 1971), developed by the USDA Forest Service, is recognized as one of the first widely used harvest scheduling models in forestry. Timber RAM focused primarily on allocating forest resources for timber production with traditional forestwide regulation constraints included to control (sustain) the area and/or timber volume harvested each planning period. Timber RAM was followed by FORPLAN (Iverson and Alston 1986) and later SPECTRUM (USDA Forest Service 2008) a name emphasizing the broad range of forest resources involved. These economic models address forest resource allocation possibilities, recognizing how stand-level decisions, when viewed in combination, impact overall forestwide conditions and flows of resource inputs and outputs. Harvest scheduling models go beyond what we think of as stand-level analysis by coordinating management of the stands in the “forest” to address concerns about resource flows and conditions of the forest as a whole.

The choice of objective function can be critical

Most forest harvest scheduling applications in this country have used an estimate of the net present value of the forest in the objective function. This includes most applications for USDA National Forest planning. Constraints are typically used to address objectives of achieving desired forest age class distributions or sustaining a relatively even flow of harvest volumes and/or harvest area over time. Recent unpublished comparisons done jointly by the MnDNR and MFI showed substantial differences in scheduling results for MnDNR-managed lands based on the objective function used. LP benchmark runs showed substantial differences in optimal harvest timings between a “maximize harvest volume” objective and a “maximize net present value” objective. These benchmark runs were otherwise unconstrained and should be viewed as unrealistic schedules, as they do not address the need to coordinate management across the forest. Yet they can be helpful a starting point in planning to help better understand the initial forest conditions in relation to the objective function assumed. Later in the analysis process, constraints can be added to address important forestwide management objectives and concerns. For example, constraints might be added to force stand age class distributions to move towards a more balanced, steady-state condition.

The “Maximize Volume” LP benchmark run for MnDNR lands suggested it best to hold a large portion of the forest for harvest in the last 5-year planning period (years 70 to 75), as the volume of these stands is not declining over time, and it is not possible to regeneration harvest most of these stands twice within the 75-year planning horizon. Forestwide (MnDNR-wide), the estimated harvest volume for the last period was more than 6 million cords/year! For general comparison purposes, most would agree that the sustainable harvest volume from MnDNR lands is less than 1 million cords annually unless management activities for timber production are intensified substantially. The Maximum Volume LP benchmark thus suggests that the forest, in terms of maximizing total volume harvested during the next 75 years, is best suited for harvesting wood in the future, compared to opportunities today.

This Maximize Volume benchmark is essentially an economic analysis that used a 0% discount rate with no value differences recognized between products such as high-valued saw logs or low-valued pulpwood. It is not surprising that this objective is too simplified from forest industry's perspective. Yet to the MnDNR's defense, political discussions are often simplified, emphasizing the total harvest volume and whether the MnDNR is harvesting their "fair share" of what is needed by existing industry. Maximizing harvest volume is certainly not an objective reflecting environmental values. And any large fluctuations in harvest volume flow levels over time would also likely be a concern. And one must also be concerned about offering for sale large volumes of species that are not currently in great demand in the market. For example, the relatively old and imbalanced age classes for the birch and tamarack forest cover types are a result of a lack of markets for birch and tamarack over the last 50 years or more.

The "Maximum Net Present Value (NPV)" LP benchmark run painted a substantially different picture of the existing conditions of MnDNR lands in terms of timber production values. Results suggest that much of the forest is already financially mature, with it desirable to harvest more than 2.5 million cords/year in the very first 5-year planning period. This NPV benchmark reflects the fact that many MnDNR stands currently have value growth rates well below the assumed discount rate. The high first-period harvest results of the Maximize NPV benchmark also suggest questions about the extent in which harvest levels in earlier periods could be higher than the long-term sustainable level without causing a substantial drop in the long-term sustainable harvest levels. Forest industries in northern Minnesota have struggled to survive in recent years, with several major mill closures. Local communities wonder whether harvesting on public lands could be increased to help mitigate the associated local economic problems without causing environmental problems or long-term timber supply problems. To address this potential opportunity well, it is likely important to consider specific opportunities for specific tree species and products, recognizing the mixed-species nature of stands.

Constraints override the objective function

Often, only one objective is included in the objective function, as it is generally difficult to assign weights for summing values of multiple objectives. So some objectives are typically addressed in the form of model constraints. The objective function is then maximized, only within the context of satisfying these constraints. As such, constraints take priority over the objective function in that if a constraint is not satisfied, then a solution is considered infeasible. Forestwide constraints might be used to: (1) move the forest towards potential long-term desired future conditions for each forest cover type, (2) control or limit the area harvested each period, (3) control or limit the timber volume each period, (4) keep volume or area flows from varying substantially (even-flows) between planning periods, (5) keep scheduled activities within the estimated budget, and (6) maintain, for each period, an acceptable amount of the forest in a condition identified as desirable for quality wildlife habitat. These are only examples of the type of constraints that could be included.

Models can easily become overconstrained

With multiple management objectives and concerns, there may be a tendency to include many constraints or set constraints at very restrictive levels. It is not surprising that there have been many proposed LP formulations where the proposed formulation does not have a single feasible solution. LP experts often encourage not adding specific constraints until it becomes apparent from modeling results that such constraints are necessary. For example, one may start with a simple unconstrained benchmark and sequentially add constraints to it only after it is clear that such constraints are needed or desirable. A real strength of LP is its ability to sort through and compare many potential management combinations for the many stands in the forest. There are usually many plausible combinations at the start of the planning process. Adding constraints, only as needed, avoids locking into prespecified solutions that stakeholders may not agree to be the best solution. If a proposed hard-wired partial solution is found by the model without hard-wiring, then the model helps confirm the solution is a good one. One is not utilizing the strength of LP by going into an analysis with many decisions already made through constraints forced on the model.

Multiple model runs help explore key assumptions

Stakeholders seldom agree on a single best model formulation, especially before seeing the modeling results for a formulation. LP is typically used as a learning tool, where multiple model runs are used to test impacts of potential management policies or strategies. Often, planners explore multiple scenarios for addressing the management situation, trying to make each alternative the best it can be. For example, in the latest forest planning process for Minnesota's National Forests, each National Forest developed seven alternatives that were modeled, with results presented in detail in the public planning documents (USDA Forest Service 2004). Although some of these alternatives were proposed initially by external stakeholders, the planning team refined each so that it could be implemented should it end up as the selected alternative. Comparing modeling results of multiple alternatives can help decision makers better understand tradeoffs between objectives and the consequences of potential forest management policies. The IIC has modeled and compared multiple alternatives to support forest planning for county land departments in Minnesota (Hoganson and Reese 2010).

Shadow prices help to better understand the cost of constraints

A real strength of LP is the shadow price information generated by the solution process for each constraint included in the model. Each shadow price is an estimated rate of change in the objective function for a per unit change in its associated constraint level. For example, suppose the objective is to maximize NPV and a constraint is included that forces storing at least 1000 units of carbon in the forest inventory in period 5. The shadow price associated with this constraint estimates how much the estimated NPV would change for a unit change in this constraint level. Essentially it is also an estimate of the carbon credit needed for period 5 in associated stand-level economic analyses so that 1000 units of carbon are present in the forest in period 5 when the best management schedules for all the stands in the forest are selected and then summed. Too low (high) of a carbon credit would produce less (more) than 1,000 units of carbon. In effect, LP solutions help find the per unit credit needed to achieve the targeted level. Interpreting shadow price estimates as penalties or credits to use in a stand-level economic analysis can be a helpful way for better understanding LP model results (Hoganson and Rose 1984, Davis and Johnson 1987, Paredes and Brodie 1988, Paredes and Brodie 1989). Shadow price estimates help show which constraints are the costly constraints to achieve and which constraints might be made more demanding at relatively low cost in terms of the impact on the optimal value of the objective function. Often, shadow price information is underutilized in forest planning.

Downward sloping demand curves can help balance flows and forest conditions

Questions often come up about whether management objectives should be addressed in the objective function or in the constraints. A disadvantage of including an objective in a constraint is that once the constraint is satisfied, there is no direct incentive to produce more than the minimum required. A potential compromise is to use a downward sloping demand curve (or marginal revenue curve) for a specific objective. This suggestion has been made frequently by stakeholders for USDA Forest Service planning. A downward sloping demand curve allows the user to recognize that the value of each additional unit produced has lower marginal value. For environmental conditions, a downward sloping demand curve can place a relatively high value on forest conditions that are relatively rare or uncommon, and yet place lower value on these conditions as more is added to the solution. For timber market flows, it can help recognize the relatively limited demand for some products. Walker (1971) with his widely recognized ECHO (EConomic Harvest Optimization) model (Walker 1976, Walker 1990), suggested using downward sloping demand curves instead of even-low constraints, emphasizing that by modeling demand/price relationships, results will move the forest towards a more balanced age class distribution without the need for specific even-flow constraints. Hrubes and Navon (1976) present a very straightforward easy-to-understand approach for including demand curves. Duloy and Norton (1975) show how a demand curve can be included by using only a single added constraint for each planning period.

Valuing ending inventory is important

Few disciplines outside of forestry have long production periods that emphasize planning for fifty years or more. Yet simply truncating an analysis for a forest management plan at the “end of the planning horizon” can lead to unintended results. Stand-level economic analyses typically value management options using an infinite planning horizon. This is commonly referred to as a Faustmann approach (Newman 2002), with all proposed management options compared for the same length of time. If one truncates recognition of benefits to be within the planning horizon with a LP model, then the model considers ways of adjusting harvest timings to take advantage of this simplification. For example, the model might suggest holding a stand to a period late in the planning horizon because a second rotation is not possible within the specified planning horizon. Or it might want to harvest a stand early so it can work in two rotations during the planning horizon. One approach often used is to estimate ending inventory values assuming periods beyond the end of the planning horizon are unconstrained. Generally this may overvalue ending inventory, depending on the forestwide constraints used during the planning horizon. For example, ending inventory will likely be overvalued if it is assumed that all of the mature forest in the ending inventory can be cut soon after the end of the planning horizon when in fact at least some of that harvesting would need to be delayed. Ending inventory considerations tend to have greater impact when planning horizons are shorter or when a lower discount rate is used. For example, if the objective is simply to Maximize Volume over the planning horizon with timber harvested in the later planning periods weighted heavily in the objective function, then adjusting harvest timings to harvest heavily in later periods might be an expected result. Another strategy sometimes used is to constrain the model to achieve a specified desirable stand age class distribution at the end of the planning horizon. Concerns could arise with this approach if the planning horizon is short or if there is not general agreement on how to specify the desired ending condition prior to analysis.

Limiting stand-level treatment options recognized can impact results substantially

It is not surprising that with investments in stand treatments (management), stand growth can be increased and such increases can increase long-term sustainable harvest levels for the forest. There is not simply one correct “allowable cut” value. Stand-level management investment options (treatments) might include site restoration or forest cover type conversion options or more stand entries via commercial thinnings or timber stand improvement work. Similarly, more restrictions on minimum and maximum rotation age assumptions can potentially impact allowable cut estimates substantially. It is important to recognize that optimal rotation ages, even without forest-wide constraints, may vary between stands within the same cover type because of differences in site quality (site index), species mix or relative stand density. Numerous intermediate harvest options are also likely plausible with overall forestwide results potentially suffering when such options are omitted from the model. Recognizing stand detail can be important, especially with new silvicultural systems designed to recognize the mixed species nature of stands and the potential to modify that mix. With forestwide constraints and with forest cover types having initial age class distributions that are generally imbalanced initially, optimal management strategies for specific stand types can vary substantially during the planning horizon. For example, stand age class imbalances will almost certainly cause rotation ages to fluctuate as the age class distribution is adjusted over time.

Short-term increases in harvesting (departures) may have little impact on long-term sustainable harvest levels

Historically, “volume control” has often been a focal point forest management plans. With volume control, emphasis is on an “allowable cut” controlling the volume harvested during each planning period. Historically, both LP models and simulation models for forest planning have considered fluctuations in volume harvest levels over time because of differences between existing conditions and the steady state age class distributions of a fully regulated forest (Johnson and Scheurman 1977, Davis and Johnson 1987). Generally, it is assumed too restrictive to constrain harvest volume initially to an estimate of the long-term steady state harvest volume. With a forest that is “old” in terms of financial maturity, it is often found that substantial short-term departures above the estimated long-term level increase the value of the objective function substantially (Johnson 1976). Questions often develop about the opportunity costs of spreading such departures out somewhat evenly over at least part of the planning horizon. Generally, it is considered undesirable to have harvest levels drop substantially below the long-term sustainable level anytime during any period in the planning horizon. If the existing forest age class distribution is imbalanced with more young stands, some relatively low harvest levels may be necessary in some periods. Situations can also benefit substantially if coordinating harvest flows variations across forest cover types is acceptable. For example, a forest with substantial older aspen and substantial recently planted red pine might provide a substantially higher flow of revenue over time if some fluctuations on volumes by species are acceptable between planning periods.

The economics may be substantially different when viewed from a forestwide perspective

Historically, stand-level financial analyses of forestry investments have often appeared marginal at best. Generally these investments involve large up-front stand establishment costs with most financial benefits not occurring until the end of the rotation—many years into the future. In a stand-level financial analysis, the impact of discounting the future returns to present value can make the overall NPV estimate low if not negative. But when considered from a forestwide perspective, investments can potentially increase forestwide allowable cuts well before the end of the rotation, making the NPV estimates of some investment options substantially greater. This change in value is referred to as an allowable cut effect (Schweitzer et al. 1972). Similar forestwide effects of investments are plausible involving forestwide management constraints in addition to allowable cut constraints. Recognizing these potential forestwide impacts is important. One would not want to reduce management choices substantially based on simple stand-level economic analyses that don't consider potential impacts of treatment options within a forestwide perspective.

Potential gains from thinking “big” in defining “the forest”

Not all that long ago, the MnDNR developed forest plans separately for major MnDNR forest administration units. Those units did not follow ecological land type boundaries so it was difficult to address the condition of specific ecological units that crossed administrative boundaries. As a result, changes were made in the planning process. Currently, the MnDNR develops forest management plans based on large ecological subsections. An entire subsection is considered together, as then ecological conditions of the subsection in specific planning periods can be addressed directly in the model. Forest industry suggests that industry needs do not partition nicely into ecology subsections, with concerns about wood supplies for specific mills.

Mill wood procurement zone boundaries shift over time for specific mills and tree species. Addressing specific market concerns would almost certainly be easier if ecological subsections are all considered simultaneously. That does not mean that constraints could not be included to address specific ecological conditions of each ecological subsection. The potential gain is that specific timber harvest volume flows could be coordinated across ecological subsections, without assuming that flows by ecological subsection need to be an even flow over time. The Minnesota statewide generic environmental impact statement on timber harvesting and forest management (Jaakko Pöyry Consulting, Inc.1994) was developed because it is difficult to assess wood supply situations for specific markets without considering interactions between markets.

Large potential inefficiencies from not coordinating management policies across forest ownerships

A challenge of forest management is to integrate multiple management objectives. Forest management organizations, whether private or public, tend to have different perspectives on the relative value of each management objective. Yet stands and underlying soils and locational characteristics are inherently different in terms of their potential to help achieve specific forest objectives. It is potentially quite inefficient for simplifications like, “the USDA Forest lands should emphasize the environmental benefits and the industry lands should emphasize the timber benefits.” Much can be gained from coordinating production in terms of relative values assumed in planning (Bixby 2006). Certainly some value differences between forest ownerships seem appropriate, but it is inefficient when one ownership must assume high values for a condition to reach relatively large production levels when another ownership might be capable of producing similar conditions at substantially lower per unit cost. Coordinating at a broader policy level can have large benefits (Hoganson et al. 2003, Bixby 2006).

Although likely time-consuming and expensive, the overall value of detailed, forestwide analyses can be large

It takes time to develop a solid forestwide management scheduling model for a forest. Data needs are large. Substantial detail is typically involved, including a range of potential management investments and intensities for a wide variety of stand types and species mixes. Models need to be complex enough to adequately describe the situation. Typically, every stand in the forest is unique in at least some important dimension. Fortunately, advances in computer technologies allow modeling to recognize enormous detail. Linkages to GIS tools help make interpretation of results, implementation, and revision easier, with planning often needing to be more of a continuous process. LP formulations can easily involve tens of thousands of stands and literally millions of unique stand-level management options. Forest planning is about much more than just timber production and allowable cut estimates. The number of possible unique stand management combinations for the forest as a whole is almost limitless. In simple terms, it is much like picking a set of lottery numbers. However, rather than just pick five or six numbers, managers are often picking “numbers” for thousands of stands. Models can help lay out a systematic way for addressing, searching through and integrating the many plausible combinations. Forests cover large areas and support local communities in multiple ways.

Caution needs to be exercised about "writing off" planning by considering estimated gains only in simple percentage terms. Often even only a 1% increase in forest value from detailed planning can translate into millions of dollars in improvement. This should make it an easy decision to invest in modeling, assuming organizations have the budget and time to plan ahead.

It is also important to realize that problems are more complex than even our best models. "Optimization" models will not replace the manager. Rather models are valuable tools to help find good solutions and to help better understand likely consequences of plausible policies and associated tradeoffs between alternative management decisions. With increases in world populations and associated increases in demands on forests and landscapes in general, the value and importance of detailed decision-support tools are almost certain to increase. Forest organizations need to realize the challenge of developing good model formulations, with it seldom plausible to just follow a simple modeling guide to develop a good forest plan based on a single model formulation.

Forest planning today uses computer modeling tools from operations research. Operations research is the science of decision-making. Management situations often fit a general operations research problem structure like a "LP problem" or a "dynamic programming problem." Yet operations research texts describe the process of developing operations research applications as much an art as a science. This certainly applies to forestry, with all the complexities and long-term nature of forest management situations.

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