Evaluating FVS Predictions in Red Pine Stands at Cloquet, MN

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Abstract:

Data from a remeasured *Pinus resinosa* (red pine) spacing study were used to compare Forest Vegetation Simulator (FVS) predictions of diameter at breast height (DBH) and volume growth at both the stand and the individual tree level. Four scenarios were run, differing by whether FVS was used to predict DBH growth, height and/or volume. The results from the scenarios differed depending on whether the evaluation was at the stand or tree level. Using FVS heights instead of modelled heights produced a negative bias in volume prediction at the tree and stand level. Using FVS predicted DBHs produced a very large error in volume and DBH at the tree level, but its effect at the stand level varied by spacing; FVS usually over-predicted DBH growth. This led to over-prediction of volumes at narrower spacings. This was particularly true for trees of smaller DBH. FVS did not show bias in the middle spacings. The bias across all spacings and reps was 1.3% of initial DBH, and 8.6% of mean DBH growth. Across all spacings, FVS underestimated the spread of DBH growth.

Introduction:

Red pine is commonly planted in rows of various spacing on lands managed for forest products. A currently recommended spacing is around 6 by 8 feet (USDA, 2005). Several growth and yield models have been used to estimate red pine volume and growth, a common model being the Forest Vegetation Simulater (FVS). Whenever such growth models are used, it is important to evaluate their accuracy, and the nature of their errors. One application of such a model would be to estimate future volume of a current stand. In this study, DBH and height were measured for the years 2004 and 2011 of a planting done in 1981, providing data to compare what growth a model would predict over that remeasured interval. The Lake States variant of the FVS growth model was used in this study, and it has previously been found to be biased towards over-prediction of DBH growth. (Pokharel and Froese, 2008; Smith-Mateja and Ramm, 2002). Pokharel and Froese reported an over prediction of 52% of growth over a

10 year period, and Smith-Mateja and Ramm reported an over-prediction of 0.45 inches on the red pine control treatment in Kellogg forest over a period of 8 years.

Methods:

The study site was at the University of Minnesota's Cloquet Forestry Center near Cloquet, MN, with approximate coordinates 46°42'N and 92°32'W. The soil type, as classified by the USDA is omega loamy sand (USDA, 2014). In May of 1981, bare-root stock red pine were planted in three reps of five spacing treatments. The different spacings were 6x7, 8x8, 8x10, 10x10, and 13x13. The trees were planted in a grid pattern, and the spacings were labeled in feet of distance in one direction by feet of distance the other direction. Hence, in the grid of trees in the 8x10 spacing, rows of trees would be 8 feet apart, "columns" of trees would be 10 feet apart. Every planted red pine tree more than two rows from the edge was tagged. Following large first-year mortality, a replant was done in 1982. After the growing seasons of 2004 and 2011, DBH was measured on all live trees in the area. DBH was measured with a DBH tape and 4.5 foot pole. When a deformity or fork occurred at 4.5 feet, DBH was measured just above the deformity or fork. Tree height and crown height were measured in every rep of every spacing for a subset of trees chosen by systematically selecting trees from a list sorted by DBH, always including both the tree of smallest and largest DBH. Trees that field personnel thought to have an unusual height for their DBH had both height and DBH recorded separately. All 2004 heights were measured with either a Criterion 400 or Impulse 200 instrument (Table A1 in the appendix).

When using both the Criterion 400 and Impulse 200, the instrument was mounted on a monopod and a reflector was used to increase the accuracy of horizontal distance measurements, being placed directly under the top of the tree. Measurements were taken from a location where the reflecter, top of the tree, and the base of the crown were visible. In instances where the base of the tree was not visible, the height to the reflector was recorded and added to the instrument's height to get the actual tree height. For leaning trees, the direction from the tree to the measuring location was always perpendicular to the direction of the tree's lean. The 'base of the crown' was defined using the accumulation method. Two live branches were necessary for a whorl to be considered alive. If the whorl had only one live branch, that branch was moved up to the next whorl when considering whether the next whorl was alive. The lowest live whorl would then count as the base of the crown. If there was a whorl with no live branches above a whorl with one live branch, the midpoint between the completely dead whorl and the next live whorl was used. Dead branches of some study trees were pruned in the January 2005. Some plots also had dead branches removed in 2003 and 2004.

The DBH and height data were entered into Excel and R. In R, a linear, mixed-effects model between the natural log of (height - 4.5) and the inverse of DBH was used to estimate the heights of the trees that did not have measured heights. The trees which were recorded separately due to an unusual DBH-height relationship were not used in model fitting. Many mixed-effects models are possible by changing which coefficients are random effects, and whether they are different by spacing or rep. The best mixed-effects model was selected by lowest AIC. The model used to estimate volume from height and DBH is given in Ek (1985). Quadratic mean DBH was also calculated in R. In Excel, per-acre statistics were calculated defining each rep within each spacing as a fixed area plot. Two FVS simulations, one with 2004 heights from the mixed-effects model and measured heights, and a simulation with only 2004 measured heights were first run, treating each tree as a proportionally small plot. The results from the second were compared to results from the first FVS simulation. In all FVS simulations, each rep in each spacing was considered a stand. FVS's predicted DBHs, heights, and volumes for each tree were used to develop four scenarios for arriving at 2011 tree volume of a tree from 2004 measured DBH (Table 1). This allowed determining the magnitude of the sources of error in 2011 FVS predictions.

Table 1. The four scenarios used to predict 2011 volume for a tree from 2004 DBH:

Scenario	2004 diameter → 2011 diameter	2011 Diameter → Height	Diameter + Height → Volume
V1	FVS	FVS	FVS
V2	FVS	FVS	Ek, 1985
V3	FVS	Mixed-effects Model	Ek, 1985
V4	Measured	Mixed-effects Model	Ek, 1985

The V4 scenario was treated as "observed" for the purposes of this study. The effect of using FVS to predict DBH can be quantified by comparing V3 to V4. The effect of using FVS to predict height can be quantified by comparing V2 to V3. Similarly, comparing V1 to V2 can reveal differences between FVS and the equation in Ek (1985). Additionally, the combined effects of using FVS to predict both DBH and height can be quantified by comparing V4 to V2, and the difference between using FVS for all predictions versus not using FVS can be observed by comparing V1 to V4.

Results:

The mixed-effects model with the lowest AIC had random effects for each rep within each spacing. The model coefficients (R code in appendix) indicate that a tree of the same DBH would be tallest in the tightest spacings. In general, tighter spacings had higher per-acre volumes, wider spacings had larger quadratic mean DBHs, and survival and growth were highest in the middle spacings (Table 2). The distributions of DBHs were fairly Normal, with skew in the 2011 DBHs being more negative than the skew in the 2004 DBHs (Figure 1).

Adding the modeled 2004 heights to the measured height for un-measured trees into FVS made the

FVS height predictions more similar to the mixed effects model for 2011, having a root mean square height difference of 4.93 ft versus 6.35 ft across all spacings. This is consistent with findings of Trincado et al. (2007), Temesgen et al. (2008), Huang et al. (2009) and VanderSchaaf (2011) in concluding that calibrated mixed-effects models yield more accurate predictions than regionwide height models. In all of the V1, V2, V3, and V4 scenarios, FVS used data with both measured and modeled 2004 heights.

Table 2. Summary statistics of the plots and reps.

Spacing	Rep	% Alive		Trees/Acre		BA(ft^2/acre)		QMD		Volume(ft^3/acre)	
		2004	2011	2004	2011	2004	2011	2004	2011	2004	2011
6 x 7	1	76%	73%	759	730	159	196	6.2	7.0	2538	3819
6 x 7	2	62%	62%	709	666	159	195	6.4	7.3	2614	4031
6 x 7	3	68%	66%	720	682	186	234	6.9	7.9	3062	4850
6 x 7	**w.a.m.[a]**	**68%**	**67%**	**728**	**691**	**168**	**208**	**6.5**	**7.4**	**2734**	**4226**
8 x 8	1	74%	73%	710	622	129	164	5.8	7.0	1859	3095
8 x 8	2	87%	87%	789	670	177	220	6.4	7.8	2670	3382
8 x 8	3	69%	67%	574	476	158	192	7.1	8.6	2579	4052
8 x 8	**w.a.m.[a]**	**76%**	**75%**	**687**	**587**	**154**	**191**	**6.4**	**7.7**	**2359**	**3514**
8 x 10	1	76%	72%	454	413	119	149	6.9	8.1	1803	2838
8 x 10	2	84%	84%	457	457	136	176	7.4	8.4	2242	3554
8 x 10	3	70%	68%	438	399	155	200	8.1	9.6	2452	4148
8 x 10	**w.a.m.[a]**	**77%**	**75%**	**449**	**423**	**137**	**175**	**7.5**	**8.7**	**2170**	**3521**
10 x 10	1	76%	76%	570	384	136	163	6.6	8.8	1935	3227
10 x 10	2	76%	76%	596	498	120	149	6.1	7.4	1682	2737
10 x 10	3	69%	69%	363	332	141	181	8.5	10.0	2315	3798
10 x 10	**w.a.m.[a]**	**74%**	**74%**	**514**	**409**	**132**	**163**	**6.9**	**8.6**	**1962**	**3227**
13 x 13	1	84%	84%	247	227	85	116	8.0	9.7	1084	2252
13 x 13	2	68%	68%	237	217	83	109	8.0	9.6	1232	2048
13 x 13	3	56%	56%	144	144	62	90	8.9	10.7	970	1726
13 x 13	**w.a.m.[a]**	**69%**	**69%**	**210**	**196**	**77**	**105**	**8.2**	**9.9**	**1095**	**2008**

[a] w.a.m. stands for weighted (by area of the rep) arithmetic mean.

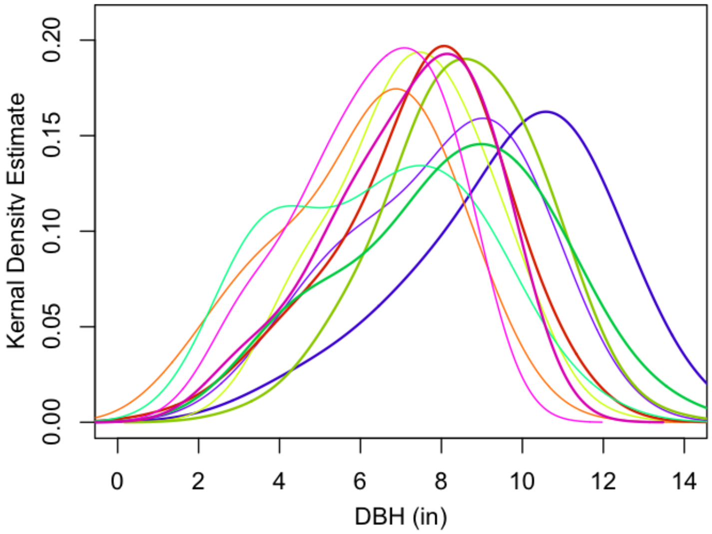


Figure 1. DBH density curves for live trees by spacing and year. Lighter, thinner lines are 2004, thicker, darker lines are 2011. Colors correspond to spacings as follows; 6x7, magenta, 8x8, red, 8x10, tan, 10x10, green, and 13x13, blue. For curves separated by spacing, see figure A1 in the appendix.

At the stand level, the largest differences in volume prediction were between scenarios V3 and V2 (Table 3): the difference between using FVS's height model and the mixed-effects height model.

Table 3. Comparison of 2011 stand-level volume ft^3/acre estimations for the different scenarios in Table 1.

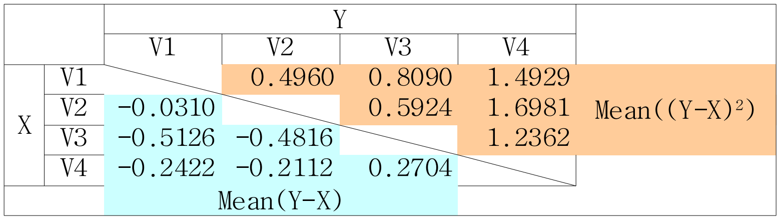
Spacing	Rep	V1	V2	V3	V4
6 x 7	1	3941	3999	4267	3819
6 x 7	2	3978	4001	4328	4031
6 x 7	3	4428	4406	4816	4850
6 x 7	w.a.m.[a]	4111	4130	4465	4226
6 x 7	D.f. V4[b]	-116	-96	239	0
8 x 8	1	2950	3005	3317	3095
8 x 8	2	4086	4046	3521	3382
8 x 8	3	3706	3637	3996	4052
8 x 8	w.a.m.	3563	3546	3615	3514
8 x 8	D.f. V4	50	32	101	0
8 x 10	1	2885	2801	3003	2838
8 x 10	2	3528	3421	3620	3554
8 x 10	3	3615	3487	3862	4148
8 x 10	w.a.m.	3348	3242	3501	3521
8 x 10	D.f. V4	-173	-280	-20	0
10 x 10	1	2895	2803	3214	3227
10 x 10	2	2477	2580	2828	2737
10 x 10	3	3288	3183	3437	3798
10 x 10	w.a.m.	2865	2841	3142	3227
10 x 10	D.f. V4	-362	-386	-85	0
13 x 13	1	1773	1698	2136	2252
13 x 13	2	1960	1896	2015	2048
13 x 13	3	1595	1535	1634	1726
13 x 13	w.a.m.	1776	1710	1928	2008
13 x 13	D.f. V4	-232	-299	-80	0
All	w.a.m.	3155	3117	3356	3322
All	D.f. V4	-167	-204	34	0

[a] w.a.m. refers to weighted (by acre size) arithmetic mean.

[b] D.f. V4 refers to the difference (in means) from V4.

Table 4 presents a comparison among scenarios at the tree level. Comparing scenario V1 to V2 in terms of both mean and mean squared difference in Table 4 indicates that the Ek (1985) volume equation predicted nearly the same volume as FVS for a tree of the same height and DBH. The large negative mean difference between V2 and V3 indicates that the mixed-effects model for 2011 height tended to lead to higher volume prediction than FVS for a tree of the same DBH. Overwhelmingly, the largest source of prediction error in tree-level volume was the prediction error in 2011 DBH. This is evidenced by the mean squared difference between V4 and all of the other scenarios being two to three times larger than the other squared differences (Table 4).

Table 4. Comparison of volume estimates for individual trees across all reps and spacing between the scenarios described in Table 1.



		Y				
		V1	V2	V3	V4	
X	V1		0.4960	0.8090	1.4929	Mean((Y-X)^2)
	V2	-0.0310		0.5924	1.6981	
	V3	-0.5126	-0.4816		1.2362	
	V4	-0.2422	-0.2112	0.2704		
		Mean(Y-X)				

FVS generally predicted greater DBH growth than what was observed for narrower spacings and predicted close to what was observed at wider spacings. For nearly all spacings, FVS predicted DBH growth had a narrower spread than what was observed (Table 5).

Table 5. Statistics for FVS's predicted DBH growth and observed DBH growth from 2004 to 2011.

Spacing	Rep	Mean		St. Dev		IQR	
		FVS	Obs.	FVS	Obs.	FVS	Obs.
6 x 7	1	1.06	0.66	0.10	0.30	0.10	0.50
6 x 7	2	1.03	0.73	0.15	0.34	0.20	0.40
6 x 7	3	0.91	0.88	0.13	0.38	0.20	0.45
8 x 8	1	1.15	0.90	0.26	0.53	0.20	0.43
8 x 8	2	0.98	0.86	0.16	0.26	0.20	0.30
8 x 8	3	1.02	1.04	0.18	0.38	0.10	0.40
8 x 10	1	1.18	0.98	0.20	0.27	0.20	0.30
8 x 10	2	1.09	1.01	0.13	0.23	0.20	0.30
8 x 10	3	0.95	1.25	0.22	0.40	0.20	0.60
10 x 10	1	1.18	1.13	0.14	0.43	0.20	0.40
10 x 10	2	1.05	0.97	0.37	0.33	0.70	0.40
10 x 10	3	0.98	1.38	0.25	0.59	0.20	0.65
13 x 13	1	1.32	1.52	0.25	0.42	0.18	0.30
13 x 13	2	1.28	1.36	0.34	0.64	0.40	0.40
13 x 13	3	1.44	1.80	0.26	0.31	0.18	0.30

Figure 2 indicates that FVS over-predicted growth of trees in the 6 by 7 spacing, especially trees of small initial DBH. The negative correlation between the differences and 2004 DBH presented in Figure 2 is not particularly strong, but is statistically significant, with a Kendall's tau of -0.17 and p-value less than 10^{-5}. The average of the difference is 0.08. The t-distribution 95% confidence interval for this bias is from 0.0528 to 0.1162 inches.

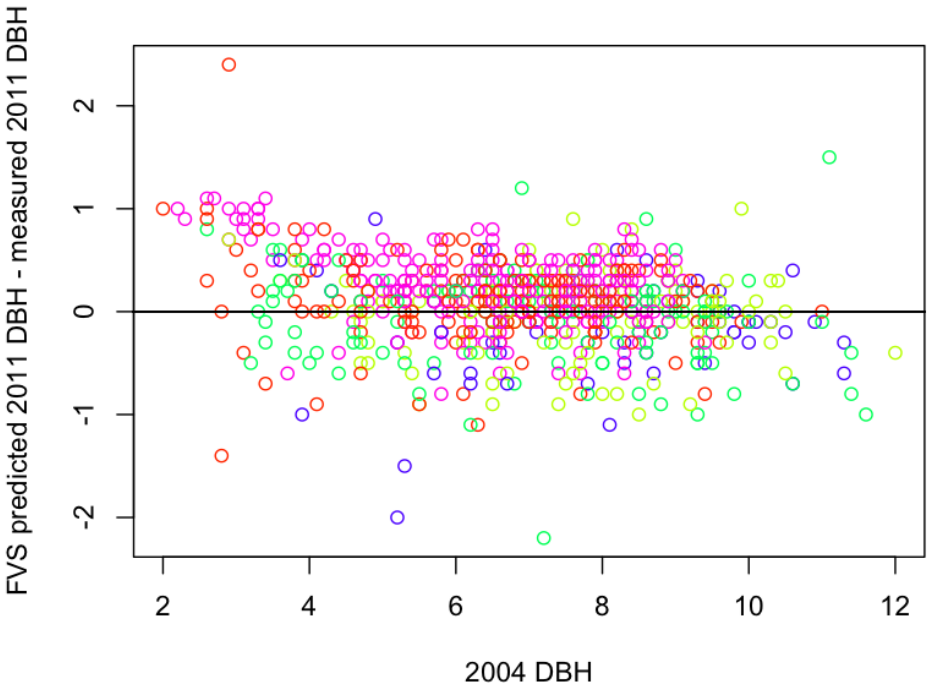


Figure 2. FVS predicted 2011 DBH - measured 2011 DBH, color-coded by spacing; magenta is 6x7, red is 8x8, yellow is 8x10, green is 10x10, and blue is 13x13.

Despite the error in predicting individual tree DBH growth, the predicted DBH distribution was quite accurate in the center, but much less accurate in the left tail (Figure 3).

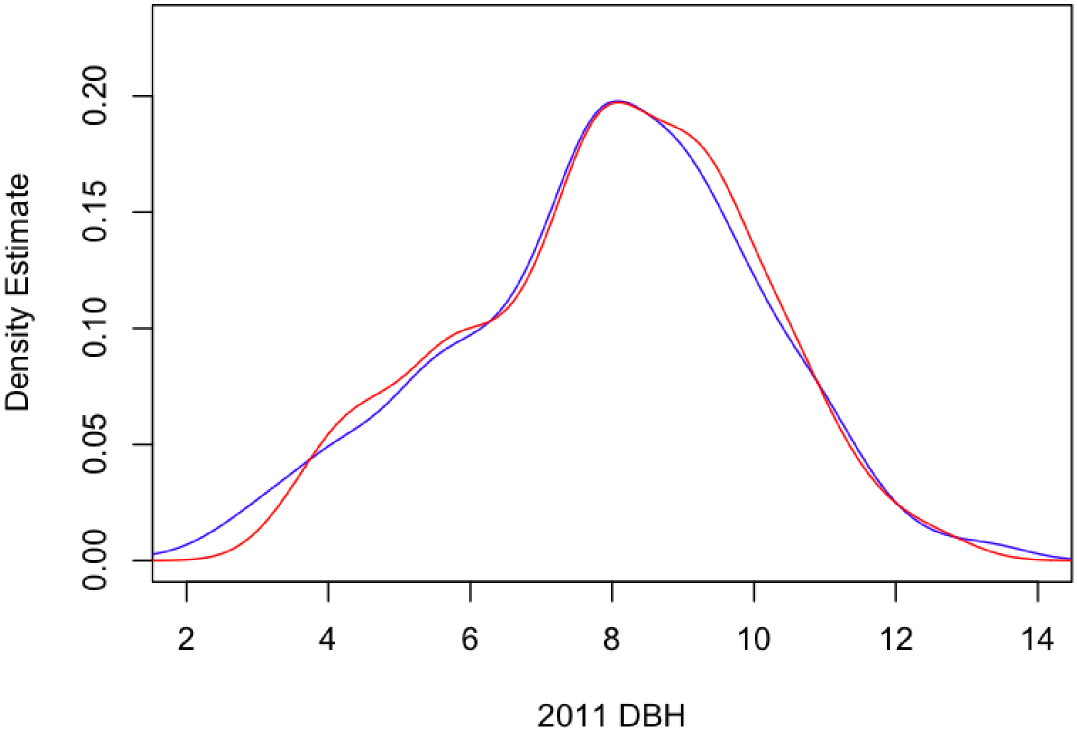


Figure 3. FVS's predicted 2011 DBH distribution across all spacings and reps in red with the measured DBH distribution in blue.

Discussion:

The coefficients for the mixed-effects model being different by spacing and rep is expected; higher accuracy of plot-level mixed effects models has been observed by Temesgen et al. (2008) and Huang et al. (2009). Trees of the same DBH being the tallest in the tightest spacing is also what would be expected; in tighter spacings, competition for light would be more intense and drive trees to devote a larger portion of their resources to height growth. The DBH distributions being close to a Normal distribution is also sensible at the young ages studied, but usually they increase in positive skew because the larger-DBH trees tend to increase their DBH more than the middle-DBH trees. The DBH distributions in this study had a tendency to increase in negative skew (Figure 1). Higher volumes in the tighter spacings initially suggests that tighter spacings may be more profitable, however, larger-DBH wood has more uses and can be more profitable per unit volume, which would also be an important concern for companies managing red pine for profit.

Having FVS predict future DBH introduced by far the largest mean squared difference in volume predictions between the scenarios at the tree level, but this discrepancy did not influence volume predictions at the stand level as much as FVS's height model. These DBH predictions have a small spacing-dependent bias component, but are predominantly not systematic, evident from the mean squared difference between V4 and any other model being much larger than any of the squared mean differences; most numbers in the last column in Table 4 are all above 1, versus numbers in the last row, which are all be less than 0.3, hence, when squared to equalize units would be even smaller. The small effect on volume bias is evident in the mean difference between scenarios V3 and both V1 and V2 being larger than the differences between V4 and the other scenarios (Table 4).

FVS's DBH predictions showed little bias in the middle spacings, but the predictions showed a stronger bias at narrower spacings. FVS tended to predict greater DBH growth than what was observed for narrower spacings. This somewhat counteracted the difference in volume predictions from the height model, as shown by scenario V4 being closer to V1 and V2 than to V3 in Table 4 and at the narrow spacing in Table 3. This is somewhat similar to the DBH bias in even-aged stands found by Ex and Smith (2014). While the Ex and Smith study focused primarily on bias of prediction, in our study FVS considerably underestimated the spread of DBH growth in all spacings; this means that when simulating a stand of trees, when FVS well predicts a reasonably close average future DBH, FVS will not necessarily accurately predict how fast the fastest growing trees, or the slowest growing trees grow. This effect on the future DBH distribution is seen in the tails of the distributions in Figure 3; still, the prediction of the middle DBHs in the future DBH distribution was reasonably accurate.

FVS's over-prediction of DBH growth would lead a manager to conclude a faster growth rate than what would actually occur. This is evidenced by the stand-level volumes predictions in Table 3 for scenarios V3 and V4, although systematic differences in height models counteracted this effect to an extent, as evidenced by comparing the stand-level V2 and V4 estimates. FVS predicted a closer-to-mean growth rate for the fastest and slowest growing trees. This would cause FVS's volume predictions to be better suited to forests of uniform DBH, where most of the stand-level volume is in the trees of average DBH and growth, as opposed to forests where volume is predominantly in the faster growing, larger trees, such as the uneven aged forests in Ex and Smith (2014). For species such as red pine, this could make a difference in estimating the increase in volume of sawlog versus pulp trees in a stand if the break point DBH for sawlog trees is just within reach of the fastest growing trees. This should be researched further in cover types where larger trees are worth much more than smaller trees such as in hardwoods.

Citations:

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Appendix:

Table A1. Instrument used for tree and crown height by rep and year.

Spacing	Rep	2004	2011
6 x 7	1	Criterion 400	Impulse 200
6 x 7	2	Impulse 200	Impulse 200
6 x 7	3	Impulse 200	Impulse 200
8 x 8	1	Impulse 200	Impulse 200
8 x 8	2	Impulse 200	Impulse 200
8 x 8	3	Criterion 400	Impulse 200
8 x 10	1	Impulse 200	Impulse 200
8 x 10	2	Criterion 400	Impulse 200
8 x 10	3	Impulse 200	Impulse 200
10 x 10	1	Criterion 400	Impulse 200
10 x 10	2	Impulse 200	Impulse 200
10 x 10	3	Criterion 400	Impulse 200
13 x 13	1	Criterion 400	Impulse 200
13 x 13	2	Impulse 200	Impulse 200
13 x 13	3	Impulse 200	Impulse 200

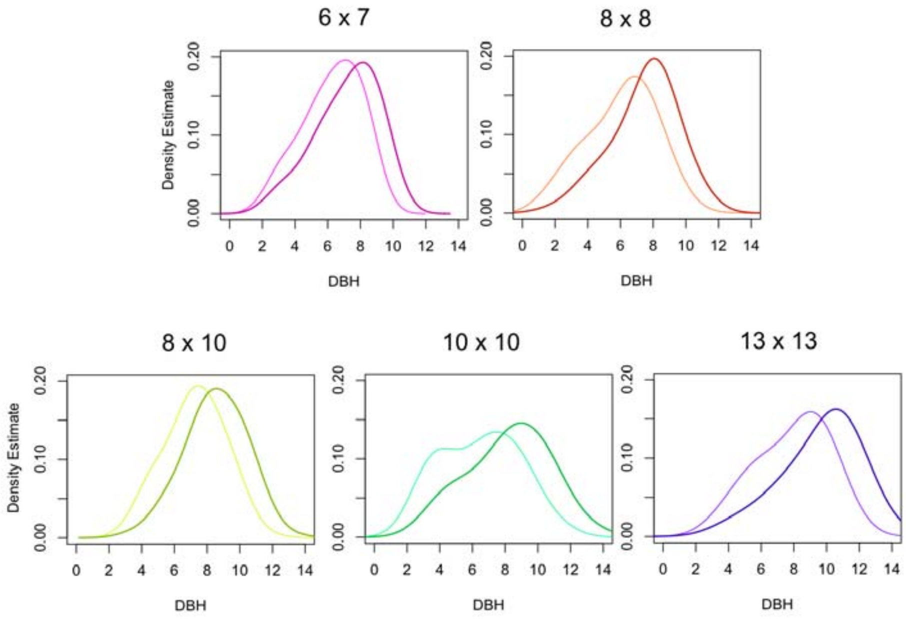

Figure A1: DBH distributions of the individual spacings; thicker line is 2011 DBH, thinner line is 2004 DBH.

R code:

```
> acm=data.frame()
> for(i in 1:3){
+ for(j in 1:3){
+ mod1=lmer(formula=y1~x1+(1|rl[[i]])+(-1+x1|rl[[j]]))
+ mod2=lmer(formula=y2~x2+(1|rl[[i]])+(-1+x2|rl[[j]]))
+ acm[j+3*(i-1),1]=j
+ acm[j+3*(i-1),2]=i
+ acm[j+3*(i-1),3]=AIC(mod1)
+ acm[j+3*(i-1),4]=BIC(mod1)
+ acm[j+3*(i-1),5]=AIC(mod2)
+ acm[j+3*(i-1),6]=BIC(mod2)}}
> for(j in 1:3){
+ mod1=lmer(formula=y1~x1+(-1+x1|rl[[j]]))
+ mod2=lmer(formula=y2~x2+(-1+x2|rl[[j]]))
+ acm[9+j,1]=j
+ acm[9+j,2]=0
+ acm[9+j,3]=AIC(mod1)
+ acm[j+9,4]=BIC(mod1)
+ acm[j+9,5]=AIC(mod2)
+ acm[j+9,6]=BIC(mod2)}
> for(j in 1:3){
+ mod1=lmer(formula=y1~x1+(1|rl[[j]]))
+ mod2=lmer(formula=y2~x2+(1|rl[[j]]))
+ acm[12+j,1]=0
+ acm[12+j,2]=j
+ acm[12+j,3]=AIC(mod1)
+ acm[j+12,4]=BIC(mod1)
+ acm[j+12,5]=AIC(mod2)
+ acm[j+12,6]=BIC(mod2)}
> for(i in 1:3){
+ for(j in 1:3){
+ mod1=lmer(formula=y1~(1|rl[[i]])+(-1+x1|rl[[j]]))
+ mod2=lmer(formula=y2~(1|rl[[i]])+(-1+x2|rl[[j]]))
+ acm[15+j+3*(i-1),1]=j
+ acm[15+j+3*(i-1),2]=i
+ acm[15+j+3*(i-1),3]=AIC(mod1)
+ acm[15+j+3*(i-1),4]=BIC(mod1)
+ acm[15+j+3*(i-1),5]=AIC(mod2)
+ acm[15+j+3*(i-1),6]=BIC(mod2)}}
> for(j in 1:3){
+ mod1=lmer(formula=y1~(1|rl[[j]]))
+ mod2=lmer(formula=y2~(1|rl[[j]]))
+ acm[24+j,1]=0
+ acm[24+j,2]=j
+ acm[24+j,3]=AIC(mod1)
```

```
+ acm[j+24,4]=BIC(mod1)
+ acm[j+24,5]=AIC(mod2)
+ acm[j+24,6]=BIC(mod2)}
> for(j in 1:3){
+ mod1=lmer(formula=y1~(-1+x1|rl[[j]]))
+ mod2=lmer(formula=y2~(-1+x2|rl[[j]]))
+ acm[27+j,1]=j
+ acm[27+j,2]=0
+ acm[27+j,3]=AIC(mod1)
+ acm[j+27,4]=BIC(mod1)
+ acm[j+27,5]=AIC(mod2)
+ acm[j+27,6]=BIC(mod2)}
> acm // matrix of AIC's and BIC's for models with different random effects
   V1 V2        V3        V4        V5        V6
1   1  1 -500.6892 -483.1019 -434.2388 -416.6515
2   2  1 -489.2128 -471.6255 -457.8100 -440.2227
3   3  1 -586.2239 -568.6366 -639.0546 -621.4674
4   1  2 -536.4467 -518.8594 -470.4271 -452.8398
5   2  2 -362.7569 -345.1696 -402.1138 -384.5265
6   3  2 -587.0646 -569.4773 -644.0066 -626.4193
7   1  3 -584.1103 -566.5231 -634.9582 -617.3710
8   2  3 -555.2371 -537.6498 -630.0841 -612.4968
9   3  3 -586.2239 -568.6366 -646.5208 -628.9336
10  1  0 -502.1044 -488.0346 -436.2388 -422.1690
11  2  0 -356.5910 -342.5212 -397.7632 -383.6934
12  3  0 -588.2239 -574.1541 -640.0846 -626.0148
13  0  1 -466.0550 -451.9851 -428.6444 -414.5746
14  0  2 -364.7569 -350.6871 -404.1138 -390.0440
15  0  3 -557.2371 -543.1673 -632.0841 -618.0143
16  1  1 -491.9100 -477.8402 -420.8252 -406.7554
17  2  1 -479.1747 -465.1049 -450.3811 -436.3113
18  3  1 -552.0308 -537.9610 -608.4516 -594.3818
19  1  2 -525.0346 -510.9648 -457.2475 -443.1777
20  2  2 -353.9911 -339.9213 -393.3020 -379.2322
21  3  2 -552.0308 -537.9610 -607.5196 -593.4498
22  1  3 -573.1575 -559.0877 -621.7308 -607.6610
23  2  3 -542.6670 -528.5971 -618.5457 -604.4759
24  3  3 -552.1806 -538.1108 -614.6703 -600.6005
25  0  1 -296.9975 -286.4452 -329.9876 -319.4353
26  0  2 -311.9070 -301.3546 -349.0209 -338.4685
27  0  3 -355.5436 -344.9912 -462.0845 -451.5321
28  1  0 -490.6110 -480.0587 -422.8252 -412.2729
29  2  0 -351.0586 -340.5062 -392.4886 -381.9362
30  3  0 -554.0308 -543.4785 -609.5196 -598.9672
> amod1=lme(fixed=y1~x1,random=~x1|rs,data=drp)
> amod2=lme(fixed=y2~x2,random=~x2|rs,data=drp)
> coef(amod2) // prediction coefficients for 2011 heights based on spacing and rep
```

```
      (Intercept)        x2
6.481    3.932787 -1.434194
6.482    3.995202 -1.487692
6.483    4.055039 -1.948852
8.941    3.968997 -2.176884
8.942    3.934188 -1.422126
8.943    4.009208 -2.011575
81       3.949116 -1.838930
82       3.866419 -3.264123
83       3.984476 -1.459767
101      3.987702 -2.145849
102      3.928394 -2.060046
103      4.001818 -2.010487
131      3.980740 -2.483332
132      3.953672 -2.577265
133      3.927359 -2.112706
> coef(amod1) // prediction coefficients for 2004 heights based on spacing and rep
      (Intercept)        x1
6.481    3.782923 -1.767120
6.482    3.802673 -1.742685
6.483    3.810854 -1.929533
8.941    3.802273 -2.594627
8.942    3.799879 -1.918945
8.943    3.799145 -2.498026
81       3.803013 -2.550789
82       3.801078 -2.488196
83       3.798630 -1.945828
101      3.792522 -3.045797
102      3.797034 -2.963460
103      3.803464 -2.380881
131      3.811074 -4.869150
132      3.804239 -3.183294
133      3.798376 -2.785511
> rpd2=read.table(file="rpt2.csv",sep=",",header=TRUE)
> qmd=function(x){return(sqrt(mean(x^2)))}
> tapply(rpd2[,3],rpd2[,1],FUN=qmd)
   6.481    6.482    6.483    8.941    8.942    8.943       81       82       83
6.366952 6.789519 6.946892 7.266026 7.379514 8.687539 6.654849 7.209968 8.050528
     101      102      103      131      132      133
8.053842 7.536890 9.194216 8.214447 9.215747 9.603503
> tapply(rpd2[,5],rpd2[,1],FUN=qmd)
    6.481     6.482     6.483     8.941     8.942     8.943        81        82
 7.406504  7.820166  7.838420  8.455646  8.464859  9.676575  7.911626  8.163967
       83       101       102       103       131       132       133
 9.113383  9.216493  8.848496 10.306425  9.581232 10.635413 11.140181
> rpdN=read.table("rpdN.csv",header=TRUE, sep=",")
> error=data.frame()
```

```
> treat=list(rpdN$X11hv,rpdN$X11ovfdfh,rpdN$X11ovfdoh,rpd$X11ov)
> for(i in 1:4){
+ for(j in 1:4){
+ if(i>j){
+ error[i,j]=mean(treat[[j]]-treat[[i]])
+ }
+ else{
+ error[i,j]=mean((treat[[j]]-treat[[i]])^2)}
+ }}
> error // generates the numbers in Table 4.
            [,1]       [,2]      [,3]     [,4]
[1,]  0.00000000  0.4960019 0.8090410 1.492926
[2,] -0.03101729  0.0000000 0.5924212 1.698126
[3,] -0.51263104 -0.4816137 0.0000000 1.236222
[4,] -0.24220694 -0.2111897 0.2704241 0.000000

> for(i in 1:2){
+ q4[1:15,i]=tapply(rp2[,c(13+i)],INDEX=rp2[,1],FUN=mean)
+ q4[1:15,c(2+i)]=tapply(rp2[,c(13+i)],INDEX=rp2[,1],FUN=sd)
+ q4[1:15,c(4+i)]=tapply(rp2[,c(13+i)],INDEX=rp2[,1],FUN=IQR)
+ }
> q4  // numbers for Table 5
           [,1]      [,2]      [,3]      [,4]  [,5]  [,6]
 [1,] 1.0578947 0.6631579 0.1036187 0.3036850 0.100 0.500
 [2,] 1.0324675 0.7259740 0.1516936 0.3396603 0.200 0.400
 [3,] 0.9112676 0.8774648 0.1347477 0.3761448 0.200 0.450
 [4,] 1.1829268 0.9780488 0.1960898 0.2660002 0.200 0.300
 [5,] 1.0851064 1.0148936 0.1301817 0.2340371 0.200 0.300
 [6,] 0.9463415 1.2536585 0.2214696 0.4031734 0.200 0.600
 [7,] 1.1484375 0.8968750 0.2569616 0.5273380 0.200 0.425
 [8,] 0.9790323 0.8596774 0.1631022 0.2633079 0.200 0.300
 [9,] 1.0183673 1.0387755 0.1787380 0.3762846 0.100 0.400
[10,] 1.1810811 1.1324324 0.1430576 0.4346480 0.200 0.400
[11,] 1.0517857 0.9696429 0.3668000 0.3340766 0.700 0.400
[12,] 0.9750000 1.3781250 0.2540003 0.5906445 0.200 0.650
[13,] 1.3181818 1.5227273 0.2500216 0.4219364 0.175 0.300
[14,] 1.2809524 1.3571429 0.3370742 0.6352727 0.400 0.400
[15,] 1.4428571 1.8000000 0.2622808 0.3063432 0.175 0.300

> r2=read.table(file="rpt2.csv",sep=",",header=TRUE)
> res=r2$X2011dbh-r2$act2011d
> cor.test(x=r2$dbh,y=res,method="kendall") // Kendall's Tau for 2004 DBH and measured DBH growth

        Kendall's rank correlation tau

data:  r2$dbh and res
```

```
z = -6.2585, p-value = 3.887e-10
alternative hypothesis: true tau is not equal to 0
sample estimates:
       tau
-0.1733418
```

`t.test(x=rp2$fd11-rp2$md11)` ***// t-test for significance of DBH bias***

```
        One Sample t-test

data:  rp2$fd11 - rp2$md11
t = 5.2395, df = 709, p-value = 2.125e-07
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.05284129 0.11617280
sample estimates:
 mean of x
0.08450704
```