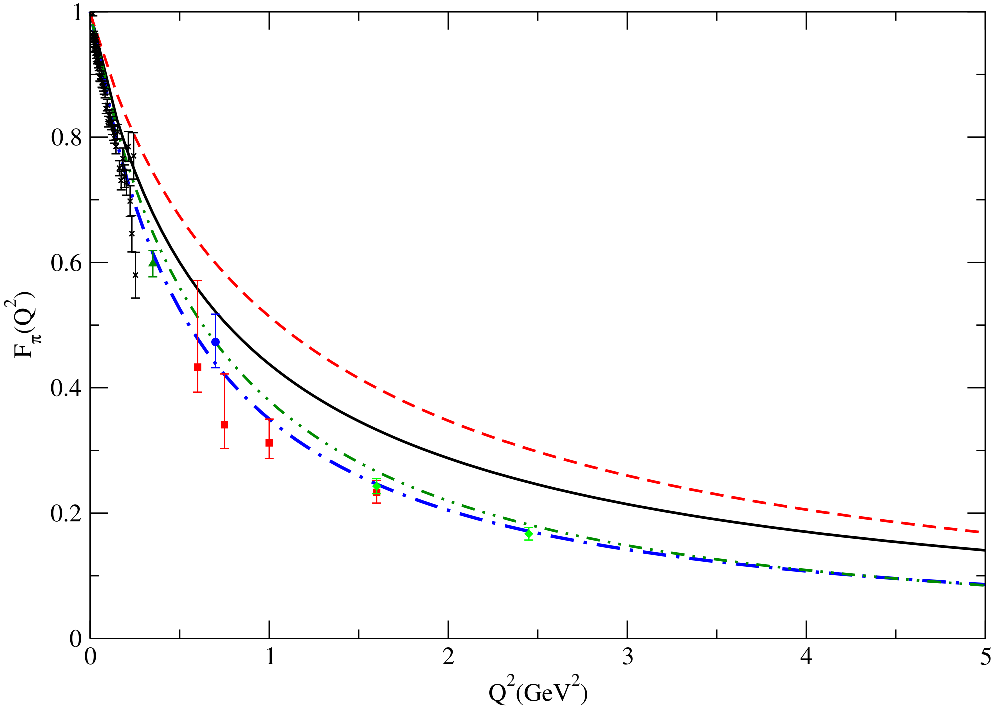
The Pion Form Factor in AdS/QCD



Richard Lebed
Arizona State University

CAQCD 08

May 22, 2008

In collaboration with Herry J. Kwee

- JHEP **0801**, 027 (2008) [arXiv:0708.4054]
- arXiv:0712.1811 [accepted by Phys. Rev. D]

Outline

1) Holographic QCD and chiral symmetry breaking
2) Meson masses, decay constants, form factors
3) Numerical vs. analytic results
4) Hard-wall and soft-wall results
5) A semi-hard wall model
6) Perfecting the holographic model for $F_{\pi}(Q^2)$

AdS/QCD on one slide

- Conjecture: To every strongly-coupled Yang-Mills theory corresponds a weakly coupled gravity theory on curved spacetime
- "Sliced" 5D Anti-de Sitter metric:

$$ds^2 = g_{MN}\, dx^M dx^N = \frac{1}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

- UV limit of YM theory (conformal) corresponds to $z = \varepsilon \to 0$ limit
- Crudely speaking, z corresponds to $1/Q$ (momentum)
- Every YM operator $\mathcal{O}(x)$ is associated with (*i.e.*, sources) a field $\Phi(x,z)$ that is uniquely determined by $\Phi(x,\varepsilon)$ ("holography")
- IR behavior of YM theory: Behavior of $\Phi(x,z)$ for $z > 0$ ("bulk")
- For QCD: Global isospin symmetry → gauged SU(2) in bulk

Holographic Correspondence

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$
$\bar{q}^\alpha_R q^\beta_L$	$(2/z) X^{\alpha\beta}$

$$V^M \equiv \tfrac{1}{2}(A^M_L + A^M_R)$$

$$A^M \equiv \tfrac{1}{2}(A^M_L - A^M_R)$$

$$X = X_0 \exp(2i\pi^a t^a)$$

$$F_V^{MN} \equiv \partial^M V^N - \partial^N V^M - i\left([V^M, V^N] + [A^M, A^N]\right)$$

$$F_A^{MN} \equiv \partial^M A^N - \partial^N A^M - i\left([V^M, A^N] + [A^M, V^N]\right)$$

$$D^M X = \partial^M X - i[V^M, X] - i\{A^M, X\}$$

$X_0 \equiv \tfrac{1}{2} v(z)$: Determined by pattern of chiral symmetry breaking

$$S = \int d^5x \, e^{-\Phi(z)} \sqrt{g} \, \mathrm{Tr}\left\{ |DX|^2 + 3|X|^2 - \frac{1}{2g_5^2}(F_V^2 + F_A^2) \right\}$$

Equations of motion (Fourier transforms), axial-like gauges $V_z = A_z = 0$,

$A_\mu = A_{\mu\perp} + \partial_\mu \varphi$

$$\partial_z \left(\frac{e^{-\Phi(z)}}{z} \, \partial_z V_\mu^a \right) + \frac{q^2 e^{-\Phi(z)}}{z} V_\mu^a = 0 \, ,$$

$$\left[\partial_z \left(\frac{e^{-\Phi(z)}}{z} \, \partial_z A_\mu^a \right) + \frac{q^2 e^{-\Phi(z)}}{z} A_\mu^a - \frac{g_5^2 \, v(z)^2 e^{-\Phi(z)}}{z^3} A_\mu^a \right]_\perp = 0 \, ,$$

$$\partial_z \left(\frac{e^{-\Phi(z)}}{z} \, \partial_z \varphi^a \right) + \frac{g_5^2 \, v(z)^2 e^{-\Phi(z)}}{z^3} (\pi^a - \varphi^a) = 0 \, ,$$

$$- q^2 \partial_z \varphi^a + \frac{g_5^2 \, v(z)^2}{z^2} \, \partial_z \pi^a = 0 \, ,$$

$$\partial_z \left(\frac{e^{-\Phi(z)}}{z^3} \partial_z X_0 \right) + \frac{3 e^{-\Phi(z)}}{z^5} X_0 = 0 \, .$$

Meson masses & form factors

- Meson masses/wave functions are obtained as Sturm-Liouville eigenvalues/eigenvectors of equations of motion → Kaluza-Klein towers reminiscent of Regge trajectories
  - *e.g.*, for hard wall, V^μ eqn. of motion is solved by Bessel functions
  - Masses are related to zeroes of Bessel functions, decay constants to (derivatives of) wave functions as $Q^2 \equiv -q^2 \rightarrow 0$
- Source currents of given quantum numbers are free-field solutions to equations of motion
  - *e.g.*, for hard wall, V^μ source is modified Bessel (Macdonald) function
- Form factors are overlap integrals (in z) of mode solutions (for external particles) with source current solution

Pion EM form factor $F_\pi(Q^2)$

- Pion wave function is lowest-mass mode of field $\pi(q^2,z)$, which appears in coupled equations of motion with axial vector longitudinal mode $\varphi(q^2,z)$. Then

$$F_\pi(q^2) \equiv F_{11}(q^2) = \int dz\, e^{-\Phi(z)}\, \frac{V(q,z)}{f_\pi^2} \left\{ \frac{1}{g_5^2 z} [\partial_z \varphi(z)]^2 + \frac{v(z)^2}{z^3} [\pi(z) - \varphi(z)]^2 \right\}$$

- Or, in the timelike region, $F_\pi(q^2) = -\sum_{n=1}^{\infty} \frac{f_n g_{n\pi\pi}}{q^2 - M_n^2}$

  where $g_{n\pi\pi}$ are Yukawa couplings given by

$$g_{n\pi\pi} = \frac{g_5}{f_\pi^2} \int dz\, \psi_n(z)\, e^{-\Phi(z)} \left\{ \frac{1}{g_5^2 z} [\partial_z \varphi(z)]^2 + \frac{v(z)^2}{z^3} [\pi(z) - \varphi(z)]^2 \right\}$$

Background field choices

$$e^{-\Phi(z)} = H(z_0 - z)$$

Hard-wall background
J. Polchinski and M.J. Strassler
PRL **88**, 031601 (2002)

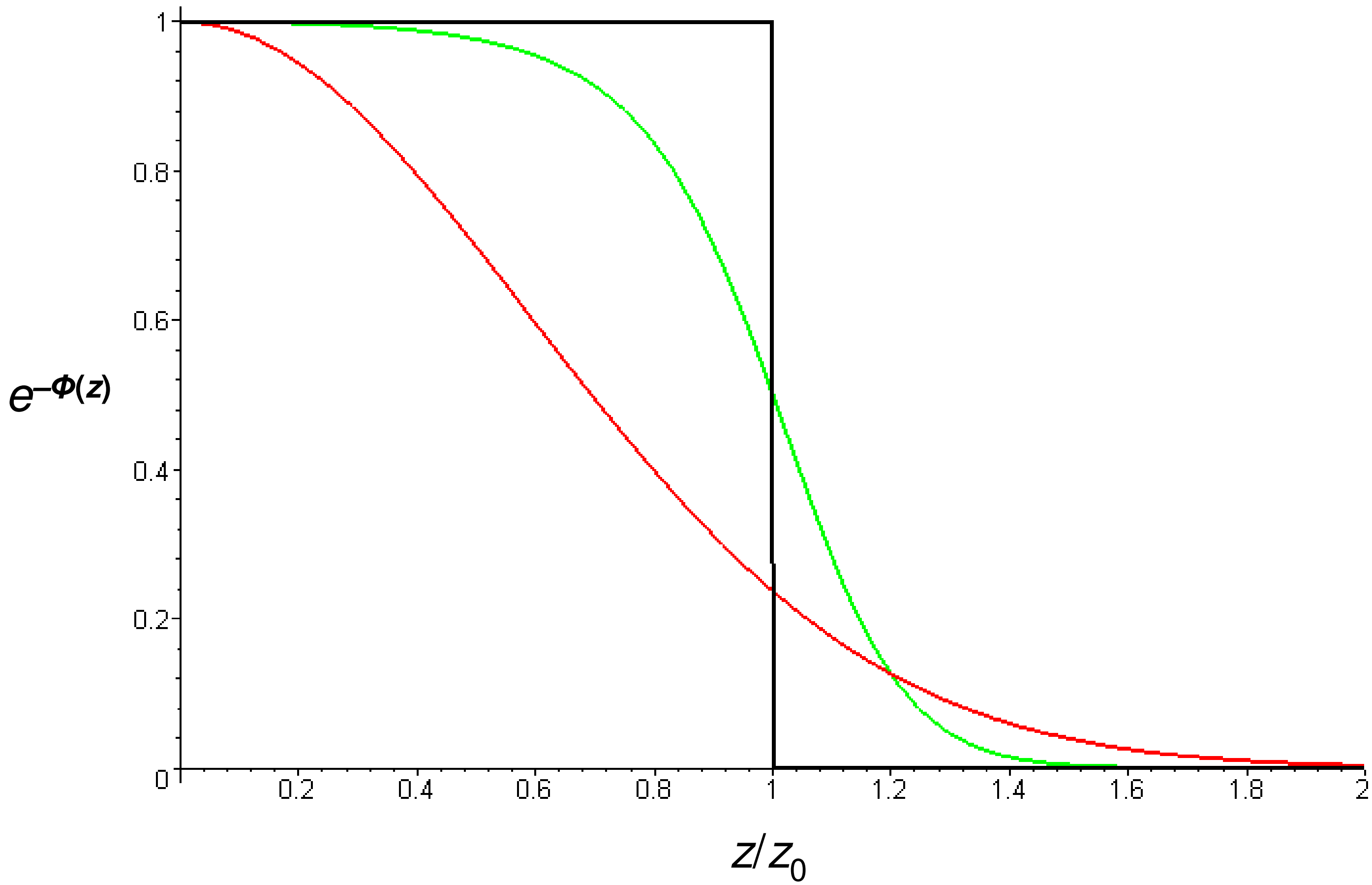
$$e^{-\Phi(z)} = e^{-\kappa^2 z^2}$$

Soft-wall background
A. Karch, E. Katz, D.T. Son, M.A. Stephanov
PRD **74**, 015005 (2006)

$$e^{-\Phi(z)} = \frac{e^{\lambda^2 z_0^2} - 1}{e^{\lambda^2 z_0^2} + e^{\lambda^2 z^2} - 2}$$

Semi-hard or "Saxon-Woods" background
HJK and RFL
JHEP **0801**, 027 (2008)

Hard, soft, semi-hard background fields



Advantages: hard vs. soft wall

- Behavior of meson masses for hard wall model: $m_n^2 \sim n^2$
- Semiclassical flux tube QCD reasoning: $m_n^2 \sim n^1$ [M. Shifman, hep-ph/0507246]
  - Supported by empirical evidence
- Soft-wall model: $m_n^2 \sim n^1$, *by construction*
- On the other hand, the hard-wall model tends to give a better numerical fit to low-energy observables [H.R. Grigoryan and A.V. Radyushkin, Phys. Rev. D**76** 095007 (2007)]
  - *e.g.*, m_ρ^2/f_ρ = 5.02±0.04 (expt); 5.55 (hard wall); 8.89 (soft wall)

- Do we really need a fully soft wall, or just one with a tail?

Explicit chiral symmetry breaking

J. Erlich, E. Katz, D.T. Son, M.A. Stephanov, PRL **95** (2005) 261602

- For the hard wall, solutions to X_0 equation of motion are z, z^3
- In the gauge/gravity correspondence:
  - Operator source ↔ Non-normalizable mode (more singular as $z \to 0$)
  - States, vevs ↔ Normalizable mode (less singular as $z \to 0$)
- Chiral symmetry breaking: Source is quark mass and vev is the sigma parameter:

$$X_0(z) \equiv \frac{1}{2} v(z) = \frac{1}{2} m_q z + \frac{1}{2} \sigma z^3$$

- Soft wall: Exact solutions are Kummer (confluent hypergeometric) functions, but only one solution vanishes asymptotically (satisfying boundary conditions), and hence has fixed z, z^3 coefficients → Fixed m_q to σ ratio? [A. Karch, E. Katz, D.T. Son, M.A. Stephanov PRD **74**, 015005 (2006)]

Patches to the soft-wall model

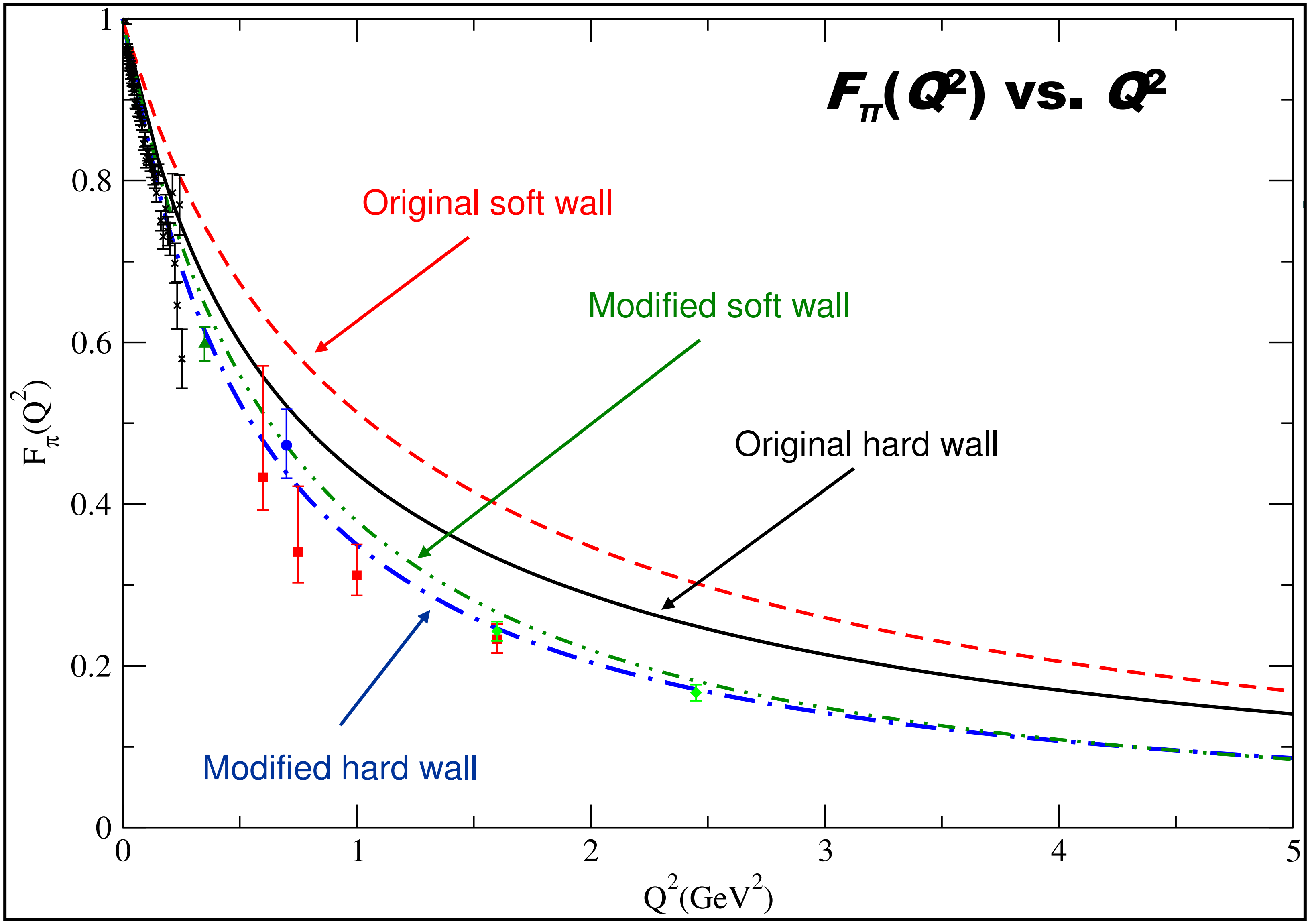
- Karch *et al*. comment: Higher-order terms in potential allow for independent low-z (z and z^3) coefficients
- Practical implementation #1 (HJK & RFL, JHEP): Background field $\exp(-\kappa^2 z^2)$ suppresses distinction between true $X_0(z)$ and $\frac{1}{2}(m_q z + \sigma z^3)$ as $z \to \infty$ ⇨ Use hard-wall $X_0(z)$
- Practical implementation #2 (HJK & RFL,PRD): Choose a background field that behaves like $\frac{1}{2}(m_q z + \sigma z^3)$ for small z but like asymptotic behavior of Kummer function for $z \to \infty$:

$$2X_0(z) = \left(m_q\, z + \sigma\, z^3\right)\left[1 - \exp\left(-\frac{A_c}{\kappa^4\, z^4}\right)\right] + B_c \exp\left(-\frac{3}{4\,\kappa^2\, z^2}\right)$$

- In our simulations, this "exact" $X_0(z)$ never gives results better than the hard-wall $X_0(z)$ (and agree for optimal parameters)

Numerical or analytic solution?

- Coupled differential equations
- At least 3 adjustable parameters, z_0 or κ, m_q, σ
  - No closed-form analytic solutions exist
- If m_q is set to 0, then analytic solutions have been found in hard-wall model [H.R. Grigoryan and A.V. Radyushkin, PRD **76**,115007 (2007)]
- If $m_q \neq 0$, one must resort to numerical solutions [HJK & RFL]
  - *Numerical Recipes*: Shooting method, numerical integration, etc.
- We have done this for hard, soft, semi-hard wall backgrounds



$F_\pi(Q^2)$ vs. Q^2
Original soft wall
Modified soft wall
Original hard wall
Modified hard wall
$F_\pi(Q^2)$
$Q^2(\mathrm{GeV}^2)$
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5

Rules of thumb for parameters

- z_0 (hard wall) or κ (soft wall) is completely fixed by m_ρ
- Given z_0 or κ, f_π is primarily determined by σ
- $m_q\sigma$ is fixed by Gell-Mann–Oakes–Renner formula:

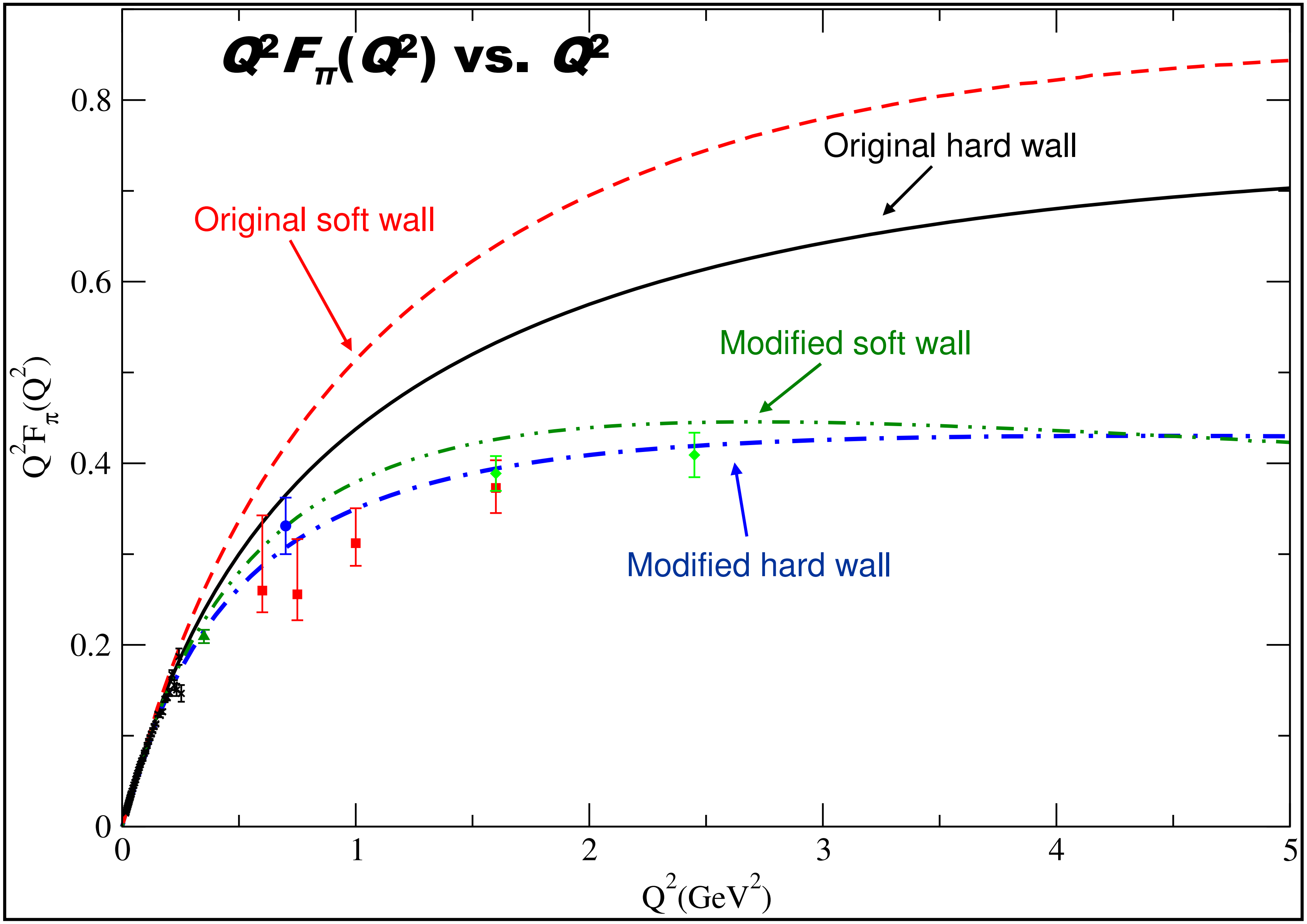
$$m_\pi^2 f_\pi^2 = 2m_q\sigma.$$

→ m_π is fixed by scaling m_q

- Empirical fact: Shape of $F_\pi(Q^2)$ driven most by σ

Model fit parameters

- Original hard wall:
  - z_0 = 1/(322 MeV), σ = (326 MeV)^3, m_q = 2.30 MeV
  - m_ρ = 775.3 MeV, m_π = 139.6 MeV, f_π = 92.1 MeV
- Original soft wall:
  - κ = 389 MeV, σ = (368 MeV)^3, m_q = 1.45 MeV
  - m_ρ = 777.4 MeV, m_π = 139.6 MeV, f_π = 87.0 MeV
- Modified hard wall:
  - z_0 = 1/(322 MeV), **σ = (254 MeV)^3**, m_q = 2.32 MeV
  - m_ρ = 775.3 MeV, m_π = 139.6 MeV, **f_π = 64.2 MeV**
- Modified soft wall:
  - κ = 389 MeV, **σ = (262 MeV)^3**, m_q = 1.47 MeV
  - m_ρ = 777.4 MeV, m_π = 139.6 MeV, **f_π = 52.2 MeV**

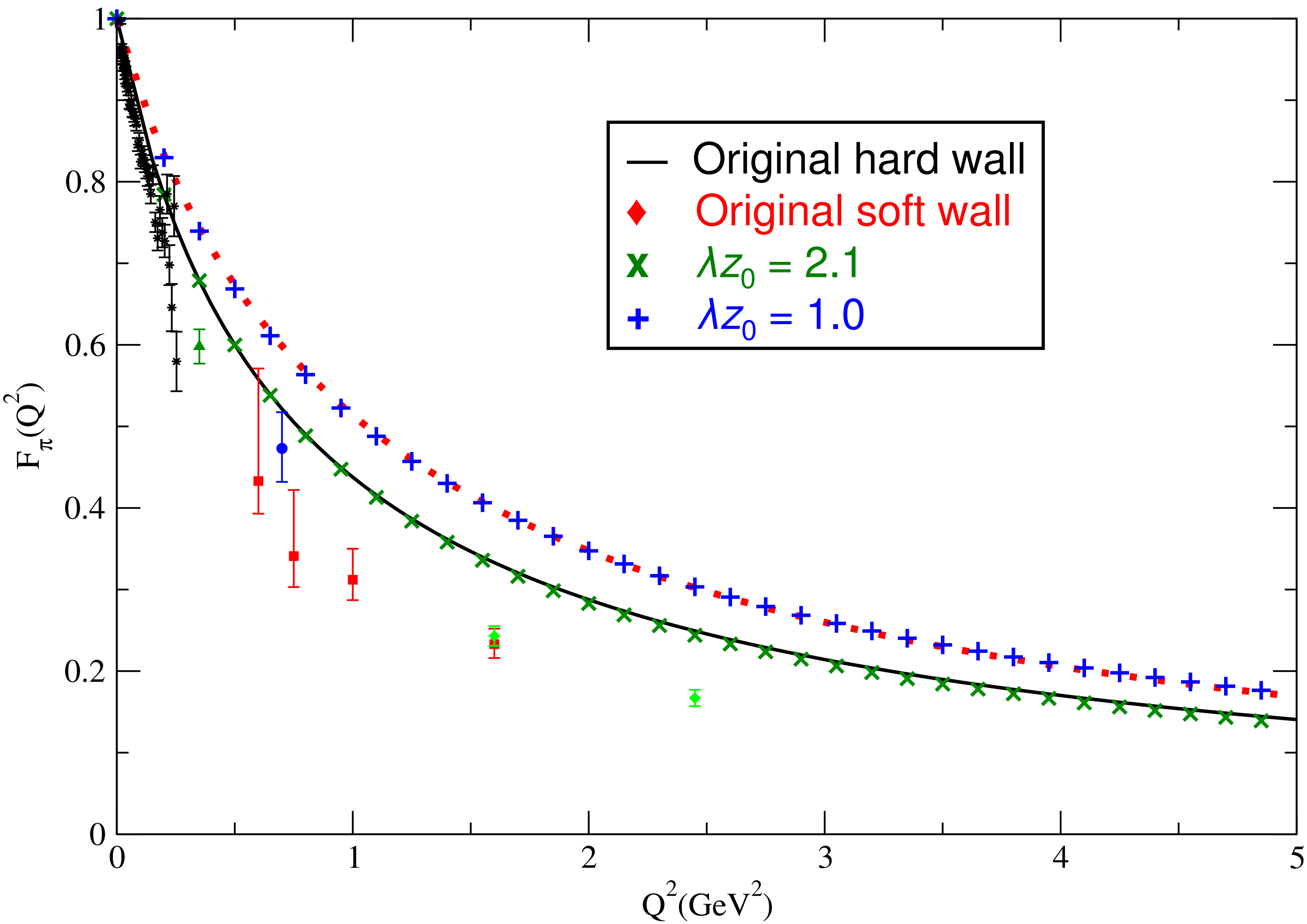


$Q^2 F_\pi(Q^2)$ vs. Q^2
Original soft wall
Original hard wall
Modified soft wall
Modified hard wall
$Q^2 F_\pi(Q^2)$
$Q^2 (GeV^2)$
0
0.2
0.4
0.6
0.8
0
1
2
3
4
5

Asymptotic behavior of $F_\pi(Q^2)$

- Define the conventional parameter $s_0 \equiv 8\pi^2 f_\pi^2 \sim 0.67 \text{ GeV}^2$, where $f_\pi = 92.4$ MeV
- *To the eye*, it appears that $Q^2 F_\pi(Q^2) \to \sqrt{2}\, s_0$ (constant)
- Very different from the partonic behavior, where $Q^2 F_\pi(Q^2)$ scales as $\alpha_s(Q^2) f_\pi^2$ [G.P. Lepage and S.J. Brodsky, PLB **87**, 359 (1979); A.V. Efremov and A.V. Radyushkin, Theor. Math. Phys. **42**, 97 (1980)]

  [Note: Grigoryan & Radyushkin find $Q^2 F_\pi(Q^2) \to s_0$, but they use $m_q = 0$, and also they find that this asymptote is *overshot* and not obtained until very large Q^2.]



Original hard wall
Original soft wall
λz_0 = 2.1
λz_0 = 1.0
$F_\pi(Q^2)$
$Q^2(GeV^2)$
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5

Semi-hard approx. to hard wall

Observable	Experiment	Hard wall	$\lambda z_0 = 2.1$
m_π	139.6±0.0004 [25]	139.6	139.6
m_ρ	775.5±0.4 [25]	775.3	777.5
m_{a_1}	1230±40 [25]	1358	1343
f_π	92.4±0.35 [25]	92.1	88.0
$f_\rho^{1/2}$	346.2±1.4 [26]	329	325
$f_{a_1}^{1/2}$	433±13 [27, 28]	463	474
$g_{\rho\pi\pi}$	6.03±0.07 [25]	4.48	4.63

Semi-hard approx. to soft wall

Observable	Experiment	Soft-wall	$\lambda z_0 = 1$
m_π	139.6±0.0004 [25]	139.6	139.6
m_ρ	775.5±0.4 [25]	777.4	779.2
m_{a_1}	1230±40 [25]	1601	1596
f_π	92.4±0.35 [25]	87.0	92.0
$f_\rho^{1/2}$	346.2±1.4 [26]	261	283
$f_{a_1}^{1/2}$	433±13 [27, 28]	558	576
$g_{\rho\pi\pi}$	6.03±0.07 [25]	3.33	3.49

Vector mass trajectories

Original hard wall:
$m_n^2 \sim n^2$

Semi-hard wall, $\lambda z_0 = 2.1$:
$m_n^2 \sim n^1$

m_ρ	m_ρ
775.6	777.5
1780.2	1608.1
2790.8	2226.8
3802.8	2637.5
4815.2	2986.6

Why are all $F_\pi(Q^2)$ curves too shallow?

- This holographic model flawed because no partonic degrees of freedom?
  - *But we are only looking at Q^2 data out to* ~3 GeV^2
- EKSS treatment of chiral symmetry breaking flawed?
  - *Possible, but it is the most realistic treatment available*
- Some of the numerical inputs are inconsistent with the assumptions of the holographic method
  - *In particular, the implicit assumption of large* N_C
  - *Which fit observable is most sensitive to varying* N_C*?*

$1/N_c$ to the rescue

- Meson masses m_π , m_ρ are $O(N_C^{\,0})$, while the decay constant f_π is $O(N_C^{\,1/2})$
- Recall that hard-wall fit to $F_{\boldsymbol{\pi}}(Q^2)$ at low Q^2 is excellent if f_π is ~64 MeV, i.e., about $1\text{-}1/N_C$ smaller
- Has such an $O(1/N_C)$ effect been seen before?
  - Yes! Adkins, Nappi, Witten: *Nucl. Phys.* ***B228*** *(1983) 552*
  - Goldberger-Treiman relation: $f_\pi = M_N g_A / g_{\pi NN}$ (our norm)
  - Predicts f_π ~ 61MeV (ANW actually predict $g_{\pi NN}$)

The “perfect” model

- The hard-wall model gives a better fit to most static observables, but has the wrong meson mass trajectories
- The semi-hard wall gives a fit just as good as the hard wall model but the correct meson mass trajectories
- However, both of these give the same, too-shallow fit to $F_{\pi}(Q^2)$.
  - But this can be cured by allowing f_π to be smaller by a $1/N_C$ correction
- Together, these two modifications give good agreement with the low- and medium-energy meson data
- But “perfect” is still not perfect...Where are the partons? (The only matching to real QCD enters through the asymptotic vector current correlator, and gives $g_5 = 2\pi$.)

Summary and Prospects

1) Holographic methods give a compelling conceptual framework for studying hadronic quantities, including $F_\pi(Q^2)$

2) Choice of background field behavior has a strong effect on observables, but it is possible to retain many of the best features of each model while adjusting this background

3) The Erlich *et al*. treatment of chiral symmetry breaking appears suitable for describing hadronic observables, but by itself appears to predict too shallow behavior for $F_\pi(Q^2)$

4) It appears that $1/N_C$ corrections are necessary for a fully satisfactory comparison with data

How shall we knit the essential partonic physics onto this promising hadronic start?